9

Mathematics

Learner's Material

Module 6:
Similarity

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Republic of the Philippines

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MODULE 6

Similarity

I. INTRODUCTION AND FOCUS QUESTIONS

Is there a way we can measure tall structures and difficult-to-obtain lengths without using direct measurement? How are sizes of objects enlarged or reduced? How do we determine distances between two places using maps? How do architects and engineers show their clients how their projects would look like even before they are built?

In short, how do concepts of similarity of objects help us solve problems related to measurements? You would be able to answer this question by studying this module on similarity in geometry.

II. LESSONS AND COVERAGE

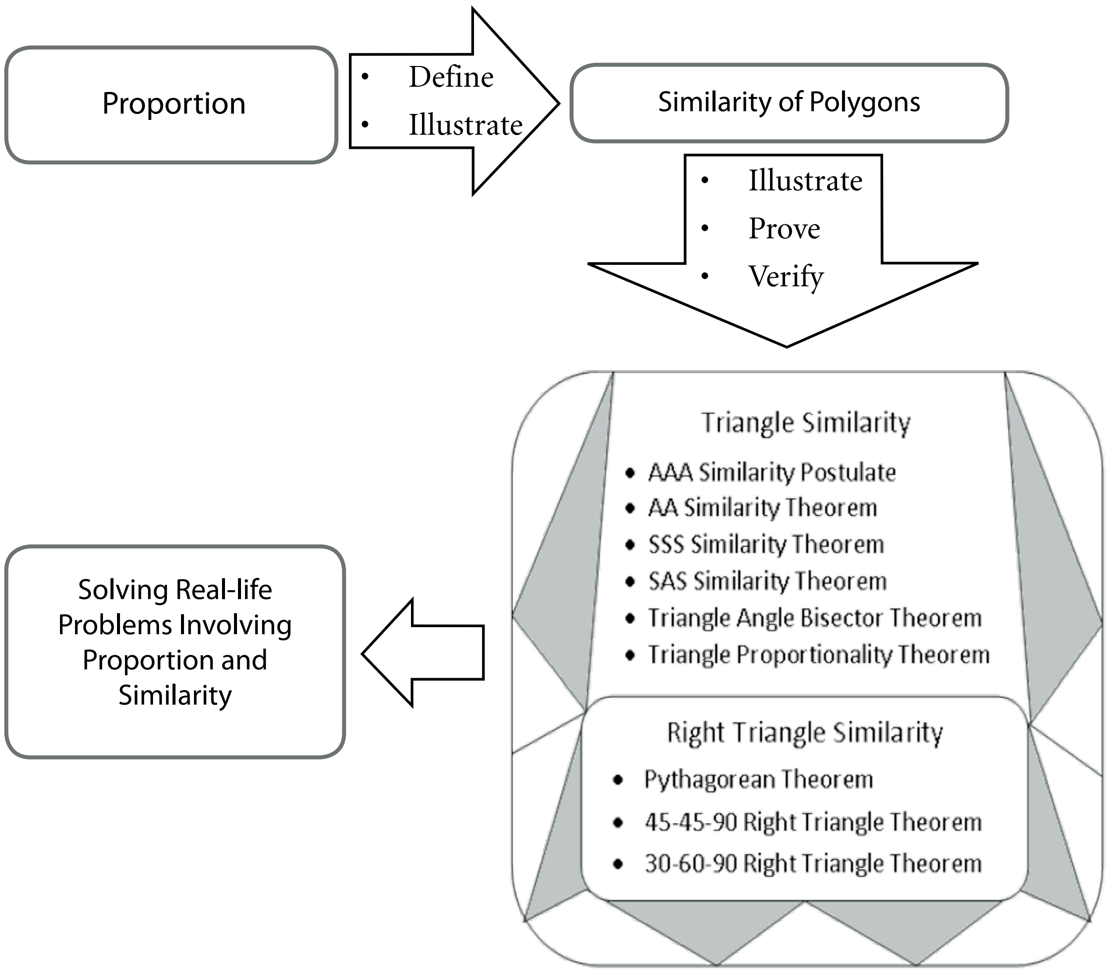
In this module, you will examine this question when you take this lesson on similarity.

In this lesson, you will learn to:

- describe a proportion
- illustrate similarity of polygons
- prove the conditions for
  - similarity of triangles
    a. AA Similarity Theorem
    b. SAS Similarity Theorem
    c. SSS Similarity Theorem
    d. Triangle Angle Bisector Theorem
    e. Triangle Proportionality Theorem
  - similarity of right triangles
    a. Right Triangle Similarity Theorem
    b. Pythagorean Theorem
    c. 45-45-90 Right Triangle Theorem
    d. 30-60-90 Right Triangle Theorem
- apply the theorems to show that triangles are similar
- apply the fundamental theorems of proportionality to solve problems involving proportions
- solve problems that involve similarity

Module Map

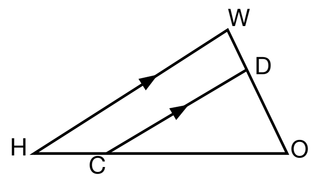
Here is a simple map of the lesson that will be covered in this module.



III. PRE-ASSESSMENT

Let's find out how much you already know about this topic. On a separate sheet, write only the letter of the choice that you think best answers the question. Please answer all items. During the checking, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. $\Delta COD \sim \Delta HOW$ Because $\overline{CD \parallel HW}$, which of the following is not true?



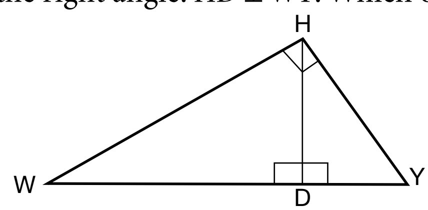
a. $\frac{OD}{DW} = \frac{OC}{CH} = \frac{CD}{HW}$

b. $\frac{OD}{OW} = \frac{OC}{OH} = \frac{CD}{HW}$

c. $\frac{DW}{OW} = \frac{CH}{OH} = \frac{HW - CD}{HW}$

d. $\frac{OD}{DW} = \frac{OC}{CH} = \frac{CD}{HW - CD}$

2. ΔWHY is a right triangle with $\angle WHY$ as the right angle. $\overline{HD} \perp \overline{WY}$. Which of the following segments is a geometric mean?



I. $\overline{HD}$ IV. $\overline{DW}$

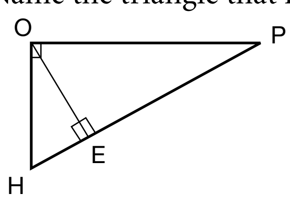
II. $\overline{DY}$ V. $\overline{HW}$

III. $\overline{HY}$ VI. $\overline{WY}$

a. II, IV, VI c. I only

b. I, III, V d. All except VI

3. In the figure, there are three similar right triangles by Right Triangle Proportionality Theorem. Name the triangle that is missing in this statement: $\Delta HOP \sim$ __________ ΔOEP.



a. ΔHOE c. ΔHOP

b. ΔOEH d. ΔHEO

4. If m:n = 3:2, what is the correct order of the steps in determining $m^2 - n^2 : m^2 - 2n^2$?

I. $m = 3k;\ n = 2k$

II. $m^2 - n^2 : m^2 - 2n^2 = 5:1$

III. $\frac{(3k)^2 - (2k)^2}{(3k)^2 - 2(2k)^2}$

IV. $\frac{m}{3} = \frac{n}{2} = k$

a. I, IV, III, II c. I, IV, II, III

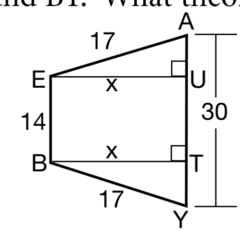
b. IV, I, III, II d. I, III, II, I

5. The ratio of the volumes of two similar rectangular prisms is 125 : 64. What is the ratio of their base areas?

a. 25:16 c. 4:5

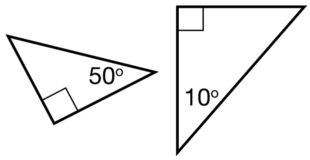
b. 25:4 d. 5:4

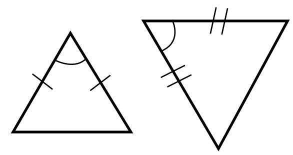
6. The lengths of the sides of a triangle are 6 cm, 10 cm, and 13 cm. What kind of a triangle is it?
   a. Regular Triangle
   b. Acute Triangle
   c. Right Triangle
   d. Obtuse Triangle
7. What is the perimeter of a 30-60-90 triangle whose shorter leg is 5 inches long?
   a. $5\sqrt{3}$ cm
   b. $15 + 5\sqrt{3}$ cm
   c. $15 + \sqrt{3}\ cm$
   d. $10 + 5\sqrt{3}$ cm
8. The hypotenuse of an isosceles right trapezoid measures 7 cm. How long is each leg?
   a. $7\sqrt{2}$ cm
   b. 3.5 cm
   c. $\frac{7\sqrt{2}}{2}$ cm
   d. $\frac{7\sqrt{3}}{3}$ cm
9. Study the proof in determining the congruent lengths EU and BT. What theorem justifies the last statement?
   a. Right Triangle Proportionality Theorem
   b. Geometric Mean
   c. Pythagorean Theorem
   d. Triangle Angle Bisector

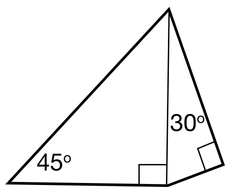


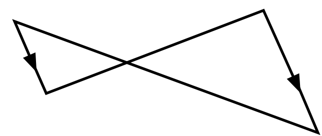
Statement	Reasons
$EA \perp YA$; $BT \perp YA$; $EU = BT$; $BY = EA$	Given
∠EUA, ∠EUT, ∠BTU, ∠BTY are right angles.	Definition of Perpendicular Lines
m ∠EUA = m ∠EUT = m ∠BTU = m ∠BTY = 90	Definition of Right Angles
$\overline{EU} \parallel \overline{BT}$	Corresponding angles EUA and BTU are congruent.
BEUT is a parallelogram	$\overline{EU}$ and $\overline{BT}$ are both parallel and congruent.
BE=TU	Opposite side of a parallelogram are congruent.
YT + TU + UA + = YA	Segment Addition Postulate
YT + BE + UA = 30	Substitution Property of Equality
△EUA and △BTY are right triangles.	Definition of Right Triangles
△EUA ≅ △BTY	Hypotenuse-Leg Right Triangle Congruence Theorem
	Substitution Property of Equality
YT + 14 + YT = 30	Subtraction Property of Equality
$\frac{2YT}{2} = \frac{16}{2} \rightarrow YT = 8$	Division Property of Equality
EU = BT = 15	?

10. Which of the following pairs of triangles cannot be proved similar?

a. 

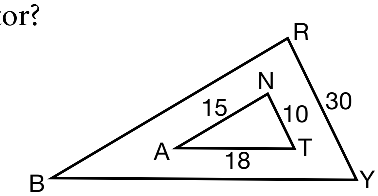
c. 

b. 

d. 

11. The ratio of the sides of the original triangle to its enlarged version is 1 : 3. The enlarged triangle is expected to have
    a. sides that are thrice as long as the original
    b. an area that is thrice as large as the original
    c. sides that are one-third the lengths of the original
    d. angles that are thrice the measurement of the original

12. $\triangle BRY \sim \triangle ANT$ .Which ratio of sides gives the scale factor?



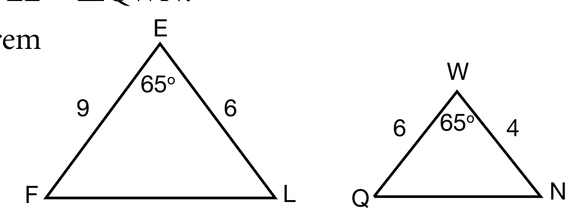
a. $\frac{NT}{AN}$

c. $\frac{AT}{BY}$

b. $\frac{NT}{RY}$

d. $\frac{NT}{AT}$

13. What similarity concept justifies that $\triangle FEL \sim \triangle QWN$?



a. Right Triangle Proportionality Theorem
b. Triangle Proportionality Theorem
c. SSS Similarity Theorem
d. SAS Similarity Theorem

14. A map is drawn to the scale of 1 cm : 150 m. If the distance between towns A and B measures 8. 5 cm on the map, determine the approximate distance between these towns.

a. 2175 m

c. 1275 m

b. 1725 m

d. 2715 m

15. The length of the shadow of your one-and-a-half-meter height is 2.4 meters at a certain time in the morning. How high is a tree in your backyard if the length of its shadow is 16 meters?

a. 25.6 m

c. 38.4 m

b. 10 m

d. 24 m

16. The smallest square of the grid you made on your original picture is 6 cm. If you enlarge the picture on a 15-cm grid, which of the following is not true?

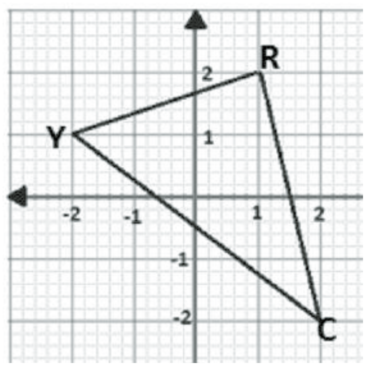
    I. The new picture is 250% larger than the original one.

    II. The new picture is two and a half time larger than the original one.

    III. The scale factor between the original and the enlarged picture is 2:5.

    a. I only   b. I and II   c. III only   d. I, II and III

For Nos. 17 and 18, use the figure shown.



17. You would like to transform △YRC by dilation such that the center of dilation is the origin and the scale factor is $\frac{1}{2}$. Which of the following is not the coordinates of a vertex of the reduced triangle?

    a. $(-1, 1)$ c. $(1, -1)$

    b. $\left(1, \frac{1}{2}\right)$ d. $\left(\frac{1}{2}, 1\right)$

18. You also would like to enlarge △YRC. If the corresponding point of C in the new triangle △Y'R'C' has coordinates (4, -4), what scale factor do you use?

    a. 4   c. 2

    b. 3   d. 1

19. A document is 80% only of the size of the original document. If you were tasked to convert this document back to its original size, what copier enlargement settings will you use?

    a. 100%   c. 120%

    b. 110%   d. 125%

20. You would like to put a 12 ft by 10 ft concrete wall division between your dining room and living room. How many 4-inch thick concrete hollow blocks (CHB) do you need for the concrete division? Note that:

Clue 1: the dimension of the face of CHB is 6 inches by 8 inches

Clue 2: 1 foot = 12 inches

Clue 3: $\frac{\text{1 CHB}}{\text{Area of the face of CHB in sq. in.}} = \frac{\text{total no. of CHB needed}}{\text{Area of the wall division in sq. in.}}$

    a. 300 pieces   c. 316 pieces

    b. 306 pieces   d. 360 pieces

What to KNOW

Let's start the module by doing two activities that will uncover your background knowledge on similarity.

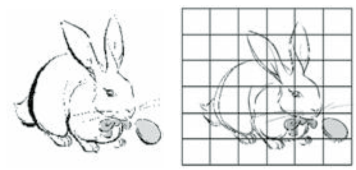
➤ Activity 1: My Decisions Now and Then Later

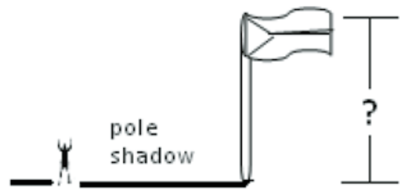
1. Replicate the table below on a piece of paper.
2. Under the my-decision-now column of the table, write **A** if you agree with the statement and **D** if you don't.
3. After tackling the whole module, you will be responding to the same statements under the My Decision-later column.

	Statement	My Decision	
		Now	Later
1	A proportion is an equality of ratios.		
2	When an altitude is drawn to the hypotenuse of a given right triangle, the new figure comprises two similar right triangles.		
3	The Pythagorean Theorem states that the sum of the squares of the legs of a right triangle is equal to the square of its hypotenuse.		
4	Polygons are similar if and only if all their corresponding sides are proportional.		
5	If the scale factor of similar polygons is m:n, the ratios of their areas and volumes are $m^2 : n^2$ and $m^3 : n^3$, respectively.		
6	The set of numbers {8, 15, and 17} is a Pythagorean triple.		
7	The hypotenuse of a 45-45-90 right triangle is twice the shorter leg.		
8	Scales are ratios expressed in the form 1:n.		
9	If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.		
10	Two triangles are similar if two angles of one triangle are congruent to two angles of another triangle.		

➤ Activity 2: The Strategy: Similarity!

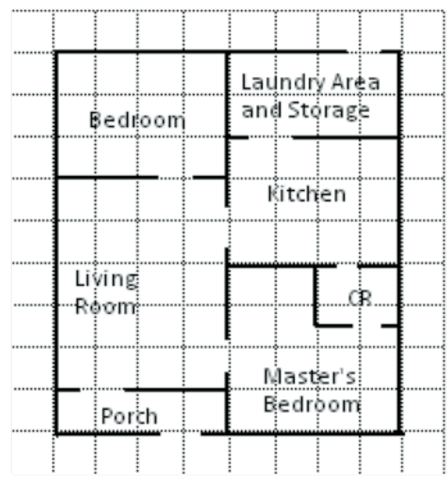
Study the pictures and share your insights about the corresponding questions.

<table>
<tr><th>Blow up my pet, please!</th><th>Tell me the height, please!</th></tr>
<tr><td>


What strategy will you use to enlarge or reduce the size of the original rabbit in this drawing?
</td><td>


Do you know how to find the height of your school's flagpole without directly measuring it?
</td></tr>
<tr><th>Tell me how far, please!</th><th>My Practical Dream House, what's yours?</th></tr>
<tr><td>


http://www.openstrusmap.org/#map=17/14.61541/120.998883

What is the approximate distance of Ferdinand Blumentritt Street from Cavite Junction to the Light Rail Transit Line 1?
</td><td>


What is the total lot area of the house and the area of its rooms given the scale **0.5 cm : 1 m**
</td></tr>
</table>

Are you looking forward to the idea of being able to measure tall heights and far distances without directly measuring them? Are you wondering how you can draw a replica of an object such that it is enlarged or reduced proportionately and accurately to a desired size? Are you excited to make a floor plan of your dream house? The only way to achieve all these is by doing all the activities in this module. It is a guarantee that with focus and determination, you will be able to answer this question: *How useful are the concepts of similarity of objects in solving measurement-related problems?*

The next lesson will also enable you to do the final project that requires you to draw the floor plan of a house and make a rough estimate of the cost of building it based on the current prices of construction materials. Your output and its justification will be rated according to these rubrics: accuracy, creativity, resourcefulness, and mathematical justification.

What to PR0CESS

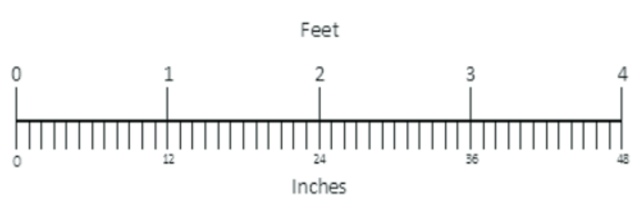
In this section, you will use the concepts and skills you have learned in the previous grades on ratio and proportion and deductive proof. You will be amazed with the connections between algebra and geometry as you will illustrate or prove the conditions of principles involving similarity of figures, especially triangle similarity. You will also realize that your success in writing proofs involving similarity depends upon your skill in making accurate and appropriate representation of mathematical conditions. In short, this section offers an exciting adventure in developing your logical thinking and reasoning—21st century skills that will prepare you to face challenges in future endeavors in higher education, entrepreneurship or employment.

➤ Activity No. 3: Let's Be Fair – Proportion Please!

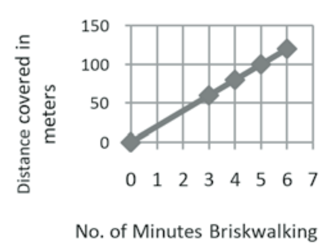
Ratio is used to compare two or more quantities. Quantities involved in ratio are of the same kind so that ratio does not make use of units. However, when quantities are of different kinds, the comparison of the quantities that consider the units is called ***rate***.

The figures that follow show ratios or rates that are proportional. Study the figures and complete the table that follows by indicating proportional quantities on the appropriate column. Two or more proportions can be formed from some of the figures. Examples are shown for your guidance.

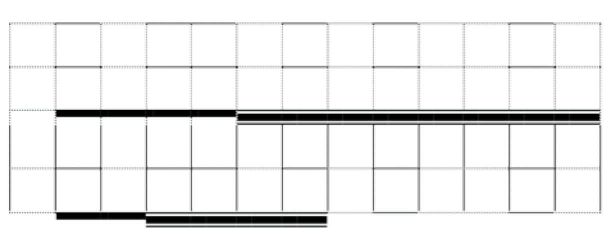
A.



C.



B.



D.

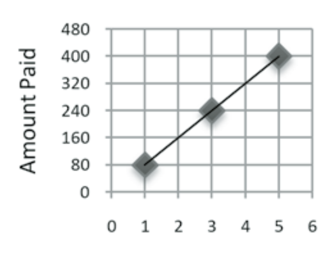


Fig.	Ratios or Rates	Proporional Quantities
A	*Feet* : *Inches*	$3\ ft : 36\ in = 4\ ft : 48\ in.$
B	*Shorter Segment* : *Thicker Segment*	
C	*Minutes* : *Meters*	$3\ min : 60\ m = 6\ min : 120\ m$
D	*Kilograms of Mango* : *Amount Paid*	$1 : 80\ pesos = 3 : 240\ pesos.$

Let us verify the accuracy of determined proportions by checking the equality of the ratios or rates. Examples are done for you. Be reminded that the objective is to show that the ratios or rates are equivalent. Hence, solutions need not be in the simplest form.

	Proportional Quantities	Checking the equality of ratios or rates in the cited proportions
A	$3\ ft. : 36\ in. = 4\ ft. : 48\ in.$	**Solution 1: Simplifying Ratios** $\frac{3}{36} ? \boxed{\frac{4}{48}} \rightarrow \frac{3}{3(12)} ? \frac{4}{4(12)} \rightarrow \frac{1}{12} = \boxed{\frac{1}{12}}$
		Solution 2: Simplifying Cross Multiplied Factors $\frac{3}{36} ? \frac{4}{48} \rightarrow 3(48) ? 36(4)$ $3(4)(12) = (3)(12)(4)$
		Solution 3: Cross Products $\frac{3}{36} ? \frac{4}{48} \rightarrow 3(48) ? 36(4) \rightarrow 144 = 144$
		Solution 4: Products of Means and Extremes $\underbrace{3\ ft. : \underbrace{36\ in. = 4\ ft.}_{144} : 48\ in.}_{144}$
B	Shorter Segment : Thicker Segment	
C	$3\ min : 60\ m = 6\ min : 120\ m$	$\frac{3}{60} ? \frac{6}{120} \rightarrow \frac{3}{60} ? \frac{6}{2(60)} \rightarrow \frac{3}{60} = \frac{3}{60}$
D	$1 : 80\ pesos = 3 : 240\ pesos.$	$\frac{1}{80} ? \frac{3}{240} \rightarrow \frac{1}{80} ? \frac{3}{3(80)} \rightarrow \frac{1}{80} = \frac{1}{80}$

The solution in the table that follows shows that corresponding quantities are proportional. In short, they form a proportion because the ratios are equal.

3 *ft.* : 36 *in.* = 4 *ft.* : 48 *in.*	Solution: $\frac{3}{36} \stackrel{?}{=} \frac{4}{48} \rightarrow \frac{3}{3(12)} \stackrel{?}{=} \frac{4}{4(12)} \rightarrow \frac{1}{12} = \frac{1}{12}$

With the aforementioned explanation, complete the definition of proportion.

Proportion is the ________________________ of two ratios.

➤ Activity No. 4: Certainly, The Ratios Are Equal!

The properties that follow show several ways of rewriting proportions that do not alter the meaning of their values.

Fundamental Rule of Proportion If $w:x = y:z$, then $\frac{w}{x} = \frac{y}{z}$ provided that $x \neq 0; z \neq 0$. Properties of Proportion	
Cross-multiplication Property	If $\frac{w}{x} = \frac{y}{z}$, then $wz = xy$; $x \neq 0$, $z \neq 0$
Alternation Property	If $\frac{w}{x} = \frac{y}{z}$, then $\frac{w}{y} = \frac{x}{z}$; $x \neq 0$, $y \neq 0$, $z \neq 0$
Inverse Property	If $\frac{w}{x} = \frac{y}{z}$, then $\frac{x}{w} = \frac{z}{y}$; $w \neq 0$, $x \neq 0$, $y \neq 0$, $z \neq 0$
Addition Property	If $\frac{w}{x} = \frac{y}{z}$, then $\frac{w+x}{x} = \frac{y+z}{z}$; $x \neq 0$, $z \neq 0$
Subtraction Property	If $\frac{w}{x} = \frac{y}{z}$, then $\frac{w-x}{x} = \frac{y-z}{z}$; $x \neq 0$, $z \neq 0$
Inverse Property	If $\frac{u}{v} = \frac{w}{x} = \frac{y}{z}$, then $\frac{u}{v} = \frac{w}{x} = \frac{y}{z} = \frac{u+w+y}{v+x+z} = k$; where k is a constant at proportionality and $v \neq 0$, $x \neq 0$, $z \neq 0$.

Rewrite the given proportions according to the property indicated in the table and find out if the ratios in the rewritten proportions are still equal.

		Use the cross-multiplication property to verify that ratios are equal. Simplify if necessary. One is done for you.
Original Proportion	$\frac{y}{3} = \frac{a}{4}$	$4y = 3a$
Alternation Property of the original proportion		
Inverse Property of the original proportion		
Addition Property of the original proportion		
Subtraction Property of the original proportion		
Sum Property of the original proportion		***Hint:*** Create two separate proportions without using ***k*** • Is $\frac{y}{3}$ equal to $\frac{y+a}{7}$? • Is $\frac{a}{4}$ equal to $\frac{y+a}{7}$?

When ***k*** is considered in the sum property of the original proportion, the following proportions can be formed: $\frac{y}{3} = k \rightarrow \boldsymbol{y = 3k}$ and $\frac{a}{4} = k \rightarrow \boldsymbol{a = 4k}$. When we substitute the value of ***y*** and ***a*** to the original proportion, all ratios in the proportion are equal to ***k***, representing the equality of ratios in the proportion.

$$\frac{y}{3} = \frac{a}{4} = \frac{y+a}{7} = k$$

$$\frac{3k}{3} = \frac{4k}{4} = \frac{3k+4k}{7} = k$$

$$\frac{3k}{3} = \frac{4k}{4} = \frac{7k}{7} = k$$

➤ Activity 5: Solving Problems Involving Proportion

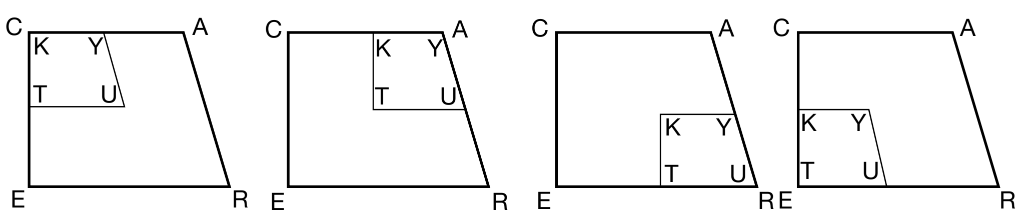
Study the examples on how to determine indicated quantities from a given proportion, then solve the items labeled as *Your Task.*

Examples	Your Task
1. If $m : n = 4 : 3$, find $3m - 2n : 3m + n$ **Solution** $\frac{m}{n} = \frac{4}{3} \rightarrow \mathbf{m = \frac{4n}{3}}$ **Using** $\mathbf{m = \frac{4n}{3}}$ $\frac{3m - 2n}{3m + n} = \frac{3\left(\frac{4n}{3}\right) - 2n}{3\left(\frac{4n}{3}\right) + n} = \frac{4n - 2n}{4n + n} = \frac{2n}{5n} = \frac{2}{5}$ **Therefore,** $3m - 2n : 3m + n = 2:5$	Find $\frac{y}{s}$ if $5y - 2s : \underline{10} = 3y - s = 7$.
2. If e and b represent two non-zero numbers, find the ratio $e : b$ if $2e^2 + eb - 3b^2 = 0$. **Solution** $2e^2 + eb - 3b^2 = 0$ $(2e + 3b)(e - b) = 0$ $\mathbf{2e + 3b = 0}$ or $\mathbf{e - b = 0}$ $2e = -3b$ $\frac{2e}{2b} = \frac{-3b}{2b}$ $\frac{e}{b} = \frac{-3}{2}$ $e = b$ $\frac{e}{b} = \frac{b}{b}$ $\frac{e}{b} = \frac{1}{1}$ Hence, $e : b = -3 : 2$ or $1 : 1$	Solve for the ratio $u{:}v$ if $u^2 + 3uv - 10v^2 = 0$.
3. If r, s and t represent three positive numbers such that $r : s : t = 4 : 3 : 2$ and $r^2 - s^2 - t^2 = 27$. Find the values of r, s and t. **Solution** Let $\frac{r}{4} = \frac{s}{3} = \frac{t}{2} = k, k \neq 0$ So, $r = 4k$; $s = 3k$; $t = 2k$ $r^2 - s^2 - t^2 = 27$ $(4k)^2 - (3k)^2 - (2k)^2 = 27$ $16k^2 - 9k^2 - 4k^2 = 27$ $16k^2 - 13k^2 = 27$ $3k^2 = 27$ $\frac{3k^2}{3} = \frac{27}{3}$ $k = 9$ $k = \{3 - 3\}$ Notice that we need to reject -3 because r, s and t are positive numbers. Therefore: • $r = 4k = 4(3) = 12$ • $s = 3k = 3(3) = 9$ • $t = 2k = 2(3) = 6$	if $g : h = 4 : 3$, evaluate $4g + h : 8g + h$

4. If $\frac{q}{2}=\frac{r}{3}=\frac{s}{4}=\frac{5q-6r-7s}{x}$. Find x. **Solution** Let $\frac{q}{2}=\frac{r}{3}=\frac{s}{4}=\frac{5q-6r-7s}{x}=k$. Then $q = 2k$, $r = 3k$, $s = 4k$, and $5q - 6r - 7s = kx$. $5(2k) - 6(3k) - 7(4k) = kx$ $10k - 18k - 28k = kx$ $-36k = kx$ $\boldsymbol{x = -36}$	Find the value of m if $\frac{e}{1}=\frac{f}{2}=\frac{g}{3}=\frac{5e-6f-2g}{m}$

➤ Activity 6: How are polygons similar?

Each side of trapezoid *KYUT* is ***k*** times the corresponding side of trapezoid *CARE*. These trapezoids are similar. In symbols, *KYUT* ~ *CARE*. One corresponding pair of vertices is paired in each of the figures that follow. Study their shapes, their sizes, and their corresponding angles and sides carefully.



Questions

1. What do you observe about the shapes of polygons *KYUT* and *CARE*?

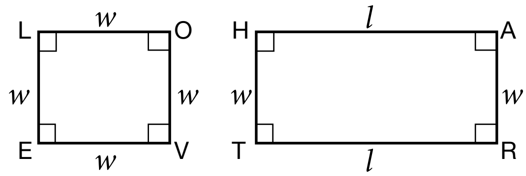
____

2. What do you observe about their sizes?

____

Aside from having the same shape, what makes them similar? Let us answer this question after studying their corresponding sides and angles. Let us first study non-similar parallelograms *LOVE* and *HART* and parallelograms *YRIC* and *DENZ* before carefully studying the characteristics of polygons *CARE* and *KYUT*.

Let us consider Parallelograms LOVE and HART.



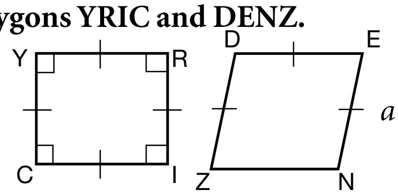
Observe the corresponding angles and corresponding sides of parallelograms LOVE and HART by taking careful note of their measurements. Write your observations on the given table. Two observations are done for you.

Corresponding Angles	Ratio of Corresponding Sides	Simplified Ratio/s of the Sides
$m\angle L = m\angle H = 90$	$\frac{LO}{HA} = \frac{w}{l}$	
$m\angle E = m\angle T = 90$	$\frac{EL}{TH} = \frac{w}{w} = 1$	

3. Are the corresponding angles of parallelograms LOVE and HART congruent?
4. Do their corresponding sides have a common ratio?
5. Do parallelograms LOVE and HART have uniform proportionality of sides?

   Note: Parallelograms LOVE and HART are not similar.
6. What do you think makes them not similar? Answer this question later.

This time, we consider polygons YRIC and DENZ.

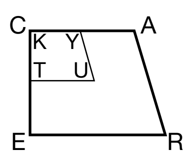
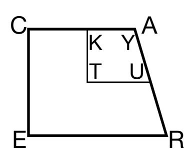
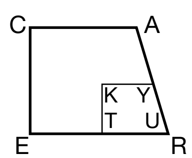
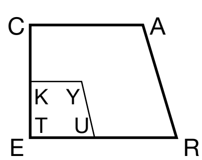


Observe the corresponding angles and corresponding sides of parallelograms YRIC and DENZ, taking careful note of their measurements. Write your observations using the given table. The first observation is done for you.

Corresponding Angles	Ratio of Corresponding Sides	Simplified Ratio/s of the Sides
$m\angle Y \neq m\angle D$	$\frac{YR}{DE} = \frac{a}{a}$	1

7. Are the corresponding angles congruent?
8. Do parallelograms *YRIC* and *DENZ* have uniform proportionality of sides?

   Note: *YRIC* and *DENZ* are not similar.
9. What do you think makes them not similar? Answer this question later.
10. Now consider again the similar polygons *KYUT* and *CARE* ($KYUT \sim CARE$). Notice that by pairing their corresponding vertices, corresponding angles coincide perfectly. It can be observed also that corresponding angles are congruent. In the following table, write your observations about the corresponding overlapping sides as each pair of corresponding vertices is made to coincide with each other.

Ratios of the corresponding sides that overlap			How do you express the proportionality of the overlapping sides using their ratios?	Corresponding Angles
	$KT:CE$	$KY:CA$	$KT:CE = KY:CA = k:1$	$\angle K \cong \angle C$
	$\frac{KT}{CE}$	$\frac{KY}{CA}$	$\frac{KT}{CE} = \frac{KY}{CA} = k$	
	$KY:CA$	$YU:AR$	$KY:CA = YU:AR = k:1$	$\angle Y \cong \angle A$
	$\frac{KY}{CA}$	$\frac{YU}{AR}$	$\frac{KY}{CA} = \frac{YU}{AR} = k$	
	$YU:AR$	$UT:RE$	$YU:AR = UT:RE = k:1$	$\angle U \cong \angle R$
	$\frac{YU}{AR}$	$\frac{UT}{RE}$	$\frac{YU}{AR} = \frac{UT}{RE} = k$	
	$UT:RE$	$KT:CE$	$UT:RE = KT:CE = k:1$	$\angle T \cong \angle E$
	$\frac{UT}{RE}$	$\frac{KT}{CE}$	$\frac{UT}{RE} = \frac{KT}{CE} = k$	

11. Observe that adjacent sides overlap when a vertex of *KYUT* is paired with a vertex of *CARE*. It means that for *KYUT* and *CARE* that are paired at a vertex, corresponding angles are ______________. Moreover, the ratios of corresponding sides are equal. Hence, the corresponding sides are _________________.

Big question: Do *KYUT* and *CARE* have uniform proportionality of sides like *YRIC* and *DENZ*? Let us study carefully the proportionality of the corresponding adjacent sides that overlap.

When the following vertices are paired:			
K & *C*	*Y* & *A*	*U* & *R*	*T* & *E*
$\frac{KT}{CE} = \frac{\mathbf{KY}}{\mathbf{CA}}$	$\frac{KY}{CA} = \frac{\mathbf{YU}}{\mathbf{AR}}$	$\frac{YU}{AR} = \frac{\mathbf{UT}}{\mathbf{RE}}$	$\frac{\mathbf{UT}}{\mathbf{RE}} = \frac{KT}{CE}$

12. Notice that $\frac{KY}{CA}$ is found in the pairing of vertices *K* & *C* and *Y* & *A*. It means that

$$\frac{KT}{CE} = \frac{\mathbf{KY}}{\mathbf{CA}} = \frac{YU}{AR}.$$

13. Observe that $\frac{YU}{AR}$ is found in the pairing of vertices *Y* & *A* and *U* & *R*. It means that

$$\frac{\square}{\square} = \frac{\boxed{\mathbf{YU}}}{\boxed{\mathbf{AR}}} = \frac{\square}{\square}.$$

14. Notice also that $\frac{UT}{RE}$ is found in the pairing of vertices *U* & *R* and *T* & *E*. It means that

$$\frac{\square}{\square} = \frac{\boxed{\mathbf{UT}}}{\boxed{\mathbf{RE}}} = \frac{\square}{\square}$$

15. Still we can see that $\frac{KT}{CE}$ is found in the pairing of vertices *T* & *E* and *K* & *C*. It means that

$$\frac{\square}{\square} = \frac{\boxed{\mathbf{KT}}}{\boxed{\mathbf{CE}}} = \frac{\square}{\square}$$

16. Therefore, we can write the proportionality of sides as

$$\frac{\boxed{\mathbf{KT}}}{\boxed{\mathbf{CE}}} = \frac{\boxed{\mathbf{KY}}}{\boxed{\mathbf{CA}}} = \frac{\boxed{\mathbf{YU}}}{\boxed{\mathbf{AR}}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

17. If $\frac{KT}{CE} = \frac{KY}{CA} = \boldsymbol{k}$, can we say that the ratios of the other corresponding adjacent sides are also equal to *k*? Explain your answer.

Since the ratios of all the corresponding sides of similar polygons KYUT and CARE are equal, it means that they have **uniform proportionality of sides**. That is, all the corresponding sides are proportional to each other.

The number that describes the ratio of two corresponding sides of similar polygons such as polygons KYUT and CARE is referred to as the ***scale factor***. This scale factor is true to all the rest of the corresponding sides of similar polygons because of the ***uniformity of the proportionality of their sides***.

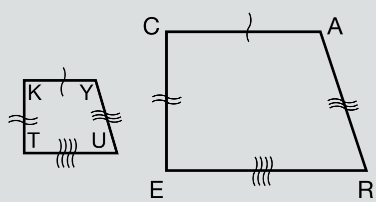
18. Express the uniform proportionality of the sides of similar polygons *KYUT* and *CARE* in one mathematical sentence using the scale factor *k*.

$$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \square$$

19. The conditions observed in similar polygons *KYUT* and *CARE* help us point out the characteristics of similar polygons.

> **Two polygons are similar if their vertices can be paired so that**
> corresponding angles are ____________________, and
> corresponding sides are ____________________.

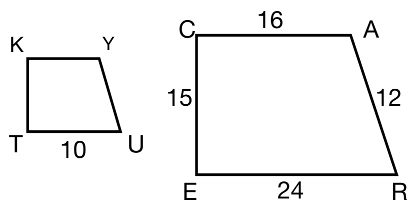
Curved marks can be used to indicate proportionality of corresponding sides of figures such as shown in parallelograms *KYUT* and *CARE* below:



20. Now that you know what makes polygons similar, answer the following questions

Why are parallelograms *LOVE* and *HART* not similar?	
Why are parallelograms *YRIC* and *DENZ* not similar?	

KYUT ~ *CARE*. Given the lengths of their sides in the figure, and their proportional sides on the table, answer the following questions:



Proportional Sides	
$\frac{KT}{CE} = \frac{KY}{CA}$	$\frac{KT}{15} = \frac{KY}{16}$
$\frac{KT}{CE} = \frac{TU}{ER}$	$\frac{KT}{15} = \frac{10}{24}$
$\frac{TU}{ER} = \frac{UY}{RA}$	$\frac{10}{24} = \frac{UY}{12}$
$\frac{UY}{RA} = \frac{KY}{CA}$	$\frac{UY}{12} = \frac{KY}{16}$

21. The scale factor of similar figures can be determined by getting the ratio of corresponding sides with given lengths. Which of the ratios of corresponding sides give the scale factor *k*?
22. What is the ratio of the corresponding sides with given lengths?
23. What is the simplified form of scale factor *k*?
24. Solve for *KT* by equating the ratio of corresponding sides containing *KT* with the scale factor *k*?

$$\frac{KT}{15} = \frac{5}{12} \rightarrow 12(KT) = 5(15) \rightarrow \boldsymbol{KT} = \frac{5(15)}{12} = \frac{5(5)}{4} = \frac{\mathbf{25}}{\mathbf{4}}$$

25. Solve for *KY* by equating the ratio of corresponding sides containing *KY* with the scale factor *k*?

26. Solve for *UY* by equating the ratio of corresponding sides containing *UY* with the scale factor *k*?

27. Trapezoids *CARE* and *KYUT*, although having the same shape, differ in size. Hence, they are not congruent, only similar. Let us remember: What are the two characteristics of similar polygons?

    (1.)____

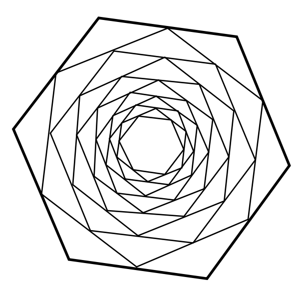
    (2.) ____

28. What can you say about the two statements that follow:

    I. All congruent figures are similar.
    II. All similar figures are congruent.

    a. Both are true.
    b. Only I is true.
    c. Only II is true.
    d. Neither one is true.

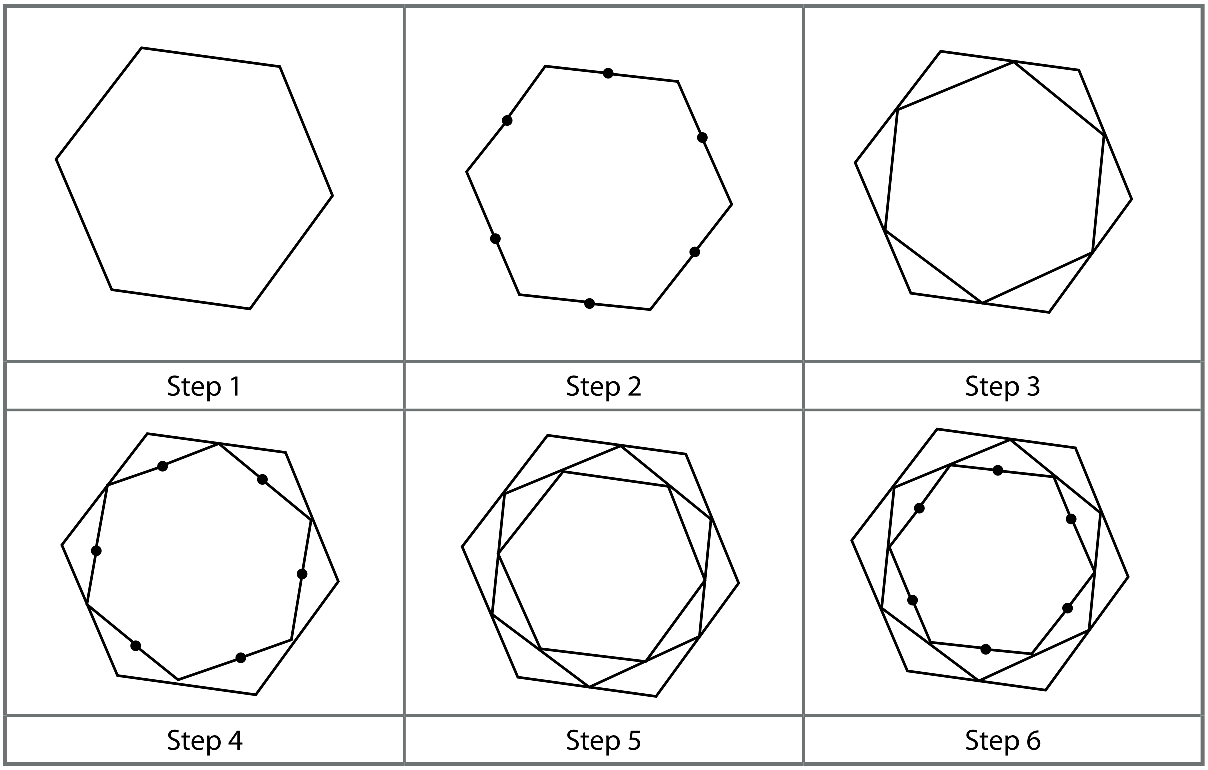
➤ Activity 7: Self–Similarity



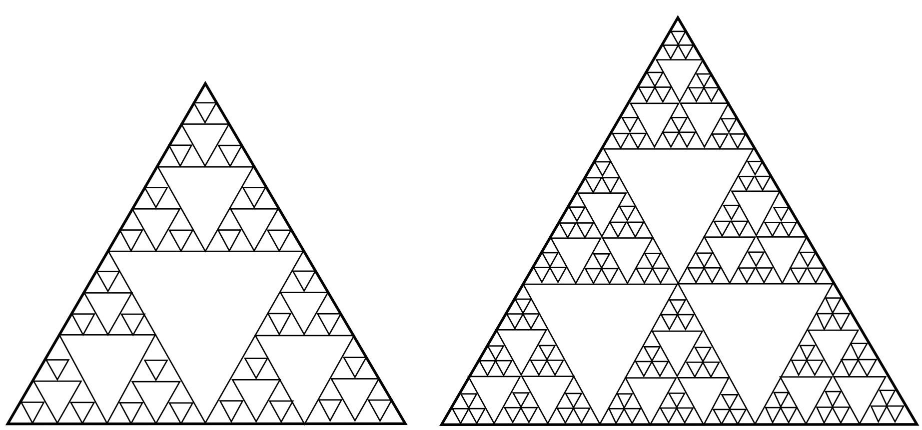
The figure shows similar regular hexagons of decreasing sizes. Being regular, all the hexagons are equiangular. Because their sizes are decreasing proportionally, corresponding sides are also proportional.

Questions

1. How many self-similar hexagons are there?
2. Do you know how this figure is formed? Study the initial steps on how the hexagon is replicated many times in decreasing sizes. Describe each step in words.



3. Notice that step 2 is repeated in steps 4 and 6 and step 3 is repeated in step 5. Perform these steps repeatedly until you have replicated the original figure of self-similar regular hexagons.
4. Aside from regular hexagons, what do you think are the other polygons that can indefinitely regenerate self-similar polygons?
5. Guided by the activity, list down the steps on how the Sierpinski triangles shown below can be constructed.



- Are the triangles of each of the Sierpinski triangles similar? Explain.
- What is the scale factor used to reduce each triangle of the Sierpinski triangle to the next one in size? Explain.

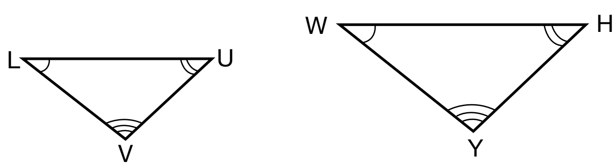
- Read more and watch a video about Sierpinski triangle from http://new-to-teaching.blogspot.com/2013/03/chaos-games-and-fractal-images.html and write an insight of what you have learned. Note that it is more beneficial if you widen your exploration to other websites on the topic.

> Your knowledge on the definition of similarity of polygons and your skill in determining the scale factors of similar polygons is useful in dealing with similarity of triangles. In this subsection, you will be illustrating and proving theorems involving triangle similarity.

Triangle Similarity

> **AAA Similarity Postulate**
> If the three angles of one triangle are congruent to three angles of another triangle, then the two triangles are similar.

Illustration

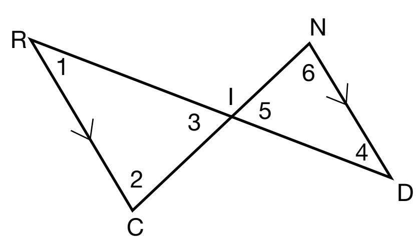


If:	$\angle L \cong \angle W$; $\angle U \cong \angle H$ $\angle V \cong \angle Y$
Then:	$\Delta LUV \sim \Delta WHY$

The illustration demonstrates the conditions of AAA Similarity Postulate using markings to show congruence of three angles of ΔLUV and ΔWHY.

Quiz on AAA Similarity Postulate

Given the figure, prove that ΔRIC ~ ΔDIN



Hints:		Statements	Reasons
1	Based on their markings, describe $\overline{RC}$ and $\overline{DN}$		Given
2	Based on statement 1, describe alternate interior angles if $\overline{CN}$ and $\overline{RD}$ are transversals		Alternate interior angles are congruent.
3	Describe the vertical angles		Vertical angles are congruent.
4	Conclude using statements 1, 2, & 3		Similarity Postulate

You have learned in Grade 8 that theorems and statements can also be proven using paragraph proof or flowchart proof. Paragraph proof is preferred in higher mathematics. The proof that follows is the paragraph version of the columnar proof of the quiz on AAA Similarity Postulate.

Proof:
The figure shows that $\overline{RC}$ of ΔRIC and $\overline{DN}$ of ΔDIN are parallel. It follows that the alternate interior angles $(\angle 1 \& \angle 4$ and $\angle 2 \& \angle 6)$ determined by these parallel lines and their transversals $(\overline{DR}$ and $\overline{CN})$ are congruent. That is, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 6$. By the vertical angles theorem, $\angle 3 \cong \angle 5$. Since all their corresponding angles are congruent, $\Delta RIC \sim \Delta DIN$ by AAA Similarity Postulate.

A paragraph proof does not have restrictions on how the proof is presented. You have the freedom to present your proof as long as there is logic and system in the presentation of the statements and corresponding reasons or justifications.

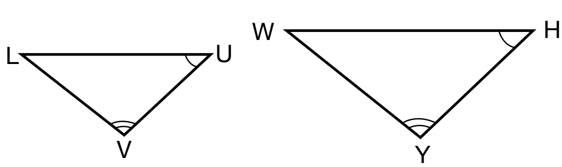
Proofs of theorems in this module use columnar proof to give you hints on how to proceed with the proof. It is recommended, however, that you try to produce a paragraph proof on all the theorems after proving them using the columnar proof.

➤ Activity 8: AA Similarity Theorem and Its Proof

Write the statements or reasons that are left blank in the proof of AA Similarity Theorem. Refer to the hints provided.

AA Similarity Theorem
Two triangles are similar if two angles of one triangle are congruent to two angles of another triangle.

Illustration



If:	$\angle U \cong \angle H$; $\angle V \cong \angle Y$
Then:	$\Delta LUV \sim \Delta WHY$

Given: ∠U ≅ ∠H; ∠V ≅ ∠Y

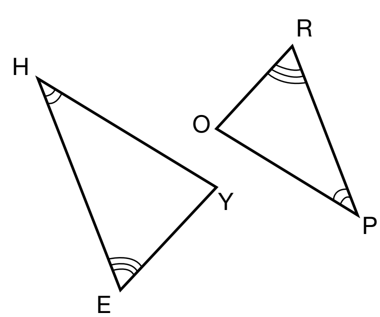
Prove: ΔLUV ~ ΔWHY

Proof:

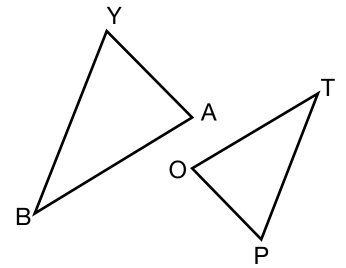
Hints		Statements	Reasons
1	Write all the given.		
2	Describe the measure of the congruent angles in statement 1.		Definition of congruent angles
3	Add m ∠V to both sides of m ∠U = m ∠H in statement 2.		Addition property of equality
4	Substitute m ∠V on the right side of statement 3 using statement 2.		Substitution
5	Add the measures of all the angles of triangles LUV and WHY.		The sum of the measures of the three angles of a triangle is 180.
6	Equate the measures of the angles of triangles LUV and WHY from statement 5.		Transitive Property of Equality
7	Substitute m ∠H on the right side of statement 6 using statement 2.		Substitution
8	Simplify statement 7.		Subtraction Property of Equality
9	Are triangles LUV and WHY similar? Reason should be based from statements 2 and 8.		Similarity Postulate

Quiz on AA Similarity Theorem

A. Use the AA Similarity Theorem in writing an if-then statement to describe the illustration or in completing the figure based on the if-then statement.

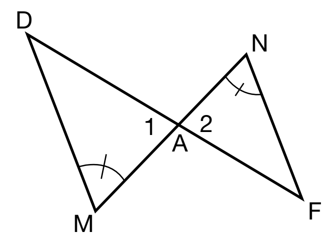


If:	
Then:	ΔHEY ~____________



If:	∠A ≅ ∠O; ∠B ≅ ∠T
Then:	ΔBAY ~ ΔTOP

B. Prove that ΔDAM ~ ΔFAN.



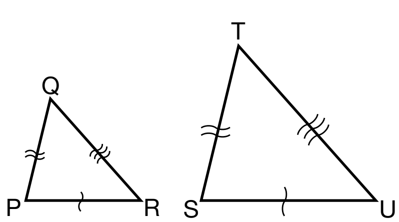
Hints:		Statements	Reasons
1	Congruent angles with markings		
2	Congruent angles because they are vertical		
3	Conclusion based on statement 1 and 2		

➤ Activity 9: SSS Similarity Theorem and Its Proof

Write the statements or reasons that are left blank in the proof of SSS Similarity Theorem. Refer to the hints provided to help you.

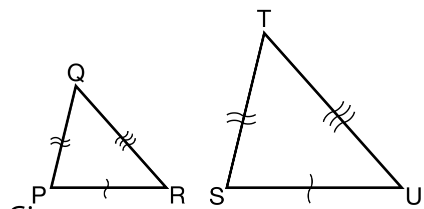
> **SSS Similarity Theorem**
> Two triangles are similar if the corresponding sides of two triangles are in proportion.

Illustration



If:	$\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$
Then:	ΔPQR ~ ΔSTU

Proof



Given

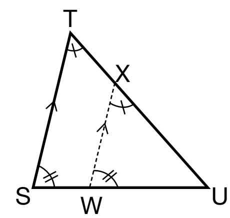
$$\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$$

Prove

$\triangle PQR \sim \triangle STU$

Proof

- Construct X on $\overline{TU}$ such that $\overline{XU} \cong \overline{QR}$.
- From X, construct $\overline{XW}$ parallel to $\overline{TS}$ intersecting $\overline{SU}$ at W.



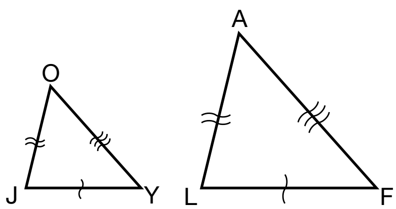
	Hints	Statements	Reasons
1	Which sides are parallel by construction?		By construction
2	Describe angles WXU and STU and XWU and TSU based on statement 1.		Corresponding angles are congruent
3	Are WXU and STU similar?		____ Similarity Theorem
4	Write the equal ratios of similar triangles in statement 3.		Definition of similar polygons
5	Write the given.		Given
6	Write the congruent sides that resulted from construction.		By construction
7	Use statement 6 in statement 5.		Substitution
8	If $\frac{PQ}{ST} = \frac{XU}{TU}$ (statement 7) and $\frac{WX}{ST} = \frac{XU}{TU}$ (statement 4), then		Transitive Property of Equality
	If $\frac{XU}{TU} = \frac{PR}{SU}$ (statement 7) and $\frac{XU}{TU} = \frac{WU}{SU}$ (statement 4), then		
9	Multiply the proportions in statement 8 by their common denominators and simplify.		Multiplication Property of Equality
10	Are triangles PQR and WXU congruent? Base your answer from statements 9 and 6.		SSS Triangle Congruence Postulate
11	Use statement 10 to describe angles WUX and SUT.		Definition of congruent triangles
12	Substitute the denominators of statement 4 using the equivalents in statements 9 and 6, then simplify.	$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \square$	Substitution

13	Using statements 2, 11, and 12, what can you say about triangles PQR and WXU?		Definition of Similar Polygons
14	Write a conclusion using statements 13 and 3.		Transitivity

Notice that we have also proven here that congruent triangles are similar (study statement 10 to 13) and the uniform proportionality of their sides is equal to ______.

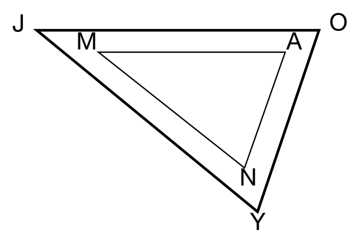
Quiz on SSS Similarity Theorem

A. Use the SSS Similarity Theorem in writing an if-then statement to describe an illustration or in completing a figure based on an if-then statement.

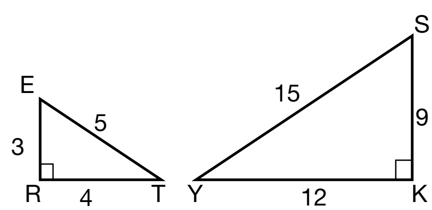


If:	
Then:	

If:	$\frac{OY}{AN} = \frac{OJ}{AM} = \frac{JY}{MN}$
Then:	ΔJOY ~ ΔMAN



B. Prove that ΔERT ~ ΔSKY.



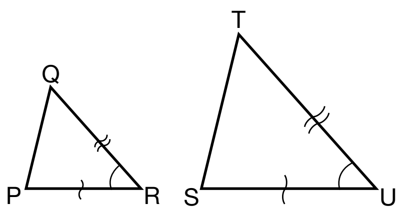
Hints:		Statements	Reasons
1	Do all their corresponding sides have uniform proportionality? Verify by substituting the lengths of the sides. Simplify afterwards.	$\frac{\square}{\square}$? $\frac{\square}{\square}$? $\frac{\square}{\square}$: $\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \square$	By computation
2	What is the conclusion based on the simplified ratios?		

➤ Activity 10: SAS Similarity Theorem and Its Proof

Write the statements or reasons that are left blank in the proof of SAS Similarity Theorem. Refer to the hints provided to help you.

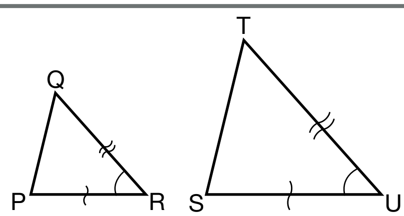
> **SAS Similarity Theorem**
> Two triangles are similar if an angle of one triangle is congruent to an angle of another triangle and the corresponding sides including those angles are in proportion.

Illustration



If:	$\frac{QR}{TU} = \frac{PR}{SU}$; $\angle R \cong \angle U$
Then:	$\triangle PQR \sim \triangle STU$

Proof



Given:

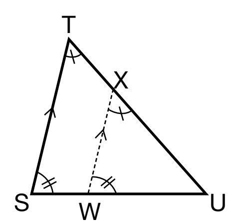
$\frac{QR}{TU} = \frac{PR}{SU}$; $\angle R \cong \angle U$

Prove:

$\triangle PQR \sim \triangle STU$

Proof:

- Construct X on $\overline{TU}$ such that $\overline{XU} = \overline{QR}$.
- From X, construct $\overline{XW}$ parallel to $\overline{TS}$ intersecting $\overline{SU}$ at W

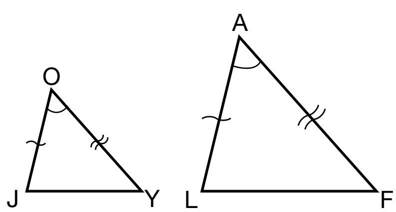
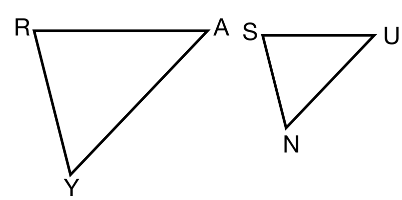


No.	Hints	Statements	Reasons
1	Which sides are parallel by construction?		By construction
2	Describe angles WXU & STU and XWU and TSU based on statement 1.		Corresponding angles are congruent
3	Are triangles WXU and STU similar?		AA Similarity Theorem
4	Write the equal ratios of similar triangles in statement 3		Definition of Similar polygons
5	Write the congruent sides that resulted from construction.		By construction
6	Write the given related to sides.		Given

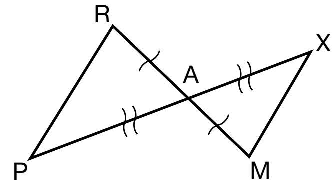
7	Use statement 5 in statement 6.		Substitution Property of Equality
8	If $\frac{XU}{TU} = \frac{PR}{SU}$ (statement 7) and $\frac{XU}{TU} = \frac{WU}{SU}$ (statement 4), then		Transitive Property of Equality
	If $\frac{XU}{TU} = \frac{PR}{SU}$ (statement 7) and $\frac{QR}{TU} = \frac{PR}{SU}$ (statement 6), then		
9	Multiply the proportions in statement 8 by their common denominators and simplify.	 	Multiplication Property of Equality
10	Write the given related to corresponding angles.		Given
11	What can you say about triangles PQR and WXU based on statements 9 and10.		______Triangle Congruence Postulate
12	Write a statement when the reason is the one shown.		Congruent triangles are similar.
13	Write a conclusion using statements 12 and 3.		Substitution Property

Quiz on SAS Similarity Theorem

A. Use the SAS Similarity Theorem in writing an if-then statement to describe an illustration or in completing a figure based on an if-then statement.

	If: Then:
If: $\angle A \cong \angle U$; $\frac{AR}{US} = \frac{AY}{UN}$ Then: $\triangle RAY \sim \triangle SUN$	

B. Given the figure, use SAS Similarity Theorem to prove that ΔRAP ~ ΔMAX.



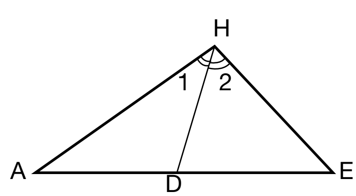
Hints:		Statements	Reasons
1	Write in a proportion the ratios of two corresponding proportional sides		
2	Describe included angles of the proportional sides		
3	Conclusion based on the simplified ratios		

➤ Activity 11: Triangle Angle Bisector Theorem (TABT) and Its Proof

Write the statements or reasons that are left blank in the proof of Triangle Angle-Bisector Theorem. Refer to the hints provided.

> **Triangle Angle-Bisector Theorem**
> If a segment bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

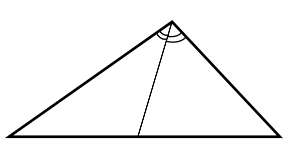
Illustration



If:	$\overline{HD}$ bisects ∠AHE,
Then:	Then: $\frac{DA}{DE} = \frac{AH}{EH}$

Notice: that sides on the numerators are adjacent. The same is true with the denominators.

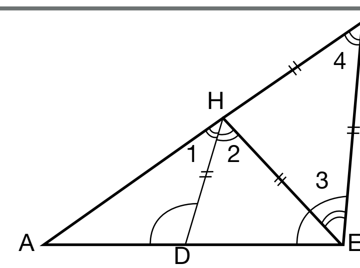
Proof



Given
$\overline{HD}$ bisects ∠AHE.

Prove
$\frac{DA}{DE} = \frac{AH}{EH}$

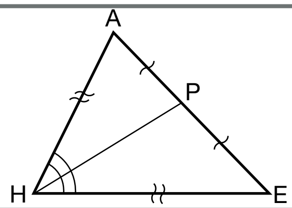
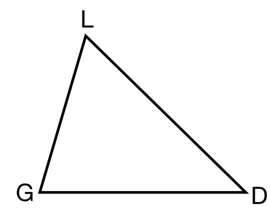
Proof
Extend $\overline{AH}$ to P such that $\overline{EP} \parallel \overline{HD}$.



No.	Hints	Statements	Reasons
1	List down the given		
2	What happens to the bisected $\angle AHE$?	$\angle 1 \cong$ ____________	Definition of angle bisector
3	What can you say about $\overline{HD}$ and $\overline{EP}$?		By ____________
4	What can you conclude about $\angle ADH$ & $\angle DEP$ and $\angle 1$ & $\angle 4$?	____________ 	Corresponding angles are congruent.
5	What can you conclude about $\angle 2$ & $\angle 3$?		Alternate interior angles are congruent.
6	What can you say about $\angle 3$ & $\angle 4$ based on statements 2, 4, and 5?		Transitive Property
7	What kind of triangle is $\triangle HEP$ based on statement 6?	$\triangle HEP$ is ____________.	Base angles of isosceles triangles are congruent.
8	What can you say about the sides opposite $\angle 4$ & $\angle 3$?		Definition of isosceles triangles
9	What can you say about $\triangle AHD$ & $\triangle APE$ using statement 4?		AA Similarity Theorem
10	Using statement 3, write the proportional lengths of $\triangle APE$.	$\frac{\boxed{AH}}{\boxed{}} = \frac{\boxed{}}{\boxed{AE}}$	Definition of Similar Polygons
11	Use Segment Addition Postulate for AP and AE.	$\frac{AH}{AH + AP} = \frac{AD}{AD + DE}$	Segment Addition Postulate
12	Use Inversion Property of Proportion statement 11.		Inversion Property of Proportion
13	Decompose the fractions in statement 12 and simplify.	$\frac{AH}{AH} + \frac{HP}{AH} = \frac{AD}{AD} + \frac{DE}{AD}$ $\_\_ + \frac{HP}{AH} = \_\_ + \frac{DE}{AD}$	Principles in the operations of fractions
14	Simplify statement 13.		Subtraction Property of Equality
15	Use statement 8 in statement 14.		Substitution
16	Use symmetric Property in statement 15.		Symmetric Property of Equality
17	Use Inversion Property in statement 16.		Inversion Property of Proportion

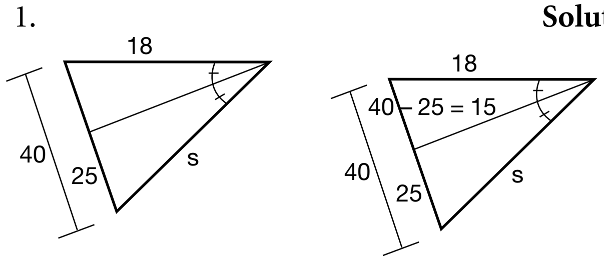
Quiz on Triangle Angle-Bisector Theorem

A. Use the TABT in writing an if-then statement to describe an illustration or completing a figure based on an if-then statement.

	If: Then:
If: $\overline{DS}$ bisects $\angle D$ Then: $\frac{SG}{SL}=\frac{GD}{LD}$	

B. Solve for the unknown side applying the Triangle Angle-Bisector Theorem. The first one is done for you. Note that the figures are not drawn to scale.

1.



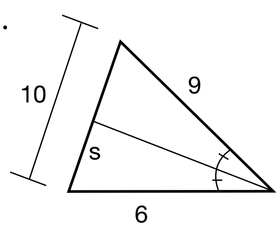
Solution $\frac{25}{15}=\frac{s}{18} \rightarrow 15s=25(18)$

$$s=\frac{25(18)}{15}$$

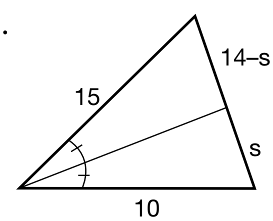
$$s=\frac{[(5)(5)][(2)(3)(3)]}{(3)(5)}$$

$$s=(5)(2)(3)=30$$

2.



3.

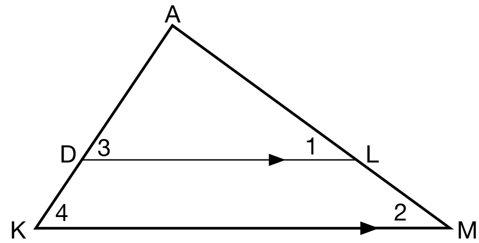


➤ Activity 12: Triangle Proportionality Theorem (TPT) and Its Proof

Write the statements or reasons that are left blank in the proof of Triangle Proportionality Theorem.

> **Triangle Proportionality Theorem**
> If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

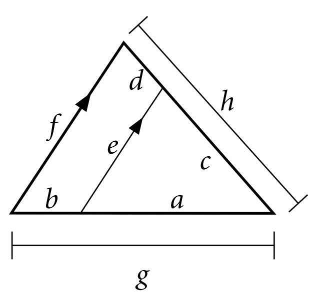
Proof

<table>
<tr><td>
</td><td>Given
$\overline{DL} \parallel \overline{KM}$

Prove
$\dfrac{AD}{AK} = \dfrac{AL}{AM}$</td></tr>
<tr><td colspan="2">Proof</td></tr>
<tr><td>Statements</td><td>Reasons</td></tr>
<tr><td>1. $\overline{DL} \parallel \overline{KM}$</td><td>1.</td></tr>
<tr><td>2.</td><td>2. Corresponding angles are congruent.</td></tr>
<tr><td>3. $\triangle DAL \sim \triangle KAM$</td><td>3. _________Similarity Theorem</td></tr>
<tr><td>4.</td><td>4. Definition of similar polygons</td></tr>
</table>

➤ Activity 13: Determining Proportions Derived from TPT

Write the proportion of the sides derived from Triangle Proportionality Theorem. One set is done for you. Note that the boxes with darker shades are those that require you to answer or respond.

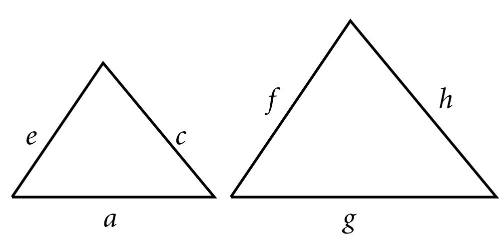


Observe that

1. $d = h - c$
2. $b = g - a$

Therefore, there is also a length that is ***f–e***

Separating the triangles:



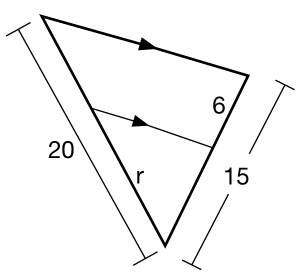
Considering the ratio of the **sides of a smaller triangle** to the **sides of a larger triangle**	
Considering the ratio of the **sides of a smaller triangle** to the **differences** between sides of a larger triangle and smaller triangle	
Considering the ratio of the **differences** between sides of a larger triangle and a smaller triangle and the **side of a larger triangle**	$\dfrac{b}{g} = \dfrac{d}{h} = \dfrac{f-e}{f}$

Be reminded that using the properties of proportion, there would be plenty of possible proportions available. For instance, in $\frac{a}{g} = \frac{c}{h}$, we could have $\frac{a}{c} = \frac{g}{h}$ by the alternation property of proportion or $\frac{g}{a} = \frac{h}{c}$ by the inverse property of proportion.

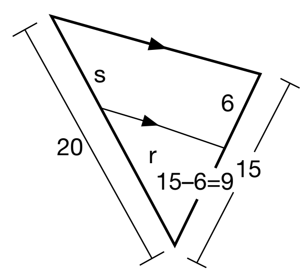
Quiz on Triangle Proportionality Theorem

Solve for the unknown sides in the figures. The first one is done for you. Note that the figures are not drawn to scale.

1.



Solutions



Solving for r

$$\frac{r}{20} = \frac{9}{15} \rightarrow 15r = 9(20)$$

$$r = \frac{9(20)}{15}$$

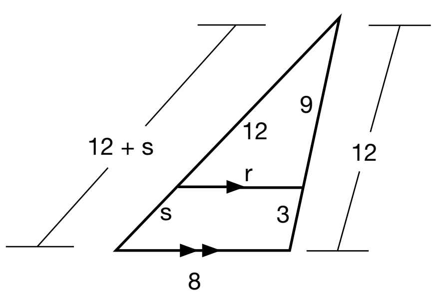
$$r = \frac{[(3)(3)][(4)(5)]}{(3)(15)} = 12$$

Solving for s

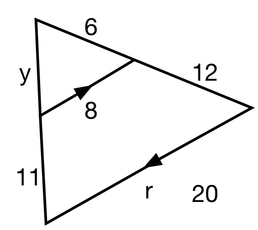
$s = 20 - r = 20 - 12 = 8$

Your Task

2.



3.

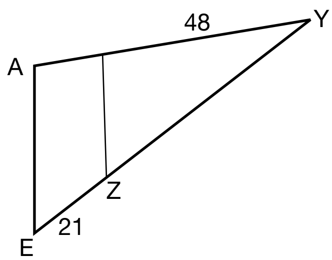


B. Fill in the blanks.

Triangle Proportionality Theorem states that if a segment divides two adjacent sides of a triangle ________________, then it is _________________to the third side of the triangle.

C. Determine whether a segment is parallel to one side of a triangle. The first one is done for you. Note that the illustrations are not drawn to scale.

1.



Solution

check whether $\frac{YM}{YA} = \frac{YZ}{YE}$

$$\frac{48}{48+16} \mathrel{?} \frac{56}{56+21}$$

$$\frac{48}{64} \mathrel{?} \frac{56}{77}$$

$$\frac{2^4(3)}{2^6} \mathrel{?} \frac{2^3(7)}{(11)(7)}$$

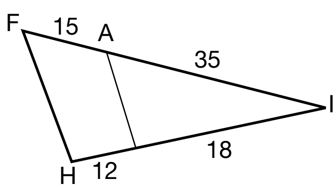
$$\frac{3}{2^2} \mathrel{?} \frac{8}{11}$$

$$\frac{3}{4} \neq \frac{8}{11}$$

Therefore, $\overline{MZ} \parallel \overline{AE}$

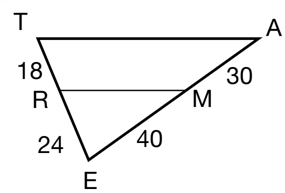
Is $\overline{AT} \parallel \overline{FH}$?

2.



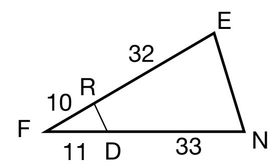
Is $\overline{AT} \parallel \overline{FH}$?

3.



Is $\overline{MR} \parallel \overline{AT}$?

4.



Is $\overline{DR} \parallel \overline{EN}$?

D. Use the figure to complete the proportion.

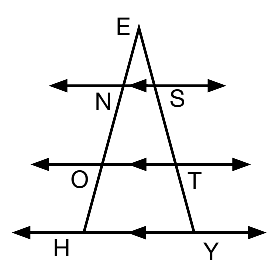
1. $\frac{EN}{EH} = \frac{\square}{EY}$

2. $\frac{NH}{\square} = \frac{SY}{EY}$

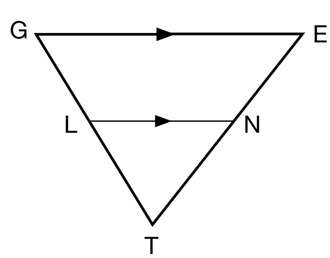
3. $\frac{EN}{EO} = \frac{\square}{OT} = \frac{ES}{ET}$

4. $\frac{ST}{ET} = \frac{NO}{\square}$

5. $\frac{\square}{SY} = \frac{OH}{NH}$

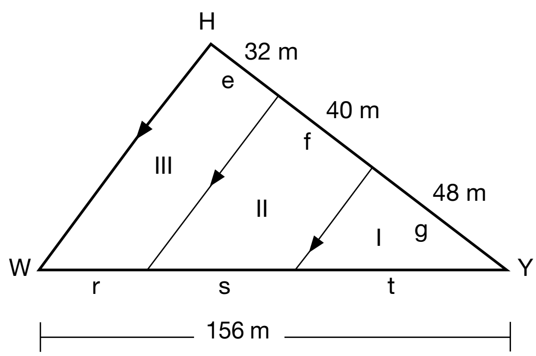


E. Tell whether the proportion is right or wrong.



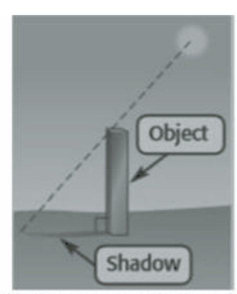
Proportion		Response
1.	$\frac{TN}{NE} = \frac{TL}{LG}$	
2.	$\frac{LN}{EG} = \frac{NT}{ET}$	
3.	$\frac{TL}{TG} = \frac{TN}{TE} = \frac{LN}{EG}$	
4	$\frac{LG}{TG} = \frac{NE}{TE} = \frac{LN}{EG}$	

F. Solve for *r, s*, and *t*.



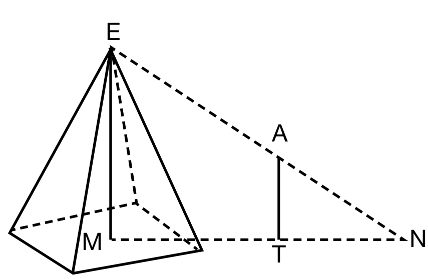
Indirect Measurement

It has been believed that the first person to determine the difficult-to-obtain heights without the aid of a measuring tool existed even before Christ, from 624-547 BC. The Greek mathematician *Thales* determined the heights of pyramids in Egypt by the method called *shadow reckoning*. The activity that follows is a version of how Thales may have done it.



http://www.augusta.k12.va.us/cms/lib01/VA01000173/Centricity/Domain/766/chap06%20Geometry.pdf

➤ Activity 14: Determining Heights without Actually Measuring Them



The sun shines from the western part of the pyramid and casts a shadow on the opposite side. Analyze the figure and answer the following questions

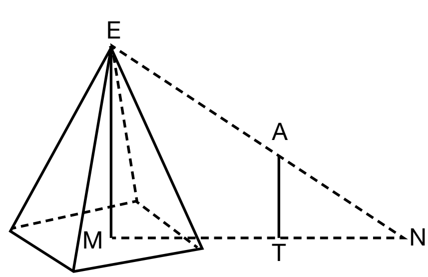
1. ME is the unknown ____________of the pyramid.
2. MN is the length of the shadow of the ________________.
3. ______ is the height of a vertical post.
4. TN is the length of the ________ of the vertical post.
5. Which of the following can be measured directly with the use of a measuring tool? If it can be measured directly, write YES, otherwise, write NO.

Lengths	Answer	Lengths	Answer
ME		MN	
AT		TN	

6. Why is the height of the pyramid difficult to measure using a measuring tool?
7. Like the post, the height of the pyramid is also vertical. What can you conclude about $\overline{ME}$ and $\overline{AT}$?
8. If $\overline{ME} \parallel \overline{AT}$, what can you say about $\triangle EMN$ and $\triangle ATN$?
9. What theorem justifies your answer?
10. The figure is not drawn to scale. Which of the following situations is true or false?

If the length of the shadow of the pyramid is greater than the height of the pyramid, the possibility is that the measurement of the shadow was done	True or False?
early in the morning	
early in the afternoon	
late in the morning	
late in the afternoon	

11. If MN = 80 ft., NT = 8 ft., and AT = 6ft., what is the height of the pyramid in this activity?
12. If the post was not erected to have its top to be along the line of shadow cast by the building such as shown, will you still be able to solve the height of the pyramid? Explain.



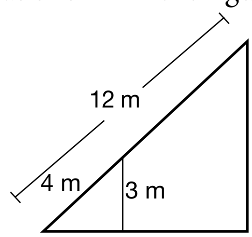
13. How long is the height of a school flagpole if at a certain time of day, a 5-foot student casts a 3-feet shadow while the length of the shadow cast by the flagpole is 12 ft? Show your solution on a clean sheet of paper.

a. 20 ft. c. 16 ft.

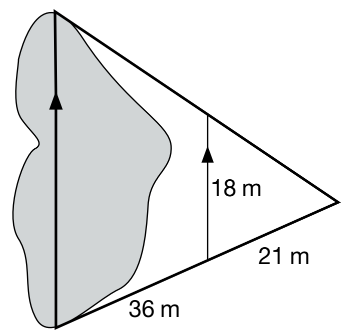
b. 18 ft. d. 15 ft.

Let us extend the activity to other cases of indirect measurement.

14. A 12-meter fire truck ladder leaning on a vertical fence also leans on the vertical wall of a burning three-storey building as shown in the figure. How high does the ladder reach?



15. Solve for the indicated distance across the lake.



➤ Activity 15: Ratios of Perimeters, Areas and Volumes of Similar Solids

Study the table that shows the base perimeters, base areas, lateral surface areas, total surface areas, and volumes of cubes.

Cube		Larger Cube	Smaller Cube	Ratio (Larger Cube: Smaller Cube)
Side	s	5	3	5 : 3
Perimeter P of the Base	$P = 4s$	20	12	20 : 12
Base Area	$BA = s^2$	25	9	25 : 9
Lateral Area	$LA = 4s^2$	100	36	100 : 36 = 25 : 9
Total Surface Area	$TA = 6s^2$	150	54	150 : 54 = 25 : 9
Volume	$V = s^3$	125	27	125 : 27

Questions

1. What do you observe about the ratio of the sides of the cubes and the ratio of their perimeters?
2. What do you observe about the ratio of the sides of the cubes and the ratio of their base areas? Lateral surface areas? Total surface areas? **Hint:** *Make use of your knowledge on exponents.*
3. What do you observe about the ratio of the sides of the cubes and the ratio of their volumes? **Hint:** *Make use of your knowledge on exponents.*
4. The ratio of the sides serves as a scale factor of similar cubes. From these scale factors, ratio of perimeters, base areas, lateral areas, total areas, and volumes of similar solids can be determined. From the activity we have learned that if the scale factor of two similar cubes is $m : b$, then

(1) the ratio of their perimeters is	
(2) the ratio of their base areas, lateral areas, or total surface areas is	
(3) the ratio of their volumes is	

Investigate the merits of the cube findings by trying it in similar spheres and similar rectangular prisms.

A. Sphere

Sphere		Smaller Sphere	Larger Sphere	Ratio (Larger Sphere : Smaller Sphere)
radius	r	3	6	
Total Surface Area	$A = 4\pi r^2$	36π	144π	
Volume	$A = \frac{4}{3}\pi r^3$	36π	288π	

B. Rectangular Prism

Rectangular Prism		Smaller Prism	Larger Prism	Ratio (Larger Prism : Smaller Prism)
Length	l	2	6	
Width	w	3	9	
Height	h	5	15	
Perimeter of the Base	$P = 2\,(l + w)$			
Base Area	$BA = lw$			
Lateral Area	$LA = 2h\,(l + w)$			

Total Surface Area	$TA = 2(lw + lh + wh)$			
Volume	$V = lwh$			

Question

1. Are the ratios for perimeters, areas, and volumes of similar cubes true also to similar spheres and similar rectangular prisms?
2. Do you think the principle is also true in all other similar solids? Explain.

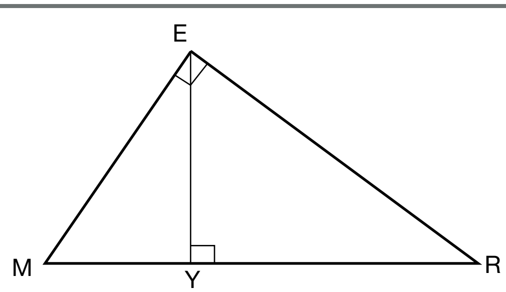
Investigation

1. Are all spheres and all cubes similar?
2. What solids are always similar aside from spheres and cubes?

➤ Activity 16: Right Triangle Similarity Theorem and Its Proof

Write the statements or reasons that are left blank in the proof of Right Triangle Similarity Theorem.

> **Right Triangle Similarity Theorem (RTST)**
> If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



Given

$\angle MER$ is a right triangle with $\angle MER$ as the right angle and $\overline{MR}$ as the hypotenuse.

$\overline{EY}$ is an altitude to the hypotenuse of ΔMER.

Prove

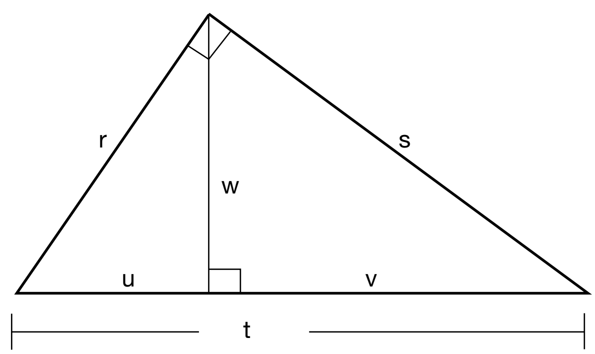
$\Delta MER \cong \Delta EYR \cong \Delta MYE$

Proof

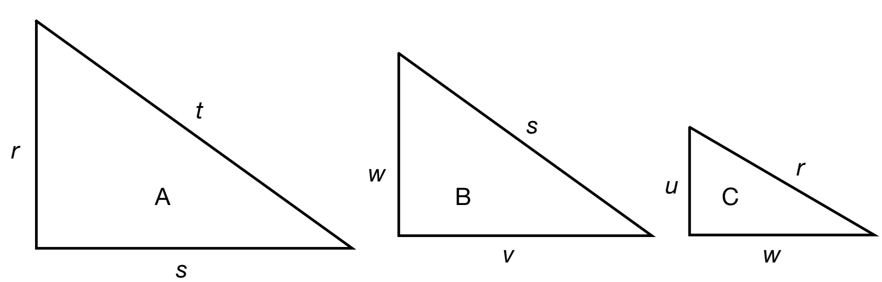
Statements	Reasons
1.1 ΔMER is a right triangle with $\angle MER$ as right angle and $\overline{MR}$ as the hypotenuse. 1.2 $\overline{EY}$ is an altitude to the hypotenuse of ΔMER.	1. ______________________

2. $\overline{EY} \perp \overline{MR}$	2. Definition of ______________
3. ∠MYE and ∠EYR are right angles.	3. Definition of _______________Lines
4. ∠ MYE ≅ ∠EYR ≅ ∠MER	4. Definition of ______________Angles
5. ∠YME ≅ ∠EMR ; ∠YRE ≅ ∠ERM	5. __________________ Property
6. ΔMYE ~ ΔMER ; ΔMER ~ ΔEYR	6. _______Similarity Theorem
7. ΔMER ≅ ΔEYR ≅ ΔMYE	7. __________________ Property

Special Properties of Right Triangles
When the altitude is drawn to the hypotenuse of a right triangle,
1. the length of the altitude is the geometric mean between the segments of the hypotenuse; and;
2. each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



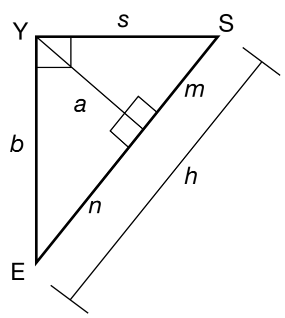
Separating the new right triangles formed from the original triangle:

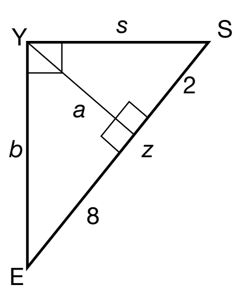


	Using the definition of Similar Polygons in Right Triangles:	
Altitude ***w*** is the geometric mean between ***u*** and ***v***.	*B* and *C*	$\frac{v}{w} = \frac{w}{u} \rightarrow w^2 = uv \rightarrow \boldsymbol{w = \sqrt{uv}}$
Leg ***r*** is the geometric mean between ***t*** and ***u***.	*A* and *C*	$\frac{t}{r} = \frac{r}{u} \rightarrow r^2 = ut \rightarrow \boldsymbol{r = \sqrt{ut}}$
Leg ***s*** is the geometric mean between ***t*** and ***v***.	*A* and *B*	$\frac{v}{s} = \frac{s}{t} \rightarrow s^2 = vt \rightarrow \boldsymbol{s = \sqrt{vt}}$

Quiz on Right Triangle Similarity Theorem

Fill in the blanks with the right lengths of the described segments and solve for the unknown sides of the similar triangles.

Figure	Description	Proportion
	The altitude of ΔYES, is the geometric mean between ____ and ____.	$\frac{m}{a} = \frac{a}{n}$
	The shorter leg ____ is the geometric mean between ____ and ____.	
	The longer leg ____ is the geometric mean between ____ and ____.	

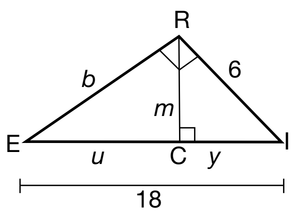


1. The corresponding sides of the similar triangles

	Original Triangle	New Larger Triangle	New Smaller Triangle
Hypotenuse			
Longer leg			
Shorter leg			

Solve for the geometric means *a*, *b*, and *s*.

Geometric Means	Proportion	Answer
Altitude *a*		
Shorter leg *s*		
Longer leg *b*		



1. The corresponding sides of the similar triangles

	Original Right Triangle	Larger New Triangle	Smaller New Triangle
Hypotenuse			
Longer leg			
Shorter leg			

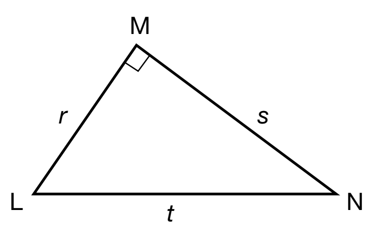
2. Solve for y, u, and b.

y	u	b	m

➤ Activity 17: The Pythagorean Theorem and Its Proof

Pythagorean Theorem
The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

Write the statements or reasons that are left blank in the proof of the Pythagorean Theorem.

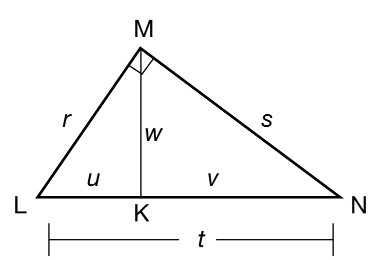


Given
- LM = r and MN = s as the legs;
- LN = t as the hypotenuse
- ∠LMN is a right angle.

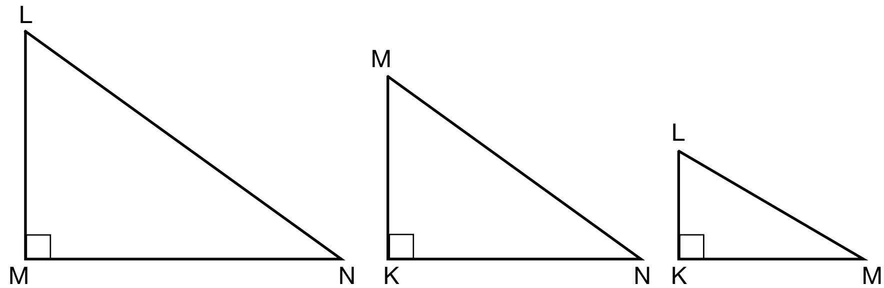
Prove

$$r^2 + s^2 = t^2$$

Proof
- Construct altitude MK = w to the hypotenuse LN = t, dividing it to LK = u and KN = c



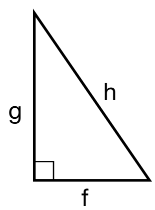
Separating the Right Triangles



	Hints	Statements	Reasons
1	Describe triangles LMN, MKN, and LKM when an altitude MK is drawn to its hypotenuse.	$\triangle LMN \square \triangle MKN \square \triangle LKM$	Right Triangle Similarity Theorem
2	Write the proportions involving the geometric means r and s.	$\frac{\square}{r} = \frac{r}{\square}$ $\frac{\square}{s} = \frac{s}{\square}$	Special Properties of Right Triangles
3	Cross-multiply the terms of the proportions in statement 2.		Cross-Multiplication Property of Proportions
4	Add s^2 to both sides of $r^2 = ut$ in statement 3.		Addition Property of Equality
5	Substitute s^2 on the right side of statement 4 using its equivalent from statement 3.		Substitution
6	Factor the right side of statement 5.		Common Monomial Factoring
7	Substitute $u + v$ in statement 6 by its equivalent length in the figure.		Segment Addition Postulate
8	Simplify the right side of statement 7.		Product Law of Exponents

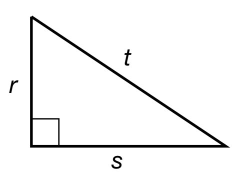
Quiz on the Pythagorean Theorem

A. Use the Pythagorean Theorem to find the unknown side of the given right triangle if two of its sides are given. Note that these lengths are known as Pythagorean triples. The last one is done for you.

Figure	Right Triangle	Shorter Leg f	Longer Leg g	Hypotenuse h	Solution
 	A	3		5	
	B	5	12		
	C		24	25	
	D	8	15		
	E	9		41	$f^2 + g^2 + = h^2$ $9^2 + g^2 = 41^2$ $81 + g^2 = 1681$ $g^2 = 1681 - 81$ $g^2 = 1600$ $g^2 = 16(100)$ $g = 4(10) = 40$

Questions

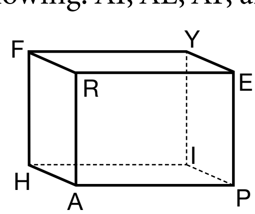
1. What do you observe about Pythagorean triples?
2. Multiply each number in a Pythagorean triple by a constant number. Are the new triples still Pythagorean triples? Explain.
3. For the right triangle shown, $t^2 = r^2 + s^2$. Which is equivalent to this equation?



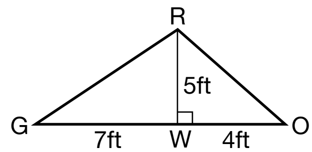
a. $r^2 = s^2 + t^2$
b. $t = r + s$
c. $r^2 = (t + s)(t - s)$
d. $s^2 = r^2 - t^2$

B. Solve the following problems using the Pythagorean Theorem.

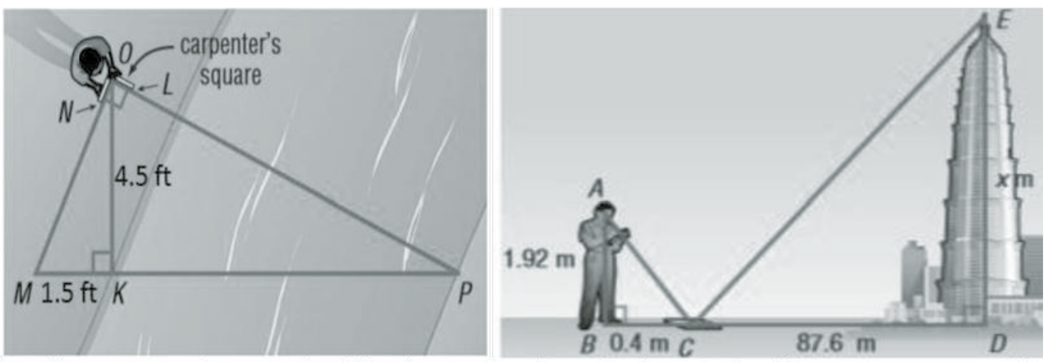
1. The size of a TV screen is given by the length of its diagonal. If the dimension of a TV screen is 16 inches by 14 inches, what is the size of the TV screen?
2. A 20-foot ladder is leaning against a vertical wall. If the foot of the ladder is 8 feet from the wall, how high does the ladder reach? Include an illustration in your solution.
3. The figure of rectangular prism shown is not drawn to scale. If AH = 3 cm, AP = 7 cm, and AR = 5 cm, find the following: AI, AE, AF, and AY?



4. The figure of the A-frame of a house is not drawn to scale. Find the lengths GR and OR.



5. Solve for the distance across the river and the height of the skyscraper whose top is reflected on the mirror.



http://www.augusta.k12.va.us/cms/lib01/VA01000173/Centricity/Domain/766/chap06%20Geometry.pdf

➤ Activity 18: Is the Triangle Right, Acute, or Obtuse?

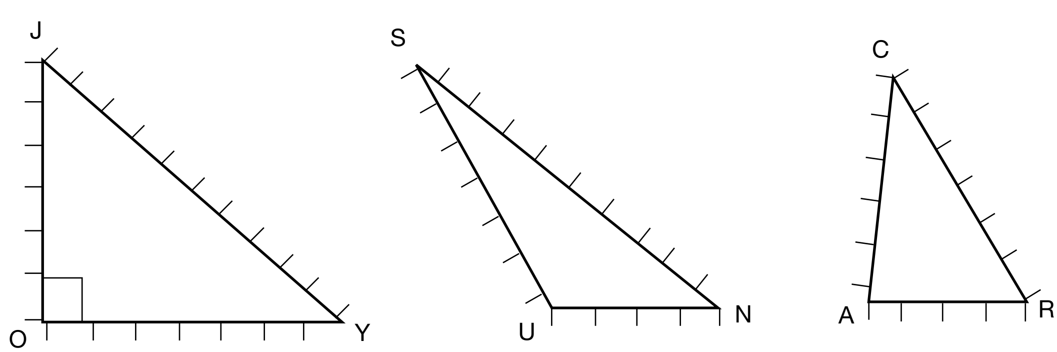
From the Pythagorean Theorem, you have learned that the square of the hypotenuse is equal to the sum of the squares of the legs. Notice that the hypotenuse is the longest side of a right triangle. What do you think is the triangle formed if the square of the longest side is:

- greater than the sum of the squares of the shorter sides?
- less than the sum of the squares of the shorter sides?

Try to predict the answer by stating two hypotheses:

- Hypothesis 1: If the square of the longest side is greater than the sum of the squares of the shorter sides, the triangle is __________________.
- Hypothesis 2: If the square of the longest side is less than the sum of the squares of the shorter sides, the triangle is __________________.

Test your hypotheses in this activity.



Complete the table that follows based on the length of the sides of the triangles in the figures.

Kind of Triangle	Name of Triangle	Lengths of			Squares of the Lengths of			
		Shorter sides		Longest Side	Shorter sides			Longest Side
		s_1	s_2	l	s_1^2	s_2^2	Sum	l^2
Right	JOY	6	8	10	36	64	100	100
Obtuse	SUN							
Acute	CAR							

Observations

1. A triangle is right if the square of the longest side is ____________the sum of the squares of the shorter sides.
2. A triangle is obtuse if the square of the longest side is ____________the sum of the squares of the shorter sides.
3. A triangle is acute if the square of the longest side is ____________the sum of the squares of the shorter sides.

Questions:

1. Do your hypotheses and results of your verification match? Why or why not?
2. How do you find predicting or hypothesis-making and testing in the activity?

Conclusion

Given the lengths of the sides of a triangle, to determine whether it is right, acute, or obtuse; there is a need to compare the square of the _________ side with the _________ of the squares of the two __________ sides.

Quiz on Determining the Kind of Triangle according to Angles

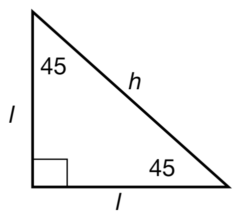
Triangle No.	Lengths of Sides of Triangles			Squares of the Lengths of			Kind of Triangle (Right , Acute, Obtuse Triangle)
	Shorter Sides		Longest Side	Shorter Sides		Longest Side	
	s_1	s_2	l	s_1^2	s_2^2	l_2	
1	7	8	10				
2	9	12	15				
3	3	6	7				

➤ Activity 19: 45-45-90 Right Triangle Theorem and Its Proof

45-45-90 Right Triangle Theorem
In a 45-45-90 right triangle:

- each leg is $\frac{\sqrt{2}}{2}$ times the hypotenuse; and
- the hypotenuse is $\sqrt{2}$ times each leg l

Write the statements or reasons that are left blank in the proof of 45-45-90 Right Triangle Theorem. Refer to the hints provided.



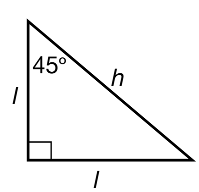
Given:
Right Triangle with
- leg = $\boldsymbol{l}$,
- hypotenuse = $\boldsymbol{h}$,

Prove:
- $\boldsymbol{h = \sqrt{2}\,l}$
- $\boldsymbol{l = \frac{\sqrt{2}}{2}h}$

	Hints	Statements	Reasons
1	List down all the given.	Right triangle with leg = $\boldsymbol{l}$, hypotenuse = $\boldsymbol{h}$	Given
2	Write an equation about the measures of the legs and the hypotenuse and simplify.	$l^2 + l^2 + = h^2 \rightarrow \boldsymbol{2l^2 = h^2}$	
3	Solving for h in statement 2		$\sqrt[e]{b^e} = b$
4	Solving for l in statement 3		
			Rationalization of Radicals

Quiz on 45-45-90 Right Triangle Theorem

A. Fill in the blanks with their measures using the formulas derived from the proof of the 45-45-90 right triangle theorems.

Figure	Formula	If	Then
	Leg = $\frac{\sqrt{2}}{2}$ hyp.	$h = 5$	$l =$ ____________
	Hyp. = $\sqrt{2}$ hyp.	$l = 12$	$h =$ ____________

B. Solve the following problems using the 45-45-90 Right Triangle Theorem.

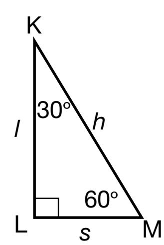
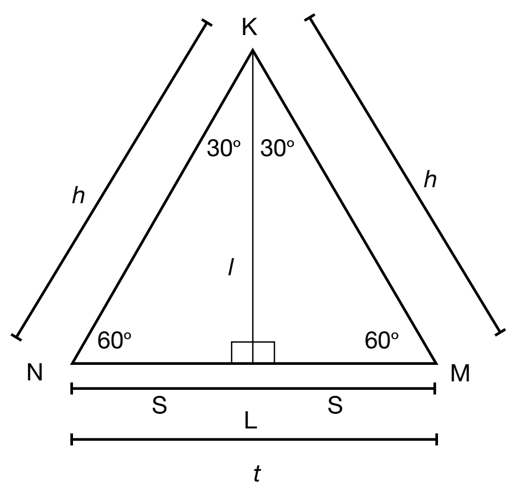
1. A square-shaped handkerchief measures 16 inches on each side. You fold it along its diagonal so you can tie it around your neck. How long is this tie?
2. You would like to put tassel around a square table cloth. If its diagonal measures 8 feet, what is the length of the tassel you need to buy?

➤ Activity No. 20: 30-60-90 Right Triangle Theorem and Its Proof

30-60-90 Right Triangle Theorem
In a 30-60-90 right triangle:

- the shorter leg is $\frac{1}{2}$ the hypotenuse h or $\frac{\sqrt{2}}{2}$ times the longer leg ;
- the longer leg l is $\sqrt{3}$ times the shorter leg s; and
- the hypotenuse h is twice the shorter leg

Directions: Write the statements or reasons that are left blank in the proof of 30-60-90 Right Triangle Theorem. Refer to the hints provided to help you.

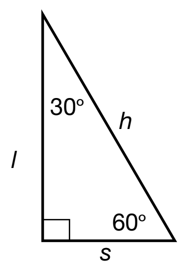
<table>
<tr>
<td>
</td>
<td>Given
Right ΔKLM with
• hypotenuse KM = h,
• shorter leg LM = s,
• longer leg KL = l
• m ∠LKM = 30
• m ∠KNL = 60</td>
<td>Prove
• $h = 2s$ *
• $s = \frac{1}{2}h$ **
• $l = \sqrt{3}s$ ***
• $s = \frac{\sqrt{3}}{3}l$ ****</td>
</tr>
<tr>
<td colspan="3">Proof: Construct a right triangle equivalent to the given triangle with the longer leg l as the line of symmetry such that: ∠IKM = 30 and ∠KNL = 60; KN = h , and LN = s.

</td>
</tr>
</table>

Clues		Statements	Reasons
1	List down all the given.	Right ΔKLM with m ∠LMK = 60; m ∠LKM = 30; KM = ____ ; LM =____ ; KL = ____	
2	List down all constructed angles and segments and their measures.	ΔKLM = ΔKLN; m ∠LKN = 30; m ∠KNL = 60; *KN* = _____, and LN = ______	
3	Use Angle Addition Postulate to ∠LKM and ∠MKN.	m∠______ = m∠LKM + m ∠LKN	
4	What is m∠MKN? Simplify.	m ∠MKN =	Substitution
5	What do you observe about △MKN considering its angles?	ΔMKN is _____________ triangle.	Definition of Equiangular Triangle
6	What conclusion can you make based from statement 5?	ΔMKN is ________________.	Equiangular Triangle is also equilateral.
7	With statement 6, what can you say about the sides of △MKN?	*KM* = *KN* = *MN* = ____	Definition of Equilateral Triangle
8	Use Segment Addition Postulate for LN and ML	*LN* + *ML* = ____	Segment Addition Postulate
9	Replace LN, ML, and MN with their measurements and simplify.	____ + ____ = ____→ 2 ____= ____	
10	What is the value of *h*?	*	__________ Property of Equality
11	Solve for *s* using statement 9.	**	___________ Property of Equality
12	What equation can you write about *s*, *l*, and *h*?		Pythagorean Theorem

13	Use statement 10 in statement 13.	$s^2 + l^2 = (\_\_\_\_\_\_\_\_\_)^2$	Substitution
14	Simplify the right side of statement 13.	$s^2 + l^2 = \_\_\_\_\_\_\_\_\_$	Power of a Product Law of Exponents
15	Solve for l^2.		Subtraction Property of Equality
16	Solve for l and simplify.	$l = \sqrt{3s^2} \rightarrow \_\_\_\_\_\_\_\_\_\_$ ***	$\sqrt[e]{b^e} = b$ law of radicals
17	Solve for s in statement 16.	$s = \frac{l}{\sqrt{3}} = \frac{l}{\sqrt{3}}(\quad) = \frac{\sqrt{3}l}{3}$****	Division Property of Equality and Rationalization of Radicals

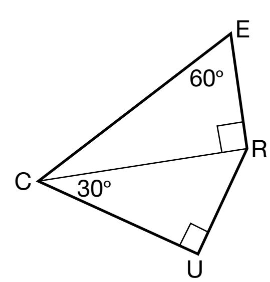
Quiz on 30-60-90 Right Triangle Theorem

A. Fill in the blanks with their measures using the formulas derived from the proof of the 30-60-90 right triangle theorems.

Figure		If	Then
	$s = \frac{h}{2}$	Shorter leg $s = 6$	Longer leg $l =$ ________ Hypotenuse $h =$ ________
	$s = \frac{\sqrt{3}}{3}l$	Hypotenuse $h = 10$	Shorter leg $s =$ ________ Longer leg $l =$ ________
	$l = 7\sqrt{3}s$ $h = 2s$	Longer leg $l = 7\sqrt{3}$	Shorter leg $s =$ ________ Hypotenuse $h =$ ________

B. Solve the following problems using the 30-60-90 Right Triangle Theorem.

1. A cake is triangular in shape. Each side measures 1 foot. If the cake is subdivided equally into two by slicing from one corner perpendicular to the opposite side, how long is that edge where the cake is sliced?
2. IF CR = 8 cm, find CU, RU, ER, and EC?



You have successfully helped in illustrating, proving, and verifying the theorems on similarity of triangles. All the knowledge and skills you've learned in this section will be useful in dealing with the next section's problems and situations that require applications of these principles.

What to REFLECT and UNDERSTAND

Having illustrated, proved, and verified all the theorems on similarity in the previous section, your goal in this section is to take a closer look at some aspects of the topic. This entails more applications of similarity concepts.

Your goal in this section is to use the theorems in identifying unknown quantities involving similarity and proportion.

Your success in this section makes you discover math-to-math connections and the role mathematics, especially the concepts of similarity, plays in our real-world experiences.

➤ Activity 21: Watch Your Rates

A 6-inch-by-5-inch picture is a copy that was reduced from the original one by reducing each of its dimensions by 40%. In short, each dimension of the available copy is 60% of the original one. You would like to enlarge it back to its original size using a copier. What copier settings would you use?

If each dimension of the available picture is 60% of the original one, then we can make the following statements to be able to determine the dimensions of the original picture:

1. The length of 6 inches is 60% of the original length L. Mathematically, it means that $6 = 60\%\ (L)$. That is, $L = \frac{6}{0.6} = 10$ inches.
2. The width of 5 inches is 60% of the original width W. Mathematically, it means that $5 = 60\%\ (W)$. That is, $W = \frac{5}{0.6} = 8\frac{1}{3}$ inches.

To determine the copier settings to use to be able to increase the 6-inch-by-5-inch picture back to the 10-inch-by- $8\frac{1}{3}$ -inch, the following statements should also be used:

1. The original length of 10 inches is what percent of 6? Mathematically, it means that $10 = \textit{rate}\ R\ (6)$. That is, $R = \frac{10}{6} = \frac{5}{3} \approx 1.67 \approx 167\%$.
2. The original width of $8\frac{1}{3}$ inches is what percent of 5? Mathematically, it means that $8\frac{1}{3} = \textit{rate}\ R(5)$. That is, $R = \frac{8\frac{1}{3}}{5} = \frac{\frac{25}{3}}{5}$

$$= \left(\frac{25}{3}\right)\left(\frac{1}{5}\right) = \frac{5}{3} \approx 1.67 \approx 167\%.$$

Therefore, the copier should be set at 167% the normal size to convert the picture back to its original size.

Questions

1. What is the scale factor used to compare the dimensions of the available picture and original? (Hint: Get the ratio of the lengths or the widths.)
2. If it was reduced by 40% before, why is it that we are not using the copier settings of 140% and use 167% instead?

   The differences in the dimensions are the same. However, the rate of conversion from the original size to the __________ size and the reduced size back to the __________ size differ because the initial __________ used in the computation are different.

The conversion factor is the quotient between the target dimension D_t and the initial dimension D_i. That is, $R = \frac{D_t}{D_i}$.

To validate the 60%, the computation is as follows:

$$R = \frac{D_t}{D_i} = \frac{L_t}{L_i} = \frac{6}{10} \approx 0.6 \approx 60\%$$

To validate the 167%, the computation is as follows:

$$R = \frac{D_t}{D_i} = \frac{L_t}{L_i} = \frac{10}{6} \approx 1.67 \approx 167\%$$

Note however that the rate of increase or decrease $\left(R_{\uparrow} \text{ or } R_{\downarrow}\right)$ is simply the quotient of the difference of compared dimensions (target dimension minus initial dimension) divided by the initial dimension. That is, $R = \frac{D_t - D_i}{D_i}$.

To validate the rate of increase from a length of 6 to the length of 10,

$$R_{\uparrow} = \frac{D_t - D_i}{D_i} = \frac{L_t - L_i}{L_i} = \frac{10 - 6}{6} = \frac{4}{6} = \frac{2}{3} \approx 0.67 = 67\%.$$

To validate the rate of decrease from a length of 10 to the length of 6,

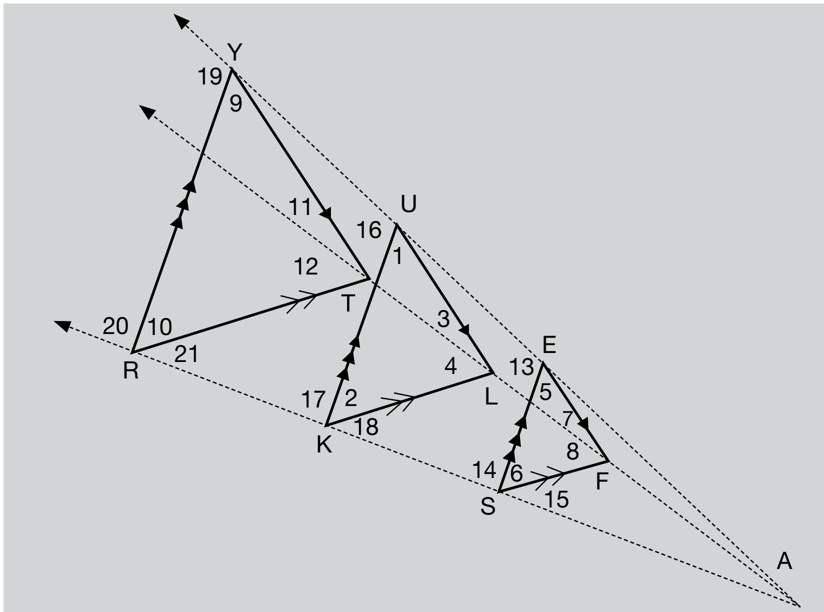
$$R_{\downarrow} = \frac{D_t - D_i}{D_i} = \frac{L_t - L_i}{L_i} = \frac{6 - 10}{6} = \frac{-4}{10} \approx -0.4 = -40\% = 40\%.$$

Be reminded that the negative sign signifies a reduction in size. However, the negative sign should be ignored.

Problem

Instead of enlarging each dimension of a document by 20%, the dimensions were erroneously enlarged by 30% so that the new dimensions are now 14.3 inches by 10.4 inches. What are the dimensions of the original document? What are the desired enlarged dimensions? What will you do to rectify the mistake if the original document is no longer available?

➤ Activity 22: Dilation: Reducing or Enlarging Triangles

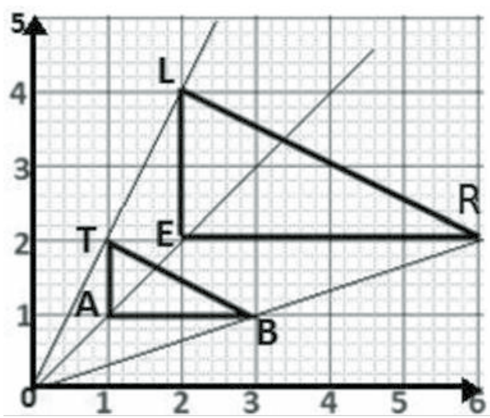


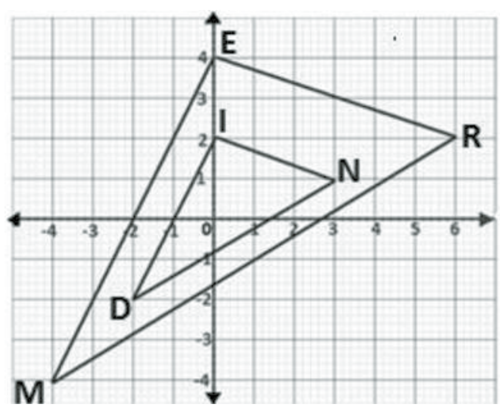
Triangles KUL and RYT are similar images of the original triangle SEF through dilation, the extending of rays that begin at a common endpoint A. The point A is called the center of dilation. Give justifications to the statements of its proof.

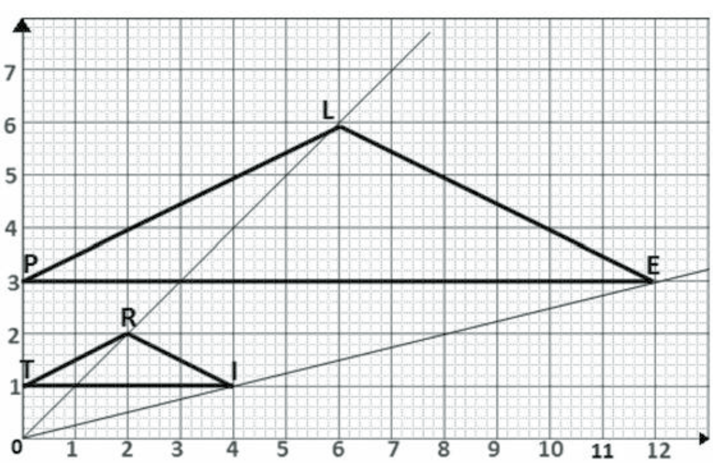
	Statements	Reasons
1.	• $\overline{RY} \parallel \overline{KU} \parallel \overline{SE}$ • $\overline{YT} \parallel \overline{UL} \parallel \overline{EF}$ • $\overline{RT} \parallel \overline{KL} \parallel \overline{SF}$	___________
2.	• ∠20 ≅ ∠17 ≅ ∠14 and ∠19 ≅ ∠16 ≅ ∠13 • ∠TYU ≅ ∠LUE ≅ ∠FEA • ∠21 ≅ ∠18 ≅ ∠15 and ∠12 ≅ ∠4 ≅ ∠8	___________ angles are congruent.
3.1	• m ∠20 + m ∠21 + m ∠10 = 180 • m ∠17 + m ∠18 + m ∠2 = 180 • m ∠14 + m ∠15 + m ∠6 = 180	___________ on a Straight Line Theorem
3.2	• m ∠19 + m ∠TYU + m ∠9 = 180 • m ∠16 + m ∠LUE + m ∠1 = 180 • m ∠13 + m ∠FEA + m ∠5 = 180	

4.	• m ∠20 + m ∠21 + m ∠2 = 180 • m ∠20 + m ∠21 + m ∠6 = 180	Substitution
	• m ∠19 + m ∠TYU + m ∠1 = 180 • m ∠19 + m ∠TYU + m ∠5 = 180	
5.	m ∠20 + m ∠21 + m ∠10 = m ∠20 + m ∠21 + m ∠2 = m ∠20 + m ∠21 + m ∠6	________________ Property of Equality
	m ∠19 + m ∠TYU + m ∠9 = m ∠19 + m ∠1 = m ∠19 + m ∠TYU + m ∠5	
6.	m ∠10 = m ∠2 = m ∠6	____________ Property of Equality
	m ∠9 = m ∠1 = m ∠5	
7.	△RYT ~ △KUL ~ △SEF	_____Similarity Theorem

Write the coordinates of each point of the following similar triangles in the table provided.







Triangles	Coordinates of		Triangles	Coordinates of		Triangles	Coordinates of	
$\triangle TAB$	T		$\triangle MER$	M		$\triangle TRI$	T	
	A			E			R	
	B			R			I	
$\triangle LER$	L		$\triangle DIN$	D		$\triangle PLE$	P	
	E			I			L	
	R			N			E	

Questions

1. Compare the abscissas of the corresponding vertices of the triangles. What do you observe?
   The abscissas of the larger triangles are ______________ of the abscissas of the smaller triangles.
2. Compare the ordinates of the corresponding vertices of the triangles. What do you observe?
   The ordinates of the larger triangles are ______________ of the ordinates of the smaller triangles.
3. What is the scale factor of $\triangle TAB$ and $\triangle LER$? $\triangle MER$ and $\triangle DIN$? $\triangle TRI$ and $\triangle PLE$?

Scale Factor	Similar Triangles		
	$\triangle TAB$ and $\triangle LER$	$\triangle MER$ and $\triangle DIN$	$\triangle TRI$ and $\triangle PLE$

4. How is scale factor used in the dilation of figures on a rectangular coordinate plane?

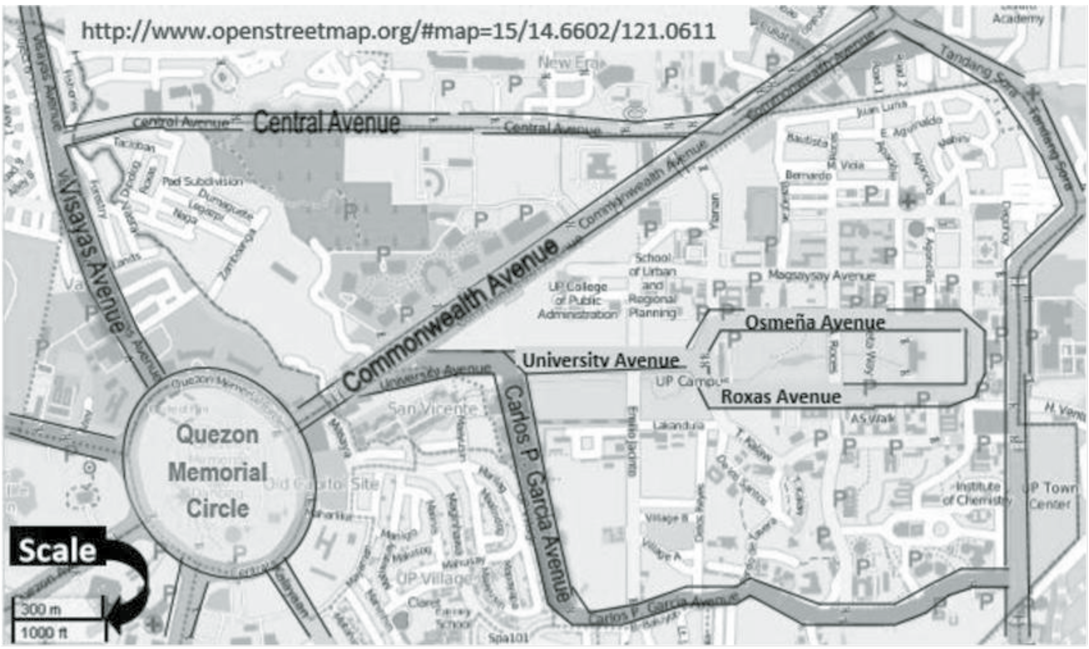
Scale drawing

> Scale drawing is ensuring that the dimensions of an actual object are retained proportionally as the actual object is enlarged or reduced in a drawing.
>
> You have learned that scale factor is the uniform ratio of corresponding proportional sides of similar polygons. Scale, on the other hand, is the ratio that compares dimensions like length, width, altitude, or slant height in a drawing to the corresponding dimensions in the actual object.
>
> The most popular examples of scale drawing are maps and floor plans.

➤ Activity 23: Avenues for Estimation

Estimation is quite important in finding distances using maps because streets or boulevards or avenues being represented on maps are not straight lines. Some parts of these streets may be straight but there are always bends and turns.

The map shows a portion of Quezon City, Philippines. The elliptical road on the map bounds the Quezon Memorial Circle. The major streets evident on the map include the following: (1) part of Commonwealth Avenue from Quezon Memorial Circle to Tandang Sora Avenue, (2) University Avenue that leads to University of the Philippines (UP) in Diliman and branch out to Carlos P. Garcia Avenue and the Osmeña and Roxas Avenues inside the UP Campus, (3) Central Avenue, and (4) part of Visayas Avenue from Quezon Memorial Circle to Central Avenue. Notice that the scale used in this map is found in the lowest left hand corner of the map. You can view this map online using the link found on the map.



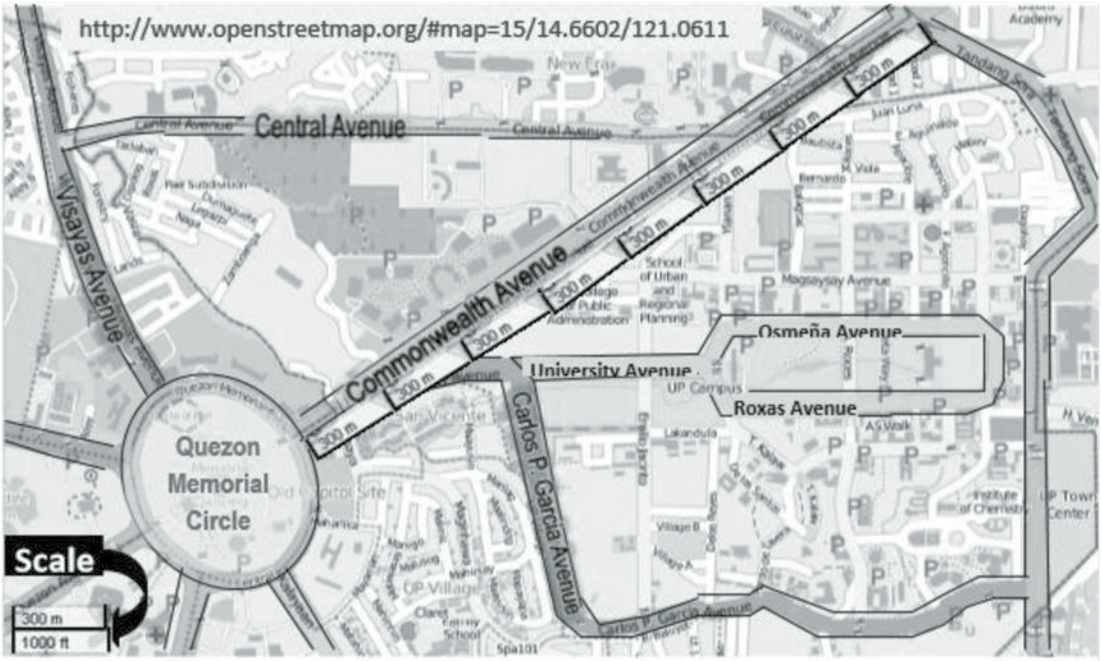
The length of the scale is equivalent to 300 meters. That is, 1:300 m. That ratio can also be written as $\frac{1}{300\,\text{m}}$. Using the scale, the approximate distance of Commonwealth Avenue from Quezon Memorial Circle to Tandang Sora Avenue is estimated on the next map.

Observe that more than eight lengths of the scale make up Commonwealth Avenue starting from the Quezon Memorial Circle to Tandang Sora junction. With several copies of the length of the scale, the distance d in meters (m) of this part of Commonwealth Avenue can already be computed as shown on the right:

$$\frac{1}{300} = \frac{8}{d}$$
$$d = 8(300)$$
$$d \approx 2400 \text{ meters}$$

Instead of using the equality symbol, we use the symbol $\approx$ for approximate equality in the final answer. The reason is that distances determined using maps are approximate distances. There is always a margin of error in these estimated distances. However, ensuring that errors in estimating distances are tolerable should always be observed.

One part of Commonwealth Avenue is approximately equal to 2400 meters. Suppose a fun run includes Commonwealth Avenue and you are at Tandang Sora junction, how long will it take you to reach the Quezon Memorial Circle if your speed while running is 120 meters per minute? To answer this problem, distance formula should be used.



Note that distance *d* covered while traveling is the product of the speed or rate *r* and time *t*. That is, *d*= *rt*. It takes 20 minutes to reach the Quezon Memorial Circle with that speed, as shown in the solution on the right?

$$D = rt$$
$$2400 = 120t$$
$$\frac{2400}{120} = \frac{120t}{120}$$
$$t = 20min$$

Quiz on estimating Distances Using Maps

A. Using the length *l* of the scale on the map, calibrate a meter of white thread. Use this in estimating the distances of the streets listed on the table shown.

Streets	Distance on the Map	Actual Street Distance
Elliptical road around the Quezon Memorial Circle		
University Avenue (from Commonwealth Avenue to UP Campus)		
Carlos P. Garcia Avenue		
Central Avenue (from Visayas Avenue to Commonwealth Avenue)		
Visayas Avenue (from Quezon Memorial Circle to Central Avenue)		

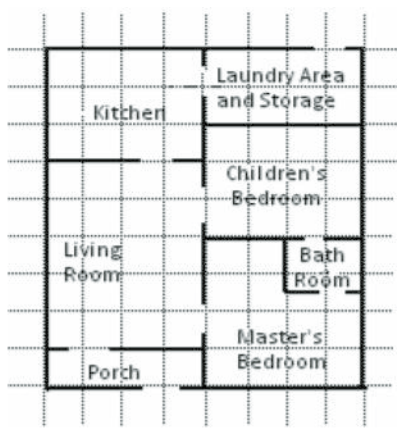
B. If you walk at 60 meters per minute, how long will it take you to cover the distance around Roxas and Osmeña Avenues inside the UP Campus?

C. Questions

1. Do you think that maps are important?
2. Have you ever tried estimating distances using the scale on the map before this lesson?
3. How do you find the exercise of estimating distances using maps?

➤ Activity 24: Reading a House Plan

Directions: Given the floor plan of the house, accomplish the table that gives the floor areas of the parts of the house. Use the scale **1 s : 1 m**. Note that the length and width of a square are congruent.



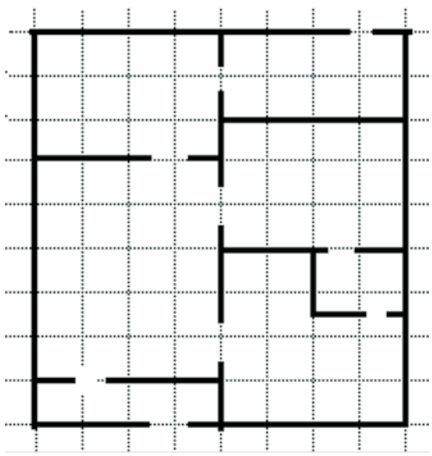
The floor plan shown is a proportional layout of a house that a couple would like to build. Observe that the floor plan is drawn on a square grid. Each side *s* of the smallest square in the square grid measures 0.5 cm and corresponds to 1 meter in actual house. Hence, the scale used in the drawing is **0.5 cm : 1 m** or **1s : 1 m.**

Parts of the House	Scale Drawing Dimensions		Actual House Dimensions		Floor Area
	Length	Width	Length	Width	
Porch					
Master's Bedroom with Bathroom					
Bathroom Alone					
Living Room					
Kitchen					
Children's Bedroom					
Laundry Area and Storage					
Whole House					

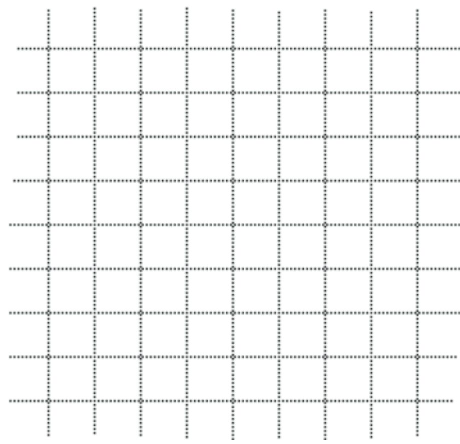
Questions

1. Which room of the house has the largest floor area?
2. Which rooms have equal floor areas?
3. Without considering the area of the bathroom, which is larger: the master's bedroom or the other bedroom? How much larger is it (use percent)?

4. Why do you think that the living room is larger?
5. Gaps in the layout of the parts of the house represent doors. Do you agree with how the doors are placed? Explain.
6. Do you agree with how the parts of the house are arranged? Explain.
7. If you were to place cabinets and appliances in the house, how would you arrange them? Show it on a replicated Grid A.
8. If you had to redesign the house, how would you arrange the parts if the dimensions of the whole house remain the same? You may eliminate and replace other parts. Use a replicated Grid B.



Grid A



Grid B

9. Make a floor plan of your residence. If your house is two- or three-storey, just choose a floor to layout. Don't forget to include the scale used in the drawing.

Quiz on Scale Drawing

A. The scale of a drawing is 3 in : 15 ft. Find the actual measurements for:

1. 4 in.	2. 6 in.	3. 9 in.	4. 11 in.

B. The scale is 1 cm : 15 m. Find the length each measurement would be on a scale drawing.

5. 150 m	6. 275 m	7. 350 m	8. 400 m

C. Tell whether the scale reduces, enlarges or preserves the size of an actual object.

9. 1 m = 10 cm	10. in. = 1 ft.	11. 100 cm = 1 m

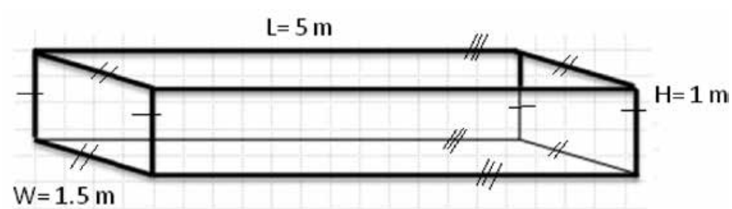
Problem Solving

12. On a map, the distance between two towns is 15 inches. The actual distance between them is 100 kilometers. What is the scale?
13. Blueprints of a house are drawn to the scale of 3 in. : 1 m. Its kitchen measures 9 inches by 6 inches on the blueprints. What is the actual size of the kitchen?
14. A scale model of a house is 1 ft. long. The floor of the actual house is 36 ft. long. In the model, the width is 8 inches. How wide is the actual house?
15. A model of a skyscraper is 4 cm wide, 7 cm long, and 28 cm high. The scale factor is 20 cm : 76 m. What are the actual dimensions of the skyscraper?

> Your transfer task requires you to sketch a floor plan of a couple's house. You will also make a rough cost estimate of building the house. In order that you would be able to do the rough cost estimate, you need knowledge and skills in proportion, measurement, and some construction standards. Instead of scales, these standards refer to rates because units in these standards differ.

➤ Activity 25: Costimation Exercise!

In aquaculture, culturing fish can be done using a fish tank. How much does it cost to construct a rectangular fish tank whose dimension is 5 m × 1.5 m × 1 m?



Complete the following table of bill of materials and cost estimates:

Materials		Quantity	Unit Cost	Total
1	CHB 4" × 8" × 16"			
2	Gravel			
3	Sand			
4	Portland Cement			
5	Steel Bar (10 mm.)			
6	Sahara Cement			
7	PVC ¾"	5 pcs		
8	PVC Elbow ¾"	6 pcs		
9	PVC 4"	1 pc		
10	PVC Solvent Cement	1 small can		

11	Faucet	1 pc		
12	G.I. Wire # 16	1 kg		
13	Hose 5 mm	10 m		
			Grand Total	

The number of bags of cement, cubic meters of sand and gravel, and number of steel bars can be computed using the following construction standards:

Table 1

QUANTITY FOR 1 CUBIC METER (cu m or m^3)									
Using 94 Lbs Portland Cement					Using 88 Lbs Portland Cement				
Class	Proportion	Cement in bags	Sand by cu m	Gravel by cu m	Class	Proportion	Cement in bags	Sand by cu m	Gravel by cu m
AA	1 : 2 : 3	10.50	0.42	0.84	A	1 : 2 : 4	8.20	0.44	0.88
A	1 : 3 : 4	7.84	0.44	0.88	B	1 : 2 : 5	6.80	0.46	0.88
B	1 : 2.5 : 5	6.48	0.44	0.88	C	1 : 3 : 6	5.80	0.47	0.89
C	1 : 3 : 6	5.48	0.44	0.88	D	1 : 3.5 : 7	5.32	0.48	0.90
D	1 : 3.5 : 7	5.00	0.45	0.90					

Table 2

Size of CHB	No. of CHB Laid Per Bag of Cement	Volume of Cement Per CHB	CHB Finish Per Square Meter		
				No. of Bags of Cement	Volume of Sand
4" × 8" × 16"	55 to 60 pieces	0.001 cu m	Tooled Finish	0.125	0.0107 m^3
6" × 8" × 16"	30 to 36 pieces	0.003 cu m	Plaster Finish	0.250	0.0213 m^3
8" × 8" × 16"	25 to 30 pieces	0.004 cu m			

Table 3

REQUIREMENTS FOR MORTAR			
Kinds	Mix	Cement	Sand
Plain Cement Floor Finish	1 : 2	0.33 bag/sq m	0.00018 cu m/sq m
Cement Plaster Finish	1 : 2	0.11 bag/sq m	0.006 cu m/sq m
Pebble Wash Out Floor Finish	1 : 2	0.43 bag/sq m	0.024 cu m/sq m
Laying of 6" CHB	1 : 2	0.63 bag/sq m	0.37 cu m/sq m
4" Fill All Holes and Joints	1 : 2	0.36 bag/sq m	0.019 cu m/sq m
Plaster Perlite	1 : 2	0.22 bag/sq m	0.12 cu m/sq m
Grouted Riprap	1 : 2	4 bag/sq m	0.324 cu m/sq m

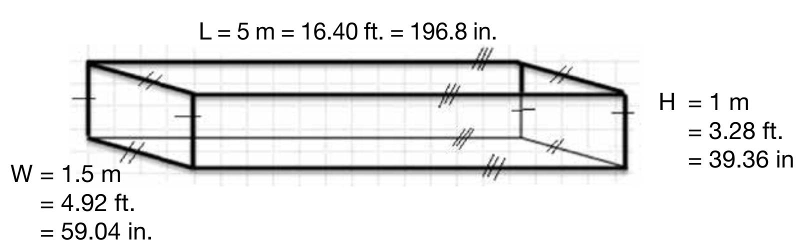
Table 4

CHB-Reinforcement					
Spacing of Vertical Bars (in meters)	Length of Bars (in meters)		Horizontal Bars for Every no. of Layers Per block	Length of Bars (in meters)	
	Per block	Per sq m		Per block	Per sq m
0.4	0.25	3.0	2	0.22	2.7
0.6	0.17	2.1	3	0.15	1.9
0.8	0.12	1.5	4	0.13	1.7
			5	0.11	1.4

Solution

Since the concrete hollow blocks (CHB) are measured in inches, there is a need to convert the dimensions from meter to feet, then to inches such as shown in the next figure.

From	To	Conversion Strategy
meter	feet	Divide by 0.3048
feet	inches	Multiply by 12



Reminders

The initial steps of the solutions are shown. Your task is to finish the solutions to get the required answers.

A. Computing for the number of concrete hollow blocks (CHB)

Important things to note:

- Whatever the thickness of the CHB (4", 6", or 8"), the W × L dimension of the face is always 8" × 16".
- The fish tank is open at the top and the base is not part of the wall. Hence, lateral area includes the rectangular faces with dimensions height H and length L (2 faces) and height and width *W* (2 faces).

Solving for the number of CHB needed:

$$\frac{1\text{ CHB}}{\text{Area of the face of CHB in sq. in.}} = \frac{\text{total no. of CHB needed}}{\text{Lateral Area of the fish tank in sq. in.}}$$

$$\frac{1\text{ CHB}}{\text{LW}} = \frac{\text{total no. of CHB}}{2\text{LH} + 2\text{WH}}$$

$$\frac{1\text{ CHB}}{\text{LW}} = \frac{\text{no. of CHB}}{2\text{H}(\text{L} + \text{W})}$$

B. Computing for the number of bags of cement needed for laying 160 CHB

Table 2 shows that the standard is that 1 bag of cement is enough to lay down 55 to 60 pieces of 4" × 8" × 16" CHB. Using this standard, the total number of bags of cement can be computed as follows:

$$\frac{1\text{ bag of cement}}{55\text{ CHB}} = \frac{\text{total no. of bags of cement}}{160\text{ CHB}}$$

$$(55\text{ CHB})(\text{total no. of bags of cement}) = 160\text{ CHB}$$

C. Computing for the number of bags of cement and volume of sand for CHB plaster finish

Solving for the number of bags of cement needed for CHB plaster finish:

$$\frac{0.25\text{ bag of cement}}{1\text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{\text{Lateral Area of the Fish Tank inside and outside in sq. m.}}$$

$$\frac{0.25}{1\text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{2\left[2\text{H}(\text{L} + \text{W})\right]}$$

$$\frac{0.25}{1\text{ sq. m.}} = \frac{\text{total no. of bags of cement}}{2\left[2(1)(5 + 1.5)\right]}$$

Solving for the volume of sand in cu. m. needed for CHB plaster finish:

$$\frac{0.0213 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{\text{Lateral area of the fish tank inside and outside in sq m}}$$

But lateral area in square meters is already known upon determining the total number of bags of cement needed for CHB plaster finish:

$$\frac{0.0213 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{26 \text{ sq m}}$$

D. Computing for volume of concrete (the no. of bags of 94 lbs cement and volume of sand and gravel) for fish tank flooring using Class A

Flooring should be 4 inches deep. Since sand and gravel are bought by cubic meter (cu. m.), 4-inch depth has to be converted to meter.

Depth of concrete flooring in meters = $4 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} \times \frac{1 \text{m}}{100 \text{ cm}} = 0.10 \text{ m}$

Table 1 shows that the standards for flooring using Class A are as follows:

- 7.84 bags of 94-lbs Portland cement per cu m
- 0.44 cu m of sand per cu m
- 0.88 cu m of gravel per cu m

Solving for the number of number of bags of cement needed for Class A flooring:

$$\frac{7.84 \text{ bags of cement}}{1 \text{ cu m}} = \frac{\text{total number of bags of cement}}{\text{Volume of concrete}}$$

$$\frac{7.84}{1 \text{ cu m}} = \frac{\text{Total number of bags of cement}}{\text{Floor Length} \times \text{Floor Width} \times \text{Depth of Concrete}}$$

$$\frac{7.84}{1 \text{ cu m}} = \frac{\text{Total number of bags of cement}}{(5)(1.5)(0.10)}$$

Solving for the volume of sand in cu m needed for Class A flooring:

$$\frac{0.44 \text{ cu m}}{1 \text{ cu m}} = \frac{\text{Volume of sand in cu m}}{\text{Volume of concrete in cu m}}$$

$$\frac{0.44 \text{ cu m}}{1 \text{ cu m}} = \frac{\text{Volume of sand in cu m}}{0.75 \text{ cu m}}$$

Solving for the volume of gravel in cu m needed for Class A flooring:

$$\frac{0.88 \text{ cu m}}{1 \text{ cu m}} = \frac{\text{Volume of gravel in cu m}}{\text{Volume of concrete in cu m}}$$

$$\frac{0.88 \text{ cu m}}{1 \text{ cu m}} = \frac{\text{Volume of gravel in cu m}}{0.75 \text{ cu m}}$$

E. Computing for the number of bags of cement and volume of sand for mortar of the walls using 4" Fill All Holes and Joints.

Table 3 shows that the standards for 4-Inch-Fill-All-Holes-and-Joints mortar are as follows:

- 0.36 bag of cement per sq m
- 0.019 cu m of sand per sq m

Solving for the number of bags of cement needed for mortar:

$$\frac{0.36 \text{ bag of cement}}{1 \text{ sq m}} = \frac{\text{total number of bags of cement}}{\text{Lateral area of the inside wall}}$$

$$\frac{0.36 \text{ bag of cement}}{1 \text{ sq m}} = \frac{\text{total number of bags of cement}}{13 \text{ sq m}}$$

Solving for the volume of sand in cu. m. needed for mortar:

$$\frac{0.019 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{\text{Lateral area of the inside wall}}$$

$$\frac{0.019 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{13 \text{ sq m}}$$

F. Computing for the number of bags of cement and volume of sand for plain cement floor finish using Class A 94-lbs cement

Table 3 shows that the standards for plain cement floor finish using Class A 94-lbs cement are as follows:

- 0.33 bag of cement per sq m.
- 0.00018 cu m of sand per sq m

Solving for the number of number of bags of cement needed for mortar:

$$\frac{0.33 \text{ bag of cement}}{1 \text{ sq m}} = \frac{\text{total number of bags of cement}}{\text{floor area}}$$

$$\frac{0.33 \text{ bag of cement}}{1 \text{ sq m}} = \frac{\text{total number of bags of cement}}{LW}$$

$$\frac{0.33 \text{ bag of cement}}{1 \text{ sq m}} = \frac{\text{total number of bags of cement}}{(5)(1.5)}$$

Solving for the volume of sand in cu m needed for mortar:

$$\frac{0.00018 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{\text{floor area}}$$

$$\frac{0.00018 \text{ cu m}}{1 \text{ sq m}} = \frac{\text{Volume of sand in cu m}}{7.5 \text{ sq m}}$$

G. **Computing for the number of needed steel bars**

The number of steel bars needed is the quotient between the sum of the lengths of horizontal bars (HB), vertical bars (VB) and floor bars (FB) divided by the ***standard length of each bar***, which is **20 ft.** or **6.096 m**.

Table 4 shows that:

- If horizontal bar is placed for every two layers, 2.7 m bar per sq m
- If 0.4 spacing is used for vertical bars, 3 m bar per sq m
- If 0.4 spacing is used for floor bars, 3 m per sq m

Solving for the Total Length of Horizontal Bars (for every 2 layers):

$$\frac{2.7 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of horizontal bar}}{\text{Lateral area of fish tank}}$$

$$\frac{2.7 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of horizontal bar}}{13 \text{ sq m}}$$

Solving for the Total Length of Vertical bars (at 0.4 spacing):

$$\frac{3 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of vertical bars}}{\text{Lateral area of fish tank}}$$

$$\frac{3 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of vertical bars}}{13 \text{ sq m}}$$

Solving for the Total Length of Floor bars (at 0.4 spacing):

$$\frac{3 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of floor bars}}{\text{Floor area of fish tank}}$$

$$\frac{3 \text{ m}}{1 \text{ sq m}} = \frac{\text{Total length of floor bars}}{7.5 \text{ sq m}}$$

Solving for the Number of Steel Bars needed:

$$\text{Number of steel bars needed} = \frac{\text{HB} + \text{VB} + \text{FB}}{\text{Standard length of bar}}$$

Questions

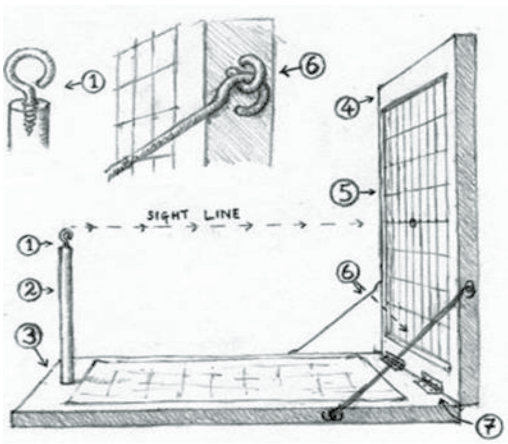
1. What is the total number of bags of cement needed to construct the fish tank?
2. If the ratio of the number of bags of Portland cement to the number of bags of Sahara cement for water proofing purpose is 1:1, how many bags of Sahara cement is needed for CHB plaster finish and floor finish?
3. What is the total volume of sand (in cubic meters) required for the fish tank construction?
4. What is the total volume of gravel (in cubic meters) required for the fish tank construction?
5. How much will it cost to construct a fish tank 5 m × 1.5 m × 1 m? Write all the quantities of materials in the table; canvass the unit cost for each item; compute for the total price of each item; and get the grand total.

Materials		Quantity	Unit Cost	Total
1	CHB 4” × 8” × 16”			
2	Gravel			
3	Sand			
4	Portland Cement			
5	Steel Bar (10 mm)			
6	Sahara Cement			
7	PVC ¾”	5 pcs		
8	PVC Elbow ¾”	6 pcs		
9	PVC 4”	1 pc		
10	PVC Solvent Cement	1 small can		
11	Faucet	1 piece		
12	G.I. Wire # 16	1 kg		
13	Hose 5 mm	10 m		
			Grand Total	

6. How do you find the activity of preparing the bill of materials and cost estimate?
7. Are the concepts and skills learned in this activity useful to you in the future? How?
8. What values and attitudes a person should have in order to be successful in preparing bill of materials and cost estimates?
9. How has your knowledge on proportion helped you in performing the task in this activity?
10. Why are standards set for construction?
11. What will happen if standards are not followed in any construction project?

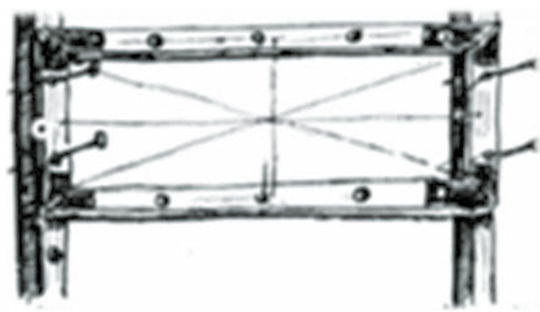
Mathematics in Drawing

Even before the invention of camera, artists and painters were already able to picture the world around them through their sketches and paintings. Albrecht Dürer and Leon Battista Alberti used a mathematical drawing tool with square grid to aid them in capturing what they intended to paint.



http://www.npg.org.uk/assets/migrated_assets/images/learning/digital/arts-techniques/perspective-seeing-where-you-stand/perspectivedraw.jpg

Drawing Tool Used by Albrecht Dürer and Leon Battista Alberti



http://www.webexhibits.org/vangogh/letter/11/223.htm

Drawing Tool Used by Vincent Van Gogh

Vincent Van Gogh, on the other hand, instead of using square grid, the frame of his drawing tool consisted only of one vertical bar, one horizontal bar, and two diagonals connecting opposite corners. With his sight focused at the intersection of the bars, he sketched his subject by region. Once all eight regions were done, the whole picture was already complete for coloring.

With the use of drawing tools, it is already possible for everyone to draw. Combined with the mathematical concepts on similarity, enlarging or reducing the size of a picture would no longer be a problem.

➤ Activity 26: Blowing Up a Picture into Twice Its Size

Copying machine, pen, ruler, bond paper, pencil, rubber eraser

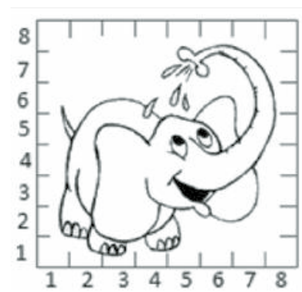
Procedure:

Step 1

Make a machine copy of this original picture of an elephant.

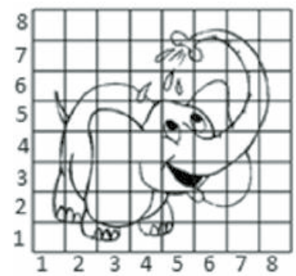
Step 2

With a pencil, enclose the elephant with a rectangle. Using a ruler, indicate equal magnitudes by making marks on the perimeter of the rectangle and number each space.



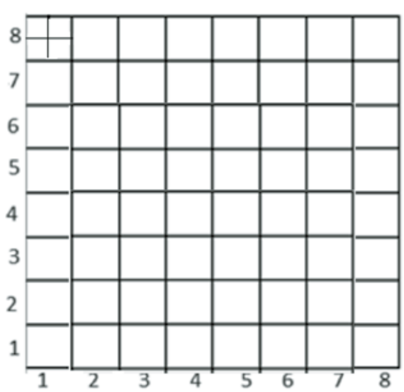
Step 3

Using a pencil, connect the marks on opposite sides of the rectangle to produce a grid.



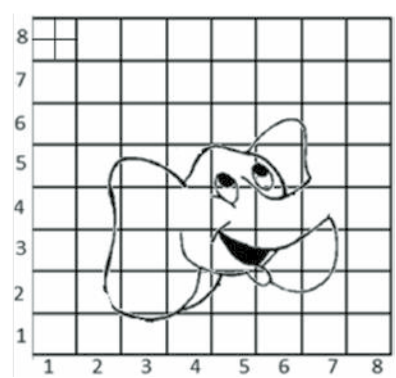
Step 4

Using a pencil, produce a larger square grid on a piece of bond paper. To make it twice as large as the other grid, see to it that each side of each smallest square is double the side of each smallest square in step 3. See the square on column 1, row 8.



Step 5

Still using a pencil, sketch the elephant square by square until you are able to complete an enlarged version of the original one.



Step 6

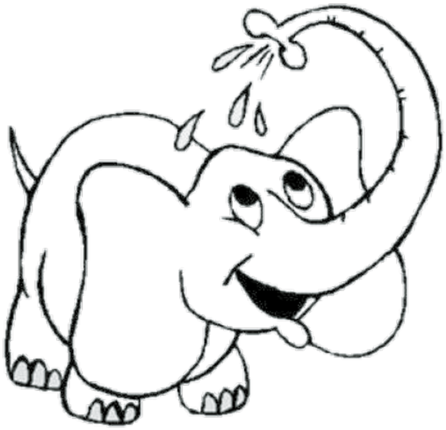
Trace the sketch of the elephant using a pen.

Step 7

Use rubber eraser to remove the penciled grid.

Step 8

This is twice as large as the original picture in step 1.



Questions:

1. What insight can you share about the grid drawing activity?
2. Do you agree that the use of grid makes it possible for everyone to draw?
3. Is the enlarged version of the picture in step 8 similar to the original one in step 1? Explain.
4. What is the scale used in enlarging the original into the new one? Why?

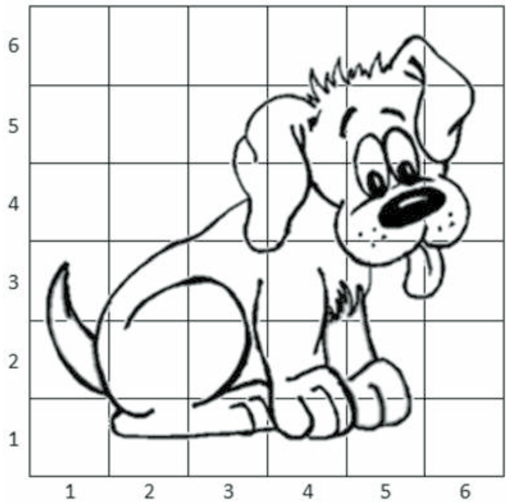
   The scale used to enlarge the original picture in this activity is ___________because the length l of the side of the smallest square in the new grid is _____________ that of the grid of the original picture.
5. What scale will you use to enlarge a picture three times the size of the original?

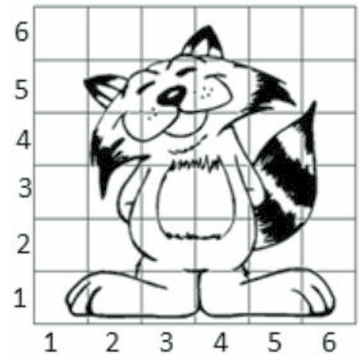
   The scale to use to enlarge a picture three times its size is __________.
6. A large picture is on a square grid. Each side of the smallest square of the grid measures 5 centimeters. You would like to reduce the size of the picture by 20%. What would be the length of the side of each smallest square of the new grid that you will use?

   To reduce the size of a picture by 20%, it means that the size of the new picture is only _____% of the size of the original. Therefore, the length l of the side of the smallest square in the new grid is the product of __________ and 5 cm. Hence, length l is equal to ______ cm.
7. A picture is on a square grid. Each side of the smallest square of the grid measures 10 millimeters. You would like to increase the size of the picture by 30%. What would be the length of the side of each smallest square of the new grid that you will use?

   To increase the size of a picture by 30%, it means that the size of the new picture is _____% of the size of the original. Therefore, the length l of the side of the smallest square in the new grid is the product of __________ and 10 mm. Hence, length l is equal to __________ cm.
8. Following steps 4 to 7 in grid drawing, draw the pictures of the dog and the cat on a piece of bond paper. See to it that your drawing of the dog is half as large as the original and your drawing of the cat is 50% larger than the original. You may color your drawing.

Note: You may choose your own pictures to blow up or reduce but follow all the steps in grid drawing, not only steps 4 to 7.





What to TRANSFER

> Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will enable you to demonstrate your understanding of proportion and similarity.

➤ Activity 27: Sketchtimating Endeavor

Goal: To sketch the floor plan of a house and make a rough estimate of the cost of building the house

Role: contractor

Audience: couple

Situation: A young couple has just bought a 9 m by 9 m rectangular lot. They would like to build a one-storey house with 49-square-meter floor area so that there is adequate outdoor space left for vehicle and gardening. You are one of the contractors asked to design and estimate the cost of their house with a master's bedroom, one guest room, kitchen, bathroom, and a non-separate living room and dining room area. How should the parts of the house be arranged and what are their dimensions? If they only want you to concrete the outside walls and use jalousies for the windows, what is the rough cost estimate in building the house?

Product: floor plan of the house, cost estimate of building the house, and presentation of the floor plan and cost estimate

Standards: accuracy, creativity, resourcefulness, mathematical justification

Rubric

CRITERIA	Excellent 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	Dimensions in the house plan and quantities & computations in the cost estimate are accurate and show a wise use of similarity concepts.	Dimensions in the house plan and quantities & computations in the cost estimate have few errors and show the use of similarity concepts.	Dimensions in the house plan and quantities & computations in the cost estimate have plenty of errors and show the use of some similarity concepts.	Dimensions in the house plan and quantities & computations in the cost estimate are all erroneous and do not show the use of similarity concepts.	
Creativity	The overall impact of the presentation of the sketch plan and cost estimate is highly impressive and the use of technology is highly commendable.	The overall impact of the presentation of the sketch plan and cost estimate is impressive and the use of technology is commendable.	The overall impact of the presentation of the sketch plan and cost estimate is fair and the use of technology is evident.	The overall impact of the presentation of the sketch plan and cost estimate is poor and the use of technology is non-existent.	
Resourcefulness	Dimensions and placement of all parts of the house follow construction standards and prices of all construction materials reflect the average current market prices.	Dimensions and placement of a few parts of the house do not follow construction standards and prices of a few construction materials do not reflect the average current market prices.	Dimensions and placement of many parts of the house do not follow construction standards and prices of many construction materials do not reflect the average current market prices.	Dimensions and placement of the parts of the house do not follow construction standards and prices of construction materials do not reflect the average current market prices.	

Mathematical Justification	Justification is logically clear, convincing, and professionally delivered. The similarity concepts learned are applied and previously learned concepts are connected to the new ones.	Justification is clear and convincingly delivered. Appropriate similarity concepts are applied.	Justification is not so clear. Some ideas are not connected to each other. Not all similarity concepts are applied.	Justification is ambiguous. Only few similarity concepts are applied.	

Questions:

1. How do you find the experience of sketching a house plan?
2. What insights can you share from the experience of making a rough estimate of the cost of building a house?
3. Has your mathematical knowledge and skills on proportion and similarity helped you in performing the task?
4. Why is it advisable to canvass prices of construction materials in different construction stores or home depots?
5. Have you asked technical advice from a construction expert to be able to do the task? Is it beneficial to consult or refer to experts in doing a big task for the first time? Why?

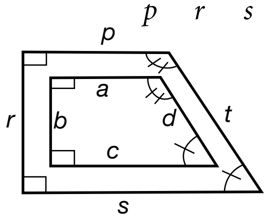
Summary

To wrap up the main concepts on Similarity, revisit your responses in Activity No. 1 under What-to-Know section for the last time and see if you want to make final revisions. After that, perform the last activity that follows.

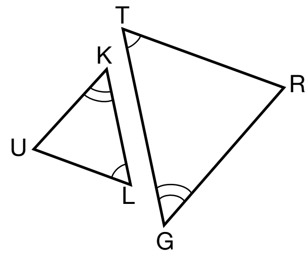
➤ Activity 28: Perfect Match

Match the illustrations of similarity concepts with their names. Write only the numbers of the figures that correspond to the name of the concept.

$$\frac{a}{p} = \frac{b}{r} = \frac{c}{s} = \frac{d}{t}$$

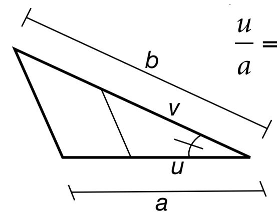


1. Smaller Trapezoid is similar to Larger Trapezoid.



2. $\triangle KUL \sim \triangle GRT$

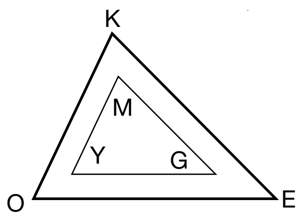
$$\frac{u}{a} = \frac{v}{b}$$



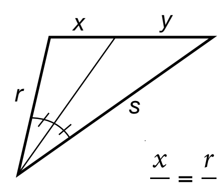
3. Smaller Triangle is similar to Larger Triangle.

$$\frac{OK}{YM} = \frac{EK}{MG} = \frac{EO}{GY}$$

$\triangle YMG \sim \triangle OKE$



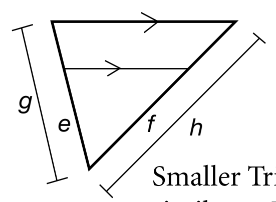
4.



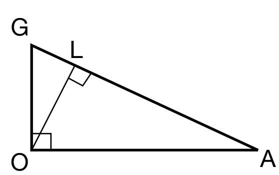
$$\frac{x}{y} = \frac{r}{s}$$

5.

$$\frac{e}{g} = \frac{f}{h}$$

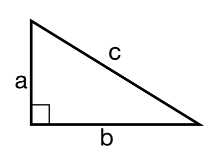


6. Smaller Triangle is similar to Larger Triangle.



$\triangle GOA \sim \triangle GLO \sim \triangle OLA$

7.

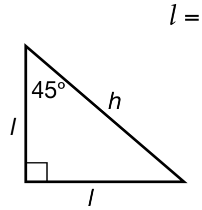


$a^2 + b^2 = c^2$

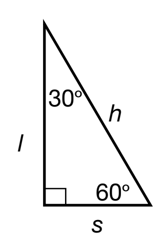
8.

$h = l\sqrt{2}$

$l = \frac{\sqrt{2}}{2} h$



9.



- $s = \frac{h}{2}$
- $s = \frac{\sqrt{3}}{3} l$
- $l = \sqrt{3}\, s$
- $h = 2\, s$

10.

Figure Number	Similarity Concept	Figure Number	Similarity Concept
	30-60-90 Right Triangle Theorem		Right Triangle Similarity Theorem
	Triangle Angle Bisector Theorem		SSS Similarity Theorem
	Pythagorean Theorem		Definition of Similar Polygons
	Triangle Proportionality Theorem		45-45-90 Right Triangle Theorem
	SAS Similarity Theorem		AA Similarity Theorem

You have completed the lesson on *Similarity*. Before you go to the next lesson, Trigonometric Ratios of Triangles, you have to answer a post-assessment.

Glossary of Terms

A. Definitions, Postulates, and Theorems

Similar polygons – are polygons with congruent corresponding angles and proportional corresponding sides.

AAA Similarity Postulate – If the three angles of one triangle are congruent to three angles of another triangle, then the two triangles are similar.

SSS Similarity Theorem – Two triangles are similar if the corresponding sides of two triangles are in proportion.

SAS Similarity Theorem – Two triangles are similar if an angle of one triangle is congruent to an angle of another triangle and the corresponding sides including those angles are in proportion.

Triangle Angle-Bisector Theorem – If a segment bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

Triangle Proportionality Theorem – If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

Right Triangle Similarity Theorem – If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Pythagorean Theorem – The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

45-45-90 Right Triangle Theorem – In a 45-45-90 right triangle: each leg l is $\frac{\sqrt{2}}{2}$ times the hypotenuse h; and the hypotenuse h is $\sqrt{2}$ times each leg l.

30-60-90 Right Triangle Theorem – In a 30-60-90 right triangle, the shorter leg *s* is $\frac{1}{2}$ the hypotenuse *h* or $\frac{\sqrt{3}}{3}$ times the longer leg *l*; the longer leg *l* is $\sqrt{3}$ times the shorter leg *s*; and the hypotenuse is twice the shorter leg *s*.

B. Important Terms

Dilation – is the reduction or enlargement of a figure by multiplying all coordinates of vertices by a common scale factor.

Geometric mean – When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse; and each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Grid drawing – makes use of grids of proportional sizes in drawing an enlarged or reduced version of irregularly shaped objects.

Proportion – is the equality of two ratios.

Rate – compares two or more quantities with different units.

Ratio – compares two or more quantities with the same units.

Scale drawing – uses scale to ensure that dimensions of an actual object are retained proportionally as the actual object is enlarged or reduced in a drawing.

Scale factor – is the uniform ratio *k* of the corresponding proportional sides of polygons.

Scale – is the ratio that compares dimensions in a drawing to the corresponding dimensions in the actual object.

Sierpinski Triangle – is a triangle formed by self-similar triangles.

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