UNIT 7

Module 21

Proving Theorems about Lines and Angles

COMMON CORE GPS

Contents

MATHEMATICAL PRACTICES

The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unpacking the Standards

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Multilingual Glossary

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.

COMMON CORE GPS **MCC9-12.G.CO.9**

Prove theorems about lines and angles.

Key Vocabulary

proof (demostración)
An argument that uses logic to show that a conclusion is true.

theorem (teorema)
A statement that has been proven.

line (línea)
An undefined term in geometry, a line is a straight path that has no thickness and extends forever.

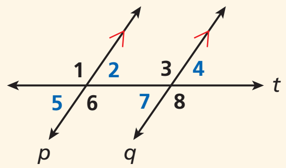
angle (ángulo)
A figure formed by two rays with a common endpoint.

What It Means For You

A line crossing a pair of parallel lines forms pairs of angles that are either congruent or supplementary. You can use simple proofs to show these relationships.

EXAMPLE

In the diagram, line p is parallel to line q.



You can show that:

(1) Any pair of black numbered angles is a pair of congruent angles.

(2) Any pair of blue numbered angles is a pair of congruent angles.

(3) Any pair of one black and one blue numbered angle is a pair of supplementary angles.

21-1 Technology TASK

Explore Parallel Lines and Transversals

Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

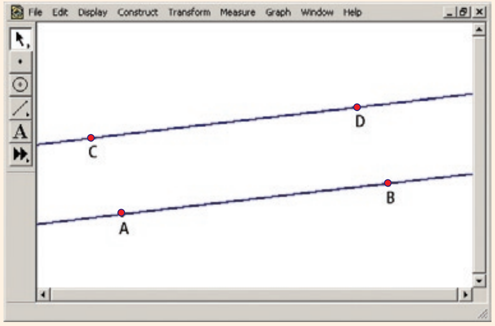
Use with Angles Formed by Parallel Lines and Transversals

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

MCC9-12.G.CO.9 Prove theorems about lines and angles.

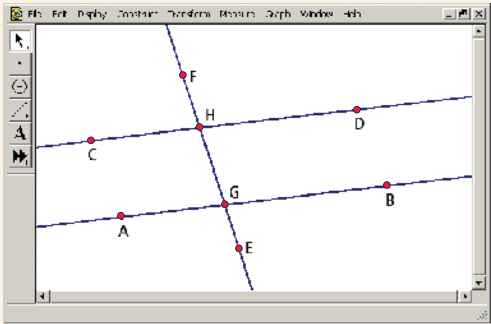
Activity

1. Construct a line and label two points on the line *A* and *B*.



2. Create point *C* not on $\overleftrightarrow{AB}$. Construct a line parallel to $\overleftrightarrow{AB}$ through point *C*. Create another point on this line and label it *D*.

3. Create two points outside the two parallel lines and label them *E* and *F*. Construct transversal $\overleftrightarrow{EF}$. Label the points of intersection *G* and *H*.



4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point *E* or *F* and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

Angle	∠*AGE*	∠*BGE*	∠*AGH*	∠*BGH*	∠*CHG*	∠*DHG*	∠*CHF*	∠*DHF*
Measure								
Measure								

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.
2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.
3. Try dragging point *C* to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?

21-1 Angles Formed by Parallel Lines and Transversals

Essential Question: How can you prove and use theorems about angles formed by transversals that intersect parallel lines?

Objective
Prove and use theorems about the angles formed by parallel lines and a transversal.

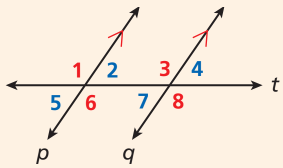
Who uses this?
Piano makers use parallel strings for the higher notes. The longer strings used to produce the lower notes can be viewed as transversals. (See Example 3.)

When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary.

Know it! Note

Animated Math

Postulate 21-1-1 Corresponding Angles Postulate

POSTULATE	HYPOTHESIS	CONCLUSION
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$ $\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$

COMMON CORE GPS **EXAMPLE 1** MCC9-12.G.CO.9

Using the Corresponding Angles Postulate

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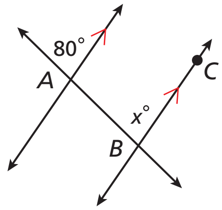
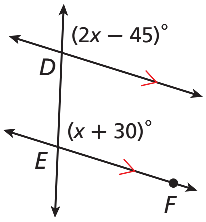
Find each angle measure.



A $m\angle ABC$

$x = 80$ *Corr. ∠s Post.*

$m\angle ABC = 80°$



B $m\angle DEF$

$(2x - 45)° = (x + 30)°$ *Corr. ∠s Post.*

$x - 45 = 30$ *Subtract x from both sides.*

$x = 75$ *Add 45 to both sides.*

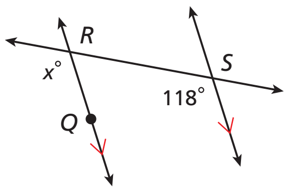
$m\angle DEF = x + 30$

$= 75 + 30$ *Substitute 75 for x.*

$= 105°$

CHECK IT OUT!

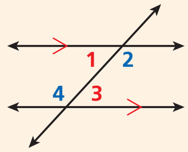
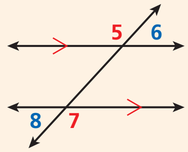
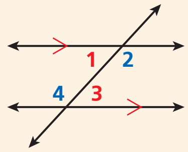
1. Find $m\angle QRS$.



Remember that postulates are statements that are accepted without proof. Since the Corresponding Angles Postulate is given as a postulate, it can be used to prove the next three theorems.

Know it! Note

Theorems — Parallel Lines and Angle Pairs

THEOREM	HYPOTHESIS	CONCLUSION
21-1-2 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.		$\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$
21-1-3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of alternate exterior angles are congruent.		$\angle 5 \cong \angle 7$ $\angle 6 \cong \angle 8$
21-1-4 Same-Side Interior Angles Theorem If two parallel lines are cut by a transversal, then the two pairs of same-side interior angles are supplementary.		$m\angle 1 + m\angle 4 = 180°$ $m\angle 2 + m\angle 3 = 180°$

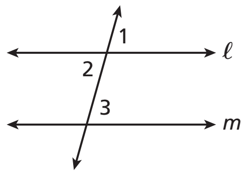
Helpful Hint

If a transversal is perpendicular to two parallel lines, all eight angles are congruent.

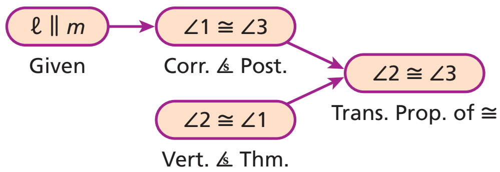
You will prove Theorems 21-1-3 and 21-1-4 in Exercises 25 and 26.

PROOF

Alternate Interior Angles Theorem



Given: $\ell \parallel m$
Prove: $\angle 2 \cong \angle 3$
Proof:



COMMON CORE GPS **EXAMPLE 2** MCC9-12.G.CO.9

Finding Angle Measures

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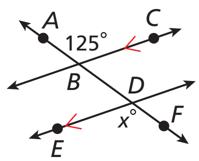
Find each angle measure.

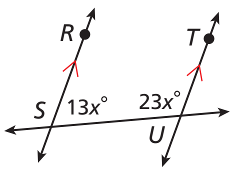


A $m\angle EDF$

$x = 125$

$m\angle EDF = 125°$ *Alt. Ext. ∠s Thm.*

B $m\angle TUS$



$13x° + 23x° = 180°$ *Same-Side Int. ∠s Thm.*

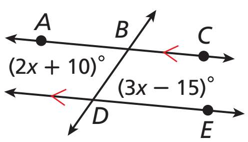
$36x = 180$ *Combine like terms.*

$x = 5$ *Divide both sides by 36.*

$m\angle TUS = 23(5) = 115°$ *Substitute 5 for x.*

CHECK IT OUT!

2. Find $m\angle ABD$.

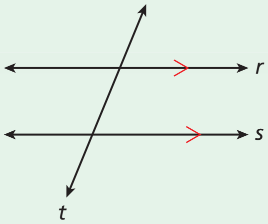


Student to Student

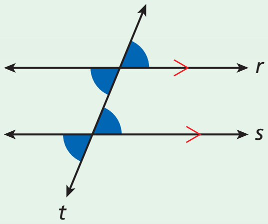
Parallel Lines and Transversals

Nancy Martin
East Branch
High School

When I solve problems with parallel lines and transversals, I remind myself that every pair of angles is either congruent or supplementary.



If r ∥ s, all the acute angles are congruent and all the obtuse angles are congruent. The acute angles are supplementary to the obtuse angles.



COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.CO.9

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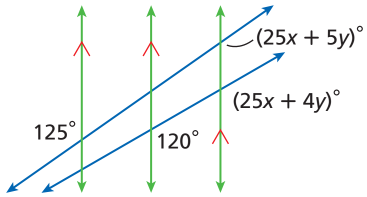
Music Application

The treble strings of a grand piano are parallel. Viewed from above, the bass strings form transversals to the treble strings. Find x and y in the diagram.



By the Alternate Exterior Angles Theorem, $(25x + 5y)^\circ = 125^\circ$.

By the Corresponding Angles Postulate, $(25x + 4y)^\circ = 120^\circ$.

$$25x + 5y = 125$$

$$-(25x + 4y = 120) \qquad \textit{Subtract the second equation from the first equation.}$$

$$y = 5$$

$$25x + 5(5) = 125 \qquad \textit{Substitute 5 for y in } 25x + 5y = 125. \textit{ Simplify and solve for x.}$$

$$x = 4,\ y = 5$$

CHECK IT OUT! **3.** Find the measures of the acute angles in the diagram.

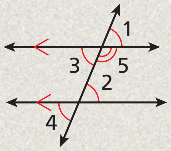
MCC.MP.1 MATHEMATICAL PRACTICES

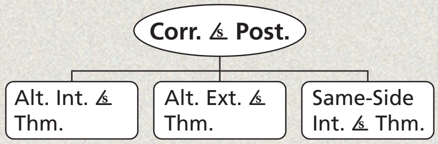
THINK AND DISCUSS

1. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

Know it! Note

2. GET ORGANIZED Copy the diagram and graphic organizer. Complete the graphic organizer by explaining why each of the three theorems is true.





21-1 Exercises

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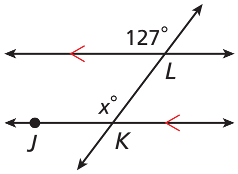
Homework Help

GUIDED PRACTICE

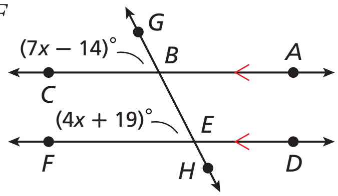
SEE EXAMPLE 1

Find each angle measure.

1. m∠*JKL*

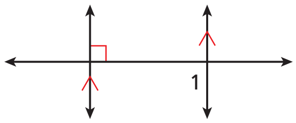


2. m∠*BEF*

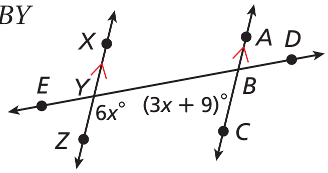


SEE EXAMPLE 2

3. m∠1

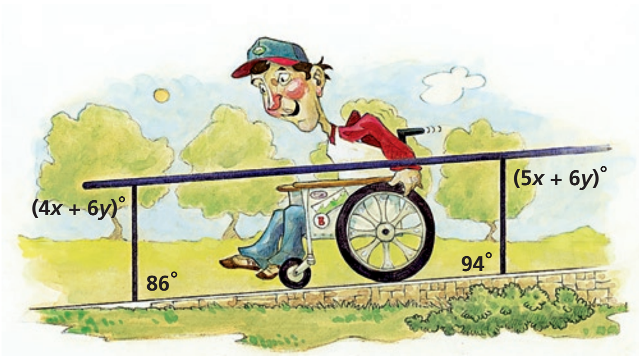


4. m∠*CBY*



SEE EXAMPLE 3

5. **Safety** The railing of a wheelchair ramp is parallel to the ramp. Find *x* and *y* in the diagram.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
6–7	1
8–11	2
12	3

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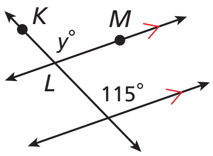
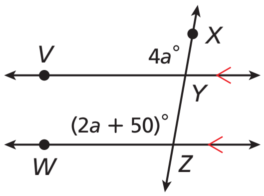
Online Extra Practice

Find each angle measure.

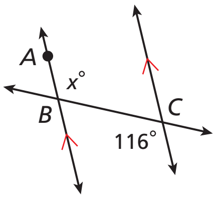
6. m∠*KLM*



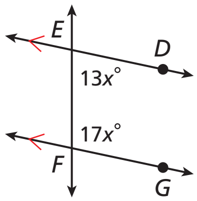
7. m∠*VYX*



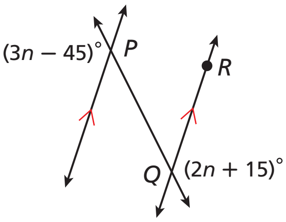
8. m∠*ABC*



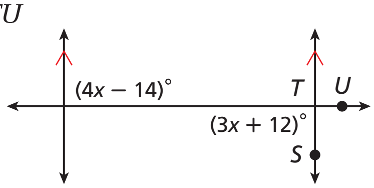
9. m∠*EFG*



10. m∠*PQR*



11. m∠*STU*

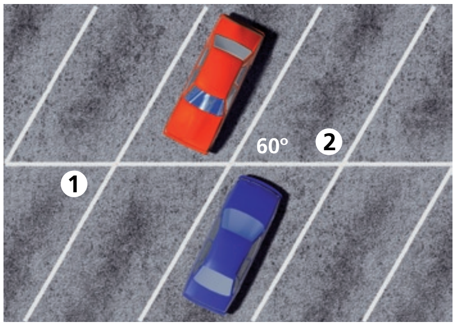


12. Parking In the parking lot shown, the lines that mark the width of each space are parallel.

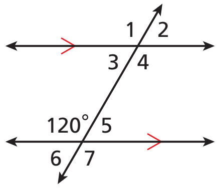
$m\angle 1 = (2x - 3y)°$

$m\angle 2 = (x + 3y)°$

Find x and y.



Find each angle measure. Justify each answer with a postulate or theorem.

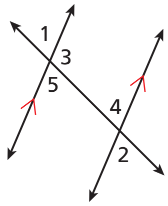


13. $m\angle 1$ **14.** $m\angle 2$ **15.** $m\angle 3$

16. $m\angle 4$ **17.** $m\angle 5$ **18.** $m\angle 6$

19. $m\angle 7$

Algebra **State the theorem or postulate that is related to the measures of the angles in each pair. Then find the angle measures.**



20. $m\angle 1 = (7x + 15)°$, $m\angle 2 = (10x - 9)°$

21. $m\angle 3 = (23x + 11)°$, $m\angle 4 = (14x + 21)°$

22. $m\angle 4 = (37x - 15)°$, $m\angle 5 = (44x - 29)°$

23. $m\angle 1 = (6x + 24)°$, $m\angle 4 = (17x - 9)°$

LINK

Architecture

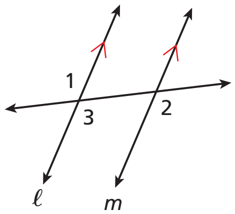
The Luxor hotel is 600 feet wide, 600 feet long, and 350 feet high. The atrium in the hotel measures 29 million cubic feet.

24. Architecture The Luxor Hotel in Las Vegas, Nevada, is a 30-story pyramid. The hotel uses an elevator called an inclinator to take people up the side of the pyramid. The inclinator travels at a 39° angle. Which theorem or postulate best illustrates the angles formed by the path of the inclinator and each parallel floor? (*Hint:* Draw a picture.)

25. Complete the two-column proof of the Alternate Exterior Angles Theorem.

Given: $\ell \parallel m$

Prove: $\angle 1 \cong \angle 2$



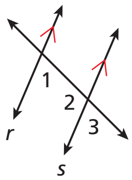
Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** Given
2. a. ___?___	**2.** Vert. ∠s Thm.
3. $\angle 3 \cong \angle 2$	**3. b.** ___?___
4. c. ___?___	**4. d.** ___?___

H.O.T. 26. Write a paragraph proof of the Same-Side Interior Angles Theorem.

Given: $r \parallel s$

Prove: $m\angle 1 + m\angle 2 = 180°$



H.O.T. Draw the given situation or tell why it is impossible.

27. Two parallel lines are intersected by a transversal so that the corresponding angles are supplementary.

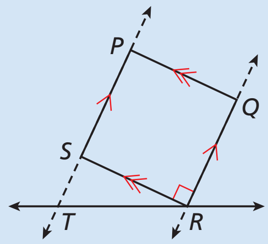
28. Two parallel lines are intersected by a transversal so that the same-side interior angles are complementary.

Real-World Connections

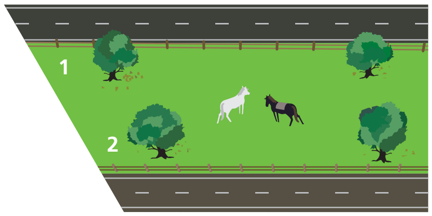
29. In the diagram, which represents the side view of a mystery spot, m∠*SRT* = 25°. $\overleftrightarrow{RT}$ is a transversal to $\overleftrightarrow{PS}$ and $\overleftrightarrow{QR}$.

a. What type of angle pair is ∠*QRT* and ∠*STR*?

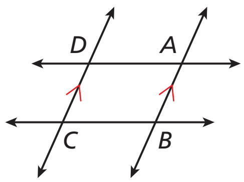
b. Find m∠*STR*. Use a theorem or postulate to justify your answer.



H.O.T. **30. Land Development** A piece of property lies between two parallel streets as shown. $m\angle 1 = (2x + 6)^\circ$, and $m\angle 2 = (3x + 9)^\circ$. What is the relationship between the angles? What are their measures?



H.O.T. **31. ERROR ANALYSIS** In the figure, $m\angle ABC = (15x + 5)^\circ$, and $m\angle BCD = (10x + 25)^\circ$. Which value of $m\angle BCD$ is incorrect? Explain.



A

$15x + 5 = 10x + 25$
$-10x \quad -10x$
$5x + 5 = 25$
$-5 \quad -5$
$5x = 20$
$x = 4$

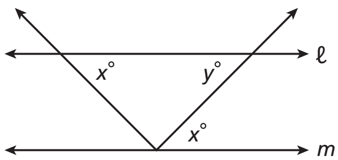
$m\angle BCD = 10(4) + 25 = 65^\circ$

B

$(15x + 5) + (10x + 25) = 180$
$25x + 30 = 180$
$-30 \quad -30$
$25x = 150$
$x = 6$

$m\angle BCD = 10(6) + 25 = 85^\circ$

32. Critical Thinking In the diagram, $\ell \parallel m$. Explain why $\frac{x}{y} = 1$.



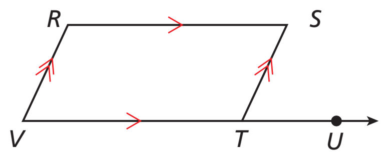
H.O.T. **33. Write About It** Suppose that lines ℓ and m are intersected by transversal p. One of the angles formed by ℓ and p is congruent to every angle formed by m and p. Draw a diagram showing lines ℓ, m, and p, mark any congruent angles that are formed, and explain what you know is true.

TEST PREP

34. $m\angle RST = (x + 50)^\circ$, and $m\angle STU = (3x + 20)^\circ$. Find m∠*RVT*.

Ⓐ 15° Ⓒ 65°

Ⓑ 27.5° Ⓓ 77.5°



35. For two parallel lines and a transversal, m∠1 = 83°. For which pair of angle measures is the sum the least?

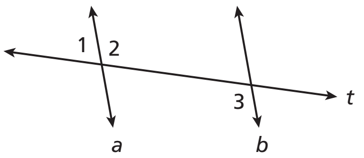
(F) ∠1 and a corresponding angle

(G) ∠1 and a same-side interior angle

(H) ∠1 and its supplement

(J) ∠1 and its complement

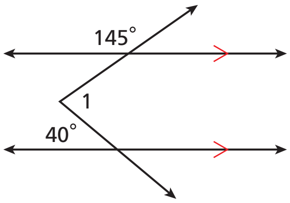
36. **Short Response** Given $a \parallel b$ with transversal t, explain why ∠1 and ∠3 are supplementary.



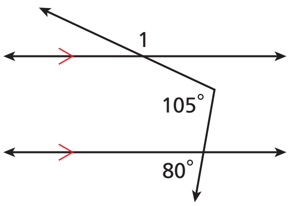
CHALLENGE AND EXTEND

Multi-Step **Find m∠1 in each diagram. (*Hint:* Draw a line parallel to the given parallel lines.)**

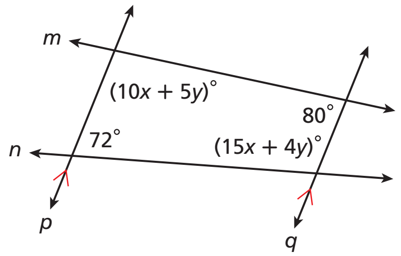
37.



38.



39. Find x and y in the diagram. Justify your answer.

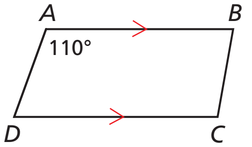


H.O.T. 40. Two lines are parallel. The measures of two corresponding angles are $a°$ and $2b°$, and the measures of two same-side interior angles are $a°$ and $b°$. Find the value of a.

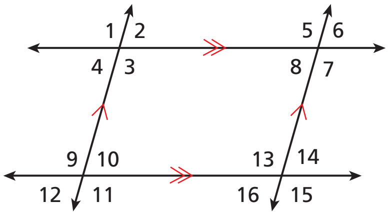
MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

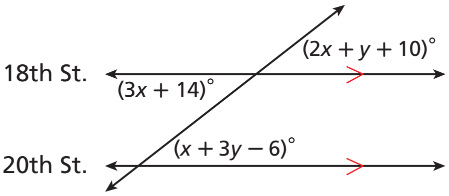
H.O.T. 41. **Error Analysis** Sarah found that $m\angle D = 70°$ and $m\angle B = 70°$. Explain her error.



H.O.T. 42. **Reasonableness** Write a convincing argument that ∠1 is congruent to ∠15 in the figure.



H.O.T. 43. **Problem Solving** 18th Street and 20th Street both cross the canal as shown in the figure. Find x and y. Show your work.



21-2 Proving Lines Parallel

Essential Question: How can you prove lines are parallel?

Objective

Use the angles formed by a transversal to prove two lines are parallel.

Who uses this?

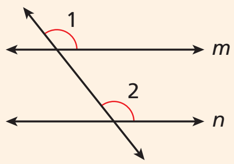
Rowers have to keep the oars on each side parallel in order to travel in a straight line. (See Example 4.)

Ken Hawkins/Mira.com

Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

Know it! Note

Postulate 21-2-1 Converse of the Corresponding Angles Postulate

POSTULATE	HYPOTHESIS	CONCLUSION
If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$

COMMON CORE GPS **EXAMPLE 1** MCC9-12.G.CO.9

Using the Converse of the Corresponding Angles Postulate

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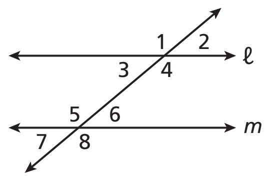
Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.



A $\angle 1 \cong \angle 5$

$\angle 1 \cong \angle 5$ — *∠1 and ∠5 are corresponding angles.*

$\ell \parallel m$ — *Conv. of Corr. ∠s Post.*

B $m\angle 4 = (2x + 10)^\circ$, $m\angle 8 = (3x - 55)^\circ$, $x = 65$

$m\angle 4 = 2(65) + 10 = 140$ — *Substitute 65 for x.*

$m\angle 8 = 3(65) - 55 = 140$ — *Substitute 65 for x.*

$m\angle 4 = m\angle 8$ — *Trans. Prop. of Equality*

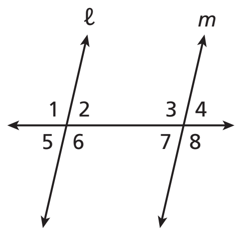
$\angle 4 \cong \angle 8$ — *Def. of ≅ ∠s*

$\ell \parallel m$ — *Conv. of Corr. ∠s Post.*

CHECK IT OUT! Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.

1a. $m\angle 1 = m\angle 3$

1b. $m\angle 7 = (4x + 25)^\circ$, $m\angle 5 = (5x + 12)^\circ$, $x = 13$



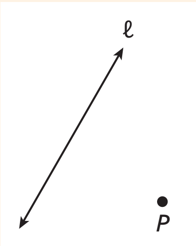
Postulate 21-2-2 Parallel Postulate

Through a point P not on line ℓ, there is exactly one line parallel to ℓ.

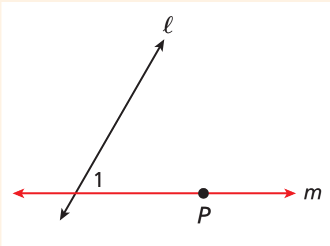
The Converse of the Corresponding Angles Postulate is used to construct parallel lines. The Parallel Postulate guarantees that for any line ℓ, you can always construct a parallel line through a point that is not on ℓ.

Construction Parallel Lines

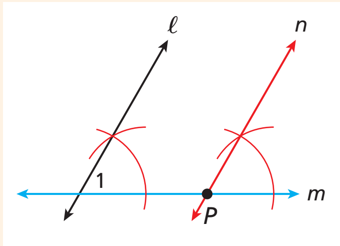
1. Draw a line ℓ and a point P that is not on ℓ.



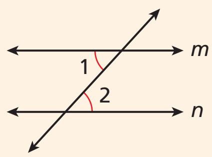
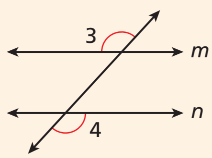
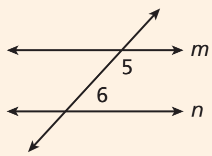
2. Draw a line m through P that intersects ℓ. Label the angle 1.



3. Construct an angle congruent to $\angle 1$ at P. By the converse of the Corresponding Angles Postulate, $\ell \parallel n$.



Theorems Proving Lines Parallel

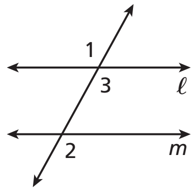
THEOREM		HYPOTHESIS	CONCLUSION
21-2-3	**Converse of the Alternate Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.	$\angle 1 \cong \angle 2$ 	$m \parallel n$
21-2-4	**Converse of the Alternate Exterior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.	$\angle 3 \cong \angle 4$ 	$m \parallel n$
21-2-5	**Converse of the Same-Side Interior Angles Theorem** If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.	$m\angle 5 + m\angle 6 = 180°$ 	$m \parallel n$

You will prove Theorems 21-2-3 and 21-2-5 in Exercises 38 and 39.

PROOF

Converse of the Alternate Exterior Angles Theorem

Given: $\angle 1 \cong \angle 2$
Prove: $\ell \parallel m$
Proof: It is given that $\angle 1 \cong \angle 2$. Vertical angles are congruent, so $\angle 1 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. So $\ell \parallel m$ by the Converse of the Corresponding Angles Postulate.



COMMON CORE GPS **EXAMPLE 2** MCC9-12.G.CO.9

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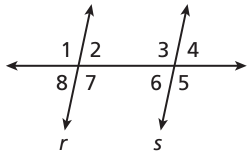
Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that $r \parallel s$.



A $\angle 2 \cong \angle 6$

$\angle 2 \cong \angle 6$ — *$\angle 2$ and $\angle 6$ are alternate interior angles.*

$r \parallel s$ — *Conv. of Alt. Int. ∠s Thm.*

B $m\angle 6 = (6x + 18)^\circ$, $m\angle 7 = (9x + 12)^\circ$, $x = 10$

$m\angle 6 = 6x + 18$

$= 6(10) + 18 = 78^\circ$ — *Substitute 10 for x.*

$m\angle 7 = 9x + 12$

$= 9(10) + 12 = 102^\circ$ — *Substitute 10 for x.*

$m\angle 6 + m\angle 7 = 78^\circ + 102^\circ$

$= 180^\circ$ — *$\angle 6$ and $\angle 7$ are same-side interior angles.*

$r \parallel s$ — *Conv. of Same-Side Int. ∠s Thm.*

CHECK IT OUT! **Refer to the diagram above. Use the given information and the theorems you have learned to show that $r \parallel s$.**

2a. $m\angle 4 = m\angle 8$ **2b.** $m\angle 3 = 2x^\circ$, $m\angle 7 = (x + 50)^\circ$, $x = 50$

COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.CO.9

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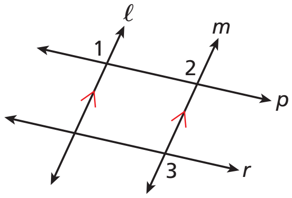
Proving Lines Parallel

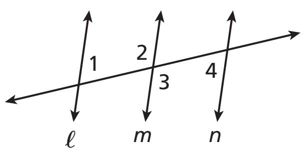
Given: $\ell \parallel m$, $\angle 1 \cong \angle 3$
Prove: $r \parallel p$



Proof:

Statements	Reasons
1. $\ell \parallel m$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Corr. ∠s Post.
3. $\angle 1 \cong \angle 3$	**3.** Given
4. $\angle 2 \cong \angle 3$	**4.** Trans. Prop. of $\cong$
5. $r \parallel p$	**5.** Conv. of Alt. Ext. ∠s Thm.

CHECK IT OUT! **3. Given:** $\angle 1 \cong \angle 4$, $\angle 3$ and $\angle 4$ are supplementary.
Prove: $\ell \parallel m$



COMMON CORE GPS **EXAMPLE 4** MCC9-12.G.CO.9

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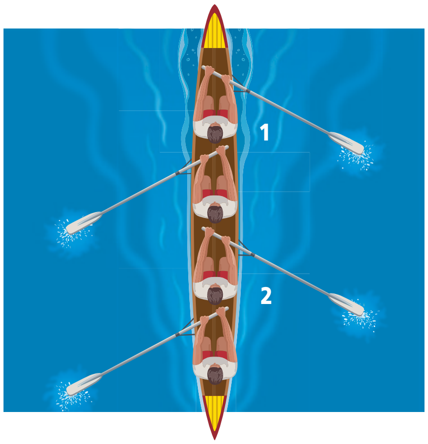
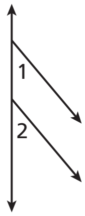
Sports Application

During a race, all members of a rowing team should keep the oars parallel on each side. If $m\angle 1 = (3x + 13)^\circ$, $m\angle 2 = (5x - 5)^\circ$, and $x = 9$, show that the oars are parallel.



A line through the center of the boat forms a transversal to the two oars on each side of the boat.



$\angle 1$ and $\angle 2$ are corresponding angles. If $\angle 1 \cong \angle 2$, then the oars are parallel.

Substitute 9 for x in each expression:

$m\angle 1 = 3x + 13$

$= 3(9) + 13 = 40^\circ$ *Substitute 9 for x in each expression.*

$m\angle 2 = 5x - 5$

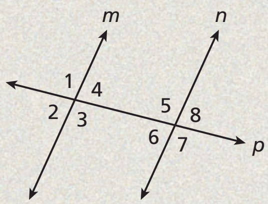
$= 5(9) - 5 = 40^\circ$ *$m\angle 1 = m\angle 2$, so $\angle 1 \cong \angle 2$.*

The corresponding angles are congruent, so the oars are parallel by the Converse of the Corresponding Angles Postulate.

CHECK IT OUT! **4. What if...?** Suppose the corresponding angles on the opposite side of the boat measure $(4y - 2)^\circ$ and $(3y + 6)^\circ$, where $y = 8$. Show that the oars are parallel.

MCC.MP.3 MATHEMATICAL PRACTICES

THINK AND DISCUSS

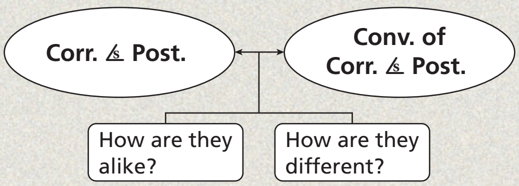


1. Explain three ways of proving that two lines are parallel.

2. If you know $m\angle 1$, how could you use the measures of $\angle 5$, $\angle 6$, $\angle 7$, or $\angle 8$ to prove $m \parallel n$?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Use it to compare the Corresponding Angles Postulate with the Converse of the Corresponding Angles Postulate.



21-2 Exercises

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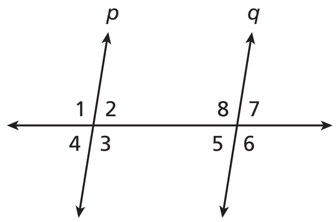
Homework Help

GUIDED PRACTICE

SEE EXAMPLE 1

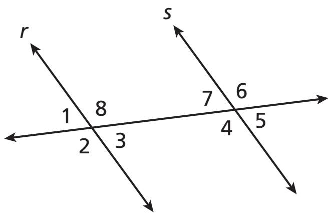
Use the Converse of the Corresponding Angles Postulate and the given information to show that $p \parallel q$.



1. $\angle 4 \cong \angle 5$
2. $m\angle 1 = (4x + 16)^\circ$, $m\angle 8 = (5x - 12)^\circ$, $x = 28$
3. $m\angle 4 = (6x - 19)^\circ$, $m\angle 5 = (3x + 14)^\circ$, $x = 11$

SEE EXAMPLE 2

Use the theorems and given information to show that $r \parallel s$.

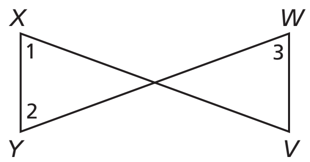


4. $\angle 1 \cong \angle 5$
5. $m\angle 3 + m\angle 4 = 180^\circ$
6. $\angle 3 \cong \angle 7$
7. $m\angle 4 = (13x - 4)^\circ$, $m\angle 8 = (9x + 16)^\circ$, $x = 5$
8. $m\angle 8 = (17x + 37)^\circ$, $m\angle 7 = (9x - 13)^\circ$, $x = 6$
9. $m\angle 2 = (25x + 7)^\circ$, $m\angle 6 = (24x + 12)^\circ$, $x = 5$

SEE EXAMPLE 3

10. Complete the following two-column proof.

Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 1$
Prove: $\overline{XY} \parallel \overline{WV}$
Proof:



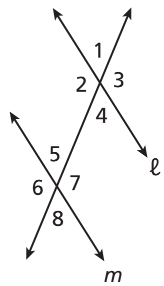
Statements	Reasons
1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 1$	1. Given
2. $\angle 2 \cong \angle 3$	2. a. ___?___
3. b. ___?___	3. c. ___?___

SEE EXAMPLE 4

11. **Architecture** In the fire escape, $m\angle 1 = (17x + 9)^\circ$, $m\angle 2 = (14x + 18)^\circ$, and $x = 3$. Show that the two landings are parallel.

PRACTICE AND PROBLEM SOLVING

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.



12. $\angle 3 \cong 7$
13. $m\angle 4 = 54^\circ$, $m\angle 8 = (7x + 5)^\circ$, $x = 7$
14. $m\angle 2 = (8x + 4)^\circ$, $m\angle 6 = (11x - 41)^\circ$, $x = 15$
15. $m\angle 1 = (3x + 19)^\circ$, $m\angle 5 = (4x + 7)^\circ$, $x = 12$

Alamy Images

Independent Practice

For Exercises	See Example
12–15	1
16–21	2
22	3
23	4

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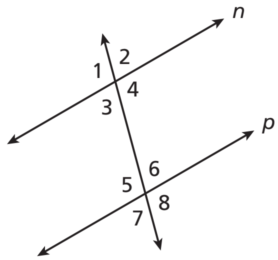
Online Extra Practice

Use the theorems and given information to show that $n \parallel p$.



16. $\angle 3 \cong \angle 6$

17. $\angle 2 \cong \angle 7$

18. $m\angle 4 + m\angle 6 = 180°$

19. $m\angle 1 = (8x - 7)°$, $m\angle 8 = (6x + 21)°$, $x = 14$

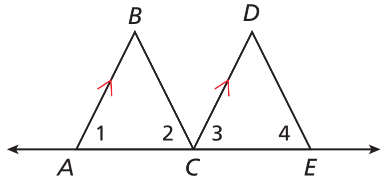
20. $m\angle 4 = (4x + 3)°$, $m\angle 5 = (5x - 22)°$, $x = 25$

21. $m\angle 3 = (2x + 15)°$, $m\angle 5 = (3x + 15)°$, $x = 30$

22. Complete the following two-column proof.

Given: $\overline{AB} \parallel \overline{CD}$, $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

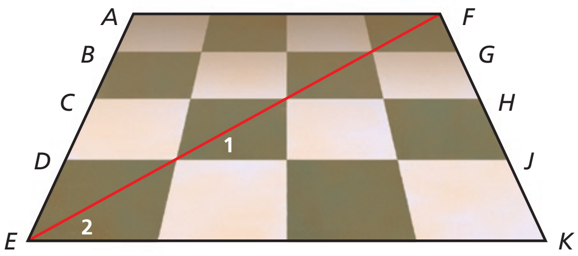
Prove: $\overline{BC} \parallel \overline{DE}$



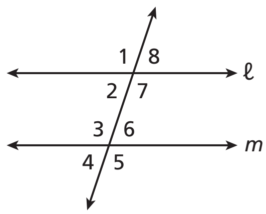
Proof:

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2. a.** ___?___
3. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**3. b.** ___?___
4. $\angle 2 \cong \angle 4$	**4. c.** ___?___
5. d. ___?___	**5. e.** ___?___

H.O.T. 23. **Art** Edmund Dulac used perspective when drawing the floor titles in an illustration for *The Wind's Tale* by Hans Christian Andersen. Show that $\overline{DJ} \parallel \overline{EK}$ if $m\angle 1 = (3x + 2)°$, $m\angle 2 = (5x - 10)°$, and $x = 6$.



Name the postulate or theorem that proves that $\ell \parallel m$.



24. $\angle 8 \cong \angle 6$

25. $\angle 8 \cong \angle 4$

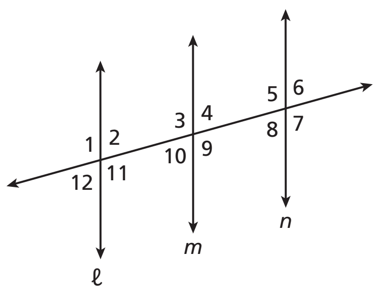
26. $\angle 2 \cong \angle 6$

27. $\angle 7 \cong \angle 5$

28. $\angle 3 \cong \angle 7$

29. $m\angle 2 + m\angle 3 = 180°$

For the given information, tell which pair of lines must be parallel. Name the postulate or theorem that supports your answer.



30. $m\angle 2 = m\angle 10$

31. $m\angle 8 + m\angle 9 = 180°$

32. $\angle 1 \cong \angle 7$

33. $m\angle 10 = m\angle 6$

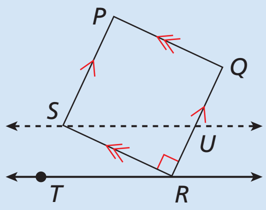
34. $\angle 11 \cong \angle 5$

35. $m\angle 2 + m\angle 5 = 180°$

36. **Multi-Step** Two lines are intersected by a transversal so that $\angle 1$ and $\angle 2$ are corresponding angles, $\angle 1$ and $\angle 3$ are alternate exterior angles, and $\angle 3$ and $\angle 4$ are corresponding angles. If $\angle 2 \cong \angle 4$, what theorem or postulate can be used to prove the lines parallel?

Real-World Connections

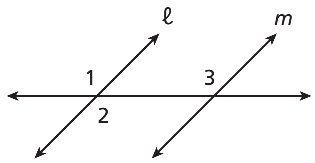
37. In the diagram, which represents the side view of a mystery spot, $m\angle SRT = 25°$, and $m\angle SUR = 65°$.



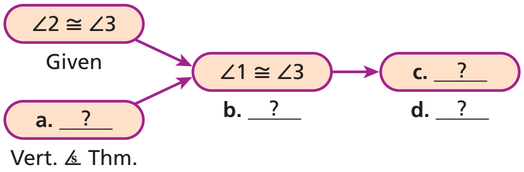
a. Name a same-side interior angle of $\angle SUR$ for lines $\overleftrightarrow{SU}$ and $\overleftrightarrow{RT}$ with transversal $\overline{RU}$. What is its measure? Explain your reasoning.

b. Prove that $\overleftrightarrow{SU}$ and $\overleftrightarrow{RT}$ are parallel.

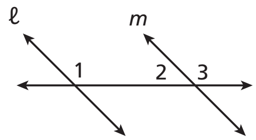
38. Complete the flowchart proof of the Converse of the Alternate Interior Angles Theorem.



Given: $\angle 2 \cong \angle 3$
Prove: $\ell \parallel m$
Proof:

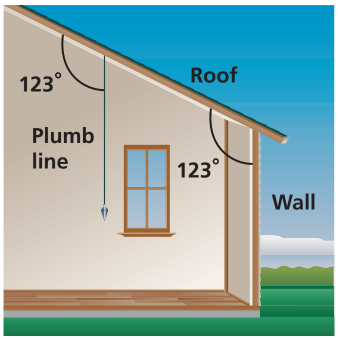


39. Use the diagram to write a paragraph proof of the Converse of the Same-Side Interior Angles Theorem.



Given: $\angle 1$ and $\angle 2$ are supplementary.
Prove: $\ell \parallel m$

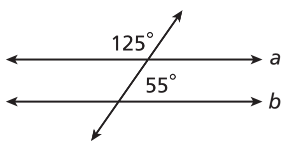
H.O.T. 40. **Carpentry** A *plumb bob* is a weight hung at the end of a string, called a *plumb line*. The weight pulls the string down so that the plumb line is perfectly vertical. Suppose that the angle formed by the wall and the roof is 123° and the angle formed by the plumb line and the roof is 123°. How does this show that the wall is perfectly vertical?



H.O.T. 41. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for parallel lines? Explain why or why not.

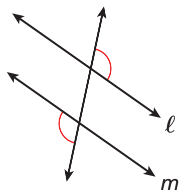
Reflexive: $\ell \parallel \ell$
Symmetric: If $\ell \parallel m$, then $m \parallel \ell$.
Transitive: If $\ell \parallel m$ and $m \parallel n$, then $\ell \parallel n$.

H.O.T. 42. **Write About It** Does the information given in the diagram allow you to conclude that $a \parallel b$? Explain.



TEST PREP

43. Which postulate or theorem can be used to prove $\ell \parallel m$?



Ⓐ Converse of the Corresponding Angles Postulate
Ⓑ Converse of the Alternate Interior Angles Theorem
Ⓒ Converse of the Alternate Exterior Angles Theorem
Ⓓ Converse of the Same-Side Interior Angles Theorem

Rick Davis/Darkhouse and Fun Enthusiast

44. Two coplanar lines are cut by a transversal. Which condition does NOT guarantee that the two lines are parallel?

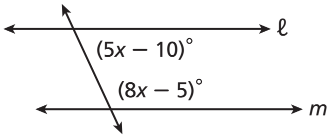
Ⓐ A pair of alternate interior angles are congruent.

Ⓑ A pair of same-side interior angles are supplementary.

Ⓒ A pair of corresponding angles are congruent.

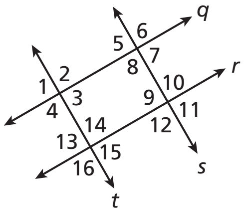
Ⓓ A pair of alternate exterior angles are complementary.

45. **Gridded Response** Find the value of x so that $\ell \parallel m$.



CHALLENGE AND EXTEND

Determine which lines, if any, can be proven parallel using the given information. Justify your answers.



46. $\angle 1 \cong \angle 15$

47. $\angle 8 \cong \angle 14$

48. $\angle 3 \cong \angle 7$

49. $\angle 8 \cong \angle 10$

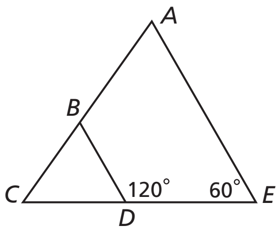
50. $\angle 6 \cong \angle 8$

51. $\angle 13 \cong \angle 11$

52. $m\angle 12 + m\angle 15 = 180°$

53. $m\angle 5 + m\angle 8 = 180°$

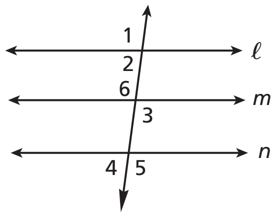
H.O.T. 54. Write a paragraph proof that $\overline{AE} \parallel \overline{BD}$.



Use the diagram for Exercises 55 and 56.

55. **Given:** $m\angle 2 + m\angle 3 = 180°$
Prove: $\ell \parallel m$

56. **Given:** $m\angle 2 + m\angle 5 = 180°$
Prove: $\ell \parallel n$

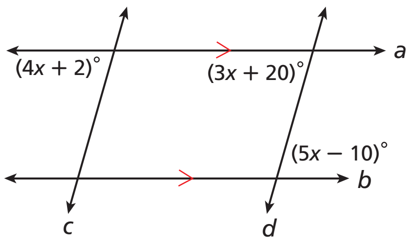


FOCUS ON MATHEMATICAL PRACTICES

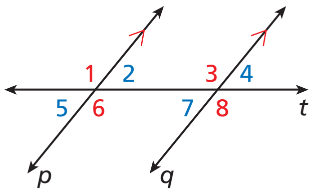
MATHEMATICAL PRACTICES

H.O.T. 57. **Communication** Explain when the Alternate Interior Angles Theorem can be applied, and when the Converse of the Alternate Interior Angles Theorem can be applied.

H.O.T. 58. **Reasoning** In the figure, line a is parallel to line b. Is line c parallel to line d? Justify your answer.



H.O.T. 59. **Justify** June named $\angle 5$ and $\angle 8$ same-side exterior angles. How are their measures related if the lines p and q are parallel? Support your answer.



21-2 Geometry TASK

Use with Proving Lines Parallel

Construct Parallel Lines

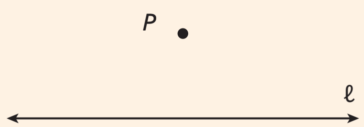
You have learned one method of constructing parallel lines using a compass and straightedge. Another method, called the rhombus method, uses a property of a figure called a *rhombus*. The rhombus method is shown below.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

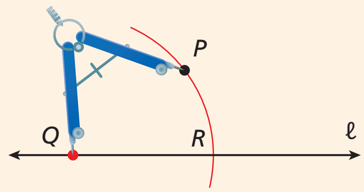
MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods …

Activity 1

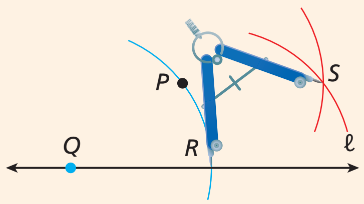
1. Draw a line ℓ and a point P not on the line.



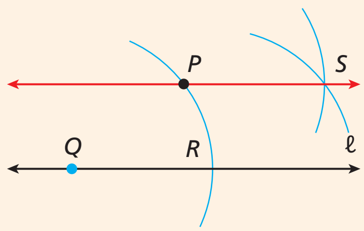
2. Choose a point Q on the line. Place your compass point at Q and draw an arc through P that intersects ℓ. Label the intersection R.



3. Using the same compass setting as the first arc, draw two more arcs: one from P, the other from R. Label the intersection of the two arcs S.



4. Draw $\overleftrightarrow{PS} \parallel \ell$.

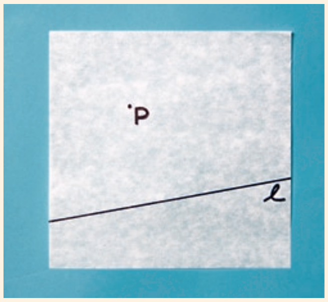


Try This

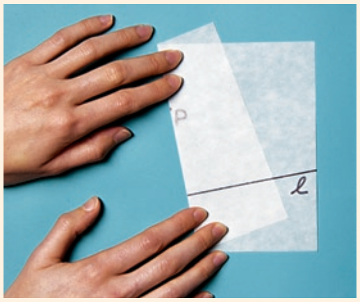
1. Repeat Activity 1 using a different point not on the line. Are your results the same?
2. Using the lines you constructed in Problem 1, draw transversal $\overleftrightarrow{PQ}$. Verify that the lines are parallel by using a protractor to measure alternate interior angles.
3. What postulate ensures that this construction is always possible?
4. A *rhombus* is a quadrilateral with four congruent sides. Explain why this method is called the rhombus method.

Activity 2

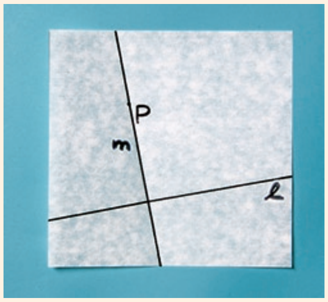
1. Draw a line ℓ and point P on a piece of patty paper.



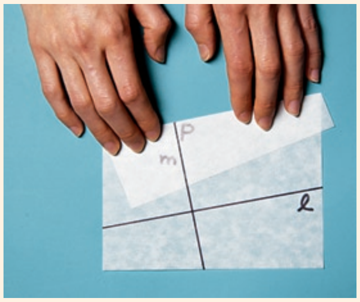
2. Fold the paper through P so that both sides of line ℓ match up



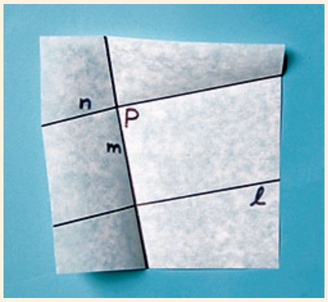
3. Crease the paper to form line m. P should be on line m.



4. Fold the paper again through P so that both sides of line m match up.



5. Crease the paper to form line n. Line n is parallel to line ℓ through P.



Try This

5. Repeat Activity 2 using a point in a different place not on the line. Are your results the same?

6. Use a protractor to measure corresponding angles. How can you tell that the lines are parallel?

7. Draw a triangle and construct a line parallel to one side through the vertex that is not on that side.

8. Line m is perpendicular to both ℓ and n. Use this statement to complete the following conjecture: If two lines in a plane are perpendicular to the same line, then ____?____ .

21-3 Perpendicular Lines

Essential Question: How can you prove and use theorems about perpendicular lines?

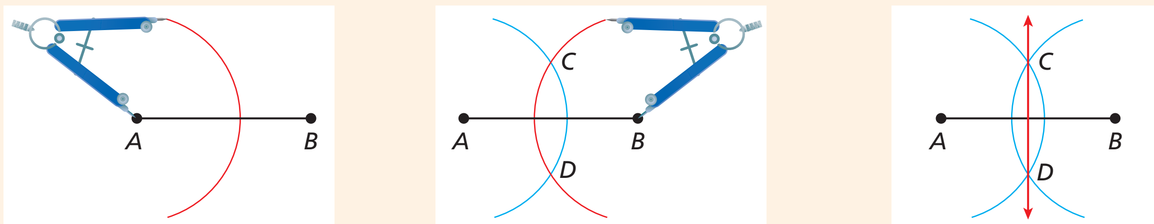
Objective
Prove and apply theorems about perpendicular lines.

Vocabulary
perpendicular bisector
distance from a point to a line

Why learn this?
Rip currents are strong currents that flow away from the shoreline and are perpendicular to it. A swimmer who gets caught in a rip current can get swept far out to sea. (See Example 3.)

The **perpendicular bisector** of a segment is a line perpendicular to a segment at the segment's midpoint. A construction of a perpendicular bisector is shown below.

Construction Perpendicular Bisector of a Segment



1. Draw $\overline{AB}$. Open the compass wider than half of AB and draw an arc centered at A.
2. Using the same compass setting, draw an arc centered at B that intersects the first arc at C and D.
3. Draw $\overleftrightarrow{CD}$. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$.

The shortest segment from a point to a line is perpendicular to the line. This fact is used to define the **distance from a point to a line** as the length of the perpendicular segment from the point to the line.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.REI.3

Distance From a Point to a Line

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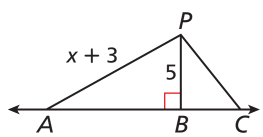
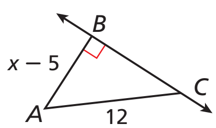
A **Name the shortest segment from P to $\overleftrightarrow{AC}$.**
The shortest distance from a point to a line is the length of the perpendicular segment, so $\overline{PB}$ is the shortest segment from P to $\overleftrightarrow{AC}$.



B **Write and solve an inequality for x.**

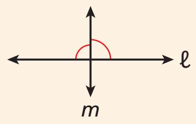
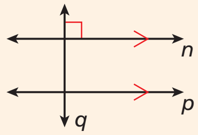
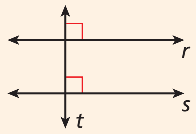
$PA > PB$	*$\overline{PB}$ is the shortest segment.*
$x + 3 > 5$	*Substitute $x + 3$ for PA and 5 for PB.*
$-3 \quad -3$	*Subtract 3 from both sides of the inequality.*
$x > 2$	

CHECK IT OUT!
1a. Name the shortest segment from A to $\overleftrightarrow{BC}$.
1b. Write and solve an inequality for x.



Know it! Note

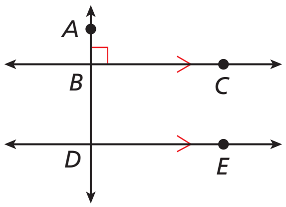
Theorems

	THEOREM	HYPOTHESIS	CONCLUSION
21-3-1	If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular. (2 intersecting lines form lin. pair of ≅ ∡ → lines ⊥.)		$\ell \perp m$
21-3-2	**Perpendicular Transversal Theorem** In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.		$q \perp p$
21-3-3	If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other. (2 lines ⊥ to same line → 2 lines ∥.)		$r \parallel s$

You will prove Theorems 21-3-1 and 21-3-3 in Exercises 37 and 38.

PROOF

Perpendicular Transversal Theorem



Given: $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

Prove: $\overleftrightarrow{AB} \perp \overleftrightarrow{DE}$

Proof:

It is given that $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$, so $\angle ABC \cong \angle BDE$ by the Corresponding Angles Postulate. It is also given that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$, so $m\angle ABC = 90°$. By the definition of congruent angles, $m\angle ABC = m\angle BDE$, so $m\angle BDE = 90°$ by the Transitive Property of Equality. By the definition of perpendicular lines, $\overleftrightarrow{AB} \perp \overleftrightarrow{DE}$.

COMMON CORE GPS **EXAMPLE 2** MCC9-12.G.CO.9

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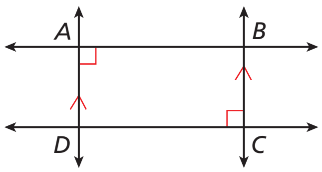
Proving Properties of Lines



Write a two-column proof.

Given: $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AD} \perp \overleftrightarrow{AB}$, $\overleftrightarrow{BC} \perp \overleftrightarrow{DC}$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

Proof:

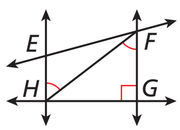
Statements	Reasons
1. $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{BC} \perp \overleftrightarrow{DC}$	**1.** Given
2. $\overleftrightarrow{AD} \perp \overleftrightarrow{DC}$	**2.** ⊥ Transv. Thm.
3. $\overleftrightarrow{AD} \perp \overleftrightarrow{AB}$	**3.** Given
4. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$	**4.** 2 lines ⊥ to same line → 2 lines ∥.

CHECK IT OUT!

2. Write a two-column proof.

Given: $\angle EHF \cong \angle HFG$, $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$

Prove: $\overleftrightarrow{EH} \perp \overleftrightarrow{GH}$



COMMON CORE GPS **EXAMPLE 3** MCC9-12.G.MG.1

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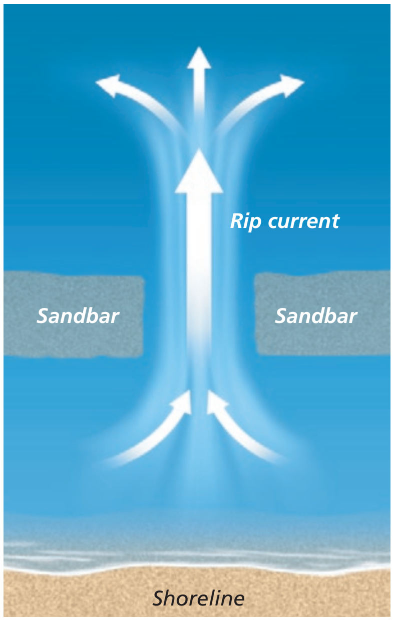
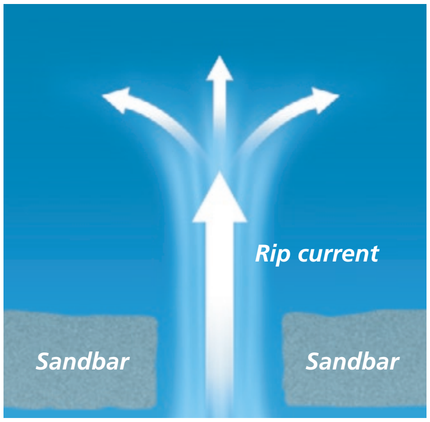
Oceanography Application



Rip currents may be caused by a sandbar parallel to the shoreline. Waves cause a buildup of water between the sandbar and the shoreline. When this water breaks through the sandbar, it flows out in a direction perpendicular to the sandbar. Why must the rip current be perpendicular to the shoreline?

The rip current forms a transversal to the shoreline and the sandbar.



The shoreline and the sandbar are parallel, and the rip current is perpendicular to the sandbar. So by the Perpendicular Transversal Theorem, the rip current is perpendicular to the shoreline.

CHECK IT OUT!

3. A swimmer who gets caught in a rip current should swim in a direction perpendicular to the current. Why should the path of the swimmer be parallel to the shoreline?

MCC.MP.3 MATHEMATICAL PRACTICES

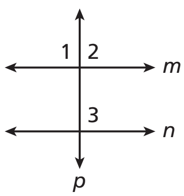
THINK AND DISCUSS

1. Describe what happens if two intersecting lines form a linear pair of congruent angles.

2. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Use the diagram and the theorems from this lesson to complete the table.

Diagram	If you are given . . .	Then you can conclude . . .
	$m\angle 1 = m\angle 2$	
	$m\angle 2 = 90°$ $m\angle 3 = 90°$	
	$m\angle 2 = 90°$ $m \parallel n$	

21-3 Exercises

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Homework Help

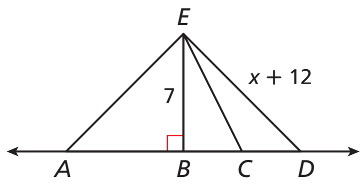
GUIDED PRACTICE

1. **Vocabulary** $\overleftrightarrow{CD}$ is the *perpendicular bisector* of $\overline{AB}$. $\overleftrightarrow{CD}$ intersects $\overline{AB}$ at C. What can you say about $\overline{AB}$ and $\overleftrightarrow{CD}$? What can you say about $\overline{AC}$ and $\overline{BC}$?

SEE EXAMPLE 1

2. Name the shortest segment from point E to $\overleftrightarrow{AD}$.

3. Write and solve an inequality for x.



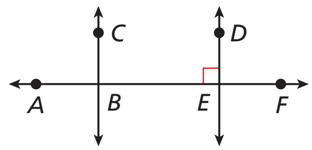
SEE EXAMPLE 2

4. Complete the two-column proof.

Given: $\angle ABC \cong \angle CBE$, $\overleftrightarrow{DE} \perp \overleftrightarrow{AF}$

Prove: $\overleftrightarrow{CB} \parallel \overleftrightarrow{DE}$

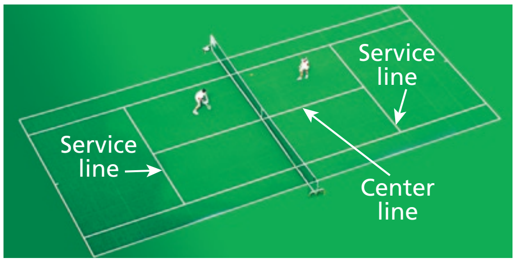
Proof:



Statements	Reasons
1. $\angle ABC \cong \angle CBE$	1. Given
2. $\overleftrightarrow{CB} \perp \overleftrightarrow{AF}$	2. a. ____?____
3. b. ____?____	3. Given
4. $\overleftrightarrow{CB} \parallel \overleftrightarrow{DE}$	4. c. ____?____

SEE EXAMPLE 3

5. **Sports** The center line in a tennis court is perpendicular to both service lines. Explain why the service lines must be parallel to each other.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
6–7	1
8	2
9	3

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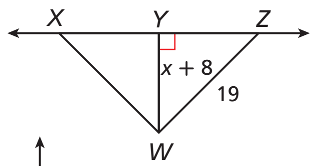
Online Extra Practice

6. Name the shortest segment from point W to $\overleftrightarrow{XZ}$.

7. Write and solve an inequality for x.

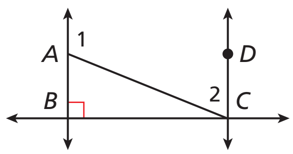


8. Complete the two-column proof below.

Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$, $m\angle 1 + m\angle 2 = 180°$

Prove: $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$

Proof:

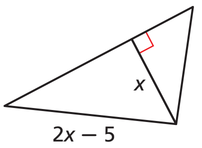


Statements	Reasons
1. $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$	1. Given
2. $m\angle 1 + m\angle 2 = 180°$	2. a. ____?____
3. $\angle 1$ and $\angle 2$ are supplementary.	3. Def. of supplementary
4. b. ____?____	4. Converse of the Same-Side Interior Angles Theorem
5. $\overleftrightarrow{BC} \perp \overleftrightarrow{CD}$	5. c. ____?____

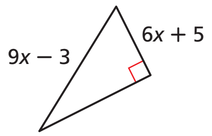
9. **Music** The *frets* on a guitar are all perpendicular to one of the strings. Explain why the frets must be parallel to each other.

For each diagram, write and solve an inequality for *x*.

10.

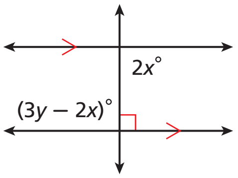


11.

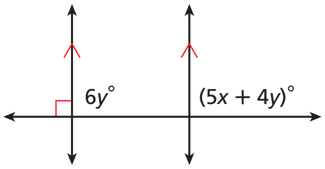


Multi-Step **Solve to find *x* and *y* in each diagram.**

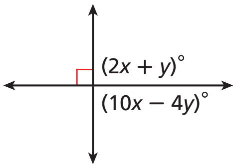
12.



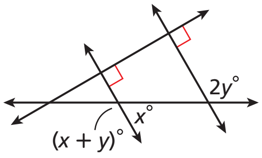
13.



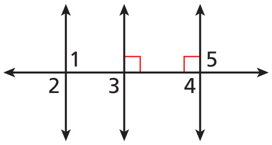
14.



15.



Determine if there is enough information given in the diagram to prove each statement.



16. $\angle 1 \cong \angle 2$ 17. $\angle 1 \cong \angle 3$

18. $\angle 2 \cong \angle 3$ 19. $\angle 2 \cong \angle 4$

20. $\angle 3 \cong \angle 4$ 21. $\angle 3 \cong \angle 5$

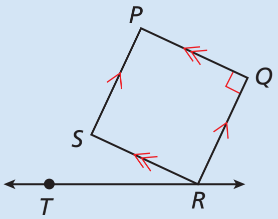
H.O.T. **22.** **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for perpendicular lines? Explain why or why not.
Reflexive: $\ell \perp \ell$
Symmetric: If $\ell \perp m$, then $m \perp \ell$.
Transitive: If $\ell \perp m$ and $m \perp n$, then $\ell \perp n$.

Real-World Connections

23. In the diagram, which represents the side view of a mystery spot, $\overline{QR} \perp \overline{PQ}$, $\overline{PQ} \parallel \overline{RS}$, and $\overline{PS} \parallel \overline{QR}$.

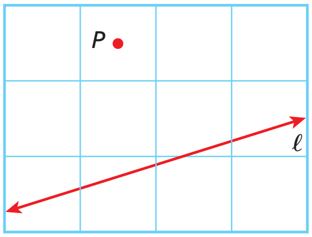
a. Prove $\overline{QR} \perp \overline{RS}$ and $\overline{PS} \perp \overline{RS}$.

b. Prove $\overline{PQ} \perp \overline{PS}$.



24. **Geography** Felton Avenue, Arlee Avenue, and Viehl Avenue are all parallel. Broadway Street is perpendicular to Felton Avenue. Use the satellite photo and the given information to determine the values of *x* and *y*.

25. **Estimation** Copy the diagram onto a grid with 1 cm by 1 cm squares. Estimate the distance from point *P* to line ℓ.



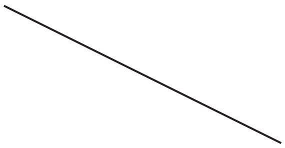
H.O.T. **26.** **Critical Thinking** Draw a figure to show that Theorem 21-3-3 is not true if the lines are not in the same plane.

27. Draw a figure in which $\overline{AB}$ is a perpendicular bisector of $\overline{XY}$ but $\overline{XY}$ is not a perpendicular bisector of $\overline{AB}$.

H.O.T. **28.** **Write About It** A ladder is formed by rungs that are perpendicular to the sides of the ladder. Explain why the rungs of the ladder are parallel.

Construction **Construct a segment congruent to each given segment and then construct its perpendicular bisector.**

29.



30.

TEST PREP

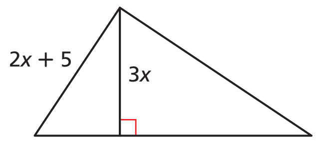
31. Which inequality is correct for the given diagram?

Ⓐ $2x + 5 < 3x$

Ⓑ $x > 1$

Ⓒ $2x + 5 > 3x$

Ⓓ $x > 5$



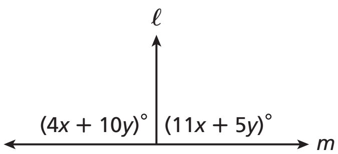
32. In the diagram, $\ell \perp m$. Find *x* and *y*.

Ⓕ $x = 5, y = 7$

Ⓖ $x = 7, y = 5$

Ⓗ $x = 90, y = 90$

Ⓙ $x = 10, y = 5$



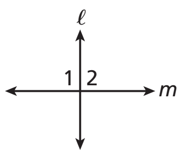
33. If $\ell \perp m$, which statement is NOT correct?

Ⓐ $m\angle 2 = 90°$

Ⓑ $m\angle 1 + m\angle 2 = 180°$

Ⓒ $\angle 1 \cong \angle 2$

Ⓓ $\angle 1 \perp \angle 2$



34. In a plane, both lines *m* and *n* are perpendicular to both lines *p* and *q*. Which conclusion CANNOT be made?

(A) $p \parallel q$

(B) $m \parallel n$

(C) $p \perp q$

(D) All angles formed by lines *m*, *n*, *p*, and *q* are congruent.

H.O.T. 35. **Extended Response** Lines *m* and *n* are parallel. Line *p* intersects line *m* at *A* and line *n* at *B*, and is perpendicular to line *m*.

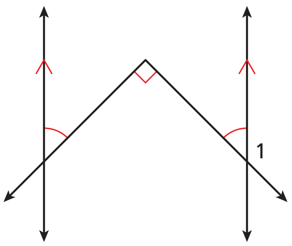
a. What is the relationship between line *n* and line *p*? Draw a diagram to support your answer.

b. What is the distance from point *A* to line *n*? What is the distance from point *B* to line *m*? Explain.

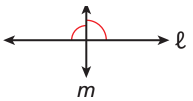
c. How would you define the distance between two parallel lines in a plane?

CHALLENGE AND EXTEND

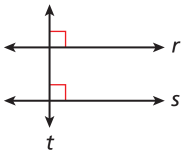
H.O.T. 36. **Multi-Step** Find m∠1 in the diagram. (*Hint:* Draw a line parallel to the given parallel lines.)



37. Prove Theorem 21-3-1: If two intersecting lines form a linear pair of congruent angles, then the two lines are perpendicular.



38. Prove Theorem 21-3-3: If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other.

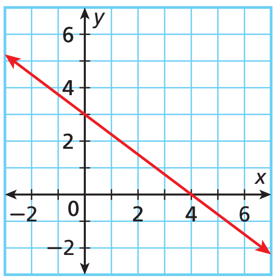


MATHEMATICAL PRACTICES FOCUS ON MATHEMATICAL PRACTICES

H.O.T. 39. **Modeling** Korey said that two lines parallel to the same line are perpendicular. Draw a figure to prove or disprove his statement.

H.O.T. 40. **Reasoning** $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$ and intersects it at point *E*. $AE = 2x + 6$, $BE = 5x - 12$, $CE = 4x + 9$, and $DE = 6x - 3$. Is $\overline{AB}$ the perpendicular bisector of $\overline{CD}$? Justify your answer.

H.O.T. 41. **Communication** Explain how you know that the distance from the origin to the line shown is less than 3.



21-3 Geometry TASK

Construct Perpendicular Lines

Use with Perpendicular Lines

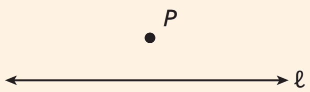
You have learned to construct the perpendicular bisector of a segment. This is the basis of the construction of a line perpendicular to a given line through a given point. The steps in the construction are the same whether the point is on or off the line.

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

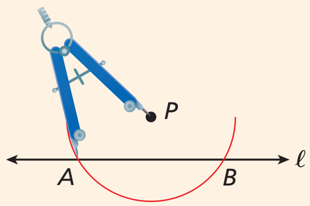
MCC9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods …

Activity

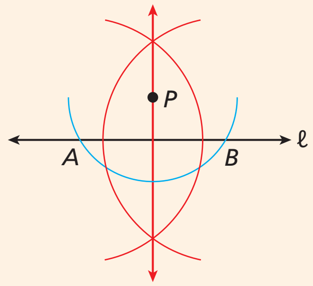
Copy the given line ℓ and point *P*.



1 Place the compass point on *P* and draw an arc that intersects ℓ at two points. Label the points *A* and *B*.



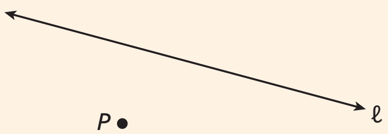
2 Construct the perpendicular bisector of $\overline{AB}$.



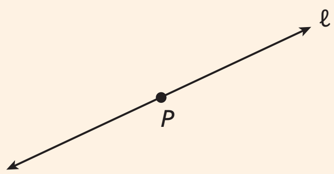
Try This

Copy each diagram and construct a line perpendicular to line ℓ through point *P*. Use a protractor to verify that the lines are perpendicular.

1.

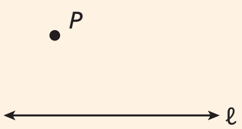


2.

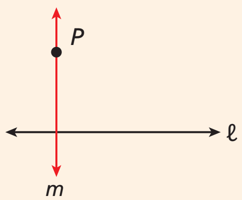


3. Follow the steps below to construct two parallel lines. Explain why ℓ ∥ *n*.

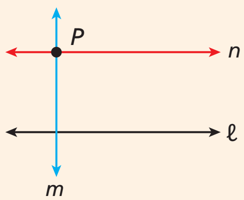
Step 1 Given a line ℓ, draw a point *P* not on ℓ.



Step 2 Construct line *m* perpendicular to ℓ through *P*.



Step 3 Construct line *n* perpendicular to *m* through *P*.



Ready to Go On?

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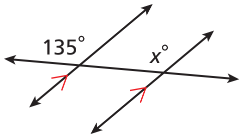
Assessment and Intervention

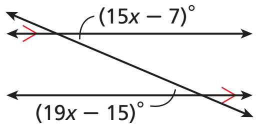
21-1 Angles Formed by Parallel Lines and Transversals

Find each angle measure.

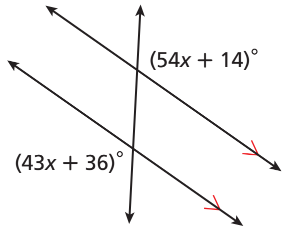
1.



2.

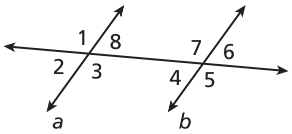


3.



21-2 Proving Lines Parallel

Use the given information and the theorems and postulates you have learned to show that $a \parallel b$.



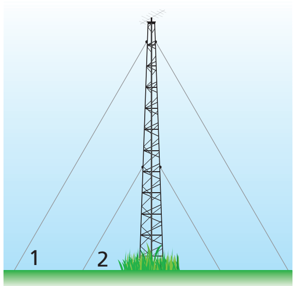
4. $m\angle 8 = (13x + 20)^\circ$, $m\angle 6 = (7x + 38)^\circ$, $x = 3$

5. $\angle 1 \cong \angle 5$

6. $m\angle 8 + m\angle 7 = 180^\circ$

7. $m\angle 8 = m\angle 4$

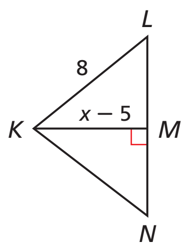
8. The tower shown is supported by guy wires such that $m\angle 1 = (3x + 12)^\circ$, $m\angle 2 = (4x - 2)^\circ$, and $x = 14$. Show that the guy wires are parallel.



21-3 Perpendicular Lines

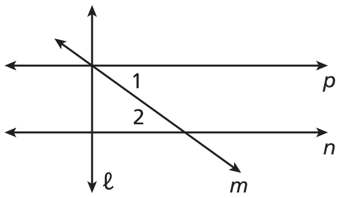
9. Name the shortest segment from point K to $\overline{LN}$.

10. Write and solve an inequality for x.



11. Write a two-column proof.

Given: $\angle 1 \cong \angle 2$, $\ell \perp n$
Prove: $\ell \perp p$

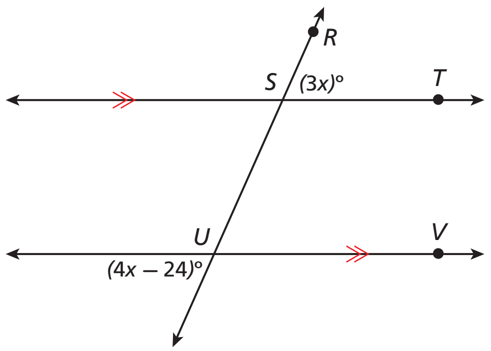


PARCC Assessment Readiness

COMMON CORE GPS

Selected Response

1. Find m$\angle RST$.



(A) m$\angle RST = 108°$
(B) m$\angle RST = 24°$
(C) m$\angle RST = 156°$
(D) m$\angle RST = 72°$

2. From the ocean, salmon swim perpendicularly toward the shore to lay their eggs in rivers. Waves in the ocean are parallel to the shore. Why must the salmon swim perpendicularly to the waves?

(F) Swimming salmon form a transversal to the shore and the waves. The shore and the waves are parallel, and the swimming salmon are perpendicular to the shore. So by the Perpendicular Transversal Theorem, the salmon are perpendicular to the waves.

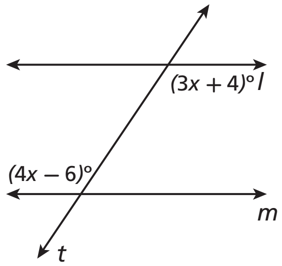
(G) Swimming salmon form a transversal to the shore and the waves. The shore and the waves are perpendicular, and the swimming salmon are parallel to the shore. So by the Perpendicular Transversal Theorem, the salmon are perpendicular to the waves.

(H) Swimming salmon form a transversal to the shore and the waves. The shore and the waves are parallel, and the swimming salmon are parallel to the shore. So by the Perpendicular Transversal Theorem, the salmon are perpendicular to the waves.

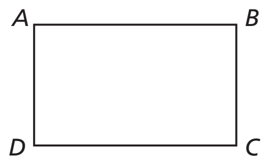
(J) Swimming salmon form a transversal to the shore and the waves. The shore and the waves are parallel, and the swimming salmon are perpendicular to the shore. So by the Parallel Transversal Theorem, the salmon are perpendicular to the waves.

Mini-Tasks

3. In a swimming pool, two lanes are represented by lines l and m. If a string of flags strung across the lanes is represented by transversal t, and $x = 10$, show that the lanes are parallel.



4. **Given:** $\overline{AD} \parallel \overline{BC}$, $\overline{AD} \perp \overline{AB}$, $\overline{DC} \perp \overline{BC}$
Prove: $\overline{AB} \parallel \overline{CD}$



5. **Given:** $m \perp p$, $\angle 1$ and $\angle 2$ are complementary.
Prove: $p \parallel q$