Momentum
How hard it is to stop a moving object.
Related to both mass and velocity.
For one particle

$\mathbf{p} = m\mathbf{v}$

For a system of multiple particles

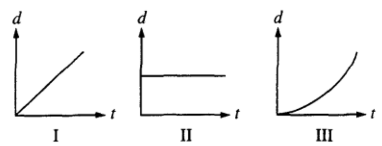
$\mathbf{P} = \Sigma \mathbf{p}_i = \Sigma m_i \mathbf{v}_i$

Units: N s or kg m/s
Momentum is a vector!

Problem: Momentum (1998)

Questions 43-44

Three objects can only move along a straight, level path. The graphs below show the position *d* of each of the objects plotted as a function of time *t*.



43. The magnitude of the momentum of the object is increasing in which of the cases?

(A) II only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III

Ans. Explain your reasoning:
Graph III is the only graph where acceleration is happening (as evidenced by a curved *d* vs. *t* graph). This means that a net force is being applied to the object of mass *m* (by Newton's Second Law). In order for momentum to increase, an impulse (*J*) needs to be applied (a Force *F* over a period of time, Δt).

$$\mathbf{\Delta p = J = F \cdot \Delta t}$$

Impulse (J)
The product of an external force and time, which results in a change in momentum
$J = F t$
$J = \Delta P$
Units: N s or kg m/s

Problem: Impulse (1984)

56. Two planets have the same size, but different masses, and no atmospheres. Which of the following would be the same for objects with equal mass on the surfaces of the two planets?

I. The rate at which each would fall freely
II. The amount of mass each would balance on an equal-arm balance
III. The amount of momentum each would acquire when given a certain impulse

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

Explain your reasoning:

Ans. The gravity would be different on both planets because gravity depends both on the mass of the planet and its radius. Since gravity is different on each planet, they would not fall at the same rate. But if you put both masses on opposite sides of an equal-arm balance they would balance since the masses are equal. Also, since they have the same mass, the same amount of impulse ($J = F \cdot \Delta t$) should produce the same results. In terms of momentum increase.

Problem: Impulse (1998)

57. A ball of mass 0.4 kg is initially at rest on the ground. It is kicked and leaves the kicker's foot with a speed of 5.0 m/s in a direction 60° above the horizontal. The magnitude of the impulse imparted by the ball to the foot is most nearly

(A) $1\ \text{N}\cdot\text{s}$

(B) $\sqrt{3}\ \text{N}\cdot\text{s}$

(C) $2\ \text{N}\cdot\text{s}$

(D) $\frac{2}{\sqrt{3}}\ \text{N}\cdot\text{s}$

(E) $4\ \text{N}\cdot\text{s}$

Show your work:

Ans. The **impulse** is simply **J = m·Δv. The mass is 0.4 kg** and the **speed** is **5.0 m/s. J = 0.4 kg × 5.0 m/s = 2 N·s. The angle is irrelevant here.**

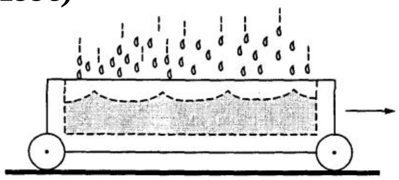
Law of Conservation of Momentum

If the resultant external force on a system is zero, then the momentum of the system will remain constant.

The sum of the momentums before a collision is equal to the sum of the momentums after a collision.

$\Sigma P_b = \Sigma P_a$

Problem: Conservation of Momentum (1998)



4. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of momentum
(B) increase because of conservation of mechanical energy
(C) decrease because of conservation of momentum
(D) decrease because of conservation of mechanical energy
(E) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

Explain your reasoning:

Ans. Mechanical energy is not conserved, in general. Total energy is, but mechanical energy is not. Momentum is ALWAYS conserved! So, suppose you had a cart that has a mass of 10kg moving at 5 m/s. It has momentum of 50 kg*m/s. Since friction does not act, the momentum will remain 50kg*m/s (Newton's law). Suppose at some later time it has filled up with 10kg of rain, so now the cart has a mass of 20kg. It still has the 50kg*m/s of momentum, so it must be moving at 2.5m/s, which means it has decreased due to conservation of momentum.

Collisions

Follow ***Newton's Third Law*** which tells us that the force exerted by body A on body B in a collision is equal and opposite to the force exerted on body B by body A.

During a collision, external forces are ignored.

The time frame of the collision is very short.

The forces are ***impulsive*** forces (high force, short duration).

Collision Types

Elastic: P is conserved, K is conserved

Inelastic: P is conserved, K is NOT conserved

Perfectly Inelastic means the bodies stick together

Problem: Collisions (1993)

10. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?

(A) The collision is elastic.
(B) All of the initial kinetic energy of the less-massive object is lost.
(C) The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.
(D) The speed of the objects that are stuck together will be less than the initial speed of the less-massive object.
(E) The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

Explain your reasoning:

Ans. Again, by **conservation of momentum,** if the **mass** increases, the **velocity** must decrease. $\mathbf{p_{in} = p_{out}}$

Problem: Collisions (1993)

11. Two objects having the same mass travel toward each other on a flat surface, each with a speed of 10 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 20 meters per second relative to the surface. Which of the following assessments of this report is most accurate?

(A) Momentum was not conserved, therefore the report is false.
(B) If potential energy was released to the objects during the collision, the report could be true.
(C) If the objects had different masses, the report could be true.
(D) If the surface was inclined, the report could be true.
(E) If there was no friction between the objects and the surface, the report could be true.

Explain your reasoning:

Ans. Momentum is conserved *provided that an external force not be applied*. If the objects have potential energy stored, this could provide the force and alter the momentum balance.

Problem: Collision (1988)

3. A railroad car of mass *m* is moving at speed υ when it collides with a second railroad car of mass *M* which is at rest. The two cars lock together instantaneously and move along the track. What is the speed of the cars immediately after the collision?

(A) $\frac{\upsilon}{2}$
(B) $\frac{m\upsilon}{M}$
(C) $\frac{M\upsilon}{m}$
(D) $\frac{(m+M)\upsilon}{m}$
(E) $\frac{mv}{m+M}$

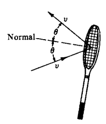
Show your work:

Ans. This is an elastic collision so the formula $m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$ applies.
Substituting our given values:

$$m\upsilon + M(0) = (m + M)v_3$$

$$\frac{m\upsilon}{m + M} = v_3$$

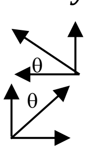
Problem: Momentum Change (1988)



7. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially, as shown above. The magnitude of the momentum change of the ball is

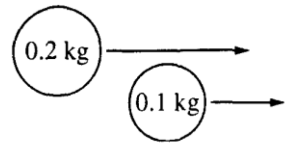
(A) 0
(B) *mv*
(C) 2*mv*
(D) 2*mv* sin θ
(E) 2*mv* cos θ

Show your work:



Ans. The horizontal components cancel each other out because they are in opposite directions. The incident component → goes to the right and cancels out the reflected component ← which goes to the left. The vertical components however add up since they are both upwards. The vertical components are

Problem: Collision (1998)



41. Two objects of mass 0.2 kg and 0.1 kg, respectively, move parallel to the x-axis, as shown above. The 0.2 kg object overtakes and collides with the 0. 1 kg object. Immediately after the collision, the y-component of the velocity of the 0.2 kg object is 1 m/s upward. What is the y-component of the velocity of the 0.1 kg object immediately after the collision?

(A) 2 m/s downward
(B) 0.5 m/s downward
(C) 0 m/s
(D) 0.5 m/s upward
(E) 2 m/s upward

Show your work:

Ans. Has to be **2 m/s downwards.** The **vertical momentum** is **zero initially.** So at the end, it also has to be **zero.**

$$p_{yin} = p_{yout}$$

$$m \cdot v_{yi} = m \cdot v_{yf}$$

$$0.2\text{ kg} \cdot 1\text{ m/s} = 0.1\text{ kg} \cdot v_{yf}$$

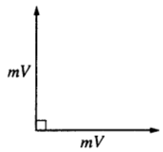
$$2\text{ m/s} = v_{yf}$$

Explosion

Mathematically, handled just like an ordinary perfectly inelastic collision.

Momentum is conserved, kinetic energy is not.

Problem: Explosion (1998)



67. A stationary object explodes, breaking into three pieces of masses *m*, *m*, and 3*m*. The two pieces of mass *m* move off at right angles to each other with the same magnitude of momentum *m*V, as shown in the diagram above. What are the magnitude and direction of the velocity of the piece having mass 3*m* ?

	Magnitude	Direction
(A)	$\frac{V}{\sqrt{3}}$	↗
(B)	$\frac{V}{\sqrt{3}}$	↙
(C)	$\frac{\sqrt{2}\,V}{3}$	↗
(D)	$\frac{\sqrt{2}\,V}{3}$	↙
(E)	$\sqrt{2}\,V$	↙

Show your work:

Ans. The magnitude of the velocity is $\frac{\sqrt{2}v}{3}$, and the direction is 45 degrees to the left of straight down. To answer the question you need to use conservation of momentum. Before the explosion there is no momentum, so there must be no net momentum after the explosion.

The two small pieces have momentum vectors pointing right and up, so in order for the large piece to cancel both of these out it must be heading down and to the left. So we take the angle θ as being the angle to the left of straight down. This eliminates answers A and C.

Looking at just the x-direction (left-right) direction to start with, the momentum of the large piece in the x-direction is

$p_x = 3 \cdot m \cdot v \cdot \sin\theta.$

And this has to be equal to the momentum of the small piece heading right.

$p_x = 3 \cdot m \cdot v \cdot \sin\theta.$

$3 \cdot m \cdot v \cdot \sin\theta = m \cdot v_1.$

Similarly in the y-direction

$p_y = 3 \cdot m \cdot v \cdot \cos\theta = m \cdot v_1$

We, can see from this that

$p_x = p_y$

$3 \cdot m \cdot v \cdot \sin\theta = 3 \cdot m \cdot v \cdot \cos\theta$

$\sin\theta = \cos\theta$

$\theta = 45°.$

So now that we know the angle we can find the velocity.

$3 \cdot m \cdot v \cdot \cos\theta = m \cdot v_1$

$3 \cdot v \cdot \cos\theta = v_1$

$$v = \frac{v_1}{3\cos\theta} = \frac{v_1}{3\cos 45^o} = \frac{v_1}{3 \cdot \frac{\sqrt{2}}{2}} = \frac{2v_1}{3 \cdot \sqrt{2}} = \frac{\sqrt{2}v_1}{3}$$

<<ADVANCED TOPIC>>

Center of Mass

Where all the mass can be considered to exist

For uniform objects, the center of mass resides at geometric center.

For collection of points, use these equations

$x_{cm} = \Sigma\, m_i x_i / \Sigma m_i$

$y_{cm} = \Sigma\, m_i y_i / \Sigma m_i$

$z_{cm} = \Sigma\, m_i z_i / \Sigma m_i$

where x_{cm}, y_{cm}, and z_{cm} are the coordinates of the center of mass, and Σm_i is the total mass of the system.

Problem: Center of Mass (1998)

63. Two people of unequal mass are initially standing still on ice with negligible friction. They then simultaneously push each other horizontally. Afterward, which of the following is true?

(A) The kinetic energies of the two people are equal.

(B) The speeds of the two people are equal.

(C) The momenta of the two people are of equal magnitude.

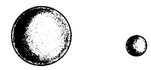
(D) The center of mass of the two-person system moves in the direction of the less massive person.

(E) The less massive person has a smaller initial acceleration than the more massive person.

Explain your reasoning:

Ans. The kinetic energies won't be the same (Ans. A) because their velocities are squared and their masses aren't the same either. By the law of conservation of momentum though, their momenta must equal zero after the collision as their momenta is zero before. Ans D, the center of mass, doesn't make sense. The center of mass would move towards the heavier person if anything.

Problem: Center of Mass (1993)



8. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?

(A) Acceleration

(B) Velocity

(C) Kinetic energy

(D) Displacement from the center of mass

(E) Gravitational force

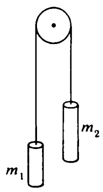
Show your work or explain your reasoning:

Ans. the gravitational force between them would be the same because of Newton's Third Law. For every action there is an equal and opposite reaction. The center of mass would be closer to the larger sphere. The acceleration due to gravity would be greater for the larger sphere.

Momentum
For one particle
$\mathbf{p} = m\mathbf{v}$
For a system of multiple particles
$\mathbf{P} = \Sigma \mathbf{p}_i = \Sigma m_i \mathbf{v}_i$
If center of mass is defined
$\mathbf{P} = M\mathbf{v}_{cm}$
Momentum is a vector!

Problem: Energy and Momentum (CM 1984)

Questions 13-14



A system consists of two objects having masses m_l and m_2 ($m_l < m_2$). The objects are connected by a massless string, hung over a pulley as shown above, and then released.

13. When the speed of each object is v, the magnitude of the total linear momentum of the system is
(A) $(m_1 + m_2)$ v **(B) $(m_2 - m_1)$ v** (C) $\frac{1}{2}(m_l+m_2)$v (D) $\frac{1}{2}(m_2 - m_1)$v (E) m_2v

14. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by
(A) $(m_2 - m_l)$gh (B) $m_2$gh (C) $(m_1 + m_2)$gh (D) $\frac{1}{2}(m_l + m_2)$gh (E) 0

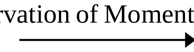
State your reasoning:
Ans.
Probl. 13. One object is rising (m_2) and the other is falling (m_1). The momenta don't cancel out because the masses are different. The resultant momentum is $\Delta p = p_1 - p_2 = m_2v - m_1v$ if we allow downwards to be the positive direction.

Probl. 14. As a system, the change in the potential energy would be the difference between the potential energies of the two masses. One block's potential energy goes up by $m_1$gh while the other's potential energy goes down by $m_2$gh. Therefore, the decrease in potential energy has a magnitude given by $(m_2 - m_1)$gh.

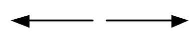
Momentum and Collisions
The same momentum exists before and after a collision.

Law of Conservation of Momentum
If the resultant external force on a system is zero, then the vector sum of the momenta of the objects will remain constant.
$\Sigma p_b = \Sigma p_a$

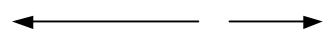
Problem: Conservation of Momentum (CM 1998)

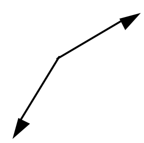


31. An object having an initial momentum that may be represented by the vector above strikes an object that is initially at rest. Which of the following sets of vectors may represent the momenta of the two objects after the collision?

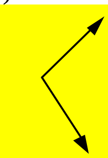
(A) 

(B) 

(C) 

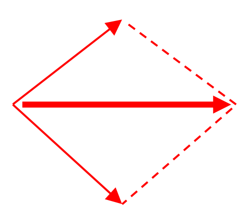


(D)



(E)

State your reasoning:
Ans. Because of momentum conservation, the momentum before has to equal the momentum afterwards. $p_i = p_f$.The momentum initially is to the right and has a large magnitude. The only answer that will give a resultant of large magnitude to the right is Ans. E. Answer A gives a resultant of zero. Answer B gives a resultant that is double the initial momentum. Answer C gives a resultant that is going left. Answer D might give the right magnitude but the direction is wrong.



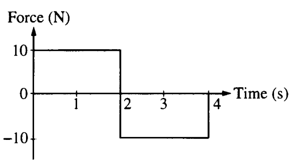
Impulse
$\Delta \mathbf{p} = \mathbf{J}$
$\mathbf{J}$ = impulse
$\mathbf{J} = \int \Sigma \mathbf{F}\, dt$

Problem: Impulse and Momentum (CM 1993)

17. If one knows only the constant resultant force acting on an object and the time during which this force acts, one can determine the
    (A) change in momentum of the object
    (B) change in velocity of the object
    (C) change in kinetic energy of the object
    (D) mass of the object
    (E) acceleration of the object

State your reasoning:
Ans. The **impulse** (J = **F**· **Δt)** is the **change of momentum.**

Problem: Impulse and Momentum (CM 1998)



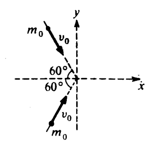
12. The graph above shows the force on an object of mass M as a function of time. For the time interval 0 to 4 s, the total change in the momentum of the object is
    (A) 40 kg m/s
    (B) 20 kg m/s
    (C) 0 kg m/s
    (D) -20 kg m/s
    (E) indeterminable unless the mass M of the object is known

Show your work:
Ans. The object experiences a positive impulse of J = **F**· **Δt** = 10 N × 2 s = 20 N·s for the first two seconds and a negative impulse of J = **F**· **Δt** = –10 N × 2 s = –20 N·s from t = 2 s to t = 4 s. These impulses cancel each other out and there is no change in momentum.

Collision Types
Elastic Collisions
Linear momentum is conserved
Total energy is conserved
Kinetic energy is conserved
Inelastic Collisions
Linear momentum is conserved
Total energy is conserved
Kinetic energy is not conserved
Explosions
Treated as an inelastic collision

2-D Collisions
Momentum change is analyzed ***by component***

Problem: Impulse and Momentum (CM 1984)



17. Two particles of equal mass m_o, moving with equal speeds v_o along paths inclined at 60° to the x-axis as shown above, collide and stick together. Their velocity after the collision has magnitude

(A) $\frac{v_0}{4}$ **(B)** $\frac{v_0}{2}$ (C) $\frac{\sqrt{2}v_0}{2}$ (D) $\frac{\sqrt{3}v_0}{2}$ (E) v_o

Show your work:
Ans. The vertical components of momentum cancel each other out since one particle is going downwards and one is going upwards and the speeds v_o are equal. The horizontal components, however, are given by:

$$p_{in} = p_{out}$$
$$m_o \cdot v_{x1} + m_o \cdot v_{x2} = (m_o + m_o)v_f$$
$$m_o \cdot v_o \cdot \cos\theta + m_o \cdot v_o \cdot \cos\theta = (m_o + m_o)v_f$$

The horizontal components before the collision are the adjacent sides of the right triangles formed by v_o, v_x, and v_y. v_x

$$m_o \cdot v_o \cdot \cos 60 + m_o \cdot v_o \cdot \cos 60 = (2m_o)v_f$$
$$m_o \cdot v_o \cdot (½) + m_o \cdot v_o \cdot (½) = (2m_o)v_f$$
$$m_o \cdot v_o \cdot (½) + m_o \cdot v_o \cdot (½) = (2m_o)v_f$$
$$m_o \cdot v_o = 2m_o v_f$$
$$\frac{\cancel{m_o} \cdot v_o}{2\cancel{m_o}} = \frac{2\cancel{m_o} v_f}{2\cancel{m_o}}$$
$$\mathbf{½v_o = v_f}$$

Center of Mass of system of particles

The point at which all of the mass of an object or system may be considered to be concentrated.

Center of Mass for collection f points

$x_{cm} = \Sigma\, m_i x_i / M$

$y_{cm} = \Sigma\, m_i y_i / Mo$

$z_{cm} = \Sigma\, m_i z_i / M$

Velocity of Center of Mass

$v_{x,cm} = \Sigma\, m_i v_{xi} / M$

$v_{y,cm} = \Sigma\, m_i v_{yi} / M$

$v_{z,cm} = \Sigma\, m_i v_{zi} / M$

Acceleration of Center of Mass

$a_{x,cm} = \Sigma\, m_i a_{xi} / M$

$a_{y,cm} = \Sigma\, m_i a_{yi} / M$

$a_{z,cm} = \Sigma\, m_i a_{zi} / M$

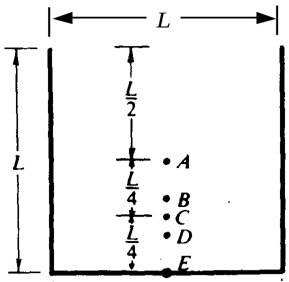
Center of Mass for simple solid object

Pick geometric center if uniform density

Center of Mass for complicated solid objects

$X_{cm} = {}^1/_M \int x\, dm$

Problem: Center of Mass (CM 1984)



29. The center of mass of a uniform wire, bent in the shape shown above, is located closest to point

(A) A **(B) B** (C) C (D) D (E) E

Show your work:

Ans. B. Take center of mass of each of the 3 line segments, and find the center of mass of those: We are only interested in the y_{cm} in this case. The horizontal center of mass is not needed since there is symmetry in this plane.

$y_{cm}=(mL/2 + mL/2)/3 = mL/3.$

Point B is located closest to the distance L/3.

We can also plug in number. Assume the length of each wire is 4 units

$$y_{cm} = \frac{m(2)+m(2)+m(0)}{3M} = \frac{4}{3}$$

or just above the 1 unit mark, but not as far as 2.0 units

Problem: Center of Mass (CM 1984)

31. Mass M_1 is moving with speed v toward stationary mass M_2. The speed of the center of mass of the system is

(A) $\frac{M_1}{M_2} v$

(B) $\left(1+\frac{M_1}{M_2}\right) v$

(C) $\left(1+\frac{M_2}{M_1}\right) v$

(D) $\left(1+\frac{M_1}{M_2}\right) v$

(E) $\left(\frac{M_1}{M_1+M_2}\right) v$

Show your work:

ANS: E

$$v_{cm} = v\frac{M_1+0}{M_1+M_2}$$

Force and Momentum

$F_{ext} = dP/dt = d(mv)/dt$

For variable mass systems, we get this

$F_{ext} = dP/dt = d(mv)/dt = ma$

For variable mass systems, we get this

$F_{ext} = mdv/dt + vdm/dt$