# MISCONCEPTIONS IN GEOMETRY

As theorized by Lev Vygotsky, knowledge is not transferred from person to person. The individual does not passively receive knowledge from the environment, but is an active participant in the construction of his/her own mathematical knowledge. The construction activity involves the reception of new ideas and the interaction of these with the pupils' existing ideas. Also, students may not be able to perceive what the teacher sees in a geometric situation if they are at a particular level of the Van Hiele model and higher levels of understanding are required. It is impossible for learners to bypass or skip a level from the model. These situations result in misconceptions arising frequently. By discussion, a teacher can get pupils to explain how they came to their answers or rules and be able to analyze faulty interaction between the pupils' extant ideas and the new concept. When the teacher is able to understand the reason behind the misconception, it is corrected by challenging or contrasting it with the faithful conception. In geometry some common misconceptions arise:

## 1. <u>Identifying the Base and Height of a Triangle</u>

Invariably pupils are habituated by the standard triangle presented to them when the area of a triangle algorithm is presented: one with horizontal base and height. When faced with any triangle they use the 'bottom' line as the base and the height 'upwards' from the base.

*Example of misconception:*

Q. Find the area of the right angled triangle
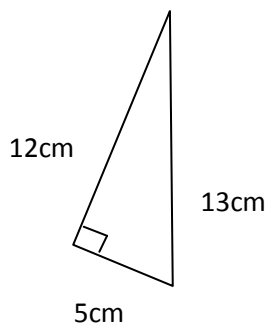
Ans.     Base = 5cm     Height = 13cm

$$\text{Area} = \frac{Base \times Height}{2}$$

$$= \frac{5 \times 13}{2}$$

$$= 32.5 \text{ sq cm}$$

12cm

13cm

5cm

*Suggested Remedy:*

- Teachers can allow students to examine different types of triangles, also with varying orientation to be better able to identify the Base and the Height in each. These can be drawn on paper or be cut out from Bristol board where students can identify the Base and height when triangles vary in their positions ( e.g. rotated)

- Students can be given the option of turning their books to analyse shapes in case of spatial problems.

- The relationship between Base and Height must be explained clearly as well: their relationship of being perpendicular to each other.

2. **Conservation Misconception**

Pupils often believe that the rules of invariance that apply to algebra also apply to geometrical shapes: there must be equality in all respects when A becomes B. This leads to the misconception that the perimeters are the same.

*Example of misconception:*

You cut rectangle A, and arrange the pieces to make a new shape B, like this:

A                    B
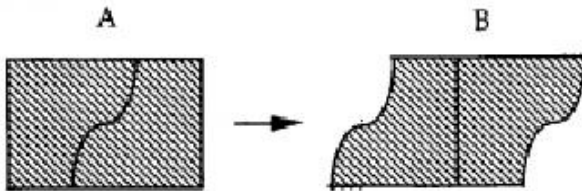
Ring two statements that are true:

The area of A is greater than the area of B

The area of A is less than the area of B

Both areas are the same

The perimeter of A is greater than the perimeter of B

The perimeter of A is less than the perimeter of B

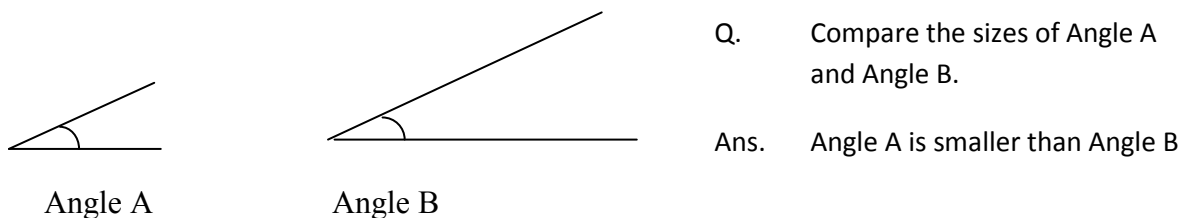Both perimeters are the same

*Suggested Remedy:*

- Different shapes of pentominoes can be used to demonstrate that same areas don't imply same perimeter. Students see that for the same area, perimeter can vary when they investigate by checking the perimeters of the pentominoes.
- Using a geoboard and rubber bands, students can construct different rectangles of varying dimensions but with the same perimeter and compare the resulting areas.

## 3. **Angles: Larger Space means Larger Angle**

This misconception is frequently held by pupils as there is perceptual illusion between a larger turn and a larger space between the two lines making the angle.
Despite the fact that similar angles are drawn on squared paper, the larger space of angle can lead a pupil to making this judgment.

*Example of misconception:*

Q.     Compare the sizes of Angle A and Angle B.

Ans.    Angle A is smaller than Angle B

Angle A            Angle B

*Suggested Remedy:*

- The teacher can differentiate between 'larger turn' and a 'larger space' between the two lines making the angle
- Geosticks can be used to show that the 'turn' does not change when the lines which make the angle are extended.
- Angles should be defined as the 'amount of turn'.

- Modeling with two stick joined together of different lengths is important to overcome this common misconceptions with angles.
- Other terms can be used to develop completeness of definition in relation to angles. eg. pivoting, rotation
- Other modeling can be used. eg. clock hands
- The teacher can allow students to measure the size of the angles with a protractor assuming that they know how to use a protractor. By doing that it creates a sense of conflict between an intuitive sense and evidence provided by the measurement.

4. **Shape Properties**

When student develop a concept image (a mental image of a shape) without a concept definition (a specified definition of the shape or its properties, they often identify examples of shapes, but will also fail to identify examples of shapes that are not identical to their own mental image of the shape or the shape prototype, i.e., the figure does not "look like" the shape. Although characteristics such as orientation and proportions are irrelevant to the defining properties of a shape, they affect whether students recognize certain shapes.
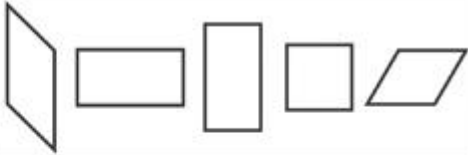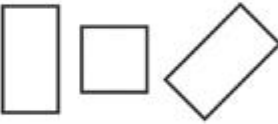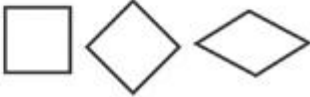
Some of the common misconceptions of triangles are as follows:

- Triangles have one point at the top and two points at the bottom
- The bottom of a triangle is flat

Some of the common misconceptions of rectangles are as follows:

- Rectangles are always long
- Rectangles have two long sides and two short sides

*Example of misconception:*



| Shape | Shape Prototype | Figures Possibly not Recognized when Reasoning with a Concept Image and not a Concept Definition |
|---|---|---|
| Parallelogram | | |
| Rectangle | | |
| Square | | |
| Rhombus | | |

*Suggested Remedy:*

- Teachers can expose students to a set of visual and/or concrete examples like shapes cut out of Bristol board.
- Teachers can introduce the properties that define a shape. Instruction can help students move towards this understanding.
- Children also need to see both examples and non-examples. For example, when introducing triangles, include many types of triangles (acute, right, obtuse, and equilateral), but also include non-triangles (three-sided objects with a wavy line, or a three-sided object with an opening).
- Teachers can also give children a variety of examples that are different sizes and that have different orientations.

## 5. <u>Orientation and Rotation of Shapes</u>

Common shapes are not recognised unless they are upright or in their usual orientation. Identifying shapes when they are positioned in a "non-standard" way can be problematic,

especially a square with a vertex at the bottom. A square with a vertex at the bottom is often identified as a rhombus or kite, and not recognized as square. A similarly non-aligned rectangle might be recognized as a parallelogram but not a rectangle.
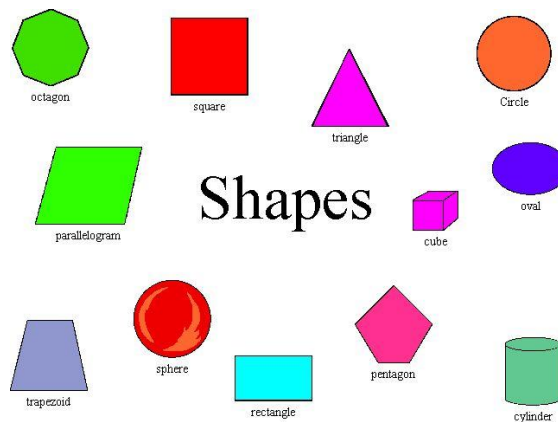
***Example of Misconception:***

In the diagram below students may not recognize the second shape as the same square, but instead a diamond or a kite.
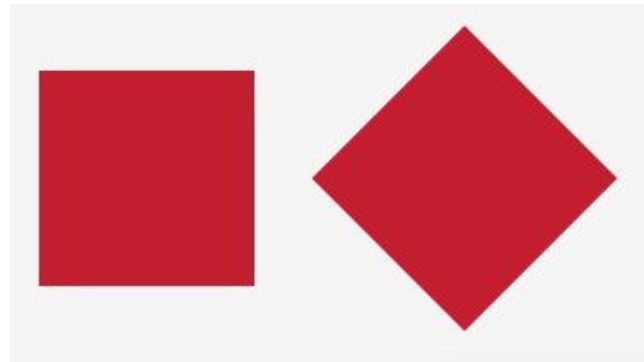


***Suggested Remedy:***

The reason for this is deeply rooted in human perception. The lesson for school children is twofold: first, it's a lesson on recognizing shapes.



Second, it's a lesson on mentally rotating objects (or physically rotating your head to look at them) in order to perceive them more clearly. This can be conducted in different ways depending on the level of geometric thinking of the student.

i)    Pentominoes can be used to introduce rotations to show that there is no change in the nature and properties of a shape. The simpler shapes can be used for

demonstrations then the use of more difficult ones. Students can turn their pages/books in order to analyse the shapes.
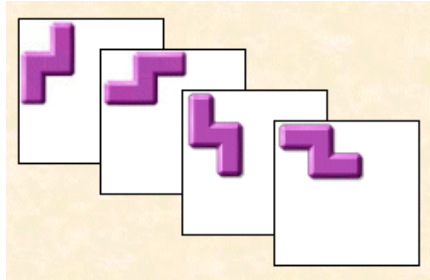


ii) When students are introduced to the Cartesian plane it can be used to show how rotation of an object does not change the general nature and shape of it as shown below.
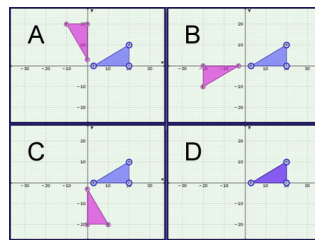


iii) Jigsaw puzzles can be a stimulating tool that is used to help students deal with the misconception associated with rotating objects. At first they will attempt to actually rotate each shape to solve the puzzles and eventually will be able to project how the pieces will look before it is rotated to work faster.
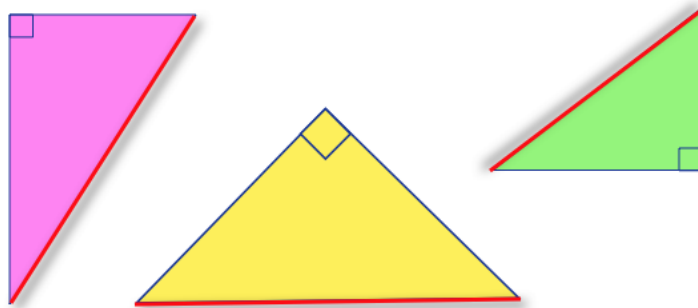
## 6. <u>Perpendicular lines</u>

Vertical and horizontal lines are easily recognized as such, and quickly perceived as perpendicular to one another. But the angles between lines oriented differently are not so easily perceived as right angles.

*Example of misconception:*

Given the right angled triangles below students may identify only the pink and green triangles as right angled triangles since they can clearly identify the right angles using the sides which are vertical and horizontal. However, since the yellow triangle does not have a vertical side meeting a horizontal one, students may not be able to acknowledge the existence of the right angle.



*Suggested Remedy:*

Again the suggested solution is twofold: first, it's a lesson on recognizing perpendicular lines. Second, it's a lesson on mentally rotating perpendicular lines in order to perceive them more clearly. Rotation of perpendicular lines produces other ways in which they appear as shown below.
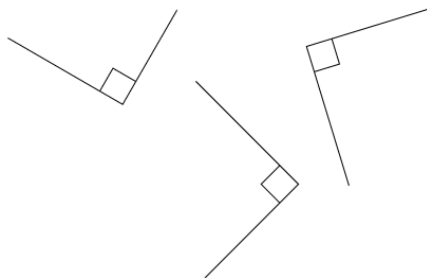
The use of manipulative like geosticks set at right angles, or cut-outs of right angles can be used to demonstrate the various ways in which perpendicular lines appear.

- The teacher first places the manipulative in the position where one arm is horizontal and the other vertical.
- The fact that a perfect square can be formed at the point where the lines meet can be established since this can helpful in confirming that two lines are perpendicular at a point.
- The teacher can then guide students to rotate the manipulative with the arms set in that position in relation to each other. They can then analyse the relationship between the position of the two arms and recognize that they are still perpendicular to each other.

## 7. **There Are Four Sorts Of Triangle: Scalene, Isosceles, Equilateral And Right-Angled**

When classifying triangles students identify any triangle which has a right angle as a right-angled triangle and fail to see that they can be classified as Scalene, Isosceles or Equilateral when required to classify them according to their sides.

***Example of misconception:***

Given the following triangles, classify them according to their sides:
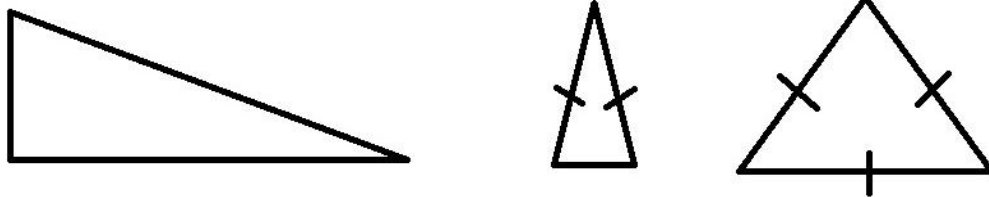


Students may classify the triangles as Right-angled, Isosceles and Equilateral respectively.

***Suggested Remedy:***

Teachers can formulate instructions appropriately and introduce classification of triangles according to their sides and according to their angles on separate occasions.

The teacher can focus on one way of classification fully by doing sufficient examples and allow students to explore in their environment and discover types of triangles using one way of

classifying. Students can express themselves verbally, using the appropriate terminology, and teachers can request them to explain the reason why they classify each triangle the way they did. On another occasion the same is done but by using the other method of classification- by their angles.

## 8. Lines of Symmetry

Students sometimes find more lines of symmetry than actually exist. Simple or convenient definitions that lines of symmetry 'chop' shapes into half do not necessarily imply that these lines must also create one half the exact mirror image of the other.

*Example of Misconception:*
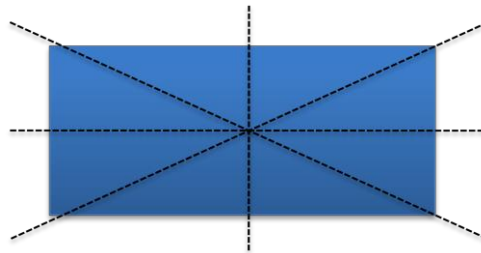


From the diagram above, the number of lines of symmetry is easily misinterpreted as 4.

*Suggested Remedy:*

- With the use of cut-out basic shapes the teacher can use the method of folding to derive lines of symmetry, as shown in the diagram below.



- Information And Communication Technology (ICT) can be used to overcome this misconception. Numerous activities are available over the internet, and other interactive software can be made available by the teacher which guides the student through activities where students discover how to find lines of symmetry.

## 9.  '…a rectangle is a long shape'…and…'a square is not a rectangle…'

When we ask a student to *circle all the rectangles* in some illustration, they do not circle the squares, demonstrating that their image of *rectangle* is too narrow. As they were introduced to the concept of rectangles as being "a long shape" they do not recognise a square to be a type of rectangle.

*Example of Misconception:*

   ❖ Circle the rectangles given the following shapes:
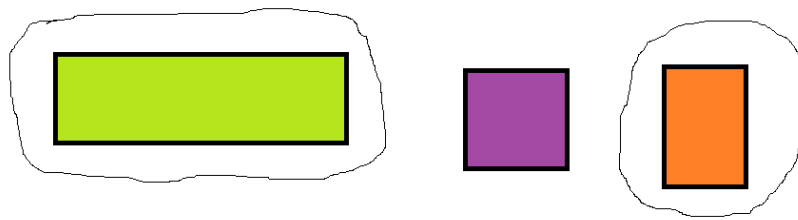


*Suggested Remedy:*

It is important for students to know that squares are rectangles: they are *special* rectangles, in that their sides are all the same length. According to van Hiele's model, geometric thinking progresses through levels. The accompanying vocabulary does as well. The formal definition of a rectangle needs to be re-introduced at the appropriate level. Students need to be told that they are parallelograms with at least one right angle.


## 10. A regular shape is one that is common.

A regular shape is not simply one that is common. A regular shape is one where all its sides and angles are equal. When students are exposed to a pentagon described as "regular" it is not because a pentagon is normally or commonly used.

*Example of misconception:*

Given the problem, "Find the size of each interior angle of a regular polygon with 5 sides." students may automatically calculate the sum of angles in a polygon and since they are unaware

of the fact that a regular pentagon means 5 equal sides and 5 equal angles, they are unable to proceed. Some students may actually divide the sum of the angles by since it seems the logical thing to do. However, if given an irregular polygon they may apply the same method.

***Suggested Remedy:***

Students need to be exposed to examples of regular polygons as shown below.



| Equilateral Triangle | Square | Pentagon | Hexagon | Octagon |
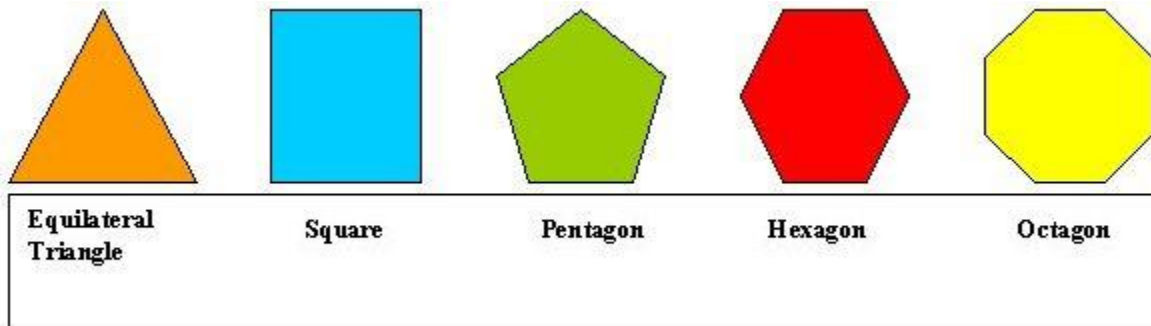
They can use a ruler, and by measuring each side, see that all sides are equal. In this process it is important that they acknowledge that a triangle is a type of polygon and the equilateral triangle would be a regular polygon

When students are exposed to non-examples of the concept as well they are better able to distinguish between regular and irregular polygons. By seeing regular and irregular polygons together comparisons can be made and differences easily seen.
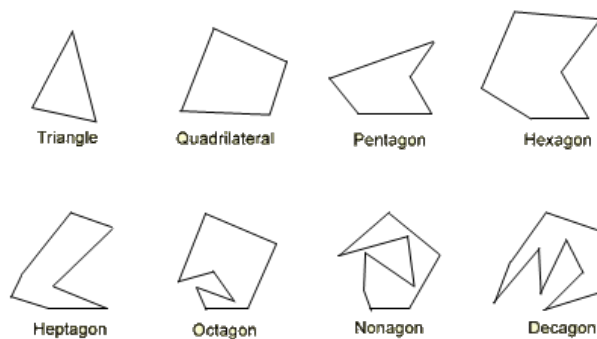
## 11. Language

Language is developmental and so therefore it is vastly different at different levels of geometric thinking.

*Example of Misconception:*
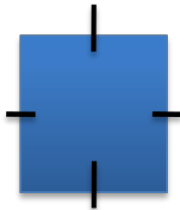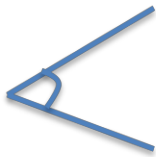
At level 1 of Van Hiele's model some examples are:

- corners, pointy

- like a square

- diamond

- a square has four sides

As opposed to the following at the higher levels:

- angles

- rectangular

- rhombus or kite

- a square has four equal sides and at least one right angle

This explains why two people sometimes cannot understand each other or follow the thought process of the other. This situation is sufficient to explain why at times teachers fail to help students in geometry learning. The students and teachers have their own languages, and often teachers use a language of a higher level, which students do not understand. The van Hieles noted that providing students with information which is above their actual thought level would not help the students to move to the next higher level. On the contrary, it will take them to a lower level.

Also, students are introduced most times to the same words in the subject of English Language. They learn their meanings to be different from when used in mathematics.

In the figure above the angles may be interpreted as "left angle" and "right angle" respectively.
Another term easily misinterpreted is "solids". Mathematically, stability or rigidity does not
define a solid. A solid is a region of space enclosed by a 3-D figure. It may be a rigid structure
but need not be. It may be open or closed. It may be regular or irregular.
If not properly introduced, geometric terms, as well as other mathematical terms may be
interpreted in the wrong way.


***Suggested Remedy:***

- When introducing new terms in geometry, and mathematics in general, especially if
  students already have knowledge of alternative meanings for them, teachers can insist on
  students using them verbally in their explanations of solutions to problems etc as much as
  possible. When students use them in context verbally as well as in their written solutions
  they become familiar with the proper terminology that is used.
- In moving through levels of the Van Hiele model, students need to be told the reasons
  why terms that they are accustomed to using at the lower level change when they
  encounter them at higher levels.
- Teachers can associate new terms with upgraded diagrams/representations/symbols etc.,
  which students can connect easily to.

  For example, both shapes below are squares introduced at different levels:
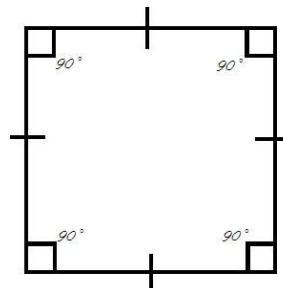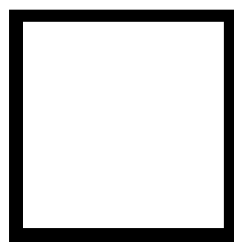
In the case of "left angle" and "right angle", the teacher can explain the origin of the term:

"An angle is called right angle as the angle between them measures a 90.,or the 2 lines which makes that angle intersects perpendicularly... The term is a literal translation of Latin angulus rectus; here rectus means "upright", referring to the vertical perpendicular to a horizontal base line."

## 12. <u>The diagonal of a square is the same length as its side.</u>

A square has four equal sides and at least one right angle. When observing the diagonals of a square, they "look" like the same length of each of the sides. This visual misinterpretation is common.

*Example of Misinterpretation:*
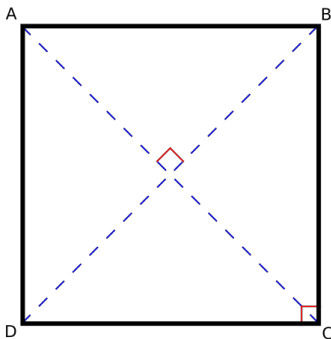


In the square ABCD students may say that AB = BD

Suggested Remedy:

- In an activity, Students can build a specific square of given dimensions using straws. They can measure and cut pieces representing the diagonals. By comparing the pieces of straws which represent a side with one which represent a diagonal they will be able to see that the lengths are different. By repeating with squares of various sizes they will conclude that the diagonal is always longer than the side of a given square.
- The misconception can also be resolved by demonstrating on a white/blackboard. Draw a square with specific dimensions and draw in the diagonals. By placing a stick on the

diagonal adjust the length of the stick to be the same. Students can now be shown that when the stick, which represents the diagonal of the square, is place on one side of the same square there is a difference in their lengths.

- Students can also experiment in their own books by constructing various squares and with the use of a ruler, measure the individual sides and diagonals. Conclusions can then be made.

13. **<u>Using a protractor:</u>**

Students have the following misconceptions when using a protractor:

- A protractor must always be placed in the horizontal position, regardless of the orientation of the angle being measured
- When measuring angles always start from the right/left.

Students need to be introduced properly to the instrument and its features, as with other instruments in mathematics. Proper use, and sufficient practice and formative assessment ensures that they are familiar with the instrument. They must be reminded that:

- The point where the perpendicular lines intersect at the centre base of the protractor must be placed at the point at which the angle to be measured is located.
- One of the arms of the angle to be measured must be aligned to the right or the left with the horizontal line at the base of the protractor. The angle is measured by looking at the graduations starting from zero.

14. **<u>In Transformations:</u>**
- The mirror line for a reflection does not need to be vertical or horizontal as misconcieved.
  - o Students can mirror shapes etc using mirror lines of different slopes.
  - o The use of miras placed in different positions/orientations demonstrates that mirror lines can be other than vertical or horizontal.
- Rotations are not always about the origin as one thinks – they can be about ANY point.
  - o Practical activities with graph paper and varying centres of rotation can resolve the misconception.