Chapter 8

Momentum, Impulse, and Collisions

PowerPoint® Lectures for
University Physics, Thirteenth Edition
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Goals for Chapter 8

- To learn the meaning of the momentum of a particle and how an impulse causes it to change
- To learn how to use the conservation of momentum
- To learn how to solve problems involving collisions
- To learn the definition of the center of mass of a system and what determines how it moves
- To analyze situations, such as rocket propulsion, in which the mass of a moving body changes

Introduction

- In many situations, such as a bullet hitting a carrot, we cannot use Newton's second law to solve problems because we know very little about the complicated forces involved.
- In this chapter, we shall introduce *momentum* and *impulse*, and the *conservation of momentum*, to solve such problems.

Impulse and Momentum

From Newton's Second Law, $\Sigma F = ma = mdv/dt$

$$\Sigma F dt = m dv$$

Let us integrate both side of the equation

$$\vec{J} = \int \overrightarrow{\Sigma F} dt = m\vec{v}_f - m\vec{v}_{i,} \quad \text{or} \quad \vec{J} = \overrightarrow{F_{avr}} \Delta t = m\vec{v}_f - m\vec{v}_{i,}$$

The integral is called the *impulse*, $\vec{J}$, of the force acting on an object over Δt, $m\vec{v}$ is called linear momentum, $\vec{p}$, and $m\vec{v}_f - m\vec{v}_{i,}$ represents a change in momentum

$\vec{J} = \overrightarrow{F_{avr}} \Delta t = m\vec{v}_f - m\vec{v}_{i,}$ represents *impulse-momentum theorem*

More About Impulse

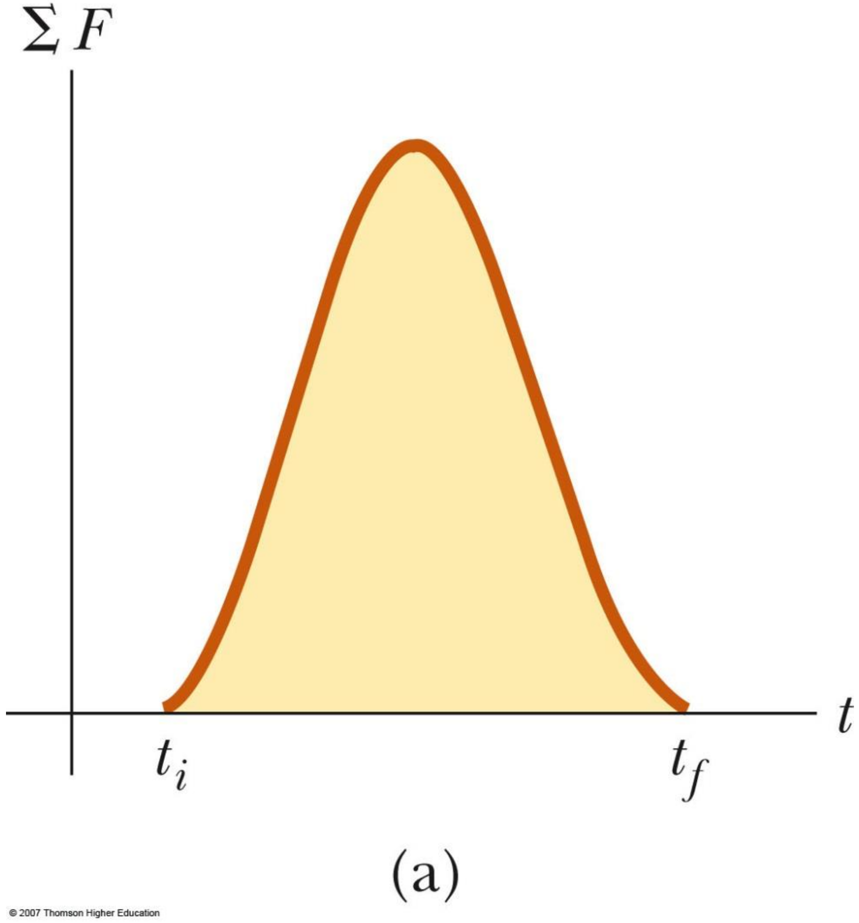
Impulse is a vector quantity

The magnitude of the impulse is equal to the area under the force-time curve

- The force may vary with time

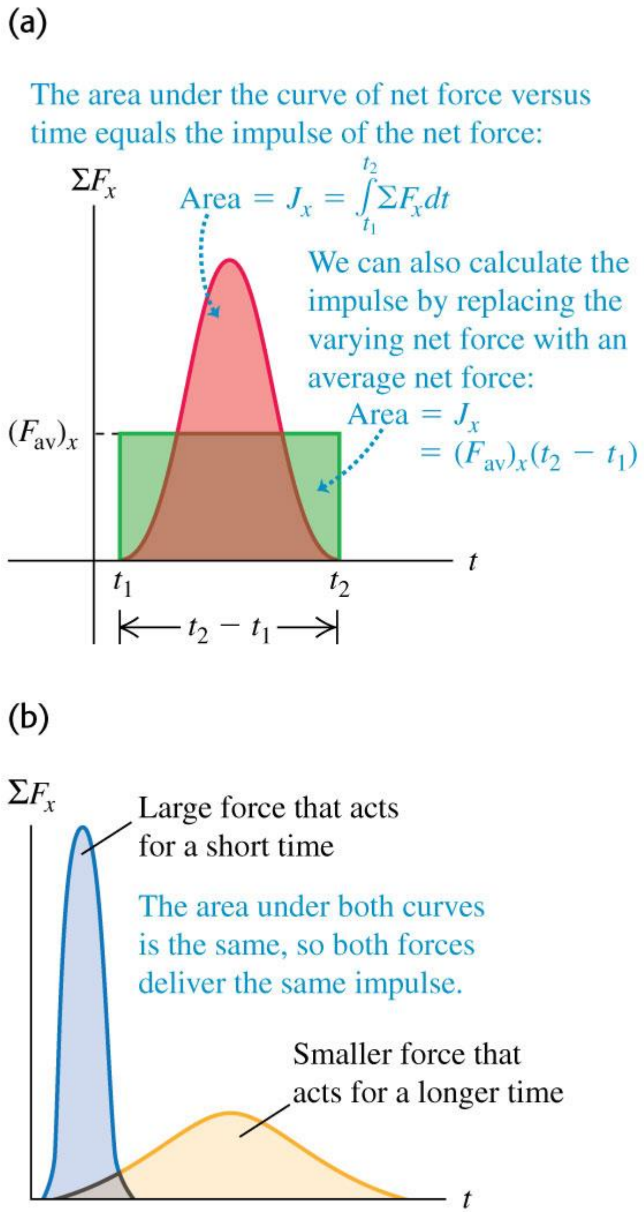
Dimension of impulse is N·s

Impulse is a measure of the change in momentum of the particle



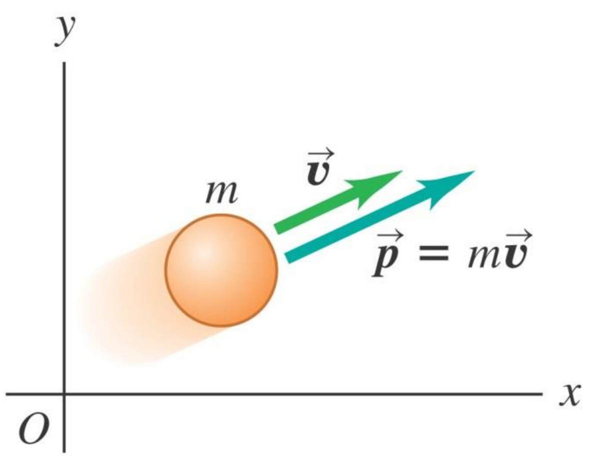
Impulse and momentum

- The *impulse* of a force is the product of the force and the time interval during which it acts.
- On a graph of ΣF_x versus time, the impulse is equal to the area under the curve, as shown in figure to the right.
- *Impulse-momentum theorem*: The change in momentum of a particle during a time interval is equal to the impulse of the net force acting on the particle during that interval.



Momentum and Newton's second law

- The *momentum* of a particle is the product of its mass and its velocity:
- Newton's second law can be written in terms of momentum as



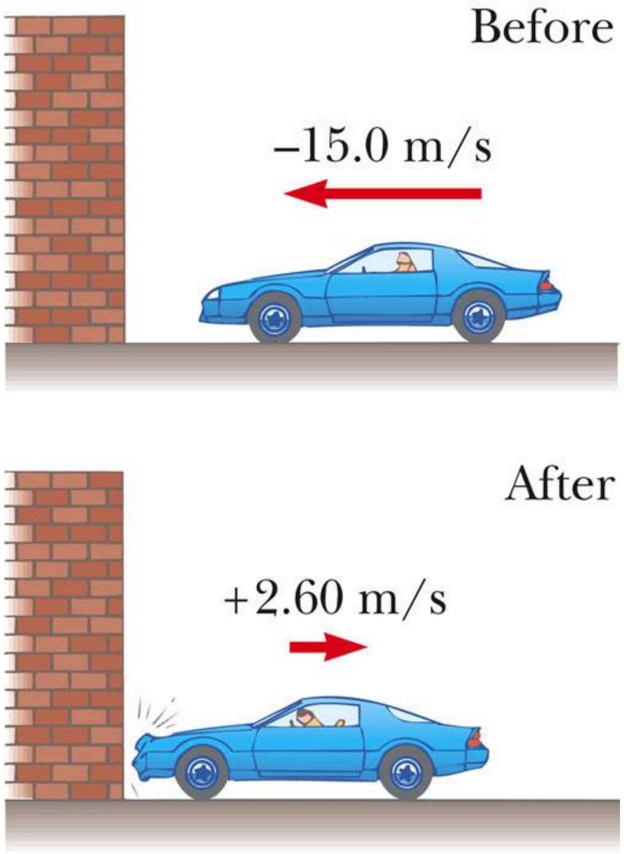
Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

Impulse-Momentum: Crash Test Example 1

In particular crash test, a car of mass 1500 kg collides with a wall. The initial and final speed of the car are v_i = 15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts 150 ms, find the average force exerted on the car.

$F\Delta t = m(v_f - (-v_i))$

$$F = \frac{1500\,kg\left(\frac{2.6m}{s} + \frac{15m}{s}\right)}{0.15s} = 1.76x10^5\text{N}$$



(a)

Example 2

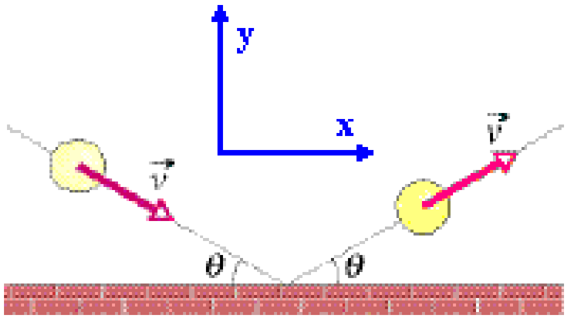
A golf ball strikes a hard, smooth floor at an angle of 25 0 and rebounds at the same angle. The mass of the ball is 0.0047 kg, and its speed is 45 m/s just before and after striking the floor.

a) What are the average magnitude and the direction of impulse on the ball from the wall?

$\Delta p_x = mv_{fx} - mv_{ix} = 0.0047$ kg(45 m/s·cos25^0 -45 m/s·cos25^0) = 0

$\Delta p_y = mv_{fy} - mv_{iy} = 0.0047$ kg(45 m/s·sin25^0 – (-45 m/ssin25^0) = 0.28 Ns

J = 0.28 Ns$\boldsymbol{j}$



- b) If the ball is in contact with the wall for 10 ms, what are the magnitude and direction of the average force on the ball?

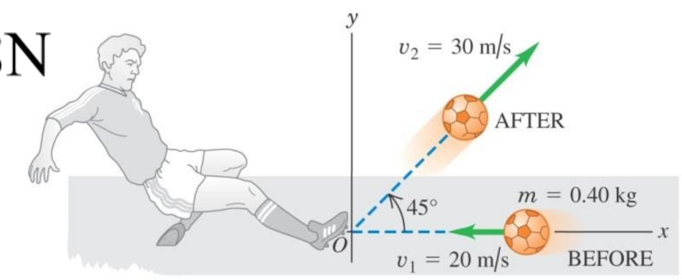
$F\Delta t = \Delta p = 0.28$ Ns$\boldsymbol{j}$ $F = \Delta p / \Delta t = 0.28$ Ns/0.01s = 28 N $\boldsymbol{j}$

Kicking a soccer ball

A kick changes the direction of a soccer ball. A 0.40 =kg soccer ball moving initially to the left at 20 m/s is kicked. After the kick it is moving at 45^0 upward and to the right with a speed of 30 m/s. Find the average net force, assuming a collision time 0.01 s.

V_{xi} = -20m/s $v_{yi} = 0$ v_{xf}=30cos45^0=21.2m/s v_{yf} =30sin45^0=21.2m/s

(a) Before-and-after diagram

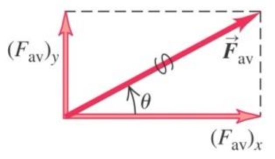


$$(F_{avr})_x = \frac{m(v_{f,x} - v_{i,x})}{\Delta t} = \frac{0.4\,kg[21.2 - (-20)]}{0.01} = 1648\text{N}$$

$$(F_{avr})_y = \frac{m(v_{f,y} - v_{i,y})}{\Delta t} = \frac{0.4\,kg[21.2 - 0]}{0.01} = 850\text{N}$$

F_{av}= 1900N $\theta = 27^0$

(b) Average force on the ball



Linear Momentum

The **linear momentum** of a particle of mass *m* moving with a velocity $\vec{\mathbf{v}}$ is defined to be the product of the mass and velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity

- Its direction is the same as the direction of the velocity

The SI units of momentum are kg · m / s

Momentum can be expressed in component form:

$p_x = m\,v_x$ $\qquad$ $p_y = m\,v_y$ $\qquad$ $p_z = m\,v_z$

Newton's Second Law can be expressed in terms of used to relate the momentum of a particle to the resultant force acting on it

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\left(m\vec{\mathbf{v}}\right)}{dt} = \frac{d\vec{\mathbf{p}}}{dt}$$

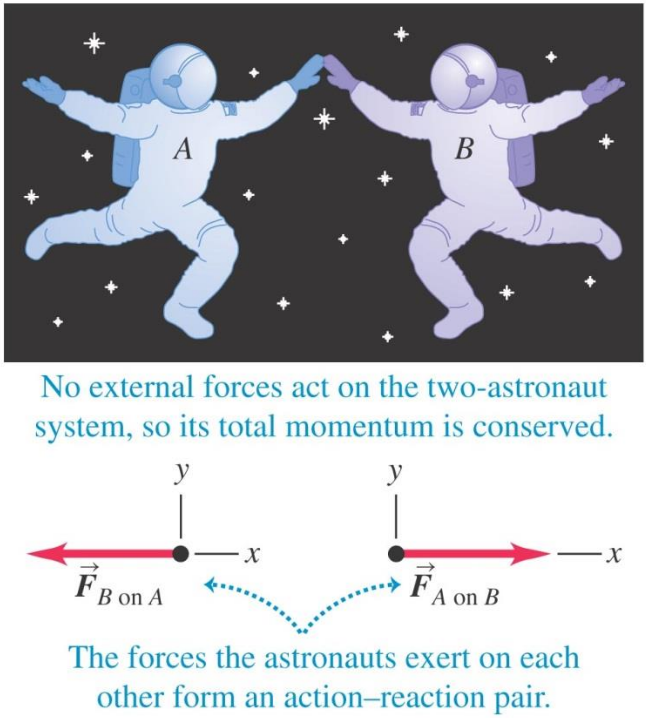
Let us now consider special case when net force is zero. If net force is zero then derivative of the total momentum in respect to time is zero. Thus we can conclude that the total momentum remains constant, *total momentum is conserved*

$$\overrightarrow{p_{tot}} = \text{constant}$$

$$p_{1i}+p_{2i}+\ldots p_{ni} = p_{1f}+p_{2f}+\ldots p_{nf}$$

An isolated system

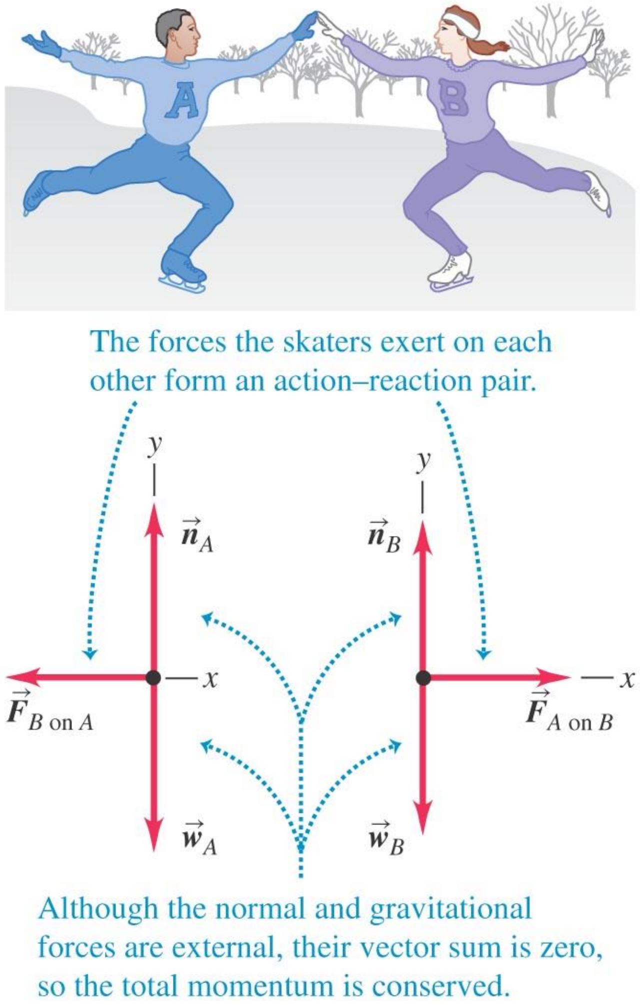
- The *total momentum* of a system of particles is the vector sum of the momenta of the individual particles.
- No *external forces* act on the *isolated system* consisting of the two astronauts shown below, so the total momentum of this system is conserved.



Conservation of momentum

- External forces (the normal force and gravity) act on the skaters shown in Figure 8.9 at the right, but their vector sum is zero. Therefore the total momentum of the skaters is conserved.

- *Conservation of momentum*: If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.



Conservation of Linear Momentum

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant

- Law of conservation of linear momentum can be applied to analyze such events as collisions, explosion and recoil

- Types of collisions : *elastic collision*

  inelastic collision

Types of Collisions

In an ***elastic*** collision, momentum and kinetic energy are conserved

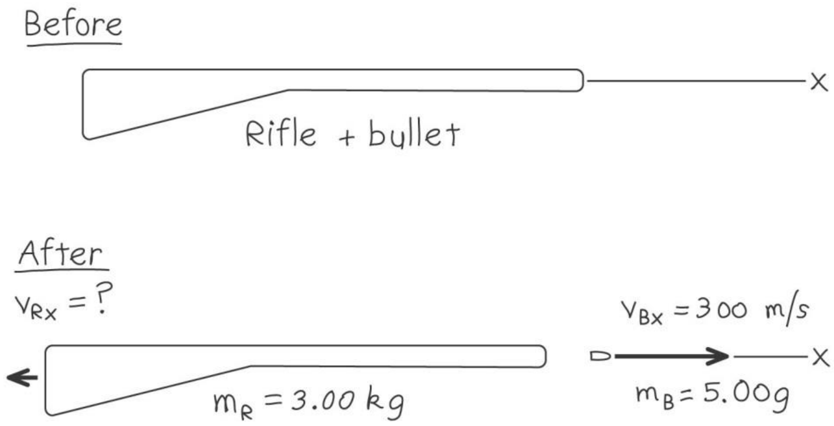
- elastic collisions occur on a microscopic level
- In macroscopic collisions, only approximately elastic collisions actually occur
  - Generally some energy is lost to deformation, sound, etc.

In an ***inelastic*** collision, kinetic energy is not conserved, although momentum is still conserved

- If the objects stick together after the collision, it is a ***perfectly inelastic*** collision

Recoil of a rifle

- A rifle fires a bullet, causing the rifle to recoil.
- $0 = m_b v_b + m_r v_r \quad v_r = -\dfrac{5\,g \cdot 300 m/s}{3000\,g} = -0.5 m/s$



Conservation of Momentum, Recoil Example 4

A 60-kg archer stands on frictionless ice and fires a 0.5 kg arrow horizontally at 50 m/s. Find the recoil velocity of the archer.

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$$\mathbf{0}= m_a v_a + m_m v_m$$

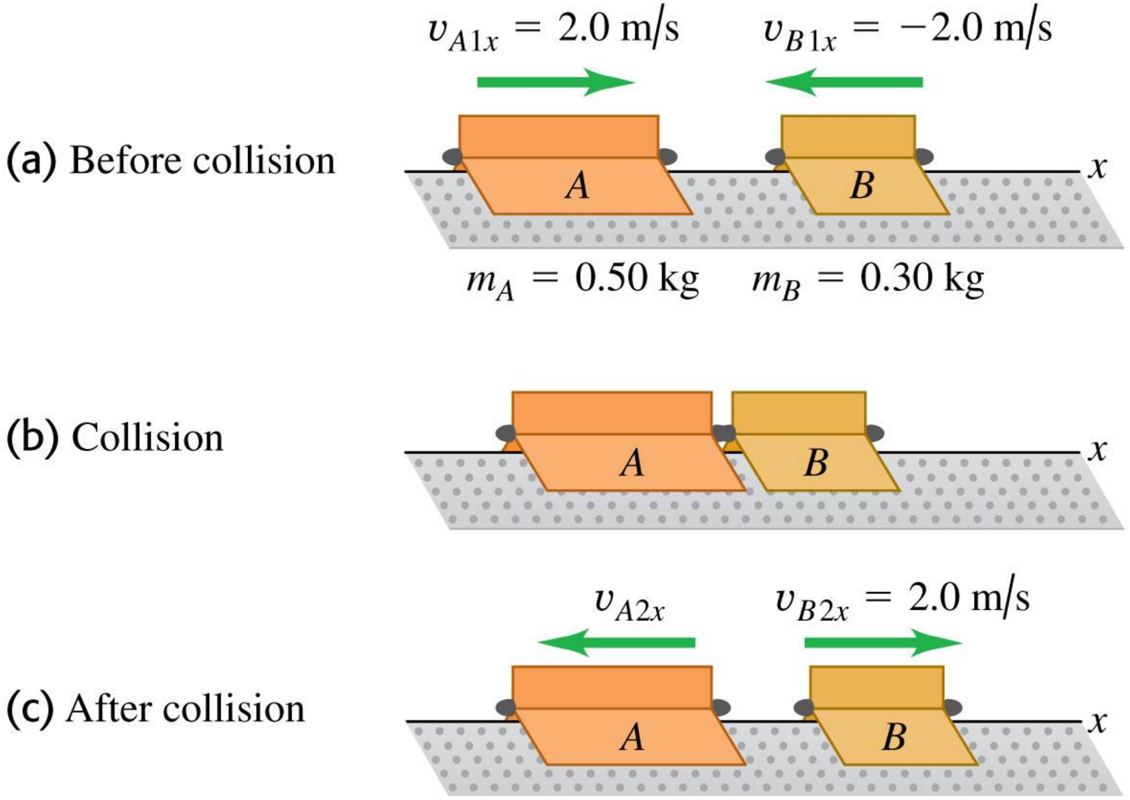
$$v_m = -\frac{0.5kg \cdot 50m/s}{60\ kg} = -0.42m/s$$

Objects colliding along a straight line

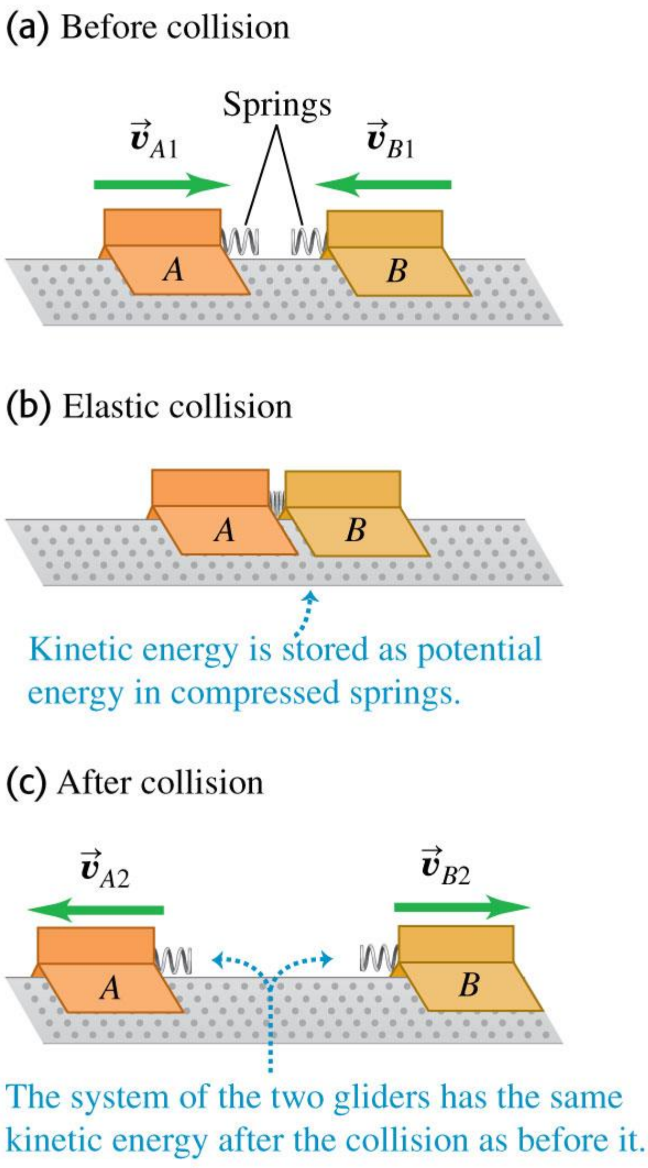
- Two gliders collide on an air track.

$0.5\text{ kg}\cdot 2.0\text{ m/s} + 0.3\text{ kg}\cdot(-2.0\text{m/s}) = -0.5\text{kg}v_{A2} + 0.3\text{kg}\cdot 2.0\text{m/s}$

v_{A2} = 0.4 m/s to the left



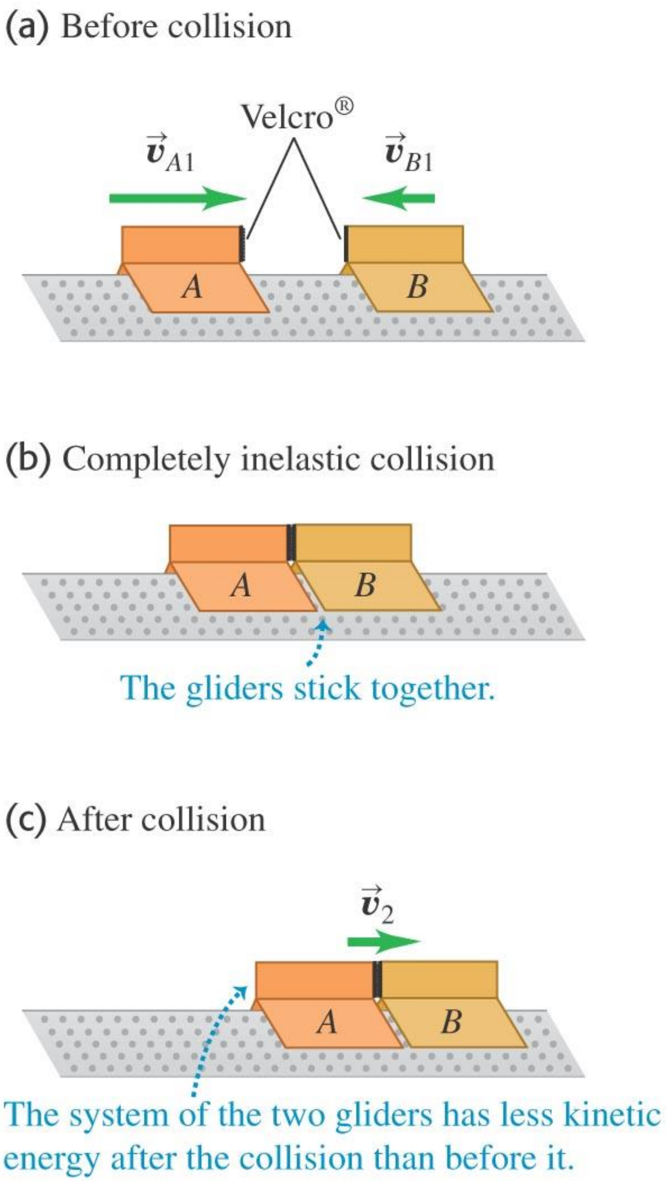
Elastic collisions



- In an *elastic collision*, the total kinetic energy of the system is the same after the collision as before.
- Figure at the left illustrates an elastic collision between air track gliders.

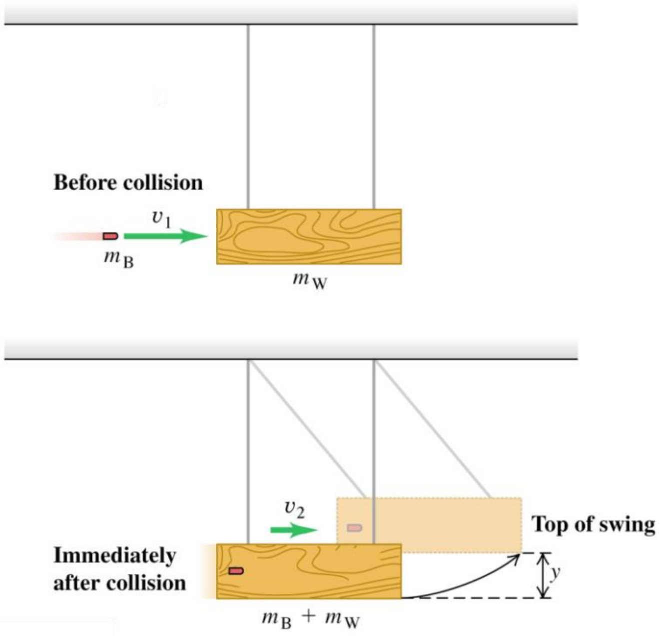
Inelastic collisions

- In an *inelastic collision*, the total kinetic energy after the collision is less than before the collision.
- A collision in which the bodies stick together is called a *completely inelastic collision* In *any* collision in which the external forces can be neglected, the total momentum is conserved.



The ballistic pendulum

Ballistic pendulums are used to measure bullet speeds. Assume m_B=5g, m_W=2.00 kg, y=3 cm



1. Perfectly inelastic collision between bullet and the block. Block is at rest.

$m_B v_1=(m_B+m_W)v_2$

$$v_1 = \frac{m_B + m_W}{m_B} v_2$$

2. Pendulum swings the block

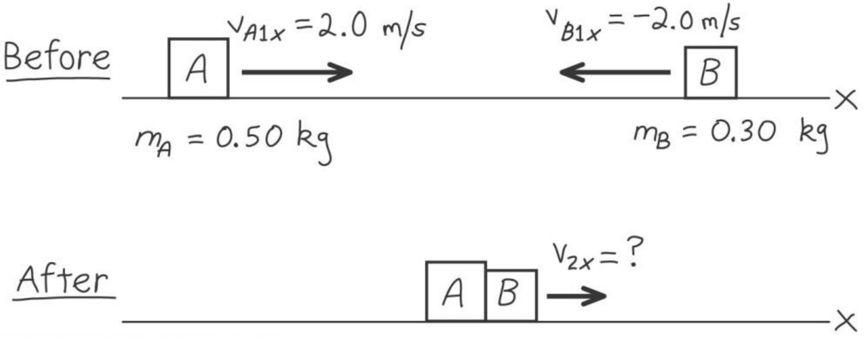
$\frac{1}{2}(m_B+m_W)(v_2)^2 = (m_B+m_W)gy$

$v_2=\sqrt{2gh} = \sqrt{2(9.8m/s^2)0.03m}$ =0.767m/s

$$v_1 = \frac{0.005kg+2.00kg}{0.005kg} 0.767m/s\text{=307 m/s}$$

Some inelastic collisions

- Cars are intended to have inelastic collisions so the car absorbs as much energy as possible.



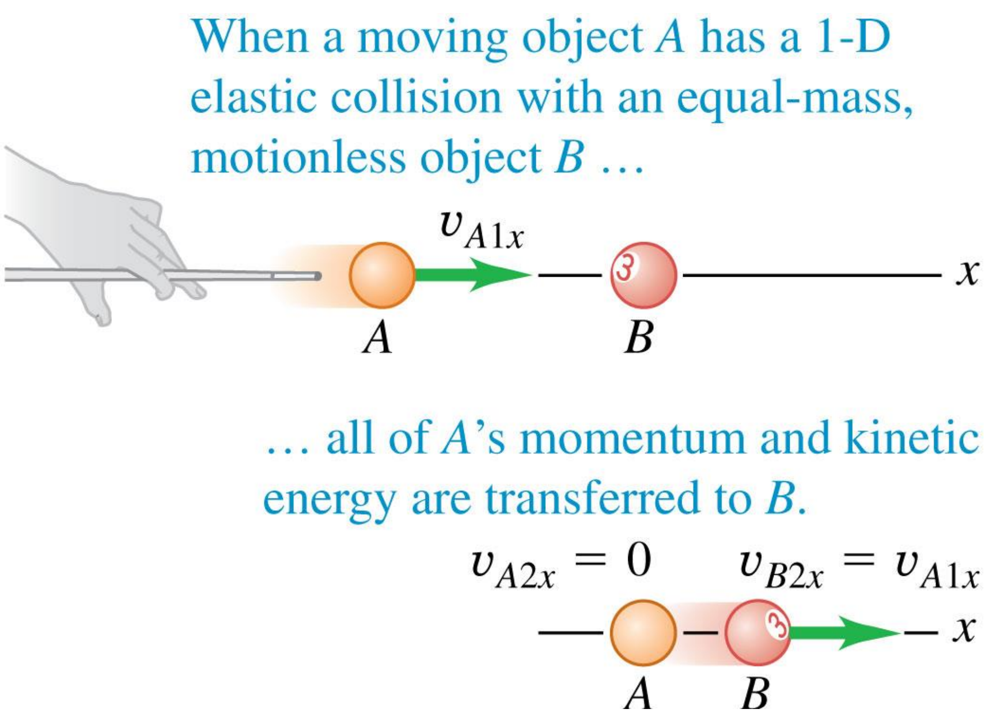
elastic collisions (one dimension)

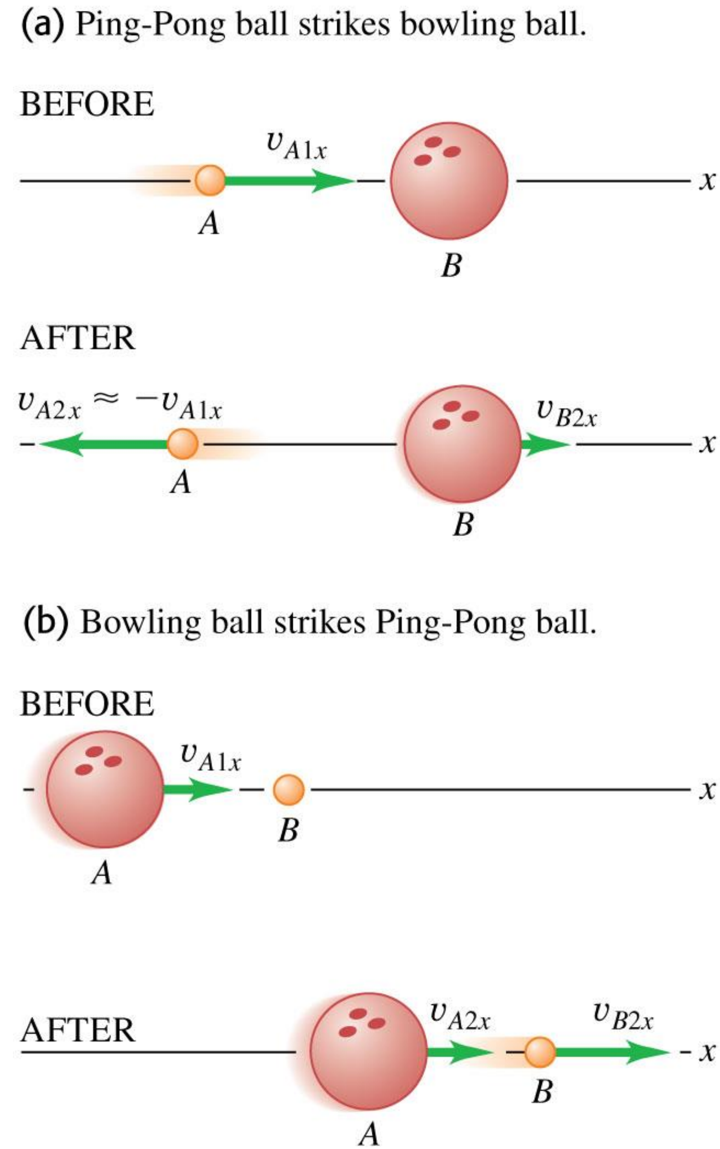
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Elastic collisions

- The behavior of the colliding objects is greatly affected by their relative masses.





elastic collisions (one dimension) x only or y only

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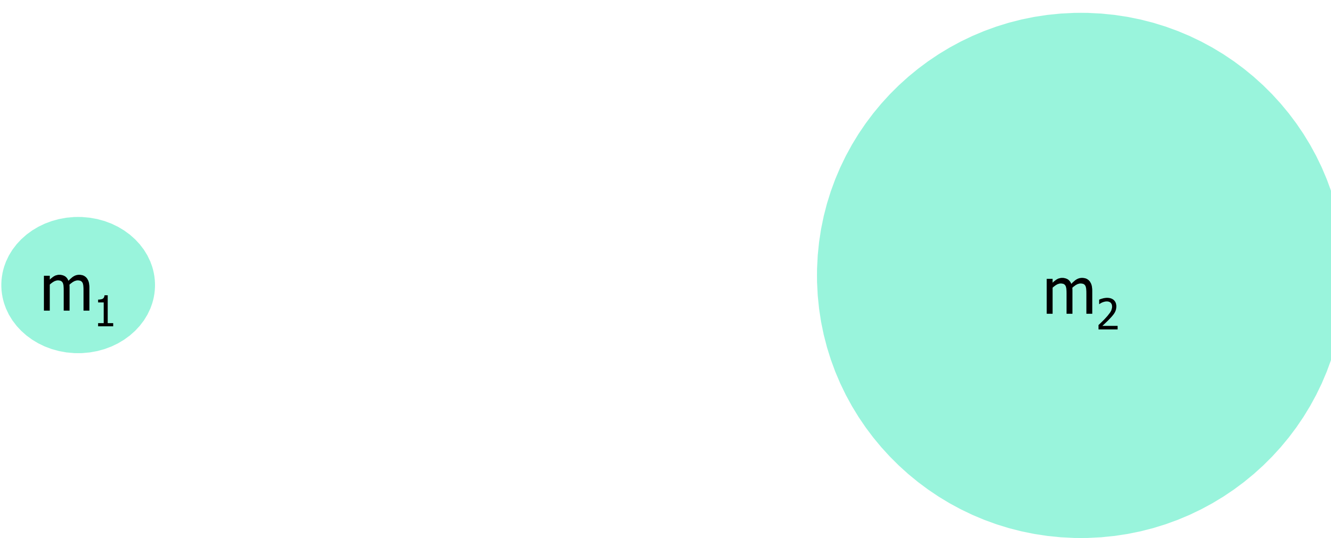
$$v_{1f} = -v_{1i}$$

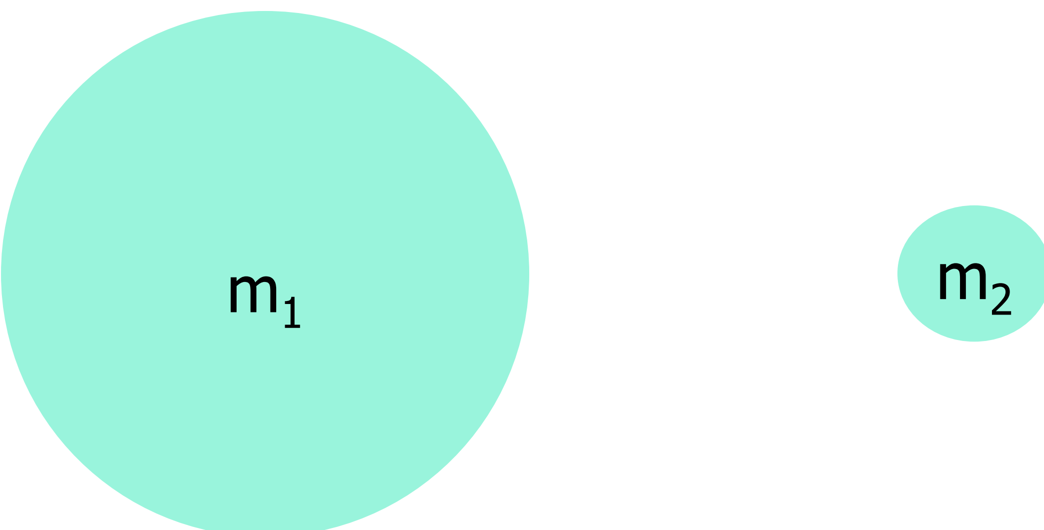
$$v_{2f} = 0$$



elastic collisions (one dimension) x only or y only

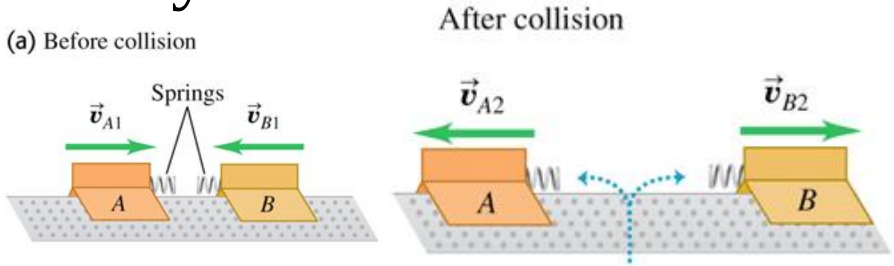
$$v_{1f} = v_{1i}$$

$$v_{2f} = 2^*v_{1i}$$



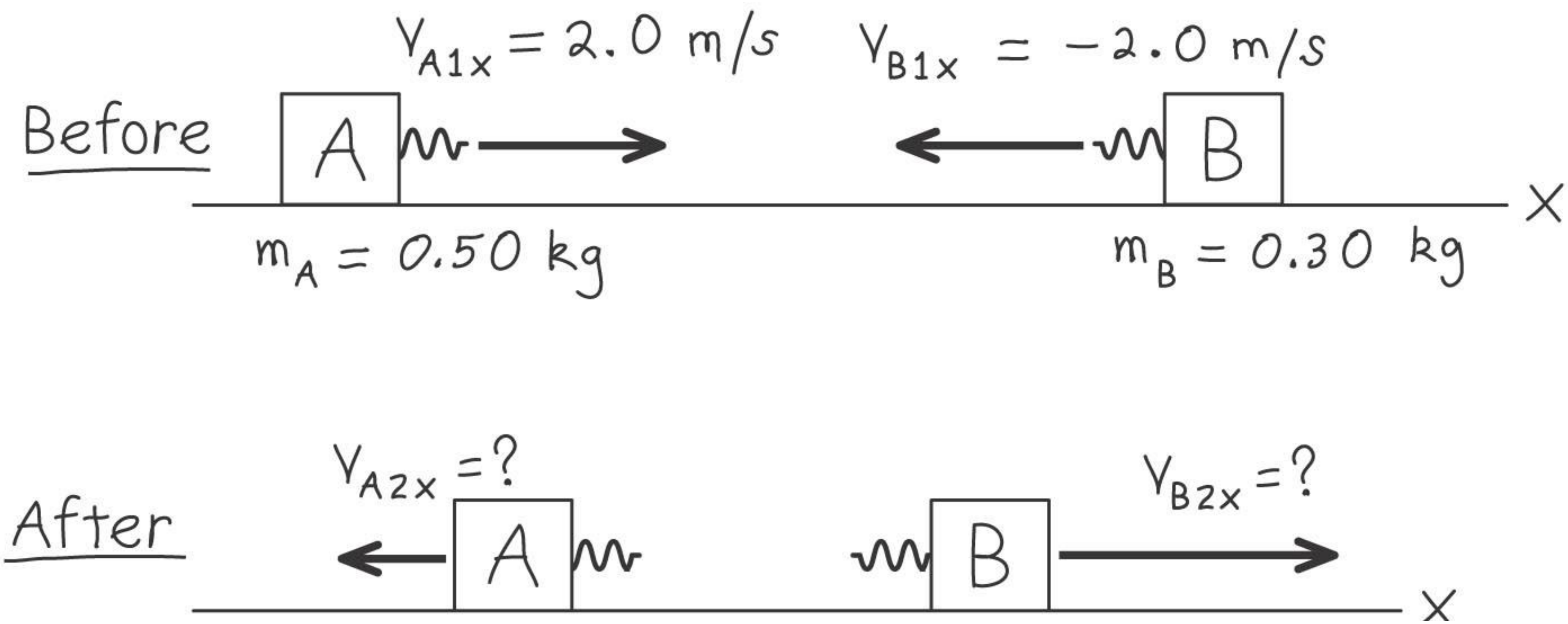
An elastic straight-line collision

Two gliders on air track undergo elastic collision. Find the velocity of each glider after the collision



$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} + \frac{2m_2}{m_1 + m_2} V_{2i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i} + \frac{m_2 - m_1}{m_1 + m_2} V_{2i}$$



Two-Dimensional Collision

For two-dimensional collision conservation of momentum has to be expressed by two component equations:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

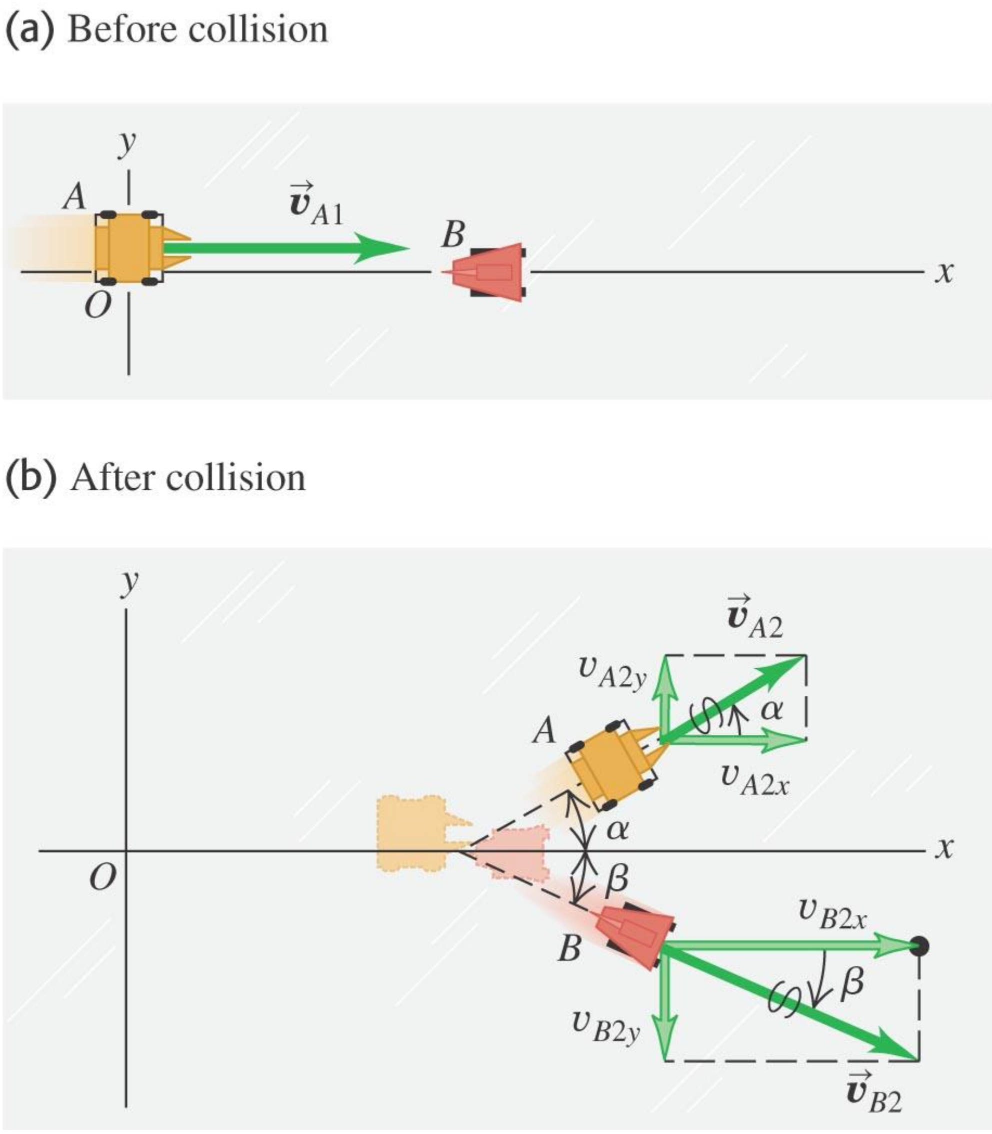
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

A two-dimensional collision

Two robots collide and go off at different angles.

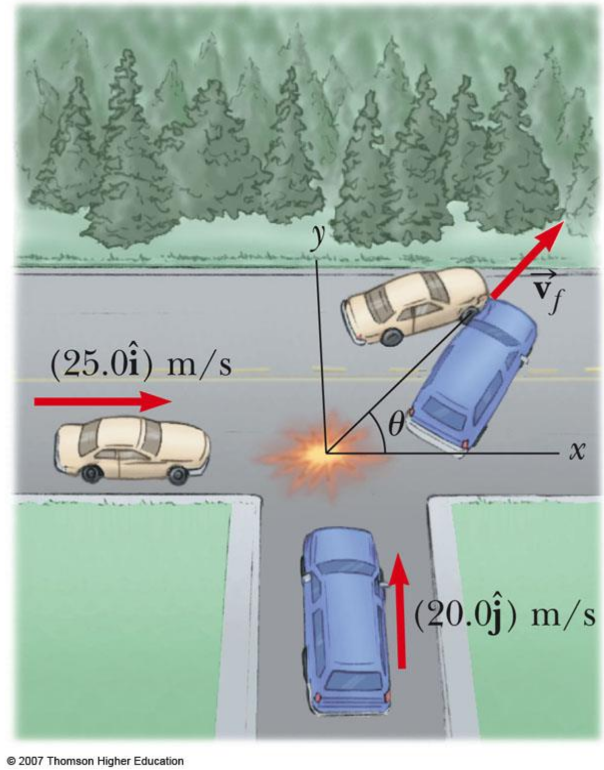
Assume m_A=20 kg, $v_{A,I}$=2m/s, m_B=12 kg,

$v_{A,f}$=1m/s at $\alpha=30^0$



A two-dimensional perfectly inelastic collision

A 1500 kg car traveling east with a speed of 25 m/s collides at an intersection with a 2500 kg van traveling north at a speed of 20 m/s. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.



Ignore friction Model the cars as particles The collision is perfectly inelasticThe cars stick together

The Center of Mass

There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point

The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass

- *M* is the total mass of the system

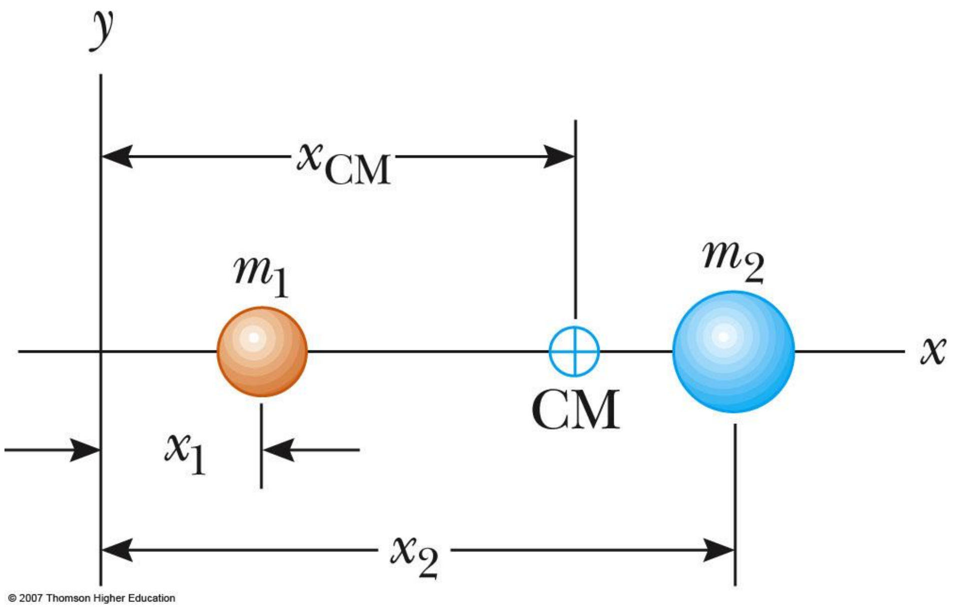
Center of Mass of System of Particles, Coordinates

The coordinates of the center of mass are

$$x_{CM} = \frac{\sum_i m_i x_i}{M}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{M}$$

$$z_{CM} = \frac{\sum_i m_i z_i}{M}$$

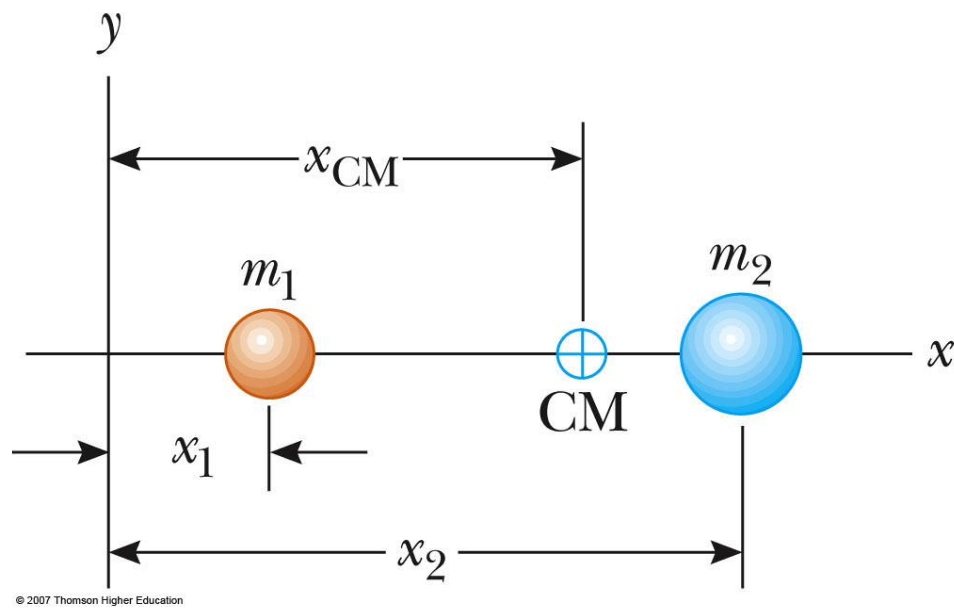


- M is the total mass of the system

Example

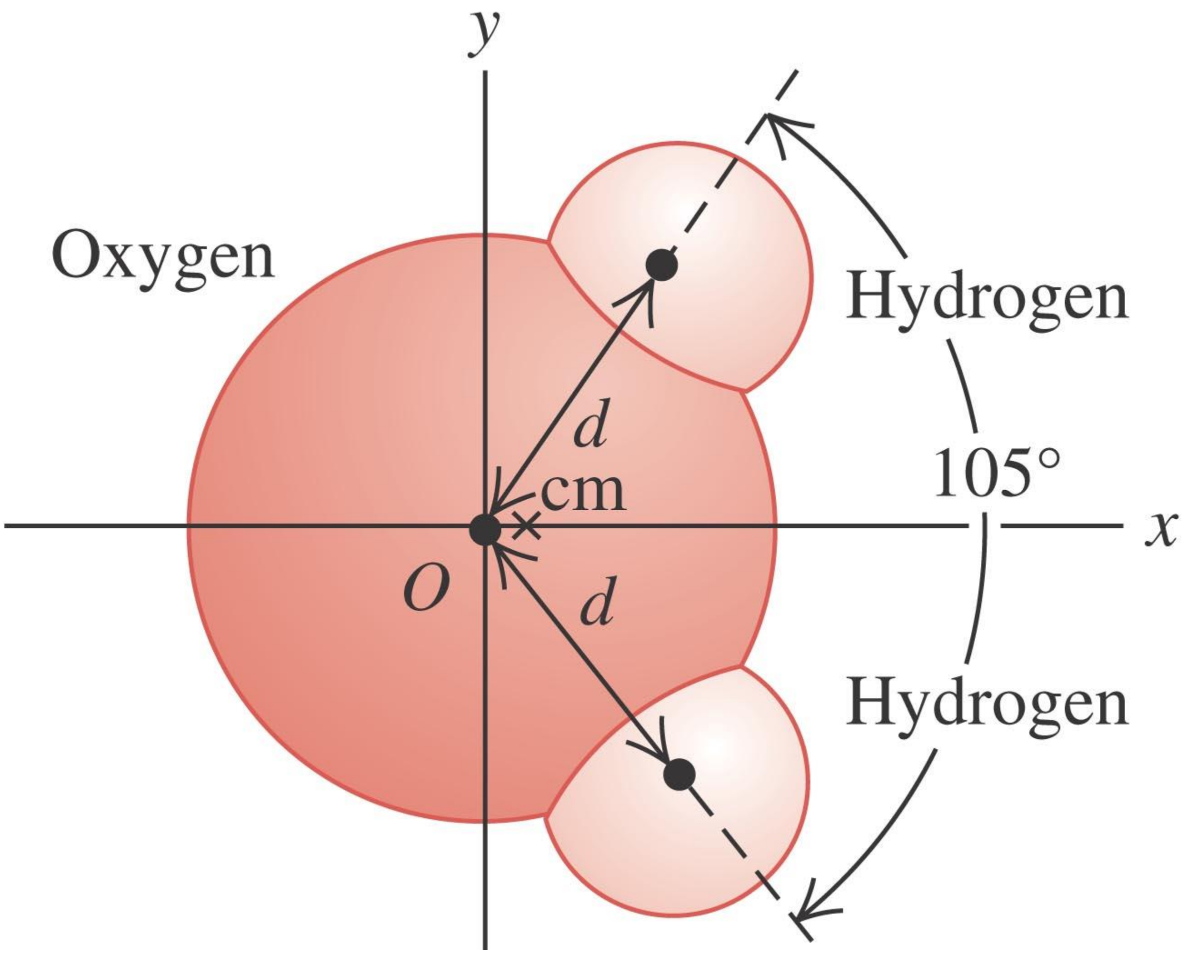
Find the position of center of mass of the system consisting of two particles m_1 = 2 kg and m_2 = 10 kg if x_1=20 cm

and x_2 =60 cm



Center of mass of a water molecule

For simple model of water molecule, find the position of center of mass if d = 0.0957nm, m_H = 1u and m_O = 16 u



Center of Mass, position

The center of mass in three dimensions can be located by its position vector, $\vec{\mathbf{r}}_{CM}$

- For a system of particles,

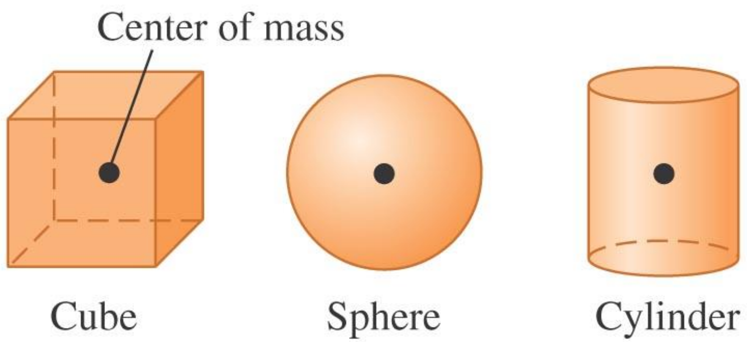
$$\vec{\mathbf{r}}_{CM} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

$$\mathbf{r}_i = x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}} + z_i\hat{\mathbf{k}}$$

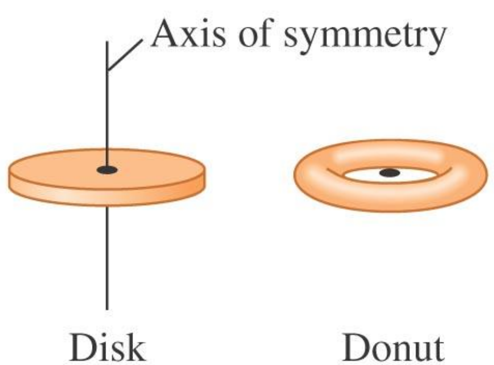
- For an extended object,

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm$$

Center of mass of symmetrical objects



If a homogeneous object has a geometric center, that is where the center of mass is located.

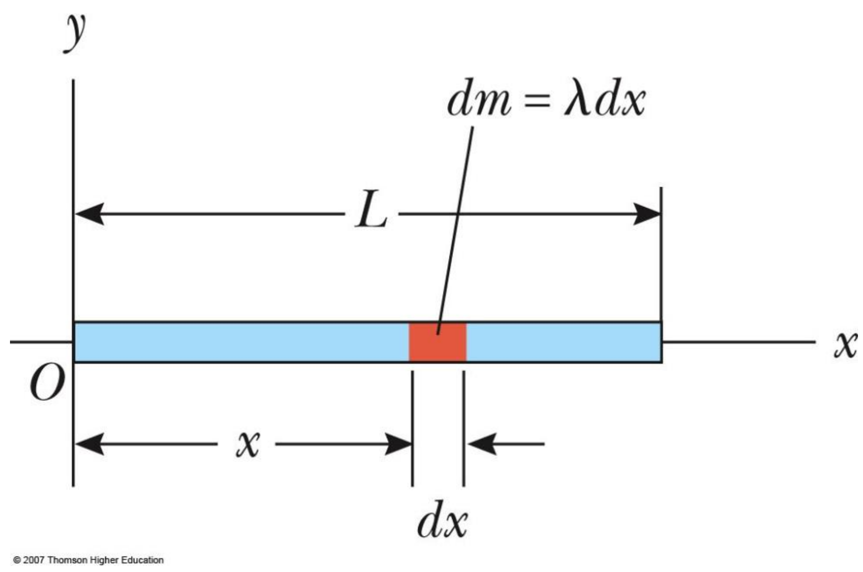


If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

- It is easy to find the center of mass of a homogeneous symmetric object, as shown in Figure 8.28 at the left.

Center of Mass, Rod

$$x_{\mathrm{CM}} = \frac{1}{M}\int_0^L x\,dm = \int_0^L x\lambda\,dx = \frac{M}{L}\left[\frac{L^2}{2}\right] = \frac{L}{2}$$



Motion of a System of Particles

Assume the total mass, M, of the system remains constant

We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system

We can also describe the momentum of the system and Newton's Second Law for the system

Velocity and Momentum of a System of Particles

The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = \frac{1}{M}\frac{d\sum_i m_i \mathbf{r}_i}{dt} = \frac{1}{M}\sum m_i \frac{d\mathbf{r}_i}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

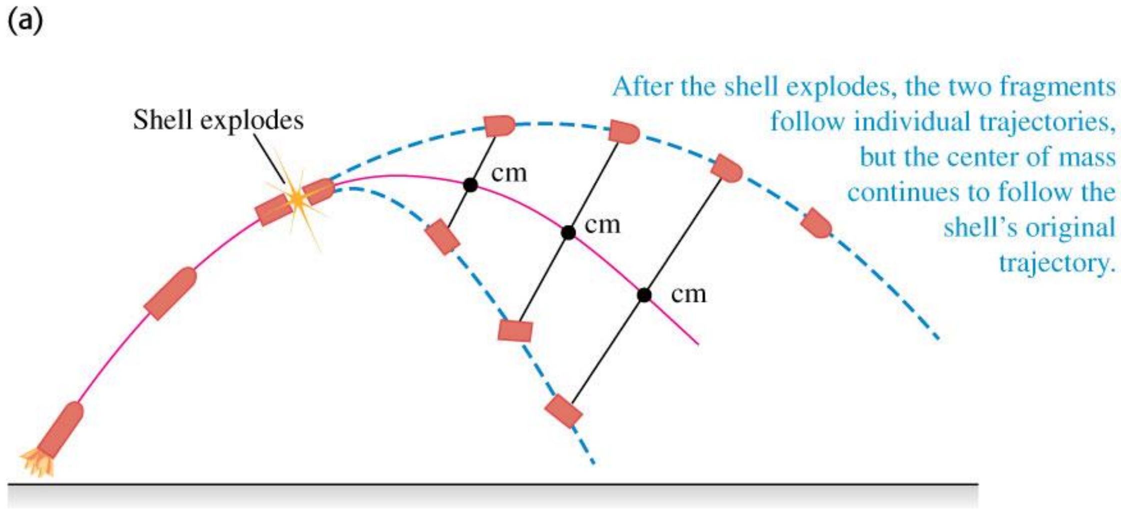
The momentum can be expressed as

$$M\vec{\mathbf{v}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\text{tot}}$$

The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass

External forces and center-of-mass motion

- When a body or collection of particles is acted upon by external forces, the center of mass moves as though all the mass were concentrated there (see Figure 8.31 below).



Rocket Propulsion

The operation of a rocket depends upon the law of conservation of linear momentum as applied to an isolated system, where the system is the rocket plus its ejected fuel.

As the rocket moves in free space, its linear momentum changes when some of its mass is ejected in the form of exhaust gases.

- Because the gases are given momentum when they are ejected out of the engine, the rocket receives a compensating momentum in the opposite direction.
- In free space, the center of mass of the system moves uniformly.

Rocket Propulsion, 2

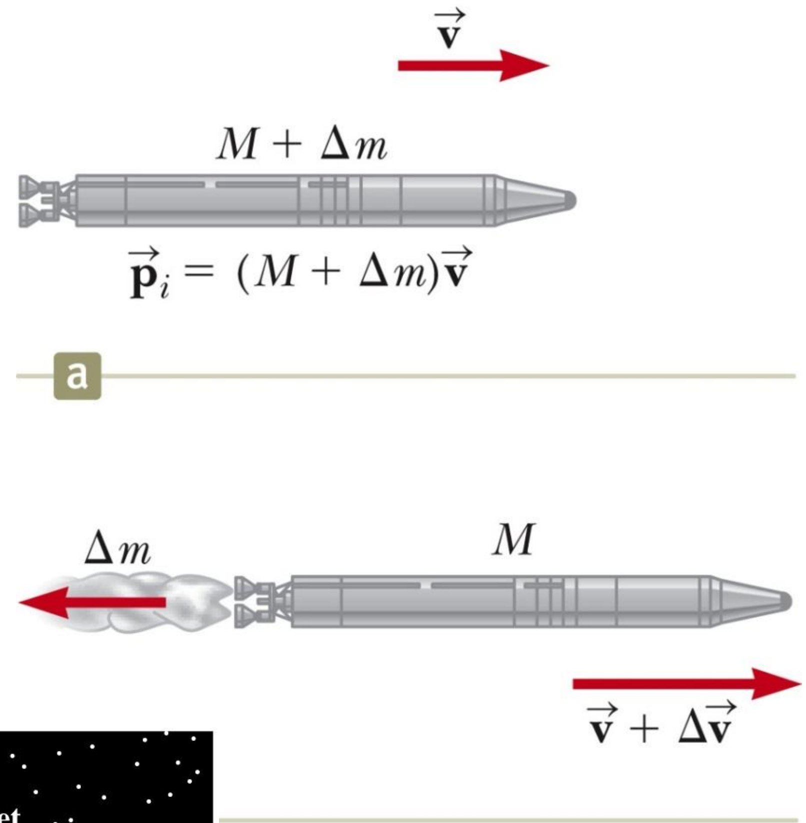
The initial mass of the rocket plus all its fuel is $M + \Delta m$ at time t_i and speed v.

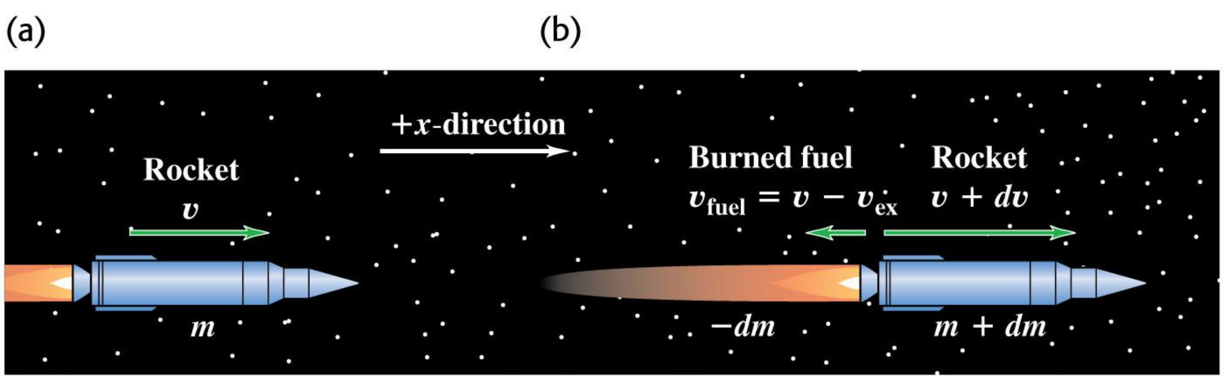
The initial momentum of the system is

$$\vec{\mathbf{p}}_i = (M + \Delta m)\vec{\mathbf{v}}$$

At some time $t + \Delta t$, the rocket's mass has been reduced to M and an amount of fuel, Δm has been ejected.

The rocket's speed has increased by Δv.

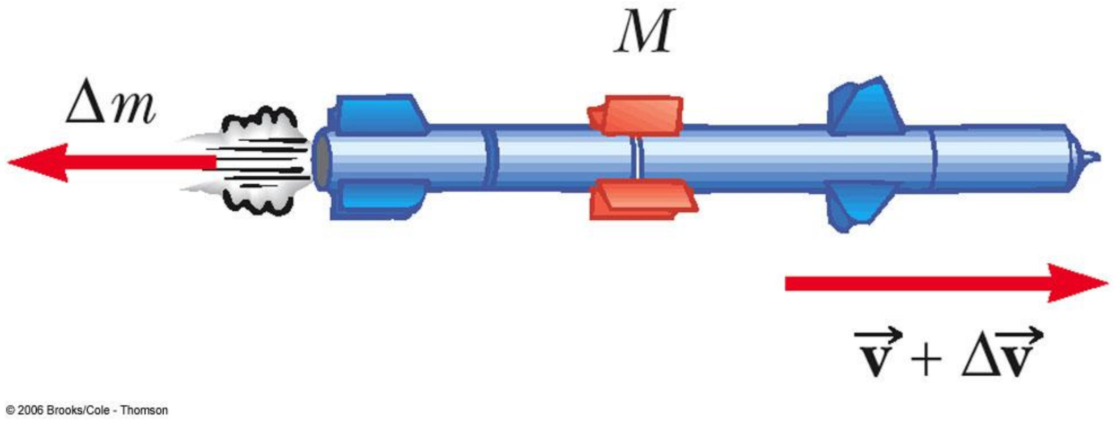




At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

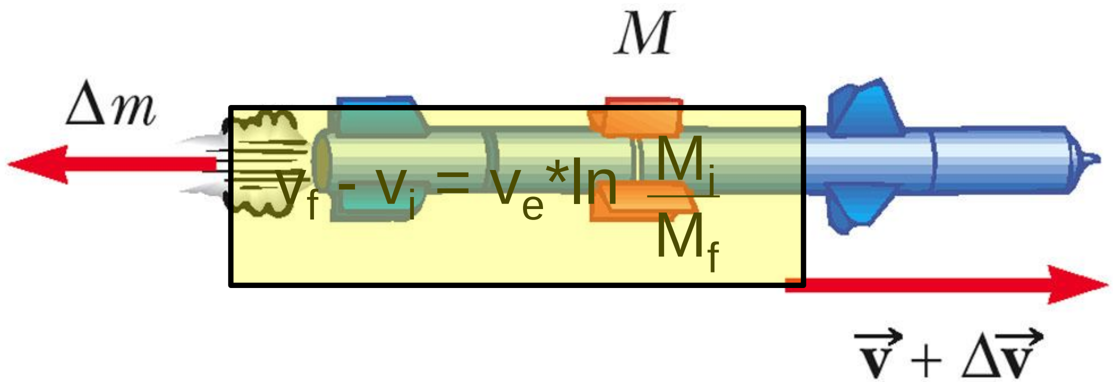
Rocket Propulsion



- The rocket's mass is M
- The mass of the fuel, Δm, has been ejected
- The rocket's speed has increased to

  $\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}}$

$$\int dv = \int - dm$$



$$v_f - v_i = v_e * \ln \frac{M_i}{M_f}$$

M_i is the initial mass of the rocket plus fuel
M_f is the final mass of the rocket plus any remaining fuel
The speed of the rocket is proportional to the exhaust speed

$$0 \qquad M\Delta v \qquad \Delta m v_e$$

Rocket Propulsion, 3

The basic equation for rocket propulsion is

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

The increase in rocket speed is proportional to the speed of the escape gases (v_e).

- So, the exhaust speed should be very high.

The increase in rocket speed is also proportional to the natural logarithm of the ratio M_i/M_f.

- So, the ratio should be as high as possible, meaning the mass of the rocket should be as small as possible and it should carry as much fuel as possible.

Thrust

The thrust on the rocket is the force exerted on it by the ejected exhaust gases.

$$thrust = M\frac{dv}{dt} = \left| v_e \frac{dM}{dt} \right|$$

The thrust increases as the exhaust speed increases.

The thrust increases as the rate of change of mass increases.

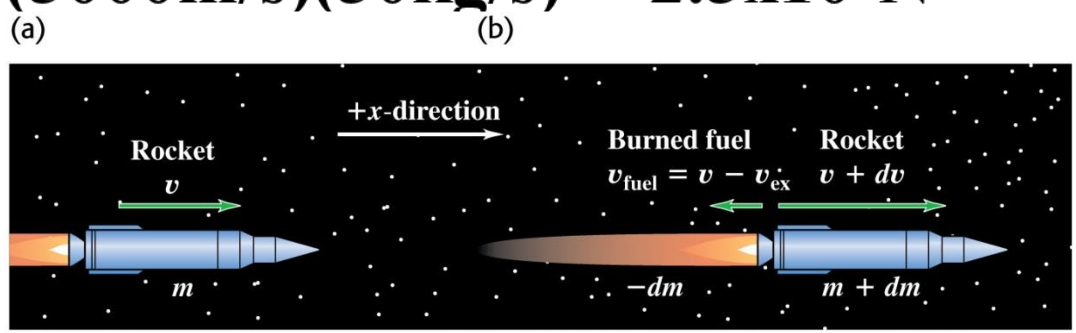
- The rate of change of the mass is called the **burn rate.**

Rocket propulsion

A rocket moving in space has a speed of 3000m/s relative to Earth. Fuel is ejected in a direction opposite the rocket's motion at a speed of 5000 m/s relative to the rocket. (a)What is the speed of the rocket relative to earth once the rocket's mass is reduced to half its mass before ignition? (b) What is the thrust on rocket if it burns at the rate of 50 kg/s.

(a) $$v_f = v_i + v_e \ln\frac{M_i}{M_f} = 3000\text{m/s} + (5000\text{m/s})\ln\frac{M_i}{0.5M_i} = 6500\text{m/s}$$

(b) $$\text{Thrust} = v_e\frac{dM}{dt} = (5000\text{m/s})(50\text{kg/s}) = 2.5\text{x}10^5\text{N}$$



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{fuel} = v - v_{ex}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)