9

Mathematics

Learner's Material

Module 3:
Variations

This instructional material was collaboratively developed and reviewed by educators from public and private schools, colleges, and/or universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at action@deped.gov.ph.

We value your feedback and recommendations.

Department of Education
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MATHEMATICS GRADE 9
Learner's Material
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UNIT II

MODULE 3

Variations

I. INTRODUCTION AND FOCUS QUESTIONS

Do you know that an increasing demand for paper contributes to the destruction of trees from which papers are made?

If waste papers were recycled regularly, it would help prevent the cutting down of trees, global warming and other adverse effects that would destroy the environment. Paper recycling does not only save the earth but also contributes to the economy of the country and to the increase in income of some individuals.

This is one situation where questions such as "Will a decrease in production of paper contribute to the decrease in the number of trees being cut?" can be answered using the concepts of variations.

There are several relationships of quantities that you will encounter in this situation. You will learn how a change in one quantity could correspond to a predictable change in the other.

In this module you will find out the relation between quantities. Remember to search for the answer to the following question(s):

- How can I make use of the representations and descriptions of a given set of data?
- What are the beneficial and adverse effects of studying variation which can help solve problems in real life?

You will examine these questions when you take the following lessons.

II. LESSONS AND COVERAGE

Lesson 1 – **Direct Variation**

Lesson 2 – **Inverse Variation**

Lesson 3 – **Joint Variation**

Lesson 4 – **Combined Variation**

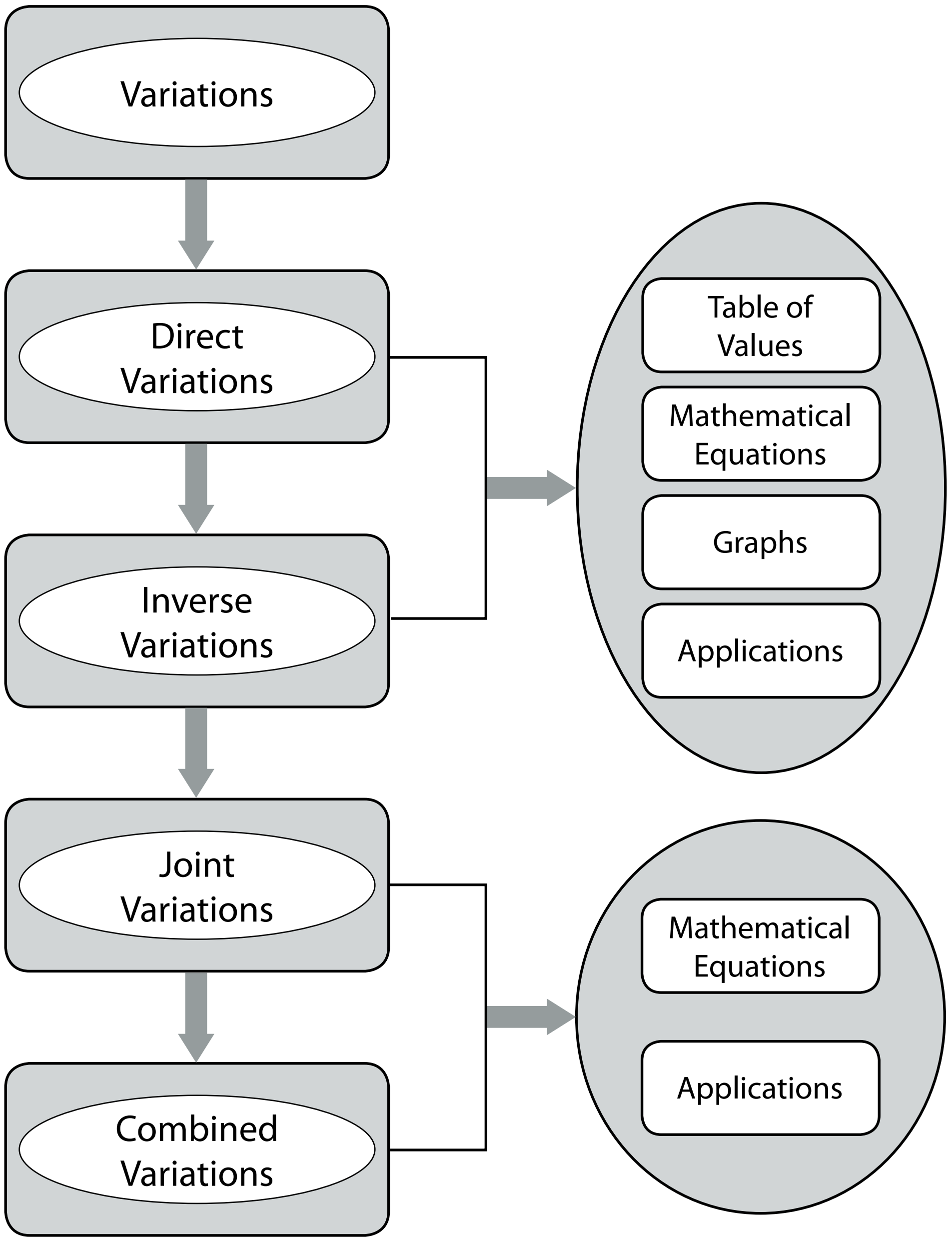
Objectives

In these lessons, you will learn the following:

Lesson 1	• illustrate situations that involve direct variation • translate into variation statement a relationship involving direct variation between two quantities given by a table of values, a mathematical equation, and a graph, and vice versa. • solve problems involving direct variations.
Lesson 2	• illustrate situations that involve inverse variation • translate into variation statement a relationship involving inverse variation between two quantities given by a table of values, a mathematical equation, and a graph, and vice versa. • solve problems involving inverse variations.
Lesson 3	• illustrate situations that involve joint variation • translate into variation statement a relationship involving joint variation between two quantities given by a mathematical equation, and vice versa. • solve problems involving joint variations.
Lesson 4	• illustrate situations that involve combined variation • translate into variation statement a relationship involving combined variation between two quantities given by a mathematical equation, and vice versa. • solve problems involving combined variations.

Module Map

Here is a simple map of the above lessons your students will cover:



To do well in this module, you will need to remember and do the following:

1. Study each part of the module carefully.
2. Take note of all the formulas given in each lesson.
3. Have your own scientific calculator. Make sure you are familiar with the keys and functions of your calculator.

III. PRE-ASSESSMENT

Part I

Let's find out how much you already know about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items. During the checking, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. The cost c varies directly as the number n of pencils is written as

   a. $c = kn$ b. $k = cn$ c. $n = \frac{k}{c}$ d. $c = \frac{k}{n}$

2. The speed r of a moving object is inversely proportional to the time t travelled is written as

   a. $r = kt$ b. $r = \frac{k}{t}$ c. $t = kr$ d. $\frac{r}{k} = r$

3. Which is an example of a direct variation?

   a. $xy = 10$ b. $y = \frac{2}{x}$ c. $y = 5x$ d. $\frac{2}{y} = x$

4. A car travels a distance of d km in t hours. The formula that relates d to t is $d = kt$. What kind of variation is it?

   a. direct b. inverse c. joint d. combined

5. y varies directly as x and $y = 32$ when $x = 4$. Find the constant of variation.

   a. 8 b. 36 c. 28 d. 128

6. Which of the following describes an inverse variation?

   a.

x	2	3	4	5
y	5	$\frac{10}{3}$	$\frac{5}{2}$	2

   b.

x	1	2	3	4
y	5	10	15	20

   c.

x	40	30	20	10
y	8	6	4	2

   d.

x	4	8	10	12
y	2	4	5	6

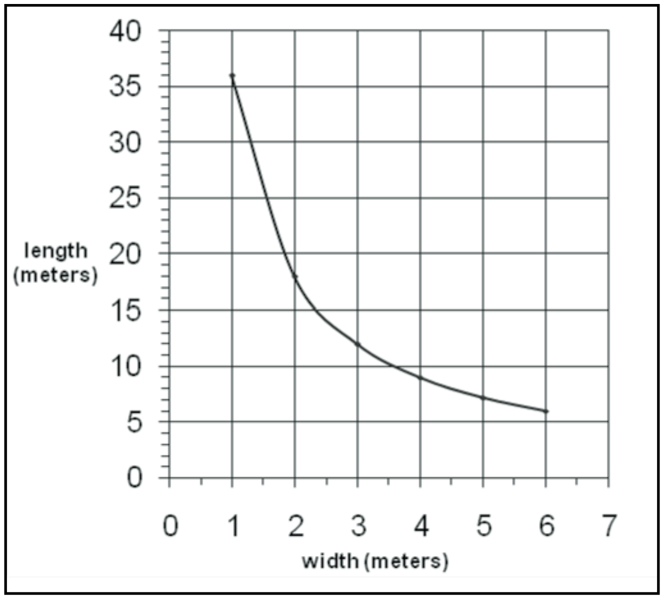
7. What happens to T when h is doubled in the equation $T = 4h$?

   a. T is halved c. T is doubled
   b. T is tripled d. T becomes zero

8. If y varies directly as x and $y = 12$ when $x = 4$, find y when $x = 12$.

   a. 3 b. 4 c. 36 d. 48

9. What mathematical statement describes the graph below?



a. $lw = 36$ b. $\frac{l}{w} = 36$ c. $\frac{l}{36} = w$ d. $\frac{w}{36} = l$

10. If y varies inversely as x and $y = \frac{1}{3}$ when $x = 8$, find y when $x = -4$.

a. $\frac{-2}{3}$ b. $\frac{2}{3}$ c. $\frac{-32}{3}$ d. $\frac{32}{3}$

11. What happens to y when x is tripled in the relation $y = \frac{k}{x}$?

a. y is tripled. c. y is halved.

b. y is doubled. d. y is divided by 3.

12. w varies directly as the square of x and inversely as p and q. If $w = 12$ when $x = 4$, $p = 2$ and $q = 20$, find w when $x = 3$, $p = 8$ and $q = 5$.

a. 10 b. 9 c. $\frac{27}{4}$ d. 5

13. If 3 men can do a portion of a job in 8 days, how many men can do the same job in 6 days?

a. 7 b. 6 c. 5 d. 4

14. If y varies inversely as x, and $y = \frac{1}{5}$ when $x = 9$, find y when $x = -3$.

a. 5 b. $\frac{1}{3}$ c. $\frac{3}{5}$ d. $-\frac{3}{5}$

15. Mackee's income varies directly as the number of days that she works. If she earns Php 8,000.00 in 20 days, how much will she earn if she worked 3 times as long?

a. Php 26,000 b. Php 24,000 c. Php 20,000 d. Php 16,000

16. If s varies directly as t and inversely as v, then which of the following equations describes the relation among the three variables s, t, and v?

a. $s = \frac{k}{tv}$ b. $s = \frac{kv}{t}$ c. $\frac{1}{s} = \frac{kt}{v}$ d. $s = \frac{kt}{v}$

17. If $(x - 4)$ varies inversely as $(y + 3)$ and $x = 8$ when $y = 2$, find x when $y = -1$.

a. 20   b. 18   c. 16   d. 14

18. The amount of gasoline used by a car varies jointly as the distance travelled and the square root of the speed. Suppose a car used 25 liters on a 100 km trip at 100 kph, about how many liters will it use on a 1000-km trip at 64 kph?

a. 100 L   b. 200 L   c. 300 L   d. 400 L

19. If y varies directly as the square of x, how is y changed if x is increased by 20%?

a. 44% decrease in y c. 0.44% decrease in y

b. 44% increase in y d. 0.44% increase in y

20. If h varies jointly as j^2 and i and inversely as g, and $h = 50$ when $j = 2$, $i = 5$, and $g = \frac{1}{2}$, find h when $j = 4$, $i = 10$, and $g = \frac{1}{4}$.

a. 25   b. 100   c. 800   d. 805

IV. LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate understanding of key concepts of variations, to formulate real-life problems involving these concepts, and to solve these using a variety of strategies. Furthermore, you should be able to investigate mathematical relationships in various situations involving variations.

Let us begin with exploratory activities that will introduce you to the basic concepts of variation and how these concepts are applied in real life.

➤ Activity 1: Before Lesson Response

Read the phrases found at the right column in the table below. If the phrase is a direct variation, place a letter **D** in the Before Lesson Response column, if it is an inverse variation, place a letter **I**. If the relationship is neither a direct nor inverse variation, mark it **N**.

Before Lesson Response	Phrase
	1. The number of hours to finish a job to the number of men working
	2. The amount of water to the space that water did not occupy in a particular container
	3. The number of persons sharing a pie to the size of the slices of the pie
	4. The area of the wall to the amount of paint used to cover it
	5. The time spent in walking to the rate at which a person walks
	6. The time a teacher spends checking papers to the number of students
	7. The cost of life insurance to the age of the insured person
	8. The age of a used car to its resale value
	9. The amount of money raised in a concert to the number of tickets sold
	10. The distance an airplane flies to the time travelling

LESSON 1 Direct Variation

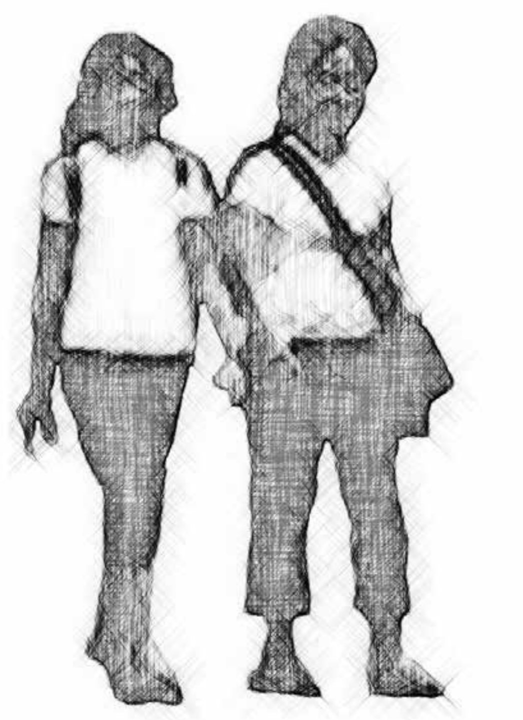
What to KNOW

> Let's start the module by doing activities that will reveal your background knowledge on direct variations. These are practical situations that you also encounter in real life.

➤ Activity 2: What's the Back Story?

Read and analyze the situation below and answer the questions that follow.

Helen and Joana walk a distance of one kilometer in going to the school where they teach. At a constant rate, it takes them 20 minutes to reach school in time for their first class.



One particular morning, the two became so engrossed in discussing an incident inside the school during the previous day that they did not notice that the pace at which they were walking slowed down.

Questions:

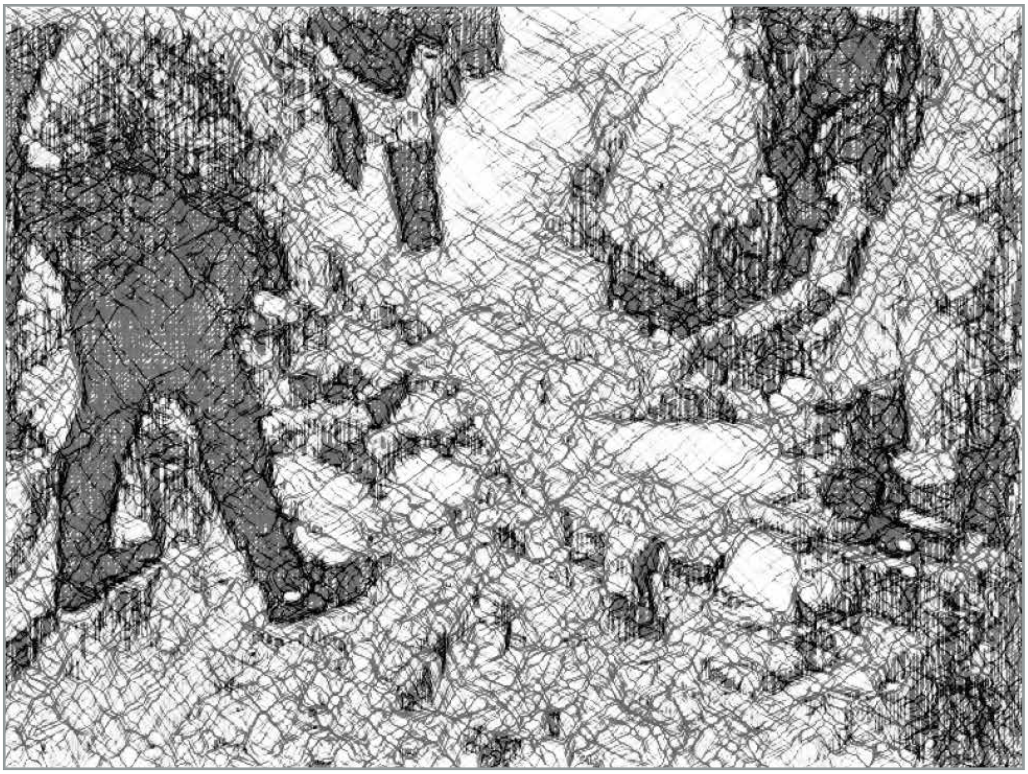
a. How will they be able to catch up for the lost number of minutes? Cite solutions.

b. How are the quantities like rate, time, and distance considered in travelling?

c. Does the change in one quantity affect a change in the other? Explain.

➤ Activity 3: Let's Recycle!

A local government organization launches a recycling campaign of waste materials to schools in order to raise students' awareness of environmental protection and the effects of climate change. Every kilogram of waste materials earns points that can be exchanged for school supplies and grocery items. Paper, which is the number one waste collected, earns 5 points for every kilo.



The table below shows the points earned by a Grade 8 class for every number of kilograms of waste paper collected.

Number of kilograms (n)	1	2	3	4	5	6
Points (P)	5	10	15	20	25	30

Questions:

1. What happens to the number of points when the number of kilograms of paper is doubled? tripled?
2. How many kilograms of paper will the Grade 8 class have to gather in order to raise 500 points? Write a mathematical statement that will relate the two quantities involved.
3. In what way are you able to help clean the environment by collecting these waste papers?
4. What items can be made out of these papers?

➤ Activity 4: How Steep Is Enough?

Using his bicycle, Jericho travels a distance of 10 kilometers per hour on a steep road. The table shows the distance he has travelled at a particular length of time.

Time (hr)	1	2	3	4	5
Distance (km)	10	20	30	40	50

Questions:

1. What happens to the distance as the length of time increases?
2. Using this pattern, how many kilometers would he have travelled in $8\frac{1}{2}$ hours?
3. How will you be able to find the distance (without the aid of the table)? Write a mathematical statement to represent the relation.
4. What mathematical operation did you apply in this case? Is there a constant number involved? Explain the process that you have discovered.

How did you find the four activities? I am sure you did not find any difficulty in answering the questions.

The next activities will help you fully understand the concepts behind these activities.

What to PROCESS

There is *direct variation* whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"

"y is directly proportional to x" and

"y is proportional to x"

may be translated mathematically as $y = kx$, where k is the constant of variation.

For two quantities, x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

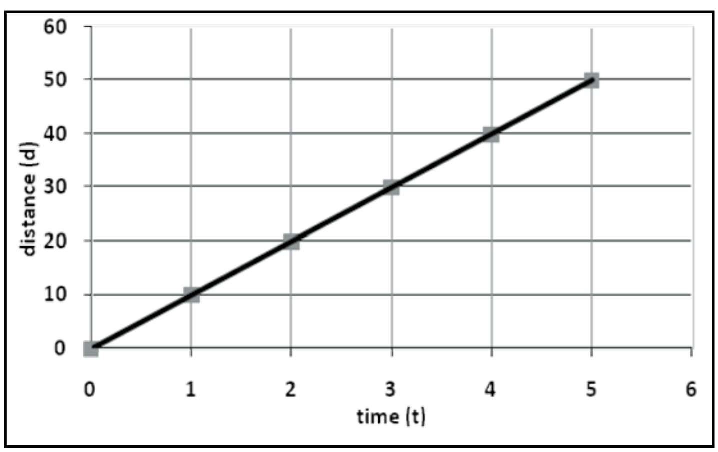
➤ Activity 5: Watch This!

If the distance d varies directly as the time t, then the relationship can be translated into a mathematical statement as $d = kt$, where k is the constant of variation.

Likewise, if the distance d varies directly as the rate r, then the mathematical equation describing the relation is $d = kr$.

In Activity 4, the variation statement that is involved between the two quantities is $d = 10t$. In this case, the constant of variation is $k = 10$.

Using a convenient scale, the graph of the relation $d = 10t$ is a line.



The graph above describes a direct variation of the form $y = kx$.

Which of the equations is of the form $y = kx$ and shows a direct relationship?

1. $y = 2x + 3$
2. $y = 3x$
3. $y = 5x$
4. $y = x^2 - 4$
5. $y = 4x^2$

Your skill in recognizing patterns and knowledge in formulating equations helped you answer the questions in the previous activities. For a more detailed solution of problems involving direct variation, let us see how this is done.

Examples:

1. If y varies directly as x and $y = 24$ when $x = 6$, find the variation constant and the equation of variation.

   Solution:

   a. Express the statement "*y varies directly as x*" as $y = kx$.

   b. Solve for k by substituting the given values in the equation.

   $$y = kx$$
   $$24 = 6k$$
   $$k = \frac{24}{6}$$
   $$k = 4$$

   Therefore, the constant of variation is 4.

   c. Form the equation of the variation by substituting 4 in the statement, $y = kx$.

   $$y = 4x$$

2. The table below shows that the distance d varies directly as the time t. Find the constant of variation and the equation which describes the relation.

Time (hr)	1	2	3	4	5
Distance (km)	10	20	30	40	50

   Solution:

   Since the distance d varies directly as the time t, then

   $$d = kt.$$

   Using one of the pairs of values, (2, 20), from the table, substitute the values of d and t in $d = kt$ and solve for k.

   $$d = kt$$
   $$20 = 2k$$
   $$k = \frac{20}{2}$$
   $$k = 10$$

   Therefore, the constant of variation is 10.

   Form the mathematical equation of the variation by substituting 10 in the statement $d = kt$.

   $$d = 10t$$

We can see that the constant of variation can be solved if one pair of values of x and y is known. From the resulting equation, other pairs having the same relationship can be obtained. Let us study the next example.

3. If x varies directly as y and $x = 35$ when $y = 7$, what is the value of y when $x = 25$?

Solution 1.

Since x varies directly as y, then the equation of variation is in the form $x = ky$.

Substitute the given values of y and x to solve for k in the equation.

$$35 = k(7)$$
$$k = \frac{35}{7}$$
$$k = 5$$

Hence, the equation of variation is $x = 5y$.

Solving for y when $x = 25$,

$$25 = 5y$$
$$y = \frac{25}{5}$$
$$y = 5$$

Hence, $y = 5$.

Solution 2.

Since $\frac{x}{y}$ is a constant, then we can write $k = \frac{x}{y}$. From here, we can establish a proportion such that $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where $x_1 = 35$, $y_1 = 7$ and $x_2 = 25$.

Substituting the values, we get

$$\frac{35}{7} = \frac{25}{y_2}$$
$$5 = \frac{25}{y_2}$$
$$y_2 = \frac{25}{5}$$
$$y_2 = 5$$

Therefore, $y = 5$ when $x = 25$.

Now, let us test what you have learned from the discussions.

➤ Activity 6: It's Your Turn!

A. Write an equation for the following statements:

1. The fare F of a passenger varies directly as the distance d of his destination.
2. The cost C of fish varies directly as its weight w in kilograms.
3. An employee's salary S varies directly as the number of days d he has worked.
4. The area A of a square varies directly as the square of its side s.
5. The distance D travelled by a car varies directly as its speed s.
6. The length L of a person's shadow at a given time varies directly as the height h of the person.
7. The cost of electricity C varies directly as the number of kilowatt-hour consumption l.
8. The volume V of a cylinder varies directly as its height h.
9. The weight W of an object is directly proportional to its mass m.
10. The area A of a triangle is proportional to its height h.

B. Determine if the tables and graphs below express a direct variation between the variables. If they do, find the constant of variation and an equation that defines the relation.

1.

x	1	2	3	4
y	-3	-6	-9	-12

2.

x	7	14	-21	-28
y	3	6	-9	-12

3.

x	-15	10	-20	25
y	-3	2	-4	5

4.

x	2	3	4	5
y	1	2	3	4

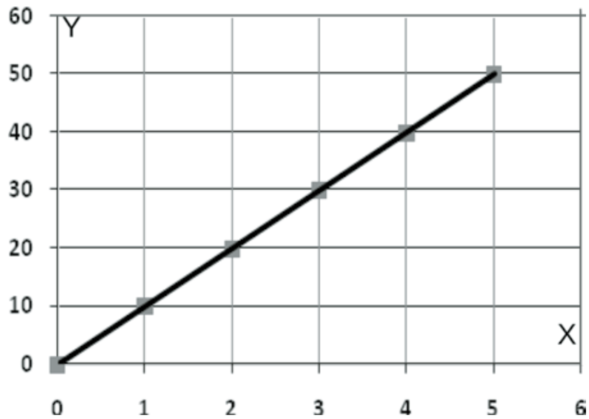
5.

x	16	20	24	28	32
y	12	15	18	21	24

6.

x	6	8	10	12
y	7	9	11	13

7.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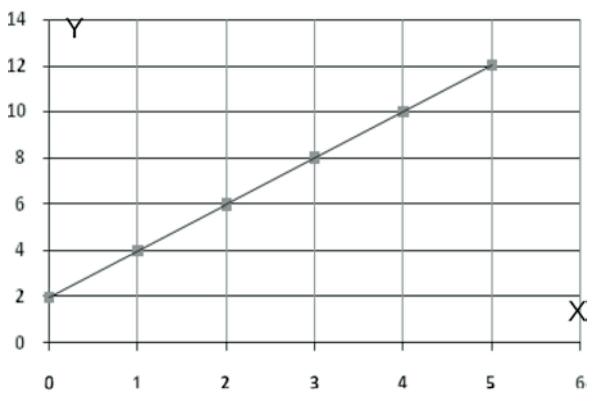
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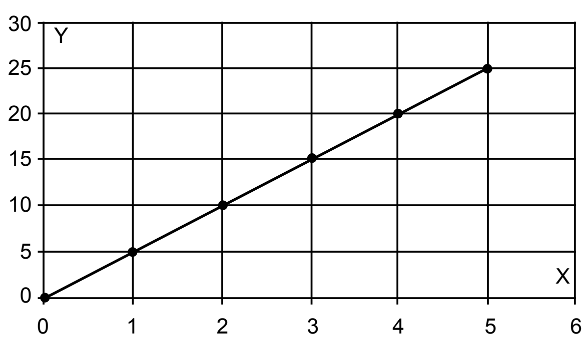
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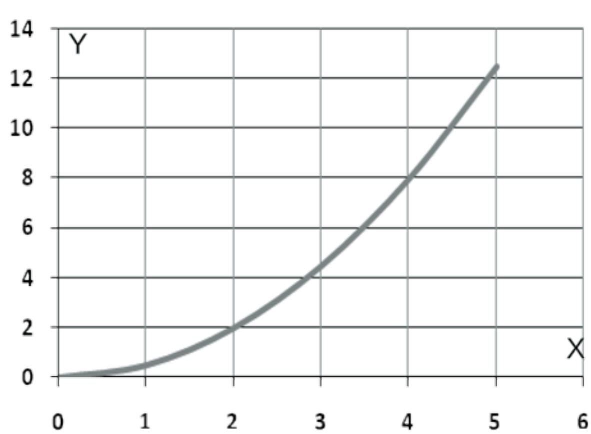
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C. Write an equation where y varies directly as x.

1. $y = 28$ when $x = 7$
2. $y = 30$ when $x = 8$
3. $y = 0.7$ when $x = 0.4$
4. $y = 0.8$ when $x = 0.5$
5. $y = 400$ when $x = 25$
6. $y = 63$ when $x = 81$
7. $y = 200$ when $x = 300$
8. $y = 1$ and $x = 2$
9. $y = 48$ and $x = 6$
10. $y = 10$ and $x = 24$

D. In each of the following, y varies directly as x. Find the values as indicated.

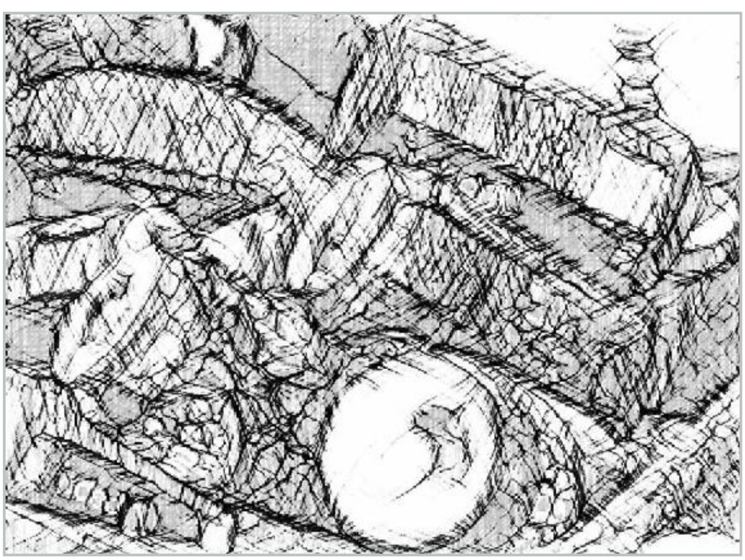
1. If $y = 12$ when $x = 4$, find y when $x = 12$
2. If $y = -18$ when $x = 9$, find y when $x = 7$
3. If $y = -3$ when $x = -4$, find x when $y = 2$
4. If $y = 3$ when $x = 10$, find x when $y = 1.2$
5. If $y = 2.5$ when $x = .25$, find y when $x = .75$

What to REFLECT and UNDERSTAND

Having developed your knowledge about the concepts of direct variation, your goal now is to take a closer look at some aspects of the lesson. This requires you to apply these concepts in solving the problems that follow

➤ Activity 7: Cans Anyone?

1. Tin cans of beverages are collected for recycling purposes in many places in the Philippines.



Junk shops pay Php 15.00 for every kilo of tin cans bought from collectors. In the following table, c is the cost in peso and n is the number of kilos of tin cans:

n	1	2	3	4	5	6
c	15	30	45	60	75	90

a. Write a mathematical statement that relates the two quantities n and c.
b. What is the constant of variation? Formulate the mathematical equation.
c. Observe the values of c and n in the table. What happens to the cost c when the number n of kilos of paper is doubled? Tripled?
d. Graph the relation.
e. How much would 20 kilos of tin cans cost if at the end of month, the cost for every kilo of tin cans will increase by 5 pesos?
f. What items can be made out of these tin cans?

2. The circumference of a circle varies directly as its diameter. If the circumference of a circle having a diameter of 7 cm is 7 π cm, what is the circumference of the circle whose diameter is 10 cm? 15 cm? 18 cm? 20 cm?
   a. Write a mathematical statement that relates the two quantities involved in the problem.
   b. What is the constant of variation? Formulate the mathematical equation.
   c. Construct a table of values from the relation.
3. The service fee f of a physical therapist varies directly as the number of hours n of service rendered. A physical therapist charges Php 2,100 for 3 hours service to patients in a home care. How much would he be paid for $6\frac{1}{2}$ hours of service?
4. The amount of paint p needed to paint the walls of a room varies directly as the area A of the wall. If 2 gallons of paint is needed to paint a 40 sq meter wall, how many gallons of paint are needed to paint a wall with an area of 100 sq meters?
5. A teacher charges Php 500 per hour of tutorial service to a second year high school student. If she spends 3 hours tutoring per day, how much would she receive in 20 days?
6. A mailman can sort out 738 letters in 6 hours. If the number of sorted letters varies directly as the number of working hours, how many letters can be sorted out in 9 hours?
7. Jessie uses 20 liters of gasoline to travel 200 kilometers, how many liters of gasoline will he use on a trip of 700 kilometers?
8. Every 3 months, a man deposits in his bank account a savings of Php 5,000. In how many years would he have saved Php 250,000?
9. The pressure p at the bottom of a swimming pool varies directly as the depth d of the water. If the pressure is 125 Pascal when the water is 2 meters deep, find the pressure when it is 4.5 meters deep.
10. The shadow of an object varies directly as its height h. A man 1.8 m tall casts a shadow 4.32 m long. If at the same time a flagpole casts a shadow 12.8 m long, how high is the flagpole?

What to TRANSFER

Your goal in this section is to apply what you have learned to real-life situations. This shall be one of your group's outputs for the second quarter. A practical task shall be given to your group where each of you will demonstrate your understanding with clarity and accuracy, and further supported through refined mathematical justifications along with your project's stability and creativity. Your work shall be graded in accordance to a rubric prepared for this task.

➤ Activity 8: GRASPS

Create a scenario of the task in paragraph form incorporating GRASPS: Goal, Role, Audience, Situation, Product/Performance, Standards. A sample has been provided for you.

Sample of GRASP

Goal or task related to understanding: Your goal is to provide jobs for skilled people in the community.

Role: Project Manager

Audience: Barangay Officials

Situation or Context of Scenario: As an aspiring businessman, you have been commissioned by a client to supply them with 10 000 pieces of hanging decors with a Christmas motif. Given two months, your problem is to find skilled persons to help you meet the deadline. You have thought of hiring people from your community. If the profit is to be considered, the task is to find the number of workers that would be hired to generate maximum profit.

Product(s) or Performances for Assessment: Prepare and discuss to the Barangay Officials the scheme with the appropriate computation of the workers' wage per piece, the profit that would be earned by the Barangay from the project. The scheme should be presented using tables showing wages generated by each at a certain rate. The proposed scheme should include the amount generated by each after a certain period. Problems/issues associated to the project should be discussed. Show the necessary plan of action that would be taken to go about the problems/issues.

Standards for Assessment: Clarity, Accuracy, and Justification

RUBRICS FOR PLAN OF ACTION

Criteria	4	3	2	1	Rating
Clarity	The presentation is very clear, precise, and coherent. It included concepts related to direct variation.	The presentation is clear, precise, and coherent. It included concepts related to direct variation.	The presentation is vague but it included concepts related to direct variation.	The presentation is vague and did not include concepts related to direct variation.	
Accuracy	The computations are accurate and show the wise use of the key concepts of direct variation.	The computations are accurate and show the use of the key concepts of direct variation.	The computations are accurate and show some use of the key concepts of direct variation.	The computations are erroneous and do not show the use of the key concepts of direct variation.	
Justification	The purpose is well justified and shows the maximum and beneficial profit that will be gained from the project.	The purpose is well justified and shows the maximum profit that will be gained from the project.	The purpose is justified and shows minimal profit that will be gained from the project.	The purpose is not justifiable and shows no profit that will be gained from the project.	

Summary/Synthesis/Generalization:

➤ Activity 9: Wrap It Up!

On a sheet of paper, summarize what you have learned from this lesson. Provide real-life examples. Illustrate using tables, graphs, and mathematical equations showing the relation of quantities.

LESSON 2 Inverse Variation

What to KNOW

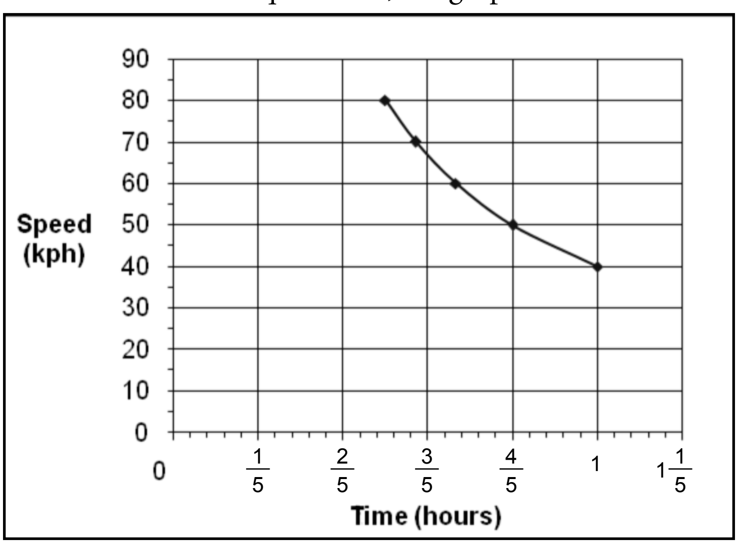
The activities on direct variation show you the behavior of the quantities involved. In one of the activities, an increase in the time travelled by a car causes an increase in the distance travelled. How will an increase in speed affect the time in travelling? Let us find out in the next activity.

➤ Activity 10:

1. Anna lives 40 km away from the office of ABC Corporation where she works. Driving a car, the time it takes her to reach work depends on her average speed. Some possible speeds and the length of time it takes her are as follows:

Time in hours	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$
Speed in kph	40	50	60	70	80

To see clearly the relation of the two quantities, the graph of the relation is shown below.



Questions:

a. How do the speed and time of travel affect each other?
b. Write a mathematical statement to represent the relation.
c. Is there a constant number involved? Explain the process that you have used in finding out.

The situation in the problem shows "an increase in speed produces a decrease in time in travelling." The situation produces pairs of numbers, whose product is constant. Here, the time t varies inversely as the speed s such that

$$st = 40 \text{ (a constant)}$$

In this situation, "the speed s is inversely proportional to the time t," and is written as $s = \frac{k}{t}$, where k is the proportionality constant or constant of variation. Hence, the equation represented in the table and graph is $s = \frac{40}{t}$; where $k = 40$.

2. Jean and Jericho who are playing in the school grounds decided to sit on a seesaw.

Jericho, who is heavier, tends to raise Jean on the other end of the seesaw. They tried to position themselves in order to balance the weight of each other.

Questions:

a. What have you noticed when the kids move closer to or farther from the center?

b. Who among the kids will have to move closer to the center in order to balance the seesaw?

c. How do the weights of the kids relate to the distance from the center?

d. Does the change in one quantity affect a change in the other? Explain.

➤ Activity 11: Observe and Compare

Consider the table of values A and B

Table A

x	-2	-1	1	2	3
y	-4	-2	2	4	6

Table B

x	80	60	40	30	20
y	6	9	12	16	24

Compare the two given table of values.

1. What have you observed about the values in both tables?
2. What do you observe about the values of y when x increases/decreases?
3. What happens to the values of y when x is doubled? tripled?
4. How do you compare the two relations?
5. Write the relationship which describes x and y.
6. How can you determine if the relationship is a direct variation or an inverse variation?

What to PROCESS

Let us now discuss the concepts behind the situations that you have encountered. These situations are examples of inverse variation.

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or $y = \frac{k}{x}$.

The statement, "y varies inversely to x," translates to $y = \frac{k}{x}$, where k is the constant of variation.

Examples:

1. Find the equation and solve for k: y varies inversely as x and $y = 6$ when $x = 18$.

   Solution:

   The relation y varies inversely as x translates to $y = \frac{k}{x}$. Substitute the values to find k:

$$y = \frac{k}{x}$$

$$6 = \frac{k}{18}$$

$$k = (6)(18)$$

$$k = 108$$

   The equation of variation is $y = \frac{108}{x}$

2. If y varies inversely as x and $y = 10$ when $x = 2$, find y when $x = 10$.

This concerns two pairs of values of x and y which may be solved in two ways.

Solution 1:

First, set the relation, and then find the constant of variation, k.

$$xy = k$$
$$(2)(10) = k$$
$$k = 20$$

The equation of variation is $y = \dfrac{20}{x}$

Next, find y when $x = 10$ by substituting the value of x in the equation,

$$y = \frac{20}{x}$$
$$y = \frac{20}{10}$$
$$y = 2$$

Solution 2:

Since $k = xy$, then for any pairs x and y, $x_1y_1 = x_2y_2$

If we let $x_1 = 2$, $y_1 = 10$, and $x_2 = 10$, find y_2.

By substitution,

$$x_1y_1 = x_2y_2$$
$$2(10) = 10(y_2)$$
$$20 = 10y_2$$
$$y_2 = \frac{20}{10}$$
$$y_2 = 2$$

Hence, $y = 2$ when $x = 10$.

➤ Activity 12: It's Your Turn!

Let us now test your skills in translating statements into mathematical equations and in finding the constant of variation.

A. Express each of the following statements as a mathematical equation.

1. The number of pizza slices p varies inversely as the number of persons n sharing a whole pizza.
2. The number of pechay plants n in a row varies inversely as the space s between them.
3. The number of persons n needed to do a job varies inversely as the number of days d to finish the job.

4. The rate r at which a person types a certain manuscript varies inversely as the time t spent in typing.
5. The cost c per person of renting a private resort varies inversely as the number n of persons sharing the rent.
6. The length l of a rectangular field varies inversely as its width w.
7. The density d of air varies inversely as the volume v of water in the atmosphere.
8. The acceleration a of a car is inversely proportional to its mass m.
9. The base b of a triangle varies inversely as its altitude h.
10. The mass m of an object varies inversely as the acceleration due to gravity g.

B. Find the constant of variation and write the equation representing the relationship between the quantities in each of the following:

1.

x	1	2	3	4
y	2	1	$\frac{2}{3}$	$\frac{1}{2}$

2.

x	2	4	6	8
y	36	18	12	9

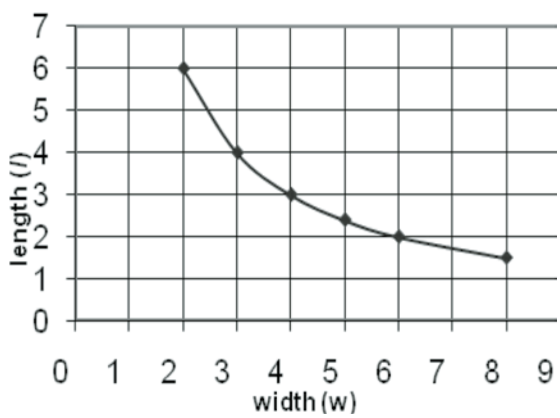
3.

x	3	6	9	12
y	2	1	$\frac{2}{3}$	$\frac{1}{2}$

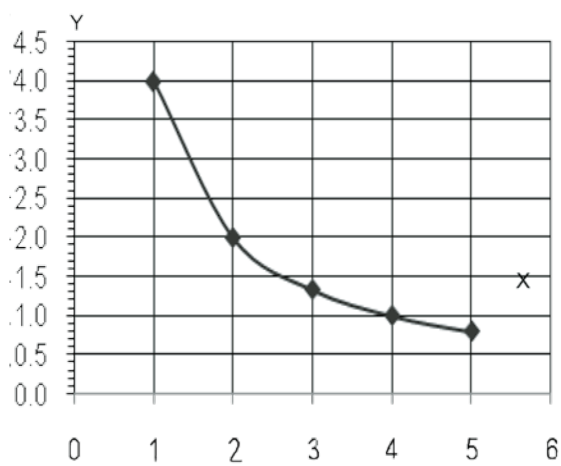
4.

x	7	5	3	1
y	$\frac{5}{7}$	1	$\frac{5}{3}$	5

5.



6.



7. y varies inversely as x and $y = 12$ when $x = 5$.
8. y varies inversely as x and $y = 18$ when $x = 3$.
9. y varies inversely as x and $y = 10$ when $x = \frac{2}{5}$.
10. y varies inversely as x and $y = 12$ when $x = \frac{2}{3}$.

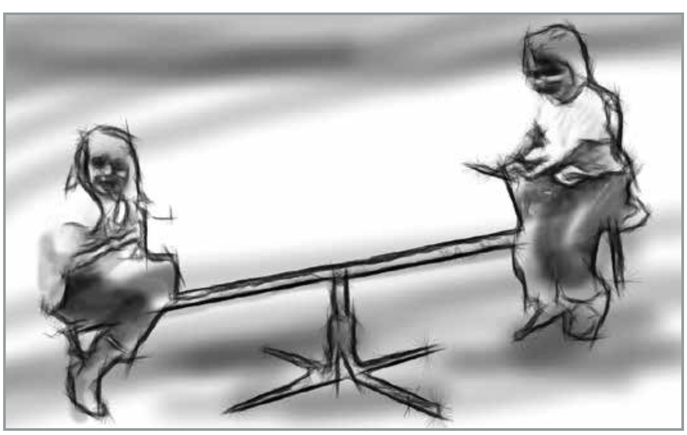
C. Solve for the indicated variable in each of the following.

1. If y varies inversely as x and $y = 3$ when $x = 4$, find y when $x = 6$.
2. If r varies inversely as s and $r = 100$ when $s = 27$, find the value of r when $s = 45$.
3. If p varies inversely as the square of q and $p = 3$ when $q = 4$, find p when $q = 16$.
4. If y varies inversely as x and $y = -2$ when $x = -8$, find x when $y = 2$.
5. If w varies inversely as y and $w = 2$ when $y = 3$, find w when $y = 6$.
6. If m varies inversely as n and $m = 8$ when $n = 3$, find m when $n = 12$.
7. If y varies inversely as x, and y is 10 when $x = 5$, find y when $x = 7$.
8. If a varies inversely as b and $a = 12$ when $b = 8$, find a when $b = 6$.
9. If w varies inversely as x and w is 10 when $x = 5$ find w when $x = 25$.
10. If y varies inversely as x and $y = 10$ when $x = 5$, find y when $x = 15$.

What to REFLECT AND UNDERSTAND

Having developed your knowledge of the concepts in this section, your goal now is to apply these concepts in various situations. As in the previous activities, there will be no guide questions for you to follow. The situation below is an example of how the problem is solved.

Jamie and Andrea are figuring out a way to balance themselves on a seewaw. Jamie who weighs 15 kilograms sits 2 meters from the fulcrum. Andrea who weighs 20 kilograms tried sitting at different distances from the fulcrum in order to balance the weight of Jamie. If you were Andrea, how far from the fulcrum should you sit?



To balance the weight of Jamie, Andrea has to sit at a distance closer to the fulcrum.

The relation shows that the distance d varies inversely as the weight w and can be transformed into a mathematical equation as

$$d = \frac{k}{w}$$

We can now solve for distance from the fulcrum where Andrea has to sit.

Solution:

Let us first solve for k.

$$k = dw$$
$$= 15(2)$$
$$k = 30$$

Solve for the distance of Andrea from the fulcrum.

$$d = \frac{k}{w}$$
$$= \frac{30}{20}$$
$$d = 1.5\text{m}$$

Hence, Andrea has to sit 1.5 meters from the fulcrum.

➤ Activity 13: Think Deeper!

Solve the following problems.

1 The number of days needed in repairing a house varies inversely as the number of men working. It takes 15 days for 2 men to repair the house. How many men are needed to complete the job in 6 days?

2. Two college students decided to rent an apartment near the school where they are studying. The nearest and cheapest apartment costs Php 5,000 a month, which they find too much for their monthly budget. How many students will they need to share the rent with so that each will pay only Php 1,250 a month?
3. At 60 kilometers per hour it takes Loida 10 hours to travel from her house to their house in the province. How long will it take her if she travels at 80 kilometers an hour?
4. The ten administrative staff of a school decided to tour the nearby towns of Laguna on a particular day. They decided to hire a van for Php 4,000. Two days before the trip, two of the staff were assigned to attend a seminar and so could no longer join the trip. How much would each share for the van rental?
5. Joseph is figuring out a way to reach Baguio at the shortest possible time. Using his car, he can reach Baguio in 6 hours at an average speed of 70 km per hour. How fast should he drive in order to reach Baguio in 5 hours?

Perform Activity 14 and check if your answers are the same as your answers in Activity 1. This will gauge how well you understood the discussions on direct and inverse variations.

➤ Activity 14: After Lesson Response

Read the phrases found at the right column in the table below. If the phrase is a direct variation, place a letter **D** in the After Lesson Response column, if it is an inverse variation, place a letter **I**. If the relationship is neither direct nor inverse variation, mark it with a letter **N**.

After Lesson Response	Phrase
	1. The number of hours to finish a job to the number of men working
	2. The amount of water to the space that water did not occupy in a particular container
	3. The number of persons sharing a pie to the number of slices of the pie
	4. The area of the wall to the amount of paint used to cover it
	5. The time spent in walking to the rate at which a person walks
	6. The time a teacher spends checking papers to the number of students
	7. The cost of life insurance to the age of the insured person
	8. The age of a used car to its resale value
	9. The amount of money raised in a concert to the number of tickets sold
	10. The distance an airplane flies to the time travelling

What to TRANSFER

You are to demonstrate your understanding of inverse variations through culminating activities that reflect meaningful and relevant problems/situations.

➤ Activity 15: Demonstrate Your Understanding!

Create a scenario of the task in paragraph form incorporating GRASP: Goal, Role, Audience, Situation, Product/Performance, Standards

G: Make a criteria for the best project related to the protection of the environment

R: Student

A: School Officials

S: The school will grant an award to the best project that promotes protection of the environment.

P: Criteria

S: Justification, Accuracy of Data, Clarity of Presentation

Summary/Synthesis/Generalization:

➤ Activity 16: Wrap It Up!

On a sheet of paper, summarize what you have learned from this lesson. Provide real-life examples. Illustrate using tables, graphs, and mathematical equations showing the relationship of quantities.

LESSON 3 Joint Variation

What to KNOW

The situations that you have studied involved only two quantities. What if the situation requires the use of more than two quantities?

This lesson deals with another concept of variation, the joint variation.

Some physical relationships, as in area or volume, may involve three or more variables simultaneously.

You will find out about this type of variation as we go along with the discussions.

What to PROCESS

At this phase, you will be provided with examples that will lead you in solving the problems you will encounter in this section. The concept of joint variation will help you deal with problems involving more than two variables or quantities.

The statement "a varies jointly as b and c" means $a = kbc$, or $k = \frac{a}{bc}$, where k is the constant of variation.

Examples:

1. Find the equation of variation where a varies jointly as b and c, and $a = 36$ when $b = 3$ and $c = 4$.

   Solution:

   $a = kbc$

   $36 = k(3)(4)$

   $k = \frac{36}{12}$

   $k = 3$

   Therefore, the required equation of variation is $a = 3bc$

2. z varies jointly as x and y. If $z = 16$ when $x = 4$ and $y = 6$, find the constant of variation and the equation of the relation.

Solution:

$$z = kxy$$

$$16 = k(4)(6)$$

$$k = \frac{16}{24}$$

$$k = \frac{2}{3}$$

The equation of variation is $z = \frac{2}{3}xy$

➤ Activity 17: What Is Joint Together?

A. Translate each statement into a mathematical sentence. Use k as the constant of variation.

1. P varies jointly as q and r.
2. V varies jointly as l, w, and h.
3. The area A of a parallelogram varies jointly as the base b and altitude h.
4. The volume of a cylinder V varies jointly as its height h and the square of the radius r.
5. The heat H produced by an electric lamp varies jointly as the resistance R and the square of the current i.
6. The force F applied to an object varies jointly as the mass m and the acceleration a.
7. The volume V of a pyramid varies jointly as the area of the base B and the altitude h.
8. The area A of a triangle varies jointly as the base b and the altitude h.
9. The appropriate length s of a rectangular beam varies jointly as its width w and its depth d.
10. The electrical voltage V varies jointly as the current I and the resistance R.

B. Solve for the value of the constant of variation k, then find the missing value.

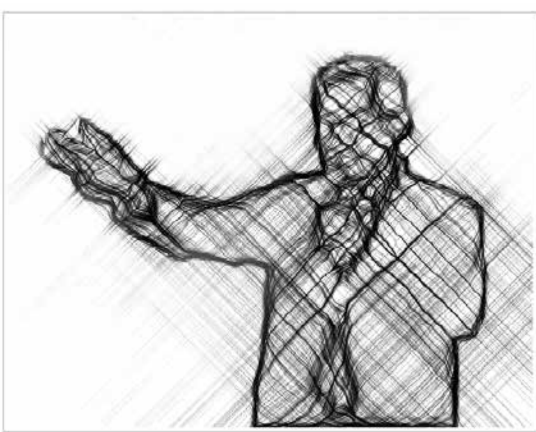
1. z varies jointly as x and y and $z = 60$ when $x = 5$ and $y = 6$.
   a. Find z when $x = 7$ and $y = 6$.
   b. Find x when $z = 72$ and $y = 4$.
   c. Find y when $z = 80$ and $x = 4$.
2. z varies jointly as x and y. If $z = 3$ when $x = 3$ and $y = 15$, find z when $x = 6$ and $y = 9$.
3. z varies jointly as the square root of the product of x and y. If $z = 3$ when $x = 3$ and $y = 12$, find x when $z = 6$ and $y = 64$.
4. d varies jointly as h and g. If $d = 15$ when $h = 14$ and $g = 5$, find g when $h = 21$ and $d = 8$.

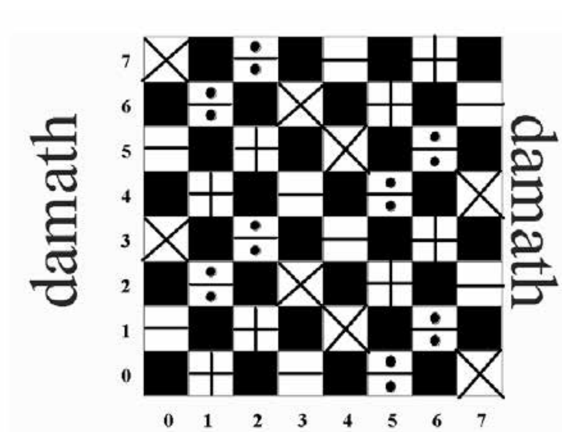
5. q varies jointly as r and s. If $q = 2.4$, when $r = 0.6$ and $s = 0.8$, find q when $r = 1.6$ and $s = .01$.
6. d varies jointly as e and l. If $d = 1.2$, when $e = 0.3$ and $l = 0.4$, find d when $e = 0.8$ and $l = .005$.
7. x varies jointly as w, y, and z. If $x = 18$, when $w = 2$, $y = 6$ and $z = 5$, find x when $w = 5$, $y = 12$ and $z = 3$.
8. z varies jointly as x and y. $z = 60$ when $x = 3$ and $y = 4$. Find y when $z = 80$ and $x = 2$.
9. The weight W of a cylindrical metal varies jointly as its length l and the square of the diameter d of its base.
   a. If $W = 6$ kg when $l = 6$ cm and $d = 3$ cm, find the equation of variation.
   b. Find l when $W = 10$ kg and $d = 2$ cm.
   c. Find W when $d = 6$ cm and $l = 1.4$ cm.
10. The area A of a triangle varies jointly as the base b and the altitude h of the triangle. If $A = 65\text{ cm}^2$ when $b = 10$ cm and $h = 13$ cm, find the area of a triangle whose base is 8 cm and whose altitude is 11 cm.

WHAT to REFLECT and UNDERSTAND

Having developed your knowledge on the concepts in the previous activities, your goal now is to apply these concepts in various real-life situations.

➤ Activity 18: Who Is This Filipino Inventor?





He is a Filipino mathematician who developed a board game called DAMATHS. The board game applies the moves used in the Filipino board game DAMA to solve problems on the different concepts in Mathematics. Who is he?

To find out, match the letter that corresponds to the answer to the numbered item on your left. The letters will spell out the name of this Filipino Mathematician.

"z varies jointly as x and y"

1. Translate into variation statement
2. If $z = 36$ when $x = 3$ and $y = 2$, find k.
3. x is 4 when $y = 3$ and $z = 2$. What is z if $x = 8$ and $y = 6$.

The area a of a triangle varies jointly as its base b and height h.

4. Express the relation as a variation statement.
5. If the area is 15 cm^2 when the base is 5 cm and the height is 6 cm, find k.
6. If $a = 65$ cm^2 when $b = 10$ cm and $h = 13$ cm, find a when $b = 8$ cm and $h = 11$ cm.

E	8
B	$a = \frac{kb}{h}$
D	$\frac{1}{2}$
F	$z = \frac{kx}{y}$
U	6
N	$a = kbh$
A	44 cm^2
H	$z = kxy$

Answer: ___ ___ ___ ___ ___ ___
1 2 3 4 5 6

➤ Activity 19: Think Deeper!

Solve the following:

1. The amount of gasoline used by a car varies jointly as the distance travelled and the square root of the speed. Suppose a car used 25 liters on a 100 kilometer trip at 100 km/hr. About how many liters will it use on a 192 kilometer trip at 64 km/hr?
2. The area of triangle varies jointly as the base and the height. A triangle with a base of 8 cm and a height of 9 cm has an area of 36 square centimeters. Find the area when the base is 10 cm and the height is 7 cm.
3. The volume of a right circular cylinder varies jointly as the height and the square of the radius. The volume of a right circular cylinder, with radius 4 centimeters and height 7 centimeters, is 112 π cm^3. Find the volume of another cylinder with radius 8 centimeters and height 14 centimeters.
4. The mass of a rectangular sheet of wood varies jointly as the length and the width. When the length is 20 cm and the width is 10 cm, the mass is 200 grams. Find the mass when the length is 15 cm and the width is 10 cm.
5. The weight of a rectangular block of metal varies jointly as its length, width and thickness. If the weight of a 13 dm by 8 dm by 6 dm block of aluminum is 18.2 kg, find the weight of a 16 dm by 10 dm by 4 dm block of aluminum.

What to TRANSFER

Journal writing and portfolio of real-life situations/pictures where concepts of joint variation are applied.

Summary/Synthesis/Generalization:

➤ Activity 20: Wrap It Up!

On a sheet of paper, summarize what you have learned from this lesson. Provide real-life examples. Illustrate situations using variation statements and mathematical equations to show the relationship of quantities.

LESSON 4 Combined Variation

What to KNOW

Combined variation is another physical relationship among variables. This is the kind of variation that involves both the direct and inverse variations.

What to PROCESS

The statement "z varies directly as x and inversely as y" means $z = \frac{kx}{y}$, or $k = \frac{zy}{x}$, where k is the constant of variation.

This relationship among variables will be well illustrated in the following examples.

Examples:

1. Translating statements into mathematical equations using k as the constant of variation.

   a. T varies directly as a and inversely as b.

   $$T = \frac{ka}{b}$$

   b. Y varies directly as x and inversely as the square of z.

   $$Y = \frac{kx}{z^2}$$

   The following is an example of combined variation where one of the terms is unknown.

2. If z varies directly as x and inversely as y, and $z = 9$ when $x = 6$ and $y = 2$, find z when $x = 8$ and $y = 12$.

   Solution:

   The equation is $z = \frac{kx}{y}$

   Solve for k by substituting the first set of values of z, x, and y in the equation.

   $$z = \frac{kx}{y}$$

   $$9 = \frac{6k}{2}$$

   $$k = \frac{9}{3}$$

   $$k = 3$$

Now, solve for z when $x = 8$ and $y = 12$.

Using the equation $z = \dfrac{3x}{y}$,

$$z = \frac{(3)(8)}{12}$$

$$z = 2$$

You may use this example to guide you in solving the activities that follow.

➤ Activity 21: DV and IV Combined!

A. Using k as the constant of variation, write the equation of variation for each of the following.

1. W varies jointly as c and the square of a and inversely as b.
2. P varies directly as the square of x and inversely as s.
3. The electrical resistance R of a wire varies directly as its length l and inversely as the square of its diameter d.
4. The acceleration A of a moving object varies directly as the distance d it travels and inversely as the square of the time t it travels.
5. The pressure P of a gas varies directly as its temperature t and inversely as its volume V.

B. Solve the following

1. If r varies directly as s and inversely as the square of u, and $r = 2$ when $s = 18$ and $u = 2$, find:
   a. r when $u = 3$ and $s = 27$.
   b. s when $u = 2$ and $r = 4$.
   c. u when $r = 1$ and $s = 36$.
2. p varies directly as q and the square of r and inversely as s.
   a. Write the equation of the relation.
   b. Find k if $p = 40$ when $q = 5$, $r = 4$ and $s = 6$.
   c. Find p when $q = 8$, $r = 6$ and $s = 9$.
   d. Find s when $p = 10$, $q = 5$ and $r = 2$.
3. w varies directly as x and y and inversely as v^2 and $w = 1200$ when $x = 4$, $y = 9$ and $v = 6$. Find w when $x = 3$, $y = 12$, and $v = 9$.
4. Suppose p varies directly as b^2 and inversely as s^3. If $p = \dfrac{3}{4}$ when $b = 6$ and $s = 2$, find b when $p = 6$ and $s = 4$.
5. If x varies as the square of y and inversely as z and $x = 12$ when $y = 3$ and $z = 6$, find x when $y = 9$ and $z = 6$.

What to REFLECT AND UNDERSTAND

Having developed your knowledge of the concepts in the previous activities, your goal now is to apply these concepts in various real-life situations.

➤ Activity 22: How Well Do You Understand?

Solve the following:

1. The current I varies directly as the electromotive force E and inversely as the resistance R. If in a system a current of 20 amperes flows through a resistance of 20 ohms with an electromotive force of 100 volts, find the current that 150 volts will send through the system.
2. The force of attraction, F of a body varies directly as its mass m and inversely as the square of the distance d from the body. When $m = 8$ kilograms and $d = 5$ meters, $F = 100$ Newtons. Find F when $m = 2$ kilograms and $d = 15$ meters.
3. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If a wire 30 meters long and 0.75 mm in diameter has a resistance of 25 ohms, find the length of a wire of the same material whose resistance and diameter are 30 ohms and 1.25 mm, respectively.
4. The acceleration a of an object varies directly as the force f exerted and inversely as its mass m. If the constant of variation is 1, find the acceleration in meters/second^2 of a 10 kg object exerting a force of 10 Newtons.
5. The maximum load m of a beam varies directly as the breadth b and the square of the depth d and inversely as the length l. If a beam 3.6 m long, 1.2 m wide, and 2.4 m deep can safely bear a load up to 909 kg, find the maximum safe load for a beam of the same material which is 3 m long, 6 m wide, and 1.8 m deep.

What to TRANSFER

Design a plan how to market a particular product considering the number of units of the product sold, cost of the product, and the budget for advertising. The number of units sold varies directly with the advertising budget and inversely as the price of each product.

Summary/Synthesis/Generalization:

➤ Activity 23: Wrap It Up?

On a sheet of paper, summarize what you have learned from this lesson. Provide real-life examples. Illustrate using variation statements and mathematical equations showing the relation of quantities.

Glossary of Terms

Direct Variation

There is *direct variation* whenever a situation produces pairs of numbers in which their ratio is constant.

The statements:

"y varies directly as x"

"y is directly proportional to x" and

"y is proportional to x"

are translated mathematically as $y = kx$, where k is the constant of variation.

For two quantities x and y, an increase in x causes an increase in y as well. Similarly, a decrease in x causes a decrease in y.

Inverse Variation

Inverse variation occurs whenever a situation produces pairs of numbers whose product is constant.

For two quantities x and y, an increase in x causes a decrease in y or vice versa. We can say that y varies inversely as x or $y = \frac{k}{x}$.

Joint Variation

The statement a varies jointly as b and c means $a = kbc$, or $k = \frac{a}{bc}$, where k is the constant of variation.

Combined Variation

The statement t varies directly as x and inversely as y means $t = \frac{kx}{y}$, or $k = \frac{ty}{x}$, where k is the constant of variation.

References

DepEd Learning Materials that can be used as learning resources for the lesson on Variation

a. EASE Modules Year II
b. Distance Learning Modules Year II

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