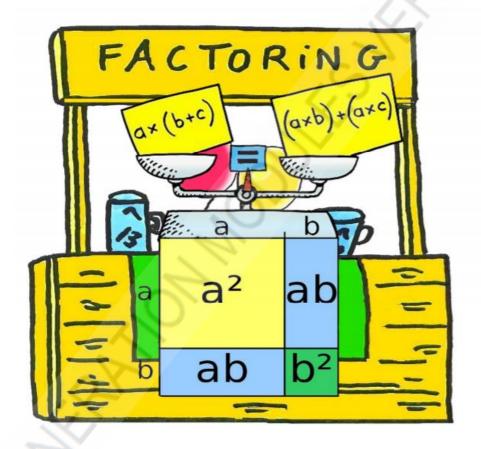


Mathematics

First Quarter - Module 10:

Factoring Polynomials





Covernment Prodective

Mathematics - Grade 10 Alternative Delivery Mode

Quarter 1 – Module 10 : Factoring Polynomials

First Edition, 2020

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First Quarter - Module 10:

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Introductory Message

This module is carefully designed to continually facilitate learners to achieve mastery on the Most Essential Learning Competencies and develop their 21st century skills. This module consists of essential components developed appropriately for self-instructional mode of learning. The components come in various developmental purposes that are designed to diagnose (pretest), recall and associate (review), discuss, explain and even provide practice activities, enrichment tasks, assessments and answer keys.

Upon taking the pretest, determine whether you need to take or skip this module. At 100% accuracy, you possess the mastery of the topic in the module; hence, you don't need to take it and you may choose to proceed to the next module. At 99% and below, you are recommended to undertake the module to acquire the necessary skills.

Though allowed, adult supervision is limited only to providing assistance in accomplishing this module. It is highly recommended that YOU, the learner, should try to engage independently in doing the different tasks for you to become a critical thinker and problem solver which are the twin goals of Mathematics.

May this module be utilized to its fullest extent in the purpose of learning the competencies construed as Most Essential for a learner in this level.

God bless and enjoy learning!

PAMELA A. RODEMIO

Education Program Supervisor - MATH



Factoring Polynomials

Most Essential Learning Competency:

The learner factors polynomials. (M10AL-Ih-1)



In Mathematics, factoring is the breaking part of a polynomial into a product of another smaller polynomials. In this lesson the learners will be able to factor polynomials using factor theorem, remainder theorem, rational root theorem or even synthetic division.

In this lesson the learner:

- 1. illustrates factorization
- 2. factors polynomials
- 3. shows reliance when working independently



Find out how much you already know about the content of this module. Write the letter that corresponds to your answer on a separate answer sheet. Take note of the items that you were not able to answer correctly and find the right solution as you go through this module.

1. Which of the following is a factor of $x^4+x^3-x^2-x-18$?

A.
$$x - 2$$

$$B.x-5$$

2. Factor $x^3+2x^2-9x-18$.

A.
$$(x+3)(x-3)(x-2)$$

B.
$$(x+3)(x+3)(x-2)$$
 C. $(x+3)(x-3)(x+2)$

C.
$$(x+3)(x-3)(x+2)$$

D.
$$(x+3)(x+3)(x+2)$$

3. Factor $8x^3 - 729$ completely.

A.
$$(2x - 9)(4x^2 - 18x + 81)$$

B.
$$(2x+9)(4x^2 - 18x + 81)$$

C.
$$(2x + 9)(4x^2 + 18x + 81)$$

D.
$$(2x - 9)(4x^2 + 18x + 81)$$

4. Factor $P(x) = x^4 + x^3 + x^2 + x$.

A.
$$x(x+1)(x^2+1)$$

B.
$$x(1) (x^2+1)$$

C.
$$x(x-1)(x^2+1)$$

D.
$$x(-1)(x^2+1)$$

5. Which of the following is a factor of $f(x) = x^3 - 5x^2 - 29x + 105$?

$$B.x-3$$

D.
$$x - 4$$

6. What is the other factor of $p(x) = x^3 + 3x^2 - 5x - 10$ when one of its factor is x - 2?

A.
$$x^2+5x-5$$

B.
$$x^2+5x+5$$

C.
$$x^2 - 5x + 5$$

D.
$$x^2$$
- 5x+5

7. Which of the following is NOT a factor of $x^4 - 4x^3 - 5x^2 + 36x + 36$?

$$A.x+2$$

B.
$$x - 2$$

D.
$$x - 3$$

8. Which polynomial is equal to $25y^2 - x^2$?

A.
$$(5y+x)(5y+x)$$

B.
$$(5y+x)(5y-x)$$

C.
$$(5x - y)(5y-x)$$

$$D.(5x+y)(5y-x)$$

- 9. Which of the following has factors (x+2)(x-2) and (x-4)?
 - A. $x^3 + 4x^2 4x + 16 = 0$

B. $x^3 - 4x^2 - x - 16 = 0$

C. $x^3 - x^2 - x + 16 = 0$

- D. $x^3 4x^2 4x + 16 = 0$
- 10. Which of the following cubic polynomials has roots $\frac{-3}{2}$, 2 and 2?
 - A. $2x^3 + 3x^2 8x 12 = 0$

B. $2x^3 + 3x^2 + 8x - 12 = 0$

C. $2x^3 + 3x^2 + 8x + 12 = 0$

- D. $2x^3 3x^2 8x 12 = 0$
- 11. Which of the following is the factor of $x^2 6x 16$?
 - A. (x 8)(x 2)

C. (x - 8)(x + 2)

B. (x + 8)(x + 2)

- D. (x + 8)(x 2)
- 12. Which of the following has the factor of (x + 8)(x 6)?
 - A. $x^2 + 2x 48$

C. $x^2 + 14x - 48$

B. $x^2 - 2x + 48$

- D. $x^2 14x 48$
- 13. Which of the following has the factor of 3xy(x+7)(x-1)?
 - A. $-3x^3y 18x^2y 21xy$
- C. $3x^3y + 18x^2y 21xy$

B. $3x^3y + 18x^2y + 21xy$

D. $3x^3y - 18x^2y - 21xy$

- 14. What is the other factor of $x^2 + 10x + 16$ when one of its factors is x + 8?
- A. x 2B. x + 2C. x - 8

- D. x + 8
- 15. Which of the following is NOT a factor of $2x^3 4x^2 6x$?
 - A. 2*x*
- B. x 3
- C. x + 1
- D. x 1



Use the Remainder Theorem to find the remainder when the given polynomial is divided by each binomial.

- 1. $P(x) = x^3 7x + 5$
 - a. x -1
- b. x+1
- c. x 2
- 2. $P(x) = 4x^4 3x^3 x^2 + 2x + 1$
 - a. x 1
- c. x 2
- 3. $P(x) = 2x^4 3x^3 + 4x^2 + 17x + 7$
 - a. 2x 3
- b. 2x+3
- c. 3x 2



What's New

Task1

- 1. Let $f(x) = 2x^3 + x^2 7x 6$. Use the factor theorem to determine whether each of the following is a factor of f(x).
 - a. x 1

- b. 2x+3
- 2. It is given that $f(x) = x^3 + 6x^2 + 5x 12$.
- a. Show that x 1 is a factor of f(x).
- b. Hence, factorize f(x) completely.

Task 2

- 1. Factorize $2x^3 5x^2 6x + 4$.
- 2. Hence, solve $2x^3 5x^2 6x + 4 = 0$



Considering the factors of the leading coefficient and the constant term, all the possible linear factors of the polynomial f(x) can be found. Let us see the following examples.

1. Let $f(x) = 2x^3 + x^2 - 7x - 6$. Use the factor theorem to determine whether each of the following is a factor of f(x).

$$b. 2x+3$$

Solution:

a.
$$f(1) = 2(1)^3 + (1)^2 - 7(1) - 6$$

= 2+1 - 7 - 6
= -10

Thus, x - 1 is not a factor of f(x).

b.
$$f(\frac{-3}{2}) = 2(\frac{-3}{2})^3 + (\frac{-3}{2})^2 - 7(\frac{-3}{2}) - 6$$

= $-\frac{27}{4} + \frac{9}{4} + \frac{21}{2} - 6$
= 0

Hence, 2x+3 is a factor of f(x).

- 2. It is given that $f(x) = x^3 + 6x^2 + 5x 12$.
- a. Show that x 1 is a factor of f(x).
- b. Hence, factorize f(x) completely.

Solution:

a. Since
$$f(1) = (1)^3 + 6(1)^2 + 5(1) - 12$$

= 1+6+5-12
= 0

Thus x - 1 is a factor of f(x).

b. By synthetic division,

The depressed equation is $x^2+7x+12 = 0$.

Therefore,
$$f(x) = (x-1)(x^2+7x+12)$$

= $(x-1)(x+3)(x+4)$.

Considering the factors of the leading coefficient and the constant term, all the possible linear factors of the polynomial f(x) can be found. Let us see the following.

1. Factorize
$$2x^3 - 5x^2 - 6x + 4$$
.
Since $f(\frac{1}{2}) = 2(\frac{1}{2})^3 - 5(\frac{1}{2})^2 - 6(\frac{1}{2}) + 4$
 $= \frac{1}{4} - \frac{5}{4} - 3 + 4$
 $= 0$

Then 2x - 1 is a factor of f(x).

By synthetic division,

Thus,
$$2x^3 - 5x^2 - 6x + 4 = (2x-1)(x^2 - 2x - 4)$$
.
2. $2x^3 - 5x^2 - 6x + 4 = 0$
 $(2x - 1)(x^2 - 2x - 4) = 0$
 $x = \frac{1}{2}$ and by quadratic formula $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$
 $= \frac{2 \pm \sqrt{20}}{2}$
 $= \frac{2 \pm 2\sqrt{5}}{2}$



A. Independent Activity 1

Find the missing factor in each of the following. Write your answers in your notebook.

1.
$$x^3 - 8 = (x-2)(\underline{\hspace{1cm}}$$

2.
$$2x^3+x^2-23x+20=(x+4)($$

3.
$$3x^3 + 2x^2 - 37x + 12 = (x - 3)($$

4.
$$x^3 - 2x^2 - x + 2 = (x - 2)$$

5.
$$2x^3 - x^2 - 2x + 1 = (2x - 1)($$
______)

B. Independent Assessment 1

Determine the factors of the following.

1.
$$x^3 - 4x^2 + 4x - 3$$

$$2. x^3 + 2x^2 - 11x + 20$$

3.
$$3x^3 - 17x^2 + 22x - 60$$

$$4.4x^3+20x^2-47x+12$$

5.
$$4x^4 - 2x^3 - 4x^2 + 16x - 7$$

C. Independent Activity 2

Factorize the following:

1.
$$x^3 - 9x$$

$$2.2x^3+x^2-8x-4$$

3.
$$x^3$$
- $7x^2$ +11x - 5

4.
$$x^3 + 2x^2 - 5x - 6$$

$$5.2x^3 - 14x - 12$$

D. Independent Assessment 2

Find the missing factor in each of the following.

1.
$$4x^4 - 2x^3 - 4x^2 + 16x - 7 = (2x-1)($$

2.
$$3x^3 - 17x^2 + 22x - 60 = (x - 5)($$
_____)

3.
$$4x^3 + 20x^2 - 47x + 12 = (2x - 3)($$
_____)

4.
$$x^3 + 2x^2 - 11x + 20 = (x+5)($$
_____)

5.
$$x^3 - 4x^2 + 4x - 3 = (x - 3)($$
_____)

E. Independent Activity 3

Read and comprehend each question. Write the letter that corresponds to the correct answer on your answer sheet.

1. Factor $x^2 - 64$.

A.
$$(x-64)(x-1)$$
 B. $(x+8)(x+8)$

B.
$$(x + 8)(x + 8)$$

C.
$$(x - 8)(x - 8)$$

D.
$$(x + 8)(x - 8)$$

2. Which of the following is the factor of $x^3 - 7x^2 + 12x$?

A.
$$x(x-4)(x-3)$$
 B $x(x+4)(x-3)$ C. $x(x+4)(x+3)$ D. $x(x-4)(x+3)$

B
$$x(x + 4)(x - 3)$$

C.
$$x(x + 4)(x + 3)$$

D.
$$x(x-4)(x+3)$$

3. Which of the following has the factors 2(5x + 1) and $(25x^2 - 5x + 1)$?

A.
$$250x^3 + 2$$

B.
$$250x^3 - 2$$

C.
$$250x^3 + 50x^2 + 2$$
 D. $250x^3 - 50x^2 - 2$

2 D.
$$250x^3 - 50x^2 - 2$$

4. What is the other factor of $x^2 - x - 30$ when one of its factors is x - 6? C. x + 5

A.
$$x - 3$$

B.
$$x + 3$$

$$\mathbf{D}$$
. $\chi = 3$

5. Which of the following is NOT a factor of $6x^3 + 48$?

$$C.x + 2$$

D.
$$x^2 - 2y + 4$$

F. Independent Assessment 3

Read and comprehend each question. Write the letter that corresponds to the correct answer on your answer sheet.

1. Factor $x^3 - 343$.

A.
$$(x + 7)(x^2 + 7x + 49)$$

C.
$$(x+7)(x^2-7x-49)$$

B.
$$(x-7)(x^2+7x+49)$$

D.
$$(x-7)(x^2+7x+49)$$

2. Which of the following is the factor of $x^3 - 14x^2 + 49x$?

A.
$$x(x+7)(x-7)$$
 B $(x+7)(x+7)$

$$D(x \mid f)(x \mid f)$$

C.
$$-x(x-7)(x+7)$$
 D. $x(x-7)^2$

3. Which of the following has the factors 7(x + y) and (x - y)?

A.
$$7x^3 + 7y^3$$

B.
$$7x^2 + 7y^2$$

C.
$$7x + 7y$$

D.
$$7x^3 + 7y^1$$

4. What is the other factor of $8x^6 - 125y^6$ when one of its factors is $2x^2 - 5y^2$?

A.
$$4x^4 + 10x^2y^2 - 25y^4$$

C.
$$4x^4 + 10x^2y^2 + 25y^4$$

B.
$$4x^4 - 10x^2y^2 + 25y^4$$

D.
$$4x^4 - 10x^2y^2 - 25y^4$$

5. Which of the following is NOT a factor of $40x^2 + 28x - 48$?

A.
$$2x + 4$$

B.
$$2x + 3$$

C.
$$5x - 4$$



What I Have Learned

The following activities will help you assess your understanding on factoring polynomials. Show your solutions on your answer sheet.

- 1. Angel states that $x^2 5x 36$ and $36 5x x^2$ has the same factors. Is she correct? Prove by showing your solution.
- 2. Why is the difference of two squares be applicable to $5x^3$ 45? What are its factors?
- 3. Determine the value of M so that (x 1) is a factor of $2x^3 + x^2 + 2Mx + 5$?



What I can Do

The following activities will help you use factoring polynomials to solve real-world problems. Show your solutions on your answer sheet.

- 1. The volume of a swimming pool is $2x^3 + 5x^2 28x 15$. If the height of the pool is (x-3) meters, what are the dimensions of the rectangular base of the pool?
- 2. Create a solid figure from a rectangular piece of paper or used folder that measures 15 inches by 9 inches.

Steps:

- 1. Cut out 2-in by 2-in square in all edges of the material.
- 2. Fold all the sides upward.
- 3. Tape the corners to form a solid figure.

Answer the following:

- a. What is the area of the material used?
- b. What is the volume of the solid figure?
- c. If the sides of the squares cut is x, give the dimensions (length, width & thickness) of the solid figure in terms of x.
- d. What is the volume of the solid figure in terms of x?



Read and comprehend each question. Write the letter that corresponds to the correct answer on your answer sheet.

1. What is the other factor of $p(x) = x^3 + 2x^2 - 20x + 15$ when one of its factor is x - 3?

A.
$$x^2+5x-5$$

B.
$$x^2+5x+5$$

C.
$$x^2 - 5x + 5$$

D.
$$x^2$$
- 5x+5

2. Factor $x^3 - 8$ completely.

A.
$$(x-2)(x^2+2x-4)$$

C.
$$(x +2)(x^2 +2x - 4)$$

B.
$$(x-2)(x^2+2x+4)$$

D.
$$(x + 2)(x^2 + 2x + 4)$$

3. Which polynomial is equal to $36y^2 - x^2$?

A.
$$(6y+x)(6y+x)$$

B.
$$(6y+x)(6y-x)$$

C.
$$(6x - y)(6y-x)$$

$$D.(6x+y)(6y-x)$$

4. Factor $P(x) = x^4 + x^3 - x^2 - x$.

A.
$$x(x+1)(x^2-1)$$

C.
$$x(x-1)(x^2+1)$$

B.
$$x(1) (x^2+1)$$

D.
$$x(-1)(x^2+1)$$

5. Which of the following is NOT a factor of $x^4 - 4x^3 - 5x^2 + 36x + 36$?

$$A.x+2$$

D.
$$x - 3$$

6. Which of the following cubic polynomials has roots -3, 3, -2 and 2?

A.
$$x^4$$
- $13x^2$ - $36 = 0$

C.
$$x^4$$
- $13x^2$ - $36 = 0$

B.
$$x^4$$
- $13x^2$ +36 = 0

D.
$$x^4 + 13x^2 - 36 = 0$$

7. Which of the following has factors (x+2)(x-2) and (x+4)?

A.
$$x^3+4x^2-4x+16=0$$

B.
$$x^3 + 4x^2 - 4x - 16 = 0$$

C.
$$x^3 - 4x^2 - 4x + 16 = 0$$

D.
$$x^3 - 4x^2 - 4x - 16 = 0$$

6

- 8. Which of the following is a factor of $x^4 x^3 4x^2 + 8x 8$?
- B. x 5
- C. x+7
- D. x+5

- 9. Factor $x^3 2x^2 9x + 18$.
 - A. (x+3)(x-3)(x-2)

C. (x+3)(x-3)(x+2)

B. (x+3)(x+3)(x-2)

- D. (x+3)(x+3)(x+2)
- 10. Which of the following is a factor of $f(x) = x^5 3x^4 5x^3 + 18x^2 11x + 6$?
 - A. x- 5
- B. x 3
- C. x+3
- D. x 4

- 11. Which of the following is the factor of $x^2 + 3x + 2$?
 - A. (x-2)(x-1)

C. (x-2)(x+1)

B. (x + 2)(x + 1)

- D. (x + 2)(x 1)
- 12. Which of the following has the factors (x 7y) and (x 6y)?
 - A. $x^2 + 13xy 42y^2$

C. $x^2 - xy + 42y^2$

B. $x^2 - 13xy + 42y^2$

- D. $x^2 + xy 42y^2$
- 13. Which of the following is equal to 5xy(x-5)(x-2)?
 - A. $5x^3y + 35x^2y + 50xy$

C. $5x^3y + 35x^2y - 50xy$

- B. $5x^3y 15x^2y + 50xy$
- D. $5x^3y 35x^2y + 50xy$ 14. What is the other factor of $x^2 - 8x - 9$ when one of its factors is x + 1?
- A. x 3B. x + 3
- C. x 9
- D. x + 9
- 15. Which of the following is NOT a factor of $2x^3 2x^2 4x$?
 - A. x + 2
- B. x 2
- D. 2*x*

For each item below, give a polynomial equation with integer coefficients that has the following roots.

- 1. 1, 3, 6
- $2.\pm 2,\pm 7$
- 3. 0, -4, -5, ± 1



What I Know

19. D	A .01	9 [.] B
14. B	a .6	A .4
13. C	8. B	3. D
12. A	A .7	2. C
J.11	8 · B	A.1

What's In

1. a. P(1)= -1
b. P(-1) = 11
c. P(2) = -1
2. a. P(1) = 3
b. P(-1) = 5
c. P(2) = 41
3. a. P(3/2) = 41
3. a. P(
$$\frac{3}{2}$$
) = 10 $\frac{1}{4}$
b. P($\frac{3}{2}$) = 10 $\frac{1}{4}$

What's More

A. Independent Activity 1

B. Independent Assessment 1

1.
$$(x-3)(x^2-x+1)$$

2. $(x+5)(x^2-3x+4)$
3. $(x-5)(3x^2-2x+12)$
4. $(2x-3)(2x^2+13x-4)$
5. $(2x-1)(2x^3-2x+7)$

C. Independent Activity 2

D. Independent Assessment 2

E. Independent Activity 3

F. Independent Assessment 3

9 [.] B	A.3
3. A 4. C 5. B	4. C
A.£	3. B
A .S	2. D
a.r	8.1

What I Have Learned

What I can Do?

1.
$$L = (x + 5) \text{ m}$$

 $W = (2x + 1) \text{ m}$
2. $a. (15) (9) = 135 \text{ sq. inches}$
b. $(11) (5) ((2) = 110 \text{ cubic inches}$
c. $L = 15 - 2x \text{ inches}$
 $W = 9 - 2x \text{ inches}$
 $W = 9 - 2x \text{ inches}$
T = x inches
d. $4x^3 - 48x^2 + 135x \text{ cubic inches}$

Assessment

A .31	10. B	A . B
14. C	A .e	A.4
13. D	A .8	3. B
12. B	A.7	2. B
11.8	9. B	A.f

Additional Activities

$$0 = 81 - x^{2} - 15 x - 18 = 0$$

$$2. x^{4} - 53 x^{2} + 196 = 0$$

$$3. x^{5} + 9x^{4} + 19x^{3} - 9x^{2} - 20 x = 0$$

$$4. 5x^{4} - 18x^{3} - 11x^{2} + 72 x - 36 = 0$$

$$4. 5x^{4} - 18x^{3} - 11x^{2} + 254x^{2} + 20x - 24 = 0$$

$$5. 21x^{5} - 62x^{4} - 89x^{3} + 254x^{2} + 20x - 24 = 0$$

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