TEACHING GUIDE

Module 2: Rational Algebraic Expressions and Algebraic Expressions with Integral Exponents

A. Learning Outcomes

1. Grade Level Standard

The learner demonstrates understanding of key concepts and principles of algebra, geometry, probability and statistics as applied, using appropriate technology, in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.

2. Content and Performance Standards

Content Standards:

The learner demonstrates understanding of key concepts and principles of rational algebraic expressions and algebraic expressions with integral exponents.

Performance Standards:

The learner is able to formulate real – life problems involving rational algebraic expressions and algebraic expressions with integral exponents and solves these with utmost accuracy using a variety of strategies.

UNPACKING THE STANDARDS FOR UNDERSTANDING

<table>
<tr><td rowspan="3">SUBJECT:
Grade 8 Mathematics
QUARTER:
Second Quarter
STRAND:
Algebra
TOPIC:
Rational Algebraic Expressions and Algebraic Expressions with Integral Exponent
Exponents
LESSONS:
1. Rational Algebraic Expressions and Algebraic Expressions with Integral Exponents
2. Operations on Rational Algebraic Expressions</td><th colspan="2">LEARNING COMPETENCIES</th></tr>
<tr><td colspan="2">Knowledge:
• Describe and illustrates rational algebraic expressions.
• Interprets zero and negative exponents.

Skill:
• Evaluates and simplifies algebraic expressions involving integral exponents.
• Simplifies rational algebraic expressions
• Performs operations on rational algebraic expressions
• Simplifies complex fractions</td></tr>
<tr><td>ESSENTIAL UNDERSTANDING:
Students will understand that rate – related problems can be modelled using rational algebraic expressions.</td><td>ESSENTIAL QUESTION:
How can rate – related problems be modelled?</td></tr>
<tr><td></td><td colspan="2">TRANSFER GOAL:
Students on their own, solve rate – related problems using models on rational algebraic expressions.</td></tr>
</table>

B. Planning for Assessment

1. Product/Performance

The following are the products and performances that students are expected to come up with in this module.

a. Simplify rational algebraic expressions correctly.
b. Perform operations on rational algebraic expressions correctly.
c. Present creatively the solution on real – life problems involving rational algebraic expression.
d. Create and present manpower plan for house construction that demonstrates understanding of rational algebraic expressions and algebraic expressions with integral exponents.

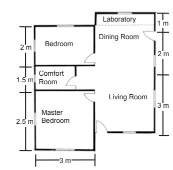
2. Assessment Matrix

TYPE	KNOWLEDGE	PROCESS/SKILLS	UNDERSTANDING	PERFORMANCE
Pre – assessment/ Diagnostic	Pre - test			
	Match It To Me, Egyptian Fraction *Explanation, Interpretation*			
			KWLH, *Self – knowledge Perspective*	
		Anticipation guide *Self – knowledge Interpretation, Explanation*		
			Picture Analysis *Interpretation, Explanation, Self – knowledge, Application, Perspective*	
Formative		My Definition Chart *Perspective, Self - knowledge*		
	Excercises *Interpretation, Explanation*	Quiz *Interpretation, Explanation*	3 – 2 – 1 Chart *Interpretation, Explanation, Self – knowledge*	

			Who's Right *Interpretation, Explanation, Self – knowledge, Empathy*	
		My Value *Interpretation, Explanation, Self – knowledge*	Quiz Constructor *Interpretation, Explanation, Self – knowledge, Empathy*	
		Match It Down *Interpretation, Explanation, Self – knowledge*		
			Circle Process *Interpretation, Explanation, Self – knowledge, Empathy*	
		How Fast *Interpretation, Explanation, Self – knowledge, Empathy, Application*		
			Chain Reaction *Interpretation, Explanation, Self – knowledge, Empathy*	
			Flow Chart *Interpretation, Explanation, Self – knowledge, Empathy*	

Summative	Presentation *Interpretation, Explanation, Self – knowledge, Application*			Manpower plan *Interpretation, Explanation, emphaty, Self – knowledge, application, Perspective*
		Reaction Guide *Self – knowledge, Interpretation, Explanation*		
	Post – test *Interpretation, Application, Self – knowledge, Emphaty*			
Self - assessment		Learned – Affirmed – Challenged *Interpretation, Explanation, Self – knowledge, Empathy, Perspective*		
	What is Wrong With Me? *Interpretation, Explanation, Self – knowledge, Empathy, Perspective*			

Assessment Matrix (Summative Test)

Levels of Assessment	What will I assess?	How will I assess?	How Will I Score?
Knowledge 15%	• Describing and illustrating rational algebraic expressions. • Interpreting zero and negative exponents. • Evaluating and simplifying algebraic expressions involving integral. • Simplifying rational algebraic expressions • Performing operations on rational algebraic expressions • Simplifying complex fractions • Solving problems involving rational algebraic expressions.	Paper and pen Test (refer to attached post – test) Items 1, 2, and 3	1 point for every correct response
Process/Skills 25%		Paper and pen Test (refer to attached post – test) Items 4, 5, 6, 7, and 8	1 point for every correct response
Understanding 30%	• Students will understand that rate – related problems can be modelled using rational algebraic expressions. • Misconception	Paper and pen Test (refer to attached post – test) Items 9, 10, 11, 12, 13, and 14	1 point for every correct response
Product 30%	**GRASPS** Apply the concepts of rational algebraic expressions to model rate – related problems	Paper and pen Test (refer to attached post – test) Items 15, 16, 17, 18, 19, and 20.	1 point for every correct response
	Students will model rate–related problems using rational algebraic expressions.	A newlywed couple plans to construct a house. The couple has already a house plan from their engineer friend. The plan of the house is illustrated below:  As a foreman of the project, you are tasked to prepare a manpower plan to be presented to the couple. The plan should include the following: number of workers needed to complete the project and their daily wages, cost and completion date.	Rubric on manpower plan. Criteria: 1. Reasoning 2. Accuracy 3. Presentation 4. Practicality 5. Efficiency

C. Planning for Teaching-Learning

Introduction:

This module covers key concepts of rational algebraic expressions and expressions with integral exponents. This module is divided into lessons. The first lesson is the introduction to rational algebraic expressions and algebraic expressions with integral exponents and the second lesson is on operations on rational algebraic expressions.

The first lesson will describe the rational algebraic expressions, interpret algebraic expressions with negative and zero exponents, evaluate and simplify algebraic expressions with integral exponents, and simplify rational algebraic expressions. In the second lesson, learner will perform operations on rational algebraic expressions, simplifies complex fraction, and solve problems involving rational algebraic expressions.

In this module, learner are given the opportunity to use their prior knowledge and skills in dealing with rational algebraic expressions and algebraic expressions with integral exponents. They are also given varied activities to process their knowledge and skills learned and deepen and transfer their understanding of the different lessons.

To introduce the lesson, let the students reflect on the introduction and focus questions in the learner's guide.

INTRODUCTION AND FOCUS QUESTIONS:

You have learned special products and factoring polynomials in Module 1. Your knowledge on these will help you better understand the lessons in this module.

Now, take a look at these pictures.


http://www.newroadcontractors.co.uk/wp-content/gallery/road-construction/dscf1702.jpg


http://planetforward.ca/blog/top-10-green-building-trends-part-one/


http://www.waagner-biro.com/images_dynam/image_zoomed/korea_small103_01.jpg

Have you ever asked yourself how many people are needed to complete a job? What are the bases for their wages? And how long can they finish the job? These questions may be answered using rational algebraic expression which you will learn in this module.

After you finished the module, you should be able to answer the following questions:

a. What is a rational algebraic expression?
b. How will you simplify rational algebraic expressions?
c. How will you perform operations on rational algebraic expressions?
d. How will you model rate – related problems?

Objectives:

At the end of the module, learner will be able to:

1. describe and illustrate rational algebraic expressions.
2. interpret zero and negative exponents.
3. evaluate and simplify algebraic expressions involving integral exponents.
4. simplifie rational algebraic expressions
5. perform operations on rational algebraic expressions.
6. simplifie complex fractions.
7. solve problems involving rational algebraic expressions.

Pre – test

1. Which of the following expressions is a rational algebraic expression?

a. $\frac{x}{\sqrt{3y}}$ b. $\frac{3c^{-3}}{\sqrt{(a+1)^0}}$ c. $4y^{-2} + z^{-3}$ d. $\frac{a-b}{a+b}$

Answer: D. Rational algebraic expression is a ratio of two polynomials

2. What is the value of a non – zero polynomial raised to 0?

a. constant b. zero c. undefined d. cannot be determine

Answer: A. Any expression raised to 0 is 1 and 1 is a constant.

3. What will be the result when a and b are replaced by 2 and -1, respectively, in the expression $(-5a - 2b)(-2a - 3b^2)$?

a. $\frac{27}{16}$ b. $-\frac{5}{16}$ c. $\frac{3}{7}$ d. $-\frac{2}{7}$

Answer: B. $\mathbf{(-5a^{-2}b)(-2a^{-3}b^2) = \frac{10b^3}{a^5} = \frac{10(-1)^3}{2^5} = \frac{-10}{32} = -\frac{5}{16}}$

4. What rational algebraic expression is the same as $\frac{x^2-1}{x-1}$?

a. $x + 1$ b. $x - 1$ c. 1 d. -1

Answer: A. $\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x + 1$

5. When a rational algebraic expression is subtracted from $\frac{3}{x-5}$, the result is $\frac{-x-10}{x^2-5x}$. What is the other rational algebraic expression?

a. $\frac{x}{4}$ b. $\frac{x}{x-5}$ c. $\frac{2}{x}$ d. $\frac{-2}{x-5}$

Answer: C. $\frac{-x-10}{x^2-5x} + \frac{3}{x-5} = \frac{-x-10}{x^2-5x} + \frac{3(x)}{(x-5)(x)} = \frac{-x-10+3x}{x^2-5x} = \frac{2x-10}{x^2-5x} = \frac{2(x-5)}{x(x-5)} = \frac{2}{x}$

6. Find the product of $\frac{a^2-9}{a^2+a-20}$ and $\frac{a^2-8a+16}{3a-9}$.

a. $\frac{a}{a-1}$ b. $\frac{a^2-1}{1-a}$ c. $\frac{a^2-7a+12}{3a+15}$ d. $\frac{a^2-1}{a^2-a+1}$

Answer: C. $\frac{a^2-9}{a^2+a-20} \cdot \frac{a^2-8a+16}{3a-9} = \frac{(a-3)(a+3)}{(a-4)(a+5)} \cdot \frac{(a-4)(a-4)}{3(a+3)} = \frac{(a-3)(a-4)}{3(a+5)} = \frac{a^2-7a+12}{3a+15}$

7. What is the simplest form of $\frac{\frac{2}{b-3}}{\frac{2}{b-3}-1}$?

a. $\frac{2}{5-b}$ b. $\frac{b+5}{4}$ c. $\frac{1}{b-1}$ d. $\frac{1-b}{3}$

Answer: A. $\frac{\frac{2}{b-3}}{\frac{2}{b-3}-1} = \frac{2}{b-3} \div \frac{2-b+3}{b-3} = \frac{2}{b-3} \cdot \frac{b-3}{5-b} = \frac{2}{5-b}$

8. Perform the indicated operation $\frac{x-2}{3} - \frac{x+2}{2}$.

a. $x + 5$ b. $x + 1$ c. $x - 6$ d. $-x - 10$

Answer: D. $\frac{x-2}{3} - \frac{x+2}{2} = \frac{2x-4-3x-6}{6} = -x - 10$

9. The volume of a certain gas will increase as the pressure applied to it decreases. This relationship can be modeled using the formula:

$$V_2 = \frac{V_1 P_1}{P_2}$$

where V_1 is the initial volume of the gas, P_1 is the initial pressure, P_2 is the final pressure and the V_2 is the final volume of the gas. If the initial volume of the gas is 500ml and the initial pressure is $\frac{1}{2}$ atm, what is the final volume of the gas if the final pressure is 5 atm?

a. 10ml   b. 50ml   c. 90ml   d. 130ml

Answer: B. $V_2 = \frac{V_1 P_1}{P_2} = \frac{(500\text{ml})(\frac{1}{2})}{5} = \frac{250\text{ml}}{5} = 50\text{ml}$

10. Angelo can complete his school project in x hours. What part of the job can be completed by Angelo after three hours?

a. $x + 3$ b. $x - 3$ c. $\frac{x}{3}$ d. $\frac{3}{x}$

Answer: D. $w = rt = \frac{1}{x}(3) = \frac{3}{x}$

11. If Maribel, a groupmate of Angelo in number 10, can do the project in three hours, which expression below represents rate of Angelo and Maribel working together?

a. $3 + x$ b. $x - 3$ c. $\frac{1}{3} - \frac{1}{x}$ d. $\frac{1}{3} + \frac{1}{x}$

Answer: D. Rate of Angelo + rate of Maribel: $\frac{1}{3} + \frac{1}{x}$

12. Aaron was asked by his teacher to simplify $\frac{a^2 - 1}{a^2 - a}$ on the board. He wrote his solution on the board this way:

$$\frac{a^2 - 1}{a^2 - a} = \frac{(\cancel{a} + 1)\cancel{(a - 1)}}{\cancel{a}\cancel{(a - 1)}} = 1$$

Did he arrive at the correct answer?

a. Yes, the expressions that he crossed out are all common factors.

b. Yes, the LCD must be eliminated to simplify the expression.

c. No, a^2 must be cancelled out so that the answer is $\frac{1}{a}$.

d. No, a is not a common factor of numerator

Answer: D. In simplifying rational algebraic expression, we can only divide out the common factor but not the common variable.

13. Your friend multiplied $\frac{x-1}{2-x}$ and $\frac{1+x}{1-x}$. His solution is presented below:

$$\frac{x-1}{2-x} \bullet \frac{x+1}{1-x} = \frac{\cancel{(x-1)}(x+1)}{(2-x)\cancel{(1-x)}} = \frac{x+1}{2-x}$$

Is his solution correct?

a. No, there is no common factor to both numerator and denominator.
b. No, the multiplier must be reciprocated first before multiplying the expressions .
c. No, common variables must be eliminated.
d. No, dividing an expression by its multiplicative inverse is not equal to one.

Answer: D. (x – 1) is additive inverse of (1 – x). If the a term is divided by the its additive inverse, quotient is - 1

14. Laiza added two rational algebraic expressions and her solution is presented below.

$$\frac{4x+3}{2} + \frac{3x-4}{3} = \frac{4x+3+3x-4}{2+3} = \frac{7x+1}{5}$$

Is there something wrong in her solution?

a. Yes, solve first the GCF before adding the rational algebraic expressions.
b. Yes, cross multiply the numerator of the first expression to the denominator of the second expression.
c. Yes, she may express first the expressions as similar fractions.
d. Yes. $4x - 4$ is equal to x

Answer: C. We may express first the expressions into similar rational algebraic expressions and follow the concepts in adding/subtracting rational expressions.

15. Your father, a tricycle driver, asked you regarding the best motorcycle to buy. What will you do to help your father?
   a. Look for the fastest motorcycle.
   b. Canvass for the cheapest motorcycle.
   c. Find an imitated brand of motorcycle.
   d. Search for fuel – efficient type of motorcycle.

   Answer: D. A, B and C are not good qualities of a motorcycle for livelihood.

16. The manager of So – In Clothesline Corp. asked you, as Human Resource Officer, to hire more tailors to meet the production target of the year. What will you look in hiring a tailor?
   a. Speed and efficiency
   b. Speed and accuracy
   c. Time conscious and personality
   d. Experience and personality

   Answer: A. To meet the deadline, you need a fast worker but an efficient one.

17. You own three hectares of land and you want to mow it for farming. What will you do to finish it at a very least time?
   a. Rent a small mower
   b. Hire three efficient laborers
   c. Do kaingin
   d. Use germicide

   Answer: B. Germicide cannot kill weeds. Kaingin is prohibited according to law. Small mower is not effective for wide area.

18. Your friend asked you to make a floor plan. As an engineer, what aspects should you consider in doing the plan?
   a. Precise and realistic
   b. Layout and cost
   c. Logical and sufficient
   d. Creative and economical

   Answer: A. The size of the parts must be realistic and should be accurate

19. Your SK Chairman planned to construct a basketball court. As a contractor, what will you do to realize the project?
   a. Show a budget proposal
   b. Make a budget plan
   c. Present a feasibility study
   d. Give a financial statement

   Answer: C. Budget proposal is for budget approval. Budget plan is like a budget proposal. Financial statement will be given after the project is completed.

20. As a contractor in number 19, what is the best action to do in order to complete the project on or before the deadline but still on the budget plan?
   a. All laborers must be trained workers.
   b. Rent more equipment and machines.
   c. Add least charge equipment and machines.
   d. Trained and amateur workers must be proportionate.

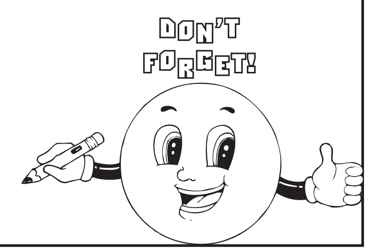
   Answer: D. A and B are expensive; C could not give the best quality of work.

Learning Goals and Targets:

In this module, learners will have the following targets:

- Demonstrate understanding of the key concepts of rational algebraic expressions and algebraic expressions with integral and zero exponents.
- Formulate real–life problems involving rational algebraic expressions and algebraic expressions with integral and zero exponents and solve these with utmost accuracy using a variety of strategies.

Teacher's Note and Reminders



What to Know

Activity 1 elicits prior knowledge of the learner in translating verbal phrases to mathematical phrases which is one of the key concepts that the student should learned in solving word problems in algebra. The result of this activity may become a benchmark on how to start facilitating word problems later on.

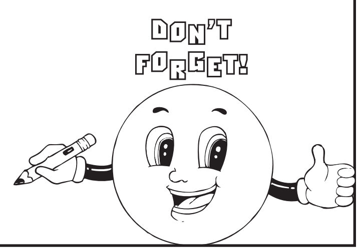
Aside from that, this also assesses the learner regarding the concepts in polynomial. They should have a firm background regarding concepts in polynomial for rational algebraic expression.

Answers Key

Activity 1

1. $\frac{x}{4} + 2$	2. $\sqrt{3}y$	3. $a^2 + 2a$	4. $b^2 - (b+2)$	5. $\frac{pq}{3}$
6. $\frac{c^2}{3}$	7. $10y + 6$	8. $z^3 - 9$	9. $w - \sqrt{9}$	10. h^4

Teacher's Note and Reminders



Lesson 1 Rational Algebraic Expressions

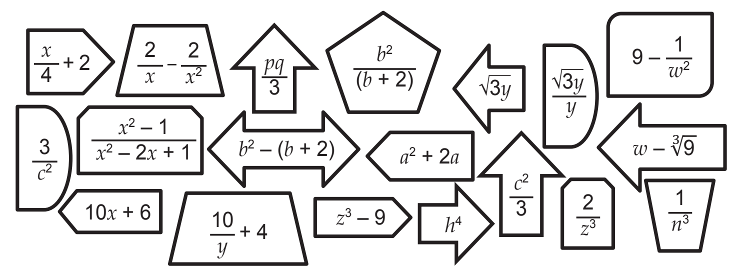
What to Know

Let's begin the lesson by reviewing some of the previous lessons and gathering your thoughts in the lesson.

Activity 1 MATCH IT TO ME

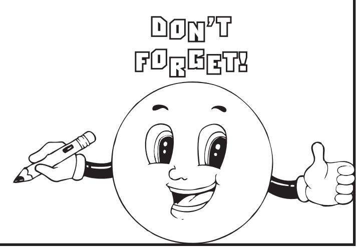
There are verbal phrases below. Look for the mathematical phrase in the figures that corresponds to the verbal phrases.

1. The ratio of number x and four added by two.
2. The product of square root of three and the number y.
3. The square of a added by twice the a.
4. The sum of b and two less than the square of b.
5. The product of p and q divided by three
6. One – third of the square of c.
7. Ten times a number y increased by six
8. Cube of the number z decreased by nine.
9. Cube root of nine less than number w.
10. Number h raised to four.



Their responses in these questions may be written in their journal notebook. As to its purpose, this activity is not meant for giving grades but a benchmark for your lesson in this module. If ever the learner has difficulty in these prerequisite concepts, try to have a short review in these concepts.

Teacher's Note and Reminders



Activity 2 How Fast
The learner is not expected to have correct answers in this activity. The aim of this activity is to find out whether he/she has a background on rational algebraic expressions applied in a real-life situation. The response to this activity could help the teaching – learning process more efficient and effective as basis for teaching – learning process. The answers may be written in a clean sheet of paper.

QUESTIONS ?

1. What did you feel in translating verbal phrases to mathematical phases?
2. What must be considered in translating verbal phases to mathematical phrases?
3. Will you consider these mathematical phases as polynomial? Why yes or why not?
4. How will you describe a polynomial?

The above activity deals with translating verbal phrases to polynomial and you encountered some of the examples of non - polynomials. Translating verbal phases to polynomial is one of the key concepts in answering worded problem.

All polynomials are expressions but not all expressions are polynomials. In this lesson you will encounter some of these expressions that are not polynomials.

Activity 2 HOW FAST

Suppose you are to print you 40 – page research paper. You observed that printer A in the internet shop finished printing it in 2 minutes.

a. How long do you think printer A can finish 100 pages?
b. How long will it take printer A finish printing the p pages?
c. If printer B can print x pages per minute, how long will printer B take to print p pages?

QUESTIONS ?

1. Can you answer the first question? If yes, how will you answer it? If no, what must you do to answer the question?
2. How will you describe the second and third questions?
3. How will you model the above problem?

Before moving to the lesson, you have to fill in the table below regarding your ideas on rational algebraic expressions and algebraic expressions with integral exponents.

MAP OF CONCEPTUAL CHANGE
Activity 3: KWHL

Aside from Activity 2, KWHL is also an activity eliciting the background of the learner regarding the rational algebraic expressions. He/She could use his/her understanding in activity 2 in doing this activity. Keep their response because at the end of this lesson, they will continue to answer this activity to track their learning.

What to Process

Topic: Introduction to Rational Algebraic Expressions

Teacher's Note and Reminders

Activity 4: Match It to Me – Revisited
Going back to activity 1, let them distinguish the polynomials from the non–polynomials in this activity by describing it. Give emphasis on the non–polynomial examples in the activity. Remind them that these non–polynomials in the activity are not the only non–polynomials. Be guided that these non–polynomials are just rational expressions and that not all non–polynomials are rational algebraic expressions.

Activity 3 KWLH

Write your ideas on the rational algebraic expressions and algebraic expressions with integral exponents. Answer the unshaded portion of the table and submit it to your teacher.

What I Know	What I Want to Find Out	What I Learned	How Can I Learn More

You were engaged in some of the concepts in the lesson but there are questions in your mind. The next lessons will answer your queries and clarify your thoughts regarding to our lesson.

What to Process

Your goal in this section is to learn and understand the key concepts on rational algebraic expressions and algebraic expressions with integral exponents.

As the concepts on rational algebraic expressions and algebraic expressions with integral exponents become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in real – life problems especially rate – related problems.

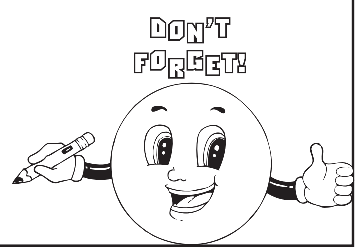
Activity 4 MATCH IT TO ME – REVISITED (REFER TO ACTIVITY 1)

1. What are the polynomials in the activity "Match It To Me"? List these polynomials under set P.
2. Describe these polynomials.
3. In the activity, which are not polynomials? List these non – polynomials under set R.
4. How do these non – polynomials differ from the polynomial?
5. Describe these non – polynomials.

Activity 5: Compare and Contrast

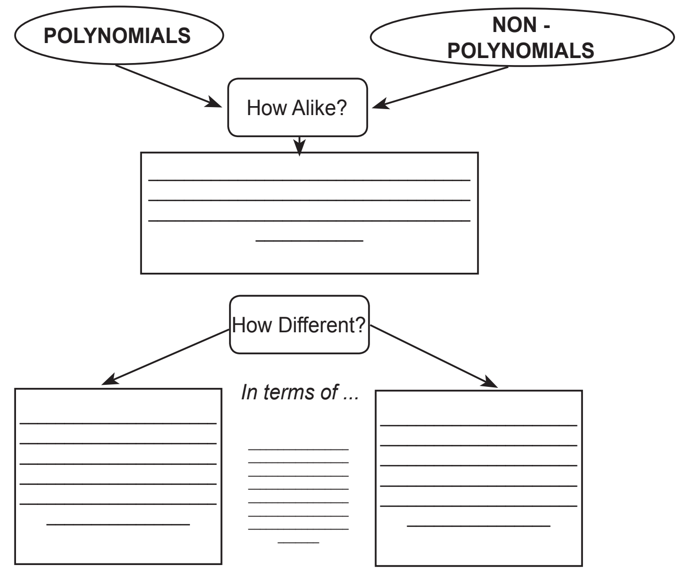
As they describe the polynomials and non–polynomials in Activity 4, they will summarize their work by completing the given graphic organizer. This activity will enable them to describe rational algebraic expressions and distinguish it from polynomials. The learner may present his/her output to the class but this is not meant for rating the learner. This activity will guide the learner to describe the rational algebraic expressions. After the presentation, discuss that these non–polynomials are rational algebraic expressions. This activity may be done individually or by group.

Teacher's Note and Reminders



Activity 5 COMPARE AND CONTRAST

Use your answers in the activity "Match It To Me – Revisited" to complete the graphic organizer compare and contrast. Write the similarities and differences between polynomials and non – polynomials in the first activity.



*In the activity "Match It to Me", the non – polynomials are called **rational algebraic expressions**. Your observations regarding the difference between polynomials and non – polynomials in activities 4 and 5 are the descriptions of rational expression. Now, can you define rational algebraic expressions? Write your own definition about rational algebraic expressions in the chart below.*

Activity 6: My Definition Chart
After they have described the rational algebraic expressions, let them define rational algebraic expression on their own. Their response may be different from the axiomatic definition of rational algebraic expressions but let it be. The purpose of this activity is to generate their ideas on rational algebraic expressions based on the examples and illustrations of rational algebraic expressions given. They can exchange their initial definitions with their classmates and discuss how they are alike or different.

Teacher's Note and Reminders

Activity 7: Classify Me
$\frac{m+2}{\sqrt{2}}$ and $\frac{c^4}{\sqrt[3]{5}}$ are the only expressions that belong to the Not Rational Algebraic Expressions column. After they classify the expressions, let them describe the expressions in each column and compare and contrast the expressions in the two columns. This activity may guide them in formulating definition similar to the axiomatic definition of rational algebraic expressions.

Activity 6 MY DEFINITION CHART

Write your initial definition on rational algebraic expressions in the appropriate box. Your final definition will be written after some activities.

My Initial Definition

My Final Definition

Try to firm up your own definition regarding the rational algebraic expressions by doing the next activity.

Activity 7 CLASSIFY ME

Classify the different expressions below into rational algebraic expression or not rational algebraic expression. Write the expression into the appropriate column.

$\frac{m+2}{\sqrt{2}}$ $\frac{k}{3k^2-6k}$ $\frac{y+2}{y-2}$ $\frac{1}{a^6}$ $\frac{1-m}{m^3}$ $\frac{a}{y^2-x^9}$ $\frac{c}{a-2}$ $\frac{c^4}{\sqrt[8]{5}}$

Rational Algebraic Expressions	Not Rational Algebraic Expressions

QUESTIONS ?

1. How many expressions did you place in the rational algebraic expression column?
2. How many expressions did you placed in the not rational algebraic expression column?
3. How did you classify a rational algebraic expression from a not rational algebraic expression?
4. Were you able to place each expression to its appropriate column?
5. What difficulty did you encounter in classifying the expressions?

Activity 8: My Definition Chart - Continuation
After Activity 7, they can now finalize their initial definitions on rational algebraic expressions. Let them exchange their final definition and discuss it with their classmate. In this stage, you can discuss further if there are questions that need to be answered.

Process their final definition. You may give emphasis on the axiomatic definition of rational algebraic expression. After they defined rational algebraic expressions, let them illustrate it and give at least three examples. You can discuss rational algebraic expression for clarification purposes.
Mathematical Investigation: Learner may investigate the concept, "polynomial divided by zero". Ask the learner why the denominator should not be equal to zero. Let him/her investigate the clue given. You can give more clues if needed to generate the pattern and will lead them to the concept of undefined numbers.

Teacher's Note and Reminders

In the previous activities, there might be some confusions to you regarding rational algebraic expressions, but this activity firmed up your idea regarding rational algebraic expressions. Now, put into words your final definition on rational algebraic expression.

Activity 8 MY DEFINITION CHART

Write your final definition on rational algebraic expressions in the appropriate box.

My Initial Definition	My Final Definition

Compare your initial definition and your final definition on rational algebraic expressions. Is your final definition clears your confusions? How? Give at least 3 rational algebraic expressions differ from your classmate.

MATH DETECTIVE
Rational algebraic expression is a ratio of two polynomials where the denominator is not equal to zero. What will happen when the denominator of a fraction becomes zero?
Clue: Start investigating in
$\frac{4}{2} = 2$ ⋙ $4 = (2)(2)$ $\frac{4}{1} = 4$ ⋙ $4 = (1)(4)$

Remember:

Rational algebraic expression is a ratio of two polynomials provided that the numerator is not equal to zero. In symbols: $\frac{P}{Q}$,where P and Q are polynomials and $Q \neq 0$.

In the activities above, you had encountered the rational algebraic expressions. You might encounter some algebraic expressions with negative or zero exponents. In the next activities, you will define the meaning of algebraic expressions with integral exponents including negative and zero exponents .

Before moving to the next activity, review the laws of exponents

Activity 9: Let the Pattern Answer It
This activity will serve as a review on laws of exponents. Let the learner complete the table to recall the concept on laws of exponents. Let the learner examine and analyze the pattern in this activity.
The pattern in this activity: the first row under in column III is divided by the base of the expression.
This activity may be done by group or individual work.

Teacher's Note and Reminders

Activity 9: Let the Pattern Answer It
Based on the pattern that they observe in the first table in this activity, let them complete the table. This will enable the learner to interpret the expressions with negative exponents. He/she will discover that the implication of negative exponents is the multiplicative inverse of the expression.

RECALL LAWS OF EXPONENTS

I – Product of Powers
If the expressions multiplied have the same base, add the exponents.
$x^a \bullet x^b = x^{a+b}$

II – Power of a Power
If the expression raised to a number is raised by another number, multiply the exponents.
$(x^a)^b = x^{ab}$

III – Power of a Product
If the multiplied expressions is raised by a number, multiply the exponents then multiply the expressions.
$(x^a y^b)^c = x^{ac} y^{bc}$ $(xy)^a = x^a y^a$

IV – Quotient of Power
If the ratio of two expressions is raised to a number, then

Case I. $\frac{x^a}{x^b} = x^{a-b}$, *where* $a > b$
Case II. $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$, *where* $a < b$

Activity 9 LET THE PATTERN ANSWER IT

Complete the table below and observe the pattern.

A	B	A	B	C	A	B	C	A	B
2•2•2•2•2	2^5	3•3•3•3•3	3^5	243	4•4•4•4•4	4^5	1,024	x•x•x•x•x	x^5
2•2•2•2		3•3•3•3			4•4•4•4			x•x•x•x	
2•2•2		3•3•3			4•4•4			x•x•x	
2•2		3•3			4•4			x•x	
2		3			4			x	

QUESTIONS ?

1. What do you observe as you answer the column **B**?
2. What do you observe as you answer the column **C**?
3. What happens to its value when the exponent decreases?
4. In the column **B**, how is the value in the each cell/box related to its upper or lower cell/box?

Now, use your observations in the activity above to complete the table below.

A	B	A	B	A	B	A	B
2^5	32	3^5	243	4^5	1,024	x^5	x•x•x•x•x
2^4		3^4		4^4		x^4	
2^3		3^3		4^3		x^3	
2^2		3^2		4^2		x^2	
2		3		4		x	
2^0		3^0		4^0		x^0	
2^{-1}		3^{-1}		4^{-1}		x^{-1}	
2^{-2}		3^{-2}		4^{-2}		x^{-2}	
2^{-3}		3^{-3}		4^{-3}		x^{-3}	

Teacher's Note and Reminders

Answer to Exercises

1. $\frac{1}{b^4}$	2. $\frac{d^8}{c^2}$	3. $\frac{1}{b^4}$	4. $\frac{no}{m^2}$	5. $\frac{df}{e^5}$
6. $x + y$	7. 1	8. 14	9. 1	10. 2

MAP OF CONCEPTUAL CHANGE

Activity 10: 3 – 2 – 1 Chart

Before moving to the next lesson, the learner should complete the 3 – 2 – 1 chart. This activity will give the learner a chance to summarize the key concepts in algebraic expressions with integral exponents. Address the question of the learner before moving to the next activity.

QUESTIONS ?

1. What do you observe as you answer the column A?
2. What do you observe as you answer the column B?
3. What happen to its value when the exponent decreases?
4. In the column A, how is the value in the each cell/box related to its upper or lower cell/box?
5. What do you observe when the number has zero exponent?
6. Do you think that when a number raised to zero is the same to another number raised to zero? Justify your answer.
7. What do you observe to the value of the number raised to a negative integer?
8. What can you say about an expression with negative integral exponent?
9. Do you think it is true to all numbers? Cite some examples?

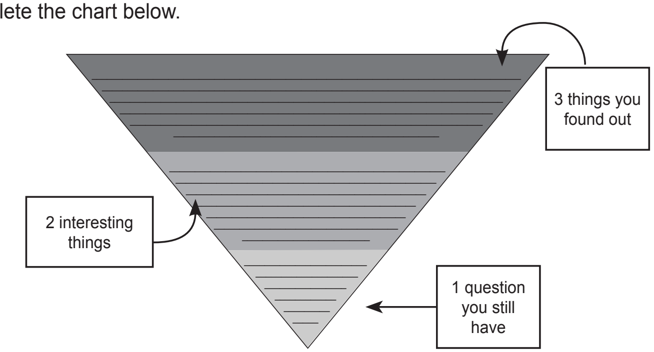
Exercises

Rewrite each item to expressions with positive exponents.

1. b^{-4}	5. $de^{-5}f$	9. $\frac{l^0}{p^0}$
2. $\frac{c^{-2}}{d^{-8}}$	6. $\frac{x+y}{(x-y)^0}$	10. $\frac{2}{(a-b+c)^0}$
3. $w^{-3}z^{-2}$	7. $\left(\frac{a^6b^8c^{10}}{a^5b^2e^8}\right)^0$	
4. $n^2m^{-2}o$	8. $14t^0$	

Activity 10 3 – 2 – 1 CHART

Complete the chart below.



Activity 11: Who is Right
Let the learner examine and analyze the solution of Allan and Gina. Let him/her decide who is correct and explain how this solution is correct and what makes the other solution wrong.
After this, explain to the learner that there is no wrong solution between the two. Explain how the concepts of laws of exponents applied to the solution.

Teacher's Note and Reminders

Activity 12: Speedy Mars
This activity aims to recall the evaluation of linear equation in grade 7. Expounding the ways of solving the problem will help in evaluating rational algebraic expressions.

Activity 11 WHO IS RIGHT?

Allan and Gina were asked to simplify $\frac{n^3}{n^{-4}}$. There solutions are shown below together with their explanation.

Allan's Solution	Gina's Solution
$\frac{n^3}{n^{-4}} = n^{3-(-4)} = n^{3+4} = n^7$	$\frac{n^3}{n^{-4}} = \frac{n^3}{\frac{1}{n^{-4}}} = n^3\,\frac{n^4}{1} = n^7$
Quotient law was used in my solution	Expressing the exponent of the denominator as positive integer, then following the rules in dividing polynomials.

Who is right? Write your explanation in a sheet of paper.

You have learned the some concepts of rational algebraic expression as you performed the previous activities. Now, let us try to put these concepts in different context.

Activity 12 SPEEDY MARS

RECALL

Speed is the rate of moving object as it transfers from one point to another. The speed is the ratio between the distance and time travelled by the object.

Mars finished the 15 – meter dash within 3 seconds. Answer the questions below.

1. How fast did Mars run?
2. At this rate, how far can Mars ran after 4 seconds? 5 seconds? 6 seconds?
3. How many minutes can Mars run 50 meters? 55 meters? 60 meters?

QUESTIONS

How did you come up with your answer? Justify your answer.

What you just did was evaluating the speed that Mars run. Substituting the value of the time to your speed, you come up with distance. When you substitute your distance to the formula of the speed, you had the time. This concept of evaluation is the same with evaluating algebraic expressions. Try to evaluate the following algebraic expressions in the next activity.

Activity 13. My Value (Answer)

You can discuss the examples in this activity to the class and give more examples, if necessary. The activity may be done in group or individual.

$a^2 + b^3 \dashrightarrow 3^2 + 43 = 73$ $\qquad \frac{a^{-2}}{b^{-3}} \dashrightarrow \frac{27}{4}$ $\qquad a^{-1}b^0 \rightarrow \frac{1}{4}$

$\dashrightarrow 2^2 + 43 = 68$ $\qquad \dashrightarrow \frac{8}{9}$

Teacher's Note and Reminders

Activity 13 MY VALUE

Find the value of each expression below by evaluation.

My Expression	Value of a	Value of b	My solution	My Value
$a^2 + b^3$	2	3	Example: $a^2 + b^2 = 2^2 + 3^3 = 4 + 9 = 13$	13
	3	4	Your solution here:	
	2	4		
$\frac{a^{-2}}{b^{-3}}$	-2	3	Example: $\frac{a^{-2}}{b^{-3}} = \frac{(-2)^{-2}}{3^{-3}} = \frac{3^3}{(-2)^2} = \frac{27}{4}$	$\frac{27}{4}$
$\frac{a^{-2}}{b^{-3}}$	3	2	Your solution here	
$a^{-1}b^0$	2	3		

QUESTIONS ?

1. What have you observed in the solution of the examples?
2. How these examples help you to find the value of the expression?
3. How did you find the value of the expression?

Answer to Exercises

1. 8 2. $\frac{16}{3}$ 3. $\frac{1}{4}$ 4. 9 5. $\frac{31}{4}$

Activity 14: Bingo

These are the expression to be evaluated by the learners.

$\frac{a^2+b^3}{c^2}, a = 1, b = 2, c = 3$ (1)	$x+y^2$, $x=4, y=(-2)$ $\left(\frac{1}{4}\right)$	$a^{-1} + a^{-2}$, $a=1$(2)
$x^{-3} - x^{-2}$, $x=(-2)$ $\left(-\frac{31}{8}\right)$	$\frac{(x-1)^{-1}}{(x+1)}, x = 4$ $\left(\frac{1}{15}\right)$	$3ab^{-2}$, $a = 1, b = 2$ $\left(\frac{3}{4}\right)$
x^2+2x+1, $x=(-2)$ (1)	$\frac{a^{-2}}{b^{-1}}, a = 3, b = 2$ $\left(\frac{2}{9}\right)$	$5x^{-2}+x^3$, $x=2$ $\left(\frac{37}{4}\right)$
$\frac{(3x+4)^0}{x^{-2}}, x = 5$ (25)	$(3x-1)^{-1}$, $x=4$ $\left(\frac{1}{11}\right)$	$\frac{(x+1)^{-1}}{(x-1)^{-1}}, x = 2$ $\left(\frac{1}{3}\right)$
$6a^2bc^{-1}$, $a=2, b=3, c=3$ $\left(\frac{3}{2}\right)$	$\frac{2x^2}{y^{-1}}, y = 1, x = 4$ (32)	$4(x^{-2}+y^{-1})$, $x=(-2), y=4$(2)
$5x^{-2}y$, $x=(-5), y=4$ $\left(\frac{1}{5}\right)$	$5a(a^2-a)^{-1}$, $a=2$(5)	$(3+x)^{-1}(3-x)$, $x=3$(0)
$6-y^{-2}$, $y=(-2)$ $\left(\frac{23}{4}\right)$	$b^{-1}(2b-a)$, $a=2$, $b=3$ $\left(\frac{4}{3}\right)$	$\frac{1}{x+1}, x = 3$ $\left(\frac{1}{4}\right)$
$\frac{1}{a^{-2}(a-2)}, a = 3$ (9)	$\frac{1}{(y^2-x^2)^{-1}}, x = (-3), y = 3$(0)	
$5x+5y^{-2}$, $x=5, y=(-5)$ $\left(\frac{126}{5}\right)$	$c-3(4x+y)^0$, $c=6$ (3)	

Activity 15: Quiz constructor

The learner will make his/her own algebraic expressions with integral exponents. The expression must have at least two variables and the expressions must be unique from his/her classmates. The learner will also assign value to the variables and he/she must show how to evaluate these values to his/her algebraic expressions.

Exercises

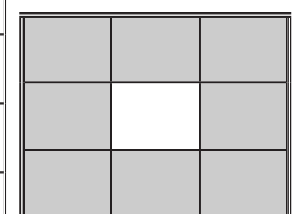
Evaluate the following algebraic expressions

1. $40y^{-1}$, $y = 5$
2. $\frac{1}{m^{-2}(m + 4)}$, $m = -8$
3. $(p^2 - 3)^{-2}$, $p = 1$
4. $\frac{(x - 1)^{-2}}{(x + 1)^{-2}}$, $x = 2$
5. $y^{-3} - y^{-2}$, $y = 2$

Activity 14 BIN - GO

Make a 3 by 3 bingo card. Choose a number to be placed in your bingo card from the numbers below. Your teacher will give an algebraic expression with integral exponents and the value of its variable. The first student can form a frame wins the game.

1	$\frac{17}{4}$	2	$-\frac{31}{8}$	$\frac{1}{15}$
1	$\frac{2}{9}$	$\frac{3}{4}$	$\frac{37}{4}$	25
$\frac{1}{11}$	$\frac{1}{3}$	$\frac{3}{2}$	32	2
$\frac{1}{5}$	5	0	$\frac{23}{4}$	$\frac{4}{3}$
$\frac{1}{4}$	9	0	$\frac{126}{5}$	6



The frame card must be like this:

Activity 15 QUIZ CONSTRUCTOR

Be like a quiz constructor. Write in a one – half crosswise three algebraic expressions with integral exponents in at least 2 variables and decide what values to be assigned in the variables. Show how to evaluate your algebraic expressions. Your algebraic expressions must be unique from your classmates.

Activity 16: Connect to my Equivalent

This activity will allow the learner to recall the steps and concepts in reducing fraction to its lowest term and relate these steps and concepts to simplifying rational algebraic expressions.

Answer to this activity

$\frac{5}{20}=\frac{1}{4}$, $\frac{8}{12}=\frac{2}{3}$, $\frac{4}{8}=\frac{1}{2}$, $\frac{5}{15}=\frac{1}{3}$, $\frac{6}{8}=\frac{3}{4}$

Teacher's Note and Reminders

Illustrative Example

You can have additional illustrative examples if necessary.

Activity 16 CONNECT TO MY EQUIVALENT

Connect column A to its equivalent simplest fraction to column B.

A	B
$\frac{5}{20}$	$\frac{1}{3}$
$\frac{8}{12}$	$\frac{1}{4}$
$\frac{4}{8}$	$\frac{3}{4}$
$\frac{5}{15}$	$\frac{1}{2}$
$\frac{6}{8}$	$\frac{2}{3}$

QUESTIONS ?

1. How did you find the equivalent fractions in column A?
2. Do you think you can apply the same concept in simplifying a rational algebraic expression?

You might wonder how to answer the last question but the key concept of simplifying rational algebraic expressions is the concept of reducing fractions to its simplest form.

Examine and analyze the following examples. Pause once in a while to answer check – up questions.

Illustrative example: Simplify the following rational algebraic expressions.

1. $\frac{4a + 8b}{12}$

Solution

$$\frac{4a + 8b}{12} = \frac{\cancel{4}(a + 2b)}{\cancel{4} \cdot 3}$$

? What factoring method is used in this step?

$$= \frac{a + 2b}{3}$$

Answer to Exercises

1. $\frac{y^2+5x+4}{y^2-3x-4} = \frac{y+4}{y-4}$

2. $\frac{-21a^2b^2}{28a^3b^3} = -\frac{3}{4ab}$

3. $\frac{x^2-9}{x^2-7x+12} = \frac{x+3}{x-4}$

4. $\frac{m^2+6m+5}{m^2-m-2} = \frac{m+5}{m-2}$

5. $\frac{x^2-5x-14}{x^2+4x+4} = \frac{x-7}{x+2}$

Answer to Activity 17

This activity may be a collaborative work or an individual performance. This may help in determining how far the learner understands the topic.

$\frac{a^2+6a+5}{a+1}$ C	$\frac{a^3+2a^2+a}{3a^2+6a+3}$ E	$\frac{3a^2-6a}{a-2}$ D	$\frac{a-1}{1-a}$ A
$\frac{(3a+2)(a+1)}{3a^2+5a+2}$ B	$\frac{3a^3-27a}{(a+3)(a-3)}$ D	$\frac{a^3+125}{a^2-25}$ $\frac{a^2-5a+25}{a-5}$	$\frac{a-8}{-a+8}$ A
$\frac{18a^2-3a}{-1+6a}$ D	$\frac{3a-1}{1-3a}$ A	$\frac{3a+1}{1+3a}$ B	$\frac{a^2+10a+25}{a+5}$ C

2. $\frac{15c^3d^4e}{12c^2d^5w}$

Solution

$$\frac{15c^3d^4e}{12c^2d^5w} = \frac{\cancel{3}\cdot 5\cancel{c^2}c\cancel{d^4}e}{\cancel{3}\cdot 4\cancel{c^2}\cancel{d^4}dw}$$

What factoring method is used in this step?

$$= \frac{5ce}{4dw}$$

3. $\frac{x^2+3x+2}{x^2-1}$

Solution

$$\frac{x^2+3x+2}{x^2-1} = \frac{\cancel{(x+1)}(x+2)}{\cancel{(x+1)}(x-1)}$$

What factoring method is used in this step?

$$= \frac{(x+2)}{(x-1)}$$

QUESTIONS ?

Based on the above examples,
1. What is the first step in simplifying rational algebraic expressions?
2. What happen to the common factors of numerator and denominator?

Web Based Booster
http://mathvids.com/lesson/mathhelp/845-rational-expressions-2---simplifying

Exercises

Simplify the following rational algebraic expressions

1. $\frac{y^2+5x+4}{y^2-3x-4}$
2. $\frac{-21a^2b^2}{28a^3b^3}$
3. $\frac{x^2-9}{x^2-x+12}$
4. $\frac{m^2+6m+5}{m^2-m-2}$
5. $\frac{x^2-5x-14}{x^2+4x+4}$

CONCEPTUAL CHANGE
Activity 18. Circle Process
The learner will write his/her understanding on the process of simplifying rational algebraic expressions. This activity will gauge the learner if he/she can really grasp the concept or not. If there are still difficulties in understanding the concept, then give another activity.

Teacher's Note and Reminders

Activity 17 MATCH IT DOWN

Match the rational algebraic expressions to its equivalent simplified expression from the top. Write it in the appropriate column. If the equivalent is not among the choices, write it in column F.

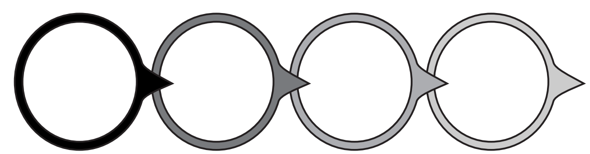
a. -1 b. 1 c. $a + 5$ d. $3a$ e. $\frac{a}{3}$

$\frac{a^2 + 6a + 5}{a + 1}$	$\frac{a^3 + 2a^2 + a}{3a^2 + 6a + 3}$	$\frac{3a^2 - 6a}{a - 2}$	$\frac{a - 1}{1 - a}$
$\frac{(3a + 2)(a + 1)}{3a^2 + 5a + 2}$	$\frac{3a^3 - 27a}{(a + 3)(a - 3)}$	$\frac{a^3 + 125}{a^2 - 25}$	$\frac{a - 8}{-a + 8}$
$\frac{18a^2 - 3a}{-1 + 6a}$	$\frac{3a - 1}{1 - 3a}$	$\frac{3a + 1}{1 + 3a}$	$\frac{a^2 + 10a + 25}{a + 5}$

A	B	C	D	E	F

Activity 18 CIRCLE PROCESS

Write each step in simplifying rational algebraic expression using the circles below. You can add or delete circle if necessary.



In this section, the discussions were all about introduction on rational algebraic expressions. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? Try to move a little further in this topic through next activities.

What to Understand

In this part of the lesson, the learner should develop the key concepts of rational algebraic expression to answer the essential question. To address the essential question, the learner should have background in solving problems involving the concept of rational algebraic expressions. He/she must be exposed to different scenarios where the rational algebraic expressions involved especially rate–related problems

Illustrative Example

As one way of solving problems, let the learner examine and analyze how the table/matrix method works. Guide the learner on how to use on table effectively.

Teacher's Note and Reminders

What to Understand

Your goal in this section is to relate the operations of rational expressions to a real – life problems, especially the rate problems.

Work problems are one of the rate – related problems and usually deal with persons or machines working at different rates or speed. The first step in solving these problems involves determining how much of the work an individual or machine can do in a given unit of time called the **rate**.

Illustrative example:

A. Nimfa can paint the wall in 5 hours. What part of the wall is painted in 3 hours?

Solution:

Since Nimfa can paint in 5 hours, then in one hour, she can paint $\frac{1}{5}$ of the wall. Her rate of work is $\frac{1}{5}$ of the wall each hour. The rate of work is the part of a task that is completed in 1 unit of time.

Therefore, in 3 hours, she will be able to paint $3 \cdot \frac{1}{5} = \frac{3}{5}$ of the wall.

You can also solve the problem by using a table. Examine the table below.

Rate of work (wall painted per hour)	Time worked	Work done (Wall painted)
$\frac{1}{5}$	1 hour	$\frac{1}{5}$
$\frac{1}{5}$	2 hours	$\frac{2}{5}$
$\frac{1}{5}$	3 hours	$\frac{3}{5}$

Illustrative Example
Another way of visualizing the problem is the part of the work done in certain time. Let them examine and analyze how this method works.
The learners should grasp the concept of rate – related problem
(rate ● time = work).
You can add more examples to strengthen their ideas regarding solving rate-related problems

Teacher's Note and Reminders

Activity 19: How Fast 2 - Revisited
Learner will fill in necessary data in this table. This will assess the learner if he/she grasps the concept of rational algebraic expressions in different context.

You can also illustrate the problem.

1st hour	2nd hour	3rd hour	4th hour	5th hour
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

So after 3 hours, nimfa only finished painting $\frac{3}{5}$ of the wall.

B. Pipe A can fill a tank in 40 minutes. Pipe B can fill the tank in x minutes. What part of the tank is filled if either of the pipes is opened in ten minutes?

Solution:

Pipe A fills $\frac{1}{40}$ of the tank in 1 minute. Therefore, the rate is $\frac{1}{40}$ of the tank per minute. So after 10 minutes,

$10 \bullet \frac{1}{40} = \frac{1}{4}$ of the tank is full.

Pipe B fills $\frac{1}{x}$ of the tank in x minutes. Therefore, the rate is $\frac{1}{x}$ of the tank per minute. So after x minutes,

$10 \bullet \frac{1}{x} = \frac{10}{x}$ of the tank is full.

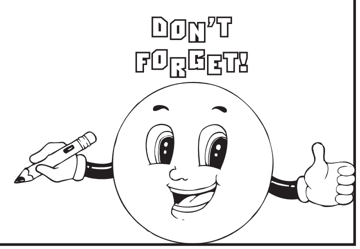
In summary, the basic equation that is used to solve work problem is:
Rate of work • time worked = work done.
r • t = w

Activity 19 HOWS FAST 2

Complete the table on the next page and answer question that follows.

You printed your 40 – page reaction paper, you observed that the printer A in the internet shop finished printing in 2 minutes. How long will it take printer A to print 150 pages? How long will it take printer A to print **p** pages? If printer B can print **x** pages per minute, how long will it take to print **p** pages? The rate of each printer is constant.

Teacher's Note and Reminders



To ensure the understanding of the learner, he/she will do this activity before moving to transfer stage. This will enable the learner to recall and reflect what has been discussed in this lesson and solicit ideas on what and how the students learned in this lesson. Try to clear his/her thought by addressing the questions regarding the topics in this lesson. Responses may be written in journal notebook.

Printer	Pages	Time	Rate
Printer A	40 pages	2 minutes	
	45 pages		
	150 pages		
	p pages		
Printer B	p pages		x ppm
	30 pages		
	35 pages		
	40 pages		

QUESTIONS

1. How did you solve the rate of each printer?
2. How did you compute the time of each printer?
3. What will happen if the rate of the printer increases?
4. How do time and number of pages affect to the rate of the printer?

The concepts of rational algebraic expressions were used to answer the situation above. The situation above gives you a picture how the concepts of rational algebraic expressions were used in solving rate – related problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Fill-in the **Learned, Affirmed, Challenged cards** given below.

Learned

What new realizations and learning do you have about the topic?

Affirmed

What new connections have you made? Which of your old ideas have veen confirmed/ affirmed?

Challenged

What questions do you still have? Which areas seem difficult for you? Which do you want to explore

What to Transfer

In this part, students will show how to transfer their understanding in a real – life situation. They will be given a task as presented in the learning guide materials. They will present their work though presentation is not part of the criteria. This may be a practice for them in presenting an output because before they finish this learning guide, they have to present an output and one of the criteria is presentation.

Teacher's Note and Reminders

What to Transfer

Your goal in this section is to apply your learning in real life situations. You will be given a practical task which will demonstrate your understanding.

Activity 20 HOURS AND PRINTS

The JOB Printing Press has two photocopying machines. P1 can print box of bookpaper in three hours while P2 can print a box of bookpaper in $3x + 20$ hours.

a. How many boxes of bookpaper are printed by P1 in 10 hours? In 25 hours? in 65 hours?

b. How many boxes of bookpaper can P2 print in 10 hours? in $120x + 160$ hours? in $30x^2 + 40x$ hours?

You will show your output to your teacher. Your work will be graded according to mathematical reasoning and accuracy.

RUBRICS FOR YOUR OUTPUT

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.	
Accuracy	All computations are correct and shown in detail.	All computations are corrects.	Most of the computations are correct.	Some the computations are correct.	
				OVERALL RATING	

What to Know

Before moving to the operation on rational algebraic expressions, review first operations of fraction and the LCD.

Activity 1: Egyptian Fraction

This activity will enhance the learner their capability in operating fractions. This is also a venuee for the learner to review and recall the concepts on operations of fractions. Their response to the questions may be written on their journal notebook.

Answer to the activity:

1. $\frac{7}{10} = \frac{1}{2} + \frac{1}{5}$
2. $\frac{8}{15} = \frac{1}{3} + \frac{1}{5}$
3. $\frac{3}{4} = \frac{1}{2} + \frac{1}{5}$
4. $\frac{11}{30} = \frac{1}{6} + \frac{1}{5}$
5. $\frac{7}{12} = \frac{1}{3} + \frac{1}{4}$
6. $\frac{13}{12} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
7. $\frac{11}{12} = \frac{1}{2} + \frac{1}{6} + \frac{1}{4}$
8. $\frac{31}{30} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5}$
9. $\frac{19}{20} = \frac{1}{2} + \frac{1}{5} + \frac{1}{4}$
10. $\frac{25}{28} = \frac{1}{7} + \frac{1}{2} + \frac{1}{4}$

Lesson 2 Operations of Rational Algebraic Expressions

What to Know

In the first lesson, you learned that rational algebraic expression is a ratio of two polynomials where the denominator is not equal to zero. In this lesson, you will be able to perform operations on rational algebraic expressions. Before moving to the new lesson, let's look back on the concepts that you have learned that are essential to this lesson.

In the previous mathematics lesson, your teacher taught you how to add and subtract fractions. What mathematical concept plays a vital role in adding and subtracting fraction? You may think of LCD or Least Common Denominator. Now, let us take another perspective in adding or subtracting fractions. Ancient Egyptians had special rules in their fraction. When they have 5 loaves for 8 persons, they did not divide it immediately by 8, they used the concept of unit fraction. Unit fraction is a fraction with 1 as numerator. Egyptian fractions used unit fractions without repetition except $\frac{2}{3}$. Like 5 loaves for 8 persons, they have to cut the 4 loaves into two and the last one will be cut into 8 parts. In short:

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

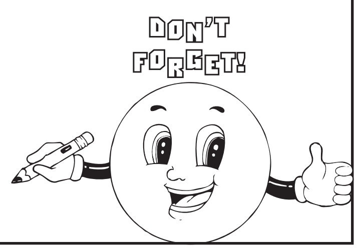
Activity 1 EGYPTIAN FRACTION

Now, be like an Ancient Egyptian. Give the unit fractions in Ancient Egyptian way.

1. $\frac{7}{10}$ using 2 unit fractions.
2. $\frac{8}{15}$ using 2 unit fractions.
3. $\frac{3}{4}$ using 2 unit fractions.
4. $\frac{11}{30}$ using 2 unit fractions.
5. $\frac{7}{12}$ using 2 unit fractions.
6. $\frac{13}{12}$ using 3 unit fractions.
7. $\frac{11}{12}$ using 3 unit fractions.
8. $\frac{31}{30}$ using 3 unit fractions.
9. $\frac{19}{20}$ using 3 unit fractions.
10. $\frac{25}{28}$ using 3 unit fractions.

Activity 2: Anticipation Guide
This activity aims to elicit background knowledge of the learner regarding operations on rational algebraic expressions. You can use the response of the learner as benchmark.

Teacher's Note and Reminders



Activity 3: Picture Analysis
Let the learner describe the picture. He/She may write his/her description and response to the questions in the journal notebook.

This picture may describe the application of operations on rational algebraic expression.

QUESTIONS ?

1. What did you do in giving the unit fraction?
2. How do you feel giving the unit fractions?
3. What difficulties do you encountered in giving unit fraction?
4. What will you do in overcoming these difficulties?

Activity 2 ANTICIPATION GUIDE

There are sets of rational algebraic expressions in the table below. Check agree if the entries in column I is equivalent to the entry in column II and check disagree if the entries in the two columns are not equivalent.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \bullet \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		
$\frac{5}{4x^2} + \frac{7}{6x}$	$\frac{15 + 14x}{12x^2}$		
$\frac{a}{b - a} - \frac{b}{a - b}$	$\frac{a + b}{b - a}$		
$\dfrac{\frac{a + b}{b} - \frac{b}{a + b}}{\frac{1}{b} + \frac{2}{a}}$	$\frac{a^2}{a + b}$		

Activity 3 PICTURE ANALYSIS

Take a close look at this picture. Describe what you see.

http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php

What to Process

Before moving to the topic, review them about operations of fraction. You can gauge their understanding on operation of fraction by letting them perform the operation of fraction.

ANSWER TO REVIEW

Perform the operation of the following fractions.

1. $\frac{1}{2} \bullet \frac{4}{3} = \frac{2}{3}$
2. $\frac{3}{4} \bullet \frac{2}{3} = \frac{1}{2}$
3. $\frac{8}{11} \bullet \frac{33}{40} = \frac{3}{5}$
4. $\frac{1}{4} \bullet \frac{3}{2} = \frac{3}{8}$
5. $\frac{1}{6} \bullet \frac{2}{9} = \frac{2}{27}$

Teacher's Note and Reminders

Illustrative Example

In every step in each illustrative example, there are ideas that are presented and there are review questions and questions to ponder. These questions will unwrap the concept in every step in the solution. Let them analyze each step. You can also give more examples to emphasize the concept.

QUESTIONS

1. What will happen if one of them will not do his job?
2. What will happen when there are more people working together?
3. How does the rate of each workers affect the entire work?
4. How will you model the rate – related problem?

The picture above shows how the operations on rational algebraic expressions can be applied to real – life scenario. You'll get to learn more rate – related problems and how operations on rational algebraic expression associate to it

What to Process

Your goal in this section is to learn and understand key concepts in the operations on rational algebraic expressions.

As the concepts of operations on rational algebraic expressions become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in solving real – life problems especially rate – related problems.

REVIEW

Perform the operation of the following fractions.

1. $\frac{1}{2} \bullet \frac{4}{3}$
2. $\frac{3}{4} \bullet \frac{2}{3}$
3. $\frac{8}{11} \bullet \frac{33}{40}$
4. $\frac{1}{4} \bullet \frac{3}{2}$
5. $\frac{1}{6} \bullet \frac{2}{9}$

Activity 4 MULTIPLYING RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the illustrative examples below. Pause once in a while to answer the check – up questions.

The product of two rational expressions is the product of the numerators divided by the product of the denominators. In symbols,

$$\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd},\ bd \neq 0$$

Illustrative example 1: Find the product of $\frac{5t}{8}$ and $\frac{4}{3t^2}$.

$$\frac{5t}{8} \bullet \frac{4}{3t^2} = \frac{5t}{2^3} \bullet \frac{2^2}{3t^2}$$

$$= \frac{(5t)\cancel{(2^2)}}{\cancel{(2^2)}(2)(3t^2)}$$

Express the numerators and denominators into prime factors as possible.

Teacher's Note and Reminders

Answers to Exercises

1. $\dfrac{4x}{u}$ 2. $\dfrac{a^2+ab}{2b}$ 3. $\dfrac{x}{x+5}$ 4. $\dfrac{x+1}{y+1}$ 5. $\dfrac{a-b}{a+1}$

$$= \frac{5}{(2)(3t)}$$
$$= \frac{5}{6t}$$

Simplify rational expression using laws of exponents.

Illustrative example 2: Multiply $\dfrac{4x}{3y}$ and $\dfrac{3x^2y^2}{10}$.

$$\frac{4x}{3y} \cdot \frac{3x^2y^2}{10} = \frac{(2^2)x}{3y} \cdot \frac{3x^2y^2}{(2)(5)}$$
$$= \frac{\cancel{(2)}(2)(x)\cancel{(3)}(x^2)(y)\cancel{(y)}}{\cancel{(3)}\cancel{(y)}\cancel{(2)}(5)}$$
$$= \frac{(2)(x^3)(y)}{(5)}$$
$$= \frac{2x^3y}{5}$$

? What laws of exponents were used in these steps?

Illustrative example 3: What is the product of $\dfrac{x-5}{(4x^2-9)}$ and $\dfrac{4x^2+12x+9}{2x^2-11x+5}$?

$$\frac{x-5}{4x^2-9} \cdot \frac{4x^2-12x+9}{2x^2-11x+5} = \frac{x-5}{(2x-3)(2x+3)} \cdot \frac{(2x+3)^2}{(2x-1)(x-5)}$$
$$= \frac{\cancel{(x-5)}(2x+3)\cancel{(2x+3)}}{(2x-3)\cancel{(2x+3)}\,(2x-1)\cancel{(x-5)}}$$
$$= \frac{2x+3}{(2x-3)(2x-1)}$$
$$= \frac{2x+3}{4x^2-8x+4}$$

? What factoring methods were used in this step?

? What are the rational algebraic expressions equivalent to 1 in this step?

QUESTIONS ?

1. What are the steps in multiplying rational algebraic expressions?
2. What do you observe from each step in multiplying rational algebraic expressions?

Exercises

Find the product of the following rational algebraic expressions.

1. $\dfrac{10uv^2}{3xy^2} \cdot \dfrac{6x^2y^2}{5u^2v^2}$
2. $\dfrac{a^2-b^2}{2ab} \cdot \dfrac{a^2}{a-b}$
3. $\dfrac{x^2-3x}{x^2+3x-10} \cdot \dfrac{x^2-4}{x^2-x-6}$
4. $\dfrac{x^2+2x+1}{y^2-2y+1} \cdot \dfrac{y^2-1}{x^2-1}$
5. $\dfrac{a^2-2ab+b^2}{a^2-1} \cdot \dfrac{a-1}{a-b}$

Answers to Activity 5: What's My Area

1. $-\frac{b}{4}$ 2. $\frac{1}{3}$ 3. $\frac{y-2}{3}$

This activity is multiplying rational algebraic expressions but in a different context. After this activity, let them sequence the steps in multiplying rational algebraic expression. Let them identify the concepts and principles for every step.

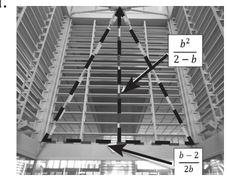
Teacher's Note and Reminders

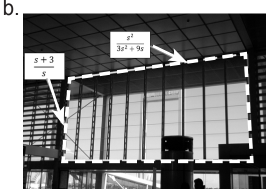
CONCEPT CHANGE MAP
Activity 6: The Circle Arrow Process

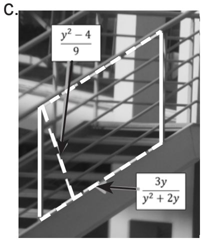
As the learner sequences the steps, he/she will identify the mathematical concepts behind each step. Place the mathematical concept inside the circle until he/she arrived at the final answer.

Activity 5 WHAT'S MY AREA?

Find the area of the plane figures below.

a.


b.


c.


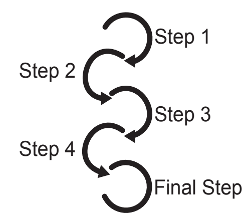
QUESTIONS

1. How did you find the area of the figures?
2. What are your steps in finding the area of the figures?

Activity 6 THE CIRCLE ARROW PROCESS

Based on the steps that you made in the previous activity, make a conceptual map on the steps in multiplying rational algebraic expressions. Write the procedure or important concepts in every step inside the circle. If necessary, add a new circle.

Web – based Booster:
Watch the videos in this web – sites for more examples. http://www.onlinemathlearning.com/multiplying-rational-expressions-help.html



QUESTIONS

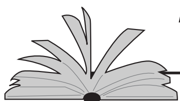
1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can the mathematical concepts used in every step be interchanged? How?
4. Can you give another method in multiplying rational algebraic expressions?

Activity 7: Dividing Rational Algebraic Expressions
The same as the illustrative examples in multiplying rational algebraic expressions, each illustrative example in this operation has key ideas, review question to unveil the concept on each step. But before they begin dividing rational algebraic expressions, they have to review how to divide fractions.

ANSWER TO REVIEW EXERCISES

Perform the operation of the following fractions.

1. $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$ 2. $\frac{5}{2} \div \frac{9}{4} = \frac{10}{9}$ 3. $\frac{9}{2} \div \frac{3}{4} = 6$ 4. $\frac{10}{16} \div \frac{5}{4} = \frac{1}{2}$ 5. $\frac{1}{2} \div \frac{1}{4} = 2$



Teacher's Note and Reminders

Activity 7 Dividing Rational Algebraic Expressions

REVIEW

Perform the operation of the following fractions.

1. $\frac{1}{2} \div \frac{3}{4}$
2. $\frac{5}{2} \div \frac{9}{4}$
3. $\frac{9}{2} \div \frac{3}{4}$
4. $\frac{10}{16} \div \frac{5}{4}$
5. $\frac{1}{2} \div \frac{1}{4}$

Examine and analyze the illustrative examples below. Pause once in a while to answer the check – up questions.

The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor. In symbols,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}, \; bc \neq 0$$

Illustrative example 4: Find the quotient of $\frac{6ab^2}{4cd}$ and $\frac{9a^2b^2}{8dc^2}$.

$$\frac{6ab^2}{4cd} \div \frac{9a^2b^2}{8dc^2} = \frac{6ab^2}{4cd} \div \frac{8dc^2}{9a^2b^2}$$

Multiply the dividend by the reciprocal of the divisor.

$$= \frac{(2)(3)ab^2}{(2)^2cd} \div \frac{(2^3)dc^2}{(3^2)a^2b^2}$$

$$= \frac{(2^2)\cancel{(2^2)}\cancel{(3)}\cancel{a}\cancel{b^2}\cancel{d}\cancel{c}c}{\cancel{(2^2)}(3)\cancel{(3)}\cancel{c}\cancel{d}\cancel{a}a\cancel{b^2}}$$

$$= \frac{(2)^2c}{(3)a}$$

$$= \frac{4c}{3a}$$

Perform the steps in multiplying rational algebraic expressions.

Illustrative example 5: Divide $\frac{2x^2 + x - 6}{2x^2 + 7x + 5}$ by $\frac{x^2 - 2x - 8}{2x^2 - 3x - 20}$.

$$\frac{2x^2 + x - 6}{2x^2 + 7x + 5} \div \frac{x^2 - 2x - 8}{2x^2 - 3x - 20}$$

$$= \frac{2x^2 + x - 6}{2x^2 + 7x + 5} \bullet \frac{2x^2 - 3x - 20}{x^2 - 2x - 8}$$

$$= \frac{(2x - 3)(x + 2)}{(2x + 5)(x + 1)} \bullet \frac{(x - 4)(2x + 5)}{(x + 2)(x - 4)}$$

Why do we need to factor out the numerators and denominators?

$$= \frac{(2x - 3)\cancel{(x + 2)}\cancel{(x - 4)}\cancel{(2x + 5)}}{\cancel{(2x + 5)}(x + 1)\cancel{(x + 2)}\cancel{(x - 4)}}$$

$$= \frac{(2x - 3)}{(x + 1)}$$

What happens to the common factors between numerator and denominator?

$$= \frac{2x - 3}{x + 1}$$

Answers to Exercises

1. Z 2. $\dfrac{a+b}{2a+2b}$ 3. -1 4. $\dfrac{x^2+2x+1}{x^2+2x-3}$ 5. $-x-1$

Answers to Activity 8

1. $\dfrac{5x-50}{4}$ 2. $\dfrac{2x^3-14x^2}{245}$

This activity may assess the learner's understanding in dividing rational algebraic expression. This may help learner consider the division of rational algebraic expressions in different context.
Let them enumerate the steps in dividing rational algebraic expressions and identify the concepts and principle involved in every step

Teacher's Note and Reminders

Exercises

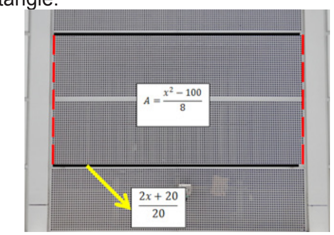
Find the quotient of the following rational algebraic expressions.

1. $\dfrac{81xz^3}{36y} \div \dfrac{27x^2z^2}{12xy}$
2. $\dfrac{2a+2b}{a^2+ab} \div \dfrac{4}{a}$
3. $\dfrac{16x^2-9}{6-5x-4x^2} \div \dfrac{16x^2+24x+9}{4x^2+11x+6}$
4. $\dfrac{x^2+2x+1}{x^2+4x+3} \div \dfrac{x^2-1}{x^2+2x+1}$
5. $\dfrac{x-1}{x+1} \div \dfrac{1-x}{x^2+2x+1}$

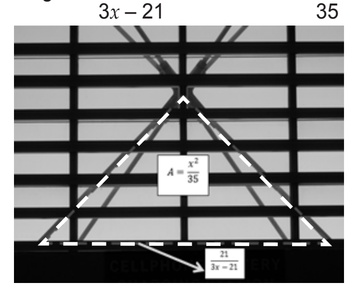
Activity 8 MISSING DIMENSION

Find the missing length of the figures.

1. The area of the rectangle is $\dfrac{x^2-100}{8}$ while the length is $\dfrac{2x^2+20}{20}$. Find the height of the rectangle.



2. The base of the triangle is $\dfrac{21}{3x-21}$ and the area is $\dfrac{x^2}{35}$. Find the height of the triangle.



QUESTIONS

1. How did you find the missing dimension of the figures?
2. Enumerate the steps in solving the problems.

MAP OF CONCEPTUAL CHANGE
Activity 9: Chain Reaction
As the learner enumerates the steps in dividing rational algebraic expression, his/her can identify mathematical concepts in each step. Place the mathematical concept inside the chamber until he/she arrived at the final answer. This activity may be individual or collaborative work.

Teacher's Note and Reminders

ANSWER TO REVIEW
Perform the operation of the following fractions.

1. $\frac{1}{2} + \frac{3}{2} = 2$
2. $\frac{5}{4} + \frac{9}{4} = \frac{7}{2}$
3. $\frac{9}{5} + \frac{3}{5} = \frac{12}{5}$
4. $\frac{10}{13} - \frac{5}{13} = \frac{5}{13}$
5. $\frac{5}{4} - \frac{1}{4} = 1$

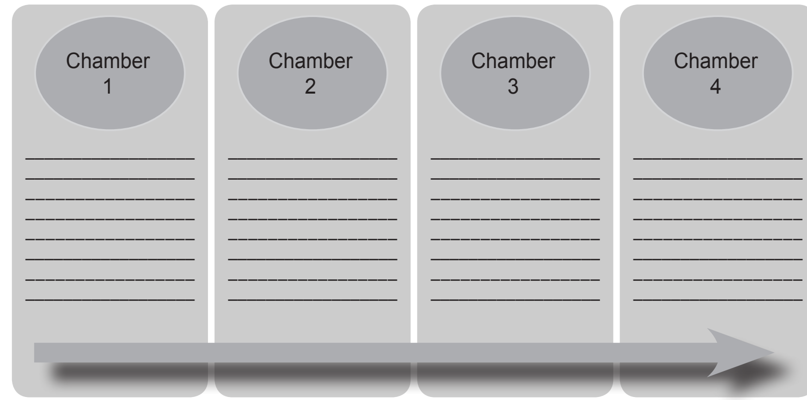
Activity 10
The illustrative examples in this topic also have ideas and questions to guide the students in identifying concepts and principle involved in every step. Before discussing and giving more examples in adding and subtracting rational algebraic expressions, review them on how to add and subtract fractions.

Activity 9 CHAIN REACTION

Web Based Booster
Click on this web site below to watch videos in dividing rational algebraic expressions

http://www.onlinemathlearning.com/dividing-rational-expressions-help.html

Use the Chain Reaction Chart to sequence your steps in dividing rational algebraic expressions. Write the process or mathematical concepts used in each step in the chamber. Add another chamber, if necessary.



QUESTIONS

1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can mathematical concept in every step be interchanged? How?
4. Can you make another method in dividing rational algebraic expressions? How?

Activity 10 ADDING AND SUBTRACTING SIMILAR RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples on the next page. Pause in a while to answer the check–up questions.

In adding or subtracting similar rational expressions, add or subtract the numerators and write it in the numerator of the result over the common denominator. In symbols,

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b},\ b \neq 0$$

REVIEW
Perform the operation of the following fractions.

1. $\frac{1}{2} + \frac{3}{2}$
2. $\frac{5}{4} + \frac{9}{4}$
3. $\frac{9}{5} + \frac{3}{5}$
4. $\frac{10}{13} - \frac{5}{13}$
5. $\frac{5}{4} - \frac{1}{4}$

Teacher's Note and Reminders

ANSWER TO EXERCISE

Perform the operation of the following fractions.

1. $\dfrac{10}{a-5}$ 2. $\dfrac{2x^2+x+2}{x^2-4}$ 3. $\dfrac{2}{4x-1}$ 4. $\dfrac{x+1}{x-1}$ 5. $\dfrac{2x-4}{x-1}$

Illustrative example 6: Add $\dfrac{x^2-2x-7}{x^2-9}$ and $\dfrac{3x+1}{x^2-9}$.

$$\frac{x^2-2x-7}{x^2-9}+\frac{3x+1}{x^2-9}=\frac{x^2-2x+3x-7+1}{x^2-9}$$

$$=\frac{x^2+x-6}{x^2-9}$$

Combine like terms in the numerator.

$$=\frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+3)}}$$

Factor out the numerator and denominator.

$$=\frac{(x-2)}{(x+3)}$$

$$=\frac{x-2}{x+3}$$

Do we always factor out the numerator and denominator? Why yes or why not?

Illustrative example 7: Subtract $\dfrac{-10-6x-5x^2}{3x^2+x-2}$ from $\dfrac{x^2+5x-20}{3x^2+x-2}$.

$$\frac{x^2+5x^2-20}{3x^2+x-2}-\frac{-10-6x-5x^2}{3x^2+x-3}=\frac{x^2+5x^2-20-(-10-6x-5x^2)}{3x^2+x-2}$$

$$=\frac{x^2+5x-20+10+6x+5x^2}{3x^2+x-2}$$

Why do we need to multiply the subtrahend by – 1 in the numerator?

$$=\frac{x^2+5x^2+5x+6x-20+10}{3x^2+x-2}$$

$$=\frac{6x^2+11x-10}{3x^2+x-2}$$

$$=\frac{\cancel{(3x-2)}(2x+5)}{\cancel{(3x-2)}(x+1)}$$

$$=\frac{2x+5}{x+1}$$

Factor out the numerator and denominator.

Exercises

Perform the indicated operation. Express your answer in simplest form.

1. $\dfrac{6}{a-5}+\dfrac{4}{a-5}$
2. $\dfrac{x^2+3x-2}{x^2-4}+\dfrac{x^2-2x+4}{x^2-4}$
3. $\dfrac{7}{4x-1}-\dfrac{5}{4x-1}$
4. $\dfrac{x^2+3x+2}{x^2-2x+1}-\dfrac{3x+3}{x^2-2x+1}$
5. $\dfrac{x-2}{x-1}+\dfrac{x-2}{x-1}$

Activity 11
Before introducing the addition/subtraction of dissimilar rational algebraic expressions, learners must review how to add/subtract dissimilar fractions. Let them perform addition/subtraction of fraction and process their answers.

ANSWER TO REVIEW
Perform the operation of the following fractions.

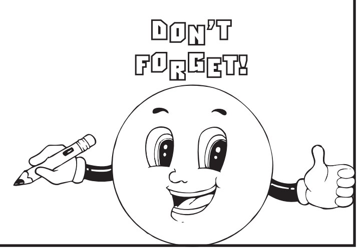
1. $\frac{1}{2} + \frac{4}{3} = \frac{11}{6}$ 2. $\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$ 3. $\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$

4. $\frac{1}{4} - \frac{3}{2} = -\frac{5}{4}$ 5. $\frac{1}{6} - \frac{2}{9} = \frac{1}{18}$

Illustrative Example 8
Each example in this topic has a box below the first step. Emphasize to them the process of finding the LCD between rational algebraic expressions. As much as possible, link this process to how LCD of fraction is being derived so that they can relate the process easily. If needed, before discussing the addition/subtraction of rational algebraic expression, give them examples of finding LCD of rational algebraic expressions.

Give more examples in adding/subtracting dissimilar rational algebraic expressions if needed. In this topic, more examples are presented in the learning guide.

Teacher's Note and Reminders



Activity 11 ADDING AND SUBTRACTING DISSIMILAR RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples below. Pause in a while to answer the check–up questions.

REVIEW
Perform the operation of the following fractions.
1. $\frac{1}{2} + \frac{4}{3}$ 4. $\frac{1}{4} - \frac{3}{2}$
2. $\frac{3}{4} + \frac{2}{3}$ 5. $\frac{1}{6} - \frac{2}{9}$
3. $\frac{3}{4} + \frac{1}{8}$

In adding or subtracting dissimilar rational expressions change the rational algebraic expressions into similar rational algebraic expressions using the least common denominator or LCD and proceed as in adding similar fractions.

illustrative example 8: Find the sum of $\frac{5}{18a^4b}$ and $\frac{2}{27a^3b^2c}$.

$$\frac{5}{18a^4b} + \frac{2}{27a^3b^2c} = \frac{5}{(3^2)(2)a^4b} + \frac{2}{(3^3)a^3b^2c}$$

Express the denominators as prime factors.

LCD of $\frac{5}{(3^2)(2)a^4b}$ and $\frac{2}{(3^3)a^3b^2c}$

$(3^2)(2)a^4b$ and $(3^3)a^3b^2c$

The LCD is $(3^3)(2)(a^4)(b^2)(c)$

Denominators of the rational algebraic expressions

Take the factors of the denominators. When the same factor is present in more than one denominator, take the factor with the highest exponent. The product of these factors is the **LCD**.

$$= \frac{5}{(3^2)(2)a^4b} \cdot \frac{3bc}{3bc} + \frac{2}{(3^3)a^3b^2c} \cdot \frac{2a}{2a}$$

$$= \frac{(5)(3)bc}{(3^3)(2)a^4b^2c} + \frac{(2^2)a}{(3^3)(2)a^4b^2c}$$

$$= \frac{15bc}{54a^4b^2c} + \frac{4a}{54a^4b^2c}$$

$$= \frac{15bc + 4a}{54a^4b^2c}$$

Find a number equivalent to 1 that should be multiplied to the rational algebraic expressions so that the denominators are the same with the **LCD**.

Teacher's Note and Reminders

Illustrative example 9: Subtract $\dfrac{t+3}{t^2-6t+9}$ by $\dfrac{8t-24}{t^2-9}$.

$$\frac{t+3}{t^2-6t+9} - \frac{8t-24}{t^2-9} = \frac{t+3}{(t-3)^2} - \frac{8t-24}{(t-3)(t+3)}$$

Express the denominators as prime factors.

LCD of $\dfrac{t+3}{(t-3t)^2}$ and $\dfrac{8t-24}{(t-3)(t+3)}$

$(t-3)^2$ and $(t-3)(t+3)$

The LCD is $(t-3)^2(t+3)$

$$= \frac{t+3}{(t-3)^2} \cdot \frac{t+3}{t+3} - \frac{(8t-24)}{(t-3)^2(t+3)} \cdot \frac{t-3}{t-3}$$

What property of equality is illustrated in this step?

$$= \frac{(t+3)(t+3)}{(t-3)^2(t+3)} - \frac{(8t-24)}{(t-3)^2(t+3)}$$

$$= \frac{t^2+6t+9}{t^3-9t^2+27t-27} - \frac{8t-48t+72}{t^3-9t^2+27t-27}$$

What special products are illustrated in this step?

$$= \frac{t^2+6t+9-(8t^2-48t+72)}{t^3-9t^2+27t-27}$$

$$= \frac{t^2+6t+9-8t^2+48t-72}{t^3-9t^2+27t-27}$$

$$= \frac{-7t^2+54t-63}{t^3-9t^2+27t-27}$$

Illustrative example 10: Find the sum of $\dfrac{2x}{x^2+4x+3}$ by $\dfrac{3x-6}{x^2+5x+6}$.

$$\frac{2x}{x^2+4x+3} + \frac{3x-6}{x^2+5x+6} = \frac{2x}{(x+3)(x+1)} + \frac{3x-6}{(x+3)(x+2)}$$

What special products are illustrated in this step?

LCD of $\dfrac{2x}{(x+3)(x+1)}$ and $\dfrac{3x-6}{(x+3)(x+2)}$

$(x+3)(x+1)$ and $(x+3)(x+2)$

The LCD is $(x+3)(x+1)(x+2)$

What property of equality was used in this step?

$$= \frac{2x}{(x+3)(x+1)} \cdot \frac{(x+2)}{(x+2)} + \frac{(3x-6)}{(x+3)(x+2)} \cdot \frac{(x+1)}{(x+1)}$$

$$= \frac{(2x)(x+2)}{(x+3)(x+1)(x+2)} + \frac{(3x-6)(x+1)}{(x+3)(x+2)(x+1)}$$

$$= \frac{2x^2+4x}{x^3+6x^2+11x+6} + \frac{3x^2-3x-6}{x^3+6x^2+11x+6}$$

ANSWER TO EXERCISE

Perform the operation of the following fractions.

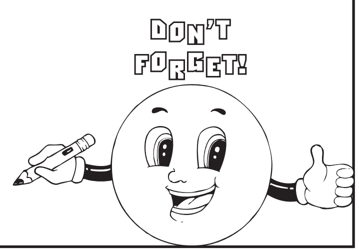
1. $\dfrac{7x + 4}{x^2 + x}$
2. $\dfrac{4x^2 + 2x + 20}{x^3 - 2x^2 - 4x + 8}$
3. $\dfrac{-x - 9}{x^2 - 9}$
4. $\dfrac{x - 11}{x^3 - 4x^2 + x + 6}$
5. $\dfrac{-x^2 + 4}{2x}$

MAP OF CONCEPTUAL CHANGE

Activity 12: Flow Chart

Let them enumerate the steps in adding/subtracting rational algebraic expressions, both similar and dissimilar expressions. Let them organize these steps by completing the flow chart below. You can validate their work by adding/subtracting rational algebraic expressions using their flow chart.

Teacher's Note and Reminders



$$= \frac{2x^2 + 3x^2 + 4x - 3x - 6}{x^3 + 6x^2 + 11x + 6}$$

$$= \frac{5x^2 + x - 6}{x^3 + 6x^2 + 11x + 6}$$

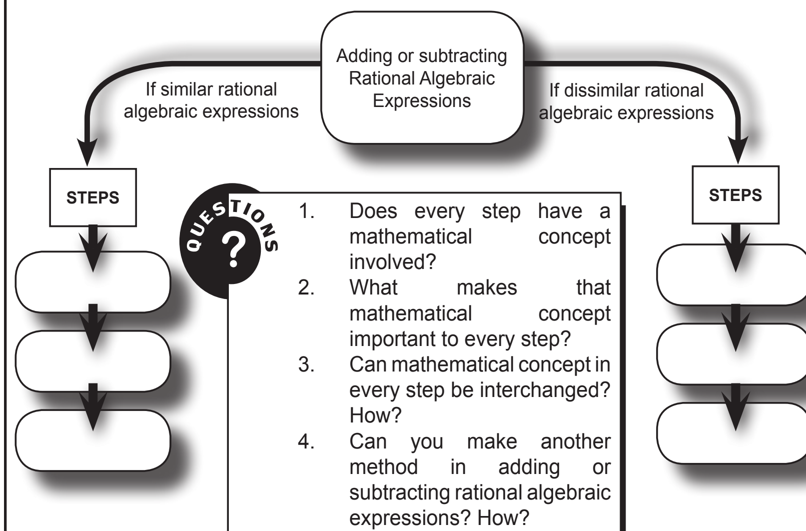
Exercises

Perform the indicated operation. Express your answer in simplest form.

1. $\dfrac{3}{x + 1} + \dfrac{4}{x}$
2. $\dfrac{x + 8}{x^2 - 4x + 4} + \dfrac{3x - 2}{x^2 - 4}$
3. $\dfrac{2x}{x^2 - 9} - \dfrac{3}{x - 3}$
4. $\dfrac{3}{x^2 - x - 2} - \dfrac{2}{x^2 - 5x + 6}$
5. $\dfrac{x + 2}{x} - \dfrac{x + 2}{2}$

Activity 12 FLOW CHART

Now that you have learned adding and subtracting rational algebraic expressions. You are now able to fill in the graphic organizer below. Write each step in adding or subtracting rational algebraic expression in the boxes below.



1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can mathematical concept in every step be interchanged? How?
4. Can you make another method in adding or subtracting rational algebraic expressions? How?

Activity 13
This activity may help students to correct their misconceptions. This may also help you gauge whether the learners learned the concept or not. If necessary, give more examples to strengthen their understanding. The response of the students in guided questions may be written in their journal notebook.

Points to be emphasize in this activity

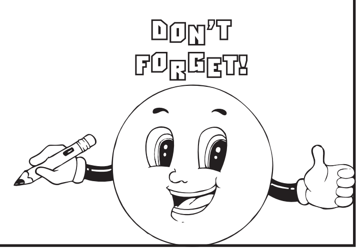
For the solution in the first box: The error in this item is the $(6 - x)$ becomes $(x - 6)$. The factor of $(6 - x)$ is $-1(x - 6)$.

For the solution in the second box: The wrong concepts here are a – 5 (a) becomes $a^2 - 5a$ and the numerator of subtrahend must be multiplied by -1. $a - 5$ (a) is equal to $a - 5a$.

For the solution in the third box: 3 must not be cancelled out. The concept of dividing out can be applied to a common factor and not to the common variable or number in the numerator and denominator.

For the solution in the fourth box: $b^2 - 4b + 4$ must be factored out as $(b - 2)(b - 2)$. The concept of factoring is essential in performing operations on rational algebraic expressions.

Teacher's Note and Reminders



Web – based Booster:
Watch the videos in these web sites for more examples.
http://www.onlinemathlearning.com/adding-rational-expressions-help.html
http://www.onlinemathlearning.com/subtracting-rational-expressions-help.html

Activity 13 WHAT IS WRONG WITH ME?

Rewrite the solution of the first box. Write your solution in the second box and in the third box, write your explanation on how your solution corrects the original one .

Original	My Solution	My Explanation
$\begin{aligned} \frac{2}{36 - x^2} - \frac{1}{x^2 - 6x} &= \frac{2}{(6 - x)(6 - x)} - \frac{1}{x(x + 6)} \\ &= \frac{2}{(x - 6)(x + 6)} - \frac{1}{x(x + 6)} \\ &= \frac{2}{(x - 6)(x + 6)} \bullet \frac{x}{x} - \frac{1}{x(x + 6)} \bullet \frac{x - 6}{x - 6} \\ &= \frac{2x}{x(x - 6)(x + 6)} - \frac{1(x - 6)}{x(x + 6)(x - 6)} \\ &= \frac{2x - (x - 6)}{x(x - 6)(x + 6)} \\ &= \frac{2x - x + 6}{x(x - 6)(x + 6)} \\ &= \frac{x + 6}{x(x - 6)(x + 6)} \\ &= \frac{1}{x(x - 6)} \\ &= \frac{1}{x^2 - 6x} \end{aligned}$		
$\begin{aligned} \frac{2}{a - 5} - \frac{3}{a} &= \frac{2}{a - 5} \bullet \frac{a}{a} - \frac{3}{a} \bullet \frac{a - 5}{a - 5} \\ &= \frac{2a}{a - 5(a)} - \frac{3(a - 5)}{a(a - 5)} \\ &= \frac{2a}{a - 5(a)} - \frac{3a - 15}{a(a - 5)} \\ &= \frac{2a - 3a - 15}{a(a - 5)} \\ &= \frac{-a - 15}{a^2 - 5a} \end{aligned}$		

Teacher's Note and Reminders

Activity 14. Complex Rational Expressions
Like on the previous topics, each illustrative example has ideas and questions to guide the learners in determining the concepts and principles in each step. For the students to relate the new topic, start the discussion by reviewing simplifying complex fraction. You can also give more examples to give emphasis on the concepts and principles involving in this topic.

Answer to the Review:
Perform the operation of the following fractions.

1. $\dfrac{\frac{1}{2}+\frac{4}{3}}{1-\frac{2}{3}}=\dfrac{11}{2}$
2. $\dfrac{\frac{1}{2}-\frac{4}{3}}{\frac{3}{4}-\frac{2}{3}}=-10$
3. $\dfrac{\frac{5}{2}-\frac{4}{3}}{\frac{2}{3}+2}=\dfrac{7}{16}$
4. $\dfrac{\frac{1}{2}+\frac{5}{4}}{\frac{4}{3}-\frac{2}{3}}=\dfrac{21}{8}$
5. $\dfrac{\frac{5}{9}+\frac{4}{3}}{1+\frac{2}{3}}=\dfrac{17}{15}$

$\frac{3x}{2x-3}+\frac{9}{3-2x}=\frac{3x}{2x-3}+\frac{9}{(-1)(2x-3)}$ $=\frac{3x}{2x-3}-\frac{9}{2x-3}$ $=\frac{3x-9}{2x-3}$ $=\frac{3(x-3)}{2x-3}$ $=\frac{x-3}{2x}$		
$\frac{4}{b-2}+\frac{b^2-4b}{b-2}=\frac{b^2-4b+4}{b-2}$ $=\frac{(b-2)(b+2)}{b-2}$ $=b+2$		

QUESTIONS ?

1. What did you feel while answering the activity?
2. Did you encounter difficulties in answering the activity?
3. How did you overcome these difficulties?

The previous activities deal with the fundamental operations on rational expressions. Let us try these concepts in a different context.

Activity 14 COMPLEX RATIONAL ALGEBRAIC EXPRESSIONS

Examine and analyze the following illustrative examples on the next page. Pause in a while to answer the check – up questions.

Rational algebraic expression is said to be in its simplest form when the numerator and denominator are polynomials with no common factors other than 1. If the numerator or denominator, or both numerator and denominator of a rational algebraic expression is also a rational algebraic expression, it is called a ***complex rational algebraic expression****. To simplify the complex rational expression, it means to transform it into simple rational expression. You need all the concepts learned previously to simplify complex rational expressions.*

REVIEW

Perform the operation of the following fractions.

1. $\dfrac{\frac{1}{2}+\frac{4}{3}}{1-\frac{2}{3}}$
2. $\dfrac{\frac{1}{2}-\frac{4}{3}}{\frac{3}{4}-\frac{2}{3}}$
3. $\dfrac{\frac{5}{2}-\frac{4}{3}}{\frac{2}{3}+2}$
4. $\dfrac{\frac{1}{2}+\frac{5}{4}}{\frac{4}{3}-\frac{2}{3}}$
5. $\dfrac{\frac{5}{9}+\frac{4}{3}}{1+\frac{2}{3}}$

Teacher's Note and Reminders

Illustrative example 11: Simplify $\dfrac{\frac{2}{a}-\frac{3}{b}}{\frac{5}{b}+\frac{6}{a^2}}$.

$$\frac{\frac{2}{a}-\frac{3}{b}}{\frac{5}{b}+\frac{6}{a^2}}$$

Main fraction —— a line that separates the main numerator and main denominator.

$$=\frac{\left(\frac{2}{a}\cdot\frac{b}{b}\right)-\left(\frac{3}{b}\cdot\frac{a}{a}\right)}{\left(\frac{5}{b}\cdot\frac{a^2}{a^2}\right)+\left(\frac{6}{a^2}\cdot\frac{b}{b}\right)}$$

Where does the $\frac{b}{b}$ and $\frac{a}{a}$ in the main numerator and the $\frac{a^2}{a^2}$ and $\frac{b}{b}$ in the main denominator came from?

$$=\frac{\left(\frac{2b}{ab}\right)-\left(\frac{3a}{ab}\right)}{\left(\frac{5a^2}{a^2b}\right)+\left(\frac{6b}{a^2b}\right)}$$

$$=\frac{\frac{2b-3a}{ab}}{\frac{5a^2+6b}{a^2b}}$$

What happens to the main numerator and main denominator?

$$=\frac{2b-3a}{ab}\div\frac{5a^2+6b}{a^2b}$$

What principle is used in this step?

$$=\frac{2b-3a}{ab}\cdot\frac{a^2b}{5a^2+6b}$$

$$=\frac{(2b-3a)a\cancel{b}}{(5a^2+6b)a\cancel{b}}$$

Simplify the rational algebraic expression.

$$=\frac{(2b-3a)a}{(5a^2+6b)}$$

What laws of exponents are used in this step?

$$=\frac{2ab-3a^2}{5a^2+6b}$$

Illustrative example 12: Simplify $\dfrac{\frac{c}{c^2-4}-\frac{c}{c-2}}{1+\frac{1}{c+2}}$.

$$\frac{\frac{c}{c^2-4}-\frac{c}{c-2}}{1+\frac{1}{c+2}}=\frac{\frac{c}{(c-2)(c+2)}-\frac{c}{c-2}}{1+\frac{1}{c+2}}$$

$$=\frac{\frac{c}{(c-2)(c+2)}-\frac{c}{(c-2)}\cdot\frac{(c+2)}{(c+2)}}{1\cdot\frac{c+2}{c+2}+\frac{1}{(c+2)}}$$

Teacher's Note and Reminders

Answer to Exercises

1. $\dfrac{x^2y + xy^2}{x^2 + y^2}$
2. $\dfrac{x^2y - 2xy^2 - y^3}{x^3 + 2x^2y - xy^2}$
3. $\dfrac{2b - 3}{b}$
4. $\dfrac{2 - 2a}{7a - 9}$
5. $\dfrac{2y - 2}{y}$

$$= \dfrac{\dfrac{c}{(c-2)(c+2)} - \dfrac{c(c+2)}{(c-2)(c+2)}}{\dfrac{c+2}{c+2} + \dfrac{1}{(c+2)}}$$

$$= \dfrac{\dfrac{c}{(c-2)(c+2)} - \dfrac{c^2+2c}{(c-2)(c+2)}}{\dfrac{c+2}{c+2} + \dfrac{1}{(c+2)}}$$

$$= \dfrac{\dfrac{c-(c^2+2c)}{(c-2)(c+2)}}{\dfrac{c+2+1}{c+2}}$$

$$= \dfrac{\dfrac{-c^2-2c+c}{(c-2)(c+2)}}{\dfrac{c+2+1}{c+2}}$$

$$= \dfrac{\dfrac{-c^2-c}{(c-2)(c+2)}}{\dfrac{c+3}{c+2}}$$

$$= \dfrac{-c^2-c}{(c-2)(c+2)} \div \dfrac{c+3}{c+2}$$

$$= \dfrac{-c^2-c}{(c-2)(c+2)} \bullet \dfrac{c+2}{c+3}$$

$$= \dfrac{(-c^2-c)\cancel{(c+2)}}{(c-2)\cancel{(c+2)}(c+3)}$$

$$= \dfrac{-c^2-c}{(c-2)(c+3)}$$

$$= \dfrac{-c^2-c}{c^2+c-6}$$

Exercises

Simplify the following complex rational expressions.

1. $\dfrac{\dfrac{1}{x} - \dfrac{1}{y}}{\dfrac{1}{x^2} + \dfrac{1}{y^2}}$

2. $\dfrac{\dfrac{x-y}{x+y} - \dfrac{y}{x}}{\dfrac{x}{y} + \dfrac{x-y}{x+y}}$

3. $\dfrac{\dfrac{b}{b-1} - \dfrac{2b}{b-2}}{\dfrac{2b}{b-2} - \dfrac{3b}{b-3}}$

4. $\dfrac{\dfrac{1}{a-2} - \dfrac{3}{a-1}}{\dfrac{5}{a-2} + \dfrac{2}{a-1}}$

5. $\dfrac{4 - \dfrac{4}{y^2}}{2 + \dfrac{2}{y}}$

Activity 15: Treasure Hunting

This activity may strengthen the understanding of the learner regarding the topic. Give extra points for correct answer.

The steps:

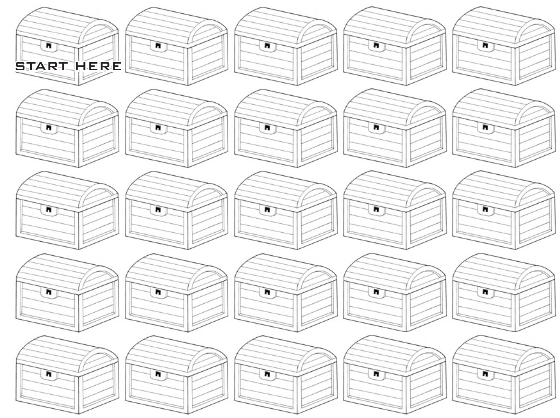
1. Down 4 steps
2. 2 steps to the right
3. Up 3 steps

Let them enumerate the steps they did in simplifying complex rational algebraic expressions and identify the principles in each step.

Teacher's Note and Reminders

Activity 15 TREASURE HUNTING

Find the box that contains treasure by simplifying rational expressions below. Find the answer of each expression in the hub. Each answer contains direction. The correct direction will lead you to the treasure. Go hunting now.



1. $\dfrac{x^2 - \dfrac{4}{x^2}}{x + \dfrac{2}{y}}$
2. $\dfrac{\dfrac{x}{2} + \dfrac{x}{3}}{\dfrac{1}{2}}$
3. $\dfrac{\dfrac{3}{x^2 + 3x + 2}}{\dfrac{x}{x + 2}}$

THE HUB

$\dfrac{5x}{3}$	$\dfrac{x^2 - 2}{x}$	$\dfrac{1}{x - 1}$	$\dfrac{x^2 + 2}{x^2 + x - 6}$	$\dfrac{3}{x^2 + x}$
2 steps to the right	Down 4 steps	3 steps to the left	4 steps to the right	Up 3 steps

QUESTIONS

Based on the above activity, what are your steps in simplifying complex rational algebraic expressions?

Activity 16: Vertical Chevron List
In the previous activity, the learner identified the steps in simplifying complex rational algebraic expressions. Let his/her organize these steps and principles using vertical chevron list.

Teacher's Note and Reminders

Activity 17: Reaction Guide
In activity 2, students were given anticipation guide. They will answer the same items in the anticipation guide, but this time they are expected to answer each item correctly. Let them compare their answer in the anticipation and reaction guide. Their answer on the questions may be written in the journal notebook. This activity will enable the students to correct their initial understanding before the lesson was presented. Let them compare their response in the anticipation guide and their response in this activity.

Activity 16 VERTICAL CHEVRON LIST

Make a conceptual map in simplifying complex rational expression using vertical chevron list. Write the procedure or important concepts in every step inside the box. If necessary, add another chevron to complete your conceptual map.

STEP 1

STEP 2

STEP 3

STEP 4

Web – based Booster:
Watch the videos in these web sites for more examples
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut11_complexrat.htm
http://www.youtube.com/watch?v=-jli9PP_4HA
http://spot.pcc.edu/~kkling/Mth_95/SectionIII_Rational_Expressions_Equations_and_Functions/Module4/Module4_Complex_Rational_Expressions.pdf

Activity 17 REACTION GUIDE

Revisit the second activity. There are sets of rational algebraic expressions in the following table. Check agree if column I is the same as column II and check disagree if the two columns are not the same.

I	II	Agree	Disagree
$\frac{x^2 - xy}{x^2 - y^2} \bullet \frac{x + y}{x^2 - xy}$	$x^{-1} - y^{-1}$		
$\frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y}$	$\frac{2y}{y + 1}$		
$\frac{5}{4x^2} + \frac{7}{6x}$	$\frac{15 + 14x}{12x^2}$		

Teacher's Note and Reminders

Activity 18: WORD PROBLEM
In this part, learner will be exposed more to how rational algebraic expressions can modelled the rate–related problems. You can discuss and give more examples similar to the items in this activity so that the students are guided on how the concepts of rational algebraic expressions modelled rate–related problems. Let them answer the activity individually or in collaborate work. Let them also enumerate the steps in solving these problems.

$\frac{a}{b-a} - \frac{b}{a-b}$	$\frac{a+b}{b-a}$		
$\dfrac{\frac{a+b}{b} - \frac{b}{a+b}}{\frac{1}{b} + \frac{2}{a}}$	$\frac{a^2}{a+b}$		

QUESTIONS

Compare your answer from the anticipation guide to the reaction guide. Do they differ from each other? Why it so?

In this section, the discussion was all about operations on rational algebraic expressions. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? The skills in performing the operations on rational algebraic expressions is one of the key concepts in solving rate – related problems.

What to Understand

Your goal in this section is to relate the operations of rational expressions to a real – life problems, especially the rate problems.

Activity 18 WORD PROBLEM

Read the problems below and answer the questions that follow.

1. Two vehicles travelled $(x + 4)$ kilometers. The first vehicle travelled for $(x^2 - 16)$ hours while the second travelled for $\frac{2}{x-4}$ hours.

   a. Complete the table below.

Vehicles	Distance	Time	Speed
Vehicle A			
Vehicle B			

Activity 19: Accent Process
Let the students enumerate the steps that they do in the previous activity. In this activity, let them organize these steps using accent process chart.

Teacher's Note and Reminders

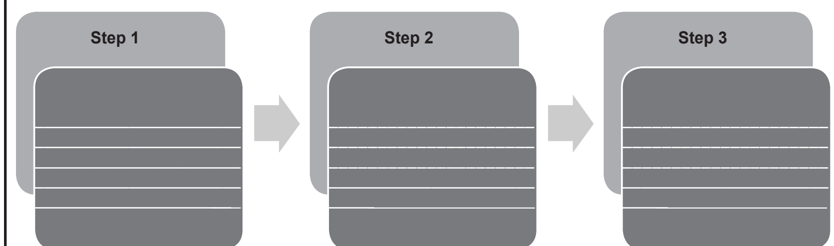
Activity 20: Presentation
In preparation for the performance task in this module, let the learner perform this activity. The learner is expected to present his/her output appropriately.

b. How did you compute the speed of the two vehicles?
c. Which of the two vehicles travelled faster? How did you find your answer?

2. Jem Boy and Roger were asked to fill the tank with water. Jem Boy can fill the tank in x minutes alone while Roger is slower by 2 minutes compared to Jem Boy if working alone.
   a. What part of the job can Jem Boy finish in 1 minute?
   b. What part of the job can Roger finish in 1 minute?
   c. Jem Boy and Roger can finish filling the tank together within certain number of minutes. How will you represent algebraically, in simplest form, the job done by the two if they worked together?

Activity 19 ACCENT PROCESS

List down the concepts and principles in solving problems involving operations of rational algebraic expressions in every step. You can add a box in necessary.



Activity 20 PRESENTATION

Present and discuss to the class the process of answering the questions below. Your output will be graded according to reasoning, accuracy, and presentation.
Alex can pour a concrete walkway in x hours alone while Andy can pour the same walkway in two more hours than Alex.

a. How fast can they pour together the walkway?
b. If Emman can pour the same walkway in one more hours than Alex and Roger can pour the same walkway in one hour less than Andy, who must work together to finish the job with the least time?

Teacher's Note and Reminders

Before moving to the transfer part, let the learner fill in the LEARNED, AFFIRMED and CHALLENGED box. This activity will solicit ideas on what and how the learner learned this lesson. Try to clear his/her thought by addressing the questions regarding in this lesson.

Rubrics for your output

CRITERIA	**Outstanding 4**	**Satisfactory 3**	**Developing 2**	**Beginning 1**
Mathematical reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail.	All computations are corrects.	Most of the computations are correct.	Some the computations are correct.
Presentation	The presentation uses appropriate and creative visual materials. It is delivered in a very convincing manner.	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganized manner.	The presentation does not use any visual materials. It is delivered in a clear manner.

In this section, the discussion was about application of operations on rational algebraic expressions. It gives you a general picture of relation between the operations of rational algebraic expressions and rate – related problems.
What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Copy the ***Learned, Affirmed, Challenged cards*** *in your journal notebook and complete it.*

Learned	**Affirmed**	**Challenge**
What new realizations and learning do you have about the topic?	What new connections have you made? Which of your old ideas have been confirmed/affirmed	What questions do you still have? Which areas seem difficult for you? Which do you want to explore

What to Transfer

This is the performance task in this module. Encourage the learner to interview skilled workers regarding their rate of work and the wage per worker. Encourage the learner to be resourceful in dealing with this performance task. They must present not only the manpower plan but also the process on how they transfer their understanding of rational algebraic expressions to this performance task. Also, after the performance task, ask the learner what difficulties they encountered and how they manage these difficulties.

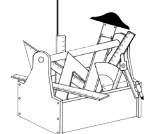
CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1
Reasoning	Explanation shows thorough reasoning and insightful justifications.	Explanation shows substantial reasoning.	Explanation shows gaps in reasoning.	Explanation shows illogical reasoning.
Accuracy	All computations are correct and shown in detail	All computations are correct.	Most of the computations are correct.	Some of the computations are correct.
Presentation	the presentation uses appropriate and creative visual materials. it is delivered in a very convincing manner	The presentation uses appropriate visual materials. It is delivered in a clear manner.	The presentation uses some visual materials. It is delivered in a disorganize manner.	The presentation does not use any visual materials. It is delivered in a clear manner.
Practically	The proposed plan will be completed at the least time.	The proposed plan will be completed in lesses time.	The proposed project will be completed with greater number of days.	The proposed plan will be completed with the most number of days.
Efficiency	The cost of the plan is minimal.	The cost of the plan is reasonable.	The cost of the plan is expensive.	The cost of the plan is very expensive.

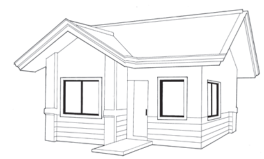
What to Transfer

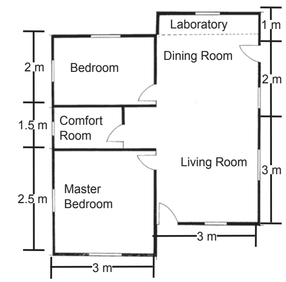
Your goal in this section is to apply your learning in real life situations. You will be given a practical task which will demonstrate your understanding.

QUESTIONS

A newly-wed couple plans to construct a house. The couple has already a house plan from their friend engineer. The plan of the house is illustrated below:







As a foreman of the project, you are task to prepare a manpower plan to be presented to the couple. Inside the plan is how many workers are needed to complete the project, daily wage of the workers, how many days can they finish the project and how much can be spend for the entire job. The man power plan will be based on reasoning, accuracy, presentation, practicality and efficiency.

POST - TEST

1. Which of the following algebraic expressions could not be considered as rational algebraic expression?

   a. $\sqrt{50}x$ b. $5x^{½}$ c. $4y^2 – 9z^2$ d. $\frac{a-b}{b+a}$

   Answer: B. The exponent in the expression in B is a fraction. Rational algebraic expression has no fractional exponent.

2. What is the rational algebraic expression equivalent to $\frac{(8kp^3)^0}{4k^{-2}p^{-3}}$?

   a. $4k^2p^3$ b. $2k^2p^3$ c. $\frac{k^2p^3}{4}$ d. $\frac{k^5p^6}{4}$

   Answer: C. The numerator is raised to 0 which means 1. The k and p are raised to a negative which means the multiplicative inverse of the expression.

3. What is the value of the expression $\frac{x^{-3}yc^8}{x^2y^{-2}c^7}$ when x is 2, y is 3 and c is -2?

   a. $-\frac{27}{16}$ b. $\frac{27}{16}$ c. $\frac{32}{27}$ d. $-\frac{32}{27}$

   Answer: A. $\frac{x^{-3}yc^8}{x^2y^{-2}c^7} = \frac{y^3c}{x^5} = \frac{(27)(-2)}{32} = -\frac{27}{16}$

4. The area of the rectangle is $x^2 – 3x – 10$. What is the length of the rectangle if the width is $x + 2$?

   a. $(x + 5)(x – 2)$ b. $\frac{(x+5)(x-2)}{x-2}$ c. $\frac{x+5}{x-2}$ d. $x – 5$

   Answer: D. $l = \frac{x^2 - 3x - 10}{x+2} = \frac{(x-5)(x+2)}{x+2} = x - 5$

5. What must be added to $\frac{3x+4}{x+2}$ so that there sum is $\frac{3x^2+x-4}{x^2-4}$?

   a. $\frac{3x+4}{x^2-4}$ b. $\frac{-3x-4}{x^2-4}$ c. $\frac{x+12}{x^2-4}$ d. $\frac{x-12}{x^2-4}$

Answer: A. $\mathbf{\frac{3x^2 + x - 4}{x^2 - 4} - \frac{3x + 4}{x + 2} = \frac{3x^2 + x - 4}{x^2 - 4} - \frac{(x - 2)(3x + 4)}{(x + 2)(x - 2)} = \frac{3x^2 + x - 4 - 3x^2 + 2x + 8}{x^2 - 4} = \frac{3x + 4}{x^2 - 4}}$

6. If one of the factors of $\frac{1}{a + 1}$ is $\frac{a - 1}{a - 2a + 1}$, find the other factor.

a. $\frac{a + 1}{a - 1}$ b. $\frac{a - 1}{1 - a}$ c. $\frac{a^2 - 2a + 1}{a^2 - 1}$ d. $\frac{a^2 - 1}{a^2 - 2a + 1}$

Answer: A. $\mathbf{\frac{a - 1}{a^2 - 2a + 1} \div \frac{1}{a + 1} = \frac{a - 1}{(a - 1)(a - 1)} \bullet \frac{a + 1}{1} = \frac{a + 1}{a - 1}}$

7. Which of the following rational algebraic expressions is equivalent to $\frac{\frac{1}{x^2 + 5x + 6}}{\frac{1}{x + 3}}$?

a. $\frac{1}{x + 2}$ b. $\frac{1}{x - 2}$ c. $\frac{1}{x + 3}$ d. $\frac{1}{x - 3}$

Answer: A. $\mathbf{\frac{\frac{1}{x^2 + 5x + 6}}{\frac{1}{x + 3}} = \frac{\frac{1}{(x + 3)(x + 2)}}{\frac{1}{x + 3}} = \frac{1}{(x + 3)(x + 2)} \bullet \frac{x + 3}{1} = \frac{1}{x + 2}}$

8. What is the difference between $\frac{m}{6}$ and $\frac{m}{3}$?

a. $\frac{m}{6}$ b. $-\frac{m}{2}$ c. $\frac{m}{2}$ d. $-\frac{m}{2}$

Answer: A . $\mathbf{\frac{m}{2} - \frac{m}{3} = \frac{3m - 2m}{6} = \frac{m}{6}}$

9. A business man invested his money and was assured that his money will increase using the formula $P(1\ \frac{r}{n})^{nt}$ where P is money invested; r is the rate of increase; n is mode of increase in a year and t is the number of years. If the business man invested Php 10 000, how much can he get at the end of the year if the rate is 50% and will increase twice a year?

a. Php 15 652 b. Php 16 552 c. Php 15 625 d. Php 15 255

Answer: C. $\mathbf{P(1 + \frac{r}{n})^{nt} = 10000\ (1 + \frac{0.5}{2})^{(2)(1)} = 10000\ (1 + \frac{1}{4})^{(2)} = 10000\ (1 + \frac{5}{4})^{(2)} = 10000\ (\frac{25}{16}) = 15\ 625}$

10. Roger can do the project in x number hours. Concepcion can do the same job in 2 hours less than Roger does. Which of the choices below is the difference of their rate?

a. $\frac{2x-2}{x^2-2x}$ b. $-\frac{2}{x^2-2x}$ c. $\frac{2}{x^2-2x}$ d. $-\frac{2x-2}{x^2-2x}$

Answer: B. $\frac{1}{x} - \frac{1}{x-2} = \frac{x-2-x}{x^2-2x} = -\frac{2}{x^2-2x}$

11. You have $(x^2 + 2)$ pesos to buy materials for your school project. You spent half of it in the first store, then you spent one – third of your money less than you spent in the first store. In the third store, you spent one – fourth of the remaining money from the two stores. What is the total cost of the materials?

a. $\frac{4x^2-8}{4}$ b. $\frac{3x^2+6}{4}$ c. $\frac{5x^2+20}{12}$ d. $\frac{7x^2+14}{12}$

Answer: B. $\frac{x^2+2}{2} + \frac{x^2-2}{2} - \frac{x^2+2}{3} + \frac{x^2+2}{12} = \frac{9x^2+18}{12} = \frac{3x^2+6}{4}$

12. James were asked to simplify $\frac{x^2+2x-8}{x^2-4}$. His solution is presented below.

$$\frac{x^2+2x-8}{x^2-4} = \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{(\cancel{x}+4)}{(\cancel{x}+2)}$$

What makes the solution of James wrong?

$$= \frac{4}{2}$$

$$= 2$$

a. Cancelling 4. b. Crossing out the $(x – 2)$. c. $x^2 – 4$ being factored out. d. Dividing out the variable x.

Answer: D. X in the $(x + 4)$ and $(x + 2)$ should not divided out because it is part of the term and it is not a common factor of the numerator and denominator.

13. Mary took the math exam. One of the problems in the exam is finding the quotient of $\frac{x^2 + 2x + 2}{4 - x^2}$ and $\frac{1 - x^2}{x^2 + x - 2}$. Her solution is shown below.

$$\frac{x^2 + 2x + 1}{4 - x^2} \div \frac{1 - x^2}{x^2 + x - 2} = \frac{(x+1)(x+1)}{(2-x)(2+x)} \div \frac{(1-x)(1+x)}{(x-1)(x+2)}$$

$$= \frac{(x+1)(x+1)}{(2-x)(2+x)} \cdot \frac{(x-1)(x+2)}{(1-x)(1+x)}$$

$$= \frac{(x+1)(x+1)(x-1)(x+2)}{(2-x)(2+x)(1-x)(1+x)}$$

$$= \frac{x+1}{2-x}$$

Did Mary arrive at the correct answer?

a. No, the dividend and divisor should be interchange.
b. No, the divisor should be reciprocated first before factoring it out.
c. No. $(2 + x)$ is not the same as $(x + 2)$.
d. No. $(x - 1)$ and $(1 - x)$ is not equal to 1

Answer: D. $(x - 1)$ is additive inverse of $(1 - x)$. If the a term is divided by the its additive inverse, quotient is -1

14. Greg simplify $\frac{\frac{2}{y+1} + 3}{\frac{3}{y+1} + 4}$ this way:

$$\frac{\frac{2}{y+1} + 3}{\frac{3}{y+1} + 4} = \frac{\left(\frac{2}{y+1} + 3\right)(y+1)}{\left(\frac{3}{y+1} + 4\right)(y+1)}$$

$$= \frac{2 + 3(y+1)}{3 + 4(y+1)}$$

$$= \frac{2 + 3y + 3}{3 + 4y + 4}$$

$$= \frac{3y + 5}{4y + 7}$$

Is there anything wrong in his solution?

a. Something is wrong with the solution. He is not following the correct process of simplifying complex rational algebraic expression.
b. None. Multiplying the numerator and denominator by the same quantity makes no difference on the given expression.
c. Something is wrong with the solution. Numerator and denominator may be multiplied by a certain number but not an algebraic expression.
d. None. The solution and answer of Greg is different but acceptable.

Answer: B. In simplifying complex rational algebraic expression, numerator and denominator can be multiplied by their LCD

15. Your Project Supervisor ask you to make a floor plan of a house. As an engineer, what must be considered in completing the plan?
    a. Reasoning and accuracy
    b. Cost and design
    c. Feasible and accurate
    d. Practical and aesthetics

    Answer: C. Dividing the parts of the house must be accurate and it must be realistic.

16. Your mother asked you to find for laborers in renovating your house. What will you look in choosing a laborer?
    a. His efficiency in doing the task.
    b. His attitude towards work.
    c. His perception in the job.
    d. His wage in a day.

    Answer: A. Though the rate/speed of the laborer counts but the quality of his work must not be compromised.

17. You need a printer in your computer shop. The list of the printers and its capacities is presented in the table. Based in the table, what printer is best to buy?

Printer	Pages to print in a minute	Capacity of the ink	Average number of wasted paper per 500 pages
HD Turbo	16	450 pages	4
IP Sun	7	500 pages	2
Bazoka	23	350 pages	12
Father’s	18	400 pages	6

    a. Father’s. it has more pages to print and good capacity of ink.
    b. Bazoka. It has the most pages to print and nice capacity of ink.

c. IP Sun. It has the best ink capacity and least number of paper wasted.
d. HD Turbo. It has lesser wasted paper and better ink capacity.

Answer: D. Though the HD Turbo is slower compared to Father's and Bazoka but it has the lesser wasted paper compared the other two printers. And the capacity of the ink is better compared to the other two printers.

18. What qualities you must look in buying a printer for personal consumption?
a. Brand and design
b. Price and pages to print
c. Cost of the printer and its efficiency.
d . Brand and the quality of the output.

Answer: C. It is better to consider the cost of the printer that will not compromise its efficiency.

19. You were tasked, as a budget officer, to give comments regarding the work plan of the engineer. What aspect of the plan should you consider?
a. The wage of the laborers and the rentals of the equipment.
b. The number of laborers and equipment needed.
c. The quality of work done by the laborers and efficiency of the equipment.
d. The job done by the laborers in one day and appropriateness of the equipment.

Answer: A. It is not necessary to look for the rate/speed and efficiency of the laborers as a budget officer because you will look for the financial aspect of the project.

20. After you give comments in the work plan in number 19, what will you do next?
A. Present a feasibility study.
B. Make a budget proposal.
c. Look for financial resources
d. Give a financial statement

Answer: B. A will be given by the engineer. C will be given after the budget plan. D will be given after the project.

SUMMARY

Now that you have completed this module, let us summarize what have you learned:

1. Rate–related problems can be modeled using rational algebraic expressions.
2. Rational algebraic expression is a ratio of two polynomials where the denominator is not equal to one.
3. Any expression raised to zero is always equal to one.
4. When an expression is raised by a negative integer, it is the multiplicative inverse of the expression.
5. Rational algebraic expression is in its simplest form if there is no common factor between numerator and denominator except 1.
6. To multiply rational algebraic expression, multiply the numerator and denominator then simplify.
7. To divide rational algebraic expression, multiply the dividend by the reciprocal of the divisor then multiply.
8. To add/subtract similar rational algebraic expressions, add/subtract the numerators and copy the common denominator.
9. To add/subtract dissimilar rational algebraic expressions, express each expression into similar one then add/subtract the numerators and copy the common denominator.
10. Complex rational algebraic expression is an expression where the numerator or denominator, or both numerator and denominator are rational algebraic expressions.

GLOSSARY OF TERMS USED IN THIS MODULE

Complex rational algebraic expression – an expression where the numerator or denominator or both numerator and denominator are rational algebraic expressions.

LCD – also known as Least Common Denominator is the least common multiple of the denominators.

Manpower plan – a plan where the number of workers needed to complete the project, wages of each worker in a day, how many days can workers finish the job and how much can be spend on the workers for the entire project.

Rate–related problems – Problems involving rates (e.g., speed, percentage, ratio, work)

Rational algebraic expression – ratio of two polynomials where the denominator is not equal to one.

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