UNIT 2

Module 6

Solving Systems of Equations

COMMON CORE GPS

Contents

MATHEMATICAL PRACTICES The Common Core Georgia Performance Standards for Mathematical Practice describe varieties of expertise that all students should seek to develop. Opportunities to develop these practices are integrated throughout this program.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unpacking the Standards

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Multilingual Glossary

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this chapter.

COMMON CORE GPS

MCC9-12.A.CED.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Key Vocabulary

equation (ecuación)
A mathematical statement that two expressions are equivalent.

What It Means For You

Creating equations in two variables to describe relationships gives you access to the tools of graphing and algebra to solve the equations.

EXAMPLE

A customer spent \$29 on a bouquet of roses and daisies.

r = number of roses in bouquet
d = number of daises in bouquet

$2.5r + 1.75d = 29$

COMMON CORE GPS

MCC9-12.A.REI.6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Key Vocabulary

system of linear equations (sistema de ecuaciones lineales) A system of equations in which all of the equations are linear.

What It Means For You

You can solve systems of equations to find out when two relationships involving the same variables are true at the same time.

EXAMPLE

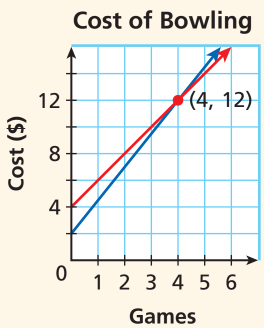
The cost of bowling at bowling alley **A** or **B** is a function of the number of games g.

Cost **A** $= 2.5g + 2$
Cost **B** $= 2g + 4$

When are the costs the same?

Cost **A** = Cost **B**
$2.5g + 2 = 2g + 4$

The cost is \$12 at both bowling alleys when g is 4.



6-1 Technology TASK

Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer "What if…?" questions. By changing one or more values, you can quickly model different scenarios.

Use with Solving Systems by Graphing

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

MCC9-12.A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Activity

Company Z makes DVD players. The company's costs are \$400 per week plus \$20 per DVD player. Each DVD player sells for \$45. How many DVD players must company Z sell in one week to make a profit?

Let n represent the number of DVD players company Z sells in one week.

$c = 400 + 20n$ *The total cost is \$400 plus \$20 times the number of DVD players made.*

$s = 45n$ *The total sales income is \$45 times the number of DVD players sold.*

$p = s - c$ *The total profit is the sales income minus the total cost.*

1. Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.

2. Under Number of DVD Players, enter 1 in cell A2.

3. Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.
   - In cell B2, enter the formula for total cost.
   - In cell C2, enter the formula for total sales income.
   - In cell D2, enter the formula for total profit.

Company Z Profit

	A	B	C	D
1	Number of DVD Players	Total Cost (\$)	Total Income (\$)	Profit (\$)
2	1	420	45	-375

= 400 + 20*A2 = 45*A2 = C2 – B2

4. Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.

5. Find the point where the profit is \$0. This is known as the breakeven point, where total cost and total income are the same.

Company Z Profit

	A	B	C	D
1	Number of DVD Players	Total Cost (\$)	Total Income (\$)	Profit (\$)
16	15	700	675	-25
17	16	720	720	0
18	17	740	765	25

Breakeven point

Profit begins.

Company Z must sell 17 DVD players to make a profit. The profit is \$25.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

1. If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?
2. Company Z makes a profit of \$225 dollars. How many DVD players did they sell?

For Exercise 3, make a spreadsheet.

3. Company Y's costs are \$400 per week plus \$20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be?

6-1 Solving Systems by Graphing

Essential Question: How can you solve systems of linear equations by using graphs?

Objectives

Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Vocabulary

system of linear equations

solution of a system of linear equations

Why learn this?

You can compare costs by graphing a system of linear equations. (See Example 3.)

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges \$2.50 per game plus \$2 for shoe rental while Bowling Pinz charges \$2 per game plus \$4 for shoe rental. A *system of linear equations* can be used to compare these charges.

A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.REI.6

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Animated Math

Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A $(4, 1); \begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

$$\begin{array}{r|l} x + 2y & 6 \\ \hline 4 + 2(1) & 6 \\ 4 + 2 & 6 \\ 6 & 6 \checkmark \end{array} \qquad \begin{array}{r|l} x - y & 3 \\ \hline 4 - 1 & 3 \\ 3 & 3 \checkmark \end{array}$$

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair $(4, 1)$ makes both equations true.

$(4, 1)$ is a solution of the system.

B $(-1, 2); \begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

$$\begin{array}{r|l} 2x + 5y & 8 \\ \hline 2(-1) + 5(2) & 8 \\ -2 + 10 & 8 \\ 8 & 8 \checkmark \end{array} \qquad \begin{array}{r|l} 3x - 2y & 5 \\ \hline 3(-1) - 2(2) & 5 \\ -3 - 4 & 5 \\ -7 & 5 \times \end{array}$$

Substitute −1 for x and 2 for y in each equation in the system.

The ordered pair $(-1, 2)$ makes one equation true, but not the other.

$(-1, 2)$ is not a solution of the system.

Helpful Hint

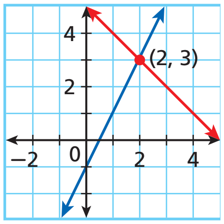
If an ordered pair does not satisfy the first equation in the system, there is no need to check the other equations.

CHECK IT OUT! **Tell whether the ordered pair is a solution of the given system.**

1a. $(1, 3); \begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$ **1b.** $(2, -1); \begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$

All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

$$\begin{cases} y = 2x - 1 \\ y = -x + 5 \end{cases}$$



The point $(2, 3)$ is where the two lines intersect and is a solution of both equations, so $(2, 3)$ is the solution of the system.

COMMON CORE GPS **EXAMPLE** MCC9-12.A.REI.6 **2**

Solving a System of Linear Equations by Graphing

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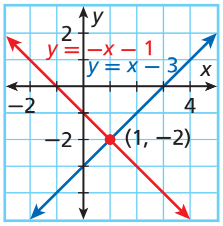
Solve each system by graphing. Check your answer.

A $\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$

Graph the system.

The solution appears to be at $(1, -2)$.



Check

Substitute $(1, -2)$ into the system.

$$\begin{array}{c|c} y & x - 3 \\ \hline -2 & 1 - 3 \\ -2 & -2 \checkmark \end{array} \qquad \begin{array}{c|c} y & -x - 1 \\ \hline -2 & -1 - 1 \\ -2 & -2 \checkmark \end{array}$$

The solution is $(1, -2)$.

Helpful Hint

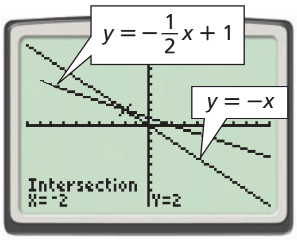
Sometimes it is difficult to tell exactly where the lines cross when you solve by graphing. It is good to confirm your answer by substituting it into both equations.

B $\begin{cases} x + y = 0 \\ y = -\frac{1}{2}x + 1 \end{cases}$

$$\begin{aligned} x + y &= 0 \\ \underline{-x} \quad &\underline{-x} \\ y &= -x \end{aligned}$$

Rewrite the first equation in slope-intercept form.

Graph using a calculator and then use the intersection command.



Check Substitute $(-2, 2)$ into the system.

$$\begin{array}{c|c} x + y & 0 \\ \hline -2 + 2 & 0 \\ 0 & 0 \checkmark \end{array} \qquad \begin{array}{c|c} y & -\frac{1}{2}x + 1 \\ \hline 2 & -\frac{1}{2}(-2) + 1 \\ 2 & 1 + 1 \\ 2 & 2 \checkmark \end{array}$$

The solution is $(-2, 2)$.

CHECK IT OUT!

Solve each system by graphing. Check your answer.

2a. $\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$

2b. $\begin{cases} y = \frac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$

COMMON CORE GPS **EXAMPLE 3** MCC9-12.A.CED.2

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Problem-Solving Application

Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental, and Bowling Pinz charges $2 per game plus $4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

1 Understand the Problem

The **answer** will be the number of games played for which the total cost is the same at both bowling alleys. **List the important information:**

- Game price: Bowl-o-Rama $2.50 Bowling Pinz: $2
- Shoe-rental fee: Bowl-o-Rama $2 Bowling Pinz: $4

2 Make a Plan

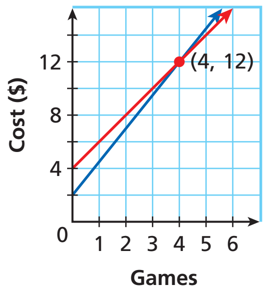
Write a system of equations, one equation to represent the price at each company. Let x be the number of games played and y be the total cost.

	Total cost	is	price per game	times	games	plus	shoe rental.
Bowl-o-Rama	y	$=$	2.5	$\cdot$	x	$+$	2
Bowling Pinz	y	$=$	2	$\cdot$	x	$+$	4

3 Solve

Graph $y = 2.5x + 2$ and $y = 2x + 4$. The lines appear to intersect at $(4, 12)$. So, the cost at both places will be the same for 4 games bowled and that cost will be $12.

Cost of Bowling



4 Look Back

Check $(4, 12)$ using both equations.

Cost of bowling 4 games at Bowl-o-Rama:
$\$2.5(4) + \$2 = 10 + 2 = 12$ ✓

Cost of bowling 4 games at Bowling Pinz:
$\$2(4) + \$4 = 8 + 4 = 12$ ✓

MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

CHECK IT OUT!

3. Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost?

MCC.MP.6 MATHEMATICAL PRACTICES

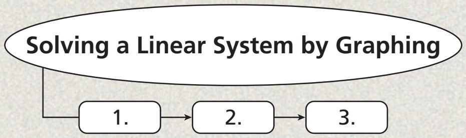
THINK AND DISCUSS

1. Explain how to use a graph to solve a system of linear equations.

2. Explain how to check a solution of a system of linear equations.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.

Know it! Note



6-1 Exercises

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Homework Help

GUIDED PRACTICE

1. **Vocabulary** Describe a *solution of a system of linear equations.*

SEE EXAMPLE 1

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2);\begin{cases}3x + y = 4\\ x - 3y = -4\end{cases}$

3. $(3, -1);\begin{cases}x - 2y = 5\\ 2x - y = 7\end{cases}$

4. $(-1, 5);\begin{cases}-x + y = 6\\ 2x + 3y = 13\end{cases}$

SEE EXAMPLE 2

Solve each system by graphing. Check your answer.

5. $\begin{cases}y = \frac{1}{2}x\\ y = -x + 3\end{cases}$

6. $\begin{cases}y = x - 2\\ 2x + y = 1\end{cases}$

7. $\begin{cases}-2x - 1 = y\\ x + y = 3\end{cases}$

SEE EXAMPLE 3

8. To deliver mulch, Lawn and Garden charges \$30 per cubic yard of mulch plus a \$30 delivery fee. Yard Depot charges \$25 per cubic yard of mulch plus a \$55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–15	2
16	3

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Online Extra Practice

Tell whether the ordered pair is a solution of the given system.

9. $(1, -4);\begin{cases}x - 2y = 8\\ 4x - y = 8\end{cases}$

10. $(-2, 1);\begin{cases}2x - 3y = -7\\ 3x + y = -5\end{cases}$

11. $(5, 2);\begin{cases}2x + y = 12\\ -3y - x = -11\end{cases}$

Solve each system by graphing. Check your answer.

12. $\begin{cases}y = \frac{1}{2}x + 2\\ y = -x - 1\end{cases}$

13. $\begin{cases}y = x\\ y = -x + 6\end{cases}$

14. $\begin{cases}-2x - 1 = y\\ x = -y + 3\end{cases}$

15. $\begin{cases}x + y = 2\\ y = x - 4\end{cases}$

16. **Multi-Step** Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be?

17. **School** The school band sells carnations on Valentine's Day for \$2 each. They buy the carnations from a florist for \$0.50 each, plus a \$16 delivery charge.
    a. Write a system of equations to describe the situation.
    b. Graph the system. What does the solution represent?
    c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

Real-World Connections

18. a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a \$50 design fee and \$5 per hat. Top Stuff charges a \$25 design fee and \$6 per hat. Write an equation for each company's pricing.
    b. Graph the system of equations from part **a.** For how many hats will the cost be the same? What is that cost?
    c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off.

Graphing Calculator **Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.**

19. $\begin{cases} y = 4.7x + 2.1 \\ y = 1.6x - 5.4 \end{cases}$

20. $\begin{cases} 4.8x + 0.6y = 4 \\ y = -3.2x + 2.7 \end{cases}$

21. $\begin{cases} y = \frac{5}{4}x - \frac{2}{3} \\ \frac{8}{3}x + y = \frac{5}{9} \end{cases}$

22. $\begin{cases} y = 6.9x + 12.4 \\ y = -4.1x - 5.3 \end{cases}$

LINK

Landscaping

Middleton Place Gardens, South Carolina, are the United States' oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s.

23. **Landscaping** The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether.

24. **Fitness** Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for x minutes. In the afternoon, Rusty will jog for x minutes. How many minutes must he jog to burn at least as many Calories y in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.

25. A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height?

26. **Critical Thinking** Write a real-world situation that could be represented by the system $\begin{cases} y = 3x + 10 \\ y = 5x + 20 \end{cases}$.

H.O.T. **27.** **Write About It** When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

TEST PREP

28. Taxi company A charges \$4 plus \$0.50 per mile. Taxi company B charges \$5 plus \$0.25 per mile. Which system best represents this problem?

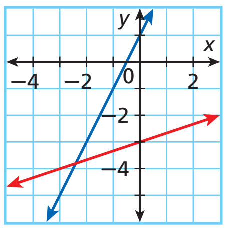
(A) $\begin{cases} y = 4x + 0.5 \\ y = 5x + 0.25 \end{cases}$

(B) $\begin{cases} y = 0.5x + 4 \\ y = 0.25x + 5 \end{cases}$

(C) $\begin{cases} y = -4x + 0.5 \\ y = -5x + 0.25 \end{cases}$

(D) $\begin{cases} y = -0.5x + 4 \\ y = -0.25x + 5 \end{cases}$

29. Which system of equations represents the given graph?



(F) $\begin{cases} y = 2x - 1 \\ y = \frac{1}{3}x + 3 \end{cases}$

(G) $\begin{cases} y = -2x + 1 \\ y = 2x - 3 \end{cases}$

(H) $\begin{cases} y = 2x + 1 \\ y = \frac{1}{3}x - 3 \end{cases}$

(J) $\begin{cases} y = -2x - 1 \\ y = 3x - 3 \end{cases}$

30. **Gridded Response** Which value of b will make the system $y = 2x + 2$ and $y = 2.5x + b$ intersect at the point $(2, 6)$?

CHALLENGE AND EXTEND

31. **Entertainment** If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be?

Total Number Sold				
Month	1	2	3	4
VCRs	500	490	480	470
DVD Players	250	265	280	295

32. Long Distance Inc. charges a \$1.45 connection charge and \$0.03 per minute. Far Away Calls charges a \$1.52 connection charge and \$0.02 per minute.

a. For how many minutes will a call cost the same from both companies? What is that cost?

b. When is it better to call using Long Distance Inc.? Far Away Calls? Explain.

c. **What if...?** Long Distance Inc. raised its connection charge to \$1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **33.** **Error Analysis** Mario says $(-1, 5)$ is a solution of the system of equations shown. Do you agree? Explain.

$$\begin{cases} x + y = 4 \\ x - y = 6 \end{cases}$$

H.O.T. **34.** **Problem Solving** Amanda cut an 8-foot length of ribbon into two pieces. One piece is three times as long as the other.

a. Write and graph a system of equations for the length of each piece of ribbon. Use x for the length of the shorter piece, and y for the longer.

b. What does the point where the lines intersect represent?

c. What is the system of equations if you define y as the length of the shorter piece and x as the longer piece? What is the solution?

Career Path

Ethan Reynolds
Applied Sciences major

Q: What math classes did you take in high school?
A: Career Math, Algebra, and Geometry

Q: What are you studying and what math classes have you taken?
A: I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

Q: How is math used in aviation?
A: I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

Q: What are your future plans?
A: I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor's degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.

6-2 Algebra TASK

Model Systems of Linear Equations

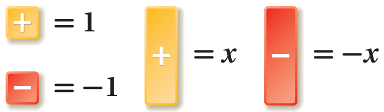
You can use algebra tiles to model and solve some systems of linear equations.

Use with Solving Systems by Substitution

MATHEMATICAL PRACTICES **Use appropriate tools strategically.**

MCC9-12.A.REI.6 Solve systems of linear equations exactly … , focusing on pairs of linear equations in two variables.

KEY

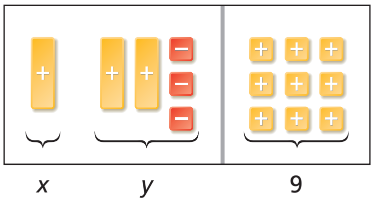
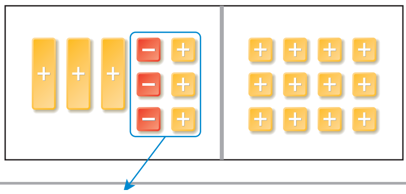
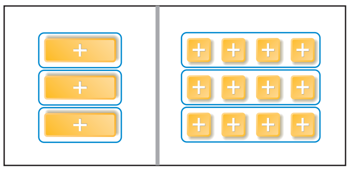


REMEMBER

When two expressions are equal, you can substitute one for the other in any expression or equation.

Activity

Use algebra tiles to model and solve $\begin{cases} y = 2x - 3 \\ x + y = 9 \end{cases}$.

MODEL		ALGEBRA
	*The first equation is solved for y. Model the second equation, x + y = 9, by substituting 2x − 3 for y.*	$x + y = 9$ $x + (2x - 3) = 9$ $3x - 3 = 9$
	*Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation.* *Remove zero pairs.*	$3x - 3 = 9$ $\underline{\quad +3 \quad +3}$ $3x \quad = 12$
	*Divide each side into 3 equal groups. Align one x-tile with each group on the right side. One x-tile is equivalent to 4 yellow tiles. x = 4*	$\frac{3x}{3} = \frac{12}{3}$ $x = 4$

To solve for y, substitute 4 for x in one of the equations:

$$\begin{aligned} y &= 2x - 3 \\ &= 2(4) - 3 \\ &= 5 \end{aligned}$$

The solution is (4, 5).

Try This

Model and solve each system of equations.

1. $\begin{cases} y = x + 3 \\ 2x + y = 6 \end{cases}$ **2.** $\begin{cases} 2x + 3 = y \\ x + y = 6 \end{cases}$ **3.** $\begin{cases} 2x + 3y = 1 \\ x = -1 - y \end{cases}$ **4.** $\begin{cases} y = x + 1 \\ 2x - y = -5 \end{cases}$

6-2 Solving Systems by Substitution

Essential Question: How can you solve systems of linear equations by using substitution?

Objective
Solve systems of linear equations in two variables by substitution.

Why learn this?
You can solve systems of equations to help select the best value among high-speed Internet providers. (See Example 3.)

Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called *substitution*.

The goal when using substitution is to reduce the system to one equation that has only one variable. Then you can solve this equation, and substitute into an original equation to find the value of the other variable.

Know it! Note

Solving Systems of Equations by Substitution	
Step 1	Solve for one variable in at least one equation, if necessary.
Step 2	Substitute the resulting expression into the other equation.
Step 3	Solve that equation to get the value of the first variable.
Step 4	Substitute that value into one of the original equations and solve.
Step 5	Write the values from Steps 3 and 4 as an ordered pair, (x, y), and check.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.REI.6

Solving a System of Linear Equations by Substitution

Solve each system by substitution.

A $\begin{cases} y = 2x \\ y = x + 5 \end{cases}$

Step 1 $y = 2x$ — *Both equations are solved for y.*
$y = x + 5$

Step 2 $y = x + 5$ — *Substitute 2x for y in the second equation.*
$2x = x + 5$

Step 3 $\begin{array}{rcl} -x & & -x \\ \hline x & = & 5 \end{array}$ — *Solve for x.*

Step 4 $y = 2x$ — *Write one of the original equations.*
$y = 2(5)$ — *Substitute 5 for x.*
$y = 10$

Step 5 $(5, 10)$ — *Write the solution as an ordered pair.*

Check Substitute $(5, 10)$ into both equations in the system.

$$\begin{array}{c|c} \multicolumn{2}{c}{y = 2x} \\ \hline 10 & 2(5) \\ 10 & 10\,\checkmark \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{y = x + 5} \\ \hline 10 & 5 + 5 \\ 10 & 10\,\checkmark \end{array}$$

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Helpful Hint
You can substitute the value of one variable into *either* of the original equations to find the value of the other variable.

Solve each system by substitution.

B $\begin{cases} 2x + y = 5 \\ y = x - 4 \end{cases}$

Step 1 $y = x - 4$	*The second equation is solved for y.*
Step 2 $2x + y = 5$	
$2x + (x - 4) = 5$	*Substitute x − 4 for y in the first equation.*
Step 3 $3x - 4 = 5$	*Simplify. Then solve for x.*
$\underline{\quad +4 \quad +4}$	*Add 4 to both sides.*
$3x = 9$	
$\frac{3x}{3} = \frac{9}{3}$	*Divide both sides by 3.*
$x = 3$	
Step 4 $y = x - 4$	*Write one of the original equations.*
$y = 3 - 4$	*Substitute 3 for x.*
$y = -1$	
Step 5 $(3, -1)$	*Write the solution as an ordered pair.*

C $\begin{cases} x + 4y = 6 \\ x + y = 3 \end{cases}$

Step 1 $x + 4y = 6$	*Solve the first equation for x by subtracting 4y from both sides.*
$\underline{\quad -4y \quad -4y}$	
$x = 6 - 4y$	
Step 2 $x + y = 3$	
$(6 - 4y) + y = 3$	*Substitute 6 − 4y for x in the second equation.*
Step 3 $6 - 3y = 3$	*Simplify. Then solve for y.*
$\underline{-6 \qquad -6}$	*Subtract 6 from both sides.*
$-3y = -3$	
$\frac{-3y}{-3} = \frac{-3}{-3}$	*Divide both sides by −3.*
$y = 1$	
Step 4 $x + y = 3$	*Write one of the original equations.*
$x + 1 = 3$	*Substitute 1 for y.*
$\underline{\quad -1 \quad -1}$	*Subtract 1 from both sides.*
$x = 2$	
Step 5 $(2, 1)$	*Write the solution as an ordered pair.*

Helpful Hint

Sometimes neither equation is solved for a variable. You can begin by solving either equation for either *x* or *y*.

CHECK IT OUT! **Solve each system by substitution.**

1a. $\begin{cases} y = x + 3 \\ y = 2x + 5 \end{cases}$ **1b.** $\begin{cases} x = 2y - 4 \\ x + 8y = 16 \end{cases}$ **1c.** $\begin{cases} 2x + y = -4 \\ x + y = -7 \end{cases}$

Sometimes you substitute an expression for a variable that has a coefficient. When solving for the second variable in this situation, you can use the Distributive Property.

COMMON CORE GPS **EXAMPLE 2** MCC9-12.A.REI.6

Using the Distributive Property

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Solve $\begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases}$ by substitution.

Step 1
$$\begin{aligned} x - 4y &= 11 \\ +4y \quad & \quad +4y \\ x &= 4y + 11 \end{aligned}$$

Solve the second equation for x by adding 4y to each side.

Step 2
$$4y - 5x = 9$$
$$4y - 5(4y + 11) = 9$$

Substitute 4y + 11 for x in the first equation.

Step 3
$$\begin{aligned} 4y - 5(4y) - 5(11) &= 9 \\ 4y - 20y - 55 &= 9 \\ -16y - 55 &= 9 \\ +55 \quad & \quad +55 \\ -16y &= 64 \\ \frac{-16y}{-16} &= \frac{64}{-16} \\ y &= -4 \end{aligned}$$

Distribute −5 to the expression in parentheses. Simplify. Solve for y.

Add 55 to both sides.

Divide both sides by −16.

Step 4
$$\begin{aligned} x - 4y &= 11 \\ x - 4(-4) &= 11 \\ x + 16 &= 11 \\ -16 \quad & \quad -16 \\ x &= -5 \end{aligned}$$

Write one of the original equations.

Substitute −4 for y.

Simplify.

Subtract 16 from both sides.

Step 5 $(-5, -4)$ *Write the solution as an ordered pair.*

Caution!

When you solve one equation for a variable, you must substitute the value or expression into the *other* original equation, not the one that has just been solved.

CHECK IT OUT! **2.** Solve $\begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases}$ by substitution.

Student to Student

Solving Systems by Substitution

Erika Chu
Terrell High School

I always look for a variable with a coefficient of 1 or −1 when deciding which equation to solve for x or y.

For the system

$$\begin{cases} 2x + y = 14 \\ -3x + 4y = -10 \end{cases}$$

I would solve the first equation for y because it has a coefficient of 1.

$$\begin{aligned} 2x + y &= 14 \\ y &= -2x + 14 \end{aligned}$$

Then I use substitution to find the values of x and y.

$$\begin{aligned} -3x + 4y &= -10 \\ -3x + 4(-2x + 14) &= -10 \\ -3x + (-8x) + 56 &= -10 \\ -11x + 56 &= -10 \\ -11x &= -66 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} y &= -2x + 14 \\ y &= -2(6) + 14 = 2 \end{aligned}$$

The solution is $(6, 2)$.

COMMON CORE GPS **EXAMPLE 3** MCC9-12.A.CED.2

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Consumer Economics Application

One high-speed Internet provider has a \$50 setup fee and costs \$30 per month. Another provider has no setup fee and costs \$40 per month.

a. In how many months will both providers cost the same? What will that cost be?

Write an equation for each option. Let t represent the total amount paid and m represent the number of months.

	Total paid	is	setup fee	plus	cost per month	times	months.
Option 1	t	$=$	50	$+$	30	$\cdot$	m
Option 2	t	$=$	0	$+$	40	$\cdot$	m

Step 1 $t = 50 + 30m$ — *Both equations are solved for t.*

$t = 40m$

Step 2 $50 + 30m = 40m$ — *Substitute 50 + 30m for t in the second equation.*

Step 3 $\underline{-30m} \quad \underline{-30m}$ — *Solve for m. Subtract 30m from both sides.*

$50 = 10m$

$\frac{50}{10} = \frac{10m}{10}$ — *Divide both sides by 10.*

$5 = m$

Step 4 $t = 40m$ — *Write one of the original equations.*

$= 40(5)$ — *Substitute 5 for m.*

$= 200$

Step 5 $(5, 200)$ — *Write the solution as an ordered pair.*

In 5 months, the total cost for each option will be the same—\$200.

b. If you plan to cancel in 1 year, which is the cheaper provider? Explain.

Option 1: $t = 50 + 30(12) = 410$ Option 2: $t = 40(12) = 480$

Option 1 is cheaper.

CHECK IT OUT!

3. One cable television provider has a \$60 setup fee and charges \$80 per month, and another provider has a \$160 equipment fee and charges \$70 per month.

a. In how many months will the cost be the same? What will that cost be?

b. If you plan to move in 6 months, which is the cheaper option? Explain.

MCC.MP.1 MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. If you graphed the equations in Example 1A, where would the lines intersect?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, solve the system by substitution using the first step given. Show that each method gives the same solution.

$\begin{cases} x + y = 8 \\ x - y = 2 \end{cases}$

Solve $x + y = 8$ for x.	Solve $x + y = 8$ for y.
Solve $x - y = 2$ for x.	Solve $x - y = 2$ for y.

6-2 Exercises

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GUIDED PRACTICE

Solve each system by substitution.

SEE EXAMPLE 1

1. $\begin{cases} y = 5x - 10 \\ y = 3x + 8 \end{cases}$ **2.** $\begin{cases} 3x + y = 2 \\ 4x + y = 20 \end{cases}$ **3.** $\begin{cases} y = x + 5 \\ 4x + y = 20 \end{cases}$

SEE EXAMPLE 2

4. $\begin{cases} x - 2y = 10 \\ \frac{1}{2}x - 2y = 4 \end{cases}$ **5.** $\begin{cases} y - 4x = 3 \\ 2x - 3y = 21 \end{cases}$ **6.** $\begin{cases} x = y - 8 \\ -x - y = 0 \end{cases}$

SEE EXAMPLE 3

7. Consumer Economics The Strauss family is deciding between two lawn-care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.

a. In how many months will both lawn-care services cost the same? What will that cost be?

b. If the family will use the service for only 6 months, which is the better option? Explain.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–10	1
11–16	2
17	3

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Online Extra Practice

Solve each system by substitution.

8. $\begin{cases} y = x + 3 \\ y = 2x + 4 \end{cases}$ **9.** $\begin{cases} y = 2x + 10 \\ y = -2x - 6 \end{cases}$ **10.** $\begin{cases} x + 2y = 8 \\ x + 3y = 12 \end{cases}$

11. $\begin{cases} 2x + 2y = 2 \\ -4x + 4y = 12 \end{cases}$ **12.** $\begin{cases} y = 0.5x + 2 \\ -y = -2x + 4 \end{cases}$ **13.** $\begin{cases} -x + y = 4 \\ 3x - 2y = -7 \end{cases}$

14. $\begin{cases} 3x + y = -8 \\ -2x - y = 6 \end{cases}$ **15.** $\begin{cases} x + 2y = -1 \\ 4x - 4y = 20 \end{cases}$ **16.** $\begin{cases} 4x = y - 1 \\ 6x - 2y = -3 \end{cases}$

17. Recreation Casey wants to buy a gym membership. One gym has a $150 joining fee and costs $35 per month. Another gym has no joining fee and costs $60 per month.

a. In how many months will both gym memberships cost the same? What will that cost be?

b. If Casey plans to cancel in 5 months, which is the better option for him? Explain.

Solve each system by substitution. Check your answer.

18. $\begin{cases} x = 5 \\ x + y = 8 \end{cases}$ **19.** $\begin{cases} y = -3x + 4 \\ x = 2y + 6 \end{cases}$ **20.** $\begin{cases} 3x - y = 11 \\ 5y - 7x = 1 \end{cases}$

21. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 6 \\ x - y = 2 \end{cases}$ **22.** $\begin{cases} x = 7 - 2y \\ 2x + y = 5 \end{cases}$ **23.** $\begin{cases} y = 1.2x - 4 \\ 2.2x + 5 = y \end{cases}$

24. The sum of two numbers is 50. The first number is 43 less than twice the second number. Write and solve a system of equations to find the two numbers.

25. Money A jar contains n nickels and d dimes. There are 20 coins in the jar, and the total value of the coins is $1.40. How many nickels and how many dimes are in the jar? (*Hint:* Nickels are worth $0.05 and dimes are worth $0.10.)

26. **Multi-Step** Use the receipts below to write and solve a system of equations to find the cost of a large popcorn and the cost of a small drink.

27. **Finance** Helene invested a total of \$1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after one year was \$58. Write and solve a system of equations to find the amount invested in each account. (*Hint:* Change the interest rates into decimals first.)

Geometry **Two angles whose measures have a sum of 90° are called complementary angles. For Exercises 28–30, x and y represent the measures of complementary angles. Use this information and the equation given in each exercise to find the measure of each angle.**

28. $y = 4x - 10$

29. $x = 2y$

30. $y = 2(x - 15)$

31. **Aviation** With a headwind, a small plane can fly 240 miles in 3 hours. With a tailwind, the plane can fly the same distance in 2 hours. Follow the steps below to find the rates of the plane and wind.

a. Copy and complete the table. Let p be the rate of the plane and w be the rate of the wind.

	Rate	•	Time	=	Distance
With Headwind	$p - w$	•		=	240
With Tailwind		•	2	=	

b. Use the information in each row to write a system of equations.

c. Solve the system of equations to find the rates of the plane and wind.

H.O.T. 32. **Write About It** Explain how to solve a system of equations by substitution.

H.O.T. 33. **Critical Thinking** Explain the connection between the solution of a system solved by graphing and the solution of the same system solved by substitution.

Real-World Connections

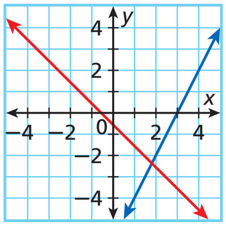
34. At the school store, Juanita bought 2 books and a backpack for a total of \$26 before tax. Each book cost \$8 less than the backpack.

a. Write a system of equations that can be used to find the price of each book and the price of the backpack.

b. Solve this system by substitution.

c. Solve this system by graphing. Discuss advantages and disadvantages of solving by substitution and solving by graphing.

35. **Estimation** Use the graph to estimate the solution to $\begin{cases}2x - y = 6\\ x + y = -0.6\end{cases}$. Round your answer to the nearest tenth. Then solve the system by substitution.



TEST PREP

36. Elizabeth met 24 of her cousins at a family reunion. The number of male cousins m was 6 less than twice the number of female cousins f. Which system can be used to find the number of male cousins and female cousins?

Ⓐ $\begin{cases}m + f = 24\\ f = 2m - 6\end{cases}$ Ⓑ $\begin{cases}m + f = 24\\ f = 2m\end{cases}$ Ⓒ $\begin{cases}m = 24 + f\\ m = f - 6\end{cases}$ Ⓓ $\begin{cases}f = 24 - m\\ m = 2f - 6\end{cases}$

37. Which problem is best represented by the system $\begin{cases}d = n + 5\\ d + n = 12\end{cases}$?

Ⓕ Roger has 12 coins in dimes and nickels. There are 5 more dimes than nickels.

Ⓖ Roger has 5 coins in dimes and nickels. There are 12 more dimes than nickels.

Ⓗ Roger has 12 coins in dimes and nickels. There are 5 more nickels than dimes.

Ⓙ Roger has 5 coins in dimes and nickels. There are 12 more nickels than dimes.

CHALLENGE AND EXTEND

38. A car dealership has 378 cars on its lot. The ratio of new cars to used cars is 5:4. Write and solve a system of equations to find the number of new and used cars on the lot.

Solve each system by substitution.

39. $\begin{cases}2r - 3s - t = 12\\ s + 3t = 10\\ t = 4\end{cases}$ 40. $\begin{cases}x + y + z = 7\\ y + z = 5\\ 2y - 4z = -14\end{cases}$ 41. $\begin{cases}a + 2b + c = 19\\ -b + c = -5\\ 3b + 2c = 15\end{cases}$

MATHEMATICAL PRACTICES FOCUS ON MATHEMATICAL PRACTICES

H.O.T. 42. **Reasoning** Examine the system. $\begin{cases}2m + 3n = 31\\ n - m = 7\end{cases}$

a. Solve the system by substitution.

b. Would you get the same answer whether you solved for m or n first? Explain.

c. Why does it make sense to solve for n in the second equation first?

H.O.T. 43. **Error Analysis** Marjorie attempted to solve the system of equations shown, but ran into trouble. What mistake did she make? $\begin{cases}3x + y = 3\\ 14x + 4y = 2\end{cases}$

Step 1: $3x + y = 3 \rightarrow y = 3 - 3x$

Step 2: $3x + y = 3 \rightarrow 3x + (3 - 3x) = 3$

Step 3: $3x + (3 - 3x) = 3$

$3 = 3$

H.O.T. 44. **Precision** Explain why it is not always possible to solve a system of linear equations by graphing, but it is always possible to solve a system using substitution.

6-3 Solving Systems by Elimination

Essential Question: How can you solve systems of linear equations by using elimination?

Objectives

Solve systems of linear equations in two variables by elimination.

Compare and choose an appropriate method for solving systems of linear equations.

Why learn this?

You can solve a system of linear equations to determine how many flowers of each type you can buy to make a bouquet. (See Example 4.)

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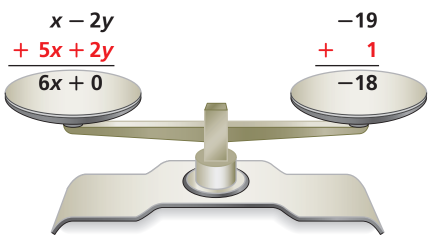
Another method for solving systems of equations is *elimination*. Like substitution, the goal of elimination is to get one equation that has only one variable.

Remember that an equation stays balanced if you add equal amounts to both sides. Consider the system $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$. Since $5x + 2y = 1$, you can add $5x + 2y$ to one side of the first equation and 1 to the other side and the balance is maintained.



Since $-2y$ and $2y$ have **opposite coefficients**, you can eliminate the y by adding the two equations. The result is one equation that has only one variable: $6x = -18$.

When you use the elimination method to solve a system of linear equations, align all like terms in the equations. Then determine whether any like terms can be eliminated because they have opposite coefficients.

Know it! Note

Solving Systems of Equations by Elimination	
Step 1	Write the system so that like terms are aligned.
Step 2	Eliminate one of the variables and solve for the other variable.
Step 3	Substitute the value of the variable into one of the original equations and solve for the other variable.
Step 4	Write the answers from Steps 2 and 3 as an ordered pair, (x, y), and check.

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.

COMMON CORE GPS **EXAMPLE 1** MCC9-12.A.REI.6

Elimination Using Addition

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Solve $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$ by elimination.

Step 1	$x - 2y = -19$	*Write the system so that like terms are aligned.*
	$+\ 5x + 2y = \quad 1$	*Notice that $-2y$ and $2y$ are opposites.*
Step 2	$6x \ + 0 = -18$	*Add the equations to eliminate y.*
	$6x = -18$	*Simplify and solve for x.*
	$\frac{6x}{6} = \frac{-18}{6}$	*Divide both sides by 6.*
	$x = -3$	
Step 3	$x - 2y = -19$	*Write one of the original equations.*
	$-3 - 2y = -19$	*Substitute -3 for x.*
	$+3 \qquad\quad +3$	*Add 3 to both sides.*
	$-2y = -16$	
	$\frac{-2y}{-2} = \frac{-16}{-2}$	*Divide both sides by -2.*
	$y = 8$	
Step 4	$(-3, 8)$	*Write the solution as an ordered pair.*

Helpful Hint

Check your answer.

$x - 2y = -19$	
$-3 - 2(8)$	-19
$-3 - 16$	-19
-19	-19 ✓

$5x + 2y = 1$	
$5(-3) + 2(8)$	1
$-15 + 16$	1
1	1 ✓

CHECK IT OUT!

1. Solve $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$ by elimination. Check your answer.

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of *each* term.

COMMON CORE GPS **EXAMPLE 2** MCC9-12.A.REI.6

Elimination Using Subtraction

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Solve $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$ by elimination.

Step 1	$3x + 4y = 18$	
	$-\ (-2x + 4y = \ 8)$	*Notice that both equations contain 4y.*
	$3x + 4y = \ 18$	*Add the opposite of each term*
	$+\ 2x - 4y = -8$	*in the second equation.*
Step 2	$5x + \ 0 = 10$	*Eliminate y.*
	$5x = 10$	*Simplify and solve for x.*
	$x = 2$	
Step 3	$-2x + 4y = \ 8$	*Write one of the original equations.*
	$-2(2) + 4y = \ 8$	*Substitute 2 for x.*
	$-4 + 4y = \ 8$	
	$+4 \qquad\quad +4$	*Add 4 to both sides.*
	$4y = 12$	*Simplify and solve for y.*
	$y = 3$	
Step 4	$(2, 3)$	*Write the solution as an ordered pair.*

Remember!

Remember to check by substituting your answer into both original equations.

CHECK IT OUT! **2.** Solve $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$ by elimination. Check your answer.

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients.

COMMON CORE GPS **EXAMPLE 3** MCC9-12.A.REI.6

Elimination Using Multiplication First

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Solve each system by elimination.

A $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$

Step 1

$$2x + y = 3$$
$$+2(-x + 3y = -12)$$

Multiply each term in the second equation by 2 to get opposite x-coefficients.

$$2x + y = 3$$
$$+(-2x + 6y = -24)$$

Add the new equation to the first equation to eliminate x.

Step 2

$$7y = -21$$
$$y = -3$$

Solve for y.

Step 3

$2x + y = 3$ — *Write one of the original equations.*

$2x + (-3) = 3$ — *Substitute −3 for y.*

$+ 3 \quad + 3$ — *Add 3 to both sides.*

$2x = 6$ — *Solve for x.*

$x = 3$

Step 4 $(3, -3)$ — *Write the solution as an ordered pair.*

Helpful Hint

In Example 3A, you could have also multiplied the first equation by −3 to eliminate *y*.

B $\begin{cases} 7x - 12y = -22 \\ 5x - 8y = -14 \end{cases}$

Step 1

$$2(7x - 12y = -22)$$
$$+(-3)(5x - 8y = -14)$$

Multiply the first equation by 2 and the second equation by −3 to get opposite y-coefficients.

$$14x - 24y = -44$$
$$+(-15x + 24y = 42)$$

Add the new equations to eliminate y.

Step 2

$$-x = -2$$
$$x = 2$$

Solve for x.

Step 3

$7x - 12y = -22$ — *Write one of the original equations.*

$7(2) - 12y = -22$ — *Substitute 2 for x.*

$14 - 12y = -22$

$-14 \quad -14$ — *Subtract 14 from both sides.*

$-12y = -36$ — *Solve for y.*

$y = 3$

Step 4 $(2, 3)$ — *Write the solution as an ordered pair.*

CHECK IT OUT! **Solve each system by elimination. Check your answer.**

3a. $\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$

3b. $\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$

COMMON CORE GPS **EXAMPLE 4** MCC9-12.A.CED.2

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Consumer Economics Application

Sam spent $24.75 to buy 12 flowers for his mother. The bouquet contained roses and daisies. How many of each type of flower did Sam buy?

Write a system. Use r for the number of roses and d for the number of daisies.

$$2.50r + 1.75d = 24.75 \quad \textit{The cost of roses and daisies totals \$24.75.}$$

$$r + d = 12 \quad \textit{The total number of roses and daisies is 12.}$$

Step 1

$$\begin{aligned} 2.50r + 1.75d &= 24.75 \\ +\ (-2.50)(r + d &= 12) \end{aligned}$$

Multiply the second equation by −2.50 to get opposite r-coefficients.

$$\begin{aligned} 2.50r + 1.75d &= 24.75 \\ +\ (-2.50r - 2.50d &= -30.00) \end{aligned}$$

Add this equation to the first equation to eliminate r.

Step 2

$$\begin{aligned} -0.75d &= -5.25 \\ d &= 7 \end{aligned}$$

Solve for d.

Step 3

$$\begin{aligned} r + d &= 12 && \textit{Write one of the original equations.} \\ r + 7 &= 12 && \textit{Substitute 7 for d.} \\ -7\ &\ \ -7 && \textit{Subtract 7 from both sides.} \\ r &= 5 \end{aligned}$$

Step 4 $(5, 7)$ *Write the solution as an ordered pair.*

Sam can buy 5 roses and 7 daisies.

CHECK IT OUT!

4. What if...? Sally spent $14.85 to buy 13 flowers. She bought lilies, which cost $1.25 each, and tulips, which cost $0.90 each. How many of each flower did Sally buy?

All systems can be solved in more than one way. For some systems, some methods may be better than others.

Know it! Note

Systems of Linear Equations

METHOD	USE WHEN…	EXAMPLE
Graphing	• Both equations are solved for y. • You want to estimate a solution.	$\begin{cases} y = 3x + 2 \\ y = -2x + 6 \end{cases}$
Substitution	• A variable in either equation has a coefficient of 1 or −1. • Both equations are solved for the same variable. • Either equation is solved for a variable.	$\begin{cases} x + 2y = 7 \\ x = 10 - 5y \end{cases}$ or $\begin{cases} x = 2y + 10 \\ x = 3y + 5 \end{cases}$
Elimination	• Both equations have the same variable with the same or opposite coefficients. • A variable term in one equation is a multiple of the corresponding variable term in the other equation.	$\begin{cases} 3x + 2y = 8 \\ 5x + 2y = 12 \end{cases}$ or $\begin{cases} 6x + 5y = 10 \\ 3x + 2y = 15 \end{cases}$

MCC.MP.3 MATHEMATICAL PRACTICES

THINK AND DISCUSS

1. Explain how multiplying the second equation in a system by -1 and eliminating by adding is the same as elimination by subtraction. Give an example of a system for which this applies.

2. Explain why it does not matter which variable you solve for first when solving a system by elimination.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a system of equations that you could solve using the given method.

Solving Systems of Linear Equations

Substitution	Elimination using addition or subtraction	Elimination using multiplication

6-3 Exercises

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GUIDED PRACTICE

Solve each system by elimination. Check your answer.

SEE EXAMPLE 1

1. $\begin{cases} -x + y = 5 \\ x - 5y = -9 \end{cases}$ **2.** $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$ **3.** $\begin{cases} 2x + 5y = -24 \\ 3x - 5y = 14 \end{cases}$

SEE EXAMPLE 2

4. $\begin{cases} x - 10y = 60 \\ x + 14y = 12 \end{cases}$ **5.** $\begin{cases} 5x + y = 0 \\ 5x + 2y = 30 \end{cases}$ **6.** $\begin{cases} -5x + 7y = 11 \\ -5x + 3y = 19 \end{cases}$

SEE EXAMPLE 3

7. $\begin{cases} 2x + 3y = 12 \\ 5x - y = 13 \end{cases}$ **8.** $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$ **9.** $\begin{cases} 2x + 4y = -4 \\ 3x + 5y = -3 \end{cases}$

SEE EXAMPLE 4

10. **Consumer Economics** Each family in a neighborhood is contributing \$20 worth of food to the neighborhood picnic. The Harlin family is bringing 12 packages of buns. The hamburger buns cost \$2.00 per package. The hot-dog buns cost \$1.50 per package. How many packages of each type of bun did they buy?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–13	1
14–16	2
17–19	3
20	4

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Online Extra Practice

Solve each system by elimination. Check your answer.

11. $\begin{cases} -x + y = -1 \\ 2x - y = 0 \end{cases}$ **12.** $\begin{cases} -2x + y = -20 \\ 2x + y = 48 \end{cases}$ **13.** $\begin{cases} 3x - y = -2 \\ -2x + y = 3 \end{cases}$

14. $\begin{cases} x - y = 4 \\ x - 2y = 10 \end{cases}$ **15.** $\begin{cases} x + 2y = 5 \\ 3x + 2y = 17 \end{cases}$ **16.** $\begin{cases} 3x - 2y = -1 \\ 3x - 4y = 9 \end{cases}$

17. $\begin{cases} x - y = -3 \\ 5x + 3y = 1 \end{cases}$ **18.** $\begin{cases} 9x - 3y = 3 \\ 3x + 8y = -17 \end{cases}$ **19.** $\begin{cases} 5x + 2y = -1 \\ 3x + 7y = 11 \end{cases}$

20. **Multi-Step** Mrs. Gonzalez bought centerpieces to put on each table at a graduation party. She spent \$31.50. There are 8 tables each requiring either a candle or vase. Candles cost \$3 and vases cost \$4.25. How many of each type did she buy?

21. **Geometry** The difference between the length and width of a rectangle is 2 units. The perimeter is 40 units. Write and solve a system of equations to determine the length and width of the rectangle. (*Hint:* The perimeter of a rectangle is $2\ell + 2w$.)

22. **ERROR ANALYSIS** Which is incorrect? Explain the error.

A

$$\begin{cases} x + y = -3 \\ 3x + y = 3 \end{cases} \qquad \begin{array}{r} x + y = -3 \\ -(3x + y = 3) \\ \hline -2x = 0 \\ x = 0 \end{array}$$

B

$$\begin{cases} x + y = -3 \\ 3x + y = 3 \end{cases} \qquad \begin{array}{r} x + y = -3 \\ -(3x + y = 3) \\ \hline -2x = -6 \\ x = 3 \end{array}$$

23. **Chemistry** A chemist has a bottle of a 1% acid solution and a bottle of a 5% acid solution. She wants to mix the two solutions to get 100 mL of a 4% acid solution. Follow the steps below to find how much of each solution she should use.

	1% Solution	+	5% Solution	=	4% Solution
Amount of Solution (mL)	x	+	y	=	
Amount of Acid (mL)	$0.01x$	+		=	0.04(100)

a. Copy and complete the table.

b. Use the information in the table to write a system of equations.

c. Solve the system of equations to find how much she will use from each bottle to get 100 mL of a 4% acid solution.

LINK **Math History**

In 1247, Qin Jiushao wrote *Mathematical Treatise in Nine Sections.* Its contents included solving systems of equations and the Chinese Remainder Theorem.

Critical Thinking **Which method would you use to solve each system? Explain.**

24. $\begin{cases} \frac{1}{2}x - 5y = 30 \\ \frac{1}{2}x + 7y = 6 \end{cases}$

25. $\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}$

26. $\begin{cases} 3x - y = 10 \\ 2x - y = 7 \end{cases}$

27. $\begin{cases} 3y + x = 10 \\ x = 4y + 2 \end{cases}$

28. $\begin{cases} y = -4x \\ y = 2x + 3 \end{cases}$

29. $\begin{cases} 2x + 6y = 12 \\ 4x + 5y = 15 \end{cases}$

30. **Business** A local boys club sold 176 bags of mulch and made a total of $520. They did not sell any of the expensive cocoa mulch. Use the table to determine how many bags of each type of mulch they sold.

Mulch Prices ($)	
Cocoa	4.75
Hardwood	3.50
Pine Bark	2.75

Real-World Connections

31. a. The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. Write a system of equations that can be used to find the individual prices for the supplies on shelf A and on shelf B.

b. Solve the system of equations by elimination.

c. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part **a**?

H.O.T. **32.** **Write About It** Solve the system $\begin{cases} 3x + y = 1 \\ 2x + 4y = -6 \end{cases}$. Explain how you can check your solution algebraically and graphically.

TEST PREP

33. A math test has 25 problems. Some are worth 2 points, and some are worth 3 points. The test is worth 60 points total. Which system can be used to determine the number of 2-point problems and the number of 3-point problems on the test?

(A) $\begin{cases} x + y = 25 \\ 2x + 3y = 60 \end{cases}$ (B) $\begin{cases} x + y = 60 \\ 2x + 3y = 25 \end{cases}$ (C) $\begin{cases} x - y = 25 \\ 2x + 3y = 60 \end{cases}$ (D) $\begin{cases} x - y = 60 \\ 2x - 3y = 25 \end{cases}$

34. An electrician charges $15 plus $11 per hour. Another electrician charges $10 plus $15 per hour. For what amount of time will the cost be the same? What is that cost?

(F) 1 hour; $25

(G) $1\frac{1}{4}$ hours; $28.75

(H) $1\frac{1}{2}$ hours; $30

(J) $1\frac{3}{4}$ hours; $32.50

35. **Short Response** Three hundred fifty-eight tickets to the school basketball game on Friday were sold. Student tickets were $1.50, and nonstudent tickets were $3.25. The school made $752.25.

a. Write a system of linear equations that could be used to determine how many student and how many nonstudent tickets were sold. Define the variables you use.

b. Solve the system you wrote in part **a.** How many student and how many nonstudent tickets were sold?

CHALLENGE AND EXTEND

H.O.T. **36.** If two equations in a system are represented by $Ax + By = C$ and $Dx + Ey = F$, where A, B, C, D, E, and F are constants, you can write a third equation by doing the following:

Multiply the second equation by a nonzero constant k to get $kDx + kEy = kF$.

Add this new equation to the first equation $Ax + By = C$ to get $(A + kD)x + (B + kE)y = C + kF$.

Prove that if (x_1, y_1) is a solution of the original system, then it is a solution of the system represented by $Ax + By = C$ and $(A + kD)x + (B + kE)y = C + kF$.

MATHEMATICAL PRACTICES

FOCUS ON MATHEMATICAL PRACTICES

H.O.T. **37.** **Reasoning** To solve the system of linear equations shown by elimination, which variable would you eliminate first? Explain your thinking, then find the solution of the system.

$$\begin{cases} 3x - 2y = 1 \\ 2x + 2y = 4 \end{cases}$$

H.O.T. **38.** **Comparison** To solve the system of linear equations shown by elimination, Mateo began by multiplying the first equation by 3. Mariana began by multiplying the second equation by −2. What will each student find as the sum of the equations? Which sum will be easier to solve?

$$\begin{cases} \frac{2}{3}x + 12y = 14 \\ -2x + 6y = 0 \end{cases}$$

Ready to Go On?

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Assessment and Intervention

6-1 Solving Systems by Graphing

Tell whether the ordered pair is a solution of the given system.

1. $(-2, 1); \begin{cases} y = -2x - 3 \\ y = x + 3 \end{cases}$

2. $(9, 2); \begin{cases} x - 4y = 1 \\ 2x - 3y = 3 \end{cases}$

3. $(3, -1); \begin{cases} y = -\frac{1}{3}x \\ y + 2x = 5 \end{cases}$

Solve each system by graphing.

4. $\begin{cases} y = x + 5 \\ y = \frac{1}{2}x + 4 \end{cases}$

5. $\begin{cases} y = -x - 2 \\ 2x - y = 2 \end{cases}$

6. $\begin{cases} \frac{2}{3}x + y = -3 \\ 4x + y = 7 \end{cases}$

7. **Banking** Christiana and Marlena opened their first savings accounts on the same day. Christiana opened her account with \$50 and plans to deposit \$10 every month. Marlena opened her account with \$30 and plans to deposit \$15 every month. After how many months will their two accounts have the same amount of money? What will that amount be?

6-2 Solving Systems by Substitution

Solve each system by substitution.

8. $\begin{cases} y = -x + 5 \\ 2x + y = 11 \end{cases}$

9. $\begin{cases} 4x - 3y = -1 \\ 3x - y = -2 \end{cases}$

10. $\begin{cases} y = -x \\ y = -2x - 5 \end{cases}$

11. $\begin{cases} x + y = -1 \\ y = -2x + 3 \end{cases}$

12. $\begin{cases} x = y - 7 \\ -y - 2x = 8 \end{cases}$

13. $\begin{cases} \frac{1}{2}x + y = 9 \\ 3x - 4y = -6 \end{cases}$

14. The Nash family's car needs repairs. Estimates for parts and labor from two garages are shown.

Garage	Parts (\$)	Labor (\$ per hour)
Motor Works	650	70
Jim's Car Care	800	55

For how many hours of labor will the total cost of fixing the car be the same at both garages? What will that cost be? Which garage will be cheaper if the repairs require 8 hours of labor? Explain.

6-3 Solving Systems by Elimination

Solve each system by elimination.

15. $\begin{cases} x + 3y = 15 \\ 2x - 3y = -6 \end{cases}$

16. $\begin{cases} x + y = 2 \\ 2x + y = -1 \end{cases}$

17. $\begin{cases} -2x + 5y = -1 \\ 3x + 2y = 11 \end{cases}$

18. It takes Akira 10 minutes to make a black and white drawing and 25 minutes for a color drawing. On Saturday he made a total of 9 drawings in 2 hours. Write and solve a system of equations to determine how many drawings of each type Akira made.

PARCC Assessment Readiness

COMMON CORE GPS

1. The Fun Guys game rental store charges an annual fee of \$5 plus \$5.50 per game rented. The Game Bank charges an annual fee of \$17 plus \$2.50 per game. For how many game rentals will the cost be the same at both stores? What is that cost?

(A) 4 games; \$27
(B) 6 games; \$38
(C) 3 games; \$22
(D) 2 games; \$16

2. If the pattern in the table continues, in what month will the number of sales of CDs and movie tickets be the same? What number will that be?

Total Number Sold				
Month	1	2	3	4
CDs	700	685	670	655
Movie tickets	100	145	190	235

(F) Month 10; 550
(G) Month 9; 580
(H) Month 8; 580
(J) Month 11; 550

3. Solve $\begin{cases} 4x - 4y = -16 \\ x - 2y = -12 \end{cases}$ by substitution. Express your answer as an ordered pair.

(A) $(8, -4)$
(B) $(4, 8)$
(C) $(-2, 4)$
(D) $(4, -8)$

4. Solve $\begin{cases} 2x - 5y = -7 \\ 5x - 3y = 11 \end{cases}$ by elimination.

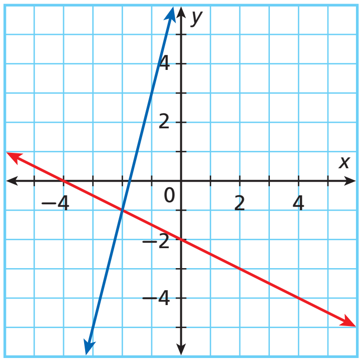
Express your answer as an ordered pair.

(F) $(3, 4)$
(G) $(3, 2)$
(H) $\left(\frac{4}{7}, \frac{8}{5}\right)$
(J) $(4, 3)$

5. At the local pet store, zebra fish cost \$2.10 each and neon tetras cost \$1.85 each. If Marsha bought 13 fish for a total cost of \$25.80, not including tax, how many of each type of fish did she buy?

(A) 5 zebra fish, 8 neon tetras
(B) 7 zebra fish, 6 neon tetras
(C) 8 zebra fish, 5 neon tetras
(D) 6 zebra fish, 7 neon tetras

6. Which system of equations is shown on the graph?



(F) $\begin{cases} x + 2y = -4 \\ y = 4x + 7 \end{cases}$

(G) $\begin{cases} y = -\frac{1}{2}x - 2 \\ y = -4x + 7 \end{cases}$

(H) $\begin{cases} x = \frac{1}{2}x - 2 \\ y = 4x + 7 \end{cases}$

(J) $\begin{cases} x + 2y = 4 \\ 4x - y = -7 \end{cases}$

7. The sum of the digits of a two-digit number is 8. If the number is multiplied by 4, the result is 104. Write and solve a system of equations. Find the number.

(A) $\begin{cases} x + y = 8 \\ 4(x + y) = 104 \end{cases}$
The number is 35.

(B) $\begin{cases} x + y = 8 \\ 4(10x + y) = 104 \end{cases}$
The number is 17.

(C) $\begin{cases} x + y = 8 \\ 4(2x + y) = 104 \end{cases}$
The number is 18.

(D) $\begin{cases} x + y = 8 \\ 4(10x + y) = 104 \end{cases}$
The number is 26.

Mini-Task

8. Gracey's little sister Eliza was born when Gracey was 7 years old. Now, Eliza is half as old as Gracey. Write and solve an equation to find the age of each sister.