Module 14.1

Rational Exponents And Radicals

How can you use rational exponents and radicals to solve real-world problems?

P. 637

What is an Exponent?

The **exponent** of a number is how many times to use that number in a multiplication. It is written as a small number to the right and above the base number. It's also called a "power" or an "index".

Exponent

$$8^{3} = 8 \times 8 \times 8$$

Base number

Be Careful About Grouping

To avoid confusion, use parentheses () in cases like this:

With () :	$(-2)^2 = (-2) \times (-2) = \mathbf{4}$
Without () :	$-2^2 = -(2^2) = -(2 \times 2) = \mathbf{-4}$

With () :	$(ab)^2 = ab \times ab$
Without () :	$ab^2 = a \times (b)^2 = a \times b \times b$

1) The Zero Exponent Rule

Nine Exponent Rules

Any number (excluding 0) to the 0 power is always equal to 1.

Examples:

- $6^0 = 1$
- $147^0 = 1$
- $55^0 = 1$

But: 0^0 is undefined.

2) The One Exponent Rule

Any number to the 1st power is always equal to that number.

Examples:

- $x^1 = x$
- $7^1 = 7$
- $53^1 = 53$
- $0^1 = 0$

3) The Negative Exponent Rule

Any number raised to a negative power is equal to
1 divided by the number raised to a positive power.

Example:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16} \qquad c^{-6} = \frac{1}{c^6}$$

And the reverse is also true. $\frac{1}{y^{-8}} = y^8$

$$\frac{2^2}{3^{-2}} = 2^2 \times 3^2 = 4 \times 9 = \mathbf{36}$$

$$\frac{2^{-3}}{5^{-4}} = \frac{5^4}{2^3} \checkmark$$

4) The Zero-To-Exponent Rule

0 to any power is 0.

Examples:

- $0^1 = 0$
- $0^7 = 0$

But: As we said, 0^0 is undefined.

Let's Practice:

5^{-2}

$y^{-4} =$

$3x^{-4} =$

$\left(\frac{3}{4}\right)^{-1}$

$2^{-5} =$

$(3x)^{-4} =$

$(-3)^{-3}$

$\frac{1}{m^{-2}} =$

$\frac{1}{x^{6}} =$

5) The Product Of Powers Rule

When a number to a power is multiplied by the same number to a power, add the powers.
Example:

$$2^2 \cdot 2^3 = (2 \cdot 2)(2 \cdot 2 \cdot 2) = 2^5 = 2^{2+3}$$

SO: $3^2 + 3^3 = 9 + 27 = 36$, not 3^5 (243)

Let's Practice:

$6^4 \cdot 6^2$

x^3x^4

$d^{-2}(d^2)$

Group Similar Components

Simplify: $3a^5b^7 \times -7a^2b^4$

$= 3 \times a^5 \times b^7 \times -7 \times a^2 \times b^4$

$= 3 \times -7 \times a^5 \times a^2 \times b^7 \times b^4$

$= -21 \times a^{5+2} \times b^{7+4}$

$= -21a^7b^{11}$

Simplify: $-2d^4k^4 \times -dk$

$= -2 \times d^4 \times k^4 \times -1 \times d^1 \times k^1$

$= -2 \times -1 \times d^4 \times d^1 \times k^4 \times k^1$

$= 2 \times d^{4+1} \times k^{4+1}$

$= 2d^5k^5$

$2y^{-1} \cdot 3xy$

$2x^3y^4 \cdot 4x^{-1}y^2 \cdot 4x^{-2}$

$-3x^4y^3z^2 \cdot -2x^3y^4$

$4u^2 \cdot 2u^{-1}v^{-3}$

$3a^{-2}b^3 \cdot a^4b^2 \cdot 2ab^{-2}$

$-3h^3j^2k^2 \cdot -2h^0j^2k^2$

6) The Power Of A Power Rule

When a number to a power is raised to another power, multiply the powers.
Example:

$$\left(2^3\right)^2 = (2\cdot2\cdot2)^2 = (2\cdot2\cdot2)(2\cdot2\cdot2) = 2^6$$

Let's Practice:

$(w^4)^2$

$(j^4)^{-3}$

$(q^2)^8$

$d^{-2}(d^2)^{-2}$

$(x^3)^4$

$m^5(g^{-3})^2m^{-5}g^7$

7) The Quotient Of Powers Rule

When a number to a power is divided by the same number to a power, subtract the two powers.

Example:

$$\frac{2^3}{2^2} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2} = 2$$

$$= 2^{3-2} = 2^1 = 2$$

Let's Practice:

1. $\frac{b^7}{b^{15}}$

2. $\frac{a^3}{a^8}$

3. $\frac{w^7}{w^5}$

4. $\frac{b^2}{t^6}$

8) The Power Of A Product Rule

When two numbers are multiplied and raised to a power, it's equal to both numbers individually raised to that power and multiplied together.

Example:

$$(2\cdot3)^2 = (2\cdot3)(2\cdot3) = (2\cdot2)(3\cdot3) = 2^2\cdot3^2$$

Let's Practice:

Apply It To Everything In The Parenthesis

$(6y)^2$

$(-2y)^4$

$x^4 \cdot (2x^2y^{-2})^{-2}$

$(5x^2)^2$

$(7x^3)^2$

$(2x^2y^2 \cdot 2yx^{-2})^3$

$(a \cdot a^2b^2)^3$

More Practice:

1) $(2x^4y^2)^3 \cdot (2y^3)^3$

2) $(2a^3)^0 \cdot 2ab^2$

3) $(2y^4)^2 \cdot (x^4y^4)^3$

4) $u^2 \cdot (2u^4v^4)^3$

5) $uv \cdot (uv)^3$

6) $hk \cdot (h^4j^2k^2)^3$

7) $m^3 \cdot (2m^3)^0$

8) $(3v^3)^3 \cdot 2v$

9) $3a \cdot (3a)^2$

10) $(3k^3)^3 \cdot k$

11) $(5x^2)^2$

12) $(3wk^3)^3$

13) $(-2y)^4$

14) $(6x^4)^2$

15) $(n^5)^2(4mn^{-2})^3$

16) $(5x^2)^2$

9) The Power Of A Quotient Rule

When two numbers are divided and raised to a power, it's equal to both numbers individually raised to that power and divided.

Example:

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3}\cdot\frac{2}{3} = \frac{2\cdot 2}{3\cdot 3} = \frac{2^2}{3^2}$$

Let's Practice:

Apply It To Everything In The Parenthesis

1. $\left(\frac{4}{7}\right)^3$

2. $\left(\frac{2}{11}\right)^2$

3. $\left(\frac{3}{4}\right)^{-2}$

Let’s Practice More – Simplify These:

1) $\dfrac{(3y^7)^3}{9y^9}$

2) $\dfrac{(2b^{-3})^{-2}}{2b^4}$

3) $\dfrac{(2x^2y^7)^3}{x^8y^9}$

4) $\dfrac{(a^4b^{-3})^{-2}}{a^{-3}b^4}$

5) $\dfrac{(m^2n^4)^2}{(m^4n^{-2})^{-1}}$

6) $\dfrac{(3x^{-2}y^7z^2)^{-1}}{x^8y^2z}$

7) $\dfrac{2(3a^4b^{-2}c^{-2})^{-2}}{3a^2b^2c^2}$

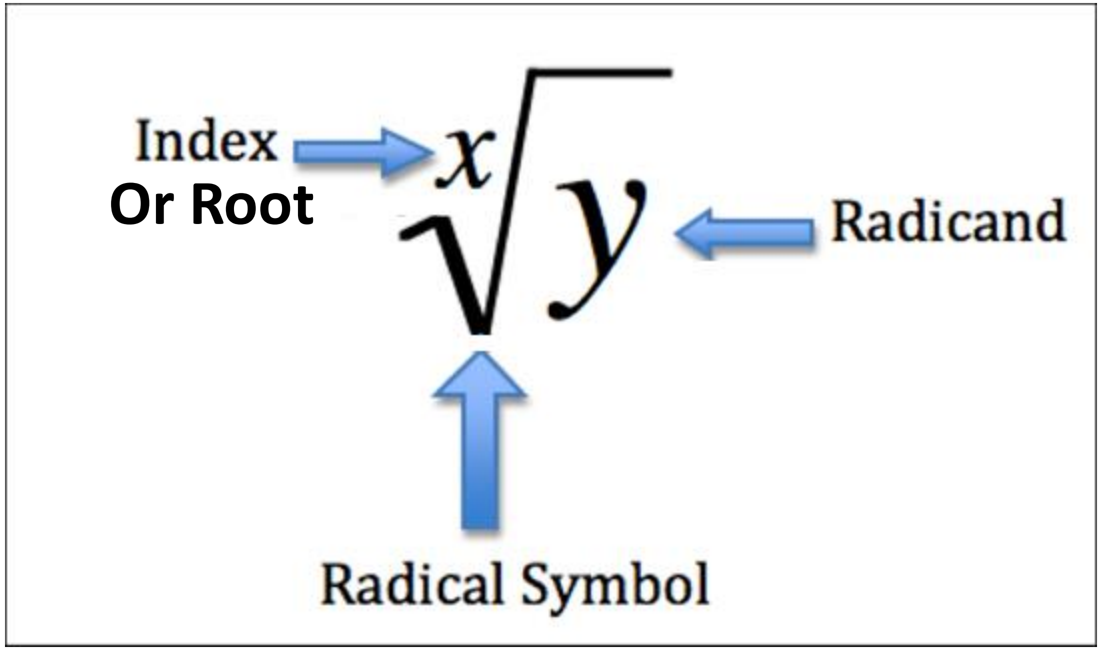
8) $-\dfrac{(6x^{-5}y^3z^{-1})^2}{(3x^4y^{-2})^2}$

$3x^{-1} \cdot 3y^{-1}$

$\dfrac{2y^{-2} \cdot 4x}{x^{-2}y^4}$

$a^{-4}b^3 \cdot 3a^2b^2 \cdot b^4$

$\dfrac{x^3}{3x^4y^{-1} \cdot 4xy^{-4}}$



We use the word “Radical” to refer to any expression within the radical symbol.

If the root doesn’t appear, it’s assumed to be 2,
and it becomes a “square root”, that is, the 2^{nd} root.

$$\sqrt{x} = \sqrt[2]{x}$$

Here are a few square roots:

$\sqrt{4} = 2$ $\sqrt{100} = 10$

Here are a few cube roots:

$\sqrt[3]{8} = 2$ $\sqrt[3]{1} = 1$ $\sqrt[3]{-64} = -4$

Simplify these:

$\sqrt[3]{8}$ $\sqrt[4]{16}$

$$\sqrt[3]{x^3} = x$$

$$\sqrt[4]{x^4} = x$$

Here's The Product Property of Radicals: $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$

Here's an example: $\sqrt{6} = \sqrt{2 \times 3} = \sqrt{2} \times \sqrt{3}$

Knowing this, here's how you can simplify a squared radical:

1) Inspect the radicand for a square factor: 4, 9, 16, 25, and so on.
2) Split the radicand into 2 parts.
3) Remove (the root of) the squared factor.

For example:

- **Simplify $\sqrt{75}$** $\sqrt{75} = \sqrt{3 \times 25} = 5\sqrt{3}$
- **Simplify $\sqrt{72}$** $\sqrt{72} = \sqrt{2 \times 36} = 6\sqrt{2}$

Product Property of Radicals

$$\sqrt{8} \cdot \sqrt{2} = \sqrt{16}$$

$$\sqrt[2]{16} \cdot \sqrt[2]{4} = \sqrt[2]{64}$$

$$\sqrt[3]{9} \ \sqrt[3]{3} = \sqrt[3]{27}$$

$$\sqrt[4]{27} \ \sqrt[4]{3} = \sqrt[4]{81}$$

Quotient Property of Radicals

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$$

$$\sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}}$$

$$\sqrt[4]{\frac{256}{16}} = \frac{\sqrt[4]{256}}{\sqrt[4]{16}}$$

Let's Practice: Simplify These…

$\sqrt{225}$ $\sqrt{300}$ $\sqrt{120}$

$\sqrt{20}$ $\sqrt{36}$ $\sqrt{32}$ $\sqrt{45}$ $\sqrt{60}$ $\sqrt{64}$

Here's The Quotient Property of Radicals: $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

Here's an example: $\sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{\sqrt{16}} = \frac{\sqrt{15}}{4}$

Simplify:

$\sqrt{\frac{1}{49}}$ $\sqrt{\frac{4}{9}}$

Here's how you can simplify a cubed radical:
1) Inspect the radicand for a cubed factor: 8, 27, 64, 125, and so on.
2) Split the radicand into 2 parts.
3) Remove (the root of) the cubed factor.
For example:

$$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\ \sqrt[3]{2} = \mathbf{3\ \sqrt[3]{2}}$$

$$\sqrt[3]{48} = \sqrt[3]{6 \times 8} = \sqrt[3]{6}\ \sqrt[3]{8} = \mathbf{2\ \sqrt[3]{6}}$$

Simplify: $\sqrt[3]{135}$ $\sqrt[3]{375}$

What if the exponent is a fraction? It's called a **Fractional Exponent**.

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\frac{m}{n} = \frac{\text{Power}}{\text{Root}}$$

Raise it to the m-th power, then take the n-th root

Take the n-th root, then raise it to the m-th power

Do Either One

Usually easier

$$4^{\frac{3}{2}} = \sqrt[2]{4^3} = \left(\sqrt[2]{4}\right)^3$$

$$= \sqrt[2]{64} = (2)^3$$

$$= 8 = 8$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\frac{m}{n} = \frac{\text{Power}}{\text{Root}}$$

Raise it to the m-th power, then take the n-th root

Take the n-th root, then raise it to the m-th power

$$27^{\frac{4}{3}} = \sqrt[3]{27^4} = \left(\sqrt[3]{27}\right)^4$$

$$= \sqrt[3]{531{,}441} = (3)^4$$

$$= 81 = 81$$

Which way is easier?

$5^{\frac{2}{3}} = \sqrt[\square]{\square^{\square}} = \left(\sqrt[\square]{\square}\right)^{\square}$

$\square = \sqrt[5]{7^2} = \left(\sqrt[\square]{\square}\right)^{\square}$

$4^{\frac{3}{4}} = \sqrt[\square]{\square^{\square}} = \left(\sqrt{\square}\right)^{\square}$

$\square^{\square} = \sqrt[\square]{\square^{\square}} = \left(\sqrt[6]{32}\right)^{3}$

$g^{\frac{h}{b}} = \sqrt[\square]{\square^{\square}} = \left(\sqrt{\square}\right)^{\square}$

$\square = \sqrt[\square]{\square^{\square}} = \left(\sqrt[t]{p}\right)^{r}$

Write each expression in radical form.

$m^{\frac{3}{5}}$ $\quad (10r)^{-\frac{3}{4}}$ $\quad (5n)^{\frac{4}{3}}$ $\quad 12a^{\frac{2}{3}}$

$(7x)^{\frac{3}{2}}$ $\quad (6b)^{-\frac{4}{3}}$ $\quad (2k)^{\frac{5}{2}}$ $\quad 6x^{\frac{5}{2}}$

$27^{2/3}$ $\quad (6x)^{-\frac{3}{2}}$ $\quad 64a^{\frac{4}{5}}$

Find the root and simplify the expression.

$64^{\frac{1}{3}}$

$27^{\frac{2}{3}}$

$81^{\frac{1}{4}} + 9^{\frac{1}{2}}$

$8^{\frac{1}{3}}$

$16^{\frac{1}{2}} + 27^{\frac{1}{3}}$

$4^{\frac{5}{2}} - 4^{\frac{3}{2}}$

$8^{\frac{2}{3}} + 8^{-\frac{2}{3}}$

$\dfrac{25^{\frac{1}{2}}}{27^{\frac{1}{3}}}$

$16^{\frac{1}{4}}$

$100^{-\frac{3}{2}}$

Rewrite in exponential form. Simplify if possible.

You must be able to convert between exponential and radical forms.

1. $\sqrt[5]{x^2}$

$\left(\sqrt[3]{5m}\right)^4$

2. $\sqrt[6]{x^3}$

$\frac{1}{\left(\sqrt[3]{x}\right)^4}$

3. $\sqrt[5]{32x^3}$

$\frac{1}{(\sqrt{6x})^3}$

$\sqrt{v}$

4. $\sqrt[3]{27d^5}$

$\frac{1}{(\sqrt[4]{n})^7}$

$\sqrt{5a}$

5. $\sqrt[4]{256a^8}$

$\sqrt[6]{10k}$

Applications

Biology The approximate number of Calories, C, that an animal needs each day is given by $C = 72m^{\frac{3}{4}}$, where m is the animal's mass in kilograms. Find the number of Calories that a 16 kilogram dog needs each day. P. 644

Biology Biologists use a formula to estimate the mass of a mammal's brain. For a mammal with a mass of m grams, the approximate mass B of the brain, also in grams, is given by $B = \frac{1}{8}m^{\frac{2}{3}}$. Find the approximate mass of the brain of a mouse that has a mass of 64 grams. P. 657

Rocket Science Escape velocity is a measure of how fast an object must be moving to escape the gravitational pull of a planet or moon with no further thrust. The escape velocity for the moon is given approximately by the equation $V = 5600 \cdot \left(\frac{d}{1000}\right)^{-\frac{1}{2}}$, where v is the escape velocity in miles per hour and d is the distance from the center of the moon (in miles). If a lunar lander thrusts upwards until it reaches a distance of 16,000 miles from the center of the moon, about how fast must it be going to escape the moon's gravity?

Geometry The volume of a cube is related to the area of a face by the formula $V = A^{\frac{3}{2}}$.

What is the volume of a cube whose face has an area of 100 cm^2?

SIMPLIFYING RADICALS

$\sqrt{160} = \sqrt{16} \cdot \sqrt{10} = 4 \cdot \sqrt{10}$ OR $4\sqrt{10}$

$\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6 \cdot \sqrt{2} = 6\sqrt{2}$

$\sqrt{4x^2} = \sqrt{4} \cdot \sqrt{x^2} = 2 \cdot x = 2x$

$\sqrt[3]{8x^3} = \sqrt[3]{8} \cdot \sqrt[3]{x^3} = 2 \cdot x = 2x$

$\sqrt[5]{32x^3} = \sqrt[5]{32} \cdot \sqrt[5]{x^3} = 2\sqrt[5]{x^3}$

$\sqrt[4]{256a^3} = \sqrt[4]{256}\ \sqrt[4]{a^3} = 4\sqrt[4]{a^3}$

Putting it all together - Simplifying Radicals with Numbers & Variables

$$\frac{\sqrt[4]{x^8}}{\sqrt[4]{x^6}}$$

$$\left(x^2y\right)^2\sqrt[4]{y^4}$$

$$\left(8x^9\right)^{\frac{2}{3}}$$

$$\left(b^3 \cdot c^5\right)^{\frac{1}{5}} =$$

$$\left(a^{\frac{3}{4}} \cdot y^{\frac{1}{2}}\right)^{\frac{2}{3}} =$$

$$\left(64x^{12}\right)^{\frac{1}{6}}$$

$$\left(\frac{1}{4x^4} \cdot x^{12}\right)^{-\frac{1}{2}}$$

$$\frac{\sqrt{8x}}{\sqrt[3]{16x}}$$

$$\sqrt[3]{\left(8x^3\right)^2}$$

$$\frac{\sqrt[4]{81c}}{\sqrt[3]{c}}$$

$$\frac{k^{\frac{4}{5}}}{k^{\frac{1}{2}}} =$$

$$b^{\frac{1}{3}} \cdot b^{\frac{3}{4}} =$$

$-4\sqrt{6p^2} \cdot \sqrt{6p^2}$

$\sqrt{6n^2} \cdot -3\sqrt{2n^3}$

$-3\sqrt{3p} \cdot -5\sqrt[3]{3p^2}$

$\sqrt[3]{4x^4} \cdot \sqrt{4x^4}$

Simplify the expression, $\left(\left(p^{-2}+\frac{1}{p}\right)^{1}\right)^{p}$, when $p=\frac{3}{4}$, in both radical and rational exponents forms.

Radical form:

Rational exponent form:

3. Write an equivalent expression in rational exponent form:

$\sqrt[8]{5^6}$

$\sqrt[4]{x^{\frac{2}{3}}}$

$\sqrt[3]{8}\left(\sqrt{8^2+8^2}\right)$

4. Determine the value of n such that $\sqrt[4]{64^{\frac{1}{3}}} = 64^{\frac{1}{n}}$.

5. Determine whether each expression is equivalent to $x^{\frac{7}{4}}$.

Expression	Yes	No
$\sqrt[7]{x^4}$	O	O
$\sqrt[4]{x^7}$	O	O
$(\sqrt[4]{x})^7$	O	O
$\sqrt{x^{\frac{7}{4}}}$	O	O
$\sqrt[4]{x^5} \cdot \sqrt[4]{x^2}$	O	O
$\sqrt[5]{x^4} \cdot \sqrt[2]{x^4}$	O	O
$\frac{(\sqrt[4]{x})^7}{(\sqrt{x})^0}$	O	O

$$\sqrt{n^6} = \sqrt[2]{(n^3)^2} = n^3$$

$$\sqrt{n^7} = \sqrt{n^6 n} = n^3\sqrt{n}$$

$$\sqrt{54n^7} = \sqrt{54}\;\sqrt{n^7} = \sqrt{9}\,\sqrt{6}\;\sqrt{n^6}\;\sqrt{n}$$

$$= 3\sqrt{6}\;n^3\sqrt{n}$$

$$= 3n^3\sqrt{6\;\sqrt{n}}$$

$$= 3n^3\sqrt{6n}$$

(A) Define rational and irrational numbers.

A rational number is any number that can be written as the ratio of two numbers, $\frac{a}{b}$, where a and b are integers and $b \neq 0$. An irrational number is a real number that is not rational.

Examples of a Rational Number:

0	
18	Integers
1.625	Decimals that end
¾	Fractions

Examples of an Irrational Number:

π	
$\sqrt{2}, \sqrt{5}$	Never ending decimals
1.2658945625692….	

Will it be Rational or Irrational?

Sums:

Rational + Rational =	Rational	$3 + 2$
Rational + Irrational =	Irrational	$3 + \pi$
Irrational + Irrational =	Usually Irrational	Exception: $\pi + (-\pi)$

Products:

Rational x Rational =	Rational	3×2
Rational x Irrational =	Irrational	$3 \times \sqrt{3}$
Irrational x Irrational =	Usually Irrational	Exception: $\sqrt{3} \times \sqrt{3}$

Determine whether each of the following is rational or irrational. Select the correct answer for each lettered part.

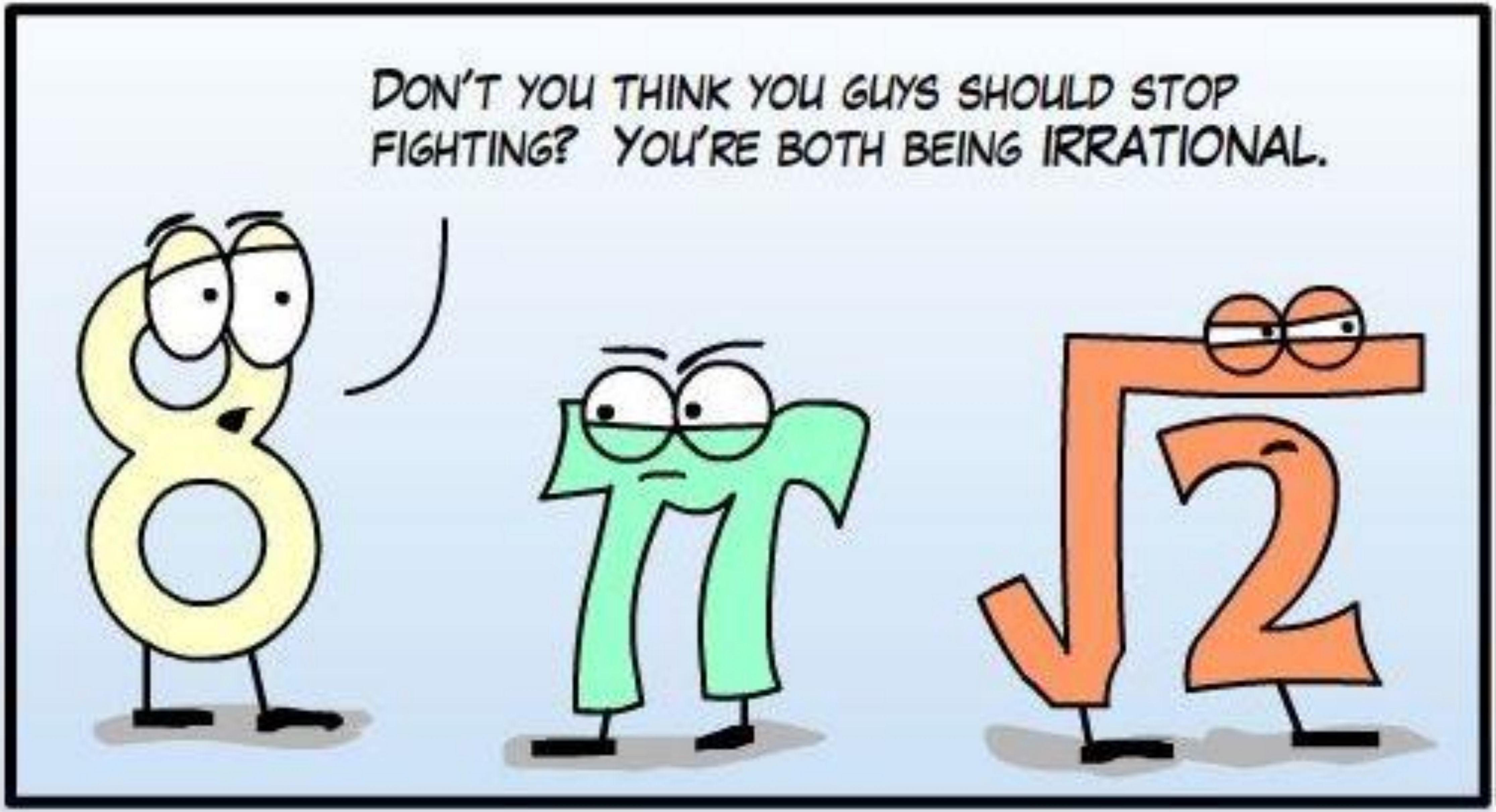
a.	The product of $\sqrt{2}$ and $\sqrt{50}$	○	Rational	○	Irrational
b.	The product of $\sqrt{2}$ and $\sqrt{25}$	○	Rational	○	Irrational
c.	$C = 2\pi r$ evaluated for $r = \pi^{-1}$	○	Rational	○	Irrational
d.	$C = 2\pi r$ evaluated for $r = 1$	○	Rational	○	Irrational
e.	$A = 2\pi r^2$ evaluated for $r = \pi^{\frac{1}{2}}$	○	Rational	○	Irrational
f.	The product of $\sqrt{\frac{2}{\pi}}$ and $\sqrt{50\pi}$	○	Rational	○	Irrational
g.	The product of $\sqrt{2}$ and $\sqrt{\frac{9}{2}}$	○	Rational	○	Irrational

Rational or Irrational?

1. Is the sum of $3\sqrt{2}$ and $4\sqrt{2}$ rational or irrational?

2. Is the sum of 4.2 and $\sqrt{2}$ rational or irrational?

3. Determine if the product of $3\sqrt{2}$ and $8\sqrt{18}$ is rational or irrational.


DON'T YOU THINK YOU GUYS SHOULD STOP FIGHTING? YOU'RE BOTH BEING IRRATIONAL.