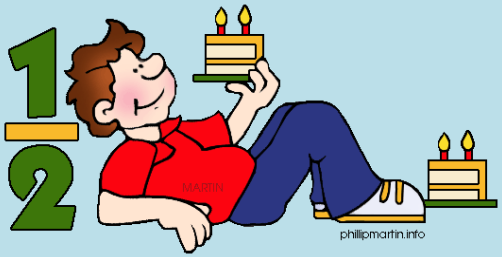
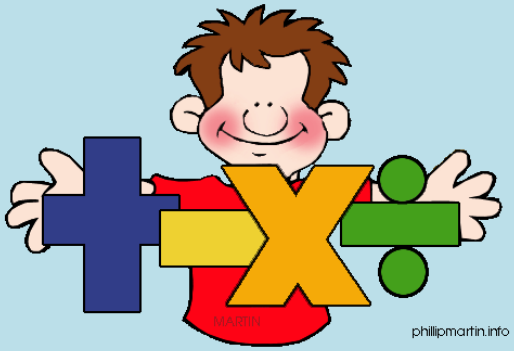


Modeling Fraction Computation



Using Visuals and Manipulatives to Deepen Conceptual Understanding

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Wake County Public Schools

Stacy Eleczko
Mathematics Specialist/ Math Coach
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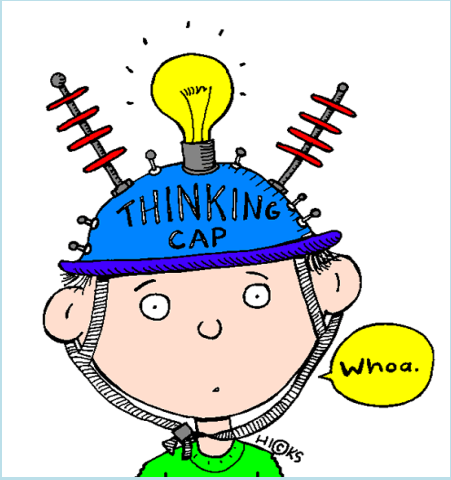
Process Goals

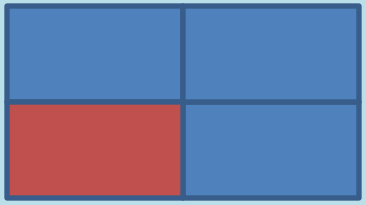
for students to:



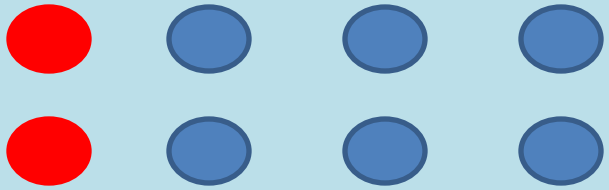
- Become mathematical problem solvers
- Communicate mathematically
- Reason mathematically
- Make mathematical connections
- Use mathematical representations to model and interpret practical situations

Models for Thinking

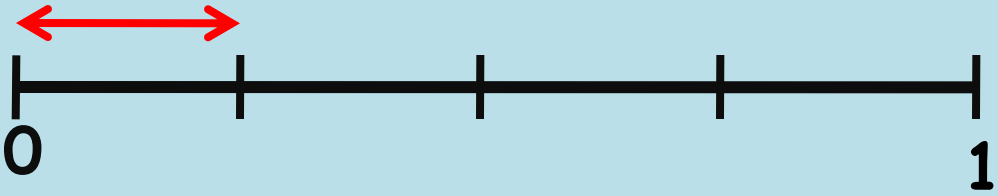
Area Model

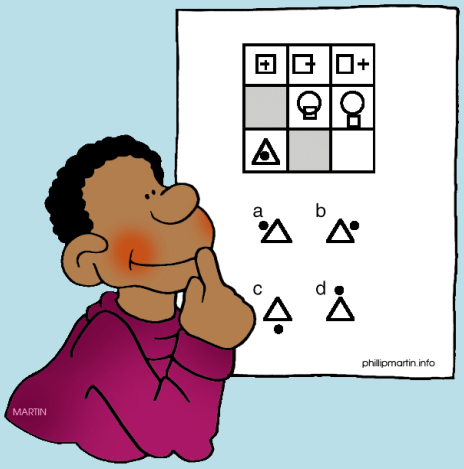


Set Model

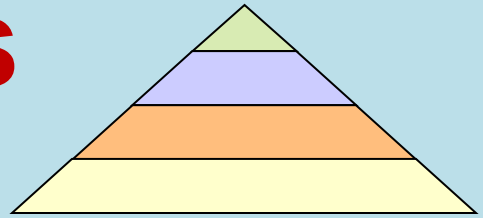


Linear Model

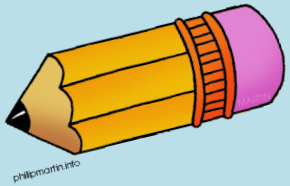


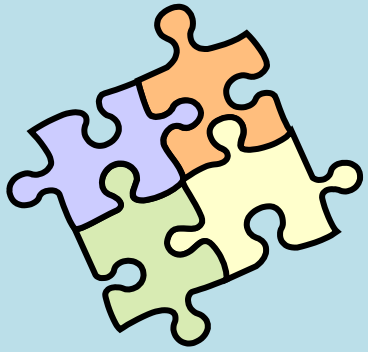


Manipulatives

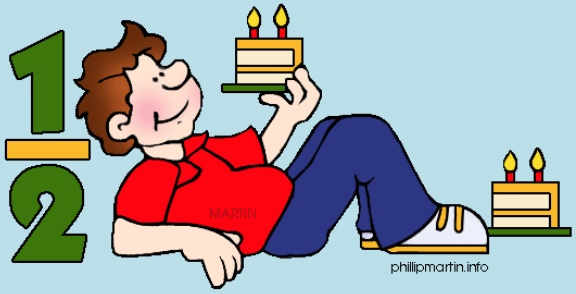


- Cuisenaire Rods
- Pattern Blocks
- Fraction Circles
- Fraction Strips
- Color Overlays
- Number Lines
- Drawing Pictures
- Fraction Bars





When adding, subtracting, multiplying, and dividing fractions, keep the following models in mind:



- **Addition- joining model**
- **Subtraction- take away model**
- **Multiplication- joining of equal groups model** ***(repeated addition)***
- **Division- taking away of equal groups model** ***(repeated subtraction)***

How Important is the WHOLE?

Question?

Students (and teachers) should

- always be mindful of **what represents the whole**.

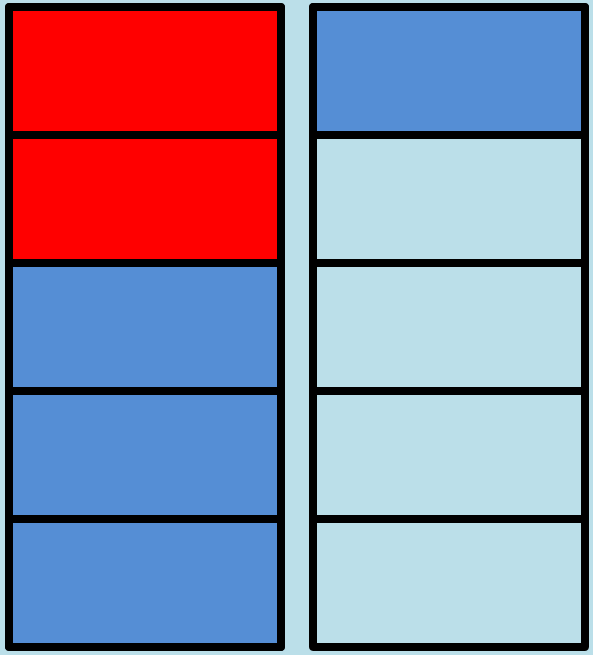
- **draw pictorial representations** to bring meaning to the mathematics involved in fraction computation.

Addition of Fractions

$$\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$$

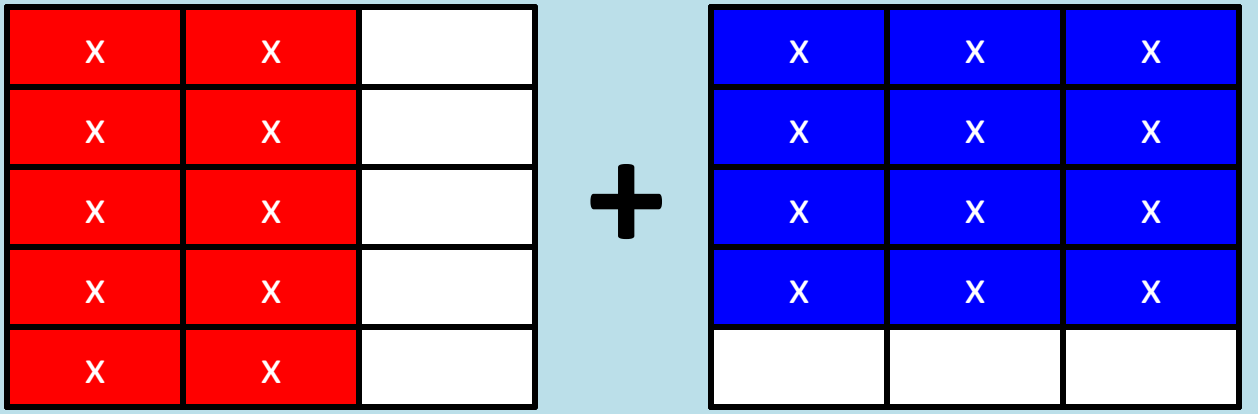
Addition of Fractions

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$$

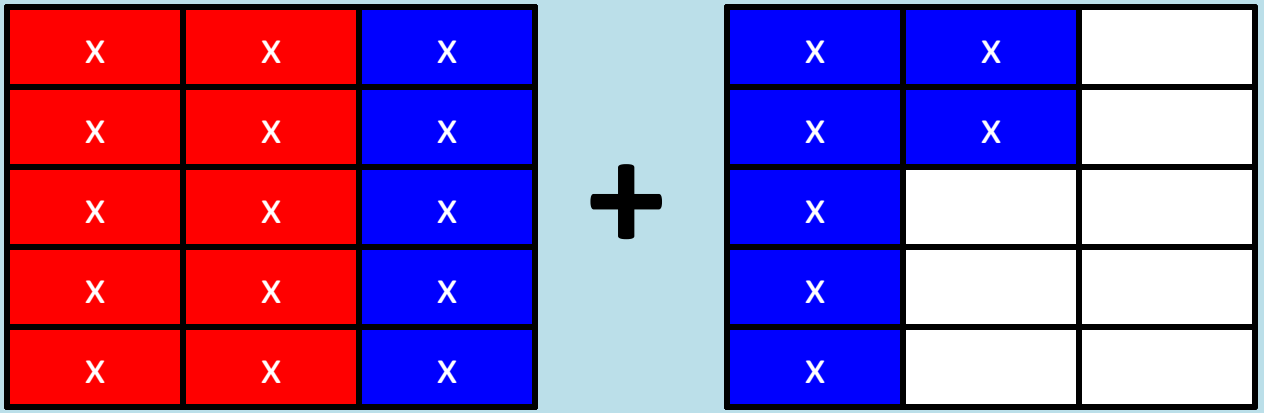


Addition of Fractions

$$\frac{2}{3} + \frac{4}{5} = \frac{22}{15}$$

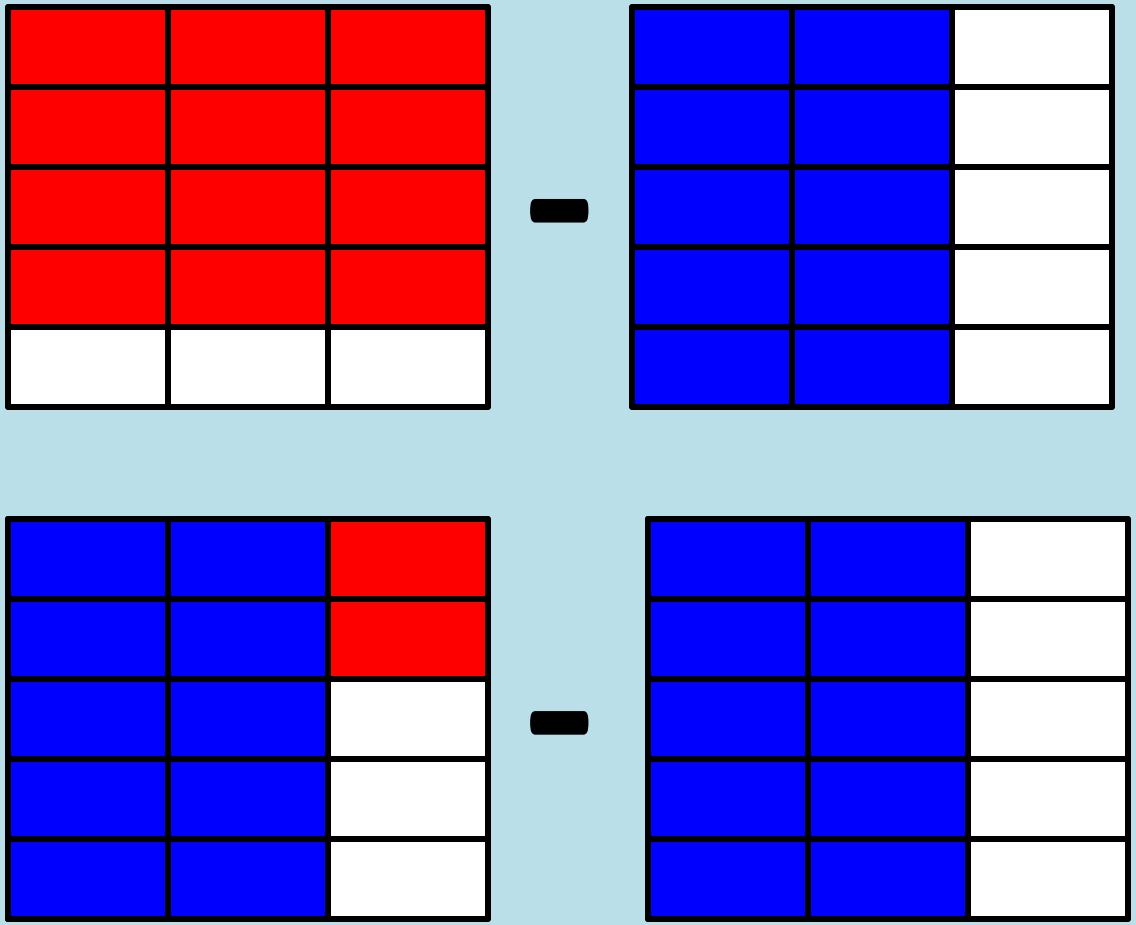


=



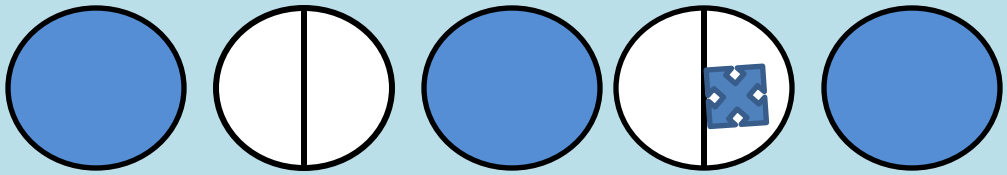
Subtraction of Fractions

$$\frac{4}{5} - \frac{2}{3} = \frac{2}{15}$$



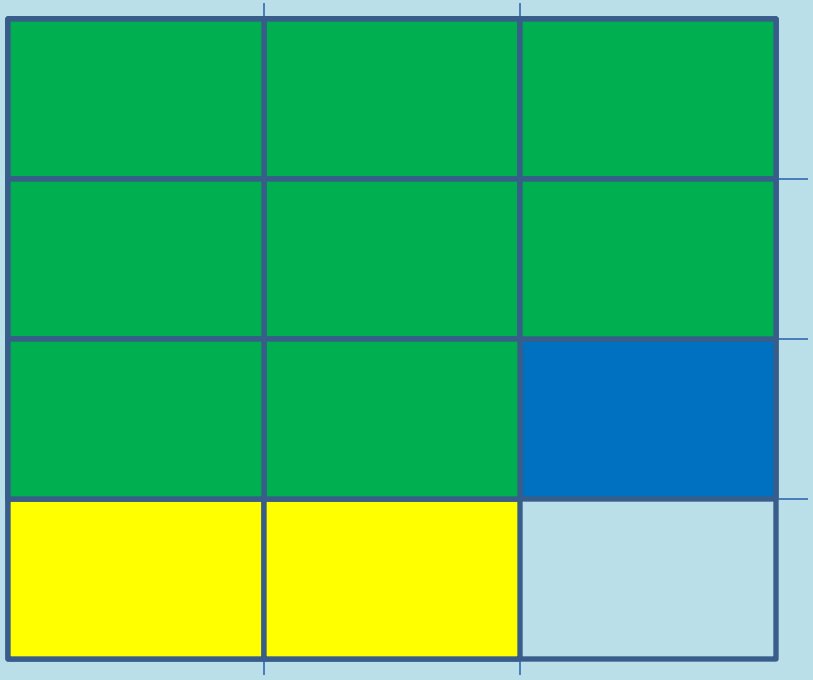
Multiplication of Fractions

1 ½ x 3 = 4 ½



Multiplication of Fractions

2/3 x 3/4



Division of Fractions

$$1 \div 2/3 =$$

- Model this problem using an area, linear, and set model.

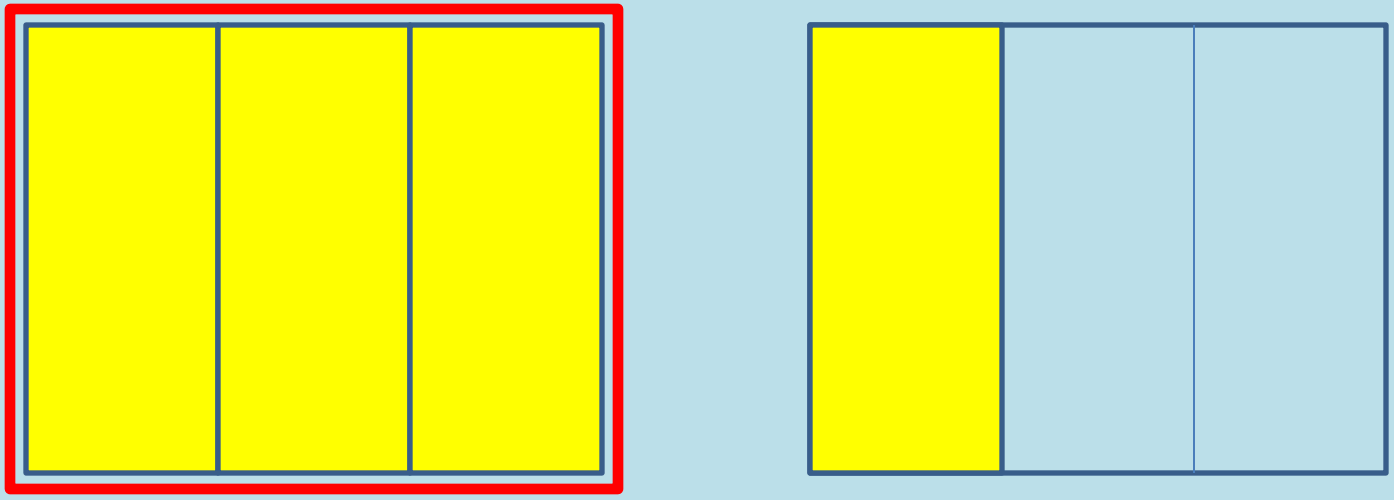
Division of Fractions

- **To divide fractions, determine how many groups of the divisor are represented in the dividend**
- **In other words, how many are in each group OR how many groups are there?**

Division of Fractions: Area Model

$$1 \div 2/3 =$$

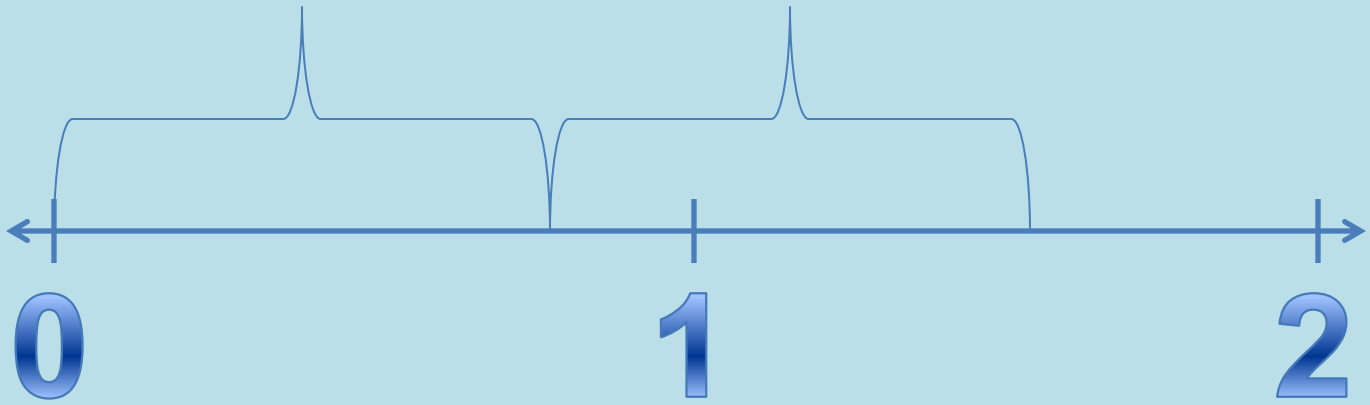
- How many sets of 2/3 are in one whole?



Division of Fractions: Linear Model

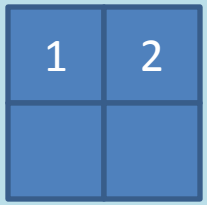
$1 \div 2/3 =$

- How many sets of 2/3 are in one whole?

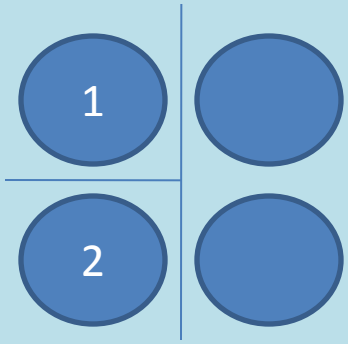


Division of Fractions

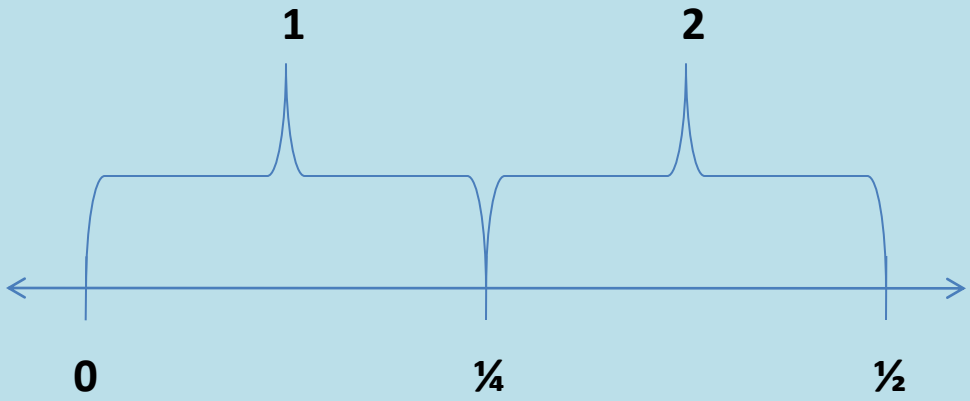
- ½ ÷ ¼ (how many quarters in a half?)
- **Area:**



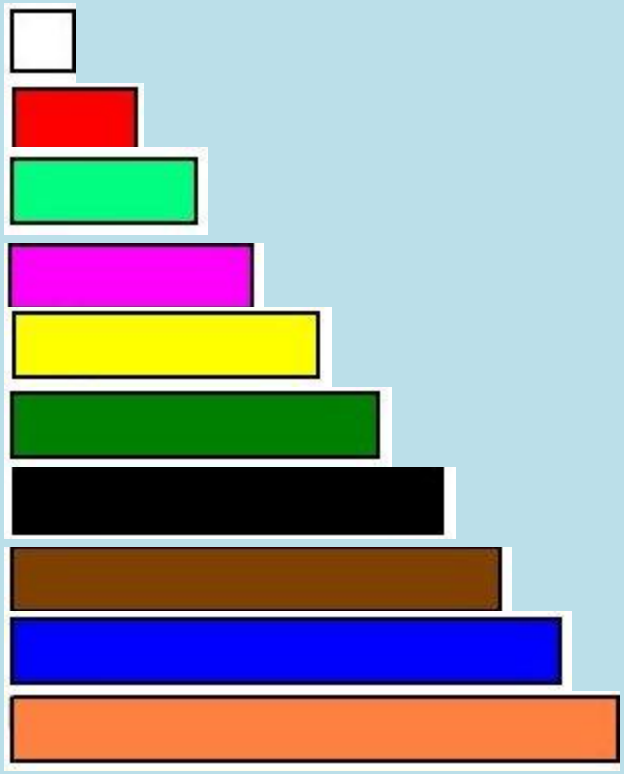
- **Set:**



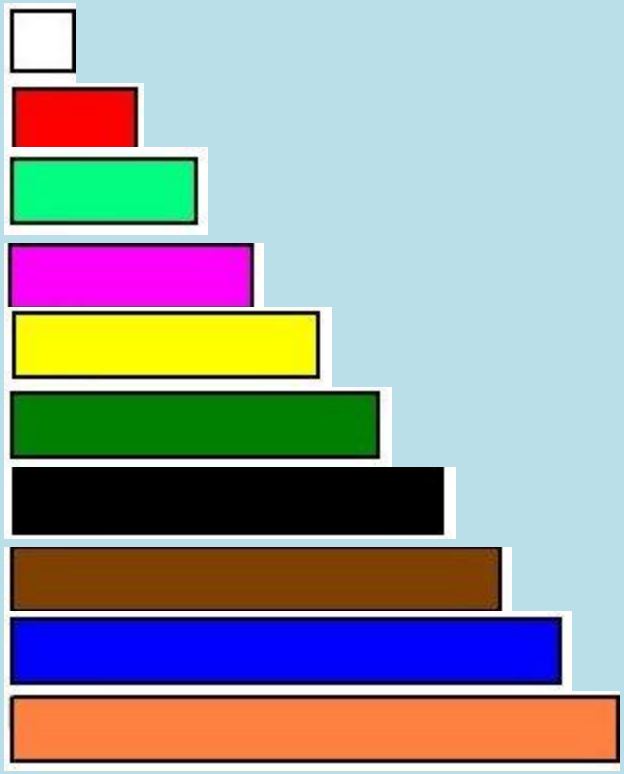
- **Linear:**



Cuisenaire Rods



Cuisenaire Rods



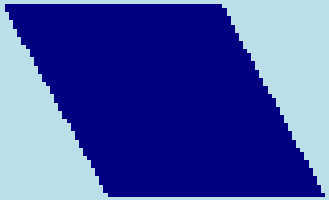
white = 1 cm.
red = 2 cm.
lime green = 3 cm.
purple = 4 cm.
yellow = 5 cm.
dark green = 6 cm.
black = 7 cm.
brown = 8 cm.
blue =9 cm.
orange = 10 cm.

Pattern Blocks

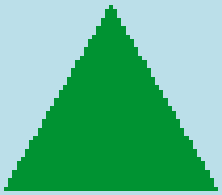
WHOLE (1)

$\frac{1}{2}$

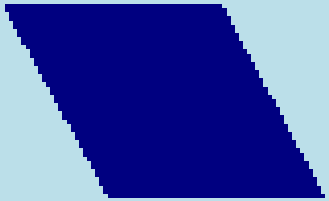
$\frac{1}{3}$



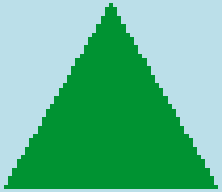
$\frac{1}{6}$



Pattern Blocks

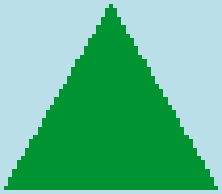


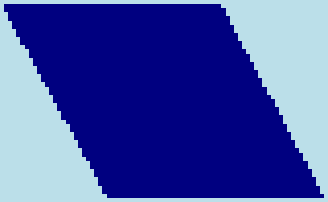
If the value of this is one, then what is ?

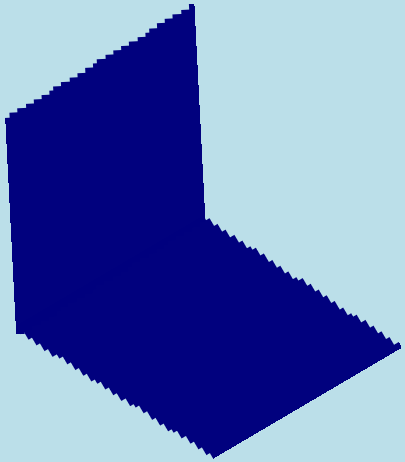


Pattern Blocks

If the value of this is one, then what is ?



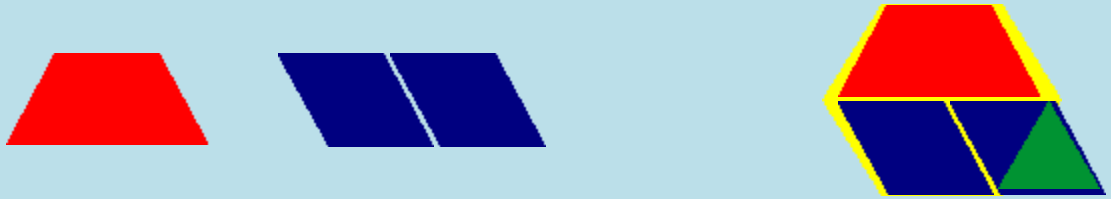




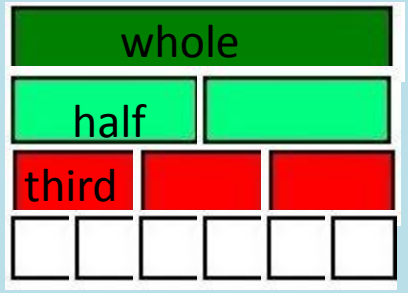
Addition of Fractions

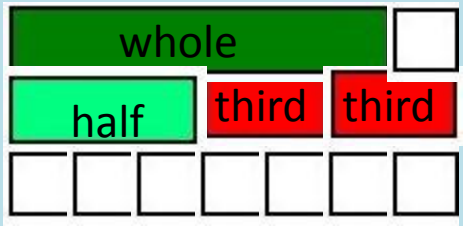
Pattern Blocks and Cuisenaires

1/2 + 2/3 = 1 and 1/6



2/3 + 1/2 = 1 and 1/6



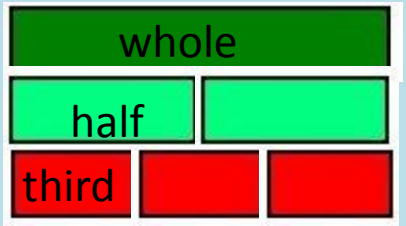


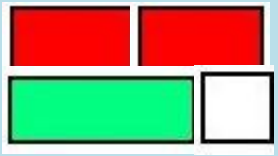
Subtraction of Fractions

Pattern Blocks and Cuisenaires

2/3 – 1/2 = 1/6

2/3 – 1/2 = 1/6

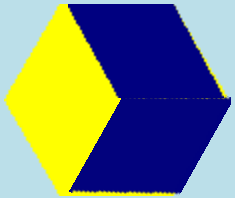


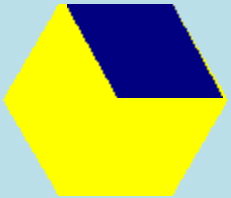


Multiplication of Fractions

Pattern Blocks and Cuisenaires

1/2 x 2/3 = 1/3



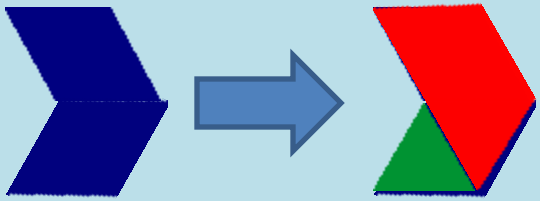


Can you use the commutative property to solve this problem backwards using both manipulatives?

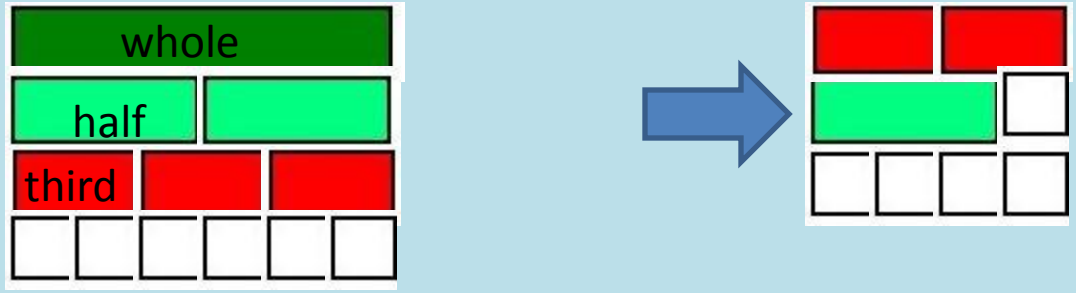
Division of Fractions

Pattern Blocks and Cuisenaires

2/3 divided by 1/2 = $1\frac{1}{3}$



2/3 divided by 1/2 =



Can you solve this problem backwards using both manipulatives?

RIGOR AND RELEVANCE

We have spent a great deal of time on the COMPUTATIONAL piece. Now it's time to put the pedal to the metal…

1 ¾ divided by ½

PLEASE CREATE A STORY PROBLEM IN WHICH ONE AND THREE QUARTERS IS DIVIDED BY ONE HALF. KINDLY SOLVE THE PROBLEM USING AT LEAST TWO DIFFERENT STRATEGIES, NEITHER OF WHICH SHOULD BE THE STANDARD ALGORITHM (KEEP, FLIP, CHANGE). ENJOY!

Li Ping Ma's take…

1 ¾ divided by ½

- Only 43% of teachers surveyed were able to compute this correctly.
- MOST confused it with dividing by 2 instead of by ½
- 6 out of 23 could NOT create a story that matched the mathematics,16 who did, yet their stories revealed rampant misconceptions. Only ONE teacher provided a conceptually correct, yet pedagogically problematic representation
- Almost EVERY SINGLE TEACHER used food: graham crackers, pizza, Twinkies, butter, pies, apples, etc. One used crayons. (YAY).

Li Ping Ma's take…

1 ¾ divided by ½

- The Chinese teachers used THREE different models of division when representing the problem
  - the MEASUREMENT(quotative) model
    - how many ½ s are there in 1 ¾
    - e.g. how many ½ foot lengths are there in something 1 ¾ feet long?
    - Cut an apple into 4 equal pieces. Take three of those pieces and join them with a whole apple. If ½ an apple is a serving size, how many servings are there?
  - the PARTITIVE model
    - finding a number such that ½ of it is 1 ¾
    - e.g. if half a length is1 and ¾ feet, how long is the whole?
    - I bought a box of candy. I have ½ of it (weighing 1 ¾ kg) to my friend. How much did the candy originally weigh?
  - PRODUCT AND FACTORS model
    - Finding a factor that, when multiplied by ½, makes 1 ¾
    - e.g. if one side of a 1 ¾ square foot rectangle is ½ feet, how long is the other side?
    - We know that the area of a rectangle is the product of length and width. Let's say that the area of a rectangular board is 1 ¾ square meters and its width is ½ meters. What is its length?

Li Ping Ma's take…

1 ¾ divided by ½

Takeaway…

- **US teachers tend to use CONCRETE models for fractions (food, money) while Chinese teachers are more apt to represent concepts in a diverse and abstract way AND relate problems to students lives using real, relevant, rigorous problems**

Websites

Models for fractions:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

http://www.visualfractions.com/

http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=212

Equivalent fractions:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Articles

10 Practical Tips for Making Fractions Come Alive and Make Sense

Doug M. Clarke, Anne Roche, and Annie Mitchell
(Vol.18 No.7 – Mathematics Teaching in the Middle School)
March 2008

Dividing Fractions: What is the Divisor's Role?

Heather A. Coughlin
(Vol.16 No.5 – Mathematics Teaching in the Middle School)
December 2010/January 2011