Global Journal of Researches in Engineering: H
Robotics & Nano-Tech
Volume 14 Issue 1 Version 1.0 Year 2014
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Modeling, Simulation and Control of 2-R Robot

By Aalim M. Mustafa & A AL-SAIF
King Fahd University of Petroleum and Minerals, Saudi Arabia

Abstract- This article presents a study of Three PID controller technique of a 2-Revelutejoint robot. First we present Denavit-Hartenberg parameters for 2-R robot. Then we studied the dynamics of the 2-R robot and derived the nonlinear equations of motion. A PID controller has been implemented for three types of modeling technique: model based on linearization about equilibrium point, model based on Autodesk Inventor and Matlab/Simulink software's, and lastly model based on feedback linearization of the robot. A comparison between the three controllers is presented showing the effectiveness of each technique.

Keywords: robotics, 2-R robot, dynamic, modeling, simulation, control and PID.

GJRE-H Classification : FOR Code: 090602p

MODELINGSIMULATIONANDCONTROLOF2RROBOT

Strictly as per the compliance and regulations of:

Modeling, Simulation and Control of 2-R Robot

Aalim M. Mustafa [α] & A AL-SAIF [σ]

Abstract- This article presents a study of Three PID controller technique of a 2-Revelutejoint robot. First we present Denavit-Hartenberg parameters for 2-R robot. Then we studied the dynamics of the 2-R robot and derived the nonlinear equations of motion. A PID controller has been implemented for three types of modeling technique: model based on linearization about equilibrium point, model based on Autodesk Inventor and Matlab/Simulink software's, and lastly model based on feedback linearization of the robot. A comparison between the three controllers is presented showing the effectiveness of each technique.

Keywords: robotics, 2-R robot, dynamic, modeling, simulation, control and PID.

I. Introduction

Robotics is the science that deals with robot's design, modeling and controlling. Nowadays robots are used everywhere in everyday life. It has accompanied people in most of industry and daily life jobs. (Gouasmi, Ouali, Fernini, & Meghatria, 2012).

The range of robot utilization is very wide. A large family of robots is used in industry and manufacturing. Robots are used in supplying the motion required in manufacturing processes such as pick and place, assembly, painting, milling, cutting, welding, drilling, etc.

Because of different types of tasks different manipulator configurations are available such as rectangular, cylindrical, spherical, revolute and horizontal jointed (Gouasmi et al., 2012).

A two revolute joint robot configuration with two degrees of freedom is generally well-suited for small parts insertion and assembly, like electronic components. Although the final goal is to design and manufacture real robotics, it is very useful to perform simulations prior to investigations with real robots. Simulations are easier to setup, less expensive, faster and more convenient to use. it allows better design exploration and helps you enhance your final real robot by selecting suitable parameters for the system you want to design (Žlajpah, 2008).

There are many control techniques used to control a robot arm. The most used ones are the PID control, optimal control, adaptive control and robust control. "There are many kinds of controllers that can be used to cause a designed robot arm to move along a desired trajectory" (Sukvichai, 2008). The simplest which we used in this paper to control the robot arm is the PID controller.

Author α: Mechanical Engineering department at King Fahd University of Petroleum and Minerals (KFUPM) Saudi Arabia.
e-mail: Aalim.motasim@hotmail.com
Author σ: System Engineering department at King Fahd University of Petroleum and Minerals (KFUPM) Saudi Arabia.

II. Problem Formulation

a) Robot Specifications

Consider the two joint sticks robot shown in figure (1) with the following specifications in Oxy coordinates:

$L_1 = 1$ m is the length of link 1

$L_2 = 1$ m is the length link 2

$m_1 = 1$ kg is the mass of link 1

$m_2 = 1$ kg is the mass of link 2

θ_1 Is the rotation angle of joint 1

θ_2 Is the rotation angle of joint 2

$L_{c1} = L_{c2} = 0.5$ m is the distance to the half of the link.

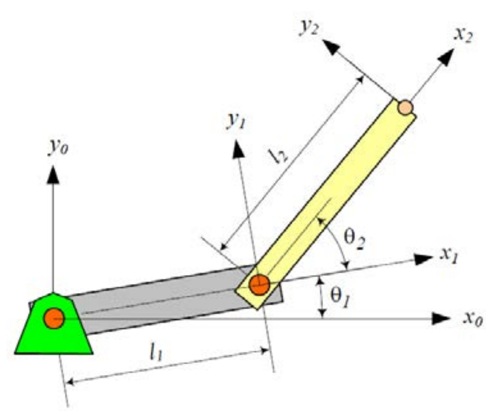

Fig. 1 : Two- joint 2-R Robot (N.Jazar, 2010)

b) Robot Kinematics

If we assigned the joints axes based on the Denavit Hartenberg representation, The (D-H) parameters for the 2-R robot will be defined as in the table below.

Table 1 : D-H parameters of 2-R Robot

Frame No.	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2

The initial position (at t = 0) from the homogeneous transformation matrix where $\theta_1 = 0^\circ \theta_2 = 0^\circ$ are shown in figure (2).

Global Journal of Researches in Engineering (H) Volume XIV Issue I Version I Year 2014

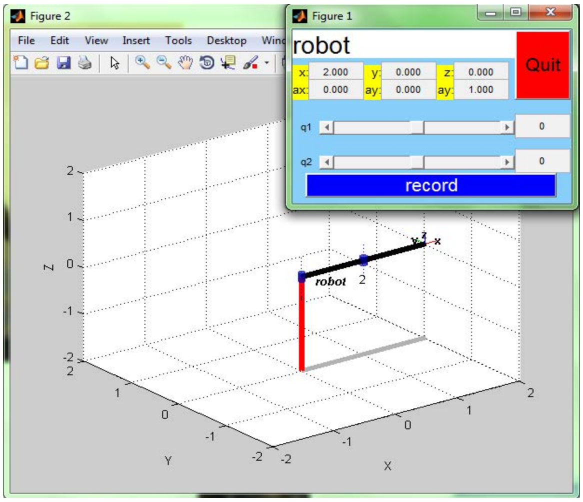

Fig. 2 : Home position of 2-R Robot

III. Robot Dynamics

Description of x and y in terms of θ_1 and θ_2 in term of linear displacement:

$x_1 = L_1 \sin\theta_1$

$y_1 = L_1 \cos\theta_1$

$x_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$

$y_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$

So, Kinetic Energy could be formed as:

$$\mathrm{KE} = \frac{1}{2} m_1 v_1{}^2 + \frac{1}{2} m_2 v_2{}^2 + \frac{1}{2} j_1 \omega_1{}^2 + \frac{1}{2} j_2 \omega_2{}^2 \qquad (1)$$

Substitute for v1 and v2

$$\mathrm{KE} \frac{1}{2} m_1 l_{g1}^2 \dot\theta_1 + \frac{1}{2} m_2 \left(l_1^2 \dot\theta_1 + 2 l_1 l_{g2} \dot\theta_1 (\dot\theta_1 + \dot\theta_2) \cos\theta_2 + l_{g2}^2 (\dot\theta_1 + \dot\theta_2)^2 \right) + \frac{1}{2} j_1 \dot\theta_1 + \frac{1}{2} j_1 (\dot\theta_1 + \dot\theta_2)^2 \qquad (2)$$

And Potential Energy is

$$\mathrm{PE} = m_1 g l_{g1} \sin\theta_1 + m_2 g (l_1 \sin\theta_1 + (l_{g2} \sin(\theta_1 + \theta_1))) \qquad (3)$$

a) *Equations of motion*

The Lagrangian of a dynamic system is defined as the difference between the kinetic and potential energy at an arbitrary instant (N.Jazar, 2010).

$$L = \mathrm{KE} - \mathrm{PE}$$

So, by Lagrange Dynamics, we form the Lagrangian

$$\mathcal{L} = l_{g1}^2 \dot\theta_1 + \frac{1}{2} m_2 \left(l_1^2 \dot\theta_1 + 2 l_1 l_{g2} \dot\theta_1 (\dot\theta_1 + \dot\theta_2) \cos\theta_2 + l_{g2}^2 (\dot\theta_1 + \dot\theta_2)^2 \right) + \frac{1}{2} j_1 \dot\theta_1 + \frac{1}{2} j_2 (\dot\theta_1 + \dot\theta_2)^2 - m_1 g l_{g1} \sin\theta_1 - m_2 g (l_1 \sin\theta_1 - (l_{g2} \sin(\theta_1 + \theta_2))) \qquad (4)$$

Using Lagrange to form generalized equations of motion in matrix form as:

$$\begin{bmatrix} m_1 l_{g1}^2 + m_2 l_1^2 + j_1 & m_2 l_1 l_{g2} \cos(\theta_1 - \theta_2) \\ m_2 l_1 l_{g2} \cos(\theta_1 - \theta_2) & m_2 l_{g2}^2 + j_2 \end{bmatrix} \begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} - (m_2 l_1 l_{g2}\, g \sin(\theta_1 - \theta_2)) \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} (m_1 l_{g1} + m_2 l_1)\, g \cos\theta_1 \\ m_2 l_{g2} g \cos\theta_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \qquad (5)$$

And the general form is:

$$H(\ddot q) + C(\dot q, q) + g(q) = M$$

IV. PID Controller based on Linear Model

We define new variables in order to convert the 2-R robot to an equivalent linear model.

$$x_1 = \theta_1 x_2 = \theta_2 x_3 = \dot\theta_1 x_4 = \dot\theta_2$$

$$\dot x_1 = \dot\theta_1 = x_3 \dot x_2 = \dot\theta_2 = x_4 \dot x_3 = \ddot\theta_1 \dot x_4 = \ddot\theta_2$$

Rewrite the equation of motion using these variables, and use new constants c_1 to c_6 function of robot specifications to make equations in simple form

$$\dot x_4 = \frac{M_2}{c_5} - \frac{c_2 M_2}{c_5} \cos(x_1 - x_2) + \frac{c_3}{c_5} \sin(x_1 - x_2)\, x_4 - \frac{c_6}{c_5} \cos x_2 \qquad (6)$$

$$\left[c_1 - \frac{M_2}{c_5} \cos^2(x_1 - x_2) \right] \dot x_3 = M_1 - \frac{c_2 M_2}{c_5} \cos(x_1 - x_2) - \frac{c_2 c_3}{c_5} \cos(x_1 - x_2) \sin(x_1 - x_2)\, x_4 + \frac{c_2 c_6}{c_5} \cos(x_1 - x_2) \cos x_3 - c_4 \cos x_1 \qquad (7)$$

$$\dot x_1 = x_3 \qquad (8)$$

$$\dot x_2 = x_4 \qquad (9)$$

Now we can write the state-space model using linearization about the equilibrium point:

$$\theta_1 = -\frac{\pi}{2} \quad \dot{\theta}_1 = 0 \theta_2 = -\frac{\pi}{2} \dot{\theta}_2 = 0 \quad M1 = 0$$
$$M2 = 0$$

We Perform Taylor series expansion of the nonlinear functions and neglect high-order terms, to get the linearized model. At equilibrium point:

Linearization of the variable x_1 with respect to other variables:

$$\frac{\partial \dot{x}_1}{\partial x_1} = 0 \quad \frac{\partial \dot{x}_1}{\partial x_2} = 0 \quad \frac{\partial \dot{x}_1}{\partial x_3} = 1 \quad \frac{\partial \dot{x}_1}{\partial x_4} = 0$$

Linearization of the variable x_1 with respect to other variables:

$$\frac{\partial \dot{x}_2}{\partial x_1} = 0 \quad \frac{\partial \dot{x}_2}{\partial x_2} = 0 \quad \frac{\partial \dot{x}_2}{\partial x_3} = 0 \quad \frac{\partial \dot{x}_2}{\partial x_4} = 1$$

Linearization of the variable x_1 with respect to other variables:

$$\frac{\partial \dot{x}_3}{\partial x_1} = \frac{c_4 c_5}{c_1 c_5 - M_2} \quad \frac{\partial \dot{x}_3}{\partial x_2} = \frac{c_2 c_6}{c_1 c_5 - M_2} \quad \frac{\partial \dot{x}_3}{\partial x_3} = 0 \quad \frac{\partial \dot{x}_3}{\partial x_4} = 0$$

Linearization of the variable x_1 with respect to other variables:

$$\frac{\partial \dot{x}_4}{\partial x_2} = \frac{-c_6}{c_5} \frac{\partial \dot{x}_4}{\partial x_3} = 0$$

$$\frac{\partial \dot{x}_4}{\partial x_4} = \frac{c_3}{c_5} \sin(x_1 - x_2) \frac{\partial \dot{x}_4}{\partial x_4} = 0$$

Linearization of the variable x_1 and x_2 with respect to input torques:

$$\frac{\partial \dot{x}_1}{\partial M_1} = 0 \quad \frac{\partial \dot{x}_1}{\partial M_2} = 0 \quad \frac{\partial \dot{x}_2}{\partial M_1} = 0 \quad \frac{\partial \dot{x}_2}{\partial M_2} = 0$$

$$\frac{\partial \dot{x}_3}{\partial M_1} = \frac{c_5}{c_1 c_5 - M_2} \frac{\partial \dot{x}_3}{\partial M_2} = \frac{-c_2}{c_1 c_5 - M_2} \frac{\partial \dot{x}_4}{\partial M_1}$$

$$= 0 \quad \frac{\partial \dot{x}_4}{\partial M_2} \quad = \frac{1 - c_2}{c_5}$$

We can write the state-space model:

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \\ \Delta \dot{x}_3 \\ \Delta \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 0 & 01 \\ \frac{c_4 c_5}{c_1 c_5 - M_2} & \frac{c_2 c_6}{c_1 c_5 - M_2} & 00 \\ 0 & \frac{-c_6}{c_5} & 00 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{c_5}{c_1 c_5 - M_2} & \frac{-c_2}{c_1 c_5 - M_2} \\ 0 & \frac{1 - c_2}{c_5} \end{bmatrix} \begin{bmatrix} \Delta M_1 \\ \Delta M_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} + [0][D]$$

a) Linearized model

We substitute values of constants c_1 to c_6 into the state-space model to get the state space matrices:

$$A = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 0 & 01 \\ -0.4568 & -0.619600 & \\ 0.2485 & -6.617400 & \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.7870 & -0.0426 \\ 0.0426 & 0.1349 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 1 & 00 & 0 \\ 0 & 10 & 0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b) Controller for Linear model

Applying state space matrices on Matlab Simulink, we formulate a model which can be controlled easily using the block (PID) in Simulink library. Figure (3) shows the linear model in Simulink. If we run the model we get the results shown in figure (4) and figure (5) for the angle θ_1 and θ_2 respectively. The input is step function with angle 45° for both links.

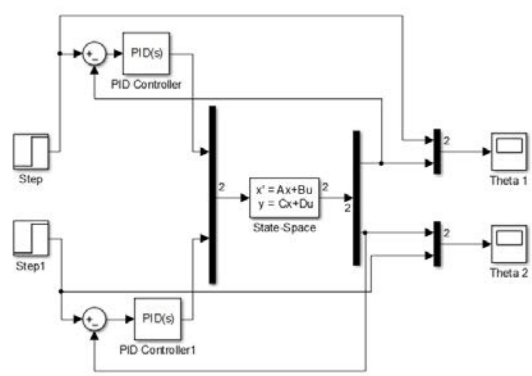

Fig. 3 : Simulink diagram for linearized model

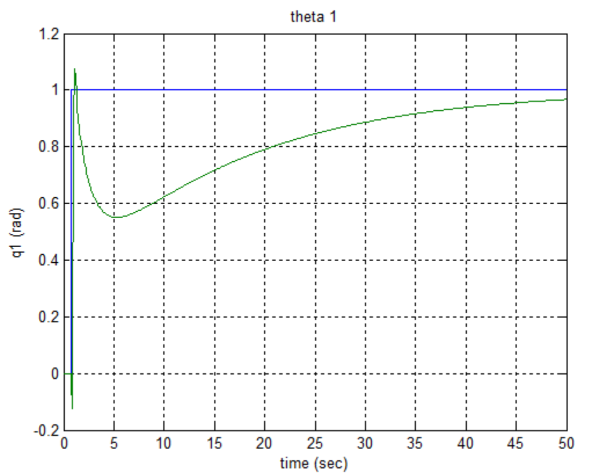

Fig. 4 : Dynamic response of link_1 (Linearized model)

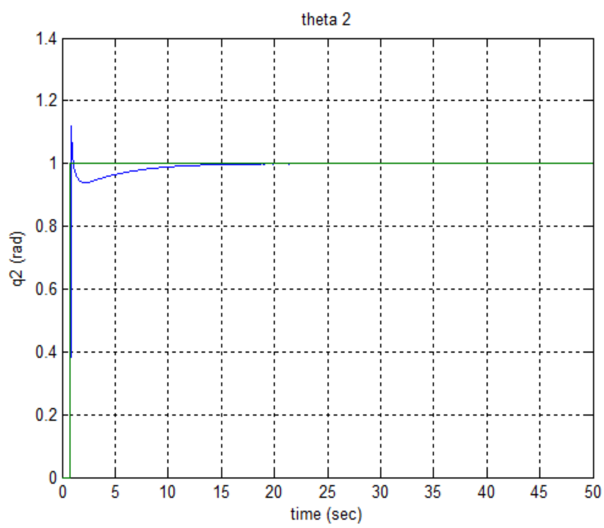

Fig. 5 : Dynamic response of link_2 (Linearized model)

It is clearly seen that the first link which have much inertia takes longer time to follow the desired trajectory. And both links have a delay in response.

V. Pid Controller based on Autodesk Inventor Model

A 2-R robot system is designed and developed using Autodesk Inventor program and MATLAB/Simulink simultaneously as shown in Figure (6) and Figure (7).Robot specifications is taken into account while modeling. After that we transform the designed model into Simulink environment and automatically block diagram has been developed for the robot.

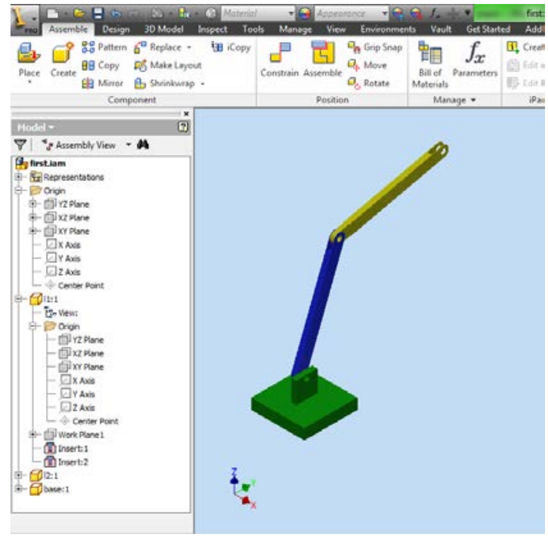

Fig. 6 : 2-R Robot in Autodesk Inventor

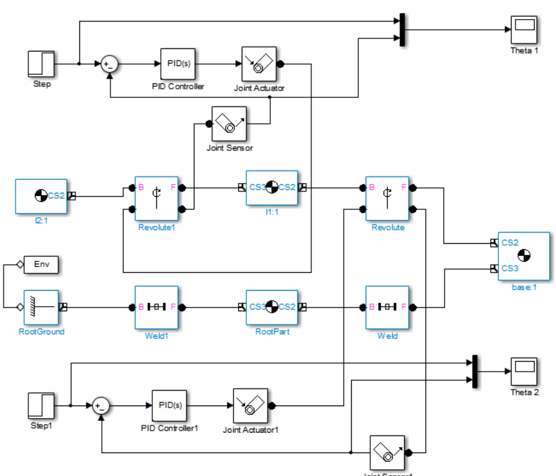

Fig. 7 : Simulink diagram for Autodesk Inventor model

a) Controller for Autodesk Inventor model

Applying a PID controller using the block (PID) in Simulink library we can control our system as shown in Figure (7) below. The results show much better response than the linearized model used in pervious part.

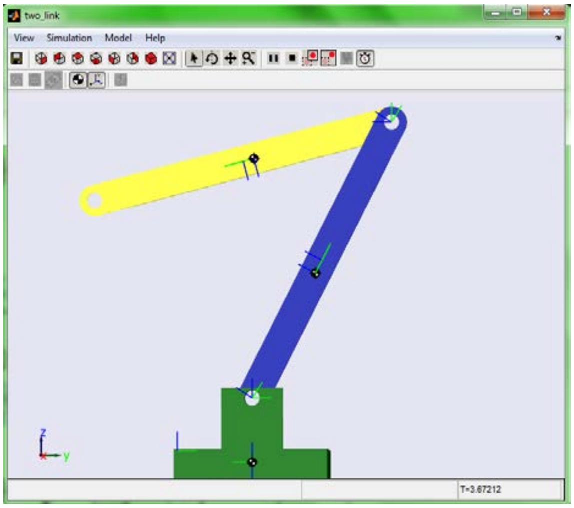

Fig. 8 : 2-R Robot simulation in Simulink

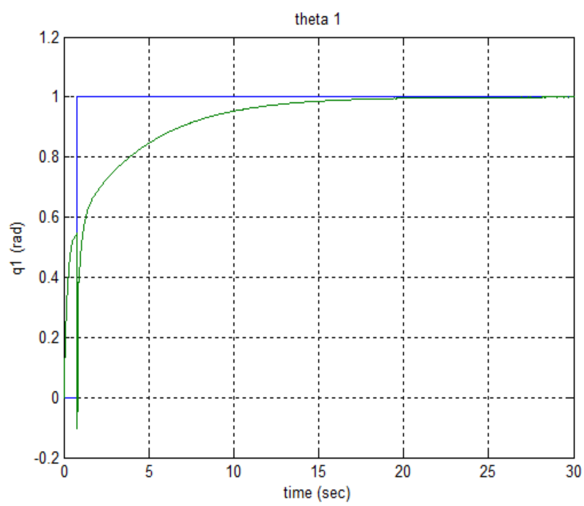

Fig. 9 : Dynamic response of link_1 (Autodesk Inventor)

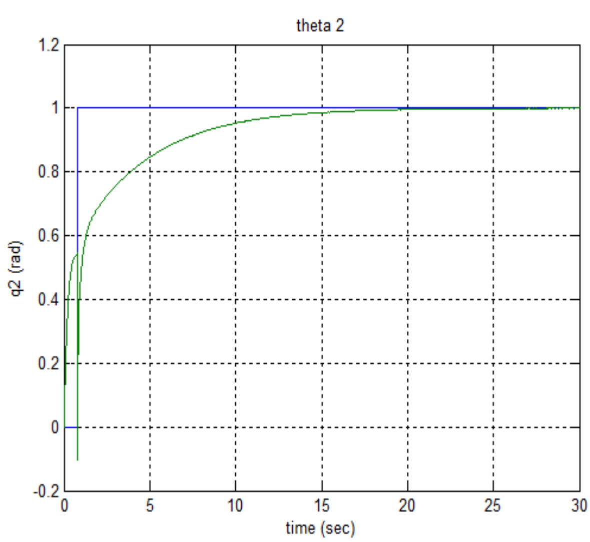

Fig. 10 : Dynamic response of link_2 (Autodesk Inventor)

VI. PID Controller based on Feedback Linearization

Having system's equation

$$H(\ddot{q}) + C(\dot{q}, q) + g(q) = M$$

$$\ddot{q} = H^{-1}[-C(\dot{q}, q) - g(q)] + \bar{M}$$

While: $\bar{M} = H^{-1}$ MAnd, $M = H^{-1}\bar{M}$

This way, we decoupled the system to have the (non-physical) torque input:

$$\bar{M} = H^{-1}\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

However, the physical torque inputs to the system are:

$$M = H\begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \end{bmatrix}$$

To design the feedback PID controller, error signals are assumed to be:

$$e\theta_1 = \theta_{1f} - \theta_1 e\theta_2 = \theta_{2f} - \theta_2$$

Assuming the final position desired is $\theta_{1f} = \frac{pi}{4}$, $\theta_{2f} = \frac{pi}{4}$ And the initial condition for the system is$\theta_{10} = 0$ $\theta_{20} = 0$ General structure of PID controller for any input would be:

$$M = K_P e + K_D \dot{e} + K_I \int e\, dt$$

$$= K_{P1}(\theta_{1f} - \theta_1) + K_{D1}\dot{\theta}_1 + K_{I1}\int(\theta_{1f} - \theta_1)\, dt \quad (10)$$

$$M_2 = K_{P2}(\theta_{2f} - \theta_2) + K_{D2}\dot{\theta}_2 + K_{I2}\int(\theta_{2f} - \theta_2)\, dt \quad (11)$$

Applying equations (10) and (11) we got results shown below in figures (11) to (14).

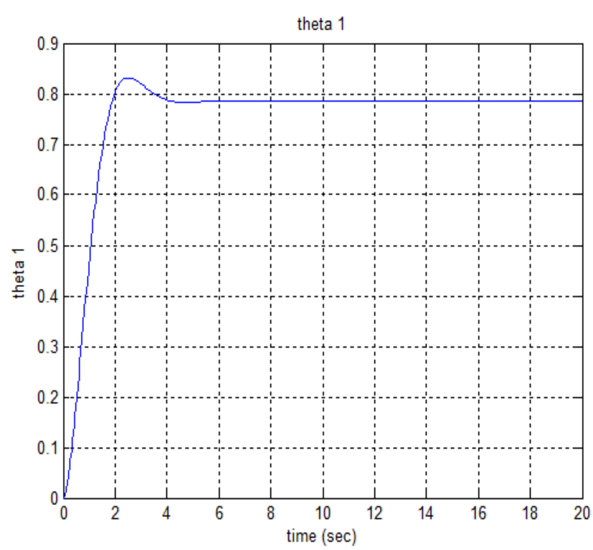

Fig. 11 : Dynamic response of link_1 (Nonlinear model)

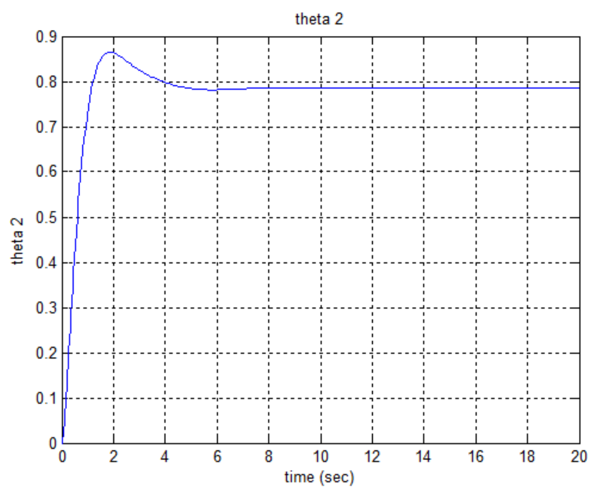

Fig. 12 : Dynamic response of link_2 (Nonlinear model)

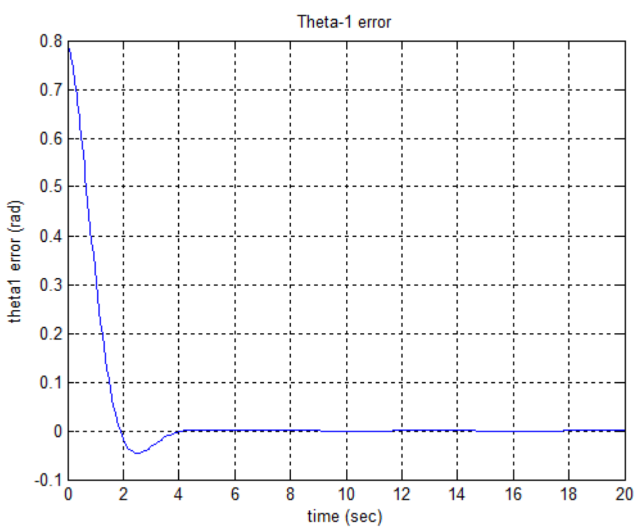

Fig. 13 : Error of theta_1 (Nonlinear model)

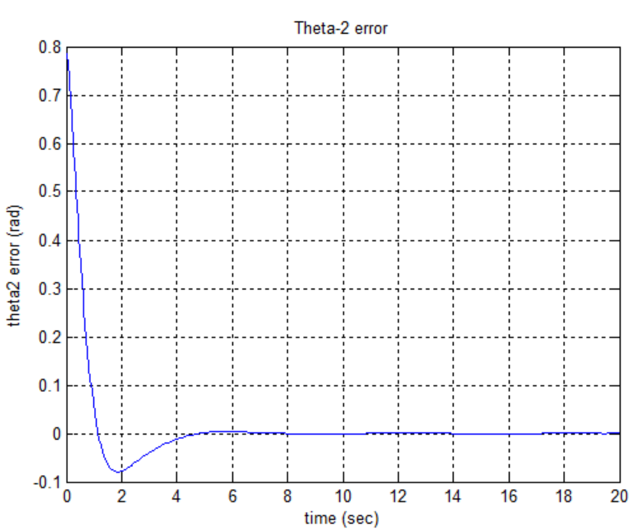

Fig. 14 : Error of theta_2 (Nonlinear model)

We notice that the response is following the control signal with relatively good manner. And errors of θ_1 and θ_2 are equal to zero in a short time.

VII. Conclusion

The main content of this paper is about modeling a 2-R robotusing two methods: first is mathematical modeling using Lagrange dynamic equations and the second is using Autodesk Inventor and Simulink software's to develop the model. After that we used PID controller to validate the models and to notice the difference in accuracy achieved by each technique. Linearization about working point is valid in one point only, while it is no longer valid for other points. The model designed from Autodesk Inventor and Simulink software's is giving better and reasonable response. Good results are found when using feedback linearization.

References Références Referencias

1. Gouasmi, M., Ouali, M., Fernini, B., & Meghatria, M. (2012). Kinematic Modelling and Simulation of a 2-R Robot Using SolidWorks and Verification by MATLAB/Simulink. International Journal of Advanced Robotic Systems, 1. doi:10.5772/50203
2. N.Jazar, R. (2010). Theory of Applied Robotics.
3. Sukvichai, K. (2008). The Application Of Nonlinear Model Reference PID Controller For a Planar Robot Arm, 637–640.
4. Žlajpah, L. (2008). Simulation in robotics. Mathematics and Computers in Simulation, 79(4), 879–897. doi:10.1016/j.matcom.2008.02.017