Modern Physics Laboratory
MP2 Blackbody Radiation

Purpose

In this experiment, you will investigate the spectrum of the blackbody radiation and its dependence on the temperature of the body.

Equipment and components

Tungsten light source, prism spectrophotometer, Science Workshop 750 interface, broad spectrum light sensor (CI-6630), rotary motion sensor (CI-6538), DC power supply, ohmmeter, and multi-meter (×2).

Background

A blackbody is an idealized body that absorbs all electromagnetic radiation incident upon it. In the laboratory, a blackbody can be approximated by a very small hole in the wall of a hollow box. Any radiation incident upon the hole enters the cavity and gets trapped by the repeated reflections from the cavity walls until it is absorbed. The cavity walls constantly absorb and emit radiation, and the emitted radiation (called blackbody radiation) can be detected by putting an external radiation detector near the hole in the cavity. The radiation from a blackbody is an electromagnetic energy converted from the thermal energy of the body at (equilibrium) temperature *T*. Therefore, the blackbody radiation is also called thermal radiation. It has a unique wavelength distribution with a maximum intensity peaked at a wavelength λ_{max}. At room temperature, the value of λ_{max} is in the infrared region. The object appears black since it does not reflect or emit visible light. From everyday experience you probably already know that, the blackbody radiates more when it is hot than when it is cold. As the temperature increases to above several hundred degrees Celsius, the blackbody starts to emit visible light, appeared red, then orange, and finally white and blue. Figure 1 shows the spectrum of the blackbody radiation at four temperatures.

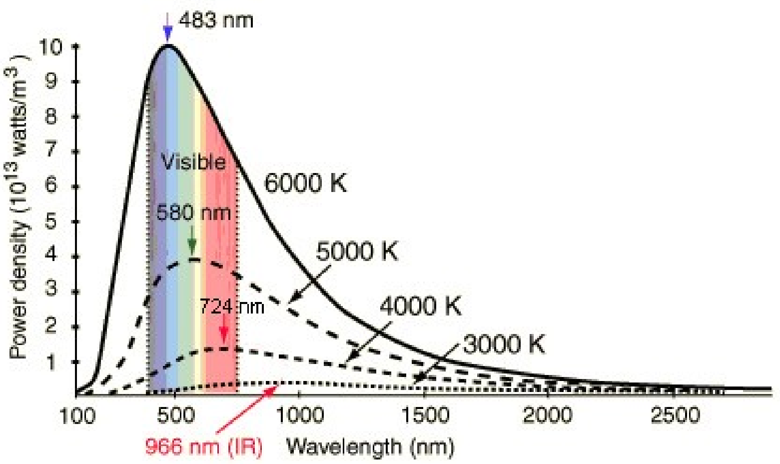


Figure 1 Spectrum of the blackbody radiation at different temperatures.[1]

In 1901 the German physicist Max Planck published his famous radiation law:

$$u(\lambda,T)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda k_B T}-1}, \quad (1)$$

where $u(\lambda,T)d\lambda$ is the total energy per unit volume in a blackbody in the wavelength interval from λ to $\lambda+d\lambda$ with units of joules per cubic meter. In the above, *c* is the speed of light in vacuum, *h* (= 6.626×10^{-34} J·s) is Planck's constant, k_B (= 1.381×10^{-23} J/K) is Boltzmann's

[1] HyperPhysics, Department of Physics and Astronomy, Georgia State University

constant, T is the absolute temperature of the body, and λ is the wavelength of the radiation.

It is seen from Fig. 1 that the spectral energy density $u(\lambda,T)$ is a singly peaked function and the maximal energy density occurs at the wavelength λ_{max}, which can be obtained by solving $\partial u(\lambda,T)/\partial\lambda = 0$ for $\lambda = \lambda_{max}$. The final result can be conveniently expressed as

$$\lambda_{\max} T = 2.898 \times 10^{-3} \mathrm{m} \cdot \mathrm{K}. \tag{2}$$

Equation (2) is called Wien's displacement law. It states the fact that the maximum of the spectral energy density of the blackbody radiation shifts to shorter wavelengths as the temperature of the body is increased.

Another interesting feature of the blackbody radiation is that by integrating Eq. (1) over all the wavelengths, we obtain the total energy density:

$$u = \int_0^\infty u(\lambda, T) d\lambda = \frac{8\pi^5 k_B^4}{15c^3 h^3} T^4 = aT^4, \tag{3}$$

where a is a universal constant. The power P radiated by an object per unit area is given by

$$P = \sigma T^4, \tag{4}$$

where $\sigma = ac/4 = 5.670\times10^{-8}$ W/(m^2·K^4) is Stefan's constant. Equation (4) can be further generalized to the Stefan-Boltzmann law:

$$P = e\sigma T^4, \tag{5}$$

where e is the emissivity of the radiating surface. For a perfect reflector, which does not radiate at all, one has $e = 0$ and for a blackbody, $e = 1$.

In this experiment, you will measure the spectrum of a tungsten light source using a prism spectrophotometer. The spectrophotometer measures the relative light intensity as a function of angle, and the wavelength corresponding to each angle is calculated using an equation converting the angle of diffraction to the wavelength. The relative light intensity is then plotted as a function of wavelength, which gives a characteristic curve of the blackbody radiation. In the second part of the experiment, the relative light intensity, which is proportional to P, emitted from the tungsten light source is measured at various temperatures. From the obtained data, you will be able to verify the relationship between the power of the thermal radiation and the temperature of the radiating body.

Procedure

It will take two lab sessions to finish the present experiment. It is suggested that Part I be finished in the first lab session and Part II be finished in the second lab session.

Experimental apparatus:

Prism Spectrophotometer: The spectrophotometer is used to measure the spectral content (spectrum) of an incident light source. As shown in Fig. 2, the collimating slits and collimating lens produce a narrow parallel light beam. The prism disperses the beam of light into a spectrum with different wavelengths (colors) at different angles. The narrow slit on the aperture disk (part of the aperture bracket) allows light with a narrow range of wavelengths to enter the light sensor. The light sensor measures the intensity of the light while the Rotary Motion Sensor measures the angle to which the light is diffracted by the prism. The prism spectrophotometer has two advantages over a grating spectrophotometer:

1. It eliminates overlap of orders observed with a grating. (When a grating is used, the second order visible wavelengths overlap the first order infrared wavelengths.)
2. The spectrum is brighter because it does not spread over several orders and/or in both directions.

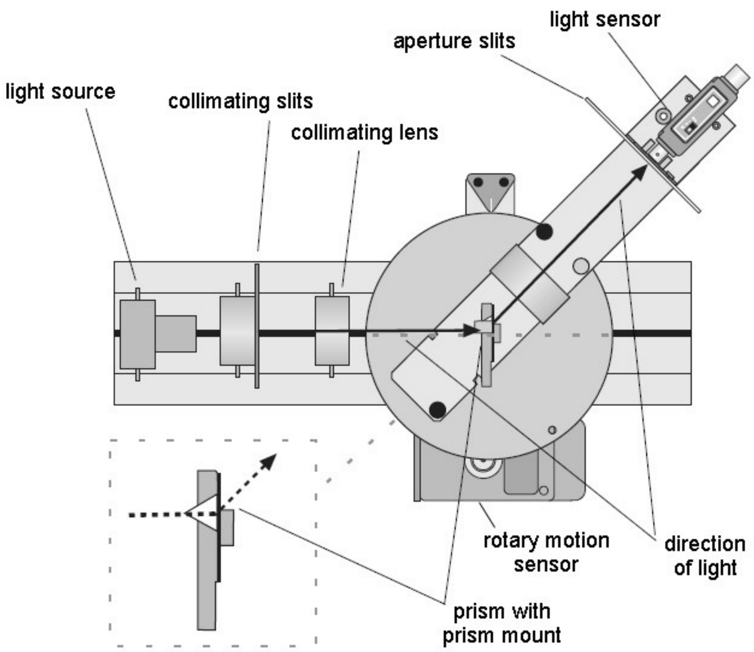


Figure 2 Sketch of the prism spectrophotometer

Broad Spectrum light sensor: The broad spectrum light sensor uses a thermopile as a sensing element. Thermopile detectors are voltage generating devices that act like a miniature array of thermocouples. The window installed in the broad spectrum light sensor is BaF_2, which has a spectral response from 300 to 10,000 nm.

Blackbody light source: The blackbody light source used in this experiment is a tungsten light bulb enclosed in a black cavity. The temperature of the tungsten filament can be calculated using the resistance value of the tungsten filament while it is lit. The resistivity of the tungsten filament is a nonlinear function of the temperature. A function that approximates the calibration curve (CRC Handbook, 45^{th} edition, page E-110) for the resistivity of tungsten is used to calculate the temperature. The resistance R of the tungsten filament is obtained using Ohm's law: $R = V / I$, where V is the voltage across the filament and I is the current through the filament. Please refer to Appendix A for details about the calculation.

Part I Spectrum of the blackbody radiation

1. Set up the prism spectrophotometer as shown in Fig. 3. Make sure that the infrared filter is installed in front of the light sensor. The filter prevents the light sensor from detecting objects in the lab (such as your hands), but still allows the sensor to detect hotter objects (such as the blackbody light source).
2. The prism should be oriented with the apex facing the light source (as shown in Fig. 2) and the prism mount bracket should be aligned with the 0 and 180 degree line on the degree plate. Check to see that the angle indicator on the degree plate is set to zero and the tab indicator on the prism mount is aligned parallel to the 180-degree line on the plane (as shown in Fig. 4). If it is not the case, please ask the technician-in-charge for help.
3. The light sensor arm should be free to rotate under the prism without moving the prism mount. If not, please ask the technician-in-charge for help.
4. Position the collimating lens such that the distance between the collimating slits and lens is 10 cm, which is the focal length of the lens.

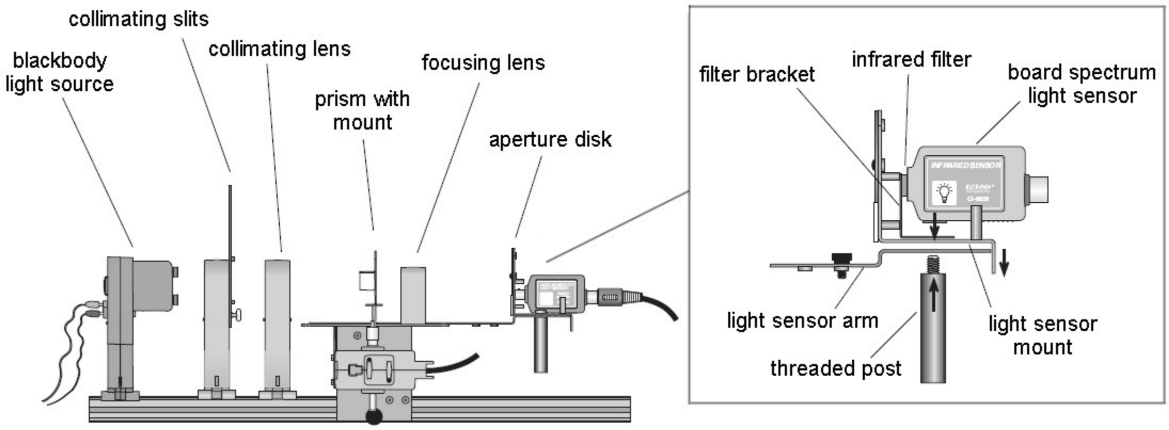


Figure 3 Experimental setup for the measurement of blackbody spectrum

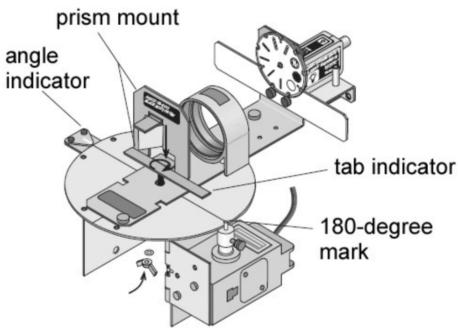


Figure 4 Correct position of prism mount on spectrophotometer table

5. Before connecting power to the blackbody light source, use an ohmmeter to measure the resistance R_0 of the tungsten light filament at room temperature. Record the value in Table 1. **HINT:** For an accurate measurement of the resistance of the tungsten light filament, the resistance of the connecting wires should be taken into account.
6. Connect the blackbody light source to a DC power supply in series with an ammeter and also connect a voltmeter across the blackbody light source. Ask a TA/Technician to check your circuit before turning on the power supply.
7. Connect the DIN-5 plug of the light sensor to Analog Channel A on the interface box. Connect the cables of the rotary motion sensor to Digital Channels 1 and 2.
8. Open the “MP2-I” *DataStudio* program in the course folder on the computer.
   - The program will open two graphic displays. Graph A is the light intensity (V) versus wavelength (nm) and Graph B is the light intensity (V) versus the angular position of the rotary motion sensor (in units of radians).
   - The Calculator window shows a set of calculations used in this program. These calculations are very helpful in this experiment. For example, the wavelength calculation helps to relate the wavelength of light with the angle of diffraction. Please refer to Appendix B for more details.
9. Set the collimating slits to slit 3 (with slit width = 0.3 mm) and turn the aperture disk to slit 3 as well. Move the light source to the collimating slits as close as possible.
10. Set the light sensor gain switch to “×100”.
11. Turn on the DC power supply and apply 3V to the blackbody light source. **CAUTION:** To save the lifetime of the light bulb, turn on the light bulb only when taking the measurements. The maximum voltage of the light source is 10V and DO NOT exceed this limit under any circumstances.
12. Rotate the scanning arm counter-clockwise until it hits the stop. This will be the starting position for all the scans.

13. Observe the light spectrum on the aperture disk screen. All the visible colors should be present.
14. Measure the voltage across and current pass through the tungsten filament (light source) and record their values in Table 1.
15. Block the light source by placing your hand between the collimating slits and the collimating lens. While the light is blocked, press the tare button on the light sensor.
16. Remove your hand and start recording data by clicking the "Start" icon in the program. As the scan begins, make sure that the recorded angle is positive. If not, reverse the rotary motion sensor plugs on the interface and re-start the data run. **Slowly** rotate the scanning arm through the spectrum and continue **all the way past zero degree** (at this position the light sensor is directly facing the light source).
17. There will be a peak in Graph B (the light intensity *vs*. angular position of the rotary sensor), when the light sensor is directly facing the light source. In this case, some light leaks through the prism without being dispersed (as shown in Fig. 5). Therefore, this peak can be used to calibrate the zero degree angle. Use the Smart Cursor to determine the angle from the starting position to the central peak where the light sensor is directly facing the light source. Click the "Calculator" in the *DataStudio* program and enter this angle as "init".
    NOTE: Graph A (the light intensity versus wavelength) is set to show the light intensity up to ~2500 nm (wavelength).
18. Finish the recording data by clicking the "Stop" icon in the program.

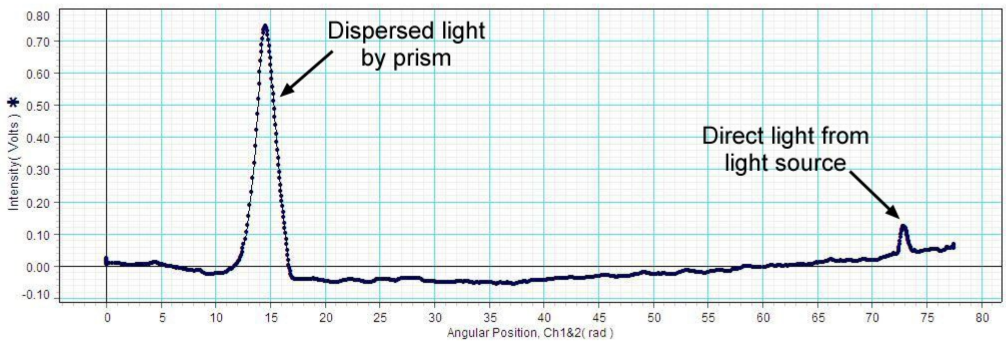


Figure 5 Determination of initial angle

19. Repeat steps 9 to 18 for increasing voltages of 5V, 7V and 9V. On these scans, it is not necessary to scan all the way past through the center as before, because the initial angle has already been calibrated. Stop the scan when the wavelength reaches 2500 nm.
    NOTE: If any scan shows flat top, it means that the light sensor reaches its maximum output (~10V), please set the collimating slits to slit 2 or smaller and turn the aperture disk to slit 2 or smaller. Repeat all the scans.
20. Save the *DataStudio* program with a different file name for your data analysis. Print the light intensity versus wavelength graph (with all runs) and attach them to the lab report.
21. Calculate the resistance of the tungsten filament (light source) and record the value in Table 1. Use the equations in Appendix A and calculate the corresponding tungsten filament temperature. Record the value in Table 1.
22. Use the Smart Cursor on the intensity *vs*. wavelength graph and find the peak wavelength on the spectrum for different temperatures of the tungsten filament. Record the values in Table 1.

Part II Temperature dependence of the blackbody radiation

1. Set up the light source and light sensor as shown in Fig. 6. Make sure that the shutter bracket is installed in front of the light sensor. The spring-loaded shutter bracket keeps

extraneous radiation from heating the sensing element either before or during the measurements.

2. Before connecting power to the blackbody light source, use an ohmmeter to measure the resistance R_0 of the tungsten filament at room temperature. Record the value in Table 2. **HINT:** For an accurate measurement of the resistance of the tungsten filament, the resistance of the connecting wires should be taken into account.

3. Connect the blackbody light source to a DC power supply in series with an ammeter and also connect a voltmeter across the blackbody light source. Before turning on the power supply, please ask a TA/Technician to check your circuit.

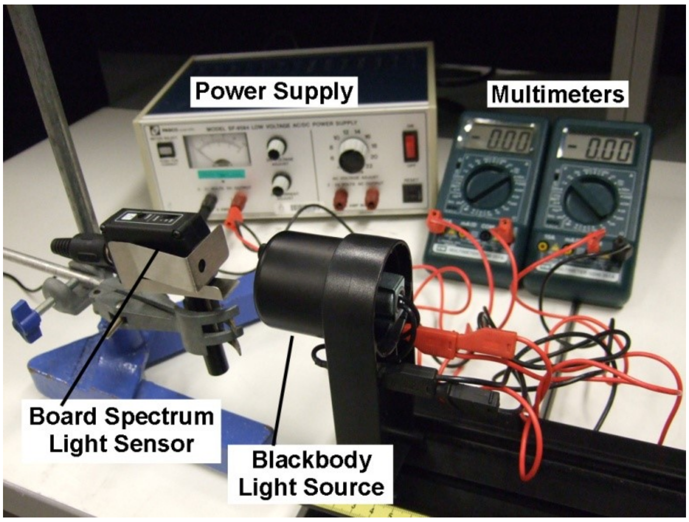


Figure 6 Experimental setup for temperature dependence of the blackbody radiation

4. Connect the DIN-5 plug of the light sensor to Analog Channel A on the interface box. Set the light sensor gain switch to “×1”.

5. Position the light sensor at the same height as the tungsten filament of the blackbody light source, with the front face of the light sensor approximately 3.5 cm away from the filament. **NOTE**: No other object other than the light source be near the light sensor.

6. Open the “MP2-II” *DataStudio* program in the course folder of the computer. The program will open with a digital display of the light intensity (V).

7. Start the measurement by clicking the “Start” icon in the program.

8. Press the tare button on the light sensor.

9. Turn on the DC power supply and apply 2V to the blackbody light source. **CAUTION:** To save the lifetime of the light bulb, turn on the light bulb only when taking the measurements. The maximum voltage of the light source is 10V and DO NOT exceed this limit under any circumstances.

10. Measure the voltage across and current pass through the tungsten filament (light source) and record the values in Table 2.

11. Open the shutter in front of the light sensor and measure the light intensity quickly and record the value in Table 2.

12. Turn off the light source after each measurement.

13. Repeat steps 8 to 12 with multiple increments of 1V each time until the voltage across the tungsten filament reaches 9V.

14. Calculate the resistance of the tungsten filament (light source) and record the values in Table 2. Use the equations in Appendix A and calculate the corresponding tungsten filament temperature. Record the values in Table 2.

Appendix A: Finding the temperature of tungsten filament

The temperature of the tungsten filament is calculated using the temperature dependence of its resistivity

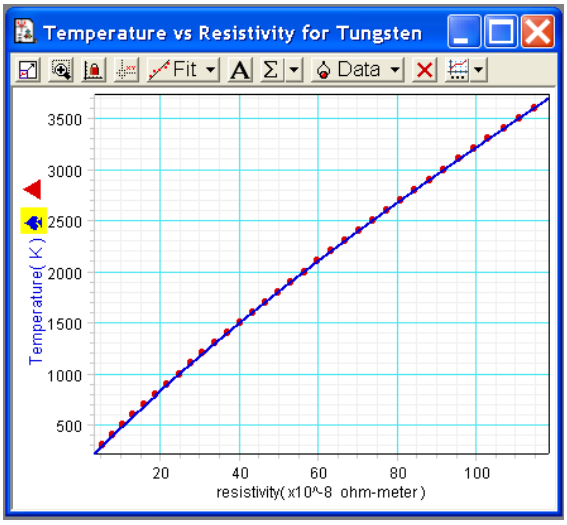
$$\rho = \rho_0[1 + \alpha(T - T_0)]. \tag{A1}$$

Equation (A1) can be rewritten as

$$T = T_0 + \frac{\rho/\rho_0 - 1}{\alpha}. \tag{A2}$$

However, the value of the temperature coefficient α is not a constant over the temperature range from room temperature to the temperature of a hot filament. The table below shows the resistivity values over a broader range of temperatures.

resistivity ($\times 10^{-8}\Omega\cdot$m)	Temperature (K)	resistivity ($\times 10^{-8}\Omega\cdot$m)	Temperature (K)
5.65	300	56.67	2000
8.06	400	60.06	2100
10.56	500	63.48	2200
13.23	600	66.91	2300
16.09	700	70.39	2400
19.00	800	73.91	2500
21.94	900	77.49	2600
24.93	1000	81.04	2700
27.94	1100	84.70	2800
30.98	1200	88.33	2900
34.08	1300	92.04	3000
37.19	1400	95.76	3100
40.36	1500	99.54	3200
43.55	1600	103.3	3300
46.78	1700	107.2	3400
50.05	1800	111.1	3500
53.35	1900	115.0	3600



The solid line in the upper right figure shows the fitted function:

$$T(in\ K) = 103 + 38.1\rho - 0.095\rho^2 + (2.48\times10^{-4})\rho^3, \tag{A3}$$

where ρ is in unit of $\times 10^{-8}\ \Omega\cdot\text{m}$. The resistivity ρ can be determined by measuring the filament resistance R using the voltage V across the filament and the current I through the filament and Ohm's law R = V/I. With the same filament, we have

$$\rho/\rho_0 = R/R_0, \tag{A4}$$

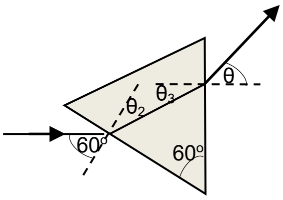
where ρ_0 and R_0 are, respectively, the resistivity and resistance of the tungsten filament at room temperature. Given that $\rho_0 = 5.50\times10^{-8}\ \Omega\cdot\text{m}$ at T = 20 °C, the resistivity ρ becomes:

$$\rho = \rho_0\frac{R}{R_0} = 5.5\frac{R}{R_0}(\times10^{-8}\ \Omega\cdot m) \tag{A5}$$

Together with Eq. (A3), one can find the temperature of the tungsten filament.

Appendix B: Calculations in the *DataStudio* program

<u>Wavelength Calculation:</u> The index of refraction n of the prism glass varies with the wavelength of the light. To determine the wavelength as a function of the angle θ, the relationship between the index of refraction and the angle is determined using Snell's Law at each surface of the prism.



Given that $\theta_2 + \theta_3 = 60^o$, it can be shown that

$$n = \sqrt{\left(\frac{2}{\sqrt{3}}\sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}}. \tag{B1}$$

The Cauchy equation gives the relationship between the index of refraction and the wavelength:

$$n(\lambda) = \frac{a}{\lambda^2} + b, \tag{B2}$$

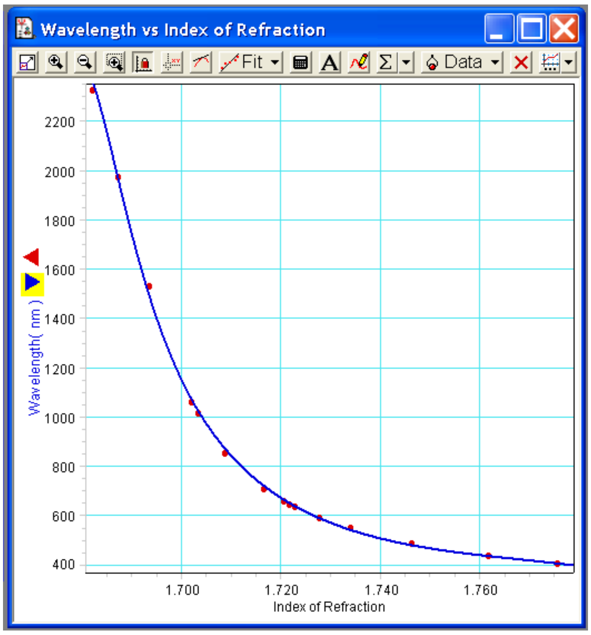
where a and b are specific to the type of glasses and are determined experimentally.

Solving Eq. (B2) for wavelength gives

$$\lambda = \sqrt{\frac{a}{n-b}}. \tag{B3}$$

However, Equation (B3) is only an approximation, which does not fit the data well in the region of interest. The following table summarizes the dependence of the index of refraction on wavelength for the prism used in this experiment (provided by the supplier of the prism):

Index of Refraction	Wavelength (nm)
1.68	2325.40
1.69	1970.10
1.69	1529.60
1.70	1060.00
1.70	1014.00
1.71	852.10
1.72	706.50
1.72	656.30
1.72	643.00
1.72	632.80
1.73	589.30
1.73	546.10
1.75	486.10
1.76	435.80
1.78	404.70



The wavelength is calculated using an empirical equation (solid line in the right figure) derived from a curve fitting to the data points,

$$\lambda = \frac{3\times 10^3}{\sqrt{A + Bn + Cn^2 + Dn^3 + Jn^4 + Fn^5 + Gn^6 + Hn^7 + In^8}} \quad \text{(B4)}$$

where

$$\begin{aligned} &A = -4.98552133 \text{ x } 10^7, \quad B = 8.60920189 \text{ x } 10^7 \\ &C = -2.998332835 \text{ x } 10^7, D = -1.435423656 \text{ x } 10^7 \\ &F = 5.64743202 \text{ x } 10^6, \quad G = 1.86343886 \text{ x } 10^6 \\ &H = -2.71922618 \text{ x } 10^6, \ I = 5.7496782 \text{ x } 10^5 \\ &J = 8.3542505 \text{ x } 10^5. \end{aligned} \quad \text{(B5)}$$

The coefficients in Eq. (B5) are determined experimentally for the type of flint glass the prism was made from. It is not necessary to change this calculation unless a different type of prism from the one supplied in the experiment is used.

Equation (B1) is expressed in the calculator of the *Datastudio* program as:

$$n = filter\left\{1.697, 10, \sqrt{\left(1.1547\sin\left(\frac{Init - filter(0,20,Angle)}{Ratio}\right) + 0.5\right)^2 + 0.75}\right\} \quad \text{(B6)}$$

where the purpose of the filter is to keep the function from getting out of range. The angle of diffraction θ is expressed as,

$$\theta = \frac{Init - Angle}{Ratio}, \quad \text{(B7)}$$

where the *Ratio* between the diameter of the degree plate and the pinion diameter of the Rotary Motion Sensor is designed to be 59.50 to 1. In other words, the pinion rotates 59.50 times for one rotation of the Degree Plate. This ratio is included in a calculation for the actual angular displacement of the Degree Plate as it turns during the measurement of a spectrum. Note that both the initial angle (*Init*) and the angular position (*Angle*) are measured in radians by the Rotary Motion Sensor. The initial angle (Init) is set as 72.4 and a more precise measurement for this value will be determined in the experiment.

Light Intensity Calculation: The light intensity is smoothed using eight data points measured by the broad spectrum light sensor to eliminate noise:

$$I = smooth(8, \text{v}) - V_0, \quad \text{(B8)}$$

where v is the voltage output of ChA (voltage output of the light sensor) and V_0 is an optional offset, in case the light sensor has not set to zero (tare) before recording the data.

Name	
Date	
Lab session (Day & time)	
Lab partner	

MP2 Blackbody Radiation Lab Report

A. Answer the following question BEFORE the first lab session (10 pts each)

1. Show that $\lambda_{max} \propto 1/T$ [see Eq. (2)] using Eq. (1). You may use the fact that when $hc/(\lambda k_B T)$ is small, one may expand $\exp\{hc/(\lambda k_B T)\}$ up to the second order terms of $hc/(\lambda k_B T)$. Find the value of λ_{max} for the human-body radiation using Eq. (2).

2. Show that $P \propto T^4$ [see Eq. (4)] using Eq. (1). You only need to show the scaling with correct units and you do not need to work out the integration constant. Find the value of P for the human-body radiation using Eq. (4).

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3. Calculate the surface temperature of the sun assuming it radiates like a blackbody. (Useful information: average radius of the earth's orbit $r_e \approx 1.5 \times 10^{11}$ m, radius of the sun $r_s \approx 7 \times 10^8$ m, and sunlight power at the earth's surface $P \approx 1.4$ kW/m^2)

B. Experimental results (92 pts)

Table 1 Spectrum of the blackbody radiation (45 pts)

Resistance R_0 of the tungsten filament at room temperature = ___________________

Run	#1	#2	#3	#4
Tungsten filament voltage ( )				
Tungsten filament current ( )				
Tungsten filament resistance ( )				
Tungsten filament temperature ( )				
Peak wavelength ( )				

Table 2 Temperature dependence of the blackbody radiation (47 pts)

Resistance R_0 of the tungsten filament at room temperature = ___________________

Tungsten filament voltage ( )	**Tungsten filament current ( )**	**Light intensity ( )**	**Tungsten filament resistance ( )**	**Tungsten filament temperature ( )**

C. Data analysis and questions (13 pts each)

4. Use the data in Table 1 and plot the peak wavelength λ_{max} as a function of the reciprocal of the tungsten filament temperature (1/T). Fit the data to a linear function and record the fitting results in your lab report. Attach your plot to the lab report.
5. Does the peak wavelength λ_{max} changes with the temperature as predicted by Eq. (2)? If so, how does the slope of the linear fit compare with the value given in Eq. (2)? What is the potential source of errors in this experiment?

6. Consider your graph of the measured intensity as a function of wavelength at the highest filament temperature. Compare the intensity in the visible part of the spectrum (area underneath the intensity *vs.* wavelength curve) with that in the infrared part of the spectrum. Which one is larger? How could a light bulb be made more efficient so that it can put out more light in the visible range?

7. Use the data in Table 2 and plot the logarithm (base 10) of the light intensity *S* as a function of the logarithm of the tungsten filament temperature [$\log_{10}(T)$]. Fit the data to a linear function and record the fitting results in your lab report. Attach your plot to the lab report.
8. From the graph and your fitting, determine the relationship between *S* and *T*. Does this relationship agree with the Stefan-Boltzmann law? What is the potential source of errors in this experiment?

9. What sources of thermal radiation, other than the lamp filament, might have influenced your measurements? What effect would you expect these sources to have on your results?

D. Summary of the experiment (20 pts)

Summarize what you have done and your interpretation of the results, especially in respect to how they match the goal of the experiment and what you have learnt from this experiment (2-3 paragraphs, less than 450 words).