MOCK MATHCOUNTS

2017
■ Chapter Competition ■
Sprint Round
Problems 1–30

HONOR PLEDGE
I pledge to uphold the highest principles of honesty and integrity. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Team 1 or Team 2 or Team 3 (circle one)

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before the time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer in included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Produced by
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- Eric Xu
- George Harpole

1. _____ Juwan has four more than three times the number of rings Dwight has. If Dwight has 14 rings, how many rings does Juwan have?

2. _____ Charlie is playing in a soccer match consisting of two 45-minute halves and a 30-minute halftime break. At the end of each half, there is also an additional five minutes of stoppage time included in the match. How many minutes long is the soccer match?

3. _____ In the following set of numbers {17, 2, 5, 21, 4, 11}, what is the positive difference between the mean and the median?

4. _____ Allen, Bernie, Carly, and Donald are standing in a line. Carly is neither first nor last in line, Donald is behind at least two people, and Bernie is in between two people. Who is last in line?

5. _____ Express the following as a common fraction in lowest terms.

$$\left(\frac{2017}{2016}\right)^{2016} \times \left(\frac{2016}{2017}\right)^{2017}$$

6. _____ What common fraction (in lowest terms) is equivalent to the repeating decimal $0.\overline{72}$ (that is $0.727272\cdots$)?

7. _____ Four times a number, n, is 14 more than twice the sum of n plus 4. What is the value of n?

8. _____ Simplify $\frac{18^5}{12^6}$ to a common fraction in lowest terms.

9. _____ Of the first 20 positive integers, what is the probability that a randomly picked integer is prime and also even? Express your answer as a fraction in lowest terms.

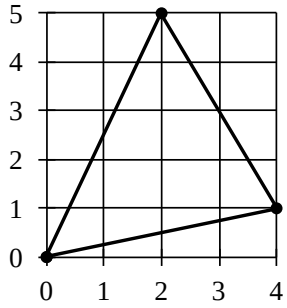
10. _____ A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle? Express your answer in simplest radical form.

11. _____ Two integers a and b have a least common multiple of 120, and their greatest common divisor is 4. What is the product of a and b?

12. _____ In a classroom of 28 students, 14 have black hair and 8 have blond hair, 16 have brown eyes and 7 have blue eyes. What is the smallest possible number of students with both black hair and brown eyes?

13. _____ In the game of Boxhead, Serge must defeat 1 monster on the first level, 6 monsters on the second level, 11 monsters on the third level, and so on. Also, he encounters devils starting from the third level. He must defeat 1 devil on the third level, 2 devils on the fourth level, 3 devils on the fifth level, and so on. What is the total number of monsters and devils Serge encounters on the 99th level?

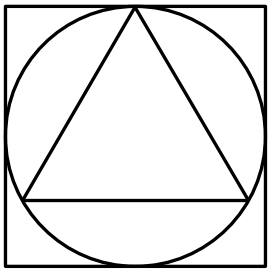
14. _____ What is the area of the triangle whose vertices are located at (0, 0), (4, 1), and (2, 5)?



15. _____ If the probability that Kevin will go to school any given day is 3/4, then what is the probability that he will go to school at least once out of 5 days? Express your answer as a common fraction in lowest terms.

16. _____ If $f(x) = \dfrac{2}{2+\dfrac{2}{x}}$, what is $f(f(f(f(f(f(f(1)))))))$?
Express your answer as a common fraction in lowest terms.

17. _____ An equilateral triangle of side length 1 is inscribed in a circle, which is then inscribed in a square. What is the area of the square? Express your answer as a common fraction in the lowest terms.



18. _____ Find the number of four digit palindromes that are multiples of 6. Palindromes are the same with the digits in reverse order, like 7337.

19. _____ If $N = 73^2 + 4\times73 + 4$, then what is the sum of the distinct prime factors of N ?

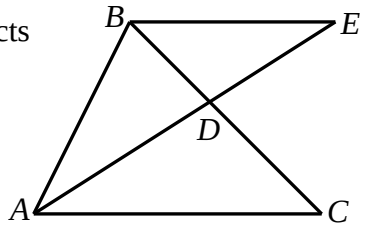
20. _____ The sum of four, distinct (none the same) positive integers is 94. What is the largest possible value of the second largest of these integers?

21. _____ What is the exponent of the largest power of 13 that divides 51! + 50! evenly with an integer result?
The "!" denotes factorial, for example, $4! = 4\times3\times2\times1 = 24$.

22. _____ If $2^7\cdot3^6 = a^2b^3$, then what is the greatest possible value of ab?

23. _____ How many ways are there to arrange 6 people around a circular table with 7 seats? (Two seatings are considered the same if one is a rotation of the other.)

24. _____ The angle bisector of $\angle BAC$ in triangle ABC intersects BC at D. Let E be a point on ray $\overline{AD}$ such that BE is parallel to AC. If $AB = 6$, $AD = 5$, and $AE = 8$, what is the length of AC?



25. _____ $2^{40} - 1$ is divisible by two integers that are between 30 and 40, inclusive. What is the sum of those two integers?

26. _____ What is the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{2017}\right)$?

27. _____ What is $x + y$ for the two, following equations? Express your answer as a common fraction. $\frac{2}{\sqrt{x}}+\frac{1}{\sqrt{y}}=13$ $\frac{3}{\sqrt{x}}-\frac{2}{\sqrt{y}}=2$

28. _____ The product of five consecutive positive integers is 95040. What is the sum of the five integers?

29. _____ What is the positive difference between the largest and the smallest four digit palindromes that are each divisible by 7?

30. _____ What is the value of $\sqrt{21\times23\times25\times27+16}$?

Forms of Answers

All answers must be expressed in simplest form. A "common fraction" means a fraction in the form $\pm\frac{a}{b}$, where *a* and *b* are natural numbers and GCF(*a*,*b*) = 1.

Examples:

Problem: Express 8 divided by 12 as a common fraction. *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: Express 12 divided by 8 as a common fraction. *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$, 1.5

Ratios shall be expressed as simplified common fractions unless otherwise specified.

Radicals must be simplified. A simplified radical must satisfy the following:
1) Radicands cannot have a factor with the root of the index (e.g., no 4 or 9 factors inside a square root).
2) No radicands contain fractions.
3) No radicals appear in the denominator of a fraction (e.g., $\frac{1}{\sqrt{2}}$ is not allowed; $\frac{\sqrt{2}}{2}$ is allowed).

Example:

Problem: Evaluate $\sqrt{15}\times\sqrt{5}$. *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Dollar amount answers shall be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The only exceptions to this rule are when *a* is zero, it may be omitted, or when *b* or *c* are both zero, they may be omitted. Examples:
Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Units of measure are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure, or when a unit of measure is provided in the answer blank, equivalent answers in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or the answer unless the problem says to do so, such as when it says to round to a specific decimal place.

Do not do any intermediate rounding (other than what the calculator performs) when calculating solutions. Any rounding should be done at the end of the calculation process, and only when the problem requests it.

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Answers must be written on the line after the problem number. Any other blank space may be used as needed to solve the problems, but will be ignored for grading.