# 5.1 Midsegment Theorem and Coordinate Proof

**Georgia Performance Standard(s)**

**MM1G1c, MM1G1e**

**Goal** • Use properties of midsegments and write coordinate proofs.

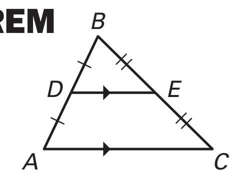
***Your Notes***

**VOCABULARY**

Midsegment of a triangle

**THEOREM 5.1: MIDSEGMENT THEOREM**

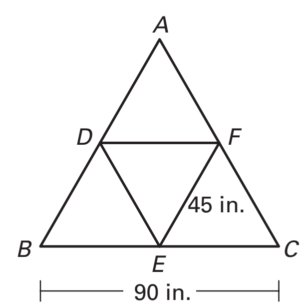
The segment connecting the midpoints of two sides of a triangle is __________ to the third side and is ______ as long as that side.




$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

## Example 1 *Use the Midsegment Theorem to find lengths*

**Windows** A large triangular window is segmented as shown. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$. Find $DF$ and $AB$.




In the diagram for Example 1, midsegment $\overline{DF}$ can be called "the midsegment opposite $\overline{BC}$."

**Solution**

$DF$ = ___ • $BC$ = ___(_______) = _______

$AB$ = ___ • $EF$ = ___(_______) = _______

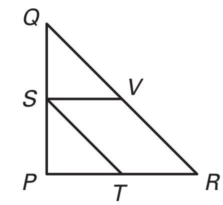
✔ ***Checkpoint*** **Complete the following exercise.**

**1.** In Example 1, consider $\triangle ADF$. What is the length of the midsegment opposite $\overline{DF}$?

***Your Notes***

**Example 2** ***Use the Midsegment Theorem***

**In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.**




**Solution**

Because $QS = SP$ and $PT = TR$, $S$ is the __________ of $\overline{QP}$ and $T$ is the __________ of $\overline{PR}$ by definition. Then $\overline{ST}$ is a ____________ of $\triangle PQR$ by definition and $\overline{QR} \parallel \overline{ST}$ by the ______________________.

**Example 3** ***Place a figure in a coordinate plane***

**Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.**

**a.** A square

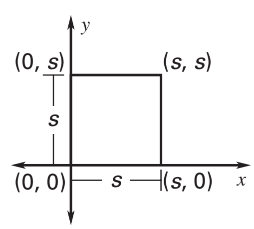
**b.** An acute triangle

**Solution**

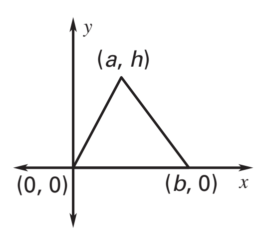
It is easy to find lengths of horizontal and vertical segments and distances from ______, so place one vertex at the _______ and one or more sides on an _____.

The square represents a general square because the coordinates are based only on the definition of a square. If you use this square to prove a result, the result will be true for all squares.

**a.** Let $s$ represent the ____________.




**b.** You need to use ______ different variables.



*Your Notes*

✔ ***Checkpoint*** **Complete the following exercises.**

**2.** In Example 2, if $V$ is the midpoint of $\overline{QR}$, what do you know about $\overline{SV}$?

**3.** Place an obtuse scalene triangle in a coordinate plane that is convenient for finding side lengths. Assign coordinates to each vertex.

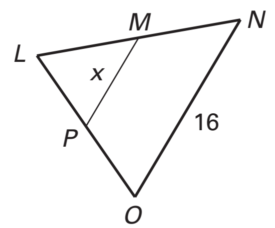
***Homework***

Name ______________________ Date ______________

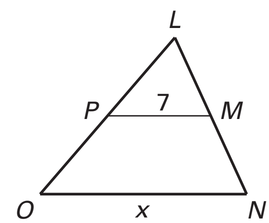
## LESSON 5.1 Practice

**$\overline{MP}$ is a midsegment of $\triangle LNO$. Find the value of $x$.**

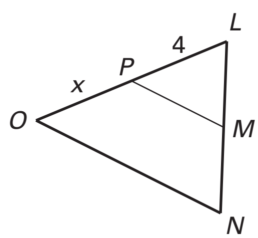
**1.**



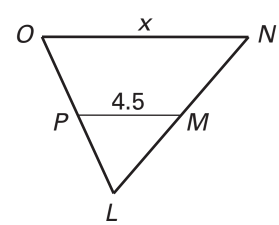

**2.**




**3.**




**4.**



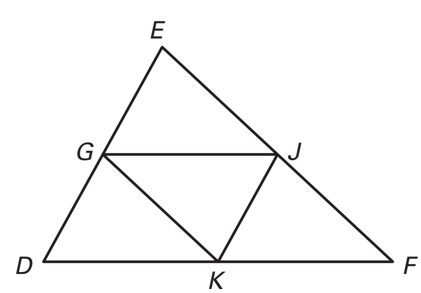

**In $\triangle DEF$, $\overline{EJ} \cong \overline{JF}$, $\overline{FK} \cong \overline{KD}$, and $\overline{DG} \cong \overline{GE}$. Complete the statement.**

**5.** $\overline{GJ} \parallel$ ____

**6.** $\overline{EJ} \cong$ ____ $\cong$ ____

**7.** $\overline{DE} \parallel$ ____

**8.** $\overline{GJ} \cong$ ____ $\cong$ ____

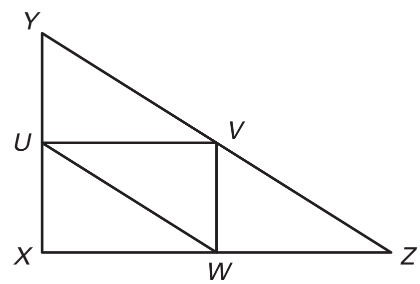


Name ______________________ Date ______________

LESSON 5.1 **Practice** *continued*

**Use the diagram of $\triangle XYZ$ where *U*, *V*, and *W* are the midpoints of the sides.**

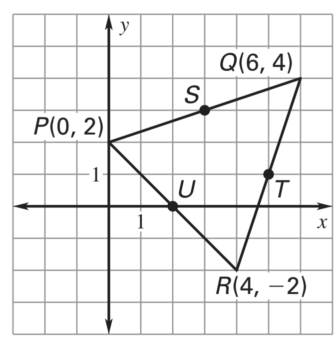
**9.** If $UW = 4x - 1$ and $YZ = 5x + 4$, what is $UW$?




**10.** Find $YV$.

**Use the graph shown.**

**11.** Find the coordinate of the endpoints of each midsegment of $\triangle PQR$.




**12.** Use the slope and the Distance Formula to verify that the Midsegment Theorem is true for $\overline{ST}$.

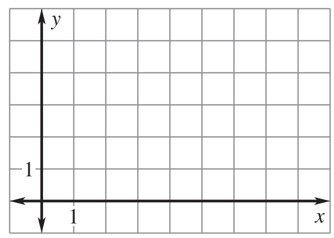
Name ______________________ Date ______________

LESSON 5.1

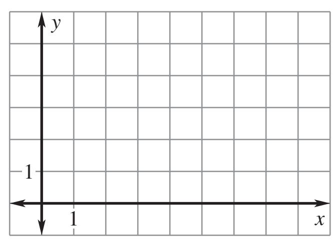
# Practice *continued*

**Place the figure in the coordinate plane. Assign coordinates to each vertex.**

**13.** A 4 unit by 7 unit rectangle with one vertex at (0, 0).

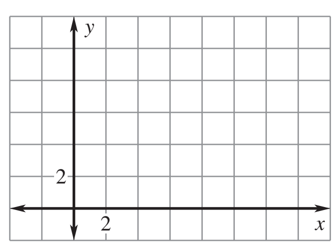



**14.** A square with side length 4 and one vertex at (4, 0).

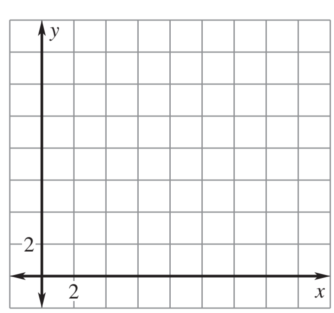



**Place the figure in the coordinate plane. Assign coordinates to each vertex. *Explain* the advantage of your placement.**

**15.** Right triangle: leg lengths are 5 units and 9 units




**16.** Isosceles right triangle: leg length is 14 units



Name ______________________ Date ______________

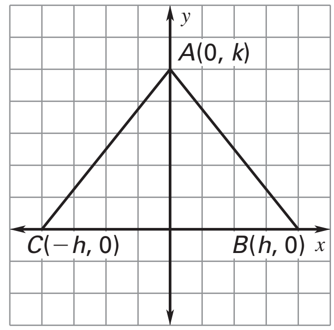
LESSON 5.1

# Practice *continued*

**In Exercises 17 and 18, describe a plan for the proof.**

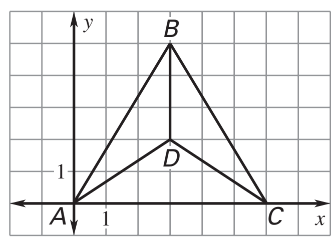
**17.** **GIVEN:** Coordinates of vertices of $\triangle ABC$

**PROVE:** $\triangle ABC$ is isosceles.




**18.** **GIVEN:** $\overline{BD}$ bisects $\angle ABC$.

**PROVE:** $\triangle BDA \cong \triangle BDC$



# 5.2 Use Perpendicular Bisectors

**Georgia Performance Standard(s)**

**MM1G3e**

***Your Notes***

**Goal** • Use perpendicular bisectors to solve problems.

**VOCABULARY**

Perpendicular bisector

Equidistant

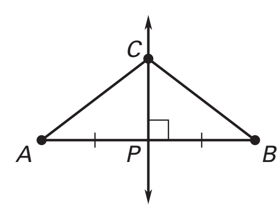
Concurrent

Point of concurrency

Circumcenter

**THEOREM 5.2: PERPENDICULAR BISECTOR THEOREM**

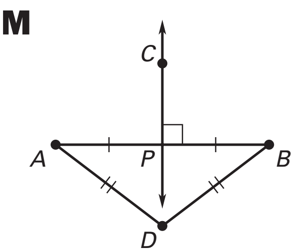
In a plane, if a point is on the perpendicular bisector of a segment, then it is ______________ from the endpoints of the segment.




If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA =$ ______.

**THEOREM 5.3: CONVERSE OF THE PERPENDICULAR BISECTOR THEOREM**

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the ________________ __________ of the segment.

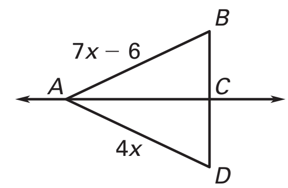



If $DA = DB$, then $D$ lies on the _____________ of $\overline{AB}$.

*Your Notes*

## Example 1 *Use the Perpendicular Bisector Theorem*

**$\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find $AD$.**




**Solution**

$AD =$ _____ **Perpendicular Bisector Theorem**

_____ $=$ _________ **Substitute.**

$x =$ ___ **Solve for *x*.**

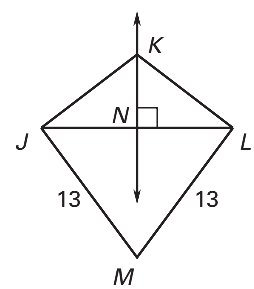
$AD =$ _____ $=$ ______ $=$ ___.

## Example 2 *Use perpendicular bisectors*

**In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.**

**a.** What segment lengths in the diagram are equal?

**b.** Is $M$ on $\overleftrightarrow{KN}$?




**Solution**

**a.** $\overleftrightarrow{KN}$ bisects $\overline{JL}$, so _____ $=$ _____. Because $K$ is on the perpendicular bisector of $\overline{JL}$, _____ $=$ _____ by Theorem 5.2. The diagram shows that _____ $=$ _____ $= 13$.

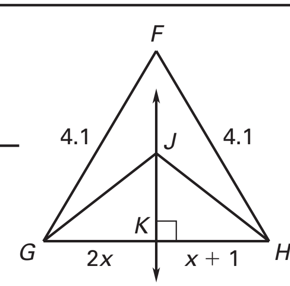
**b.** Because $MJ = ML$, $M$ is _____________ from $J$ and $L$. So, by the ________________________________________ __________, $M$ is on the perpendicular bisector of $\overline{JL}$, which is $\overleftrightarrow{KN}$.

✔ ***Checkpoint*** **In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{GH}$.**

**1.** What segment lengths are equal?

____________________________________________

**2.** Find $GH$.



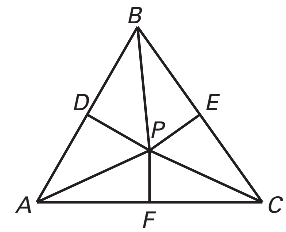

 

*Your Notes*

**THEOREM 5.4: CONCURRENCY OF PERPENDICULAR BISECTORS OF A TRIANGLE**

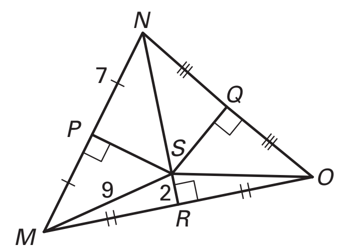
The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA$ = _____ = _____.




**Example 3** ***Use the concurrency of perpendicular bisectors***

**The perpendicular bisectors of $\triangle MNO$ meet at point $S$. Find $SN$.**




**Solution**

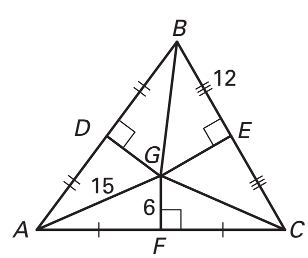
Using ______________, you know that point $S$ is ______________ from the vertices of the triangle.
So, _____ = _____ = _____ .

_____ = _____ **Theorem 5.4**

_____ = ___ **Substitute.**

✔ ***Checkpoint*** **Complete the following exercise.**

**3.** The perpendicular bisectors of $\triangle ABC$ meet at point $G$. Find $GC$.



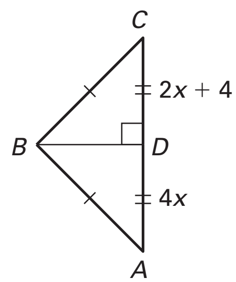

***Homework***

Name ______________________ Date ______________

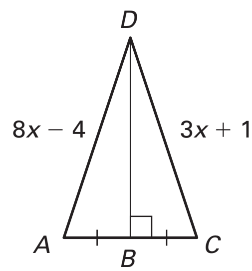
# LESSON 5.2 Practice

**Find the length of $\overline{CD}$.**

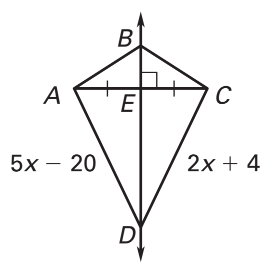
**1.**



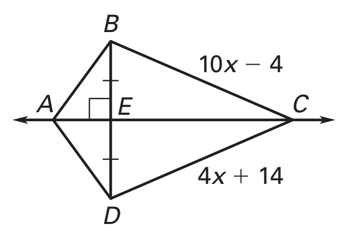

**2.**



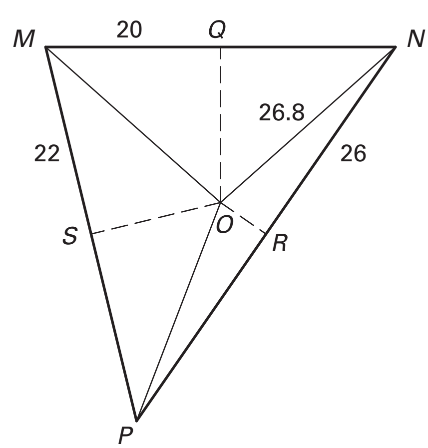

**3.**




**4.**




**In the diagram, the perpendicular bisectors of △*MNP* meet at point *O* and are shown dashed. Find the indicated measure.**




**5.** Find *MO*.

**6.** Find *PR*.

**7.** Find *MN*.

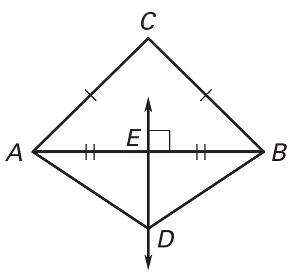
Name ______________________ Date ______________

LESSON 5.2

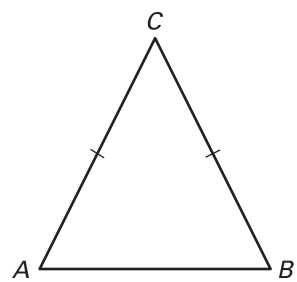
# Practice *continued*

**Tell whether the information in the diagram allows you to conclude that *C* is on the perpendicular bisector of $\overline{AB}$. *Explain*.**

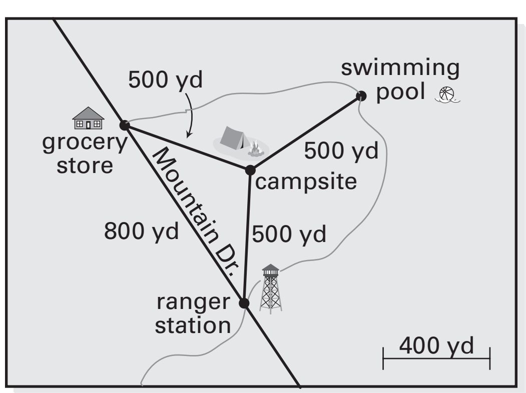
**8.**




**9.**




**10. Camping** Your campsite is located 500 yards from the ranger station, the grocery store, and the swimming pool, as shown on the map. The ranger station and the grocery store are located 800 yards apart along Mountain Drive. How far is your campsite from Mountain Drive?



# 5.3 Use Angle Bisectors of Triangles

Georgia Performance Standard(s)

MM1G3e

*Your Notes*

**Goal** • Use angle bisectors to find distance relationships.

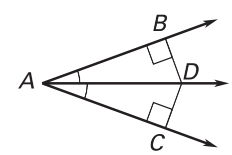
**VOCABULARY**

Angle bisector

Incenter

**THEOREM 5.5: ANGLE BISECTOR THEOREM**

If a point is on the bisector of an angle, then it is equidistant from the two _______ of the angle.

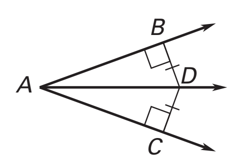



If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB =$ _____.

In Geometry, *distance* means the *shortest* length between two objects.

**THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM**

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the __________ of the angle.

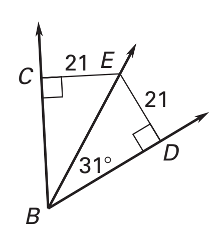



If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ _________ $\angle BAC$.

**Example 1** *Use the Angle Bisector Theorems*

**Find the measure of $\angle CBE$.**

**Solution**

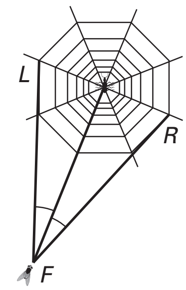



Because $\overline{EC} \perp$ _____, $\overline{ED} \perp$ _____, and $EC = ED = 21$, $\overrightarrow{BE}$ bisects $\angle CBD$ by the _________________ _________________________. So, $m\angle CBE = m\angle$______ $=$ _____.

*Your Notes*

**Example 2** ***Solve a real-world problem***

**Web** A spider's position on its web relative to an approaching fly and the opposite sides of the web form congruent angles, as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?




**Solution**

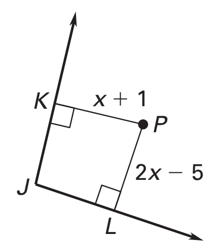
The congruent angles tell you that the spider is on the __________ of $\angle LFR$. By the ________________ __________, the spider is equidistant from $\overrightarrow{FL}$ and $\overrightarrow{FR}$.

So, the spider must move the ________________ to reach each edge.

**Example 3** ***Use algebra to solve a problem***

**For what value of $x$ does $P$ lie on the bisector of $\angle J$?**

**Solution**




From the Converse of the Angle Bisector Theorem, you know that $P$ lies on the bisector of $\angle J$ if $P$ is equidistant from the sides of $\angle J$, so when _____ = _____.

_____ = _____ **Set segment lengths equal.**

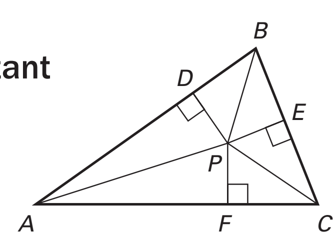
________ = ________ **Substitute expressions for segment lengths.**

___ = $x$ **Solve for $x$.**

Point $P$ lies on the bisector of $\angle J$ when $x$ = ___.

**THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE**

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

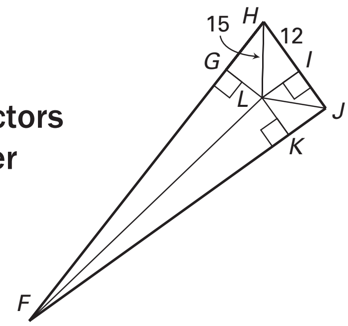



If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD$ = _____ = _____.

*Your Notes*

**Example 4** ***Use the concurrency of angle bisectors***

**In the diagram, $L$ is the incenter of $\triangle FHJ$. Find $LK$.**




By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter $L$ is ____________ from the sides of $\triangle FHJ$. So, to find $LK$, you can find ____ in $\triangle LHI$. Use the Pythagorean Theorem.

____ = _________ **Pythagorean Theorem**

_____ = ___________ **Substitute known values.**

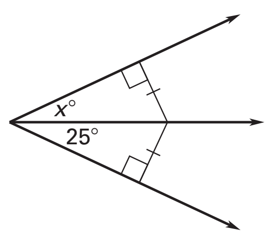
_____ = _____ **Simplify.**

__ = ____ **Take the positive square root of each side.**

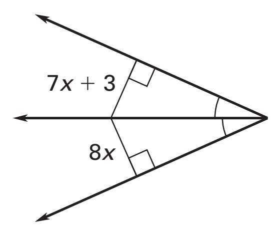
Because ____ = $LK$, $LK$ = ___.

**✔ *Checkpoint* In Exercises 1 and 2, find the value of $x$.**

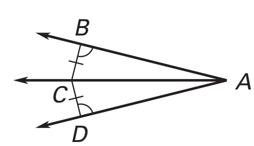
**1.**




**2.**




**3.** Do you have enough information to conclude that $\overrightarrow{AC}$ bisects $\angle DAB$? *Explain.*




**4.** In Example 4, suppose you are not given $HL$ or $HI$, but you are given that $JL = 25$ and $JI = 20$. Find $LK$.

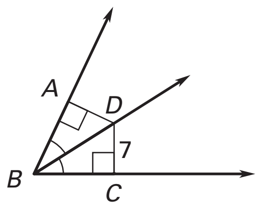
***Homework***

Name ______________________ Date ______________

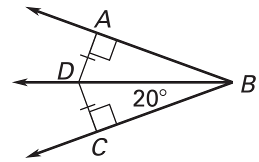
# LESSON 5.3 Practice

**Use the information in the diagram to find the measure.**

**1.** Find *AD*.

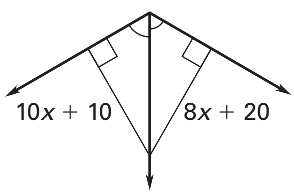



**2.** Find $m\angle DBA$.

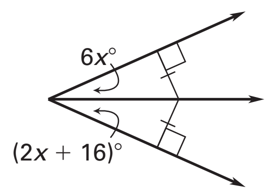



**Find the value of *x*.**

**3.**

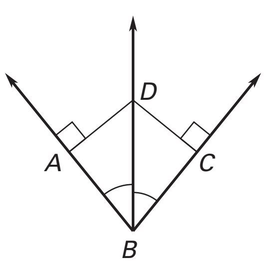



**4.**

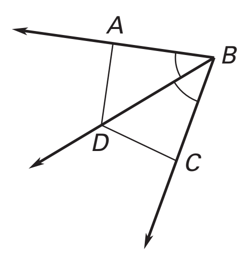



**Is *DA* = *DC*? *Explain*.**

**5.**




**6.**

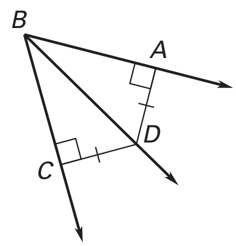


Name ______________________________ Date ______________

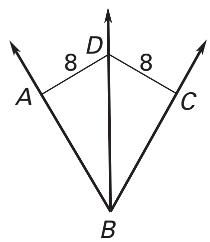
LESSON 5.3 **Practice** *continued*

**Can you conclude that $\overrightarrow{BD}$ bisects $\angle ABC$? *Explain*.**

**7.**

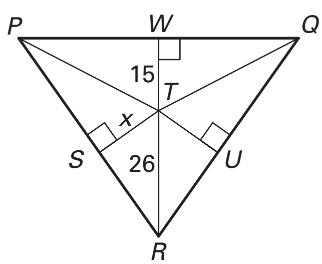



**8.**

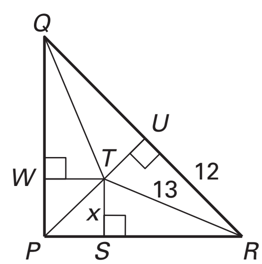



**In Exercises 9 and 10, point *T* is the incenter of △*PQR*. Find the value of *x*.**

**9.**

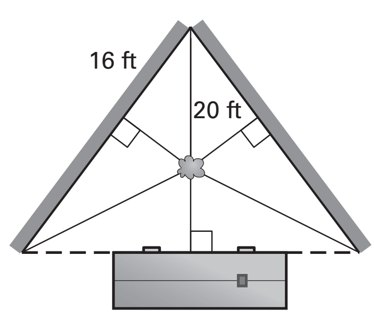



**10.**




**11. Bird Bath** Your neighbor is moving a new bird bath to his triangular back yard. He wants the bird bath to be the same distance from each edge of the yard. Where should your neighbor place the bird bath? *Explain*.

**12. Landscaping** You are planting a tree at the incenter of your triangular front yard. Use the diagram to determine how far the tree is from the house.



# 5.4 Use Medians and Altitudes

Georgia Performance Standard(s)

MM1G1c, MM1G1e, MM1G3e

***Your Notes***

**Goal** • Use medians and altitudes of triangles.

**VOCABULARY**

Median of a triangle

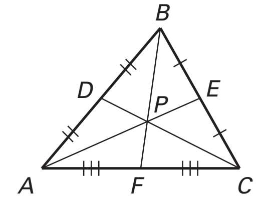
Centroid

Altitude of a triangle

Orthocenter

**THEOREM 5.8: CONCURRENCY OF MEDIANS OF A TRIANGLE**

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.



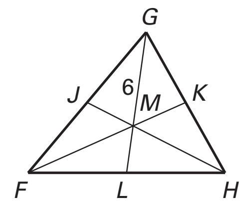

The medians of $\triangle ABC$ meet at $P$ and

$AP = \frac{2}{3}$_____, $BP = \frac{2}{3}$_____, and $CP = \frac{2}{3}$_____.

***Your Notes***

**Example 1** ***Use the centroid of a triangle***

**In $\triangle FGH$, $M$ is the centroid and $GM = 6$. Find $ML$ and $GL$.**




______ = $\frac{\quad}{\quad}$ $GL$ **Concurrency of Medians of a Triangle Theorem**

___ = $\frac{\quad}{\quad}$ $GL$ **Substitute ___ for $GM$.**

___ = $GL$ **Multiply each side by the reciprocal, $\frac{\quad}{\quad}$.**

Then $ML = GL -$ ______ = ___ − ___ = ___.

So, $ML =$ ___ and $GL =$ ___.

**✔ *Checkpoint* Complete the following exercise.**

**1.** In Example 1, suppose $FM = 10$. Find $MK$ and $FK$.

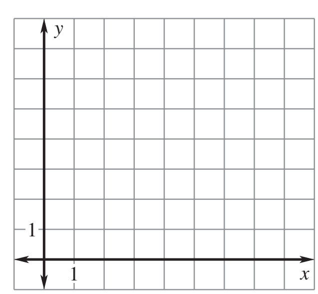
**Example 2** ***Find the centroid of a triangle***

**The vertices of $\triangle JKL$ are $J(1, 2)$, $K(4, 6)$, and $L(7, 4)$. Find the coordinates of the centroid $P$ of $\triangle JKL$.**

Sketch $\triangle JKL$. Then use the Midpoint Formula to find the midpoint $M$ of $\overline{JL}$ and sketch median $\overline{KM}$.

Median $\overline{KM}$ is used in Example 2 because it is easy to find distances on a vertical segment. You can check by finding the centroid using a different median.

$M\left(\frac{\boxed{\quad}}{2}, \frac{\boxed{\quad}}{2}\right) =$ _________




The centroid is ___________ of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex $K$ to point $M$ is 6 − ___ = ___ units. So, the centroid is

$\frac{\quad}{\quad}$(___) = ___ units down from $K$ on $\overline{KM}$.

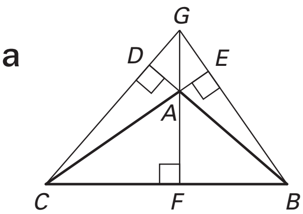
The coordinates of the centroid $P$ are (4, 6 − ___), or (______).

*Your Notes*

**THEOREM 5.9: CONCURRENCY OF ALTITUDES OF A TRIANGLE**

The lines containing the altitudes of a triangle are ____________.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at $G$.

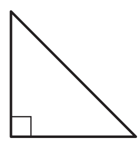



**Example 3** *Find the orthocenter*

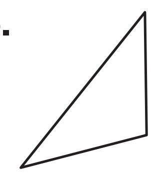
**Find the orthocenter $P$ of the triangle.**

Notice that in a right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

**a.**

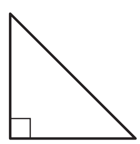


**b.**

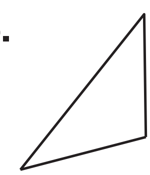


**Solution**

**a.**



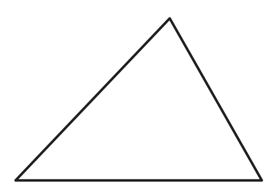
**b.**



✔ ***Checkpoint*** **Complete the following exercises.**

**2.** In Example 2, where do you need to move point $K$ so that the centroid is $P(4, 5)$?

**3.** Find the orthocenter $P$ of the triangle.



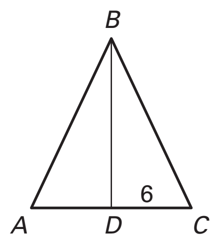
***Homework***

Name ______________________________ Date ______________

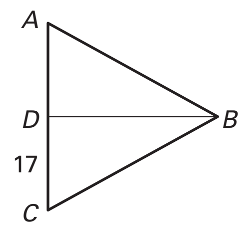
# LESSON 5.4 Practice

**$\overline{BD}$ is a median of $\triangle ABC$. Find the length of $\overline{AD}$.**

**1.**




**2.**



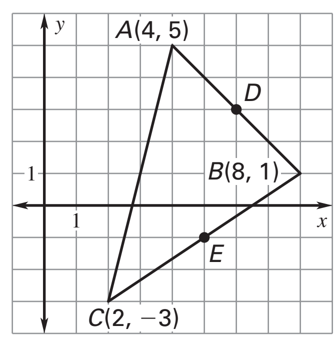

**Use the graph shown.**

**3.** Find the coordinates of $D$, the midpoint of $\overline{AB}$.

**4.** Find the length of the median $\overline{CD}$.

**5.** Find the coordinates of $E$, the midpoint of $\overline{BC}$.

**6.** Find the length of the median $\overline{AE}$.




**Complete the statement for $\triangle MNP$ with medians $\overline{MT}$, $\overline{NR}$, and $\overline{PS}$, and centroid $Q$.**

**7.** $QR =$ ____ $NR$

**8.** $MQ =$ ____ $MT$

***S* is the centroid of $\triangle RTW$, $RS = 4$, $VW = 6$, and $TV = 9$. Find the length of the segment.**

**9.** $\overline{RV}$

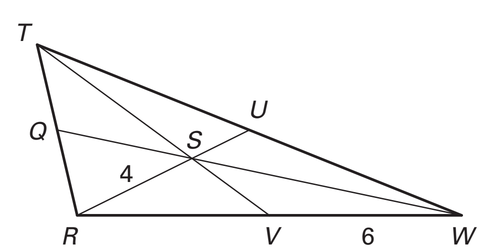
**10.** $\overline{SU}$

**11.** $\overline{RU}$

**12.** $\overline{RW}$

**13.** $\overline{TS}$

**14.** $\overline{SV}$

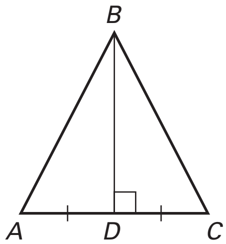


Name ______________________ Date ______________

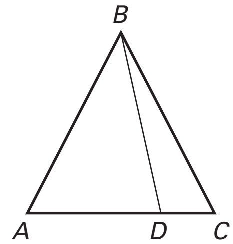
LESSON 5.4
# Practice *continued*

**Is $\overline{BD}$ a *median* of $\triangle ABC$? Is $\overline{BD}$ an *altitude*? Is $\overline{BD}$ a *perpendicular bisector*?**

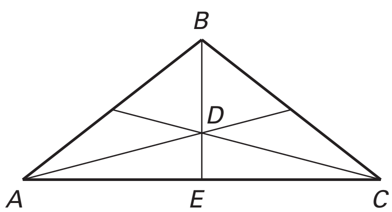
**15.**




**16.**




**17.** **Error Analysis** $D$ is the centroid of $\triangle ABC$. Your friend wants to find $DE$. The median $\overline{BE}$ has length 24. *Describe* and correct the error. *Explain* your reasoning.



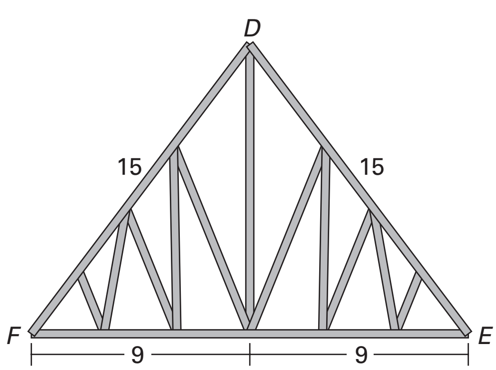

$DE = \frac{2}{3}BE$

$DE = \frac{2}{3}(24)$

$DE = 16$

**In Exercises 18 and 19, use the following information.**

**Roof Trusses** Some roofs are built using several triangular wooden trusses.




**18.** Find the altitude (height) of the truss.

**19.** How far down from $D$ is the centroid of $\triangle DEF$?

# 5.5 Use Inequalities in a Triangle

Georgia Performance Standard(s)

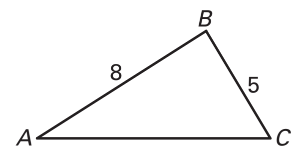
MM1G3b

***Your Notes***

**Goal** • Find possible side lengths of a triangle.

**THEOREM 5.10**

If one side of a triangle is longer than another side, then the angle opposite the longer side is ________ than the angle opposite the shorter side.

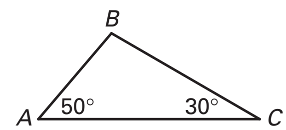



*AB* > *BC*, so

$m\angle$___ > $m\angle$___.

**THEOREM 5.11**

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is ________ than the side opposite the smaller angle.



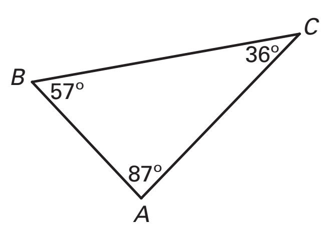

$m\angle A > m\angle C$,

so _____ > _____.

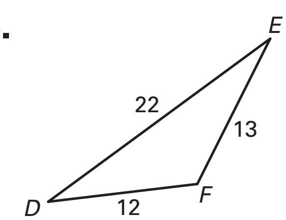
**Example 1** ***Write measurements in order from least to greatest***

**Write the measurements of the triangle in order from least to greatest.**

**a.**




**b.**




**Solution**

**a.** _______ < $m\angle B$ < _______

_____ < *AC* < _____

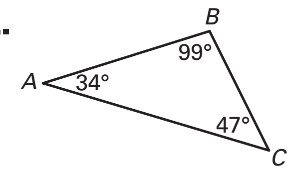
**b.** ______ < $m\angle D$ < _______

_____ < *EF* < _____

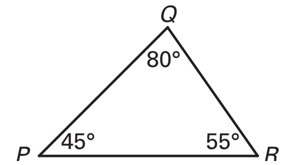
*Your Notes*

✔ ***Checkpoint*** **Write the measurements of the triangle in order from least to greatest.**

**1.**

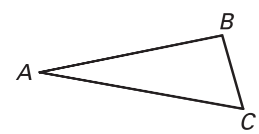



**2.**




**THEOREM 5.12: TRIANGLE INEQUALITY THEOREM**

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.




_____ + _____ > *AC*

*AC* + _____ > _____

_____ + *AC* > _____

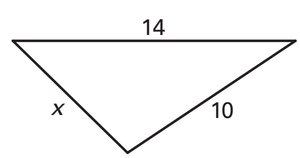
**Example 2** ***Find possible side lengths***

**A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.**

**Solution**

Let *x* represent the length of the third side. Draw diagrams to help visualize the small and large values of *x*. Then use the Triangle Inequality Theorem to write and solve inequalities.

**Small values of *x***




$x +$ _____ > _____

$x >$ ___

**Large values of *x***

_____ + _____ > $x$

_____ > $x$, or $x <$ _____

The length of the third side must be ____________________ _____________.

*Your Notes*

✔ ***Checkpoint*** **Complete the following exercise.**

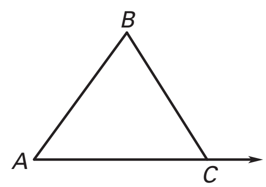
**3.** A triangle has one side of 23 meters and another of 17 meters. *Describe* the possible lengths of the third side.

**THEOREM 5.13: EXTERIOR ANGLE INEQUALITY THEOREM**

The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.

_______ > $m\angle 2$

_______ > $m\angle 3$

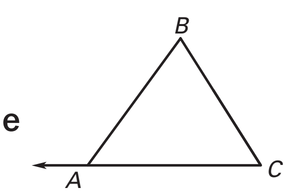



**Example 3** ***Relate exterior and interior angles***

**Write inequalities that relate $m\angle 1$ to $m\angle B$ and $m\angle C$.**

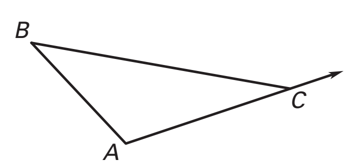
**Solution**

$\angle B$ and $\angle C$ are nonadjacent interior angles to $\angle 1$. So, by the Exterior Angle Inequality Theorem, _______ > 70° and _______ > _____.




✔ ***Checkpoint*** **Complete the following exercise.**

**4.** Write inequalities that relate $m\angle 1$ to $m\angle B$ and $m\angle C$.




***Homework***

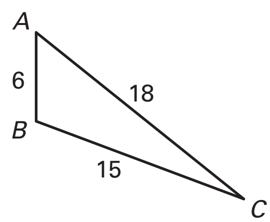
Name ______________________ Date ____________

**LESSON 5.5**

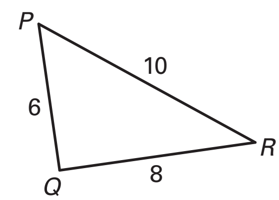
# Practice

**List the sides in order from shortest to longest and the angles in order from smallest to largest.**

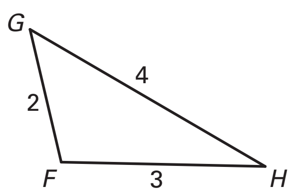
**1.**



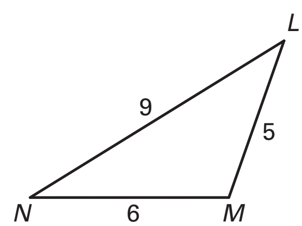

**2.**



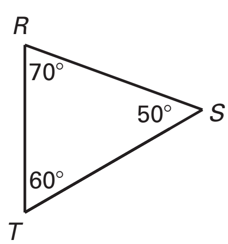

**3.**



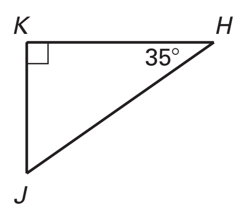

**4.**



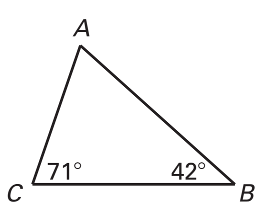

**5.**



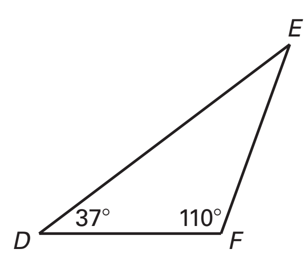

**6.**




**7.**




**8.**




**Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?**

**9.** Obtuse scalene

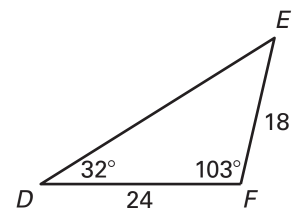
**10.** Right scalene

Name ______________________ Date ______________

LESSON 5.5

# Practice *continued*

**For Exercises 11 and 12, use the following diagram.**




**11.** Name the smallest and largest angles of $\triangle DEF$.

**12.** Name the shortest and longest sides of $\triangle DEF$.

**Is it possible to construct a triangle with the given side lengths? If not, explain why not.**

**13.** 6, 10, 15

**14.** 11, 16, 32

***Describe*** **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

**15.** 12 in., 6 in.

**16.** 3 ft, 8 ft

**17.** 12 cm, 17 cm

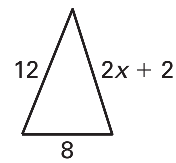
**18.** 7 yd, 13 yd

Name ______________________ Date ______________

LESSON 5.5

# Practice *continued*

**19.** Describe the possible values of $x$.




**In Exercises 20–22, you are given a 12-inch piece of wire. You want to bend the wire to form a triangle so that the length of each side is a whole number.**

**20.** Sketch two possible isosceles triangles and label each side length.

**21.** Sketch a possible scalene triangle.

**22.** List two combinations of segment lengths that will not produce triangles.

**23.** **Distance** Union Falls is 60 miles NE of Harnedville. Titus City is 40 miles SE of Harnedville. Is it possible that Union Falls and Titus City are less than 100 miles apart? *Justify* your answer.

# 5.6 Inequalities in Two Triangles and Indirect Proof

Georgia Performance Standard(s)

MM1G2a

**Goal** • Use inequalities to make comparisons in two triangles.

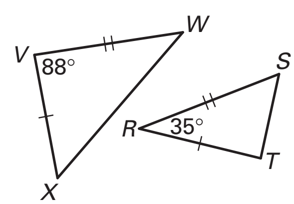
***Your Notes***

**VOCABULARY**

Indirect Proof

**THEOREM 5.14: HINGE THEOREM**

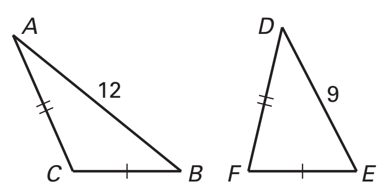
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is ________ than the third side of the second.




$WX >$ ____

**THEOREM 5.15: CONVERSE OF THE HINGE THEOREM**

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is ________ than the included angle of the second.



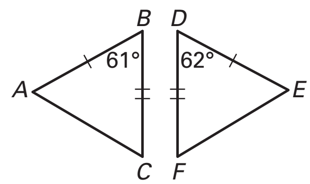

$m\angle C > m\angle$ ___

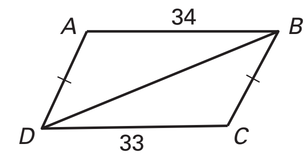
*Your Notes*

## Example 1 *Use the Hinge Theorem and its converse*

**Complete the statement with <, >, or =. Explain.**

**a.** $AC$ __?__ $EF$ **b.** $m\angle ADB$ __?__ $m\angle CBD$



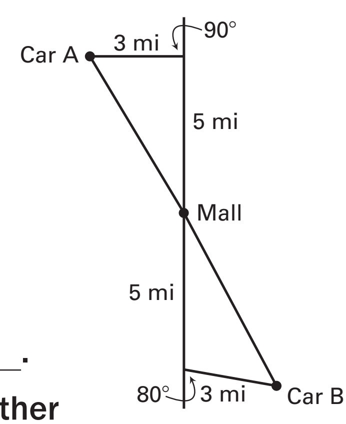

**Solution**

**a.** You are given that $\overline{AB} \cong$ _____ and $\overline{BC} \cong$ _____. Because 61° < _____, by the Hinge Theorem, $AC$ < _____.

**b.** You are given that $AD \cong$ _____ and you know that $\overline{BD} \cong$ _____ by the Reflexive Property. Because 34 > 33, _____ > _____ . So, by the Converse of the Hinge Theorem, __________ > __________.

## Example 2 *Solve a multi-step problem*

**Travel** Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall?




Draw a diagram. The distance driven and the distance back to the mall form two triangles, with ____________ 5 mile sides and ____________ 3 mile sides. Add the third side to the diagram.

Use linear pairs to find the included angles of _____ and ______.

Because 100° > 90°, Car ___ is farther from the mall than Car A by the _________________.

*Your Notes*

You have reached a contradiction when you have two statements that cannot both be true at the same time.

**Example 3** *Write an indirect proof*

**Write an indirect proof to show that an odd number is not divisible by 6.**

**Given** $x$ is an odd number.

**Prove** $x$ is not divisible by 6.

### Solution

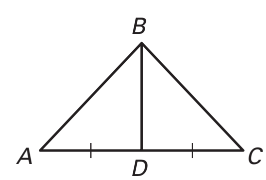
**Step 1** Assume temporarily that __________________.
This means that $\frac{\quad}{\quad} = n$ for some whole number $n$.
So, multiplying both sides by 6 gives ___ = ____.

**Step 2** If $x$ is odd, then, by definition, $x$ cannot be divided evenly by ___. However, ___ = ____ so $\frac{\quad}{\quad} = \frac{\quad}{\quad}$ = ____. We know that ____ is a whole number because $n$ is a whole number, so $x$ can be divided evenly by ___. This contradicts the given statement that _________.

**Step 3** Therefore, the assumption that $x$ is divisible by 6 is ______, which proves that ________________ _____.

✔ ***Checkpoint*** **Complete the following exercises.**

**1.** If $m\angle ADB > m\angle CDB$ which is longer, $\overline{AB}$ or $\overline{CB}$?




**2.** In Example 2, car C leaves the mall and goes 5 miles due west, then turns 85° toward south for 3 miles. Write the cars in order from the car closest to the mall to the car farthest from the mall.

**3.** Suppose you want to prove the statement "If $x + y \neq 5$ and $y = 2$, then $x \neq 3$." What temporary assumption could you make to prove the conclusion indirectly?

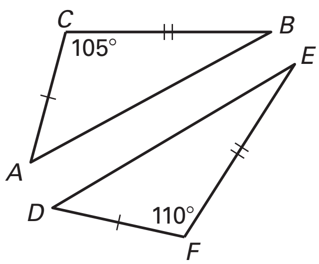
***Homework***

Name ______________________ Date ____________

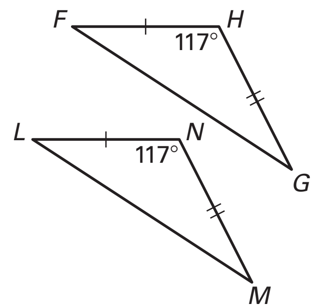
# LESSON 5.6 Practice

**Complete with <, >, or = .**

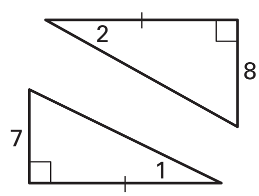
**1.** $AB$ ____ $DE$



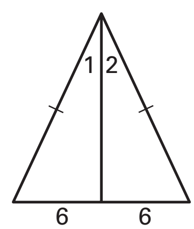

**2.** $FG$ ____ $LM$



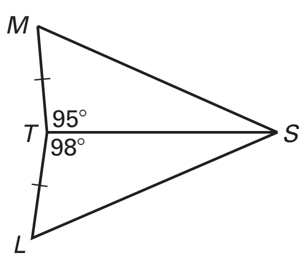

**3.** $m\angle 1$ ____ $m\angle 2$



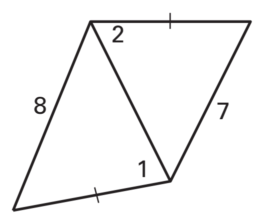

**4.** $m\angle 1$ ____ $m\angle 2$



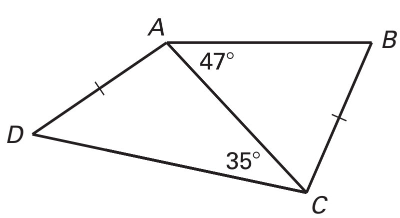

**5.** $MS$ ____ $LS$




**6.** $m\angle 1$ ____ $m\angle 2$




**7. Error Analysis** *Explain* why the student's reasoning is not correct.



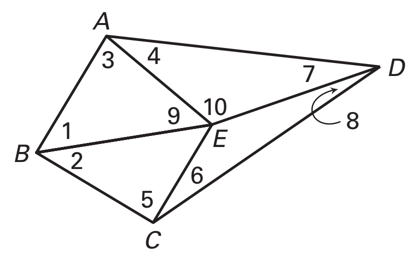

By the Hinge Theorem, $AB > DC$.

Name ________________________ Date ____________

LESSON 5.6

# Practice *continued*

**Match the conclusion on the right with the given information. *Explain* your reasoning.**




**8.** $AB = BC, m\angle 1 > m\angle 2$ **A.** $m\angle 7 > m\angle 8$

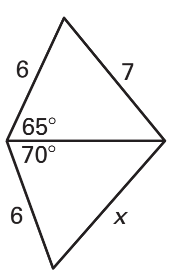
**9.** $AE > EC, AD = CD$ **B.** $AD > AB$

**10.** $m\angle 9 < m\angle 10, BE = ED$ **C.** $m\angle 3 + m\angle 4 = m\angle 5 + m\angle 6$

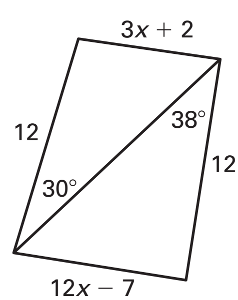
**11.** $AB = BC, AD = CD$ **D.** $AE > EC$

**Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of *x*.**

**12.**




**13.**

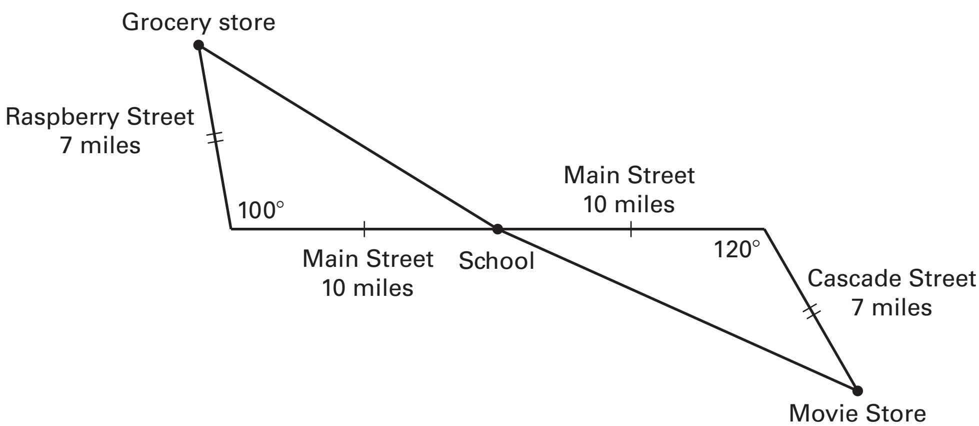


Name ____________________ Date ____________

LESSON 5.6

# Practice *continued*

**14.** **Shopping** You and a friend are going shopping. You leave school and drive 10 miles due west on Main Street. You then drive 7 miles NW on Raspberry Street to the grocery store. Your friend leaves school and drives 10 miles due east on Main Street. He then drives 7 miles SE on Cascade Street to the movie store. Each of you has driven 17 miles. Which of you is farther from your school?




**15.** Write the first statement for an indirect proof of the situation. In $\triangle MNO$, if $\overline{MP}$ is perpendicular to $\overline{NO}$, then $\overline{MP}$ is an altitude.

# 5.7 Find Angle Measures in Polygons

**Georgia Performance Standard(s)**

**MM1G3a**

**Goal** • Find angle measures in polygons.

***Your Notes***

**VOCABULARY**

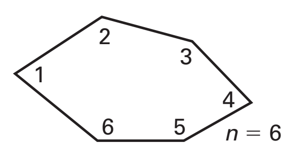
Diagonal

Interior angles of a polygon

Exterior angles of a polygon

**THEOREM 5.16: POLYGON INTERIOR ANGLES THEOREM**

The sum of the measures of the interior angles of a convex $n$-gon is $(n - \_\_\_) \cdot \_\_\_\_\_\_$.



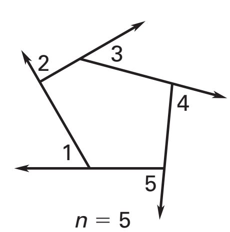

$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - \_\_\_) \cdot \_\_\_\_\_\_$

**COROLLARY TO THEOREM 5.16: INTERIOR ANGLES OF A QUADRILATERAL**

The sum of the measures of the interior angles of a quadrilateral is ______.

**THEOREM 5.17: POLYGON EXTERIOR ANGLES THEOREM**

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is ______.




$m\angle 1 + m\angle 2 + \cdots + m\angle n = \_\_\_\_\_$

***Your Notes***

**Example 1** ***Find the sum of angle measures in a polygon***

**Find the sum of the measures of the interior angles of a convex octagon.**

**Solution**

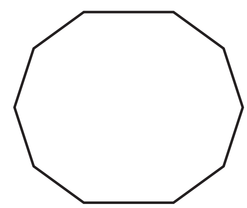
A octagon has ___ sides. Use the Polygon Interior Angles Theorem.

| | | |
|---|---|---|
| ($n$ − ___) • ______ | = (___ − ___) • ______ | **Substitute ___ for *n*.** |
| | = ___ • ______ | **Subtract.** |
| | = _______ | **Multiply.** |

The sum of the measures of the interior angles of a hexagon is _______.

✔ ***Checkpoint*** **Complete the following exercise.**

**1.** Find the sum of the measures of the interior angles of the convex decagon.



**Example 2** ***Find the number of sides of a polygon***

**The sum of the measures of the interior angles of a convex polygon is 1260°. Classify the polygon by the number of sides.**

**Solution**

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides $n$. Then solve the equation to find the number of sides.

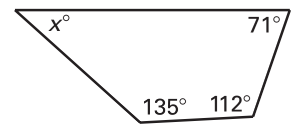
| | | |
|---|---|---|
| ($n$ − ___) • ______ = _______ | | **Polygon Interior Angles Theorem** |
| $n$ − ___ = ___ | | **Divide each side by ______.** |
| $n$ = ___ | | **Add ___ to each side.** |

The polygon has ___ sides. It is a __________.

***Your Notes***

## Example 3 *Find an unknown interior angle measure*

**Find the value of *x* in the diagram shown.**




### Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving *x*. Then solve the equation.

$x°$ + ______ + ______ + ______ = ______ **Corollary to Theorem 5.16**

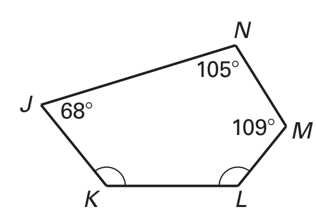
$x$ + ______ = ______ **Combine like terms.**

$x$ = ______ **Subtract ______ from each side.**

***Checkpoint*** **Complete the following exercises.**

**2.** The sum of the measures of the interior angles of a convex polygon is 1620°. Classify the polygon by the number of sides.

**3.** Use the diagram at the right. Find $m\angle K$ and $m\angle L$.

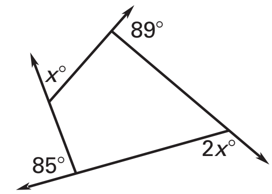


*Your Notes*

**Example 4** ***Find unknown exterior angle measures***

**Find the value of $x$ in the diagram shown.**




**Solution**

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° +$ _____ $+$ _____ $+$ _____ $=$ ______ **Polygon Exterior Angles Theorem**

___$x +$ _____ $=$ _____ **Combine like terms.**

$x =$ ____ **Solve for *x*.**

***Checkpoint*** **Complete the following exercises.**

**4.** A convex pentagon has exterior angles with measures 66°, 77°, 82°, and 62°. What is the measure of an exterior angle at the fifth vertex?

**5.** Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

***Homework***

Name ______________________ Date ______________

# LESSON 5.7 Practice

**Find the sum of the measures of the interior angles of the indicated convex polygon.**

**1.** Heptagon

**2.** 13-gon

**3.** 17-gon

**4.** 18-gon

**5.** 22-gon

**6.** 25-gon

**7.** 30-gon

**8.** 34-gon

**9.** 39-gon

**The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.**

**10.** 1260°

**11.** 2160°

**12.** 3240°

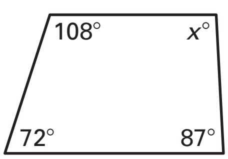
**13.** 4680°

**14.** 5400°

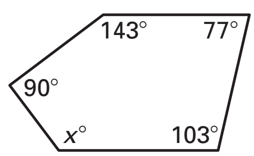
**15.** 7560°

**Find the value of *x*.**

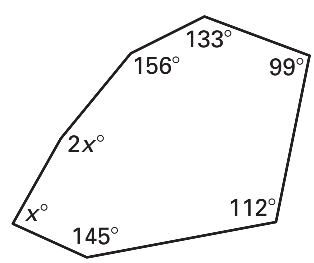
**16.**



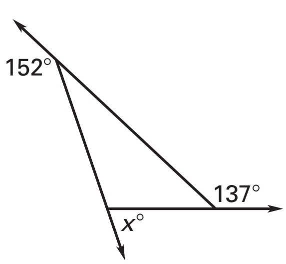

**17.**



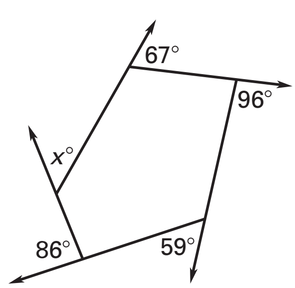

**18.**



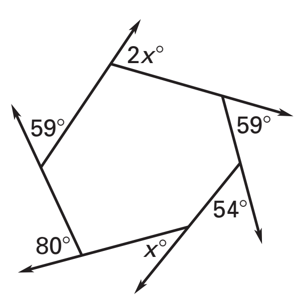

**19.**




**20.**




**21.**




**22.** The measures of the interior angles of a convex quadrilateral are $x°$, $2x°$, $4x°$, and $5x°$. What is the measure of the largest interior angle?

**23.** The measures of the exterior angles of a convex pentagon are $2x°$, $4x°$, $6x°$, $8x°$, and $10x°$. What is the measure of the smallest exterior angle?

Name ____________________ Date ____________

LESSON 5.7

# Practice *continued*

**Find the measures of an interior angle and an exterior angle of the indicated regular polygon.**

**24.** Regular hexagon

**25.** Regular decagon

**26.** Regular 15-gon

**27.** Regular 20-gon

**28.** Regular 30-gon

**29.** Regular 36-gon

**In Exercises 30–37, find the value of *n* for each regular *n*-gon described.**

**30.** Each interior angle of the regular $n$-gon has a measure of 90°.

**31.** Each interior angle of the regular $n$-gon has a measure of 108°.

**32.** Each interior angle of the regular $n$-gon has a measure of 135°.

**33.** Each interior angle of the regular $n$-gon has a measure of 144°.

**34.** Each exterior angle of the regular $n$-gon has a measure of 90°.

**35.** Each exterior angle of the regular $n$-gon has a measure of 60°.

**36.** Each exterior angle of the regular $n$-gon has a measure of 40°.

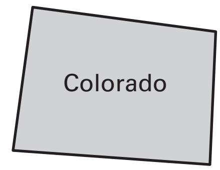
**37.** Each exterior angle of the regular $n$-gon has a measure of 30°.

Name ______________________ Date ______________

LESSON 5.7

# Practice *continued*

**38. Geography** The shape of Colorado can be approximated by a polygon, as shown.

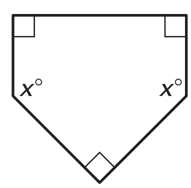



**a.** How many sides does the polygon have? Classify the polygon.

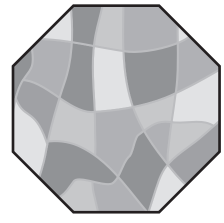
**b.** What is the sum of the measures of the interior angles of the polygon?

**c.** What is the sum of the measures of the exterior angles of the polygon?

**39. Softball** A home plate marker for a softball field is a pentagon, as shown. Three of the interior angles of the pentagon are right angles and the remaining two interior angles are congruent. What is the value of $x$?




**40. Stained Glass Window** Part of a stained-glass window is a regular octagon, as shown. Find the measure of an interior angle of the regular octagon. Then find the measure of an exterior angle.



# 5.8 Use Properties of Parallelograms

**Georgia Performance Standard(s)**

**MM1G1e, MM1G3d**

***Your Notes***

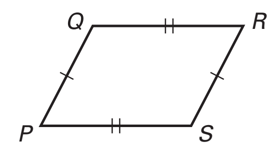
**Goal** • Find angle and side measures in parallelograms.

**VOCABULARY**

Parallelogram

**THEOREM 5.18**

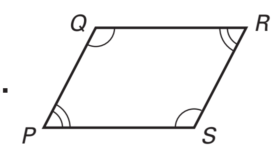
If a quadrilateral is a parallelogram, then its opposite sides are congruent.




If *PQRS* is a parallelogram, then _____ $\cong \overline{RS}$ and $\overline{QR} \cong$ _____.

**THEOREM 5.19**

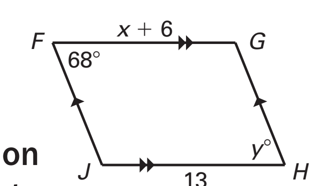
If a quadrilateral is a parallelogram, then its opposite angles are congruent.




If *PQRS* is a parallelogram, then $\angle P \cong$ _____ and _____ $\cong \angle S$.

**Example 1** ***Use properties of parallelograms***

**Find the values of *x* and *y*.**




**Solution**

*FGHJ* is a parallelogram by the definition of a parallelogram. Use Theorem 5.18 to find the value of *x*.

$FG =$ _____ **Opposite sides of a ▱ are ≅.**

$x + 6 =$ _____ **Substitute $x + 6$ for $FG$ and _____ for _____.**

$x =$ ___ **Subtract 6 from each side.**

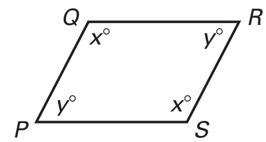
By Theorem 5.19, $\angle F \cong$ _____, or $m\angle F =$ _______. So, $y° =$ _____.

In ▱*FGHJ*, $x =$ ___ and $y =$ ____.

*Your Notes*

**THEOREM 5.20**

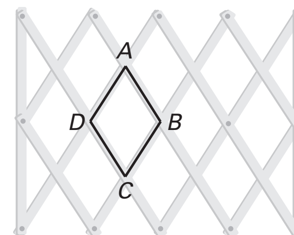
If a quadrilateral is a parallelogram, then its consecutive angles are ________________.




If *PQRS* is a parallelogram, then $x° + y° =$ ______.

**Example 2** ***Use properties of a parallelogram***

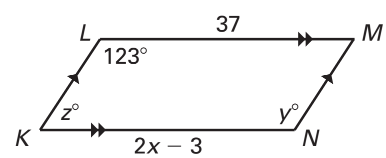
**Gates** As shown, a gate contains several parallelograms. Find $m\angle ADC$ when $m\angle DAB = 65°$.




**Solution**

By Theorem 5.20, the consecutive angle pairs in $\square ABCD$ are ________________. So, $m\angle ADC + m\angle DAB =$ ______. Because $m\angle DAB = 65°$, $m\angle ADC =$ ______ − _____ = ______.

***Checkpoint*** **Find the indicated measure in $\square KLMN$ shown at the right.**




**1.** *x*

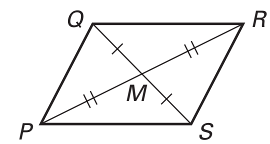
**2.** *y*

**3.** *z*

*Your Notes*

**THEOREM 5.21**

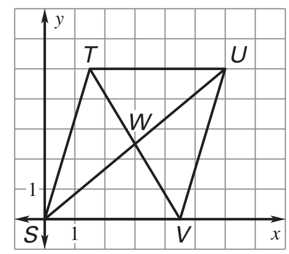
If a quadrilateral is a parallelogram, then its diagonals ________ each other.




$\overline{QM} \cong$ _____ and $\overline{PM} \cong$ _____

**Example 3** ***Find the intersection of diagonals***

**The diagonals of ▱*STUV* intersect at point *W*. Find the coordinates of *W*.**




**Solution**

By Theorem 5.21, the diagonals of a parallelogram ________ each other.
So, *W* is the ___________ of the diagonals $\overline{TV}$ and $\overline{SU}$.
Use the ____________________.

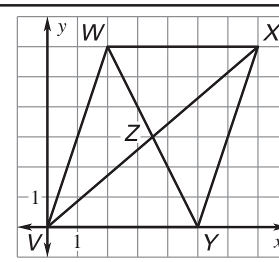
Coordinates of midpoint *W* of

$\overline{SU}$ = ( ______________ ) = ( ______ )

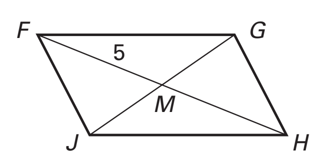
In Example 3, you can use either diagonal to find the coordinates of *W*. Using $\overline{SU}$ simplifies calculations because one endpoint is (0, 0).

✔ ***Checkpoint*** **Complete the following exercises.**

**4.** The diagonals of ▱*VWXY* intersect at point *Z*. Find the coordinates of *Z*.




**5.** Given that ▱*FGHJ* is a parallelogram, find *MH* and *FH*.



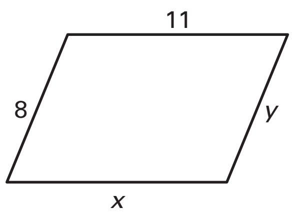

***Homework***

Name ______________________ Date ______________

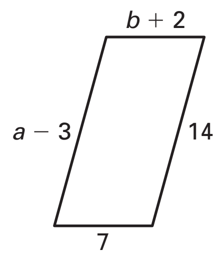
# LESSON 5.8 Practice

**Find the value of each variable in the parallelogram.**

**1.**

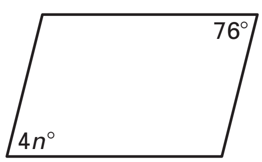



**2.**

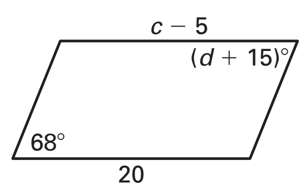



**3.**

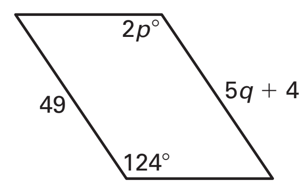

**4.**




**5.**

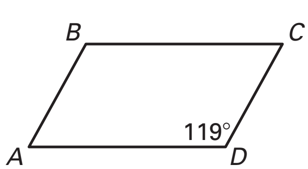



**6.**

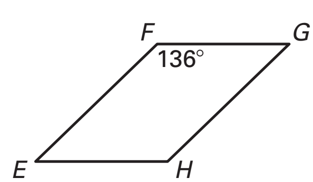



**Find the measure of the indicated angle in the parallelogram.**

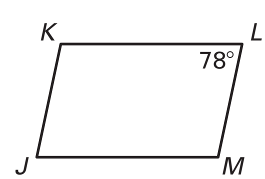
**7.** Find $m\angle C$.




**8.** Find $m\angle E$.




**9.** Find $m\angle K$.



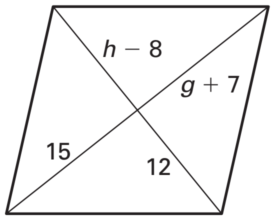
Name ______________________ Date ______________

LESSON 5.8

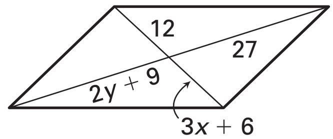
# Practice *continued*

**Find the value of each variable in the parallelogram.**

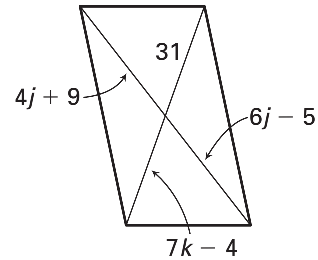
**10.**



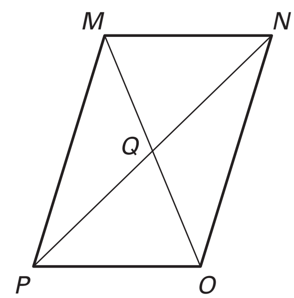

**11.**




**12.**




**Use the diagram of parallelogram *MNOP* at the right to complete the statement. *Explain.***




**13.** $\overline{MN} \cong$ ________

**14.** $\overline{MN} \parallel$ ________

**15.** $\overline{ON} \cong$ ________

**16.** $\angle MPO \cong$ ________

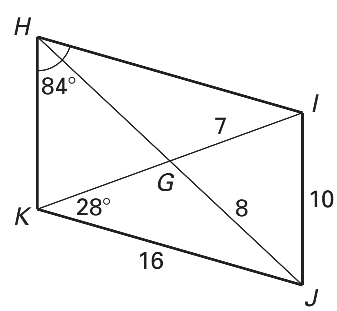
**17.** $\overline{PQ} \cong$ ________

**18.** $\overline{QM} \cong$ ________

**19.** $\angle MQN \cong$ ________

**20.** $\angle NPO \cong$ ________

**Find the indicated measure in ▱*HIJK*. *Explain.***




**21.** $HI$

**22.** $KH$

**23.** $GH$

**24.** $HJ$

**25.** $m\angle KIH$

**26.** $m\angle JIH$

**27.** $m\angle KJI$

**28.** $m\angle HKI$

Name ____________________ Date ____________

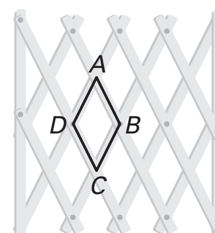
LESSON 5.8

# Practice *continued*

**29.** The measure of one interior angle of a parallelogram is twice the measure of another angle. Find the measure of each angle.

**30.** The measure of one interior angle of a parallelogram is 30 degrees more than the measure of another angle. Find the measure of each angle.

**The crossing slats of a gate form parallelograms that move together to make the gate wider. In Exercises 31–34, use the figure at the right.**




**31.** What is $m\angle A$ when $m\angle B = 110°$?

**32.** What is $m\angle D$ when $m\angle B = 130°$?

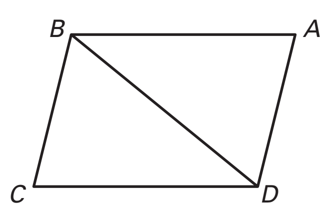
**33.** What happens to $m\angle A$ when $m\angle B$ decreases?

**34.** What happens to $AC$ when $m\angle B$ increases?

**35.** Complete the proof.

**GIVEN:** $ABCD$ is a ▱.

**PROVE:** $\triangle ABD \cong \triangle CDB$




| Statements | Reasons |
|---|---|
| **1.** $ABCD$ is a ▱. | **1.** |
| **2.** | **2.** Opposite sides of ▱ are ≅. |
| **3.** | **3.** Opposite sides of ▱ are ≅. |
| **4.** $\angle A \cong \angle C$ | **4.** |
| **5.** $\triangle ABD \cong \triangle CBD$ | **5.** |

# 5.9 Show that a Quadrilateral is a Parallelogram

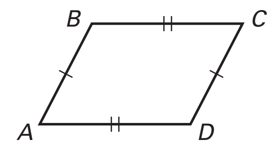
**Georgia Performance Standard(s)**

**MM1G1a, MM1G1e, MM1G3d**

***Your Notes***

**Goal** • Use properties to identify parallelograms.

**THEOREM 5.22**

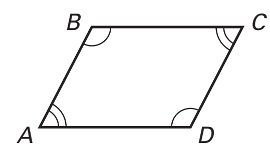



If both pairs of opposite _______ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong$ _____ and $\overline{BC} \cong$ _____, then *ABCD* is a parallelogram.

---

**THEOREM 5.23**

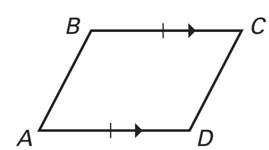



If both pairs of opposite ________ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong$ _____ and $\angle B \cong$ _____, then *ABCD* is a parallelogram.

---

**THEOREM 5.24**

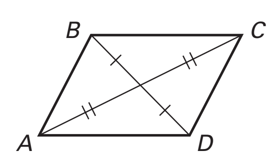



If one pair of opposite sides of a quadrilateral are ____________ and _________, then the quadrilateral is a parallelogram.

If $\overline{BC}$ ____ $\overline{AD}$ and $\overline{BC}$ ___ $\overline{AD}$, then *ABCD* is a parallelogram.

---

**THEOREM 5.25**




If the diagonals of a quadrilateral ________ each other, then the quadrilateral is a parallelogram.

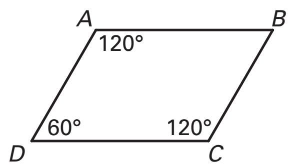
If $\overline{BD}$ and $\overline{AC}$ ________ each other, then *ABCD* is a parallelogram.

**Your Notes**

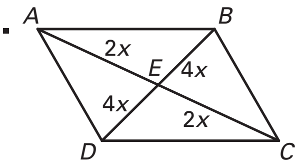
## Example 1 *Identify parallelograms*

**Explain how you know that quadrilateral *ABCD* is a parallelogram.**

**a.**




**b.**




**a.** By the ________________________ you know that $m\angle A + m\angle B + m\angle C + m\angle D =$ ______, so $m\angle B =$ _____. Because both pairs of opposite angles are ____________, then *ABCD* is a parallelogram by ______________.

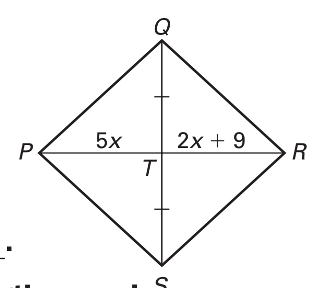
**b.** In the diagram, $AE =$ _____ and $BE =$ _____ . So, the diagonals bisect each other, and *ABCD* is a parallelogram by ______________.

***Checkpoint*** **Complete the following exercise.**

**1.** In quadrilateral *GHJK*, $m\angle G = 55°$, $m\angle H = 125°$, and $m\angle J = 55°$. Find $m\angle K$. What theorem can you use to show that *GHJK* is a parallelogram?

## Example 2 *Use algebra with parallelograms*

**For what value of $x$ is quadrilateral *PQRS* a parallelogram?**




By Theorem 5.25, if the diagonals of *PQRS* ________ each other, then it is a parallelogram. You are given that $\overline{QT} \cong$ _____. Find $x$ so that $\overline{PT} \cong$ _____.

| | |
|---|---|
| $PT =$ _____ | **Set the segment lengths equal.** |
| $5x =$ _________ | **Substitute for *PT* and for _____.** |
| ___$x =$ ___ | **Subtract _____ from each side.** |
| $x =$ ___ | **Divide each side by ___.** |

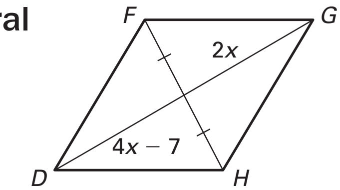
When $x =$ ___, $PT = 5($___$) =$ _____ and $RT = 2($___$) + 9 =$ _____.

Quadrilateral *PQRS* is a parallelogram when $x =$ ___.

*Your Notes*

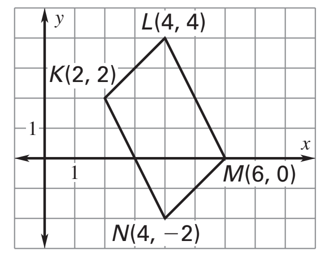
**Checkpoint** Complete the following exercise.

2. For what value of $x$ is quadrilateral $DFGH$ a parallelogram?




**Example 3** *Use coordinate geometry*

**Show that quadrilateral *KLMN* is a parallelogram.**




One way is to show that a pair of sides are congruent and parallel. Then apply ______________.

First use the Distance Formula to show that $\overline{KL}$ and $\overline{MN}$ are ____________.

$KL = \sqrt{\qquad\qquad\qquad} = \sqrt{\quad}$

$MN = \sqrt{\qquad\qquad\qquad} = \sqrt{\quad}$

Because $KL = MN = \sqrt{\quad}$, $\overline{KL}$ ____ $\overline{MN}$.

Then use the slope formula to show that $\overline{KL}$ ___ $\overline{MN}$.

Slope of $\overline{KL} = \frac{\square}{\square} =$ ___

Slope of $\overline{MN} = \frac{\square}{\square} =$ ___

$\overline{KL}$ and $\overline{MN}$ have the same slope, so they are _________.

$\overline{KL}$ and $\overline{MN}$ are congruent and parallel. So, $KLMN$ is a parallelogram by ______________.

**Checkpoint** Complete the following exercise.

***Homework***

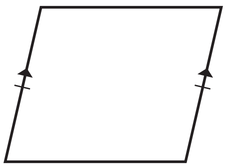
3. *Explain* another method that can be used to show that quadrilateral $KLMN$ in Example 3 is a parallelogram.

LESSON 5.9

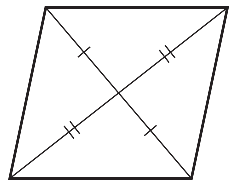
# Practice

**What theorem can you use to show that the quadrilateral is a parallelogram?**

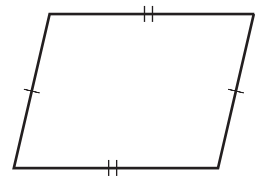
**1.**



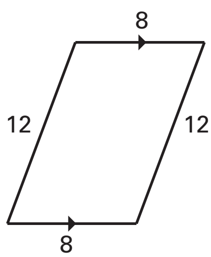
**2.**



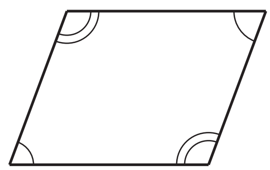
**3.**



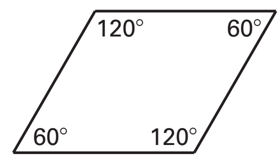
**4.**




**5.**

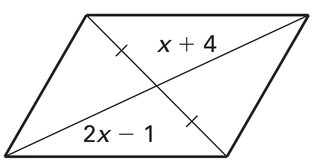


**6.**

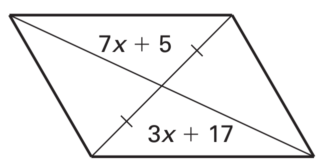



**For what value of *x* is the quadrilateral a parallelogram?**

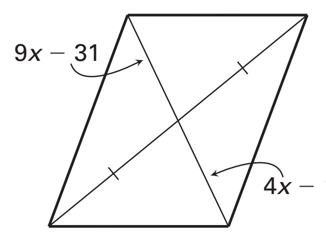
**7.**



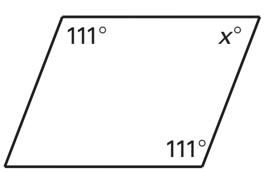

**8.**



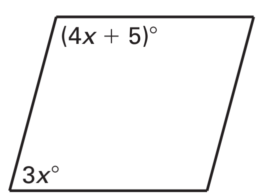

**9.**



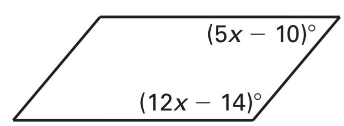

**10.**




**11.**




**12.**

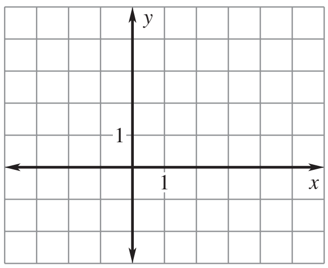


Name ______________________ Date ______________

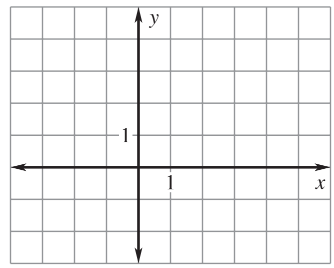
LESSON 5.9
# Practice *continued*

**The vertices of quadrilateral *ABCD* are given. Draw *ABCD* in a coordinate plane and show that it is a parallelogram.**

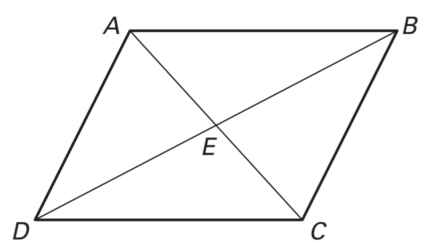
**13.** $A(-1, 3), B(4, 3), C(2, -1), D(-3, -1)$




**14.** $A(-2, 3), B(3, 2), C(3, -1), D(-2, 0)$




**What additional information is needed in order to prove that quadrilateral *ABCD* is a parallelogram?**




**15.** $\overline{AB} \parallel \overline{DC}$

**16.** $\overline{AB} \cong \overline{DC}$

**17.** $\angle DCB \cong \angle DAB$

**18.** $\overline{DE} \cong \overline{EB}$

**19.** $m\angle CDA + m\angle DAB = 180°$

**20.** $\angle DCA \cong \angle BAC$

Name ______________________ Date ______________

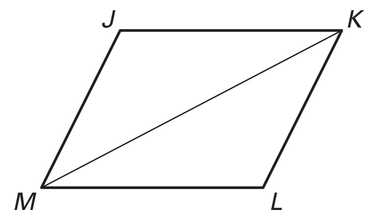
LESSON 5.9

# Practice *continued*

**In Exercises 21 and 22, use the diagram below to complete the proof using two different methods.**

**GIVEN:** $\triangle MJK \cong \triangle KLM$

**PROVE:** *MJKL* is a parallelogram.




**21.**

| Statements | Reasons |
|---|---|
| **1.** | **1.** Given |
| **2.** $\overline{JK} \cong \overline{LM}$<br>$\overline{JM} \cong \overline{LK}$ | **2.** |
| **3.** *MJKL* is a ▱. | **3.** |

**22.**

| Statements | Reasons |
|---|---|
| **1.** | **1.** Given |
| **2.** $\overline{JK} \cong \overline{LM}$<br>$\angle JKM \cong \angle KML$ | **2.** |
| **3.** | **3.** Alternate Interior $\angle$'s Converse |
| **4.** *MJKL* is a ▱. | **4.** |

# 5.10 Properties of Rhombuses, Rectangles, and Squares

**Georgia Performance Standard(s)**

**MM1G3d**

***Your Notes***

**Goal** • Use properties of rhombuses, rectangles, and squares.

**VOCABULARY**

Rhombus

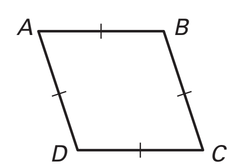
Rectangle

Square

**RHOMBUS COROLLARY**

A quadrilateral is a rhombus if and only if it has four congruent _______.

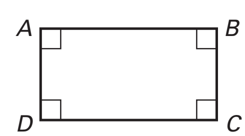
*ABCD* is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.




**RECTANGLE COROLLARY**

A quadrilateral is a rectangle if and only if it has four _____________.

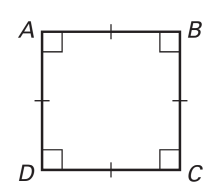
*ABCD* is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.




**SQUARE COROLLARY**

A quadrilateral is a square if and only if it is a ___________ and a ___________.

*ABCD* is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



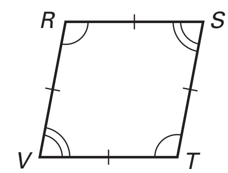
*Your Notes*

## Example 1 *Use properties of special quadrilaterals*

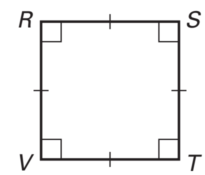
**For any rhombus *RSTV*, decide whether the statement is always or sometimes true. Draw a sketch and explain your reasoning.**

**a.** $\angle S \cong \angle V$ **b.** $\angle T \cong \angle V$

**Solution**

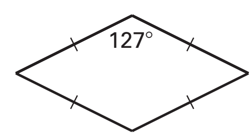



**a.** By definition, a rhombus is a parallelogram with four congruent _______. By Theorem 5.19, opposite angles of a parallelogram are ____________. So, $\angle S \cong \angle V$. The statement is _________ true.




**b.** If rhombus *RSTV* is a _________, then all four angles are congruent right angles. So $\angle T \cong \angle V$ if *RSTV* is a _________. Because not all rhombuses are also __________, the statement is _____________ true.

## Example 2 *Classify special quadrilaterals*




**Classify the special quadrilateral. Explain your reasoning.**

The quadrilateral has four congruent _______. One of the angles is not a _____________, so the rhombus is not also a _________. By the Rhombus Corollary, the quadrilateral is a ___________.

✔ ***Checkpoint*** **Complete the following exercises.**

**1.** For any square *CDEF*, is it *always* or *sometimes* true that $\overline{CD} \cong \overline{DE}$? *Explain* your reasoning.

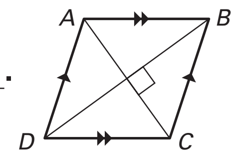
**2.** A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

*Your Notes*

**THEOREM 5.26**

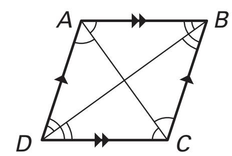
A parallelogram is a rhombus if and only if its diagonals are ______________.




$\square ABCD$ is a rhombus if and only if ______ $\perp$ ______.

**THEOREM 5.27**

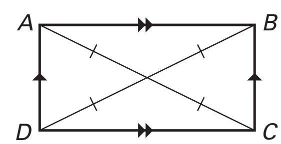
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.




$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle$______ and $\angle$______ and $\overline{BD}$ bisects $\angle$______ and $\angle$______.

**THEOREM 5.28**

A parallelogram is a rectangle if and only if its diagonals are ______________.



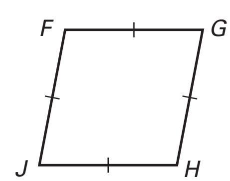

$\square ABCD$ is a rectangle if and only if ______ $\cong$ ______.

**Example 3** *List properties of special parallelograms*

**Sketch rhombus *FGHJ*. List everything you know about it.**

**Solution**

By definition, you need to draw a figure with the following properties:




- The figure is a ______________.
- The figure has four congruent ______.

Because *FGHJ* is a parallelogram, it has these properties:

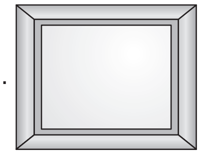
- Opposite sides are ________ and __________.
- Opposite angles are __________. Consecutive angles are ______________.
- Diagonals ______ each other.

By Theorem 5.26, the diagonals of *FGHJ* are ______________. By Theorem 5.27, each diagonal bisects a pair of ______________.

*Your Notes*

**Example 4** ***Solve a real-world problem***

**Framing** You are building a frame for a painting. The measurements of the frame are shown at the right.




**a.** The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain*.

**b.** You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

### Solution

**a.** No, you cannot. The boards on opposite sides are the same length, so they form a ________________. But you do not know whether the angles are ______________.

**b.** By Theorem 5.28, the diagonals of a rectangle are ____________. The diagonals of the frame are ____________, so the frame forms a ___________.

**Checkpoint** Complete the following exercises.

**3.** Sketch rectangle *WXYZ*. List everything that you know about it.

**4.** Suppose the diagonals of the frame in Example 4 are not congruent. Could the frame still be a rectangle? *Explain*.

***Homework***

Name ______________________ Date ____________

**LESSON 5.10**

# Practice

**For any rhombus *ABCD*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.**

**1.** $\angle A \cong \angle C$

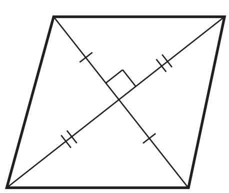
**2.** $\overline{DA} \cong \overline{AB}$

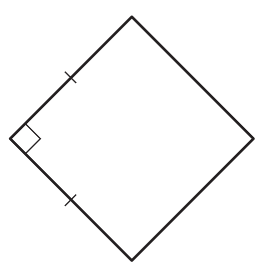
**For any rectangle *FGHJ*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and explain your reasoning.**

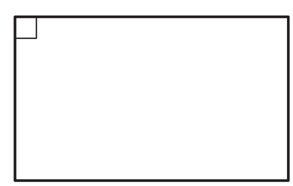
**3.** $\angle G \cong \angle H$

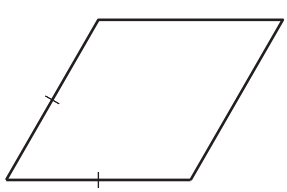
**4.** $\overline{JF} \cong \overline{FG}$

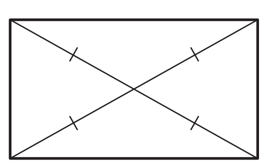
**Classify the parallelogram. *Explain* your reasoning.**

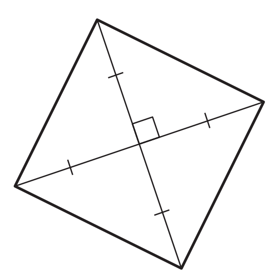
**5.**


**6.**


**7.**


**8.**


**9.**


**10.**


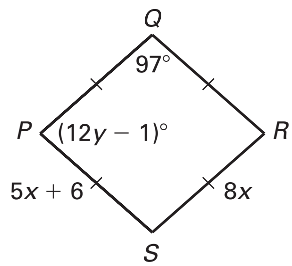
Name ________________________ Date ______________

LESSON 5.10

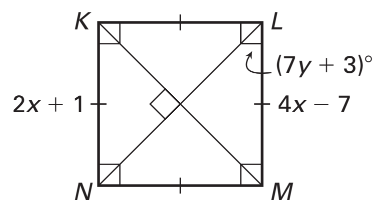
# Practice *continued*

**Classify the special quadrilateral. *Explain* your reasoning. Then find the values of *x* and *y*.**

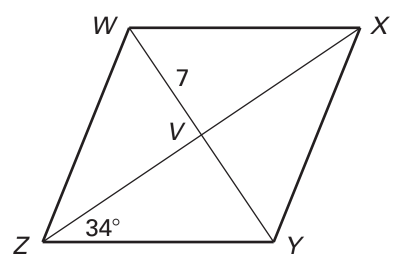
**11.**




**12.**




**The diagonals of rhombus *WXYZ* intersect at *V*. Given that $m\angle XZY = 34°$ and $WV = 7$, find the indicated measure.**



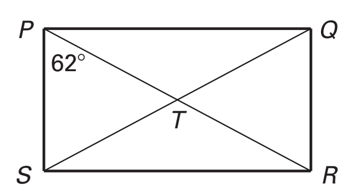

**13.** $m\angle WZV$

**14.** $m\angle XYZ$

**15.** $VY$

**16.** $WY$

**The diagonals of rectangle *PQRS* intersect at *T*. Given that $m\angle RPS = 62°$ and $QS = 18$, find the indicated measure.**




**17.** $m\angle QPR$

**18.** $m\angle PTQ$

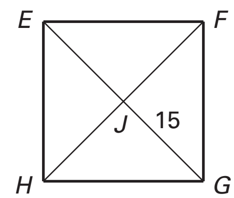
**19.** $ST$

**20.** $PR$

Name ____________________ Date ____________

**LESSON 5.10** # Practice *continued*

**The diagonals of square *EFGH* intersect at *J*. Given that *GJ* = 15, find the indicated measure.**



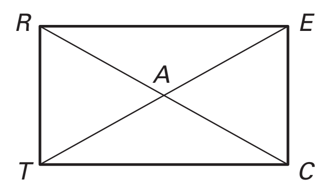

**21.** $m\angle EJF$ **22.** $m\angle JFG$

**23.** $FH$ **24.** $EJ$

**25.** Complete the proof.

**GIVEN:** *RECT* is a rectangle.

**PROVE:** $\triangle ART \cong \triangle ACE$




| Statements | Reasons |
|---|---|
| **1.** | **1.** Given |
| **2.** $\overline{RT} \cong \overline{EC}$<br>$\overline{RT} \parallel \overline{EC}$ | **2.** |
| **3.** | **3.** Alternate Interior $\angle$s are $\cong$. |
| **4.** | **4.** Vertical $\angle$s are $\cong$. |
| **5.** $\triangle ART \cong \triangle ACE$ | **5.** |

**Write the corollary as a conditional statement and its converse. Then explain why each statement is true.**

**26.** Rhombus Corollary

**27.** Rectangle Corollary

**28.** Square Corollary

# 5.11 Use Properties of Trapezoids and Kites

Georgia Performance Standard(s)

MM1G1e, MM1G3d

***Your Notes***

**Goal** • Use properties of trapezoids and kites.

**VOCABULARY**

Trapezoid

Bases of a trapezoid

Base angles of a trapezoid

Legs of a trapezoid

Isosceles trapezoid

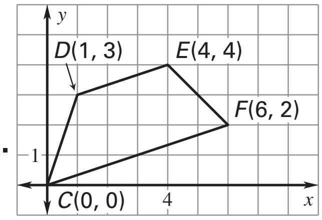
Midsegment of a trapezoid

Kite

Your Notes

## Example 1 *Use a coordinate plane*

**Show that *CDEF* is a trapezoid.**




**Solution**

Compare the slopes of opposite sides.

Slope of $\overline{DE}$ = ______ = ______

Slope of $\overline{CF}$ = ______ = ______ = ______

The slopes of $\overline{DE}$ and $\overline{CF}$ are the same, so $\overline{DE}$ ___ $\overline{CF}$.

Slope of $\overline{EF}$ = ______ = ______ = ______

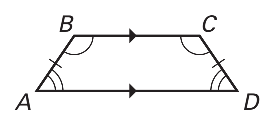
Slope of $\overline{CD}$ = ______ = ______ = ______

The slopes of $\overline{EF}$ and $\overline{CD}$ are not the same, so $\overline{EF}$ is ____________ to $\overline{CD}$.

Because quadrilateral *CDEF* has exactly one pair of ______________, it is a trapezoid.

**THEOREM 5.29**

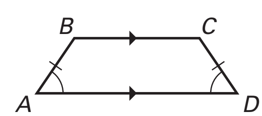
If a trapezoid is isosceles, then each pair of base angles is ____________.




If trapezoid *ABCD* is isosceles, then $\angle A \cong \angle$___ and $\angle$___ $\cong \angle C$.

**THEOREM 5.30**

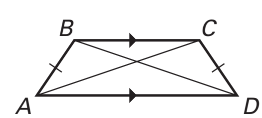
If a trapezoid has a pair of congruent ____________, then it is an isosceles trapezoid.




If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid *ABCD* is isosceles.

**THEOREM 5.31**

A trapezoid is isosceles if and only if its diagonals are ____________.

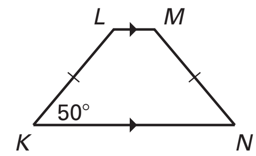



Trapezoid *ABCD* is isosceles if and only if _____ $\cong$ _____.

*Your Notes*

## Example 2 *Use properties of isosceles trapezoids*

**Kitchen** A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find $m\angle N$, $m\angle L$, and $m\angle M$.




### Solution

**Step 1** **Find $m\angle N$.** *KLMN* is an ____________________, so $\angle N$ and $\angle$___ are congruent base angles, and $m\angle N = m\angle$___ $=$ _____.

**Step 2** **Find $m\angle L$.** Because $\angle K$ and $\angle L$ are consecutive interior angles formed by $\overleftrightarrow{KL}$ intersecting two parallel lines, they are ________________. So, $m\angle L =$ ______ $-$ _____ $=$ ______.

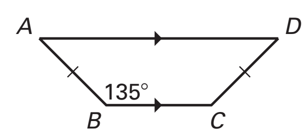
**Step 3** **Find $m\angle M$.** Because $\angle M$ and $\angle$___ are a pair of base angles, they are congruent, and $m\angle M = m\angle$___ $=$ ______.

So, $m\angle N =$ _____, $m\angle L =$ ______, and $m\angle M =$ ______.

***Checkpoint*** **Complete the following exercises.**

**1.** In Example 1, suppose the coordinates of point *E* are (7, 5). What type of quadrilateral is *CDEF*? *Explain*.

**2.** Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.

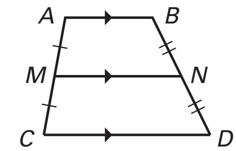


*Your Notes*

**THEOREM 5.32: MIDSEGMENT THEOREM FOR TRAPEZOIDS**

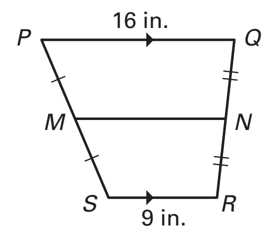
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.




If $\overline{MN}$ is the midsegment of trapezoid *ABDC*, then $\overline{MN} \parallel$ _____, $\overline{MN} \parallel$ _____, and $MN =$ ___(_____ + _____).

**Example 3** *Use the midsegment of a trapezoid*

**In the diagram, $\overline{MN}$ is the midsegment of trapezoid *PQRS*. Find *MN*.**




**Solution**

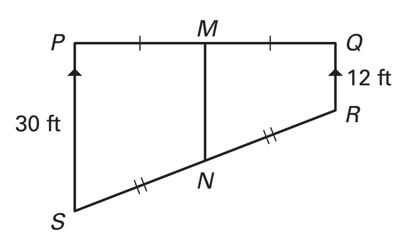
Use Theorem 5.32 to find *MN*.

| | | |
|---|---|---|
| $MN =$ | ___(_____ + _____) | **Apply Theorem 5.32.** |
| $=$ | ___(____ + ___) | **Substitute ____ for *PQ* and ___ for *SR*.** |
| $=$ | ______ | **Simplify.** |

*MN* is ______ inches.

***Checkpoint*** **Complete the following exercise.**

**3.** Find *MN* in the trapezoid at the right.

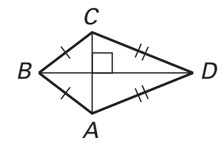


***Your Notes***

**THEOREM 5.33**

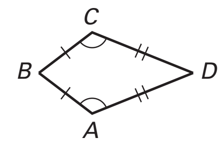
If a quadrilateral is a kite, then its diagonals are ______________.

If quadrilateral *ABCD* is a kite, then _____ $\perp$ _____.




**THEOREM 5.34**

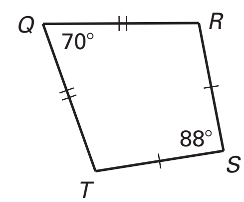
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.




If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A$ ____ $\angle C$ and $\angle B$ ____ $\angle D$.

**Example 4** ***Use properties of kites***

**Find $m\angle T$ in the kite shown at the right.**




**Solution**

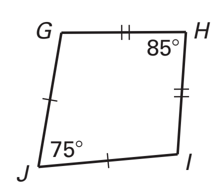
By Theorem 5.34, *QRST* has exactly one pair of ____________ opposite angles. Because $\angle Q \not\cong \angle S$, $\angle$___ and $\angle T$ must be congruent. So, $m\angle$___ $= m\angle T$. Write and solve an equation to find $m\angle T$.

| | |
|---|---|
| $m\angle T + m\angle R +$ _____ $+$ _____ $=$ ______ | **Corollary to Theorem 5.16** |
| $m\angle T + m\angle T +$ _____ $+$ _____ $=$ ______ | **Substitute $m\angle T$ for $m\angle R$.** |
| ___$(m\angle T) +$ ______ $=$ ______ | **Combine like terms.** |
| $m\angle T =$ ______ | **Solve for $m\angle T$.** |

***Homework***

✔ ***Checkpoint*** **Complete the following exercise.**

**4.** Find $m\angle G$ in the kite shown at the right.



Name ______________________________ Date ______________

**LESSON 5.11** **Practice**

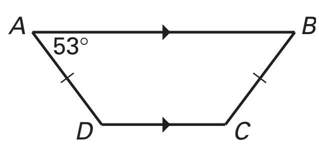
**Points *J, K, L,* and *M* are the vertices of a quadrilateral. Determine whether *JKLM* is a trapezoid.**

**1.** $J(-1, -1), K(0, 3), L(3, 3), M(4, -1)$

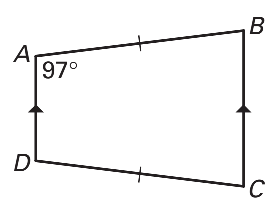
**2.** $J(-4, -2), K(-4, 3), L(2, 3), M(3, -5)$

**Find $m\angle B$, $m\angle C$, and $m\angle D$.**

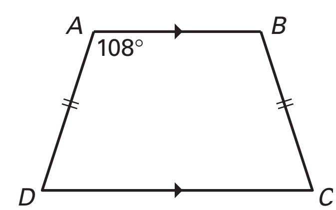
**3.**




**4.**




**5.**

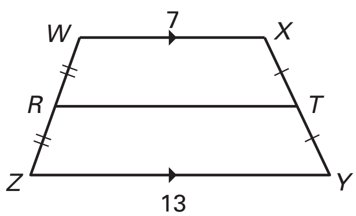


Name ____________________ Date ____________

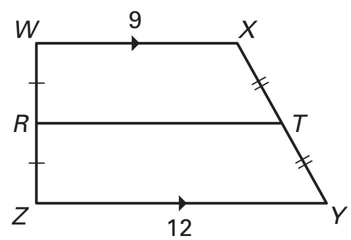
# LESSON 5.11 Practice *continued*

**Find the length of the midsegment $\overline{RT}$.**

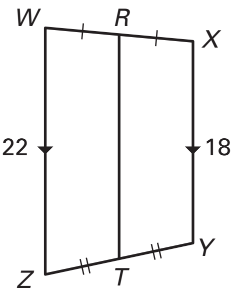
**6.**




**7.**




**8.**




**Tell whether the statement is *always*, *sometimes*, or *never* true.**

**9.** A trapezoid is a parallelogram.

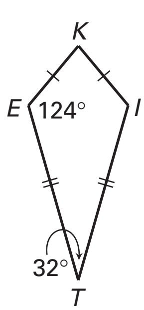
**10.** The bases of a trapezoid are parallel.

**11.** The base angles of an isosceles trapezoid are congruent.

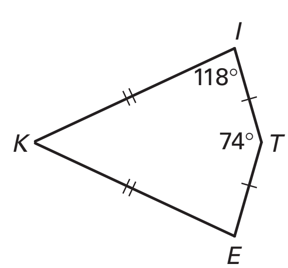
**12.** The legs of a trapezoid are congruent.

***KITE* is a kite. Find *m*∠*K*.**

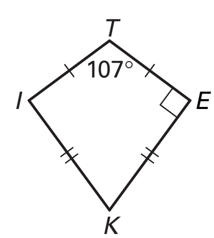
**13.**




**14.**




**15.**



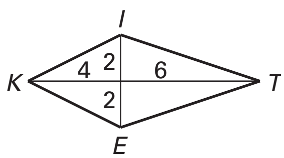
Name ______________________ Date ______________

LESSON 5.11

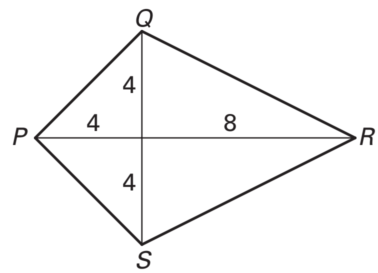
# Practice *continued*

**Use Theorem 5.33 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.**

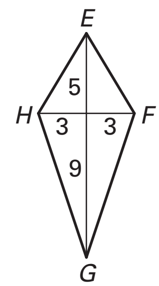
**16.**




**17.**

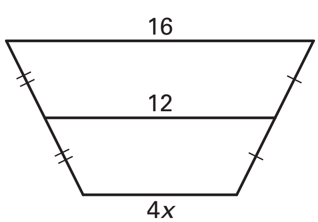



**18.**

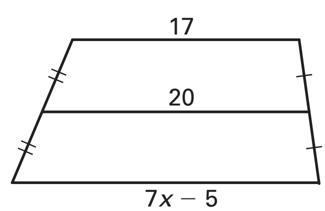



**Find the value of *x*.**

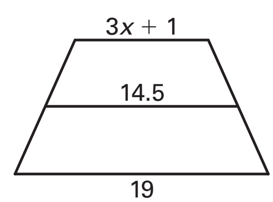
**19.**




**20.**




**21.**

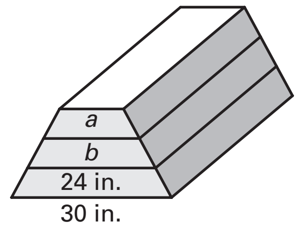


Name ______________________________ Date ______________

LESSON 5.11

# Practice *continued*

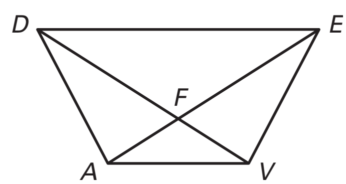
**22.** **Vaulting Box** Three vaulting boxes used by a gymnastics team are stacked on top of each other as shown. The sides are in the shape of a trapezoid. Find the lengths of *a* and *b*.




**23.** Complete the proof.

**GIVEN:** $\overline{DE} \parallel \overline{AV}$, $\triangle DAV \cong \triangle EVA$

**PROVE:** *DAVE* is an isosceles trapezoid.




| Statements | Reasons |
|---|---|
| **1.** $\overline{DE} \parallel \overline{AV}$ | **1.** |
| **2.** *DAVE* is a trapezoid. | **2.** |
| **3.** | **3.** Given |
| **4.** | **4.** Corresponding parts of ≅ ⚠ are ≅. |
| **5.** *DAVE* is an isosceles trapezoid. | **5.** |

# 5.12 Identify Special Quadrilaterals

Georgia Performance Standard(s)

MM1G3d

***Your Notes***

**Goal** • Identify special quadrilaterals.

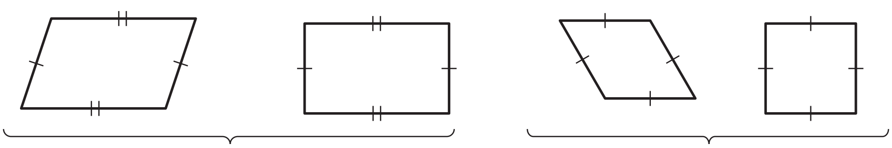
## Example 1 *Identify quadrilaterals*

**Quadrilateral *ABCD* has both pairs of opposite sides congruent. What types of quadrilaterals meet this condition?**

**Solution**

There are many possibilities.

____________ ____________ ____________ ____________



Opposite sides are congruent. All sides are congruent.

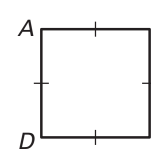
**Checkpoint** Complete the following exercise.

1. Quadrilateral *JKLM* has both pairs of opposite angles congruent. What types of quadrilaterals meet this condition?

In Example 2, *ABCD* is shaped like a square. But you must rely only on marked information when you interpret a diagram.

## Example 2 *Classify a quadrilateral*

**What is the most specific name for quadrilateral *ABCD*?**




**Solution**

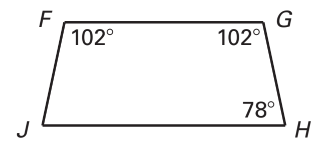
The diagram shows that both pairs of opposite sides are congruent. By Theorem 5.22, *ABCD* is a ________________. All sides are congruent, so *ABCD* is a ____________ by definition.

____________ are also rhombuses. However, there is no information given about the angle measures of *ABCD*. So, you cannot determine whether it is a __________.

*Your Notes*

**Example 3** ***Identify a quadrilateral***

**Is enough information given in the diagram to show that quadrilateral *FGHJ* is an isosceles trapezoid? Explain.**




**Solution**

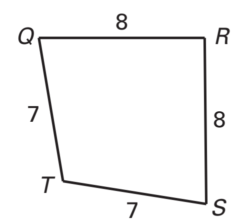
**Step 1** **Show** that *FGHJ* is a ___________. ∠*G* and ∠*H* are ________________ but ∠*F* and ∠*G* are not. So, _____ ∥ _____, but $\overline{FJ}$ is not _________ to $\overline{GH}$. By definition, *FGHJ* is a ___________.

**Step 2** **Show** that trapezoid *FGHJ* is ___________. ∠*F* and ∠*G* are a pair of congruent _____________. So, *FGHJ* is an ____________________ by Theorem 5.30.

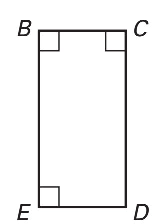
Yes, the diagram is sufficient to show that *FGHJ* is an isosceles trapezoid.

✔ ***Checkpoint*** **Complete the following exercises.**

**2.** What is the most specific name for quadrilateral *QRST*? *Explain* your reasoning.




**3.** Is enough information given in the diagram to show that quadrilateral *BCDE* is a rectangle? *Explain*.




***Homework***

Name ______________________ Date ____________

**LESSON 5.12**

# Practice

**Match the property on the left with all of the quadrilaterals that have the property.**

**1.** Both pairs of opposite sides are parallel.

**2.** Both pairs of opposite sides are congruent.

**3.** Both pairs of opposite angles are congruent.

**4.** Exactly one pair of opposite sides are parallel.

**5.** Exactly one pair of opposite sides are congruent.

**6.** Exactly one pair of opposite angles are congruent.

**7.** Diagonals are congruent.

**8.** Diagonals are perpendicular.

**A.** Parallelogram

**B.** Rectangle

**C.** Rhombus

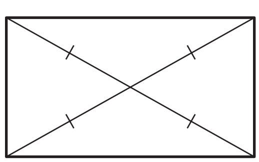
**D.** Square

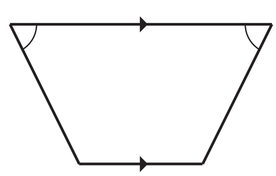
**E.** Trapezoid

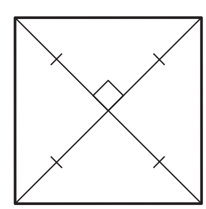
**F.** Isosceles Trapezoid

**G.** Kite

**Give the most specific name for the quadrilateral. *Explain*.**

**9.** 

**10.** 

**11.** 

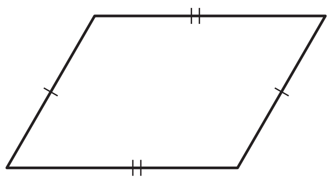
Name ______________________________ Date ______________

LESSON 5.12

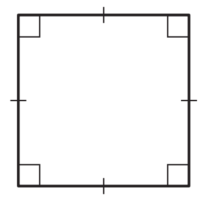
# Practice *continued*

**Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. *Explain*.**

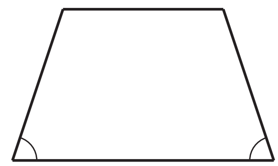
**12.** Parallelogram



**13.** Square

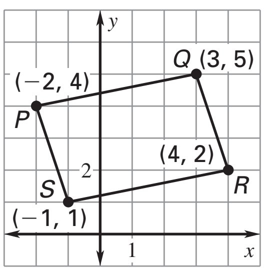


**14.** Trapezoid

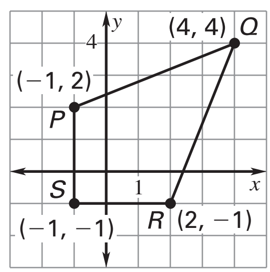


**Give the most specific name for quadrilateral *PQRS*. *Justify* your answer.**

**15.**

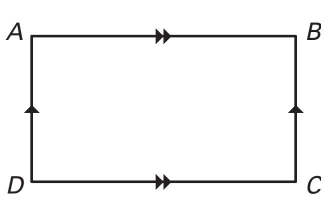



**16.**

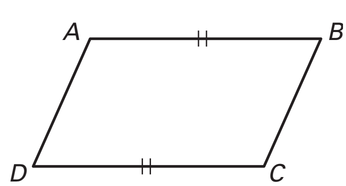



**Which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral? *Explain*. There may be more than one right answer.**

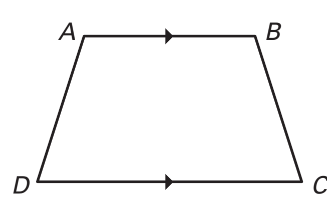
**17.** Rectangle




**18.** Parallelogram




**19.** Isosceles Trapezoid



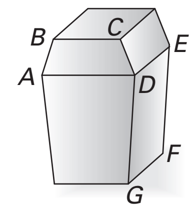
Name ______________________ Date ______________

LESSON 5.12

# Practice *continued*

**In Exercises 20 and 21, use the following information.**

**Gem Cutting** There are different ways of cutting gems to enhance the beauty of the jewel. One of the earliest shapes used for diamonds is called the *table cut*, as shown. Each face of a cut gem is called a *facet*.



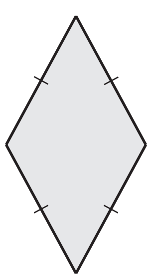

**20.** $\overline{BC} \parallel \overline{AD}$, $\overline{AB}$ and $\overline{DC}$ are not parallel. What shape is the facet labeled *ABCD*?

**21.** $\overline{DE} \parallel \overline{GF}$, $\overline{DG}$ and $\overline{EF}$ are congruent, but not parallel. What shape is the facet labeled *DEFG*?

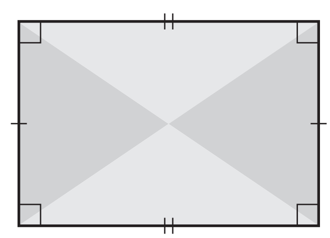
**In Exercises 22–24, use the following information.**

**Wall Hangings** Decorative wall hangings are made in a variety of shapes. What type of special quadrilateral is shown?

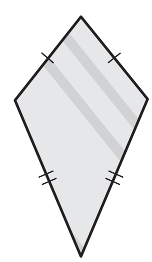
**22.**



**23.**



**24.**



# Words to Review

**Give an example of the vocabulary word.**

| | |
|---|---|
| Midsegment of a triangle | Perpendicular bisector |
| Equidistant | Concurrent |
| Point of concurrency | Circumcenter |
| Angle bisector | Incenter |

| | |
|---|---|
| **Median of a triangle** | **Centroid** |
| **Altitude of a triangle** | **Orthocenter** |
| **Indirect proof** | |
| **Diagonal** | **Interior angles of a polygon** |

| | |
|---|---|
| Exterior angles of a polygon | Parallelogram |
| Rhombus | Rectangle |
| Square | Trapezoid |
| Bases, Legs, and Base angles of a trapezoid | Isosceles trapezoid |
| Midsegment of a trapezoid | Kite |