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MODELLING OF FORGETTING CURVES IN EDUCATIONAL E-ENVIRONMENT

Abstract. Modelling of the didactical process by using educational network needs network representation of learning and forgetting curves known from the literature. The learning and forgetting curves are the solution of differential equations. The differential equations can be represented in the form of a network of connected elements in a similar way to the electrical circuits and represented in the form of an intuitive schematic. The network can be simulated using a microsystems simulator. Such an approach enables the easy creation of the macro models and their analysis. It enables the use of many advanced simulation algorithms. The use of analogy enables defining the educational environment by defining network variables and giving them meaning relative to generalized variables. In the paper, examples of representation of forgetting curves as the above-mentioned network are presented. Parameters of elements were selected in the deterministic optimisation process. The obtained results were compared with the forgetting curves known from the literature. The appropriate time constants were identified in the systems and their values were given. Time constants have their equivalents in the appropriate values in the formulas describing the forgetting curves. Based on the results, appropriate conclusions were drawn. The work also shows the concept of a model that uses behavioural modelling and variable parameters of elements depending on the state and time. The model has been used in practice. The presented approach enables a much more accurate determination of the parameters of the forgetting curves. The models can be used in the simulation of the forgetting process. The paper can be interesting for those who deal with modelling of the didactical process, especially on the e-learning platforms.

Keywords: modelling of forgetting curves; educational analogy; simulation of forgetting curves; education as microsystem.

1. INTRODUCTION

In the past work, the process of forgetting was described by so-called forgetting curves [1].Forgetting curves are appropriate equations, whose form and coefficients allow matchingvalues to measured data obtained during the experiments [2]. The form of theequationsindicates that they are the solutions of differential equations. Differential equations describe a dynamic model of the system, in this case, a model of brain activity in the sphere of learning. The best approach to the problem of modelling is to find a model whichis intuitive. The presented below educational analogy as well as the educational network commit theserequirements. The brain activity at the level of neurons is described as the electrical circuit [3].

The aim of this article is to show how the forgetting curves are modelled as the educationalnetwork and simulated using the microsystem simulator.

The use of microsystems simulators gives access to advanced simulation and optimisation techniques and behavioural modelling languages [4, 5], such as EMDL [6, 7], MDL [8], VHDL-AMS [9], Verilog-AMS [10]. Originally, simulators were developed for simulation of electronic circuits and then ported to use in other areas. This was possible by defining a generalized environment and using analogies to transform equations between different environments (electrical, mechanical). In the research, the Dero [8] simulator was used. In the research, the Dero [8] simulator was used. The Dero is equipped with Model Description Language (MDL)[8].

2. THE THEORETICALBACKGROUNDS

Description of the process of learning and forgetting was first dealt with by Hermann Ebbinghaus [1] in the 19th century. He set out levels of knowledge at 9-time points and found a mathematical function that describes the process and is best suited to the data points. Initially, the learning curve was determined at 7-time points after completion of learning: 20minutes, 1hour, 9hours, 1day, 2days, 6days and 31days (Figure 1 - Ebbinghaus 1880).

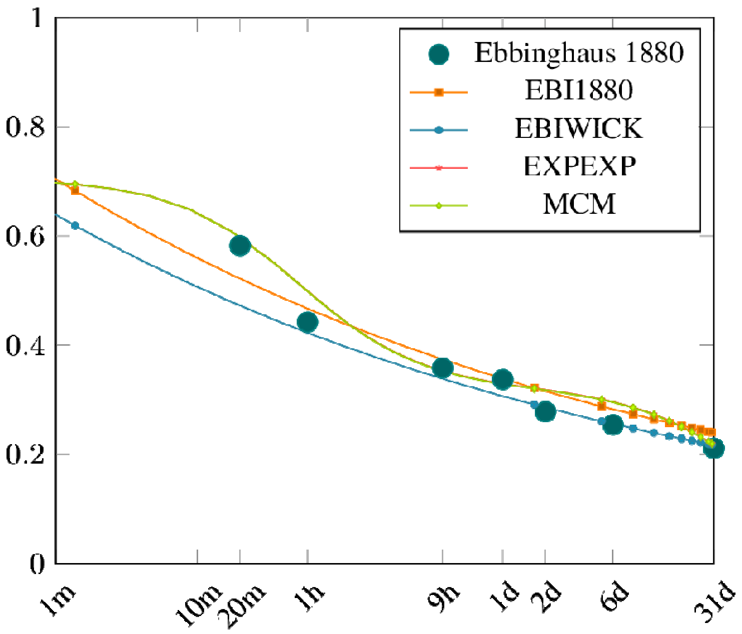


Figure 1: Forgetting curves

The forgetting curve is a description of the reverse process. The Ebbinghaus original forgetting curve was estimated using the power function (1)-(Figure1-EBI1880).

$$k(t) = (1 + \mu t)^{-1/\tau} = (1 + 0.523 \cdot t)^{-1/9.900990099} \quad (1)$$

where: k - knowledge - the amount of memorized information, t - time, μ and τ are parameters. He proposed also the forgetting curve as the logarithmic function (2).

$$k(t) = \frac{1.84}{log_{10}(t)^{1.25} + 1.84} \cdot t)^{-1/9.900990099} \quad (2)$$

where: k-knowledge-the amount of memorized information, a - memorization factor,b - forgetting factor, c - knowledge level asymptote, t - time.

Different functions are used to describe the forgetting curve [2]: power, exponential.Superposition of functions (e.g. superposition of exponential functions [11, 12]) and the Memory Chain Model (MCM) [13] are also used. However, the models are too simple to fit all Ebbinghaus points. The MCM model is the most accurate. It describes an increase in knowledge level around 24hours that was not included in other models.

Both power and exponential functions are used to model forgetting processes. In the world of nature, exponential functions more often describe different phenomena. Nevertheless, the power function is better suited to the Ebbinghaus points. On the basis of literature, it is possible to say that this exponential function better describes the process of forgetting, but its character is lost when the various short-term and long-term effects overlap [12].

Table I:

Generalized variables for the electrical and educationalenvironment

Generalized variables	Electrical environment	Educational environment
e effort	v voltage	k knowledge
f flow	i current	i information flow
p state	q charge	q information

Therefore, superposition of exponential functions is a good solution. The curve of forgetting to this day is examined and used in practice. The models areused to describe the efficiency of repetitive operations on production lines: hyperbolic and exponential models [14, 15], multiparameter and multidimensional models [16, 17, 18]. They are also used in computer programs such as SuperMemo [19] or Anki [20]. In many cases, simplified models based on one exponential function are also used. In 1974Wickelgren [21] proposed a function which, for typical conditions, can be reduced to the power form (3) and well describes the Ebbinghaus curve (Figure 1 - EBIWICK).

$$k = \lambda(1 + \beta t)^{-\frac{1}{\tau}} \stackrel{1}{=} 1 \cdot (1 + 1.4 \cdot t)^{-1/9.900990099} \tag{3}$$

where: k - knowledge - amount of information stored, λ, ψ - parameters, t - time. On the basis of the results of the research [22, 23], it was found that the forgetting curve is better approximated by the superposition of exponential curves (4)-(Figure1-EXPEXP).

$$k(t) = \mu_S \cdot e^{-\frac{t}{\tau_S}} + \mu_L \cdot e^{-\frac{t}{\tau_L}} \tag{4}$$

The Memory Chain Model [22, 23] models the changes of the knowledge as a result of rewriting the content of the short-time memory in the long-time memory (5) - (Figure 1 - MCM).

$$k(t) = \mu_S \cdot e^{-\frac{t}{\tau_S}} + \frac{\mu_S \mu_L}{\frac{1}{\tau_S} - \frac{1}{\tau_L}} \cdot \left(e^{-\frac{t}{\tau_L}} - e^{-\frac{t}{\tau_S}}\right) \tag{5}$$

where [2]: μ_S= 0.704, μ_L= 0.000145, τ_S= 3134.7962382445, τ_L= 5586592.17877095.

As shown in Figure 1, none of the models fit the Ebbinghaus curve. The values of parameters are taken from literature [2]. The curves were calculated using Dero simulator and directfunctions.

3. MODELLING OF THE FORGETTING CURVES

Simulation of the forgetting curves using a microsystems simulator enables the use of advanced algorithms implemented in the simulator. It also allows describing the model of the whole process in the relatively simple way.

Theuseofmicrosystemssimulatorrequiresdefiningnewenvironmentbydefiningvariables and giving them meaning. The predefined generalized environment enables the transformation of the equations between different environments. The educational environment definition is shown in Table I. The basic variables and equations can be formulated as (6).

$$x = [k, i, q]^T \tag{6}$$

where: : k - variables related to knowledge, i- information flows, q - variables describing unit information. The set of variables can be represented as a network. The equations can be represented by the connected elements. The education network may include elements describedbythreebasictypesofequationsdescribingthebasictypesofnetworkbranches:

1) branch describing information flow $\boldsymbol{i}=\boldsymbol{f_i}(x,x^{\cdot *},t)$,
2) branch describing the level of knowledge $k=\boldsymbol{f_k}(x,x^{\cdot *},t)$,

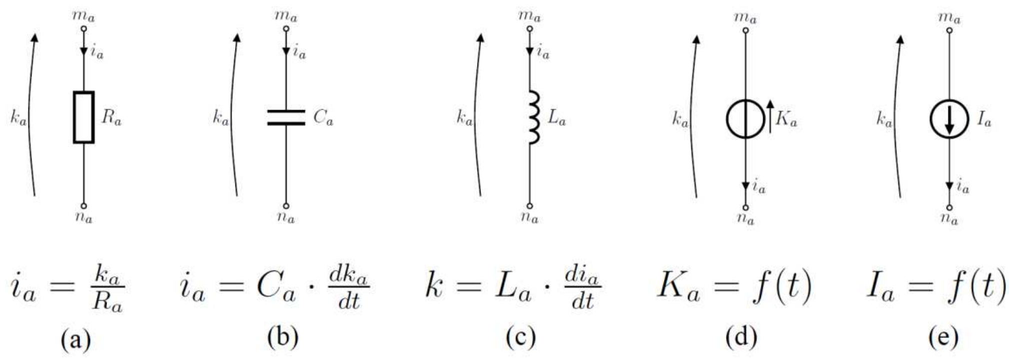


Figure 2: Basic elements of the network

3) branch describing the level of knowledge with the flow of information as unknown $q=\boldsymbol{f_q}(x,x^{\cdot *},t)$.

The basic elements of the network and its equations are shown in Figure 2. The electrical like notation is used to represent the elements of the educational network. Other notation can also be used, but they are not so intuitive and widely known as electrical schematics. The use of Modified Nodal Equations (MNE) [8, 24, 25] enables automatic formulation ofnetwork equations by using stamps. Interpretation of the elements is as follows. Information resistance (Figure 2a) is a measure of the difficulty of information flow in the information channel. The inverse of the information resistance is a measure of the ease of information flow in the information channel. Information capacity (Figure 2b) is an element of information gathering. Information inductance (Figure 2c) is an element that transforms information change into knowledge change. It models the ability to self-learning. It also allows for modelling the mutual influence of information on yourself. Knowledge and information sources are shown in Figure 2d and2e.

4. RESULTS

The educational network was used to describe the process of forgetting. In the study,models of repetition were omitted. Repetition modifies the parameters of the models due to the phenomenon of fixation of memory traces [21]. The networks discussed below were simulated using the Dero [8] simulator. The Levenberg-Marquardt algorithm [8] was used to determine the values of the parameters of theelement.

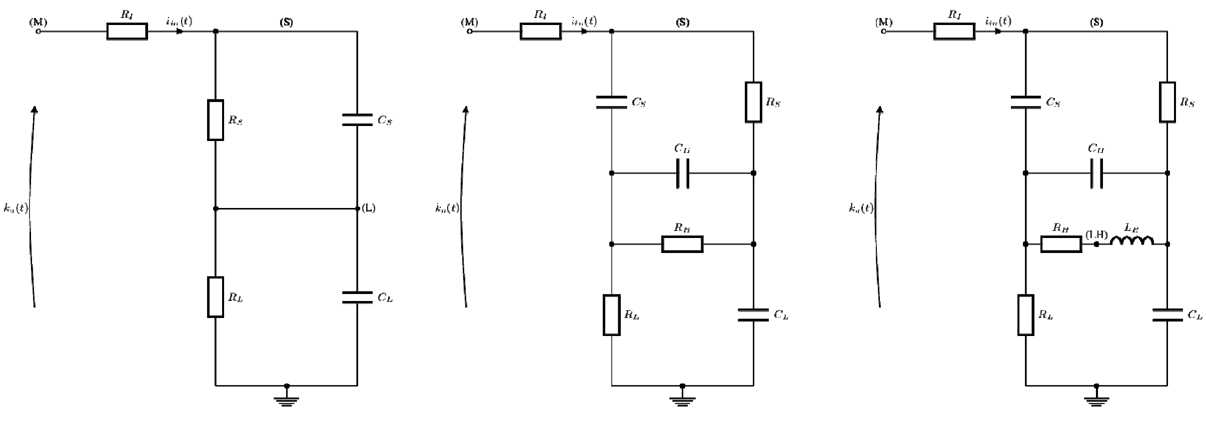
4.1 Simulation of forgetting curves by using direct formulas

The forgetting curves shown in Figure 1 were simulated using direct functions. The MDL model was defined for the curves (Listing 1).

Listing 1: Forgetting curves as a MDL model

```
.MODEL STUDENT_EQ
.EXTERNAL IN, GND;
.INTERNAL EBI1880,EBIWICK,EXPEXP,MCM;

.INPUT KNOWLEDGE(IN,GND):REAL KIN;

.OUTPUT EDUI(GND,EBI1880):REAL EBI1880;
.OUTPUT EDUR(EBI1880,GND):REAL REBI1880;
.OUTPUT EDUI(GND,EBIWICK):REAL EBIWICK;
```

```
.OUTPUT EDUR(EBIWICK,GND):REAL REBIWICK;
.OUTPUT EDUI(GND,EXPEXP):REAL EXPEXP;
.OUTPUT EDUR(EXPEXP,GND):REAL REXPEXP;
.OUTPUT EDUI(GND,MCM):REAL MCM;
.OUTPUT EDUR(MCM,GND):REAL RMCM;

.PARAM REAL RL = 1;

.PARAM REAL miS = 0.704;
.PARAM REAL tauS = 3134.7962382445;
.PARAM REAL miL = 0.000145;
.PARAM REAL tauL = 5586592.17877095;

.PARAM REAL EXPmiS = 0.383; # 0.704;
.PARAM REAL EXPtauS = 3134.7962382445;
.PARAM REAL EXPmiL = 0.321; # 0.000145;
.PARAM REAL EXPtauL = 5586592.17877095;

.PARAM REAL MCMmiS = 0.704;
.PARAM REAL MCMtauS = 3134.7962382445;
.PARAM REAL MCMmiL = 0.000145;
.PARAM REAL MCMtauL = 5586592.17877095;

.PARAM REAL miEbi1880 = 0.523;
.PARAM REAL tauEbi1880 = 9.900990099; # 1/0.101;
.MEM REAL LAST_LEARN_TIME=0;

.PARAM REAL LAMBDA = 1;
.PARAM REAL BETA = 1.4;
.PARAM REAL TAU1 = 0.101; # =1/TAU

.BEGIN

EBI1880 = 0;
IF( TIME > 0 ){ EBI1880 = POW((1+miEbi1880*TIME), -1/tauEbi1880); }
REBI1880 = RL;

EBIWICK = LAMBDA*POW((1+BETA*TIME),(-TAU1));
REBIWICK = RL;

EXPEXP = (EXPmiS)*EXPO((-TIME/EXPtauS)) + (EXPmiL) * EXPO(-TIME/EXPtauL);
REXPEXP = RL;

MCM = MCMmiS*EXPO((-TIME/MCMtauS)) + (MCMmiS*MCMmiL)*(EXPO(-TIME/MCMtauL) -
EXPO(-TIME/MCMtauS))/(1/MCMtauS-1/MCMtauL);
RMCM=RL;

.END
```



(a) EXPEXP *(b) MCM1* *(c) MCM4*

Figure 3: Network models of the forgetting functions

Function parameters are defined in lines 18..39. The default values are set. Model outputs are declared in lines 7..14. The forgetting curves assign their values in lines 47, 50, 53. The outputs are defined as the information source (EDUI type). The information is converted into knowledge by using EDUR type outputs. Conversion is not needed if the EDUK type output is used. In that case EDUR outputs are notrequired.

4.2 Network representation of the forgettingcurves

In this section, the two main groups of forgetting curves models are taken into account: superposition of exponent functions (4) and MCM models (5). They can be modelled as networks shown in Figure 3. In the examples, the model parameters were calculated in the optimisation process to fit Ebbinghaus curves (marked with dots) in five points (20minutes, 1hour, 2days, 6days, 31days).Time constants were determined in the optimisation process with an accuracy of 0.01. Schematic of the network which models the forgetting curve as the sum of the exponential functions (4) (EXPEXP) is shown in Figure 3a. Time constants τ_S and τ_L were determined in the optimisation process. The model calculated the elements R and C based on the τ constants. The initial values were determined by the values of the R_I, $R_S$and R_L. The model parameters values obtained in the optimisation process are as follows: R_I= 2.985064*e*-01, R_S= 3.650454*e*-01, R_L= 3.188979*e*- 01,τ_S= R_SC_S= 48.01*minutes*, τ_L= R_LC_L= 65.01*days*.

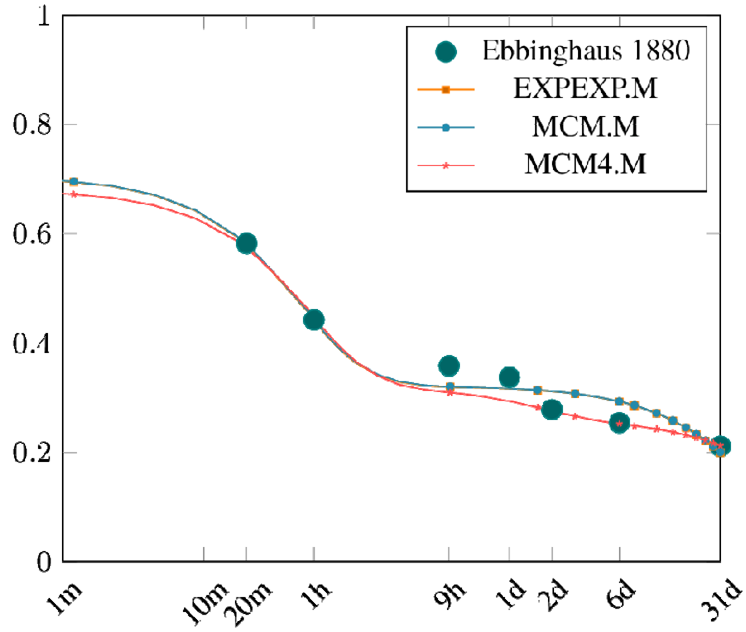


Figure 4: The results of the simulation of the circuits from Figure 3

The C parameters were calculated from τ and R. Figure 3b shows the schematic describing the MCM1 model. The initial values were determined by the values of the R_I, $R_S$and R_L.The model parameters values obtained in the optimisation process are as follows: R_I= 2.983274*e*-01, R_S= 3.748792*e*-01, R_L= 2.254302*e*-01, R_H = 9.061777*e*-02,τ_S= 48.86*minutes*, τ_L= 46.49*days*,τ_H= 16.16*hours*. As can be seen the circuit is equivalent to the previous model. Both models match exactly 3 points.

Schematic describing the MCM4 model is shows in Figure 3c. The initial values were determined by the values of the R_I, $R_S$and R_L. The model contains additional elements R_H, C_H, L_H that model changes of the knowledge in the first 24hours after learning. The model parameters values obtained in the optimisation process are as follows: R_I= 2.999696e-01, R_S = 3.823910e-01, R_L= 3.873290e-01, R_H= 8.707899e-02, R_{HH}= 3.237874e-02, τ_S = 57.61min, τ_L=121.17$days$, τ_H=1.89$days$, τ_{HH}=3.08$hours$. The simulation results are shown in Figure 4. The model fits five of seven Ebbinghaus points. However, the model cannot fit all points because it is too simple.

4.3 Simulation files

The above-mentioned simulations were done with the Dero simulator. The input file is shown in Listing 2. The task begins in line 1 and ends in line 42. Library MDL models are loaded in lines 2..11. Local MDL models are loaded in lines 13..16.

Listing 2: Simulation of forgetting curves

```
1  .TASK "Simulation of forgetting curves"
2  .LIB "types.mdl";                # Data types
3  .LIB "units.mdl";                # Units (minutes, hours, etc)
4  .LIB "math/one.mdl";             # Math function one
5  .LIB "math/pulse.mdl";           # Math function pulse
6  .LIB "edu/vars.mdl";             # Net variables types
7  .LIB "edu/bus.mdl";              # Bus variables
8  .LIB "edu/inputs.mdl";           # Inputs
9  .LIB "edu/outputs.mdl";          # Outputs
10 .LIB "edu/exam.mdl";             # Exam model
11 .LIB "edu/exercise.mdl";         # Exercise model
12
13 .INC "student-eq.mdl";           # Direct equations
14 .INC "student-ebi-data.mdl";     # Ebbinghaus points
15 .INC "student-mcm1.mdl";         # MCM1
16 .INC "student-mcm4.mdl";         # MCM4
17
18 .MODEL "eq" "STUDENT_EQ";
19 .MODEL "student-data" "STUDENT_DATA";
20 .MODEL "student-mcm1" "STUDENT_MCM1";
21 .MODEL "student-mcm4" "STUDENT_MCM4";
22
23 "ebi-data"."data"        DT 0;
24 "eq"."EQ"                EQ 0;
25 "student-mcm1"."MCM1"    MCM1 0;
26 "student-mcm4"."MCM4"    MCM4 0;
27
28 .CMD
29 .OPTI ECHO=1,ALLP=1,HMIN=2;
30 .OPTI EOPT=0.01;
31 .OP TITLE="INITIAL CONDITIONS";
32
33 .PROBE ADD TR("*");
34 .FILE LOG="sym-mod-dero.dc.optim.log";
35 .INC IFEXIST "optim-rc-rc.cmd";
36 .INC IFEXIST "optim-mcm1.cmd";
37 .INC IFEXIST "optim-mcm4.cmd";
38
39 .OPTI PROBE_FMT=CSV; #  CSV FORMAT
40 .FILE OUT="sym-mod-dero.out.optim.csv";
41 .TR STEP=02:00:00 T0=0 TMIN=0 TMAX=32days TFMT=REAL;
42 .END
```

Model lines are defined in lines 18...21. They store common model parameters. Elements describing mentioned above models are declared in lines 23...26. They call their models via the model line. Elements are connected to the net by its nodes (variables), e.g. DT, EQ, MCM1, MCM4, 0 (reference node). The command section begins in line 28. The options are set in lines 29...30. Command files containing specifications, optimisation variables as well as optimisation and analysis commands are in lines 35...37 individually for each model. Finally, the transient analysis is performed (line 42) for parameters calculated during optimisation process. Example of the optimisation command file is shown in Listing3.

Listing 3: Command file for optimisation of MCM4 model

```
1  # specifications
2  .SP  ADD SID="MCM4M"  X_TYPE=TR TYPE=ABS X=20min    S1=0.582 S2=0.582 W1=1 W2=1;
3  .SP  ADD SID="MCM4M"  X_TYPE=TR TYPE=ABS X=1hour    S1=0.442 S2=0.442 W1=1 W2=1;
4  .SP  ADD SID="MCM4M"  X_TYPE=TR TYPE=ABS X=2days    S1=0.278 S2=0.278 W1=1 W2=1;
5  .SP  ADD SID="MCM4M"  X_TYPE=TR TYPE=ABS X=6days    S1=0.254 S2=0.254 W1=1 W2=1;
6  .SP  ADD SID="MCM4M"  X_TYPE=TR TYPE=ABS X=31days   S1=0.211 S2=0.211 W1=1 W2=1;
7  # optimisation variables
8  .VAR ADD "MCM4".RI0     MIN=0.25     MAX=0.3      SCALE=0.1;
9  .VAR ADD "MCM4".RS0     MIN=0.3      MAX=0.5      SCALE=0.1;
10 .VAR ADD "MCM4".RL0     MIN=0.2      MAX=0.5      SCALE=0.1;
11 .VAR ADD "MCM4".RH0     MIN=0.01     MAX=0.5      SCALE=0.1;
12 .VAR ADD "MCM4".RHH0    MIN=1e-2     MAX=0.1      SCALE=0.1;
13 .VAR ADD "MCM4".tauS    MIN=40min    MAX=600min   SCALE=100min;
14 .VAR ADD "MCM4".tauL    MIN=31days   MAX=150days  SCALE=100days;
15 .VAR ADD "MCM4".tauH    MIN=9hours   MAX=15days   SCALE=10days;
16 .VAR ADD "MCM4".tauHH   MIN=30min    MAX=6days    SCALE=1day;
17
18 .CENT T0=0 TITLE="MCM4 optimisation";
19
20 .VAR PRI VAR;      # variables printout
```

Design specifications (constraints) are defined in lines 2..6. The optimisation variables are defined in lines 8..16. Optimisation is performed in line 18. Optimised values of the variables are printed in line 21.

4.4 Piecewise linear models

Shown above models are relatively complicated. It is possible to create simplifiedpiecewise linear model shown in Figure 5. The model enables flexible behavioural modelling by changing the *R*(*t*) and *C*(*t*) in time. It enables to change the speed of forgetting in time and simulate memory behavior. Such an approach was used by the author in practice to model the didactical process over time on the e-learning platform Quela [26].

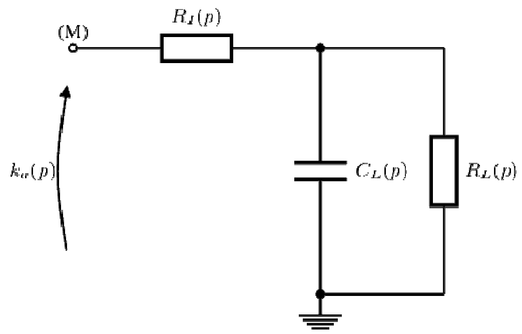


Figure 5: Piecewise linear model

5. DISCUSSION

The presented networks are composed of linear elements. The network can also include nonlinear elements and time-dependent elements (not discussed in this article). The values of the model's parameters have been selected in the optimisation process of the system. Design constraints were introduced into the first two and the last three points of the Ebbinghaus curve. The MCM4 model matched very well with the selected points. However the MCM4 makes it impossible to fit all Ebbinghaus curve points.

6. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

As shown in the article the forgetting curves can be described by the differential equations represented as the network of connected elements. The developed analogy makes it easy to create networks in a way similar to creating electrical schemas. The representation of forgetting curves in the form of schematics is intuitive. The schematics can be modified and simulated by using the microsystems simulator.

The educational analogy enables defining network variables and element models.Themodel's parameters can be easily adjusted to the measurement data in the optimisation process.This approach allows the use of microsystems simulator and gives access to advancedsimulation and optimisation methods implemented in simulators. Behavioural modellingallows creation of models based on mathematical functions, including nonlinear ones. Modeldescription language plays here an important role. Simulator and simulation techniques arepowerful tools. The networks and models described above can be used in many areas, e.g.:in simulation of the learning process on e-learning platforms, in medical research,in psychology.The problem of mapping schemas to the appropriate structures responsible for the learningprocess goes beyond the scope of the article and is not discussed. The issues presented in thearticle (in an extended form) are used in practice by the author to monitor (design, simulate, optimise) didactical processes on the Quela platform.

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МОДЕЛЮВАННЯ КРИВИХ ЗАБУВАННЯ У НАВЧАЛЬНОМУ Е-СЕРЕДОВИЩІ

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Анотація. Моделювання дидактичного процесу з використанням освітньої мережі потребує мережного представлення кривих навчання та забування, відомих з літератури. Криві навчання та забування – це розв’язок диференціальних рівнянь. Диференціальні рівняння можуть бути представлені у вигляді мережі пов'язаних елементів аналогічно електричним ланцюгам та у вигляді інтуїтивної схеми. Мережа може бути модифікована за допомогою симулятора мікросистем. Такий підхід дозволяє легко створювати макромоделі та здійснювати їхній аналіз. Це дозволяє використовувати безліч ефективних алгоритмів моделювання. Використання аналогії дозволяє визначити навчальний зміст, визначаючи мережні змінні, надаючи їм значення відносно загальних змінних. У статті наведені приклади представлення кривих забування як вищезгаданої мережі.

Параметри елементів були обрані в процесі детермінованої оптимізації. Отримані результати зіставлені з відомими з літератури кривими забування. Відповідні постійні часу були визначені в системах і надано їх значення. Константи часу мають свої еквіваленти у відповідних значеннях у формулах, що описують криві забування. На підставі отриманих результатів були зроблені відповідні висновки. У роботі також показана концепція моделі, яка використовує поведінкове моделювання та змінні параметри елементів у залежності від стану і часу. Модель була використана на практиці. Представлений підхід дозволяє набагато точніше визначити параметри кривих забування. Моделі можуть бути використані

при моделюванні процесу забування.
Стаття може бути цікавою для тих, хто займається моделюванням дидактичного процесу, особливо на платформах електронного навчання.

Ключові слова: моніторинг; дидактичний процес; моделювання; оцінка; якість; об’єктивність; демократизація; ІКТ.

МОДЕЛИРОВАНИЕ КРИВЫХ ЗАБЫВАНИЯ В ЭЛЕКТРОННОЙ ОБУЧАЮЩЕЙ СРЕДЕ

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Аннотация. Моделирование дидактического процесса с использованием образовательной сети требует сетевого представления кривых обучения и забывания, известных из литературы. Кривые обучения и забывания – это решение дифференциальных уравнений. Дифференциальные уравнения могут быть представлены в виде сети связанных элементов аналогично электрическим цепям и в виде интуитивной схемы. Сеть может быть модифицирована с помощью симулятора микросистем. Такой подход позволяет легко создавать макромодели и осуществлять их анализ. Это позволяет использовать множество эффективных алгоритмов моделирования. Использование аналогии позволяет определить учебное содержание, определяя сетевые переменные, предоставляя им значения относительно общих переменных. В статье представлены примеры представления кривых забывания как вышеупомянутой сети.
Параметры элементов были выбраны в процессе детерминированной оптимизации. Полученные результаты сопоставлены с известными из литературы кривыми забывания. Соответствующие постоянные времени были определены в системах и предоставлены их значения. Константы времени имеют свои эквиваленты в соответствующих значениях в формулах, описывающих кривые забывания. На основании полученных результатов были сделаны соответствующие выводы. В работе также показана концепция модели, которая использует поведенческое моделирование и переменные параметры элементов в зависимости от состояния и времени. Модель была использована на практике. Представленный подход позволяет гораздо точнее определить параметры кривых забывания. Модели могут быть использованы при моделировании процесса забывания.
Статья может быть интересна для тех, кто занимается моделированием дидактического процесса, особенно на платформах электронного обучения.

Ключевые слова: мониторинг; дидактический процесс; моделирование; оценка; качество; объективность; демократизация; ИКТ.