Momentum and Impulse

8.01

W05D1

Today's Reading Assignment (W05D1):

MIT 8.01 Course Notes

Chapter 10 Momentum, System of Particles, and Conservation of Momentum

Sections 10.1-10.9

Announcements

Problem Set 3 due Week 5 Tuesday at 9 pm in box outside 26-152

Math Review Week 5 Tuesday 9-11 pm in 26-152.

Add Date Friday Oct 4

Learning Resources and Suggestions

Sunday Tutoring Session 1-5 pm in 26-152

Office Hours: You can attend any office hour that fits your schedule

Join Seminar XL Study Group

TSR Tutorial Service Room 12-124:

Individual tutoring

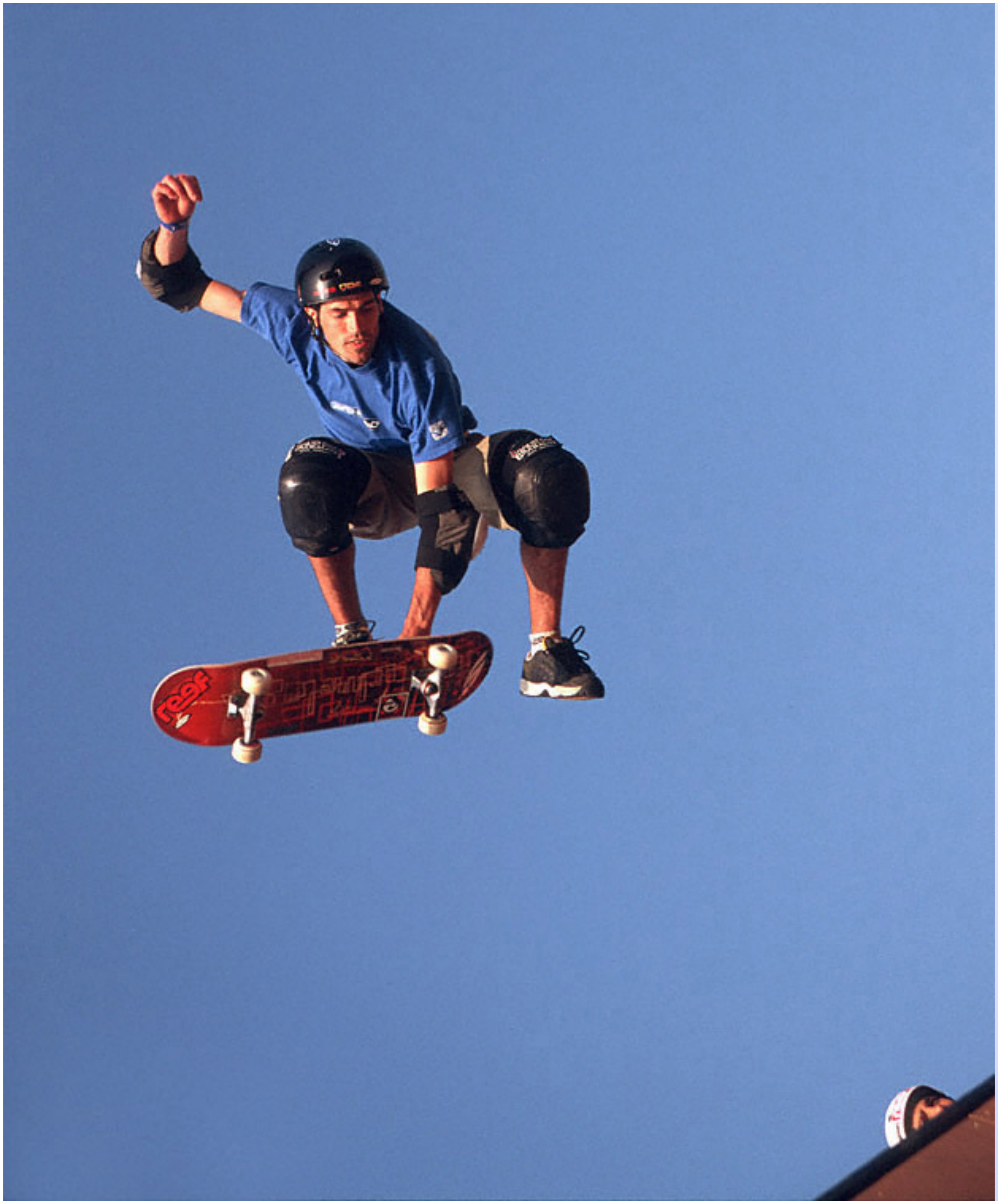
Physics Homework Nights Sunday in 12-124 from 7 to 11 p.m.

Study Suggestions:

Print Today's Presentation Slides PDF (Print) version and bring to each class

Print and work through In-class and Friday Problem Solving solutions

Print and work through Problem Set solutions if you had trouble with any particular problem



Momentum and Impulse

Obeys a conservation law

Simplifies complicated motions

Describes collisions

Basis of rocket propulsion & space travel

Momentum and Impulse: Single Particle

- Momentum $$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

  SI units $$[\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}] = [\mathrm{N}\cdot\mathrm{s}]$$

- Change in momentum $$\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$$

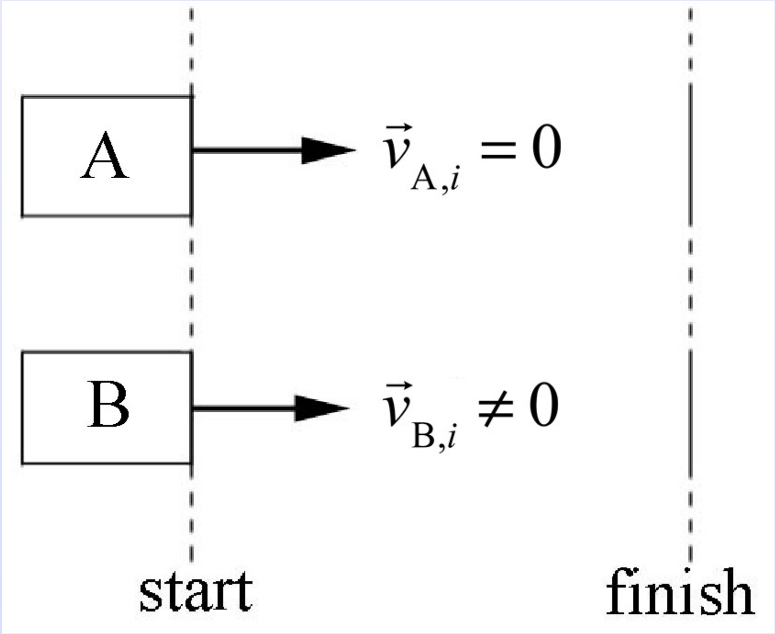
- Force $$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$$

- Impulse $$\vec{\mathbf{I}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}\,dt = \int_{t_i}^{t_f} \frac{d\vec{\mathbf{p}}}{dt}\,dt = \int_{t_i}^{t_f} d\vec{\mathbf{p}} = \Delta\vec{\mathbf{p}} \equiv \vec{\mathbf{p}}(t_{\mathrm{f}}) - \vec{\mathbf{p}}(t_i)$$

- SI units $$[\mathrm{N}\cdot\mathrm{s}]$$

Concept Question: Pushing Identical Objects

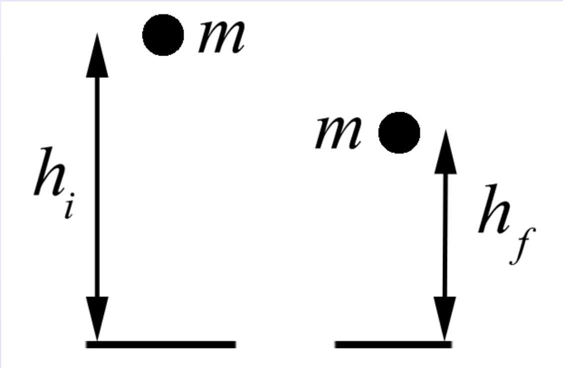
Identical constant forces push two identical objects A and B continuously from a starting line to a finish line. Neglect friction. If A is initially at rest and B is initially moving to the right,



1. Object A has the larger change in momentum.
2. Object B has the larger change in momentum.
3. Both objects have the same change in momentum
4. Not enough information is given to decide.

Table Problem: Impulse and Superball

A superball of *m*, starting at rest, is dropped from a height h_i above the ground and bounces back up to a height of h_f. The collision with the ground occurs over a time interval Δt_c. Ignore air resistance.



a) What is the momentum of the ball immediately before the collision?

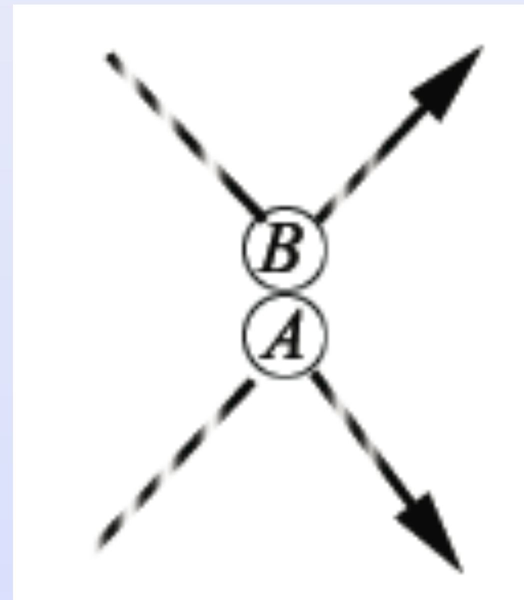
b) What is the momentum of the ball immediately after the collision?

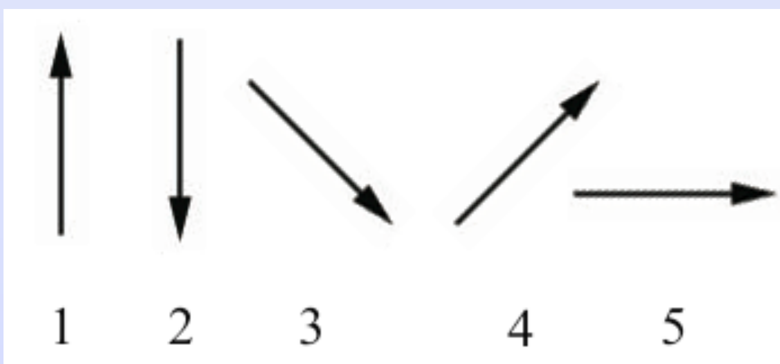
c) What impulse is imparted to the ball?

d) What is the average force of the ground on the ball?

Concept Question: Impulse

The figure to the right depicts the paths of two colliding steel balls, A and B. Which of the arrows 1-5 best represents the impulse applied to ball B by ball A during the collision?

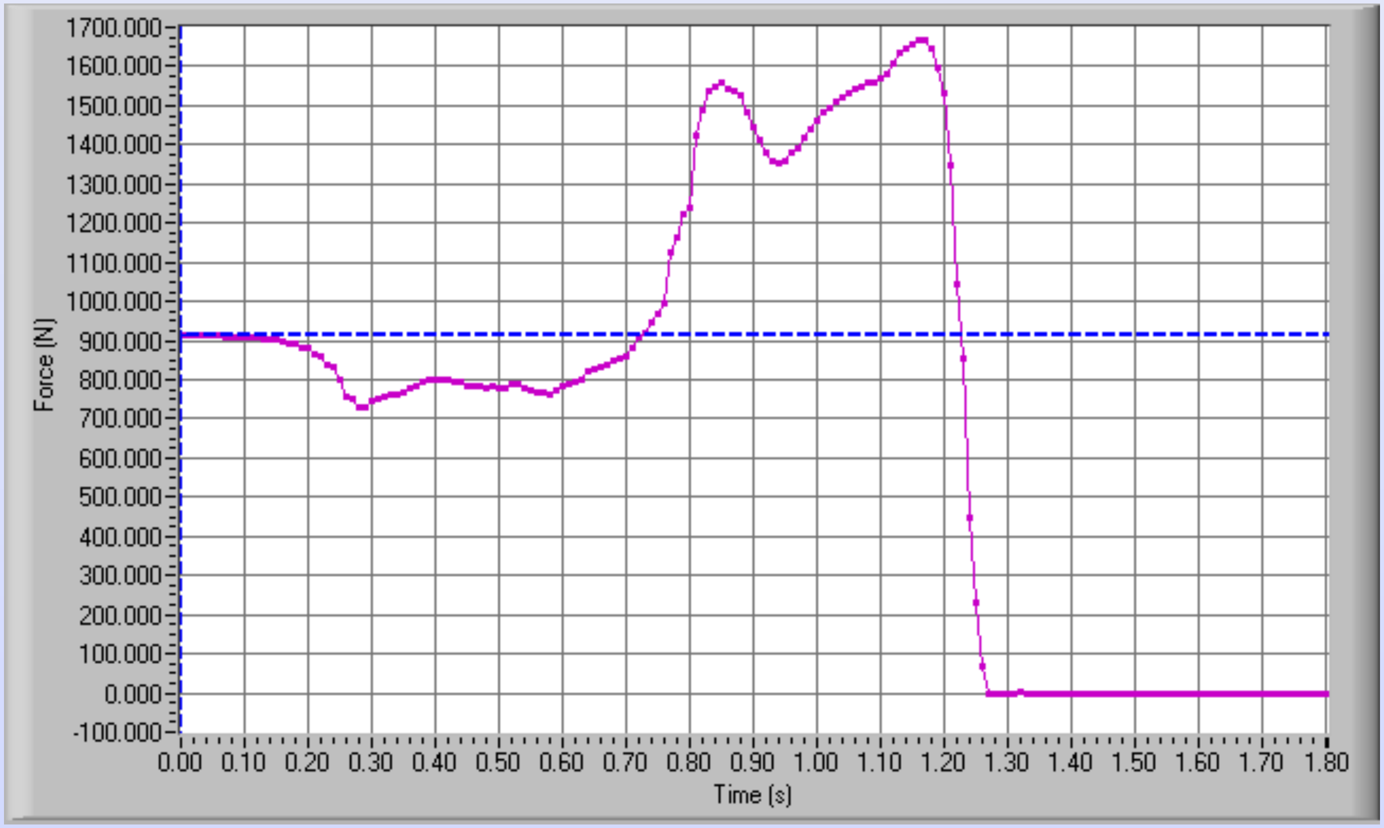




Demo:
Jumping Off the Floor
with a Non-Constant Force

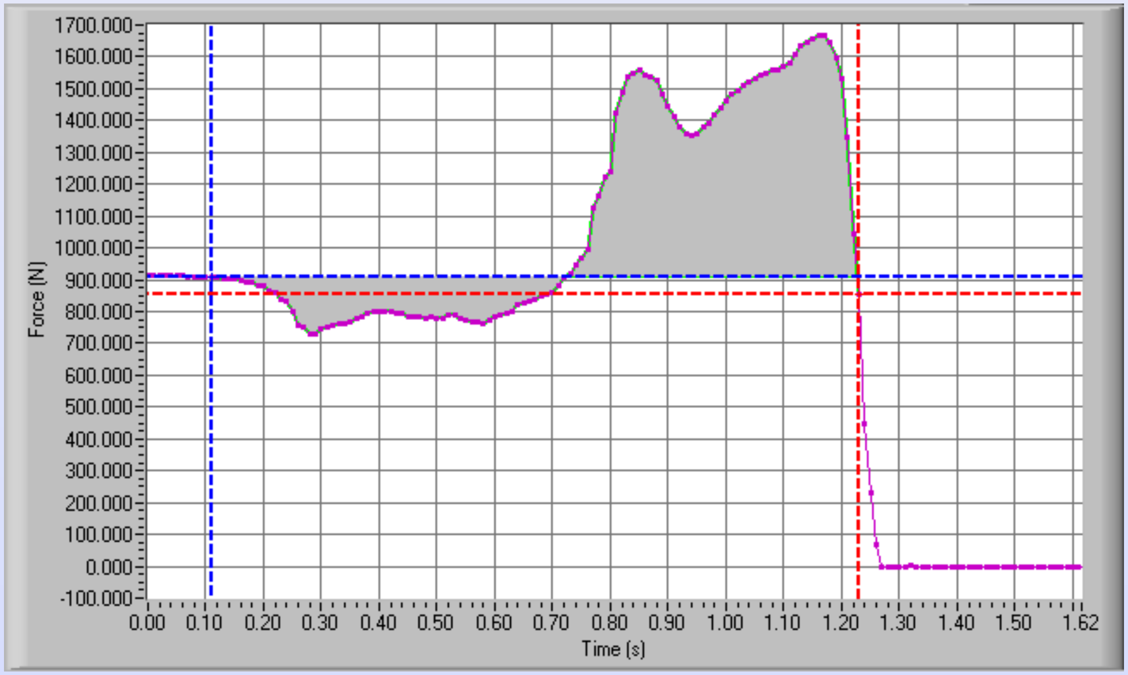
Demo Jumping: Non-Constant Force

- Plot of total external force vs. time for Andy jumping off the floor. Weight of Andy is 911 N.



Demo Jumping: Impulse

- Shaded area represents impulse of total force acting on Andy as he jumps off the floor



$$\vec{\mathbf{F}}^{\text{total}}(t) = \vec{\mathbf{N}}(t) + \vec{\mathbf{F}}_{grav}$$

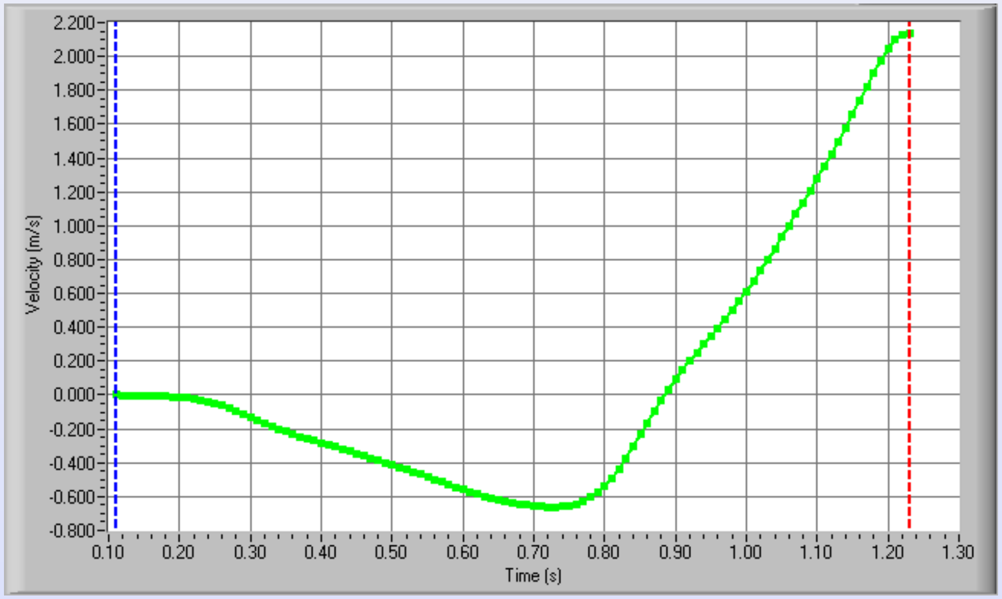
$$\vec{\mathbf{I}}[t_i, t_f] = \int_{t_i = 0.11\,\text{s}}^{t_f = 1.23\,\text{s}} \vec{\mathbf{F}}^{\text{total}}(t)\, dt = 199\ \text{N} \cdot \text{s}$$

Demo Jumping: Height

When Andy leaves the ground, the impulse is

$$I_y[0.11\ \mathrm{s}, 1.23\ \mathrm{s}] = 199\ \mathrm{N} \cdot \mathrm{s}$$

So the *y*-component of his velocity is



$$v_{y,f} = I_y[t_i, t_f]/m = (199\ \mathrm{N} \cdot \mathrm{s})(9.80\ \mathrm{m} \cdot \mathrm{s}^{-2})/(911\ \mathrm{N}) = 2.14\ \mathrm{m} \cdot \mathrm{s}^{-1}$$

Andy jumped

$$(1/2)mv_{y,f}^2 = mgh \Rightarrow h = v_{y,f}^2/2g = 23.3\ \mathrm{cm}$$

System of Particles: Center of Mass

Position and Velocity of Center of Mass

Mass for collection of discrete bodies (system):

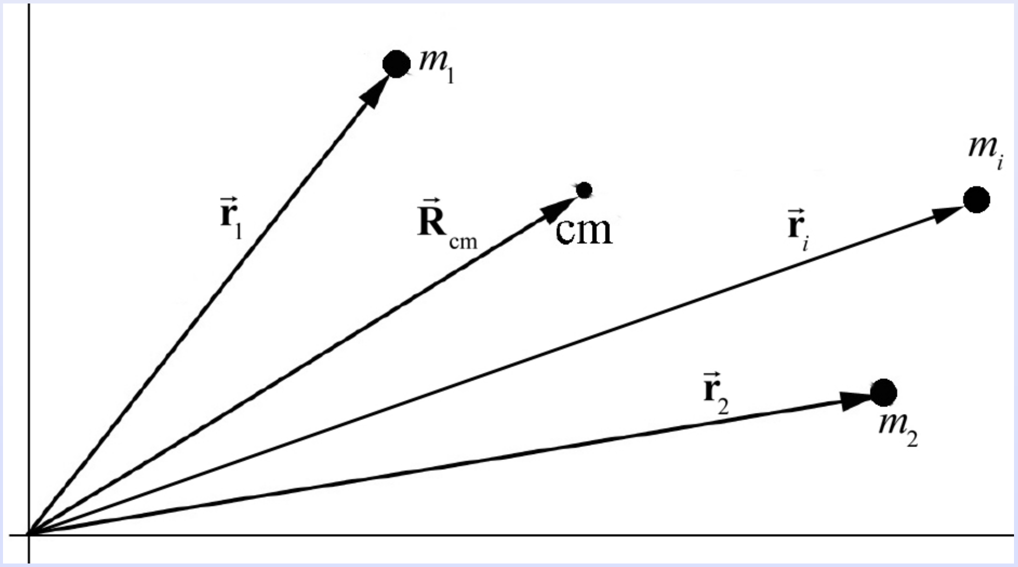
$$m_{\text{sys}} = \sum_{i=1}^{i=N} m_i$$

Momentum of system:

$$\vec{\mathbf{p}}_{\text{sys}} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i$$

Position of center of mass

$$\vec{\mathbf{R}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i$$



Velocity of center of mass

$$\vec{\mathbf{V}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i = \frac{\vec{\mathbf{p}}_{\text{sys}}}{m_{\text{sys}}} \Rightarrow \qquad \vec{\mathbf{p}}_{\text{sys}} = m_{\text{sys}} \vec{\mathbf{V}}_{cm}$$

Continuous Bodies of Center of Mass

Infinitesimal mass element

$$m_i \to dm = \begin{cases} \rho dV, & \text{volume element} \\ \sigma dA, & \text{area element} \\ \lambda ds, & \text{length element} \end{cases}$$

Mass for system: $$m_{\text{sys}} = \sum_{i=1}^{i=N} m_i \to \int_{body} dm$$

Position vector for infinitesimal element $\vec{\mathbf{r}}_i \to \vec{\mathbf{r}}$

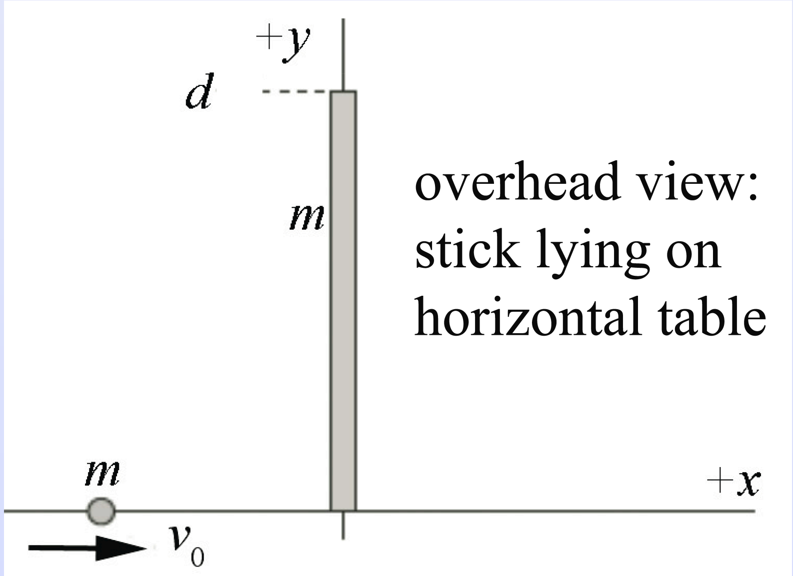
Position of center of mass $$\vec{\mathbf{R}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i \to \frac{1}{m_{\text{sys}}} \int_{body} dm \vec{\mathbf{r}}$$

Velocity vector for infinitesimal element $\vec{\mathbf{v}}_i \to \vec{\mathbf{v}}$

Velocity of center of mass $$\vec{\mathbf{V}}_{cm} = \frac{1}{m_{\text{sys}}} \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i \to \frac{1}{m_{\text{sys}}} \int_{body} dm \vec{\mathbf{v}} = \frac{\vec{\mathbf{p}}_{\text{sys}}}{m_{\text{sys}}}$$

Table Problem: Center of Mass of Rod and Particle Post- Collision

A slender uniform rod of length d and mass m rests along the x-axis on a frictionless, horizontal table. A particle of equal mass m is moving along the x-axis at a speed v_0. At $t = 0$, the particle strikes the end of the rod and sticks to it. Find a vector expression for the position of the center of mass of the system for (i) $t = 0$, (ii) $t > 0$. **The figure on the right shows an overhead view of the rod lying on the table.**



System of Particles:
Internal and External Forces,
Center of Mass Motion

System of Particles: Newton's Second and Third Laws

The momentum of a system remains constant unless the system is acted on by an **external** force in which case the acceleration of center of mass satisfies

$$\vec{\mathbf{F}}_{\text{ext}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt} = m_{\text{sys}} \frac{d\vec{\mathbf{V}}_{cm}}{dt} = m_{\text{sys}} \vec{\mathbf{A}}_{cm}$$

Demo : Center of Mass Trajectory B78

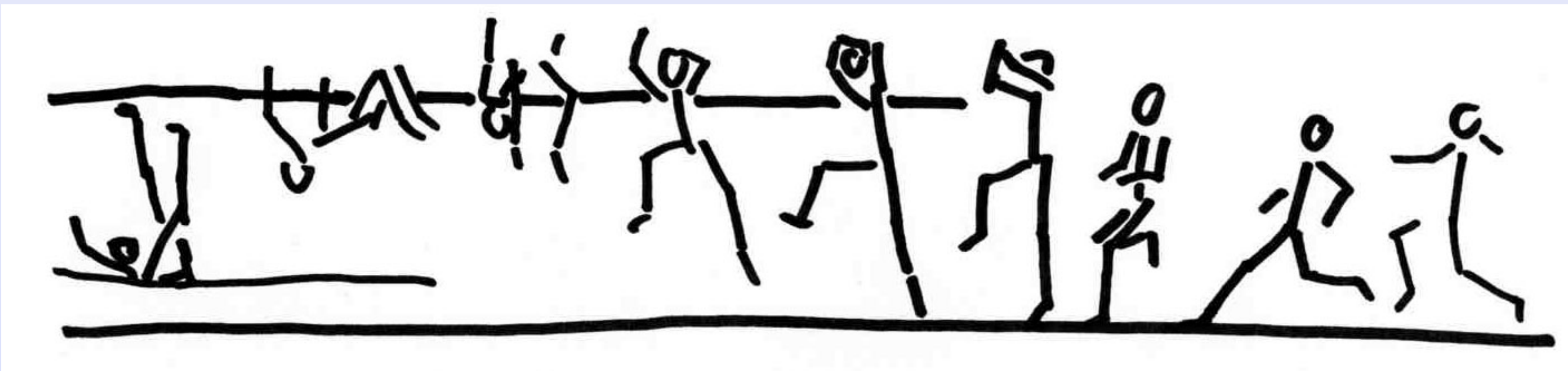
http://tsgphysics.mit.edu/front/?page=demo.php&letnum=B 78&show=0

Odd-shaped objects with their centers of mass marked are thrown. The centers of mass travel in a smooth parabola. The objects consist of: a squash racket, a 16” diameter disk weighted at one point on its outer rim, and two balls connected with a rod. This demonstration is shown with UV light.

Video link:

http://techtv.mit.edu/videos/3052-center-of-mass-trajectory

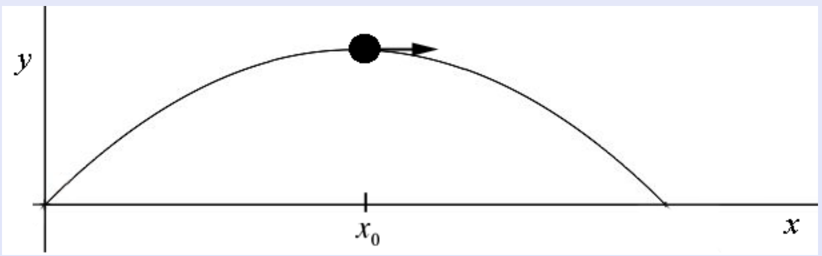
CM moves as though all external forces on the system act on the CM

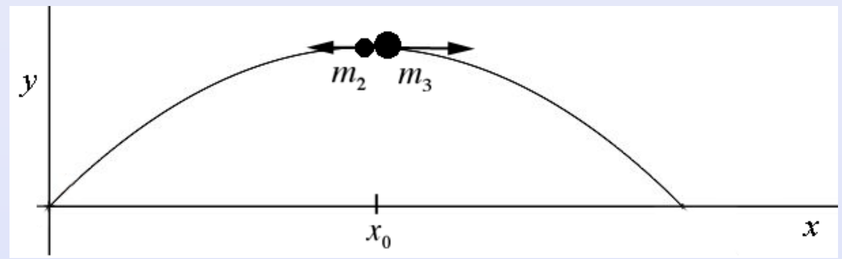


so the jumper's cm follows a parabolic trajectory of a point moving in a uniform gravitational field

Center of mass passes under the bar

Table Problem: Exploding Projectile Center of Mass Motion





An instrument-carrying projectile of mass m_1 accidentally explodes at the top of its trajectory. The horizontal distance between launch point and the explosion is x_0. The projectile breaks into two pieces which fly apart horizontally. The larger piece, m_3, has three times the mass of the smaller piece, m_2. To the surprise of the scientist in charge, the smaller piece returns to earth at the launching station.

a) How far has the center of mass of the system traveled from the launch when the pieces hit the ground?

b) How far from the launch point has the larger piece traveled when it first hits the ground?

Internal Force on a System of N Particles is Zero

- The internal force on the *i*th particle is sum of the interaction forces with all the other particles

$$\vec{\mathbf{F}}_{\text{int},i} = \sum_{\substack{j=1 \\ i \neq j}}^{j=N} \vec{\mathbf{F}}_{j,i}$$

- The internal force is the sum of the internal force on each particle

$$\vec{\mathbf{F}}_{\text{int}} = \sum_{i=1}^{i=1} \vec{\mathbf{F}}_{\text{int},i} = \sum_{\substack{j=1 \\ i \neq j}}^{j=N} \vec{\mathbf{F}}_{j,i}$$

- Newton's Third Law: internal forces cancel in pairs

$$\vec{\mathbf{F}}_{i,j} = -\vec{\mathbf{F}}_{j,i}$$

- So the internal force is zero

$$\vec{\mathbf{F}}_{\text{int}} = \vec{\mathbf{0}}$$

Force on a System of N Particles is the External Force

The force on a system of particles is the external force because the internal force is zero

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\text{ext}} + \vec{\mathbf{F}}_{\text{int}} = \vec{\mathbf{F}}_{\text{ext}}$$

External Force and Momentum Change

The momentum of a system of N particles is defined as the sum of the individual momenta of the particles

$$\vec{\mathbf{p}}_{\text{sys}} \equiv \sum_{i=1}^{i=N} \vec{\mathbf{p}}_i = m_{sys} \vec{\mathbf{V}}_{\text{cm}}$$

Force changes the momentum of the system

$$\vec{\mathbf{F}} = \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i = \sum_{i=1}^{i=N} \frac{d\vec{\mathbf{p}}_i}{dt} \equiv \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt}$$

Force equals external force

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\text{ext}}$$

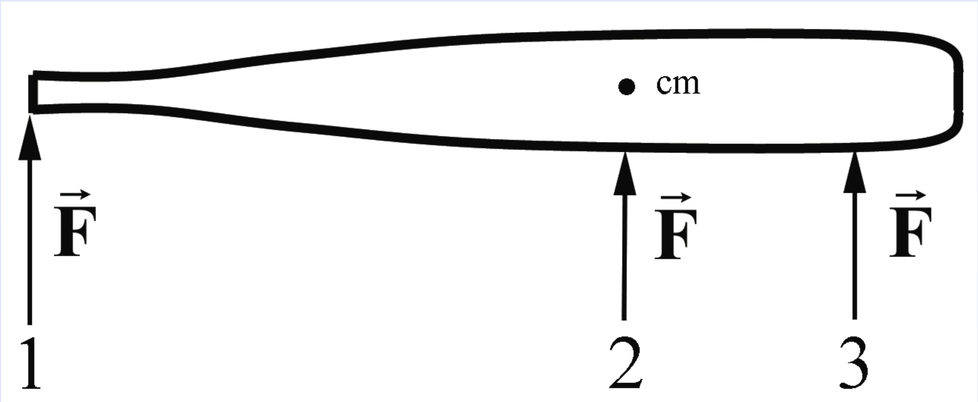
Newton's Second and Third Laws for a System of Particles:

The external force is equal to the change in momentum of the system and is proportional to the acceleration of the center of mass.

$$\vec{\mathbf{F}}_{\text{ext}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt} = \frac{d(m_{\text{sys}}\vec{\mathbf{V}}_{cm})}{dt} = m_{\text{sys}}\vec{\mathbf{A}}_{cm}$$

Concept Q.: Pushing a Baseball Bat

A baseball bat is pushed with a force F. You may assume that the push is instantaneous. Which of the following locations will the force produce an acceleration of the center of mass with the largest magnitude?



1. Pushing at Position 1.
2. Pushing at Position 2 (center of mass).
3. Pushing at Position 3.
4. Pushing at 1,2,and 3 all produce the same magnitude of acceleration of the center of mass/

Conservation of Momentum: System

For a fixed choice of system, if there are no external forces acting on the system then the momentum of the system is constant is constant.

$$\vec{\mathbf{F}}_{\text{ext}} = \vec{\mathbf{0}} \Rightarrow \Delta\vec{\mathbf{p}}_{\text{system}} = \vec{\mathbf{0}}$$

Strategy: Momentum of a System

1. Choose system

2. Identify initial and final states

3. Identify any external forces in order to determine whether any component of the momentum of the system is constant or not

i) If there is a non-zero total external force:

$$\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}_{sys}}{dt}$$

ii) If the total external force is zero then momentum is constant

$$\vec{\mathbf{p}}_{sys,0} = \vec{\mathbf{p}}_{sys,f}$$

External Forces and Constancy of Momentum Vector

The external force may be zero in one direction but not others

The component of the system momentum is constant in the direction that the external force is zero

The component of system momentum is not constant in a direction in which external force is not zero

Modeling: Instantaneous Interactions

- Decide whether or not an interaction is instantaneous.

- External impulse changes the momentum of the system.

$$\vec{\mathbf{I}}[t, t+\Delta t_{col}] = \int_{t}^{t+\Delta t_{col}} \vec{\mathbf{F}}_{ext} dt = (\vec{\mathbf{F}}_{ext})_{ave} \Delta t_{col} = \Delta \vec{\mathbf{p}}_{sys}$$

- If the collision time is approximately zero,

$$\Delta t_{col} \simeq 0$$

then the change in momentum is approximately zero.

$$\Delta \vec{\mathbf{p}}_{system} \cong \vec{\mathbf{0}}$$

Table Problem: Landing Plane and Sandbag

A light plane of mass 1000 kg makes an emergency landing on a short runway. With its engine off, it lands on the runway at a speed of 40 ms^{-1}. A hook on the plane snags a cable attached to a sandbag of mass 120 kg and drags the sandbag along. If the coefficient of friction between the sandbag and the runway is $\mu = 0.4$, and if the plane's brakes give an additional retarding force of magnitude 1400 N, how far does the plane go before it comes to a stop?