MIDTERM #2, PHYS 1211 (INTRODUCTORY PHYSICS), November 5, 2015

PART I: MULTIPLE CHOICE QUESTIONS (question 1 to 5)

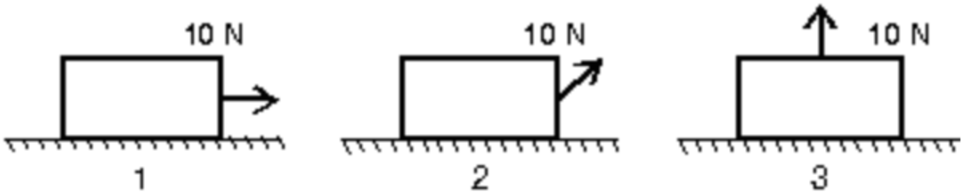
For each question **circle** the correct answer (a,b,c,d or e).

1. (**2 points**) A 1000-kg plane moves in a straight line at constant speed. The force of air friction is 1800 N. The net force on the plane is:

A) 0N B) 11600 N C) 1800 N D) 9800 N E) None of these

ANSWER: A

2. (**2 points**) A crate rests on a horizontal surface and a woman pulls on it with a 10-N force. No matter what the orientation of the force, the crate does not move. Rank the situations shown below according to the magnitude of the frictional force of the surface on the crate, least to greatest

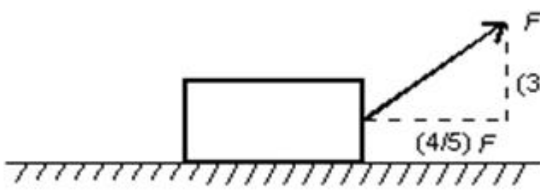


A) 1, 2, 3 B) 2, 3, 1 C) 2, 1, 3 D) 1, 3, 2 E) 3, 2, 1

ANSWER: E

3. (**2 points**) A 400-N block is dragged along a horizontal surface by an applied force $\vec{F}$ as shown. The coefficient of kinetic friction is u_k = 0.4 and the block moves at **constant velocity**. The magnitude of $\vec{F}$ is **closest** to:

A) 100 N B) 150 N C) 200 N D) 290 N E) 400 N



HINT: 1) The weight of box is mg = 400 N; 2) May be easiest to substitute the answers to see which one **obeys Newton's law**. However, the systematic method of applying Newton's Law will also give the correct answer.

Equilibrium so use **Newton's first law**.

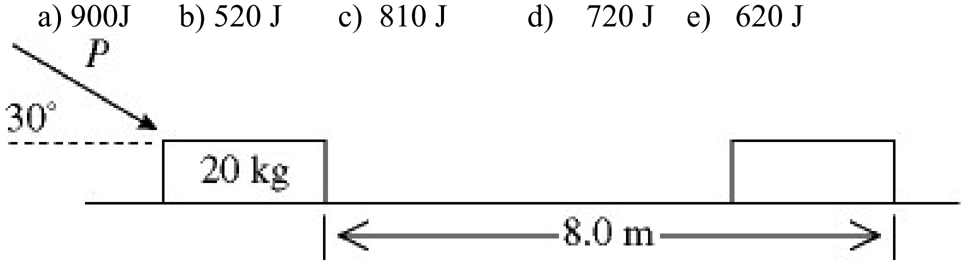
Y-com: $F_N - F_g + (3/5)F = 0 \rightarrow F_N = 400N - (3/5)F$. Kinetic Friction $f_k = F_N \mu_k$,

$f_k = 160N - 0.24F$. X-comp $f_k = (4/5)F \rightarrow 160N - 0.24F = 0.8F \rightarrow F = 154N$.

ANSWER B

4. (**2 points**) A constant external force P = 130 N is applied to a 20-kg box, which is on a rough horizontal surface. The force pushes the box a distance of 8.0 m, in a time interval of 7.0 s, and the speed changes from $v_1 = 0.6m/s$ to $v_2 = 3.2m/s$. The **work done** by the **external force *P*** is closest:

a) 900J b) 520 J c) 810 J d) 720 J e) 620 J



Warning: Read the question carefully!!

Work done by $\vec{P}$, $W = \vec{P} \bullet \vec{d} = P\cos 30^\circ = 130N\cos 30^\circ \times 8m = 900J$, **ANSWER: A**

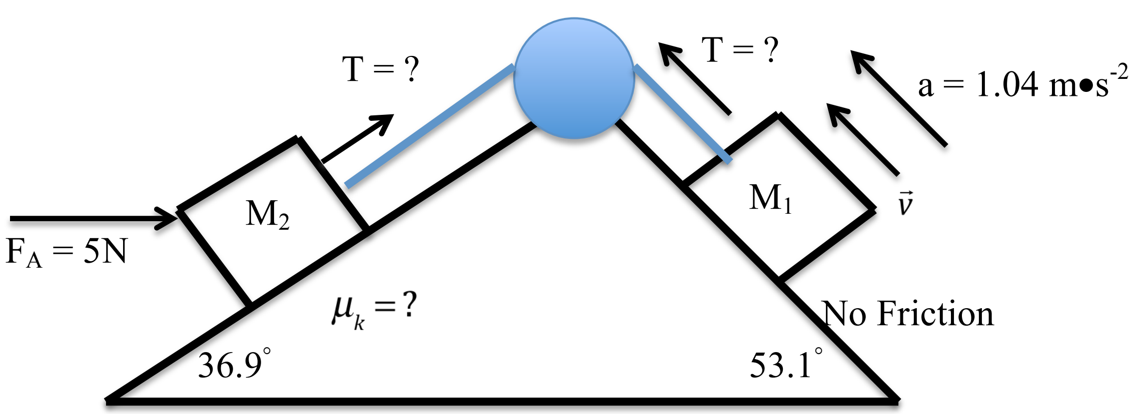
5. (**2 points**) A 3.00-kg ball swings rapidly in a complete vertical circle of radius 2.00 m by a light string. The ball moves so fast that the string is always taut. As the ball swings from its lowest point to its highest point:
   a) The work done on it by gravity and the work done on it by the tension in the string are both equal to zero.
   b) The work done on it by gravity is -118 J and the work done on it by the tension in the string is zero.
   c) The work done on it by gravity and the work done on it by the tension in the string are both equal to -118 J.
   d) The work done on it by gravity is +118 J and the work done on it by the tension in the string is -118 J.
   e) The work done on it by gravity is -118 J and the work done on it by the tension in the string is +118 J.

ANSWER: B Work by gravity is $W_g = -mg\Delta y = -3kg \times 9.8m \bullet s^{-2} \times 4m = -118J$. Since the ball travels in a circle, then tension is perpendicular to the direction of motion, and the work done by the tension is zero.

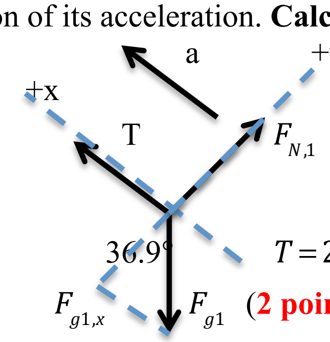
PART II: FULL ANSWER QUESTIONS (question 6 to 9)

Do all four questions on the provided space. Show all works.

6. (**10 points**) In the diagram, block 1($M_1 = 2.0kg$) lies on **frictionless incline** of 53.1°, and is **moving up** the incline with acceleration $a = 1.04m \bullet s^{-2}$. Block 1 is connected by an ideal rope through a frictionless pulley to block 2 ($M_2 = 7kg$), which rests on a 36.9° incline with friction. Block 2 is acted on by an applied horizontal force of magnitude F_A = 5N. The **tension** (T = ?) and **kinetic coefficient** ($\mu_k = ?$) are **unknown**.



a) Draw a free-body-diagram (FBD) of all forces on block 1 (M_1), which includes the direction of its acceleration. **Calculate** the **tension**, T.



(**2 points**)

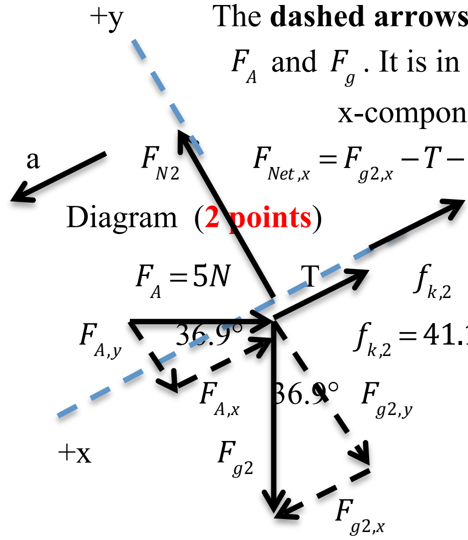
Since there is no friction on this side, only the Horizontal is important x-component:

$$F_{1,x}^{Net} = T - F_{g1,x} = M_1 a \rightarrow T = M_1 g \sin 36.9^\circ + M_1 a$$

$$T = 2kg \times 9.8m \bullet s^{-2} \times 0.8 + 2kg \times 1.04m \bullet s^{-2} = 17.76N$$ (**2 points**)

b) Draw a free-body diagram (FBD) of all forces acting on block 2 (M_2). Use this to determine the **magnitude** and **direction** of the friction force $f_{k,2}$, acting on block 2. Calculate the **coefficient** of **kinetic friction**, μ_k, between surfaces of block 2 and incline.

In diagram below the forces and acceleration are dented by the solid black arrows. The **dashed arrows** denote the direction (+ or -) of the components of F_A and F_g. It is in this solution for clarity, not for marks.



x-component

$$F_{Net,x} = F_{g2,x} - T - F_{A,x} - f_{k,2} = M_2 a \ , \ F_{g2,x} = M_2 g \sin 36.9^\circ = 41.16N$$

$$F_{A,x} = F_A \cos 36.9^\circ = 4N$$

$$f_{k,2} = F_{g2,x} - T - F_{A,x} - M_2 a$$

$$f_{k,2} = 41.16N - 17.76N - 4N - 7kg \times 1.04 m \bullet s^{-2} = 12.12N$$

(**2 points**)

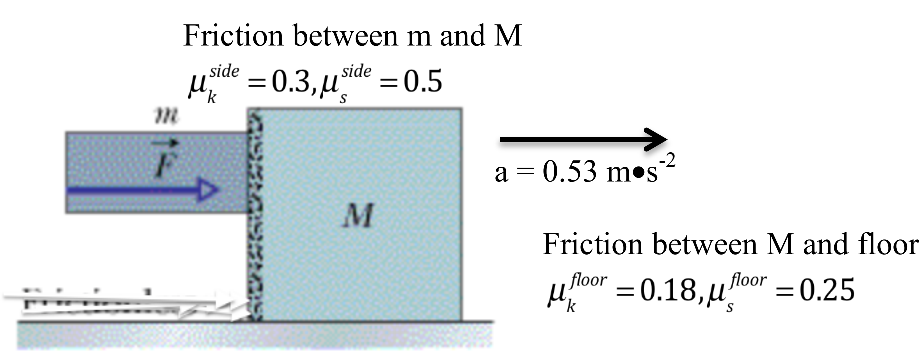
Y-component

$$F_{Net,Y} = F_{N2} - F_{A,y} - F_{g2,y} = 0$$

$$F_{N2} = 5N \times \sin 36.9^\circ + 7kg \times 9.8 m \bullet s^{-2} \cos 36.9^\circ = 57.88N$$

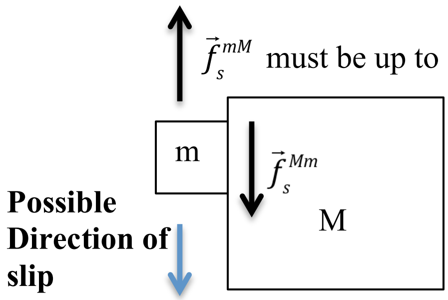
Friction: $f_{k2} = \mu_k F_{N2} \rightarrow \mu_k = 12.12N / 57.88N = 0.21$ (**2 points**)

7. (**10 points**) In the diagram below, the two blocks are accelerating at a = 0.53 m•s^{-2}, due to a **force** of **unknown magnitude** (F = ?) that is pushing block m (m = 10 kg). Block M has mass M = 99 kg. The coefficients of friction between the sides of the two blocks are $\mu_k^{side} = 0.3$ and $\mu_s^{side} = 0.5$. It is assumed that block m does not slide down the side of block M. The coefficients of friction between block M and the floor are $\mu_k^{floor} = 0.18$ and $\mu_s^{floor} = 0.25$.



A) Assume that block m does not slide down the side of M. **Calculate** the **magnitude** and **direction** of the force of static friction on m by M, $\vec{f}_s^{mM}$. Use Newton's third law to **determine** the force of static friction on M by m, $\vec{f}_s^{Mm}$. **NOTE:** direction is up or down.

$\vec{f}_s^{mM}$ must be up to support the weight of m, $f_s^{mM} = mg = 98N$

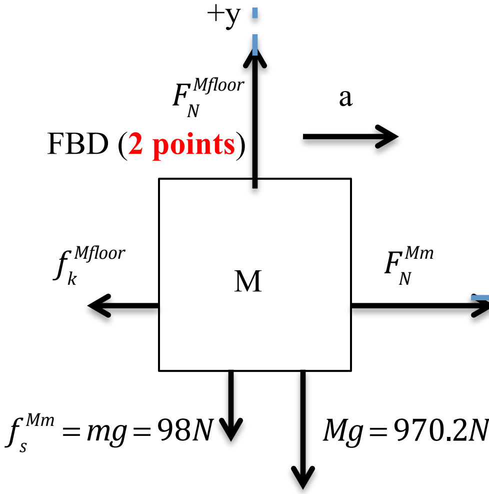


$\vec{f}_s^{Mm}$ and $\vec{f}_s^{mM}$ are action-reaction pair

By Newton's third law $\vec{f}_s^{Mm}$ must point down

$f_s^{Mm} = mg = 98N$ (**2 points**)

B) Draw a **free-body diagram** of **block M**, which includes all forces acting on it, as well as the direction of its acceleration. Use **Newton's first law** to calculate the **vertical normal force** on block M by floor's surface, F_N^{Mfloor}. Hence calculate the magnitude of force of kinetic friction on block M by floor's surface, f_k^{Mfloor}. Finally, use **Newton's second law** to calculate the **horizontal normal force** on M by m, F_N^{Mm}.



y-component

$F_{Net,y} = F_N^{Mfloor} - mg - Mg = 0 \rightarrow F_N^{Mfloor} = 1068.2N$

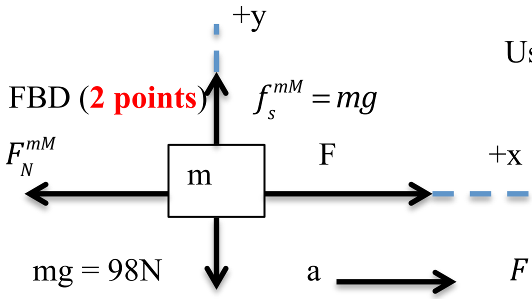
Friction $f_k^{Mfloor} = \mu_k^{floor} F_N^{Mfloor} = 192.3N$ (**2 points**)

x-component

$F_{Net,x} = F_N^{Mm} - f_k^{Mfloor} = Ma$

$F_N^{Mm} = 192.3N + 99kg\left(0.53m \bullet s^{-2}\right) = 244.77N$ (**2 points**)

C) Draw a **free-body diagram** of **block m**, which include all forces acting on it, as well as the direction of its acceleration. Use **Newton's second and third law** to calculate the magnitude of the horizontal force F that pushes block m.



Using Newton's third Law

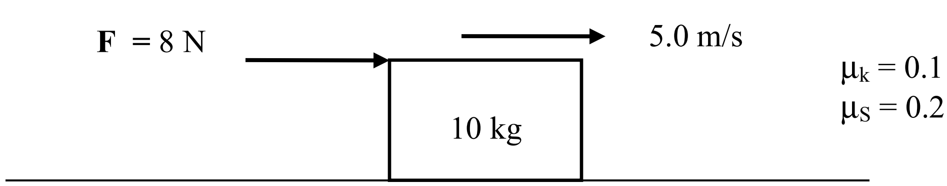
$F_N^{mM} = F_N^{Mm} = 98N$

x-component

$F_{Net,x} = F - F_N^{mM} = ma$

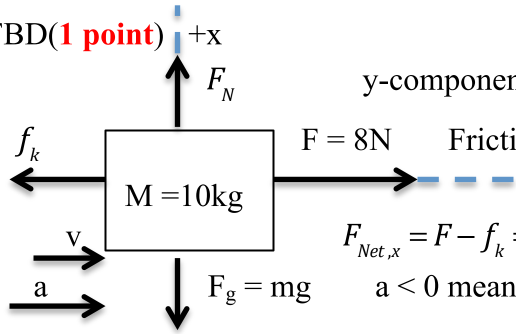
$F = 244.77N + 10kg \bullet 0.53m \bullet s^{-2} = 250.1N$ (**2 points**)

8. (**10 points**) A 10 kg crate is initially moving with a speed of 5.0 m/s, when it is acted on by a horizontal 8 N force as shown in the diagram below. The coefficient of kinetic and static friction between all surfaces are $\mu_k = 0.1$ and $\mu_S = 0.2$, respectively.



A) Draw a **free-body diagram** showing all forces on the crate. Determine the normal force, F_N, and the friction force on the crate. Find the acceleration.

FBD(**1 point**)



y-component: $F_{Net,y} = F_N - mg = 0 \rightarrow F_N = 98N$ (**1 point**)

Friction $f_k = F_N \mu_k = 9.8N$ (**1 point**)

x-component

$F_{Net,x} = F - f_k = Ma \rightarrow a = (8N - 9.8N)/10kg = -0.18m \cdot s^{-2}$

a < 0 means acceleration is left (**1 point**)

B) Use the **kinematics equation** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the speed of the crate after it has traveled 10m to the right of its initial position (above diagram).

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) = (5m \cdot s^{-1})^2 + 2(-0.18m \cdot s^{-2})(10m) \rightarrow v_x = 4.63m \cdot s^{-1} \quad (\textbf{2 points})$$

C) Calculate the work done by the F = 8N force, the normal force, gravity (weight), and friction after it has traveled 10m to the right of its initial position.

F = 8N, $W_F = \vec{F} \cdot \vec{d} = 8N \times 10m = 80J$, positive since force is parallel to displacemnet

Normal force, $W_N = \vec{F}_N \cdot \vec{d} = 0$, since $\vec{F}_N \perp \vec{d}$

Gravity, $W_g = \vec{F}_g \cdot \vec{d} = 0$, since $\vec{F}_{Ng} \perp \vec{d}$

Friction, $W_f = \vec{f}_k \cdot \vec{d} = -9.8N \times 10m = -98J$, negative since force of kinetic friction is anti-parallel to displacement. (**2 points**)

D) Use the **work-energy theorem** (and result of part C) to find the speed of the crate after it has traveled 10m to the right of its initial position. Compare this answer with that of part B.

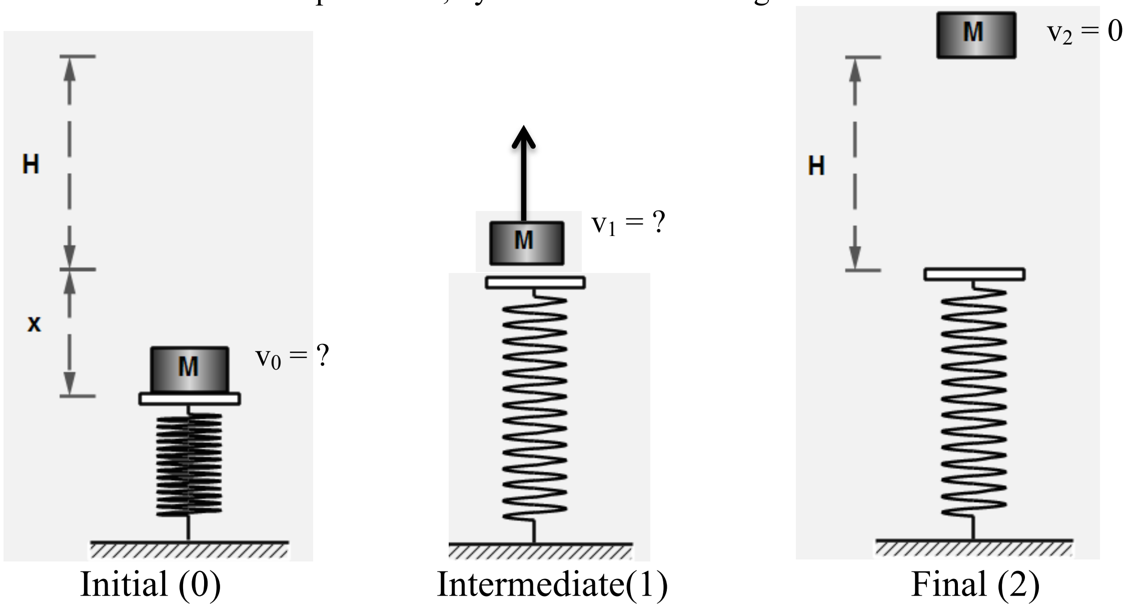
Net work $W^{Net} = W_F + W_N + W_g + W_f = -18J$

Work-Energy Theorem: $W^{net} = \Delta K = (1/2)mv_f^2 - (1/2)mv_i^2$, where $v_i = 5m \cdot s^{-1}$

$$v_f = \sqrt{\frac{2W^{Net}}{m} + v_i^2} = \sqrt{\frac{2 \times (-18J)}{10kg} + (5m \cdot s^{-1})^2} = 4.63\frac{m}{s}$$, which is identical to part b.

(**2 points**)

9. (**10 points**) In the system below a **massless ideal** spring (k = 410 N/M) is compressed at x = 0.8 m from equilibrium, by a mass of M = 2.2 kg. It is then released.



A) Calculate the **net work done** on the Mass M, from the initial (0) to the intermediate position (1), when the spring is back at equilibrium. Use the **work-energy theorem** to find the **speed** of the block at position 1, v_1.

Spring $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}\left(410\frac{N}{m}\right)(0.8m)^2 - \frac{1}{2}\left(410\frac{N}{m}\right)(0)^2 = 131.2J$ (**1.5 points**)

Gravity $W_{grav} = -mg\Delta y = -2.2kg \times 9.8\frac{m}{s^2} \times 0.8m = -17.248J$ (**1.5 points**)

Net work, $W^{Net} = W_s + W_{grav} = 113.952J$.

Work-energy $W^{Net} = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 \rightarrow v_1 = \sqrt{\frac{2W^{Net}}{m}}$, where we used $v_0 = 0$ (see diagram)

$v_1 = \sqrt{2 \times 113.952J / 2.2kg} = 10.18 m \bullet s^{-1}$ (**2 points**)

B) The block will reach its **maximum height** at the final position (2). Use the **work-energy theorem** to determine the **maximum height** H, above **spring**. **Hint:** Look at the diagram, and note that from position 1 to 2 gravity is the only force acting on the spring.

$W^{net} = W_{grav} = -mg\Delta y = -mgH$ (**1 point**)

Work energy $W^{Net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -mgH$ (**1 point**). Use $v_2 = 0$ (see diagram) (**1 point**).

$-\frac{1}{2}mv_1^2 = mgH \rightarrow H = \frac{1}{2g}v_1^2 = \frac{1}{2 \times 9.8 m \bullet s^{-2}}\left(10.18\frac{m}{s}\right)^2 = 5.28m$. (**2 points**)

USEFUL EQUATIONS

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, unit vector notation.

Scalar Product: $\vec{A}\bullet\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\phi$

Solution of quadratic equation, $ax^2 + bx + c = 0 \rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Kinematics $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$, $v_x = v_{0x} + a_xt$, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$g = 9.8m/s^2$, $F_g = mg$ **Centripetal acceleration** $a_{rad} = \dfrac{v^2}{r}$

Newton's Laws $\vec{F}^{net} = \sum\vec{F} = 0$ (Object in equilibrium)

$\vec{F}^{net} = m\vec{a}$ (Nonzero net force); third law, an applied force on object A by object B will induce an equal (in magnitude) and opposite (in direction) force on object B by object A.

Friction $f_s \le \mu_sF_N$, $f_k = \mu_kF_N$.

Work and Energy

$K = (1/2)mv^2$, $W = \vec{F}\bullet\vec{d} = (F\cos\theta)d = F_{\parallel}d$ (Straight-Line Motion, Constant Force)

Scalar product form: $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, $\vec{d} = d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$, $W = \vec{F}\bullet\vec{d} = F_xd_x + F_yd_y + F_zd_z$

Hooke's Law $F_x = -kx$. **Work by spring:** $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Gravitational Work: $W^{grav} = -mg(y_f - y_i) = mgy_i - mgy_f$

Work-Energy Theorem: $W^{net} = \Delta K = (1/2)mv_f^2 - (1/2)mv_i^2$ (valid if W^{net} is the **net** or **total work done** on the object)