

## Models for Dividing Fractions – Grade Six

### Ohio Standards Connection

#### **Number, Number Sense and Operations**

##### Benchmark H

Use and analyze the steps in standard and non-standard algorithms for computing with fractions, decimals and integers.

##### Indicator 8

Represent multiplication and division situations involving fractions and decimals with models and visual representations; e.g., show with pattern blocks what it means to take  $2\frac{2}{3}$  divided by  $\frac{1}{6}$ .

#### **Mathematical Processes**

##### Benchmarks

- H. Use representations to organize and communicate mathematical thinking and problem solutions.
- J. Communicate mathematical thinking to others and analyze the mathematical thinking and strategies of others.

### Lesson Summary:

*In this lesson students represent division of fractions using manipulatives, such as freezer pops, candy bars, and models such as drawings squares. Students develop an algorithm from these examples and solve problems using fractions.*

**Estimated Duration:** Three hours

### Commentary:

The concept of division of fractions has been greatly misunderstood and inadequately addressed in traditional mathematics instruction. Developing an understanding of what happens when you divide by a fraction prior to development of the algorithm is essential in the thought process. This is accomplished by having the student visually see and understand what dividing by a fraction means with physical examples. Additionally the use of a hands-on approach with difficult or new concepts helps to bridge the prior knowledge with the new information. Likewise, the guiding of a student to the understanding instead of telling them what they should know leads to longer retention and understanding of the concept.

### Pre-Assessment:

Assess students' informal thinking about dividing whole numbers by fractions.

- Present the following questions on the board or overhead projector to students.
  1. How many quarter-hours are in three hours?
  2. How many thirds of a cup are in two cups?
  3. How many half-meters are in five meters?
  4. How many eighths of a mile are in four miles?
- Have the students determine the number and draw a representation to show how they determined the number. Observe students and note different representations student use to determine the numbers.
- Select students to share their representations of the situations. Include a variety of representations students used.

### Scoring Guidelines:

Assess prior knowledge of understanding the number of fractional parts that go into one whole then into whole numbers larger than one. Provide intervention for students who do not



## Models for Dividing Fractions – Grade Six

understand that three-thirds, two-halves and eight-eighths is equivalent to one. Assist students with drawing visual representations. For example, draw a circle and determine the number of quarter-hours in one hour (May have to relate to money or other measurements.)

*Answers:*

1. *Twelve quarter-hours in three hours*
2. *Six thirds of a cup in two cups*
3. *Ten half-meters in five meters*
4. *Thirty-two eighths of a mile in four miles*

### **Post-Assessment:**

In the post-assessment, students display an algorithm and its representing models (visual representation) and explain how the algorithm represents the model. Students write a number sentence and compute the answer using fractions when given a real-world situation.

- Give each student a copy of Attachment A, *Division Assessment*. Collect and assess students' understanding using Attachment B, *Division Assessment*.

### **Scoring Guidelines:**

Student should be able to model, write and solve division problems that involve fractional numbers. Use Attachment B, *Division Assessment – Answers* to check students' answers and apply the scoring guidelines below to determine the need for intervention.

<i>Ready for more complex number problems.</i>	Consistently models the situation or number sentence and writes a number sentence for the situation. Shows how to use the algorithm to solve the number sentence.
<i>Intervention in understanding and applying the algorithm.</i>	Consistently models the situation and writes a number sentence. Shows misunderstanding in solving the number sentence.
<i>Intervention in modeling the situation or number sentence.</i>	Consistently writes a number sentence for the situation and can show how to use the algorithm to solve the number sentence. Shows incorrect modeling of the situation or number sentence.
<i>Re-teaching of the concept</i>	Inadequately models the situation or writing and solving the number sentence.

### **Instructional Procedures:**

#### **Part 1**

1. Ask five volunteers to come to the front of the classroom. Give each student a freezer pop (use pops with two sticks) and ask if they have ever eaten one. Then ask if they had eaten the entire freezer pop or split it in half. Because of the two sticks, one student may answer that he/she splits the freezer pop in half. Ask students to split the pops in half and have a student count the total number of halves. Use *Frozen Juice Pops*, Attachment E, as a visual representation for the situation.

## Models for Dividing Fractions – Grade Six

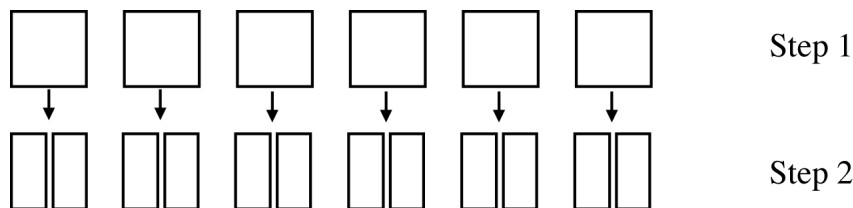
- Ask students if they notice anything about the size of the 10 pieces compared to the original 5 freezer pops. Student should note that they are smaller. Elicit that they are half the size of the original freezer pops.
- Ask a volunteer for a number sentence to represent the 5 freezer pops divided in half and the answer ( $5 \div \frac{1}{2} = 10$ ). Write the number sentence on the chalkboard for the class to see. If students need help determining this number sentence, ask “How many half-size freezer pops were contained in the original 5 whole freezer pops? Then, remind the class that when we ask how many of something is in something else, that is a division situation (e.g., if we want to know how many 3’s are in 12, we divide 12 by 3).
- Distribute a variety of chocolate bars that are made with divided sections or use *Chocolate Bar Models*, Attachment F. Ask students to describe how the bars could be divided and give a number sentence to represent this division.  $5 \div \frac{1}{4} = 20$ ,  $3 \div \frac{1}{12} = 36$ . Again write the number sentences on the chalkboard. Discuss with the students that the size of the pieces desired is not the same as the original pieces. Note: Some students may come up with a number sentence such as  $4 \div 16 = \frac{1}{4}$ , which is also a correct way to model this number sentence. Encourage these students to find a second number sentence that also models the situation (How many little pieces are in the original big piece?).

### Instructional Tip

Make sure students are able to recognize that even though they end with a greater number of pieces after completing the division, the size of the portion is less.

### Part 2

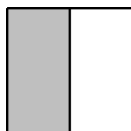
- Pose the following situation to the class.  
*I have six squares that I want to divide by one half. How many pieces would I have?*
- Ask students to draw a picture to represent the problem. A sample response should be



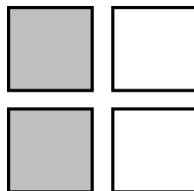
- Ask the following guiding questions;
  - How many squares did I have? (6)
  - What size did I want? ( $\frac{1}{2}$ )
  - How many pieces of that size do we have? (12)
- Ask students how this situation would be represented as a mathematical sentence. Guide the discussion to obtain the number sentence  $6 \div \frac{1}{2} = 12$

## Models for Dividing Fractions – Grade Six

- Place students in pairs and pose another situation. Ask them to model it and write a mathematical sentence that represents the situation.  
*I have one half of a square and I want to divide it by one fourth. How many pieces would I have?*
- Monitor the partners working on the task and ask the same type of guiding questions when students appear to be struggling with how to represent the situation. The solution should resemble the following example:



This represents having one half of the square.



This represents dividing the square into pieces whose size is one fourth. The students then need to answer the question of how many pieces of size one fourth do I have?

- Ask the partners to write a number sentence for the problem ( $\frac{1}{2} \div \frac{1}{4} = 2$ ). Ask for a volunteer to provide the number sentence. Ask the student why he/she placed the numbers in that order.
- Write the number sentences on the chalkboard or white board after each situation, noting the relationships among the numbers in the number sentence. Have students look for any patterns or relationships they note in the number sentences.
- Have partners make conjectures or descriptions as to what they believe is happening when they divide a number by a fraction. Ask partners to share their conjectures with the class. Record the conjectures and descriptions on the board, chart paper or on a transparency. Possible conjectures include:
  - When you divide by a fraction you get a whole number.
  - When you divide by a fraction you get a larger number.
  - When you divide by a fraction you multiply the whole number by the denominator.

### Instructional Tip

Use the conjectures to adjust the instructions as needed, determining whether students are ready to work with more complex fractions or dividing a fraction by a whole number. Students can test their conjectures and refine their descriptions. The goal is to move students to determining the algorithm for dividing by a fractional number.

### Part Two

- Present the following situation:

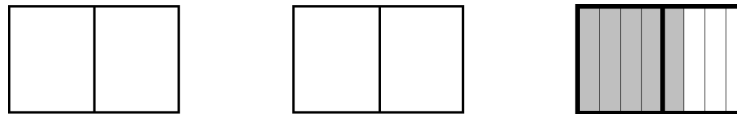
*Cierra has  $2\frac{5}{8}$  meters of yarn that she wants to cut into  $\frac{1}{2}$  meters lengths. How many  $\frac{1}{2}$  meters lengths of yarn will Cierra have?*

## Models for Dividing Fractions – Grade Six

- a. Instruct students to draw a model to solve the problem. A sample model may be



- b. Divide the pieces in half.



- c. There are 5 halves in  $2\frac{5}{8}$ . There is also  $\frac{1}{8}$  left, which is  $\frac{1}{4}$  of the remaining half of the whole. Therefore, there are  $5\frac{1}{4}$  halves in  $2\frac{5}{8}$ . Cierra will have 5 complete  $\frac{1}{2}$  meter lengths of yarn.

- d. Circulate the room as students work. Select a student to model the problem on the chalkboard. Note: there will be a piece left ( $\frac{1}{8}$ ) over after finding 5 halves. If students are not sure what this represents, ask, “What is the relationship of  $\frac{1}{8}$  to the remaining half?”

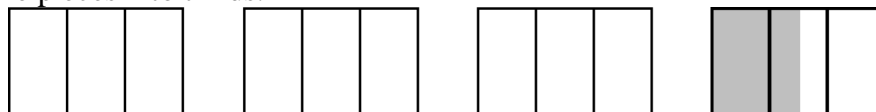
( $\frac{1}{8}$  is  $\frac{1}{4}$  of the remaining half.)

15. Present a similar situation, such as:

*Jonathan has  $3\frac{1}{2}$  cups of chocolate chips to make cookies. The recipe uses  $\frac{1}{3}$  cup of chips in each batch. How many batches of cookies can Jonathan make?*



- a. Divide the pieces into thirds.



Jonathan can make  $10\frac{1}{2}$  batches of cookies.

## Models for Dividing Fractions – Grade Six

### Instructional Tip

Students should see that once they divide the whole part(s) into the desired fractional part, the remaining part, which is a fractional part of another whole piece, needs to be divided also.

Students may see a fractional piece left over and should determine the relationship of this piece to the original piece.

16. Present the following problem:

*I have one fourth of a square and I want to divide it by one half. How many pieces would I have?*

- Have students work as partners to solve the problem. Remind students to draw a picture to represent the problem.
- Ask guiding questions used with other situations such as, “What are you looking for (How many sets of  $\frac{1}{2}$  are in  $\frac{1}{4}$ ?)?” A sample response should be

$\frac{1}{2}; \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$  ; I have one half of the desired size.



- Have students compare this problem with the problem in step 9 noting any differences and similarities. Discuss the meaning of the answer in each problem.
- Have the partners test and refine the conjectures and descriptions by completing Attachment C, *Dividing By a Fraction*. Observe the partners working on the situations and provide intervention, reminding them of the guiding questions that were used with the other situations.
- Put partners together to make groups of four. Have each group check the solutions obtained, then review answers as a class, asking each group to provide the solutions to the situations. Use questioning to modify incorrect models.
- Ask groups to review the conjectures and descriptions based on the new information. Lead students through the revision process by looking at each conjecture or description and determining if any of the situations contradicted the statement or if clarification is needed.
- Have students determine if the statements refer to the process or the results of the problem.
- Lead students in the process of creating one or two general statements related to the process and another to the results.
- Have students refine their conjectures as the class continues to work with problems that use division with fractions

### Instructional Tips

The general statement for the process should be similar to the algorithm for dividing two values when fractions are involved. Additional discussion or examples may be needed to make the statement about this relationship.



## Models for Dividing Fractions – Grade Six

### **Differentiated Instructional Support:**

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Group students that are able to accurately model and write number sentences with those students that are not having the same success.
- Assign student pairs to create a concrete example similar to the ones used to introduce the lesson. Have students present each example to the class with a division number sentence and an inverse multiplication sentence.

### **Extensions:**

- Have students test the algorithm developed in class with other types of fractions. For example, dividing a fraction by a whole number or dividing a number by an improper fraction.
- Distribute pattern blocks for the students to manipulate. Present several division problems for students to model with the pattern blocks and have students rewrite each division problem as a multiplication sentence.

### **Home Connections:**

- Plan a division celebration for the conclusion of the lesson using other items that can easily be divided.
- Draft volunteers to bake cookies or cupcakes at home to be brought to school to be divided for the classroom. Example: 8 cupcakes to be divided by  $\frac{1}{3}$ , or 6 large cookies to be divided by  $\frac{1}{4}$ .

### **Materials and Resources:**

*The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.*

*For the teacher:* Freezer pops, cookies, candy bars, whiteboard, or chalkboard

*For the student:* Paper, pencils, pattern blocks (optional)

### **Vocabulary:**

- algorithm
- conjecture



## Models for Dividing Fractions – Grade Six

- fraction
- inverse

### **Technology Connection:**

Demonstrate using an over-head calculator (if available) division with fractions so students can use individual calculators to check work.

### **Research Connections:**

Arter, Judith and Jay McTighe. *Scoring Rubrics in the Classroom: Using Performance Criteria for Assessing and Improving Student Performance*. Thousand Oaks, CA: Corwin Press, 2001.

“BSCS Science: An Inquiry Approach.” BSCS Biological Sciences Curriculum Study. 23 Dec. 2003

Cawletti, Gordon. *Handbook on Research on Improving Student Achievement*. Arlington, VA: Educational Research Service, 1999.

### **General Tip:**

Any manipulative can be used to demonstrate the division process, however, the emphasis should be on showing the students whole numbers divided by a fraction and then fractions divided by fractions, making sure to first demonstrate the division and then write its representative number sentence on the board. Cite several examples before working the division to the inverse operation.

### **Attachments:**

Attachment A, *Models for Dividing Fractions – Post-Assessment*

Attachment B, *Models for Dividing Fractions – Post-Assessment, Answer Guide*

Attachment C, *Dividing By a Fraction*

Attachment D, *Dividing By a Fraction- Answers*

Attachment E, *Frozen Juice Pops*

Attachment F, *Chocolate Bar Models*

Attachment G, *Models for Dividing Fractions*





## Models for Dividing Fractions – Grade Six

### Attachment A Models for Dividing Fractions – Post-Assessment

Name \_\_\_\_\_ Date \_\_\_\_\_

**Directions:** Read the situation, draw a picture to represent the situation and then write a number sentence to represent the situation.

1. I have a one-half gallon container of ice cream and want to divide it into one-cup servings to share with the students in my class. A cup is one sixteenth of a gallon. How many serving dishes would I need?

Model the problem situation.

Write a number sentence and show how to solve the problem

2. I also have three large chocolate candy bars that are perforated into eight sections each. If I divide the bars into these sections how many sections will I have altogether?

Model the problem situation.

Write a number sentence and show how to solve the problem

3. Becca works for the Humane Society and had to buy food for the dogs. She bought  $5\frac{1}{2}$  pounds of dog food. She feeds each dog about one-third of a pound. How many dogs can she feed?

Model the problem situation.

Write a number sentence and show how to solve the problem.



## Models for Dividing Fractions – Grade Six

### Attachment A (continued) Models for Dividing Fractions – Post-Assessment

Name \_\_\_\_\_ Date \_\_\_\_\_

4. Julie goes to the park across the street from her house several times a day and jogs a total of six miles every day. She jogs three-fourths of a mile at a time. How many times each day does she go to the park to run?

Model the problem situation.

Write a number sentence and show how to solve the problem.

**Model and solve each of the following number sentences.**

5.  $\frac{3}{4} \div \frac{1}{2}$

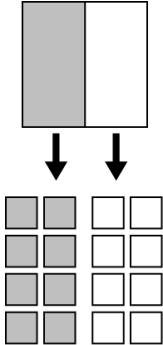
6.  $\frac{5}{3} \div \frac{1}{3}$

7.  $\frac{1}{2} \div \frac{4}{5}$

## Attachment B

### Models for Dividing Fractions - Post-Assessment, Answer Guide

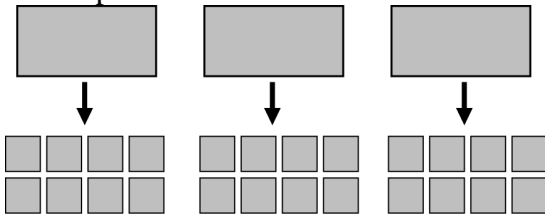
1. Model the problem situation.



Write a number sentence and show how to solve the problem.

$$\frac{1}{2} \div \frac{1}{16} = 8$$

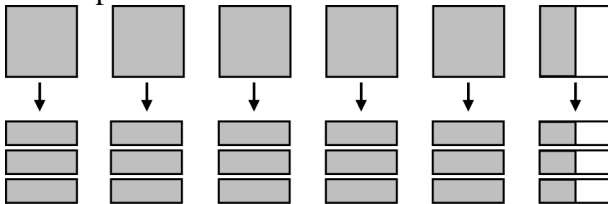
2. Model the problem situation.



Write a number sentence and show how to solve the problem.

$$3 \div \frac{1}{8} = 24$$

3. Model the problem situation.



Write a number sentence and show how to solve the problem.

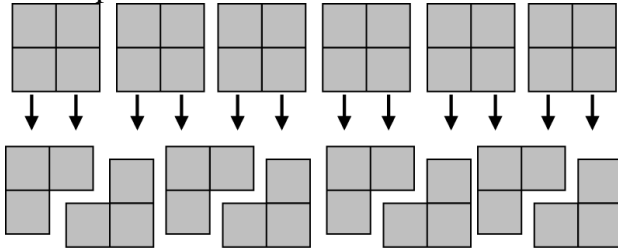
$$5\frac{1}{2} \div \frac{1}{3} = \frac{33}{2} \text{ or } 16\frac{1}{2}$$

# Models for Dividing Fractions – Grade Six

## Attachment B (continued)

### Models for Dividing Fractions - Post-Assessment, Answer Guide

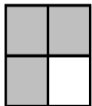
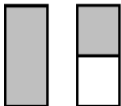
4. Model the problem situation.

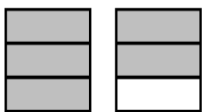
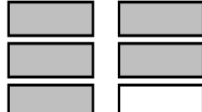


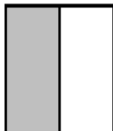
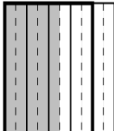
Write a number sentence and show how to solve the problem.

$$6 \div \frac{3}{4} = 8$$

Model and solve each of the following number sentences.

5.  $\frac{3}{4} \div \frac{1}{2}$    $1 \frac{1}{2}$    $\frac{3}{4} \div \frac{1}{2} = \frac{3}{2}$  or  $1 \frac{1}{2}$

6.  $\frac{5}{3} \div \frac{1}{3}$     $\frac{5}{3} \div \frac{1}{3} = 5$

7.  $\frac{1}{2} \div \frac{4}{5}$     $\frac{1}{2} \div \frac{4}{5} = \frac{5}{8}$



## Models for Dividing Fractions – Grade Six

### Attachment C Dividing By a Fraction

Name \_\_\_\_\_ Date \_\_\_\_\_

**Directions:** Model each situation using squares; write the number sentence for each.

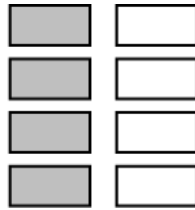
1. I have one half of a square and I want to divide it by one eighth. How many pieces would I have?
  
  
  
  
  
  
  
  
  
  
2. I have five halves of a square and I want to divide it by one fourth. How many pieces would I have?
  
  
  
  
  
  
  
  
  
  
3. I have two thirds of a square and I want to divide it by one half. How many pieces would I have?
  
  
  
  
  
  
  
  
  
  
4. I have one half of a square and I want to divide it by three fourths. How many pieces would I have?

## Models for Dividing Fractions – Grade Six

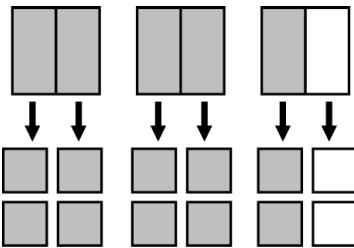
### Attachment D Dividing By a Fraction – Answers

Name \_\_\_\_\_ Date \_\_\_\_\_

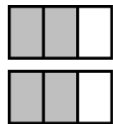
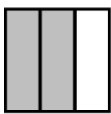
1. I have one half of a square and I want to divide it by one eighth. How many pieces would I have?  $4$  ;  $\frac{1}{2} \div \frac{1}{8} = 4$  ; I have 4 pieces that are one eighth of the square.



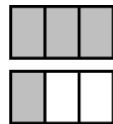
2. I have five halves of a square and I want to divide it by one fourth. How many pieces would I have?  $10$  ;  $\frac{5}{2} \div \frac{1}{4} = 10$  ; I have 10 pieces of size one fourth of a square.



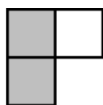
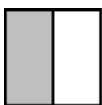
3. I have two thirds of a square and I want to divide it by one half. How many pieces would I have?  $\frac{4}{3}$  ;  $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$  ; I have four thirds of the size one half of a square.



OR

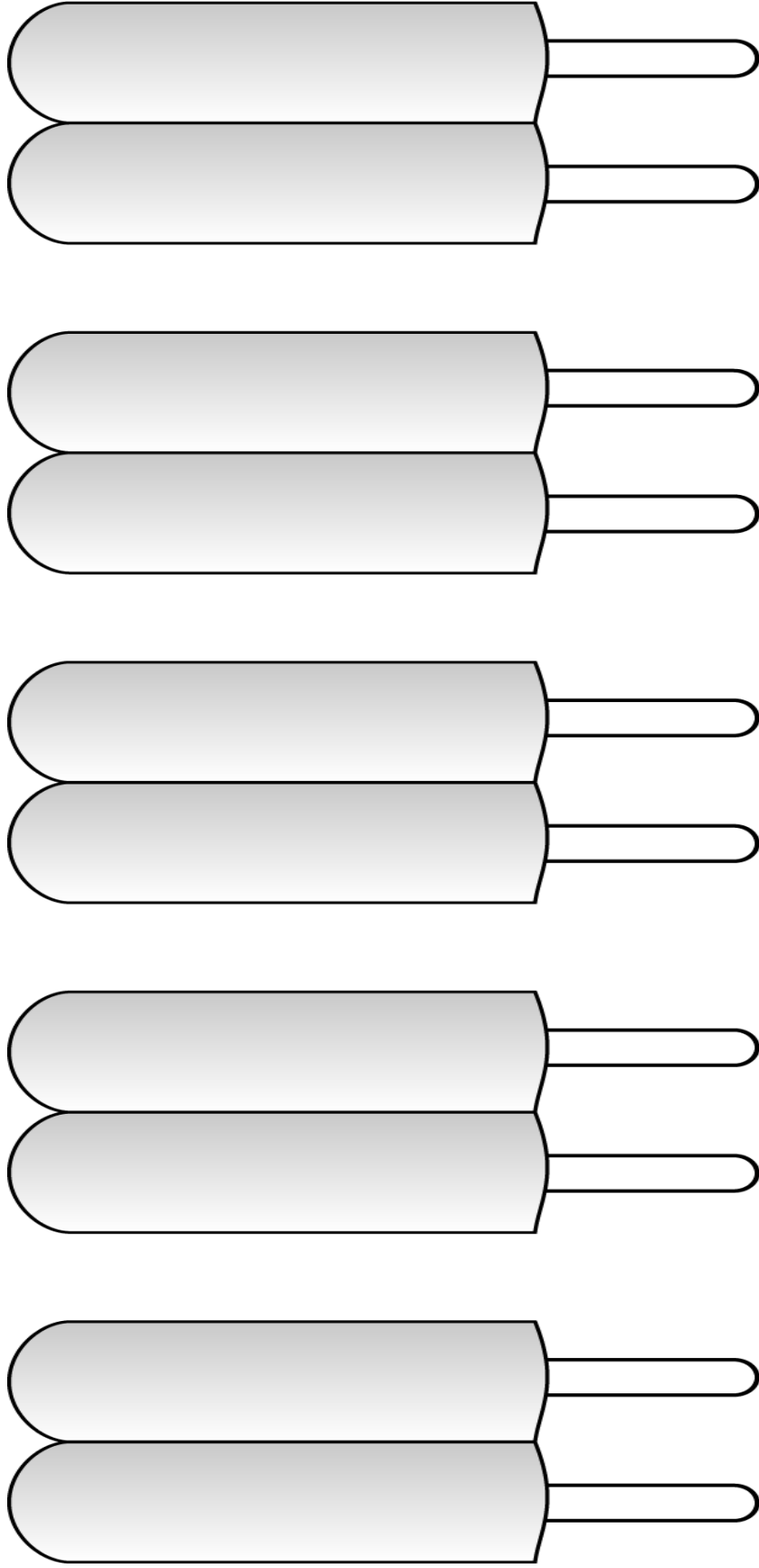


4. I have one half of a square and I want to divide it by three fourths. How many pieces would I have?  $\frac{2}{3}$  ;  $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$  ; I have two thirds of the size three fourths of a square.



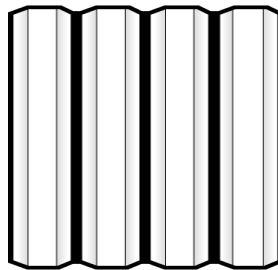
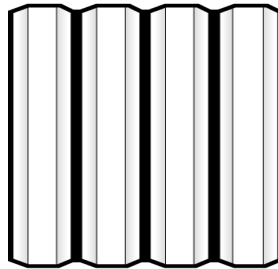
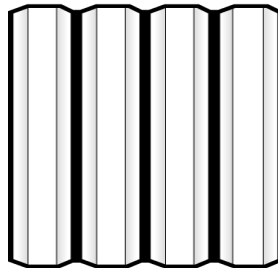
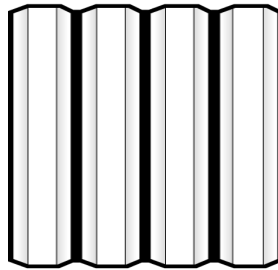
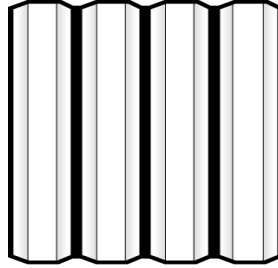
## Models for Dividing Fractions – Grade Six

### Attachment E Frozen Juice Pops



# Models for Dividing Fractions – Grade Six

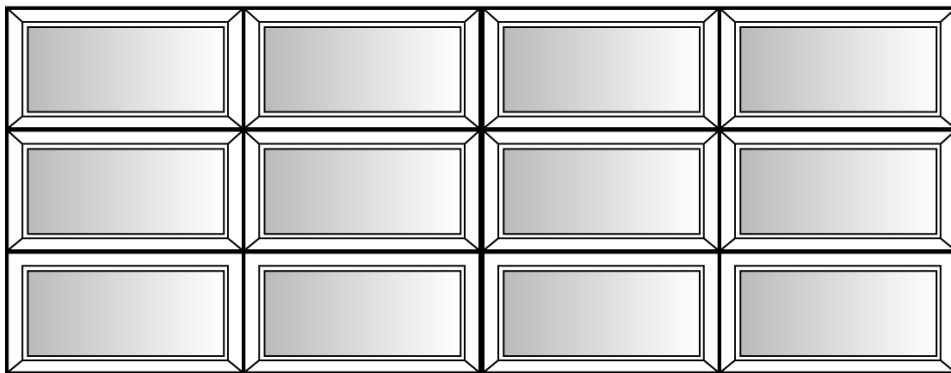
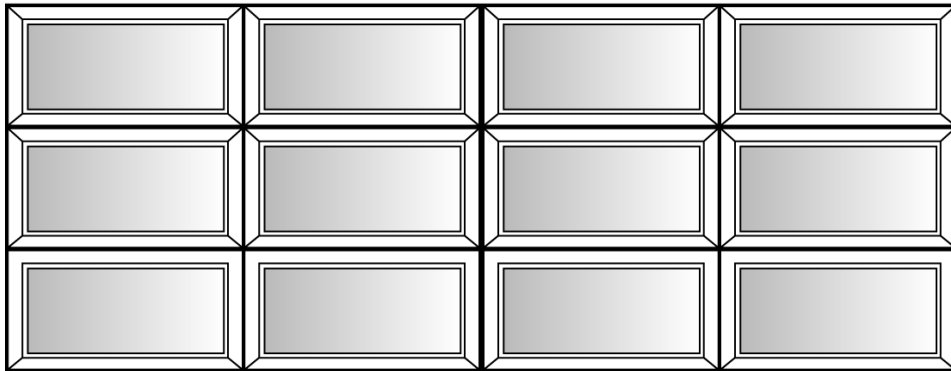
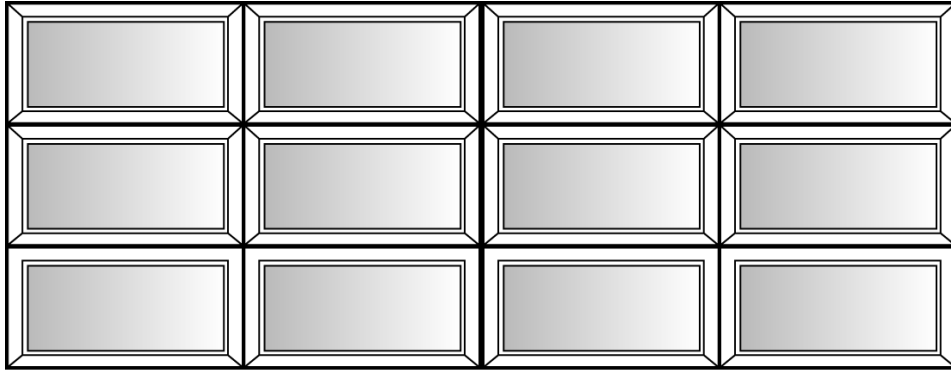
## Attachment F Chocolate Bar Models





## Models for Dividing Fractions – Grade Six

### Attachment F (continued) Chocolate Bar Models



## Models for Dividing Fractions – Grade Six

### Attachment G Models for Dividing Fractions

