Modeling and Design of Wave Spring Washers

Waheeduddin Sohaib Mohammad, Hong Zhou
Department of Mechanical Engineering
Texas A&M University-Kingsville
Kingsville, Texas, USA

***Abstract*—Spring washers are elastic washers that have irregular disk-like shapes. They are flexible axially and deflect like springs when they are compressed. Spring washers have a preload between the two fastened surfaces and are used to prevent loose fastening, absorb shocks, eliminate play and uniformize load. Wave spring washers are wavy in the axial direction and have multiple curvilinear lobes. They provide spring pressure when the wavy surface is compressed. Wave spring washers are employed when radial space is constrained and moderate load is applied in the operation. In this paper, a systematic method is introduced for modeling, analyzing and designing wave spring washers. To model a wave spring washer, a sweep surface is first employed to describe the wavy surface. The solid model of the wave spring washer is then generated through extruding the swept surface by its thickness along the axial direction. The solid model of a wave spring washer is fully decided by five geometric parameters: the number of waves, inner and outer diameters, washer thickness and wave height. The design of a wave spring washer is thus systemized as optimizing its five geometric parameters. The analysis of a wave spring washer is conducted based on its solid model that is defined by its five parameters. Examples on modeling, analyzing and designing wave spring washers are presented in the paper to verify the effectiveness and demonstrate the procedure of the introduced method.**

Keywords*—*spring washer; wave washer; modeling; design; sweep surface; geometric parameter.

I. INTRODUCTION

Washers are usually doughnut-shaped disks that serve to increase the area of contact between the bolt head or nut and the clamped part. A washer is normally used where the bolt compression load on the clamped part needs to be distributed on a larger area than what the bolt head or nut can provide. The use of a washer also prevents damage to the bearing surface adjacent to the fastener by the nut when it is tightened [1]. Washers are vital components in fastening and assembly operations. Besides acting as a seat for bolts, nuts, screws and rivets, washers perform many other functions that include insulating, sealing, locking, spacing, improving appearance, providing spring take-up, aligning, distributing loads. Washers are indispensible to the functions of many machines or devices and of importance to their operations [2].

There are many different washers for various applications. The configurations and specifications of washers are closely related to their applications and requirements. Washers are divided into six basic categories based on their configurations: flat, shoulder, tab, lock, countersunk and spring [2]. Flat washers have flat disk configuration and internal hole, but their external shapes can be round, square, rectangular, hexagonal or others based on applications. Shoulder washers are also called step or flange washers since there is an integral low cylindrical sleeve. They are mainly applied in electronic devices as insulators and are made of non-conductive materials. Tab washers have single or multiple internal or external protrusions in them. The protrusions or tabs prevent a washer from rotating under the bolt or in relation to a shaft. The internal protrusions prevent shaft rotation, whole external protrusions can lock into holes or over the edge of the assembly base. Lock washers have a bent or crimped surface to prevent a bolt or screw from turning or loosening. Lock washers provide great bolt tensioning for tight assemblies and protection against loosening resulting from vibration. Countersunk washers give a bearing surface for flat head screws. They can also provide a sealing function for flat head screws. Springs washers are elastic washers and have irregular disk-like shapes. They are flexible axially and deflect like springs when they are compressed. Spring washers have a preload between the two fastened surfaces and are used to prevent loose fastening, absorb shocks, eliminate play and uniformize load. This paper is on spring washers.

Springs washers have three basic types: Belleville, curved or cylindrically curved, and wave [2-3]. Belleville washers have truncated cone or spherical shapes and are also called as conical or cupped spring washers. Belleville washers provide a small deflection range and have a high load bearing capacity. A Belleville washer initially has the form of a cone that progressively flattens as the bolt is tightened. In the initial tightening, the load-deflection curve has a constant positive gradient. As the tightening continues, the load-deflection curve will have a negative gradient due to the large geometric change in the shape of the conic washer. A Belleville washer should operate in the region where the gradient is negative [4]. In this way, it will increase the load on the bolt if the jolt is loosened, so that the loosening is counteracted. Curved washers are cylindrically curved in one direction and provide a uniform spring rate over a large deflection range. They are also known as crescent or bowed washers. They are best suited for applications that flexibility, frequent load cycling and light loads. In order for a curved washer to function properly, the formed portion has to be free to slide and bearing surfaces should be hard enough to prevent washer corners from scraping or digging in. Wave washers are wavy in the axial direction and have multiple curvilinear lobes. They provide spring pressure when the wavy surface is compressed. Wave spring washers are employed when radial space is constrained and moderate load is applied in the operation. The wave uniformity is important because the real load deflection gradient does not start until all waves are evenly loaded. Once all waves are uniformly loaded, a relatively linear spring rate will be obtained until the washer deflects close to its flat position. This paper is focused on wave washers.

To model a wave spring washer, a sweep surface is first employed to describe the wavy surface in this paper. The solid model of the wave spring washer is then generated through extruding the swept surface by its thickness along the axial direction. The solid model of a wave spring washer is fully decided by five geometric control parameters: the number of waves, inner and outer diameters, washer thickness and height.

The analysis of a wave spring washer is conducted based on its solid model that is defined by its five geometric parameters. The design of a wave spring washer is thus systemized as optimizing its five control parameters.

The remainder of the paper is organized as follows. The modeling on wave spring washers is presented in section II. The analysis of wave spring washers is provided in section III. Section IV is on designing wave spring washers. Conclusions are derived in section V.

II. MODELING OF WAVE SPRING WASHERS

The surface of a wave spring washer can be modelled as a sweep surface (also known as swept surface), which is generated by moving a profile curve, **C**(*u*), along a trajectory curve, **B**(*v*). The general formula for a sweep surface is as follows [5].

$$\mathbf{P}(u,v) = \mathbf{B}(v) + M(v)\,\mathbf{C}(u) \quad (1)$$

In (1), **P**(*u*, *v*) is a point on the surface corresponding to the specific values of parameters *u* and *v*. *M*(*v*) is the transformation matrix that transforms the profile curve **C**(*u*) along the trajectory curve **B**(*v*) with functions of translation, rotation, scaling or shearing. A geometric transformation is a function that is both onto and one-to-one, and whose range and domain are points [6]. Transformation here refers to a geometric operation that applies to all the points of an object, which may move, rotate, scale or shear the object. Ordinary three-dimensional coordinate system (which is the Cartesian coordinate system for Euclidean geometry) is used in (1), so that a point on a surface or curve has *x*, *y*, and *z* coordinates, **P**(*u*, *v*), **B**(*v*) and **C**(*u*) in (1) are all 3x1 column vectors, and *M*(*v*) is a 3x3 transformation matrix.

When homogeneous coordinates are applied, equation (1) can be represented as (2).

$$\mathbf{P}(u,v) = \mathbf{C}(u)\,T(v) \quad (2)$$

P(*u*, *v*) and **C**(*u*) in (2) are point vectors in a homogeneous coordinate system. They are now 1x4 row vectors. *T*(*v*) is a 4x4 transformation matrix.

When a homogeneous coordinate system (which is the projective coordinate system for projective geometry) is used, one extra dimension is added into the Cartesian coordinate system, so every point (*x*, *y*, and *z*) in a three-dimensional Cartesian coordinate system has a corresponding set of homogeneous coordinates *hx*, *hy*, *hz* and *h* in a four-dimensional projective coordinate system. Because $x = hx/h$, $y = hy/h$, and $z = hz/h$ for all real *h* except $h = 0$, there are an infinite number of points in the four-dimensional homogeneous coordinate system corresponding to each point in the ordinary three-dimensional Cartesian coordinate system. All the points on a line through the origin in the four-dimensional homogeneous coordinate system have the same ordinary three-dimensional coordinates. Then, the point (*x*, *y*, and *z*) is the projection of the point (*hx*, *hy*, *hz* and *h*) onto the hyper-plane $h = 1$. The origin is the center of projection and the hyper-plane is the ordinary three-dimensional coordinate system [7]. A point vector in (2) is a 1x4 row vector that has the form of (*x*, *y*, *z*, 1).

A general transformation matrix *T* in (2) can be represented by its elements as follows.

$$T = \begin{bmatrix} t_{11} & t_{12} & t_{13} & 0 \\ t_{21} & t_{22} & t_{23} & 0 \\ t_{31} & t_{32} & t_{33} & 0 \\ t_{41} & t_{42} & t_{43} & 1 \end{bmatrix} \quad (3)$$

t_{41}, t_{42} and t_{43} in (3) represent translations along *x*, *y* and *z* directions, respectively. t_{11}, t_{22} and t_{33} in (3) are responsible for scaling and reflecting. In scaling transformations, the coordinates of the profile curve are multiplied by scaling factors. A scaling transformation can be uniform or non-uniform. A same scaling factor is used for all coordinates in uniform scaling transformations while non-uniform scaling transformations may have different scaling factors in different directions. The six off-diagonal elements (t_{21}, t_{31}, t_{32}, t_{12}, t_{13}, t_{23}) in the top left 3x3 sub-matrix in (3) are responsible for shearing. Rotation of the profile curve is from the joint effect of all the nine element in the top left 3x3 sub-matrix [8]. A general rotation is a rotation through any angle θ along an arbitrary axis. If the unit vector of an arbitrary rotation axis is denoted as $\mathbf{u} = (u_x, u_y, u_z)$, the general rotation matrix can then be represented as follows.

$$T_{rm} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} = \begin{bmatrix} u_x^2 V\theta + C\theta & u_x u_y V\theta + u_z S\theta & u_x u_z V\theta - u_y S\theta \\ u_x u_y V\theta - u_z S\theta & u_y^2 V\theta + C\theta & u_y u_z V\theta + u_x S\theta \\ u_x u_z V\theta + u_y S\theta & u_y u_z V\theta - u_x S\theta & u_z^2 V\theta + C\theta \end{bmatrix} \quad (4)$$

In (4), $C\theta = \cos\theta$, $S\theta = \sin\theta$, $V\theta = 1 - \cos\theta$. The rotation matrix represented by (4) is for a rotation whose rotation axis passes through the origin of the coordinate system. If an arbitrary rotation axis does not pass through the origin, we can first translate the axis to make it pass through the origin, then rotate the profile curve, and finally translate the rotation axis back to its original location. Similar procedure can be used for scaling transformations with arbitrary points that are not the origin.

Different transformations can be combined into one compound transformation matrix that is nothing but the product of the individual transformation matrices. In general, matrix multiplication is not commutative. The order of the individual transformation matrices matter.

If equation (2) is transposed on both side, it will be changed to equation (5).

$$\mathbf{P}(u,v) = T(v)\,\mathbf{C}(u) \quad (5)$$

$T(v)$ in (5) is the transpose of $T(v)$ in (2). Points represented by $\mathbf{P}(u, v)$ and $\mathbf{C}(u)$ in (5) are now 4x1 column vectors that has the form of $(x, y, z, 1)^T$.

The following shows some examples of sweep surfaces.

The sweep surface shown in Fig. 1 is generated by translating a sine function curve that is on the *x*-*y* plane along *z* axis. The *x*, *y* and *z* coordinates of any point on the surface are represented by the following equation.

$$\begin{pmatrix} x & y & z & 1 \end{pmatrix} = \mathbf{C}(u)\,T(v) = \begin{pmatrix} Lu & W\sin(6\pi u) & 0 & 1 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & Hv & 1 \end{bmatrix} \tag{6}$$

In (6), parameters *u* and *v* all change from 0 to 1. *L* is the length of the profile curve in the *x* direction and 2*W* is the width of the profile curve in the *y* direction. *H* is the height of the sweep surface in the *z* direction.

The surface image shown in Fig. 1 is plotted by function "surf" of MATLAB [9]. Interpolated shading is used when the surface is plotted.

The sweep surface shown in Fig. 2 is generated by rotating a sine function curve that is on the *x*-*z* plane through an angle with respect to *z* axis. The *x*, *y* and *z* coordinates of any point on the surface can be denoted by the following equation.

$$\begin{pmatrix} x & y & z & 1 \end{pmatrix} = \mathbf{C}(u)T(v) = \begin{pmatrix} R - W\sin(6\pi u) & 0 & Hu & 1 \end{pmatrix} \begin{bmatrix} \cos(\theta v) & \sin(\theta v) & 0 & 0 \\ -\sin(\theta v) & \cos(\theta v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

In (7), parameters *u* and *v* are in the interval of 0 to 1. *H* is the height of the profile curve in the *z* direction and 2*W* is the width of the profile curve in the *x* direction. *R* is the average distance of the profile curve from the rotation axis (*z*) and θ is the rotation angle of the profile curve.

The sweep surface shown in Fig. 3 is generated by simultaneously translating and rotating a line segment that is on the *x* axis. The *x*, *y* and *z* coordinates of any point on the surface can be derived by the following equation.

$$\begin{pmatrix} x & y & z & 1 \end{pmatrix} = \mathbf{C}(u)T(v) = \begin{pmatrix} R_i - u(R_o - R_i) & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \cos(\theta v) & \sin(\theta v) & 0 & 0 \\ -\sin(\theta v) & \cos(\theta v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & Hv & 1 \end{bmatrix} \tag{8}$$

Parameters *u* and *v* in (8) are the same as those in (6) and (7). R_i and R_o are the distances of the inner and outer ends of the profile line segment from the *z* axis, respective. *H* is the total translation distance along the *z* axis and θ is the total rotation angle of the profile line segment.

Fig. 4 shows two different views of the surface of a wave spring washer. The sweep surface is from the simultaneous translation and rotation of a line segment on the *x* axis. The *x*, *y* and *z* coordinates of any point on the surface can be calculated by equation (9).

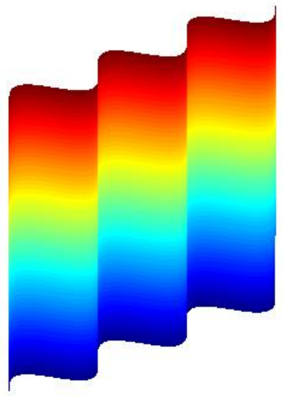

Fig. 1 A sweep surface generated from a pure translation.

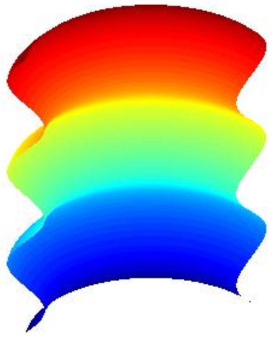

Fig. 2 A sweep surface generated from a pure rotation.

Fig. 3 A sweep surface generated from a combined translation and rotation.

$$\begin{pmatrix} x & y & z & 1 \end{pmatrix} = \mathbf{C}(u)T(v) = \begin{pmatrix} R_i - u(R_o - R_i) & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \cos(2\pi v) & \sin(2\pi v) & 0 & 0 \\ -\sin(2\pi v) & \cos(2\pi v) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & H\sin(2\pi N v) & 1 \end{bmatrix} \tag{9}$$

Parameters *u* and *v* in (9) are the interval of 0 to 1. R_i and R_o have the same meanings as those in (8). *2H* is the total translation distance along the *z* axis, which is the wave height of the surface. The total rotation angle (θ) is now 2π. *N* in (9) is the number of waves, which is 3 in Fig. 4.

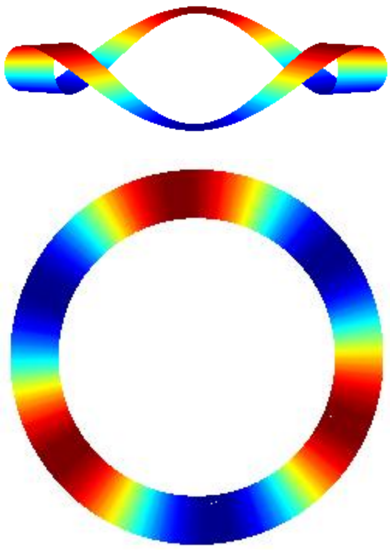

Fig. 4 The surface of a wave spring washer.

Each wave spring washer has a parameter of washer thickness. The solid model of a wave spring washer can be generated through extruding its wavy surface by its thickness along its axis. Fig. 5 shows a solid model of a wave spring washer that is from extruding the wave surface shown in Fig. 4.

The image of the 3D model shown in Fig. 5 is plotted by function "patch" of MATLAB [9]. The arguments of "FaceColor" and "EdgeColor" in patch function are set as "y" and "none", respectively. The red edge lines are added to the patch object by 3D line plotting function of "plot3."

III. ANALYSIS OF WAVE SPRING WASHERS

The analysis of wave spring washers is conducted in ANSYS, a popular finite element analysis software [10-11]. With the given five geometric parameters (the number of waves, inner and outer diameters, washer thickness and height) of a wave spring washer, its solid model can be created in ANSYS. It can also be created by a solid modeler and then imported into ANSYS. After the solid model is in ANSYS, it is discretized into elements for finite element analysis.

Fig. 6 shows a finite element model of a wave spring washer. The washer is made of carbon steel with Young's modulus (E) of 207 GPa, Poisson's ratio of 0.3, strength of 1700 MPa. There are 3 spring waves in the spring washer. Its inner and outer diameters (D_i and D_o) are 42 mm and 54 mm, respectively. The spring washer has wave height (h) of 5.0 mm and thickness (t) of 0.5 mm, so the total free height of the spring washer is 5.5 mm.

Shell element (SHELL181) in ANSYS is employed for the deflection and stress analysis of the wave spring washer. The bottom surface of the washer is placed on a fixed rigid surface while its top surface is in contact with another rigid surface that moves downward by its input displacement of 2 mm. The input displacement of 2 mm is divided into 2 even load steps and geometric nonlinearity command "NLGEOM" is turned on when the deflection and stress of the spring washer are analyzed in ANSYS.

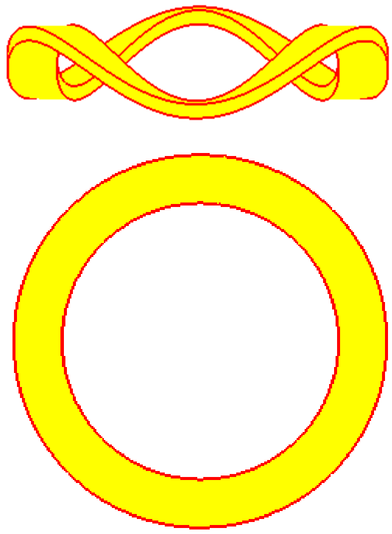

Fig. 5 The solid model of a wave spring washer.

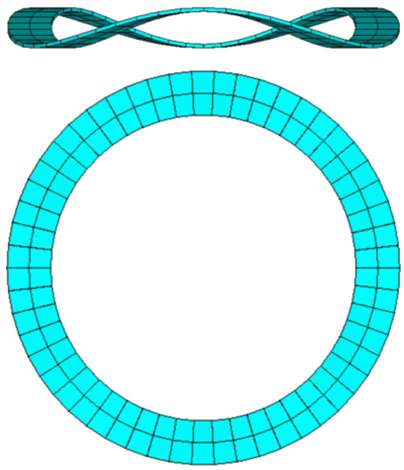

Fig. 6 The finite element discretization of the analyzed wave spring washer.

When the input displacement is 2 mm, the deflected wave spring washer and stress distribution are shown in Fig. 7 and Fig. 8, respectively. The maximum stress in Fig. 8 is 1180.2 MPa. The spring rate is 59.0 N/mm, which is calculated when its input displacement is at its middle value (1 mm in this example).

Fig. 7 The deflected wave spring washer.

When the number of spring waves is increased from 3 to 4 and other parameters are kept as the same as Fig. 6, the element discretization, the deflected spring washer and stress distribution are shown in Fig. 9, Fig. 10 and Fig. 11, respectively.

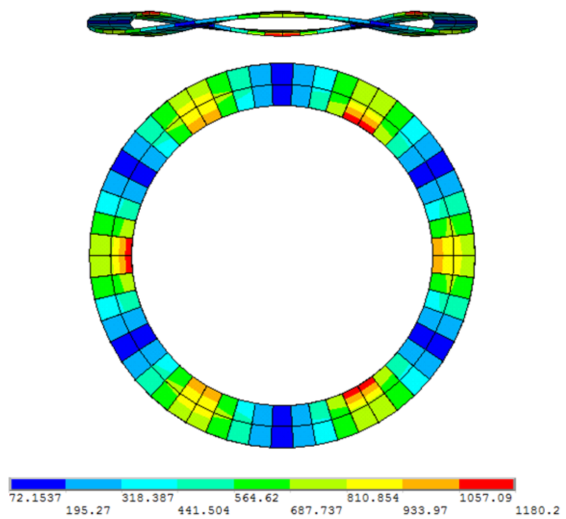

Fig. 8 The stress distribution of the wave spring washer.

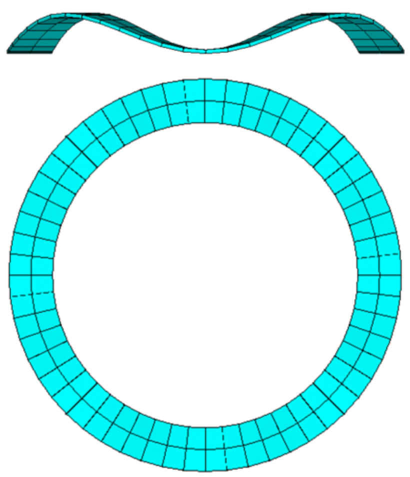

Fig. 9 The finite element discretization for N = 4.

Fig. 10 The deflected wave spring washer for N = 4.

The spring rate for N = 4 is 204.6 N/mm and is much stiffer than that of N = 3, but the maximum stress is now 2290.87 MPa as is shown in Fig. 11, which is beyond its allowable value (1700 MPa). So the number of spring waves cannot be 4 for this set of geometric parameters in this example.

IV. DESIGN OF WAVE SPRING WASHERS

A wave spring washer is designed for its application requirements that include spring rate, spring deflection, spring diameter, spring width and spring height. There are five spring geometric parameters to be selected to meet its requirements. Moreover, the maximum stress within the spring has to be below the allowable value of its material.

There is a formula shown below to calculate the spring rate approximately [3, 12].

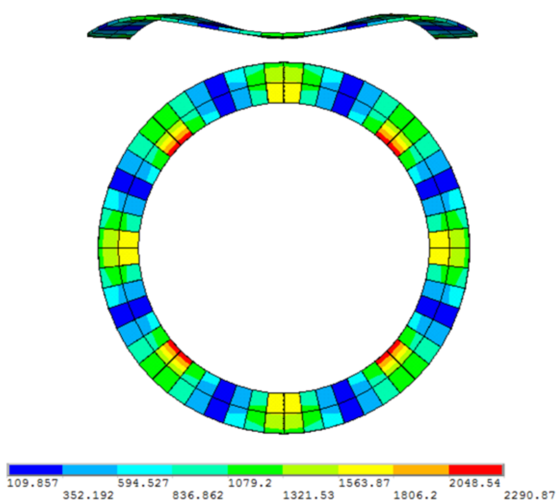

Fig. 11 The stress distribution for N = 4.

$$k = \frac{P}{f} = \frac{Ebt^3 N^4 D_o}{2.4 D^3 D_i} \tag{10}$$

P and f in (10) are the spring load and deflection, respectively. D and b are the mean diameter and width of the spring, and can be derived by D_i and D_o as $0.5(D_o + D_i)$ and $0.5(D_o - D_i)$, respectively. The formula is approximate because it is based on the equations for simple beams with correction factors [3]. There is another approximate formula to calculate the stress within the wave spring washer [3].

$$S = \frac{3\pi PD}{4bt^2 N^2} \tag{11}$$

Although equations (10) and (11) are not exact, they provide fast and convenient approaches to estimate spring rate and stress. However, they are not used in this work. Since the analysis of wave spring washers is conducted in ANSYS in this paper, the load, deflection and stress of an analyzed wave spring washer are directly from ANSYS.

The design of a wave spring washer is to optimize its five geometric parameters (that are the design variables) to meet its application needs. The optimization of the design variables in this paper is through the Global Optimization Toolbox of MATLAB [13-14]. The communications between MATLAB optimization and ANSYS analysis are based on ANSYS Parametric Design Language [15].

A design example is presented below. The spring material is the same as that used in last section. The spring rate and spring deflection are desired to be 38 N/mm and 2.5 mm, respectively. The inner and outer diameters of the wave spring washer are in the range of 38 to 46 mm. The spring wave height is from 3.7 to 4.5 mm and spring thickness is in the interval of 0.37 to 0.43 mm, so the unloaded spring height is from 4.07 to 4.93 mm.

The objective function to be minimized is as follows.

$$F_{obj} = W_1 \left| k_a - k_d \right| + W_2 \frac{\sigma_{rm}}{\sigma_{am}} \tag{12}$$

k_a and k_d are the actual and desired spring rates, and σ_{rm} and σ_{am} are the real and allowable maximum stresses, respectively. W_1 and W_2 are the weight coefficients with their sum of 1. The intention of the objective function is to make the actual spring rate close to the desired one and the maximum stress low. W_1 and W_2 are chosen in this example as 0.7 and 0.3, respectively.

Each design variable has its lower and upper bounds. The maximum stress in the spring washer is constrained to be below its allowable value as follows.

$$\frac{\sigma_{rm}}{\sigma_{am}} - 1 \leq 0 \tag{13}$$

To uniformize the spring force, the spring width is constrained to be no lower than a certain value.

$$\frac{D_o - D_i}{2b_{am}} - 1 \leq 0 \tag{14}$$

b_{am} is the allowable minimum spring width, which is 3.0 mm in this example.

The design result is shown in Fig. 12. The number of waves is 3. Its inner and outer diameters are 38 mm and 46 mm, respectively. The wave height and thickness are 4.0 mm and 0.4 mm, respectively. The spring rate is 37.9 N/mm when the spring deflection is half (1.25 mm) of its maximum deflection. The maximum stress in the wave spring washer is 1412 MPa. The deflected spring washer and stress distribution are shown in Figs. 13 and 14, respectively.

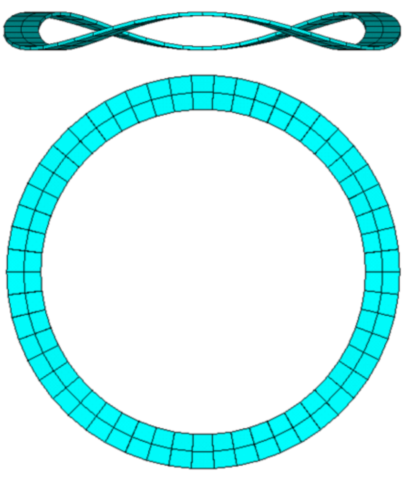

Fig. 12 The design result of the wave spring washer example.

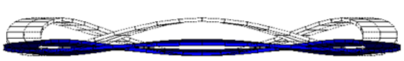

Fig. 13 The deflection of the wave spring washer design example.

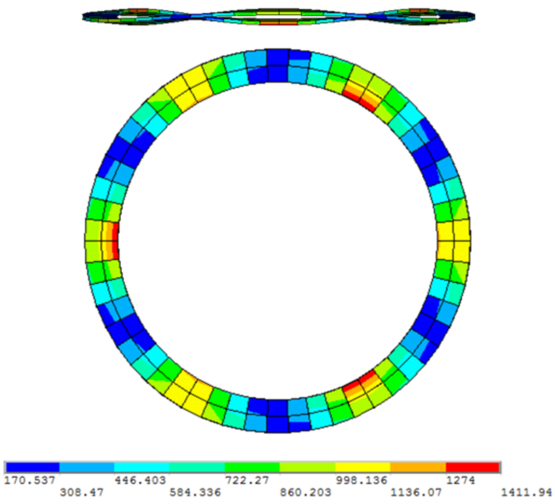

Fig. 14 The stress distribution of the wave spring washer design example.

V. CONCLUSIONS

A method for modeling, analyzing and designing wave spring washers is presented in the paper. To model a wave spring washer, its wavy surface is first generated by a sweep surface. The swept surface is then extruded by the thickness of the spring washer along its axis to produce its solid model. The solid model of a wave spring washer is discretized into elements in ANSYS for its finite element analysis. The shell element (SHELL181) in ANSYS is employed for the deflection and stress analysis with its geometric nonlinearity command (NLGEOM) turned on.

To design a wave spring washer, its five geometric parameters (the number of waves, inner and outer diameters, washer thickness and wave height) are considered as design variables and optimized by the Global Optimization Toolbox of MATLAB. The optimization objective is to make the actual spring rate to be the desired one and the maximum stress within the spring washer to be low. The deflection and stress of wave spring washer candidates are analyzed by using ANSYS during the optimization process. The data transfer between MATLAB optimization and ANSYS finite element analysis is based on the ANSYS Parametric Design Language.

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