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Modelling of Railway Vehicle-track-underground Dynamic Interaction Induced by High-speed Trains

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MODELLING OF RAILWAY VEHICLE-TRACK-UNDERGROUND DYNAMIC INTERACTION INDUCED BY HIGH-SPEED TRAINS

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SUMMARY

Mathematical models to simulate the vertical dynamic interaction of railway vehicle-track-underground system are presented. The emphasis is laid on the dynamic response of track structures due to high-speed trains and the effects of subsoil and track irregularities. An existing computer program named DIFF is enhanced by taking account of the underground subsystem so that dynamic analysis of the whole vehicle-track-ground system can be carried out. The numerical procedures adopted are based on the semi-discrete finite element method, by which the spatial domain is discretized using finite elements and then a solution technique combining modal superposition with time integration for the first-order system of state-space equations is applied in the time domain. Simulations of an X-2000 bogie running on a common Swedish track rested on normal as well as soft subsoil are carried out. It is shown that considering vehicle, track and un-

derground as an integrated system is important in investigating the high-speed train induced track-ground vibration.

Keywords: railway dynamics, vehicle-track-ground system, finite element, high-speed trains, train-related vibration

1 INTRODUCTION

One of the primary environmental impacts of railway transport comes from noise and vibration [1,2]. Having its origin at the irregular wheel-rail contact, the train-related vibration is transmitted through and damped by the railway track and ground but can also be magnified where it resonates with the natural frequency of a structure or rises through a flexibly framed building. Especially, when high-speed trains pass over areas with a low railway embankment founded on soft soil ground, where surface wave velocities may be close to or even lower than the designed train speed, high-level track and ground vibrations can become an urgent problem. This phenomenon has been observed on actual lines in several countries and is considered as similar to the supersonic boom occurring when an airplane reaches the speed of sound. Among a rich literature within this field we refer to References [3-11], which are closely related to the topic of high-speed train induced ground vibrations.

The analysis of the generation and propagation of ground vibration from trains is of interest not only to geotechnical engineers but also to track engineers. The increased train speeds are likely accompanied by increased dynamic vertical wheel-rail forces, which in turn may lead to increased track irregularities and accelerated track deterioration. Hence, it is of great importance to understand the dynamics of the railway vehicle-track-ground system and, thereby, find solutions to the increasing problems with track damage like wheel unroundness, rail corrugation, degradation and stability of ballast and embankment. In order to consider the dynamic excitation mechanism arising from sleeper-passing, track irregularities and non-linear wheel-rail contact etc, it requires more advanced train and track models than those of just a moving constant load and a beam as frequently used in the literature on train-related ground vibrations. Researches on vehicle-track dynamic interaction have been very extensive during the past decade. Several mathematical models of varying complexity have been developed to simulate vehicle-track dynamics. Reviews of the state-of-the-art and a rich literature on these issues can be found among others in [11-13].

In co-operation with the Swedish National Rail Administration (Banverket), several research projects related to the topic of vehicle-track dynamic interaction have been carried out at the competence centre CHARMEC, Chalmers University of Technology. Mathematical models to simulate the vertical vehicle-track dynamic interaction and a computational program called DIFF have been developed [14]. The DIFF program permits calculation of displacements, velocities, accelerations and forces in various track components and enables us to investigate how parameters such as train speed, axle load, track irregularities and so on affect the dynamic wheel-rail forces. Full-scale measurements were performed to verify the DIFF model and good agreements are achieved.

A disadvantage of the track model used in DIFF (and most of other track models available in the literature) is that the ballast and the subgrade are considered together only as a visco-elastic foundation. Consequently, the present version of DIFF cannot be used directly to investigate the track-ground vibration and wave propagation from high-speed trains.

The main objective of the present study is to further develop the DIFF program so as to obtain an integrated model for the whole train-track-ground system. Following [12,15,16], we consider the ballast and the subsoil separately. The ballast is modelled as an assembly of parallel rods, while layered subsoil is modelled by two-dimensional finite elements. We adopt the same numerical algorithms used in the DIFF program, which, after FE-discretization in space, applies a solution technique that combines time integration for a first-order system of state-space vector equations in conjunction with a modal superposition analysis.

An outline of the rest of the paper follows. In Section 2 we review the mathematical models for the train-track-ground system and their respective governing differential equations. We then describe the numerical algorithms used for solving the resulting system of coupled equations in Section 3. The simulation results of an X2000 bogie running on a common Swedish track founded on a ground of layered subsoil are presented in Section 4. The importance of including train and track models in modelling the generation and propagation of track-ground vibration from trains is demonstrated. Finally, conclusions and future work are discussed in Section 5.

2 MATHEMATICAL MODELS

Figure 1 illustrates the model used to study the vertical dynamics of the train-track-ground system. The model includes bogie, rail, rail pads, sleepers, ballast and layered subsoil. Wheel and rail are coupled together at two contact points, where track irregularities are introduced as a dynamic excitation.

2.1 *Vehicle*

A rigid multibody model is used to simulate the train bogie, which includes a bogie frame, two primary suspensions and two unsprung masses. Each unsprung mass represents half a wheelset and parts of a traction motor.

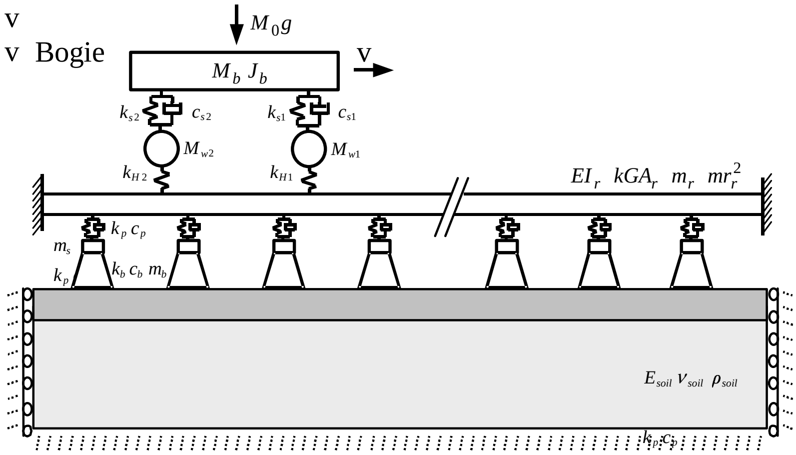

Figure 1. Dynamic model of the railway vehicle-track-underground system

Since the dynamic response of the track is of primary interest, a discrete model of the vehicle is considered adequate and the inertia of the sprung masses appearing above the secondary suspension may be neglected.

Hence, the governing equations of the vehicle model can be written as

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\mathrm{bb}}^{\mathrm{v}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_{\mathrm{a}}^{\mathrm{v}} \\ \ddot{\mathbf{x}}_{\mathrm{b}}^{\mathrm{v}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\mathrm{bb}}^{\mathrm{v}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}_{\mathrm{a}}^{\mathrm{v}} \\ \dot{\mathbf{x}}_{\mathrm{b}}^{\mathrm{v}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{aa}}^{\mathrm{v}} & \mathbf{K}_{\mathrm{ab}}^{\mathrm{v}} \\ \mathbf{K}_{\mathrm{ba}}^{\mathrm{v}} & \mathbf{K}_{\mathrm{bb}}^{\mathrm{v}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{a}}^{\mathrm{v}} \\ \mathbf{x}_{\mathrm{b}}^{\mathrm{v}} \end{bmatrix} + \begin{bmatrix} \mathbf{F}^{\mathrm{wr}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F}^{\mathrm{ext}} \end{bmatrix} \tag{1}$$

where $\mathbf{x}_{\mathrm{a}}^{\mathrm{v}}$ is the displacement vector at the massless wheel-rail contact dofs; $\mathbf{x}_{\mathrm{b}}^{\mathrm{v}}$ is the displacement vector of the vehicle model; $\mathbf{M}^{\mathrm{v}}$, $\mathbf{C}^{\mathrm{v}}$ and $\mathbf{K}^{\mathrm{v}}$ are the mass, damping and stiffness matrices of the vehicle; and $\mathbf{F}^{\mathrm{ext}}$ is the external force vector representing the part of carbody weight added to the bogie frame. The wheel-rail contact forces are assembled in the vector $\mathbf{F}^{wr}$ and will be treated as independent variables in the analysis.

We note that Equation (1) is a nonlinear state-dependent system as the stiffness matrix $\mathbf{K}^{v}$ contains contributions from the wheel-rail contact. Following the Hertz theory, the wheel-rail contact stiffness is determined by

$$k_H = \begin{cases} C_H (x_w - x_a)^{1/2} & \text{for } x_w - x_a > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where x_w is the wheel displacement, x_a is the displacement at the wheel-rail contact dof and C_H is the Hertzian constant calculated according to the elastic modulus and the geometry of wheel and rail [17].

2.2 *Track and ground*

The track model used in the study includes one rail discretely supported via rail pads by sleepers on ballast. The rail is represented as a Rayleigh-Timoshenko beam with its shear deformation and rotary inertia being considered. The rail pads are modelled as a discrete spring-damper system, while the sleepers are only rigid mass bodies.

In summary, the governing differential equations of motion for the track-ground can be outlined as follows.

For the rail,

$$m_r \ddot{w}_r + \sum_{j=-\infty}^{\infty} \delta(x - jl_s)\left[c_p\left(\dot{w}_r - \dot{w}_{sj}\right) + k_p\left(w_r - w_{sj}\right)\right] - kGA_r\left(\frac{\partial \beta}{\partial x} + \frac{\partial^2 w_r}{\partial x^2}\right) = \sum_{i=1}^{2} \delta(x - x_i) p_i \tag{3}$$

$$\mu_r \ddot{\beta}_r + kGA_r\left(\beta + \frac{\partial w_r}{\partial x}\right) - EI_r \frac{\partial^2 \beta_r}{\partial x^2} = 0 \qquad (4)$$

where w_r is the vertical displacement of rail, β_r the slope of the cross section of the rail, w_s the vertical sleeper displacement and l_s the sleeper spacing. The rail and the sleepers are connected at discrete locations via rail pads and the wheel-rail contact forces act at two locations as external forces.

For the sleepers

$$m_s \ddot{w}_{sj} + c_p\left(\dot{w}_{sj} - \dot{w}_r\right) + k_p\left(w_{sj} - w_r\right) + S_j = 0 \qquad (5)$$

where S_j denotes the vertical forces between sleeper and ballast.

For the ballast

$$m_b \ddot{w}_{bj} + c_b \dot{w}_{bj} + \frac{\partial}{\partial z}\left(EA_b \frac{\partial w_{bj}}{\partial z}\right) = 0 \qquad (6)$$

where w_{bj} is the vertical displacement of ballast, which varies only in the vertical direction.

For the subsoil, a linear elastic continuum is assumed and its differential equations of motion can be written as

$$\sigma_{ji,j} + \rho b_j = \rho \ddot{u}_i + c\dot{u}_i \qquad (7)$$

where σ_{ij} is the stress tensor, b_i the body force, ρ the density and u_i the displacement of subsoil.

3 SOLUTION ALGORITHMS

As mentioned above, the solution algorithms adopted are based on the semi-discrete finite element method. First, the spatial domain is discretized by use of finite elements, which results in a system of second-order ordinary differential equations in time. A complex modal superposition technique is then applied to the track-ground system and constraints on contact forces and accelerations between physical components of the vehicle and modal components of the track are introduced. Finally, a first-order initial value problem for the whole vehicle-track-ground dynamic system is established and solved by using a standard solver for initial value problems. In this study, we choose the standard Matlab ODE solver, ode23s, which is based on a modified Rosenbrock formula [18]. For more details of the solution algorithms we refer to [14]. For the use of more advanced adaptive discontinuous Galerkin space-time FE procedures for solving this type of problems, see [19].

3.1 *FE in space*

Figure 2 illustrates a finite element mesh used for the track-ground system. We note that FE-discretization is needed for the rail, the ballast and the subsoil. For the rail we use beam elements with standard piecewise cubic Hermite shape functions, for the ballast we use two-node rod elements and for the subsoil we use four-node bilinear isoparametric elements. By doing so, the partial differential equations described above are reduced to second-order ordinary differential equations in time. We assemble them together with other discrete components (rail pads and sleepers) and obtain the following semi-discrete dynamic equations

$$\mathbf{M}^{t}\ddot{\mathbf{x}}^{t} + \mathbf{C}^{t}\dot{\mathbf{x}}^{t} + \mathbf{K}^{t}\mathbf{x}^{t} - \mathbf{F}^{wr} = \mathbf{0} \tag{8}$$

The N coupled second-order equations of motion are reformulated as $2N$ first-order equations of the form

$$\mathbf{A}^{t}\dot{\mathbf{y}}^{t} + \mathbf{B}^{t}\mathbf{y}^{t} = \begin{Bmatrix} \mathbf{F}^{wr} \\ \mathbf{0} \end{Bmatrix} \tag{9}$$

where

$$\mathbf{y}^{t} = \begin{Bmatrix} \mathbf{x}^{t} \\ \dot{\mathbf{x}}^{t} \end{Bmatrix},\ \mathbf{A}^{t} = \begin{bmatrix} \mathbf{C}^{t} & \mathbf{M}^{t} \\ \mathbf{M}^{t} & \mathbf{0} \end{bmatrix}, \mathbf{B}^{t} = \begin{bmatrix} \mathbf{K}^{t} & \mathbf{0} \\ \mathbf{0} & \text{-}\mathbf{M}^{t} \end{bmatrix} \tag{10}$$

3.2 *Modal superposition for track-ground*

The modal superposition technique is applied by first solving the following eigenvalue problem

$$\begin{bmatrix} \mathbf{K}^{t^{-1}}\mathbf{C}^{t} & \mathbf{K}^{t^{-1}}\mathbf{M}^{t} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\rho}^{(n)} \\ i\omega_n \boldsymbol{\rho}^{(n)} \end{Bmatrix} = -\frac{1}{i\omega_n} \begin{Bmatrix} \boldsymbol{\rho}^{(n)} \\ i\omega_n \boldsymbol{\rho}^{(n)} \end{Bmatrix} \tag{11}$$

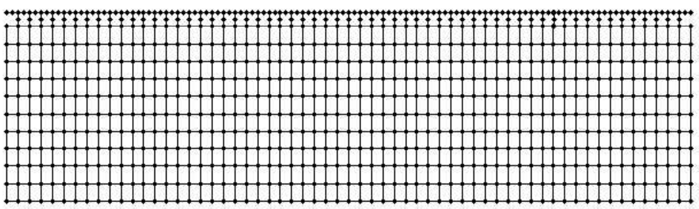

Figure 2. Finite element mesh used for the track-ground system

The solution of the above eigenvalue problem yields N pairs of complex-conjugated sets of eigenvalues $i\omega_n$ and eigenvectors $\boldsymbol{\rho}^{(n)}$. The lowest M modal pairs are accounted for in the modal synthesis and assembled in the modal matrix as

$$\mathbf{P} = \begin{bmatrix} \boldsymbol{\rho}^{(1)} & \cdots & \boldsymbol{\rho}^{(2M)} \\ i\omega_1 \boldsymbol{\rho}^{(1)} & \cdots & i\omega_{2M} \boldsymbol{\rho}^{(2M)} \end{bmatrix} \tag{12}$$

By introducing the transformation

$$\mathbf{y}^{\mathrm{t}}(t) = \mathbf{P}\,\mathbf{q}^{\mathrm{t}}(t), \quad \mathbf{Q}^{\mathrm{t}}(t) = \mathbf{P}^{\mathrm{T}} \begin{Bmatrix} \mathbf{F}^{\mathrm{t}}(t) \\ 0 \end{Bmatrix} \tag{13}$$

we obtain $2M$ uncoupled equations of motion of the form

$$\mathrm{diag}\,(a_n)\dot{\mathbf{q}}^{\mathrm{t}}(t) + \mathrm{diag}(b_n)\mathbf{q}^{\mathrm{t}}(t) = \mathbf{Q}^{\mathrm{t}}(t) \tag{14}$$

$$\mathrm{diag}\,(a_n) = \mathbf{P}^{\mathrm{T}}\mathbf{A}^{\mathrm{t}}\mathbf{P}\,, \;\; \mathrm{diag}\,(b_n) = \mathbf{P}^{\mathrm{T}}\mathbf{B}^{\mathrm{t}}\mathbf{P} \tag{15}$$

where a_n and b_n are modal normalisation constants or the so-called modal Foss damping and modal Foss stiffness.

3.3 *Wheel-rail coupling*

In order to couple the vehicle and the track-ground models together into an integrated system, constraints on contact forces, displacements, velocities and accelerations at wheel-rail contact dofs need to be introduced. The contact displacements can be written in matrix form as

$$\mathbf{x}_{\mathrm{a}}(t) = \mathbf{N}\mathbf{P}^{\mathrm{wr}}\mathbf{q}^{\mathrm{t}}(t) + \mathbf{x}^{\mathrm{irr}} \tag{16}$$

where $\mathbf{N}$ are the local finite element shape functions of the rail elements related to the contact interfacial dofs, $\mathbf{P}^{\mathrm{wr}}$ are the partition of the modal matrix in Equation (13) related to the contact dofs and $\mathbf{x}^{\mathrm{irr}}$ are the prescribed track irregularities at the surface of the rail.

By assuming a constant train speed v and introducing a local longitudinal coordinate ξ, the velocities and accelerations at the wheel-rail contact dofs are calculated as the time derivatives of $\mathbf{x}_{\mathrm{a}}(t)$ and we obtain the following two algebraic equations

$$\dot{\mathbf{x}}_{\mathrm{a}}(t) = \mathbf{T}(t)\dot{\mathbf{q}}^{\mathrm{t}}(t) + \mathbf{U}(t)\mathbf{q}^{\mathrm{t}}(t) + \dot{\mathbf{x}}^{\mathrm{irr}} \tag{17}$$

$$\ddot{\mathbf{x}}_{\mathrm{a}}(t) = \mathbf{R}(t)\dot{\mathbf{q}}^{\mathrm{t}}(t) + \mathbf{S}(t)\mathbf{q}^{\mathrm{t}}(t) + \ddot{\mathbf{x}}^{\mathrm{irr}} \tag{18}$$

where

$$\mathbf{T} = \mathbf{N}\mathbf{P}^{\mathrm{wr}}, \quad \mathbf{U} = \frac{d\mathbf{N}}{d\xi} v \mathbf{P}^{\mathrm{wr}} \tag{19}$$

$$\mathbf{R} = 2\frac{d\mathbf{N}}{d\xi} v \mathbf{P}^{\mathrm{wr}} + \mathbf{N}\mathbf{P}^{\mathrm{wr}} \mathrm{diag}(i\omega_n), \mathbf{S} = \frac{d^2\mathbf{N}}{d\xi^2} v^2 \mathbf{P}^{\mathrm{wr}} \tag{20}$$

$$\dot{\mathbf{x}}^{\mathrm{irr}} = \frac{d\mathbf{x}^{\mathrm{irr}}}{d\xi} v \quad \text{and} \quad \ddot{\mathbf{x}}^{\mathrm{irr}} = \frac{d^2\mathbf{x}^{\mathrm{irr}}}{d\xi^2} v^2 \tag{21}$$

Finally, an initial value problem for the whole train-track-ground dynamic system is formulated by assembling Equation (1) for the vehicle, Equation (14) for the track-ground together with Equations (17) and (18) for the wheel-rail contact, which in its general form can be written as

$$\mathbf{A}(\mathbf{z},t)\dot{\mathbf{z}} + \mathbf{B}(\mathbf{z},t)\mathbf{z} = \mathbf{F}(\mathbf{z},t) \tag{22}$$

where

$$\mathbf{z} = \left\{ \mathbf{q}^{\mathrm{t}} \quad \mathbf{x}_{\mathrm{a}}^{\mathrm{v}} \quad \mathbf{x}_{\mathrm{b}}^{\mathrm{v}} \quad \dot{\mathbf{x}}_{\mathrm{a}}^{\mathrm{v}} \quad \dot{\mathbf{x}}_{\mathrm{b}}^{\mathrm{v}} \quad \hat{\mathbf{F}}^{\mathrm{wr}} \right\}^{\mathrm{T}} \tag{23}$$

$$\mathbf{A} = \left[\begin{array}{c|ccccc} \mathrm{diag}(a_n) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{P}^{\mathrm{wr}\,\mathrm{T}}\mathbf{N}^{\mathrm{T}} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{\mathrm{bb}}^{\mathrm{v}} & \mathbf{0} & \mathbf{M}_{\mathrm{bb}}^{\mathrm{v}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{R} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{T} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right] \tag{24}$$

$$\mathbf{B} = \left[\begin{array}{c|ccccc} \mathrm{diag}(b_n) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{K}_{\mathrm{aa}}^{\mathrm{v}} & \mathbf{K}_{\mathrm{ab}}^{\mathrm{v}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathrm{ba}}^{\mathrm{v}} & \mathbf{K}_{\mathrm{bb}}^{\mathrm{v}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \mathbf{S} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{U} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right] \tag{25}$$

$$\mathbf{F} = \left\{ \mathbf{0}^{\mathrm{T}} \quad \mathbf{0}^{\mathrm{T}} \quad \mathbf{F}^{\mathrm{ext}\,\mathrm{T}} \quad \mathbf{0}^{\mathrm{T}} \quad -\ddot{\mathbf{x}}^{\mathrm{irr}\,\mathrm{T}} \quad -\dot{\mathbf{x}}^{\mathrm{irr}\,\mathrm{T}} \right\}^{\mathrm{T}} \tag{26}$$

In the equations above, $\mathbf{z}$ is a mixed state-space vector in the sense that it consists of not only modal displacements of the track-ground, physical displacements and velocities of the vehicle but also impulses of wheel-rail contact forces. Specifying an initial condition $\mathbf{z}(t=0)=\mathbf{z}_0$, Equation (22) can be solved by using a proper ODE-solver. As mentioned previously, the standard Matlab ODE solver ode23 is used in this study.

4 NUMERICAL RESULTS

Based on the described algorithms, the program package DIFF has been further developed and can now be used to simulate the vertical dynamics of the whole train-track-ground system. In this section, we present numerical results of an X2000 bogie running at a speed of 200 km/h on a typical Swedish track on a layered subsoil ground to demonstrate the performance of the enhanced version of DIFF program.

Table 1. Track and vehicle parameters

Parameter	Notation	Value	Unit
Rail mass per meter	m_r	60.34	kg/m
Young's modulus of the rail	E_r	2.1e11	N/m^2
Poisson's ratio	ν	0.3	
Moment of inertia of the rail	I_r	3.055e-5	m^4
Rail pad stiffness	k_p	500e6	N/m
Rail pad damping	c_p	30e3	Ns/m
Sleeper spacing	l_s	0.65	m
Sleeper mass	m_s	300	kg
Axle load	P_0	18.25	ton
Wheelset mass	m_w	2050	kg
Boggie mass	m_b	8900	kg
Rotary inertia of Boggie	J_b	135	kgm^2
Primary suspension stiffness	k_s	1.45e6	N/m
Primary suspension damping	c_s	30e3	Ns/m

The track model used here is of 100 sleeper bays and consists of UIC60 rail, Pandrol soft rail pads, concrete sleepers and a macadam ballast-bed. Some parameters of the track as well as the vehicle are listed in Table 1. A two-layer subsoil ground is considered. The top layer, representing the embankment, has a depth of 4 m and an elasticity modulus of 8e7 N/m^2. The bottom layer, representing the subsoil ground, has a depth of

4m and an elasticity modulus of 1.2e9 N/m^2 for normal ‘stiff’ ground and 1.2e7 N/m^2 for ‘soft’ ground.

Dynamic track forces due to track irregularities are considered. Figure 3 shows the data of the vertical track irregularities used in the simulation, which are taken from the results registered by Banverket’s measuring wagon STRIX. Their magnitudes have been amplified so that the maximum value is about 6 mm, which corresponds to the peak value for maintenance of Line class K0.

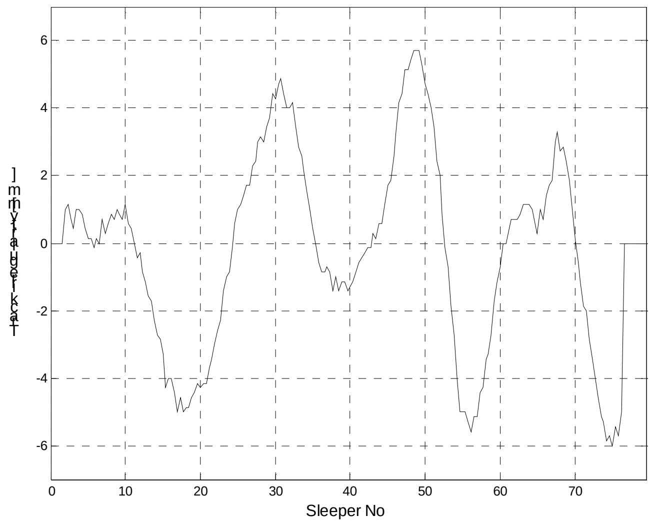

Figure 3. Vertical track irregularities used in the simulation.

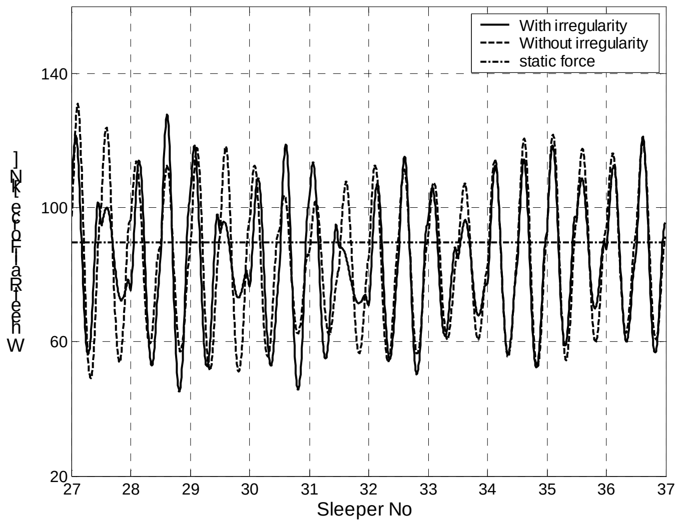

Figure 4. Calculated vertical wheel-rail contact forces with and without vertical track irregularities, stiff subsoil.

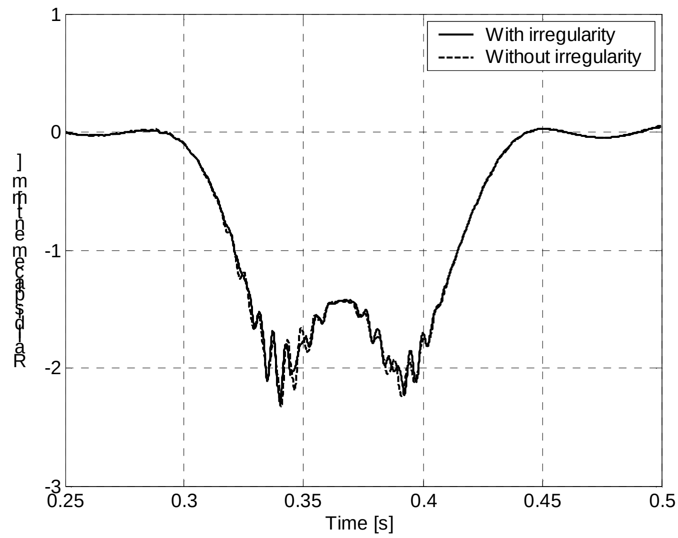

Figure 5. History of vertical rail displacement above the No. 30 sleeper with and without track irregularities, stiff subsoil.

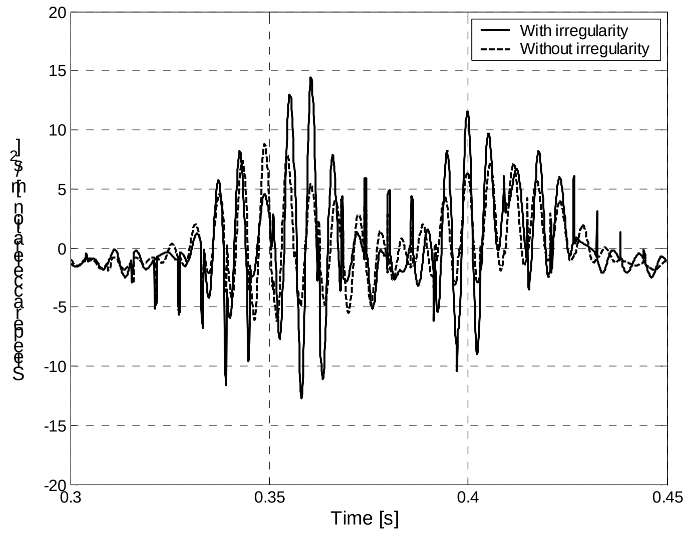

Figure 6. Calculated acceleration history of sleeper No 30 with and without track irregularities, stiff subsoil

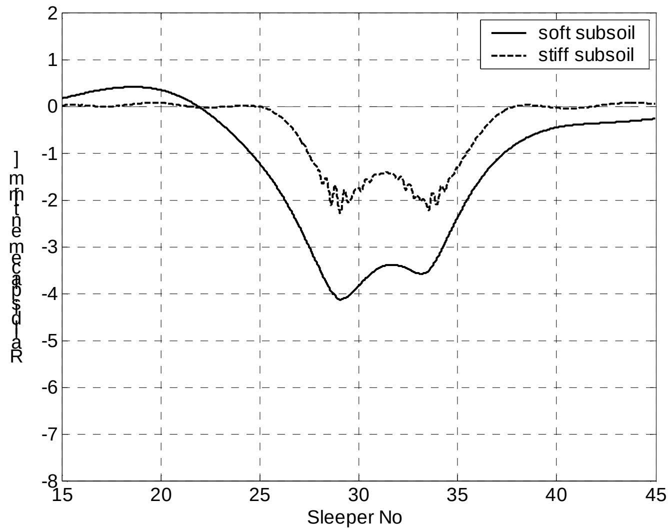

Figure 7. Displacement response of rail due to vertical track irregularities: a comparison of stiff and soft subsoil ground

Responses of the track rested on normal ‘stiff’ ground with and without the presence of vertical track irregularities are first calculated. Figure 4 shows the wheel-rail contact forces. Figure 5 shows the history of vertical rail displacements above sleeper 30 and Figure 6 the history of accelerations of sleeper 30. From these figures we can observe that the track responses show a strong dynamic character and it is not sufficient to represent the wheel-rail force as a moving constant load in the analysis of the generation and propagation of track-ground vibrations.

A comparison of the rail displacement response with ‘stiff’ and ‘soft’ subsoil ground is presented in Figure 7. It is observed that the parameters of subsoil have a substantial influence on the dynamic behaviour of track structures. A detailed representation of ballast and subsoil is thereby needed in the analysis of train-track dynamic interaction.

5 CONCLUSIONS

Mathematical models and solution algorithms to simulate the dynamic train-track-ground system have been presented. The ballast and the layered subsoil ground are modelled separately by finite elements. The existing program package DIFF has been enhanced and can now be used to analyze the vertical dynamics of the whole train-track-ground system. Numerical results demonstrate that in modelling the track dynamics it is important to use an advanced track model and to include the couplings between the train and the track and between the track and the ground. Our future work will involve further improvement of the program and more detailed parameter studies regarding the influence of ballast and subsoil on track dynamics. The effects of other irregularities such as rail corrugation and wheel flat will also be investigated.

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