# MATLAB EXPO
## 2021

**Modeling and Solving Optimization Problems**

*Mary Fenelon*



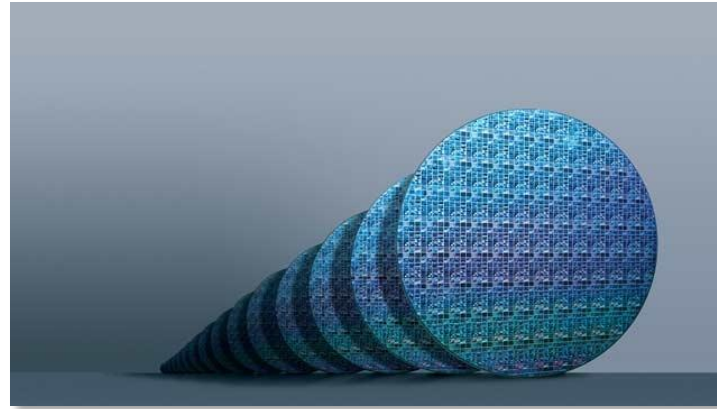MathWorks®

# Outline

- Introduction

- Nonlinear problem

- Hydroelectric dam operation

- Location planning

- Takeaways
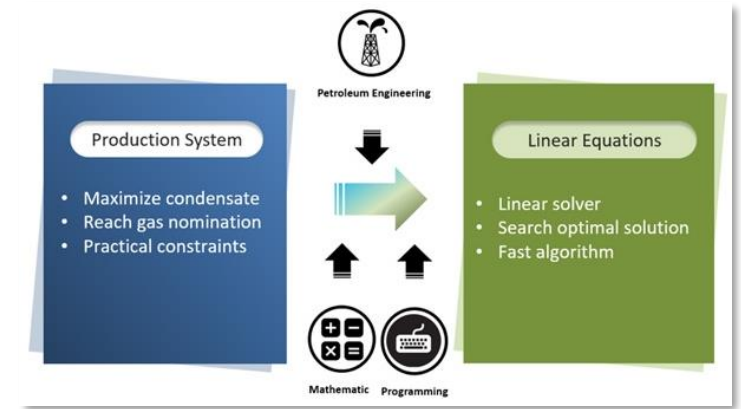
# Introduction

# What can you do with optimization?



**BuildingIQ:** Reduce energy consumption



**Realtek:** Optimize designs



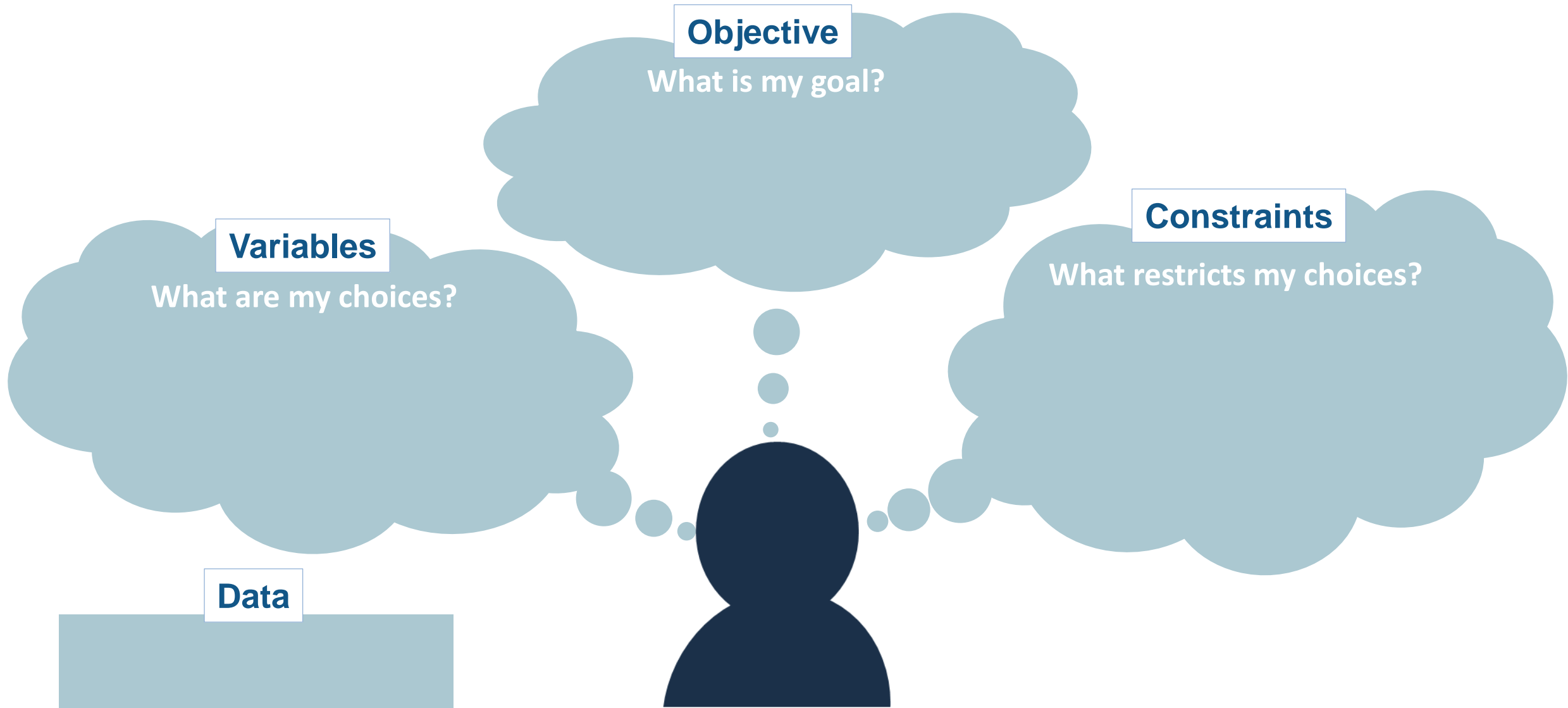**PTTEP:** Plan gas field production



**STIWA:** Optimize trajectories



**Clearpath Robotics**: Minimize travel distance
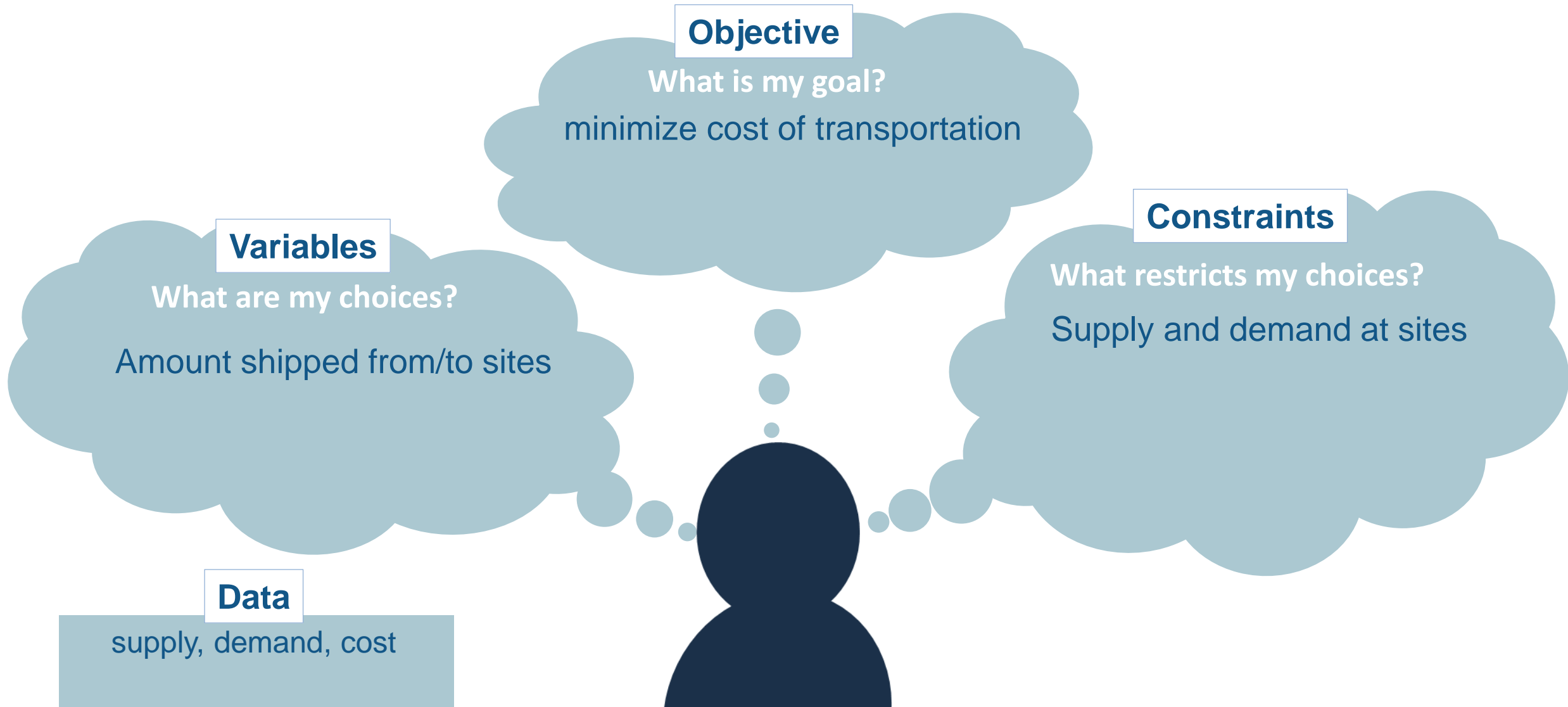


**Tessella:** Minimize propellant consumption

# Formulating an Optimization Problem: Transportation Problem

**Objective**

What is my goal?

**Constraints**

What restricts my choices?

**Variables**

What are my choices?

**Data**

# Formulating an Optimization Problem: Transportation Problem

**Objective**

What is my goal?

minimize cost of transportation

**Constraints**

What restricts my choices?

Supply and demand at sites

**Variables**

What are my choices?

Amount shipped from/to sites

**Data**

supply, demand, cost

# Formulating an Optimization Problem: Transportation Problem

**Objective**

What is my goal?

minimize cost of transportation

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

**Variables**

What are my choices?

Amount shipped from/to sites

$$x_{ij}$$

**Constraints**

What restricts my choices?

Supply and demand at sites

$$\sum_{j=1}^{m} x_{ij} = s_i \qquad \sum_{i=1}^{n} x_{ij} = d_j$$

$$x_{ij} \geq 0$$

**Data**

supply, demand, cost

$$s_i \quad d_j \quad c_{ij} \quad m \quad n$$

# Specify the problem using expressions with **Problem-Based Optimization**

$$\underset{x,y}{\text{minimize}} \quad e^x(4x^2 + 2y^2 + 4xy + 2y - 1)$$

subject to

$$\frac{xy}{2} + (x+2)^2 + \frac{(y-2)^2}{2} \leq 2$$

$$x + y \leq 1$$

```
prob = optimproblem;

x = optimvar('x');
y = optimvar('y');

prob.Objective = exp(x).*(4*x.^2 + 2*y.^2 + 4*x.*y + 2*y - 1);

prob.Constraints.c1 = x.*y/2 + (x+2).^2 + (y-2).^2/2 <= 2;
prob.Constraints.c2 = x + y <= 1;

initialPt.x = -3;
initialPt.y = 3;
[sol,fval,exitflag,output] = solve(prob,initialPt)
```

# Problem-Based Optimization makes optimization easier to use

- Familiar MATLAB syntax for expressions

- No need to write functions and build coefficient matrices

- `fcn2optimexpr` enables use of non-supported operators and functions

- Functions to help debugging `show, evaluate, infeasibility`

- Automatic solver selection

- Automatic differentiation  R2020b

# Solve many types of optimization problems with MATLAB

Optimization Toolbox
*Global Optimization Toolbox*

| Constraint Type | Objective Type | | | | | |
|---|---|---|---|---|---|---|
| | **Linear** | **Quadratic** | **Least Squares** | **Smooth nonlinear** | **Nonsmooth** | **Multiobjective** |
| **None** | | quadprog | lsqcurvefit lsqnonlin | fminsearch fminunc | fminsearch *ga* | fgoalattain fminimax *paretosearch gamultiobj* |
| **Bound** | | quadprog | lsqcurvefit lsqnonlin lsqnonneg lsqlin | fmincon | fminbnd *ga surrogatopt patternsearch particleswarm simulannealbnd* | fgoalattain fminimax *paretosearch gamultiobj* |
| **Linear** | linprog | quadprog | lsqlin | fmincon | *ga patternsearch surrogatept* | fgoalattain fminimax *paretosearch gamultiobj* |
| **Second-Order Cone** | coneprog | coneprog | | | | |
| **General smooth** | fmincon | fmincon | fmincon | fmincon | *ga patternsearch surrogateopt* | fgoalattain fminimax *paretosearch gamultiobj* |
| **General nonsmooth** | *ga patternsearch* | *ga patternsearch* | *ga patternsearch* | *ga patternsearch* | *ga patternsearch surrogateopt* | *paretosearch gamultiobj* |
| **Discrete** | intlinprog | | | | *ga surrogateopt* | |

# Exercises

# Instructions to get started with the exercises
Clickable links are in the Chat and in the handout

1. Access MATLAB Online using the link. Sign in with your MathWorks account

2. Access the shared folder on MATLAB Drive using the link
    a. Create your own unshared copy of the folder and its contents and add it to your files, by clicking ➕ **Add to my Files** and select **Copy Folder**
    b. Click Files on the left side of the page to access the folder in your MATLAB Drive.

3. Return to the MATLAB Online window and open the Live Script
    a. Double-click on **Expo2021_Optimization_Workshop** folder in the Current Folder pane
    b. Double-click on **1_Simple_Nonlinear_Problem** folder
    c. Double-click on **Work_NonlinearProblem.mlx** to open

# Exercise 1 - Nonlinear Problem



Contours of f

**Purpose:**

- Ensure MATLAB Online is running properly

- Create and solve a nonlinear optimization problem

- Include a black-box MATLAB function

**To Do:**

- Open the 1-Simple_Nonlinear_Problem folder

- Open Work_NonlinearProblem.mlx

  – Accept the prompt to open MATLAB Online

- Follow along with instructor
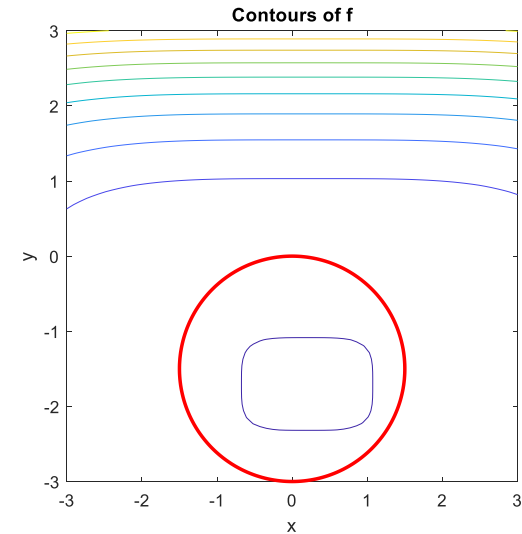
# Nonlinear Problem Review

Contours of f



## Workflow

| | |
|---|---|
| **Data** | |
| **Problem** | `optimproblem` |
| **Variables** | `optimvar` |
| **Objective** | `ObjectiveSense`<br>`Objective` |
| **Constraints** | `Constraints` |
| **Solve** | `solve` |
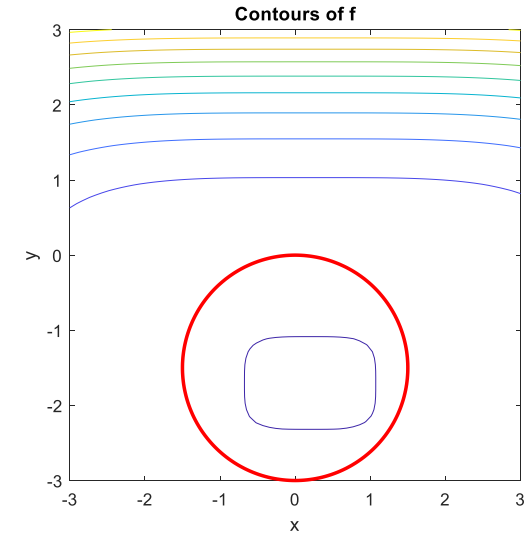
# Exercise 2 - Hydroelectric Dam Operation
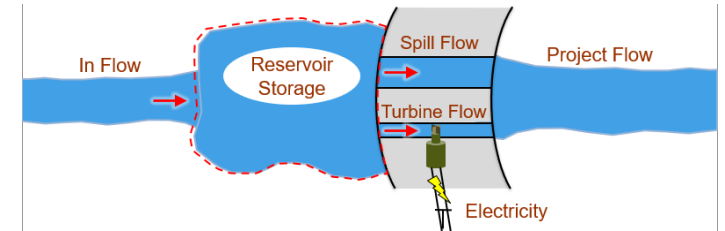


**Purpose:**

- Solve a planning problem over multiple time periods
- Use vectors of variables and constraints

**To Do:**

- Open the 2-HydroelectricDamOperation folder
- Open Work_HydroelectricDamOperation.mlx
- Follow along with instructor
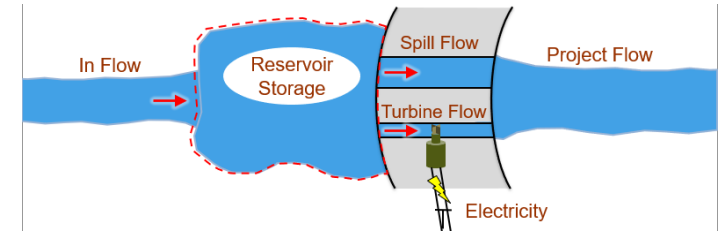
# Hydroelectric Dam Operation Review



- Constructed the model so that any size data can be used

- Created vectors of variables and constraints

- Created intermediate optimization expressions as a convenience
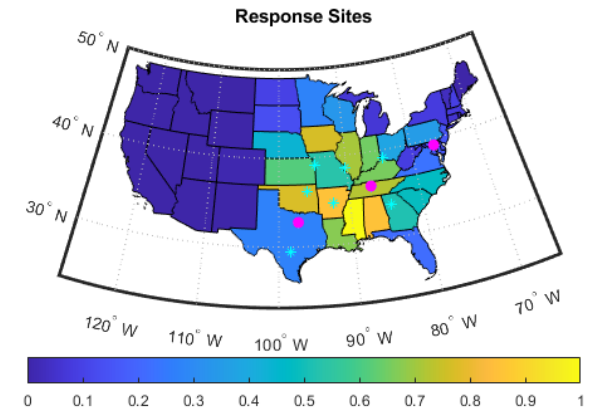
- Solved a quadratic program

# Exercise 3 - Disaster Response Facility Location



**Purpose:**

- Solve a facility location problem
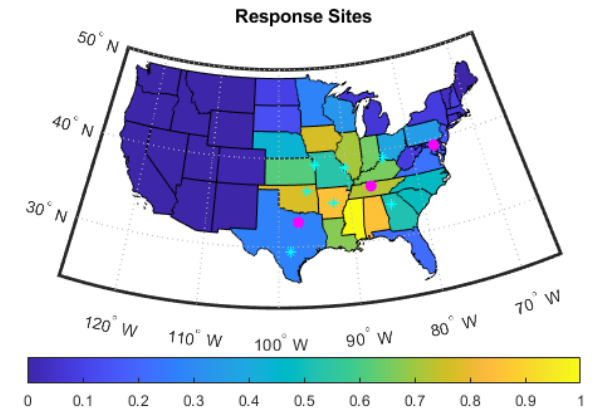
- Use arrays of variables and constraints

**To Do:**

- Open the 3-LocationPlanning folder

- Open Work_DisasterResponseFacilityLocation.mlx

- Work on your own

# Facility Location Review

- Indexed variables by strings

- Created 2-D arrays of variables

- Solved an integer linear program


Response Sites

# Takeaways

# Problem-Based Optimization makes optimization easier to use

- Familiar MATLAB syntax for expressions

- No need to write functions and build coefficient matrices

- `fcn2optimexpr` enables use of non-supported functions

- Functions to help debugging `show, evaluate, infeasibility`

- Automatic solver selection

- Automatic differentiation

- `eqnproblem` for nonlinear systems of equations

# Use these resources to learn about optimization with MATLAB

| Getting Started | | | |
|---|---|---|---|
| **Optimization Toolbox** | Overview | Video | Documentation |
| **Global Optimization Toolbox** | Overview | Video | Documentation |
| Cheat Sheets | | | |
| Training: Optimization Techniques in MATLAB | | | |

| How-To Videos | |
|---|---|
| Master Class: Solving Optimization Problems | Mathematical Modeling with Optimization |
| Design Optimization with MATLAB | Linear and Mixed-Integer Linear Programming |
| Optimize Live Task | Advanced Statistics and Optimization |
| Surrogate Optimization | Multiobjective Optimization |
| Global Search | MultiStart Optimization |

| Basics |
|---|
| Nonlinear Programming |
| Linear Programming |
| Integer Programming |
| Quadratic Programming |
| Genetic Algorithm |
| Least Squares and Nonlinear Systems of Equations |

| Examples | |
|---|---|
| Flight Path Optimization | Traveling Salesman Problem |
| Production Planning | Portfolio Optimization |
| Minimizing Electrostatic Energy | Optimal Dispatch of Power Generators |
| Antenna Design | Circuit Component Selection |

# MATLAB EXPO

## 2021

## Thank you

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