CM3110
Transport I
Part I: Fluid Mechanics

More Complicated Flows III: Boundary-Layer Flow

(plus Miscellaneous topics)

Professor Faith Morrison

Department of Chemical Engineering
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More complicated flows II

Powerful:

Solving never-before-solved problems.

Left to explore:

- What is non-creeping flow like?
  (boundary layers)
- Viscosity dominates in creeping flow, what about the flow where inertia dominates?
  (potential flow)
- What about mixed flows (viscous+inertial)?
  (boundary layers)
- What about really complex flows (curly)?
  (vorticity, irrotational+circulation)

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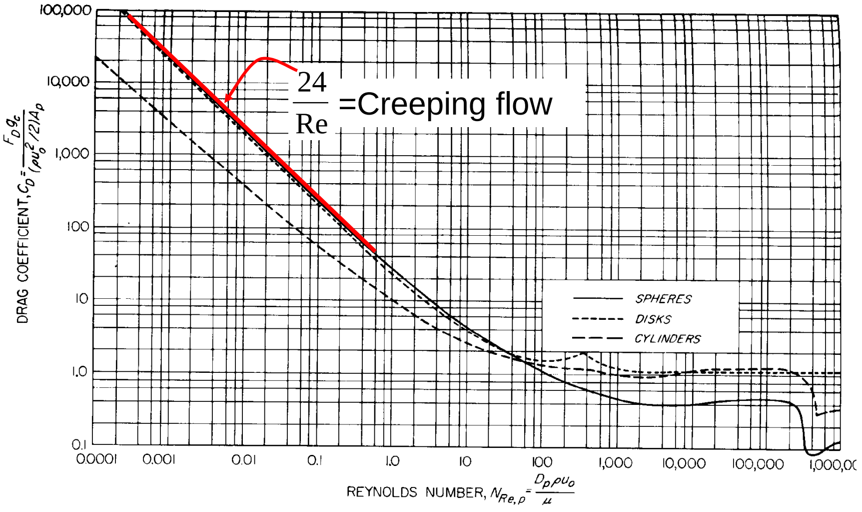
graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere

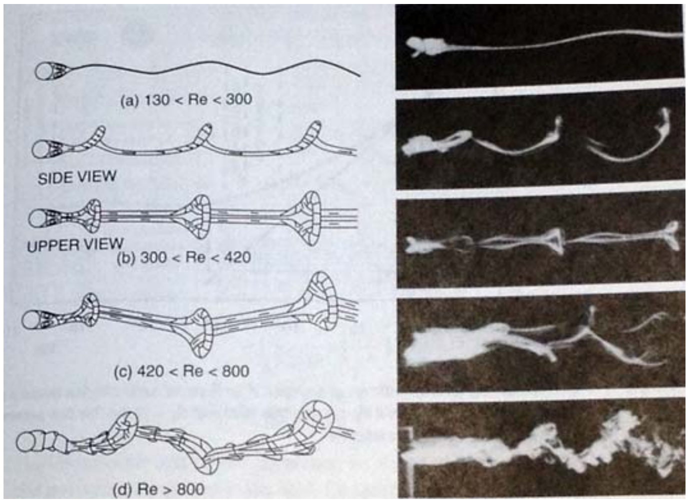


$\frac{24}{\text{Re}}$ =Creeping flow

McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

More complicated flows III

What does **non-creeping** flow look like?



Text, Figure 8.22, p649, from Sakamoto and Haniu, 1990

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graphical correlation

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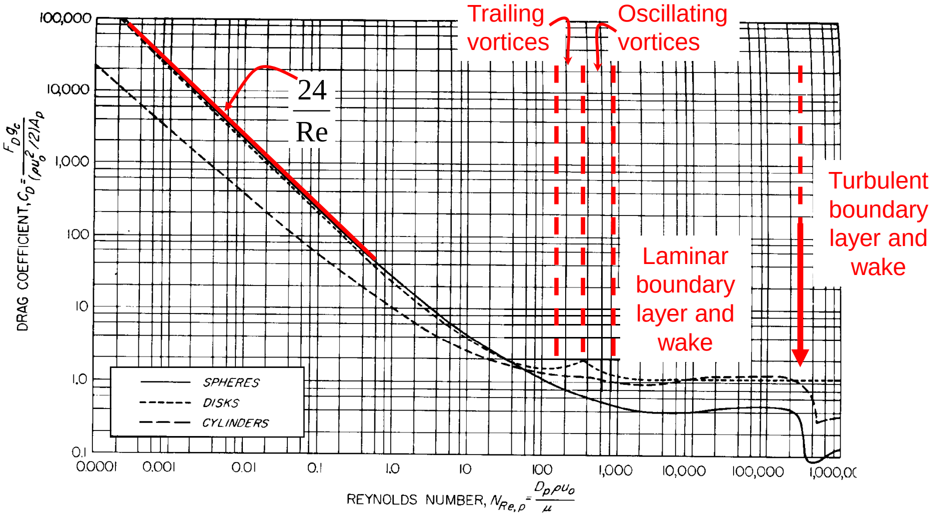

$\frac{24}{Re}$

Trailing vortices

Oscillating vortices

Laminar boundary layer and wake

Turbulent boundary layer and wake

McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

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Flow where Inertia Dominates:

Nondimensional Navier-Stokes Equation:

$$\frac{\partial \underline{v}^*}{\partial t^*} + \left(\underline{v}\cdot\nabla\underline{v}\right)^* = -\nabla^* P + \frac{\mu}{\rho V D}\left(\nabla^2 \underline{v}\right)^* + \frac{gD}{V^2}\underline{g}^*$$

With the appropriate terms in spherical coordinates

Consider the high Re limit:

$$\frac{\partial \underline{v}^*}{\partial t^*} + \left(\underline{v}\cdot\nabla\underline{v}\right)^* = -\nabla^* P + \cancel{\frac{1}{\mathrm{Re}}}\left(\nabla^2 \underline{v}\right)^* + \cancel{\frac{1}{Fr}}\underline{g}^*$$

$Re \rightarrow \infty$

No free surfaces

Now solve for a sphere

Potential flow around a Sphere (high Re, no viscosity)

(equation 8.203)

$$\nabla^* \cdot \underline{v}^* = 0$$

$$\frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v} \cdot \nabla \underline{v})^* = -\frac{\partial P^*}{\partial z^*}$$

$$C_D = \frac{2}{\pi}\int_0^{2\pi}\int_0^{\pi}\left[-P^* \cos\theta\right]_{r^*=\frac{1}{2}} \sin\theta \, d\theta d\phi$$

Solution:

$$\underline{v} = \begin{pmatrix} v_\infty\left(1-\left(\frac{R}{r}\right)^3\right)\cos\theta \\ -v_\infty\left(1+\frac{1}{2}\left(\frac{R}{r}\right)^3\right)\sin\theta \\ 0 \end{pmatrix}_{r\theta\phi}$$

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2\left(2\left(\frac{R}{r}\right)^3\left(1-\frac{3}{2}\sin^2\theta\right)-\left(\frac{R}{r}\right)^6\left(1-\frac{3}{4}\sin^2\theta\right)\right)$$

(equation 8.238-9)

Potential flow around a Sphere (high Re, no viscosity)

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How does this compare to what we see at high Re?

Solution:

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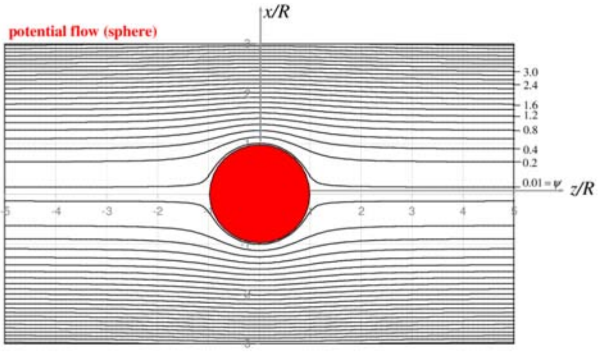

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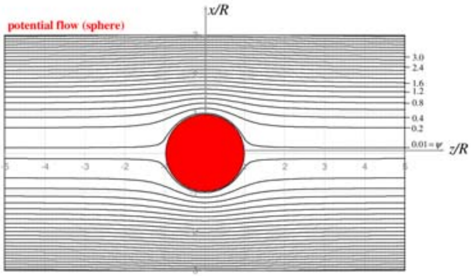

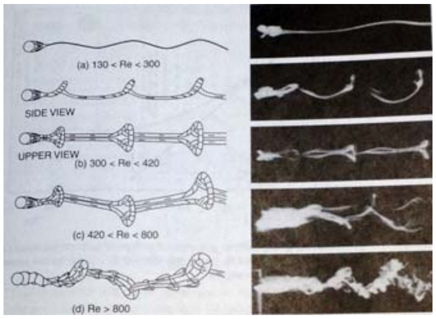

Potential flow around a Sphere (high Re iscosity)

Wrong!

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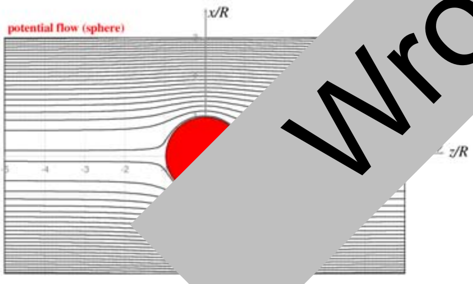

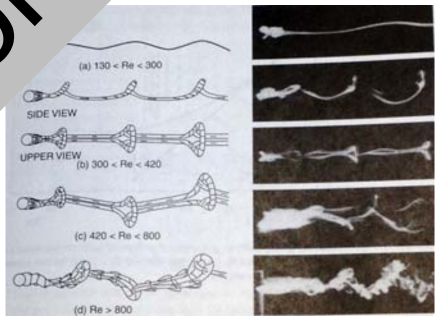

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(equation 8.20

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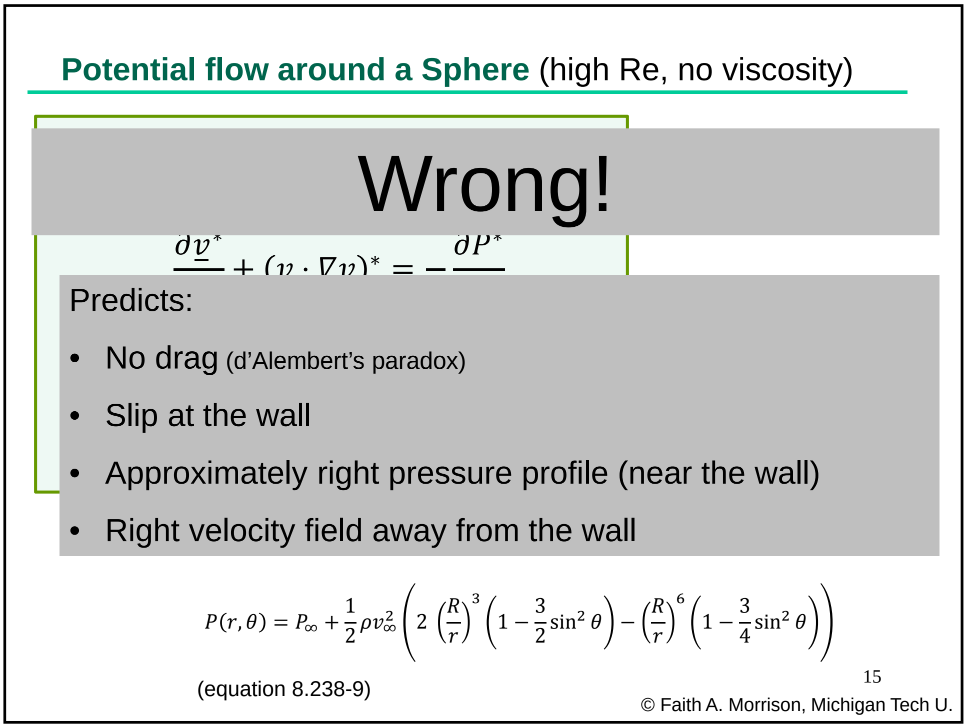

Potential flow around a Sphere (high Re, no viscosity)

Wrong!

$\frac{\partial \underline{v}^*}{} + (\underline{v}\cdot\nabla\underline{v})^* = -\frac{\partial P^*}{}$

Predicts:

- No drag (d'Alembert's paradox)
- Slip at the wall
- Approximately right pressure profile (near the wall)
- Right velocity field away from the wall

$$P(r,\theta) = P_\infty + \frac{1}{2}\rho v_\infty^2 \left(2\left(\frac{R}{r}\right)^3 \left(1 - \frac{3}{2}\sin^2\theta\right) - \left(\frac{R}{r}\right)^6 \left(1 - \frac{3}{4}\sin^2\theta\right)\right)$$

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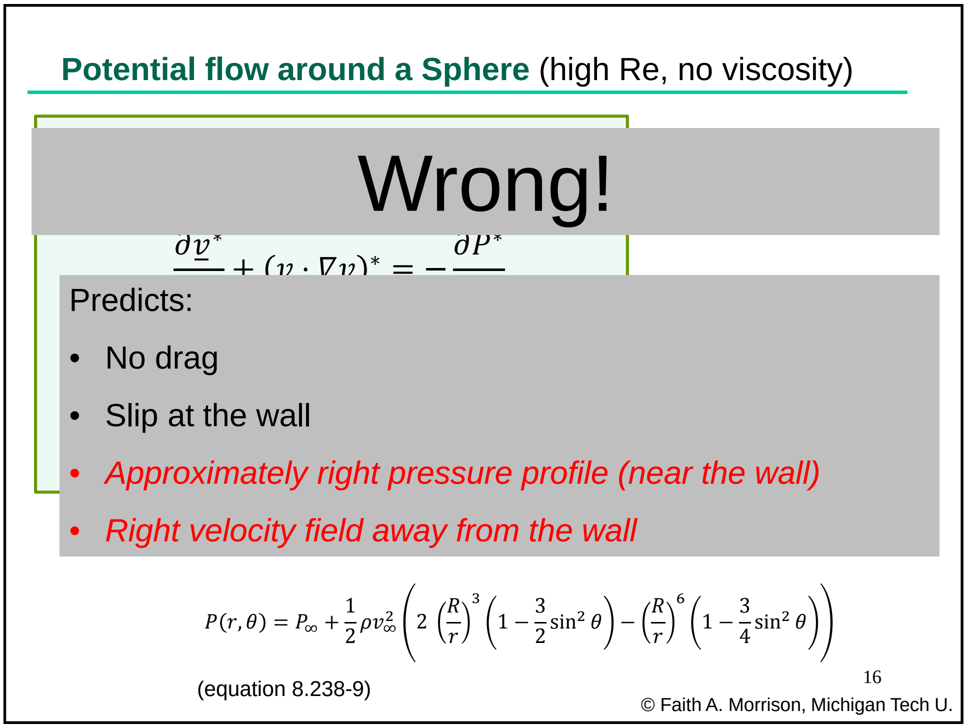

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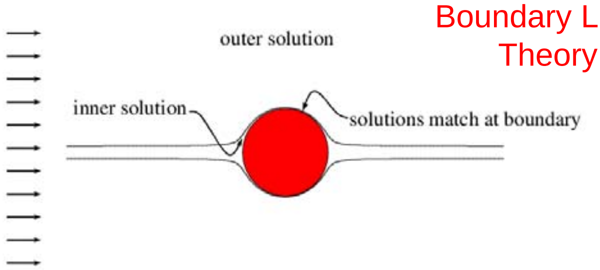
(equation 8.238-9)

More complicated flows III

Prandtl's Great Idea (1904):

- Keep the good parts of the potential flow solution: $\underline{v}$ in free stream, $p(r,\theta)$ near the surface
- Throw away the bad parts: slip at the wall
- Solve a new problem near the wall with $p(r,\theta)$ from the potential-flow solution

Boundary Layer Theory



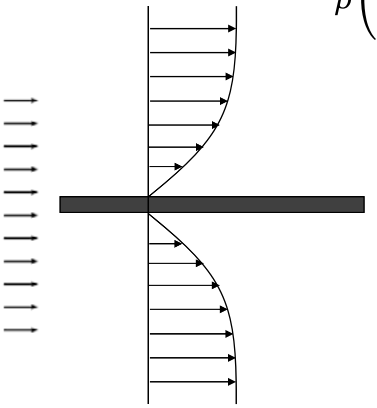
More complicated flows III

(Section 8.2)

Boundary Layer Theory

- Choose simplest boundary layer (flat plate)
- Nondimensionalize Navier-Stokes
- Eliminate small terms
- Solve

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$$



Characteristic values:

U in principal flow direction v_1

V in direction perpendicular to wall, v_2

L length of plate for x_1

δ boundary layer thickness for x_2

More complicated flows III

(Section 8.2)

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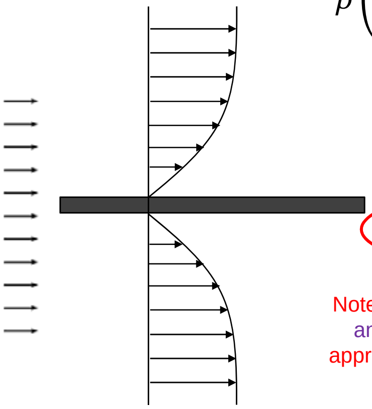

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Note that for this flow, two length scales and two velocities were found to be appropriate for the dimensional analysis.

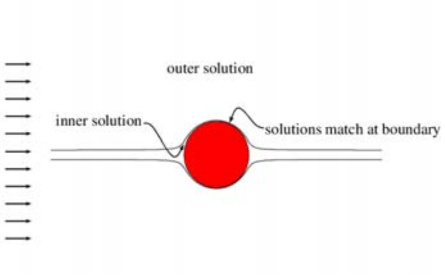
More complicated flows III

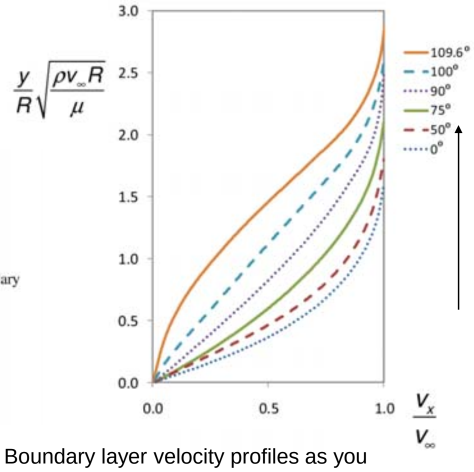
It works!

(see text, p710)

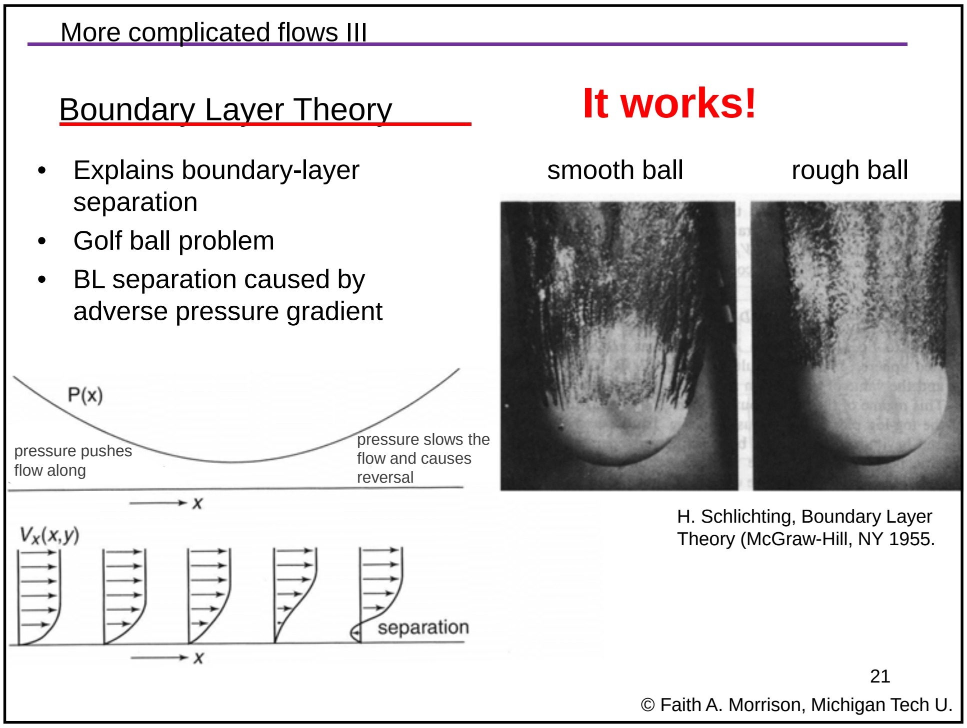
Boundary Layer Theory

- Apply to uniform flow approaching a sphere





Boundary layer velocity profiles as you progress from the stagnation point (0^o) to the top of the sphere (90^o) and beyond.



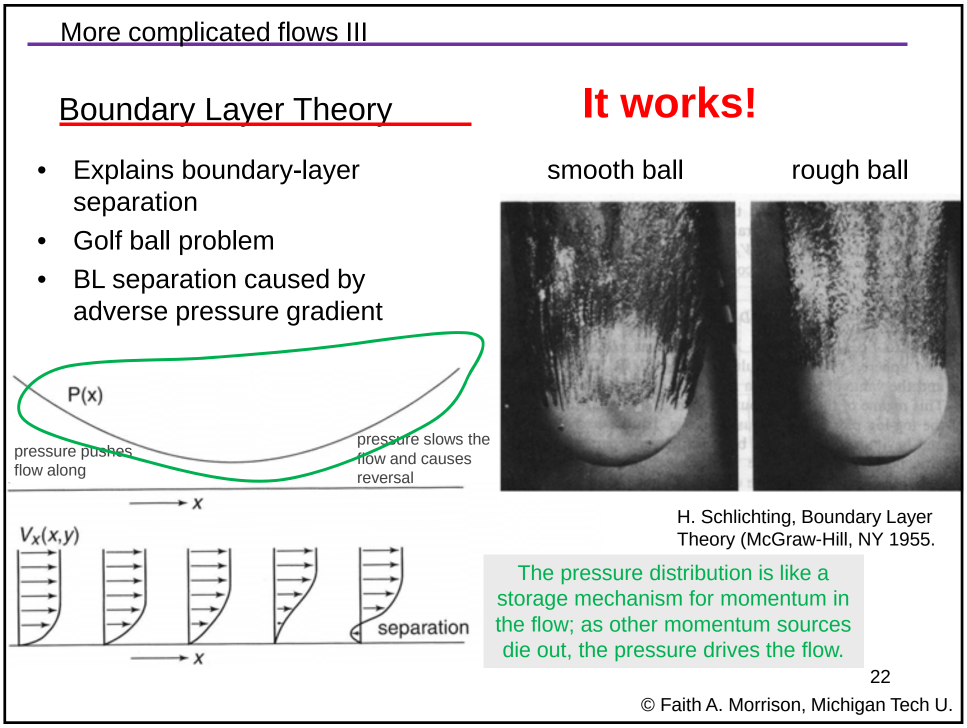
More complicated flows III

Boundary Layer Theory

- Explains boundary-layer separation
- Golf ball problem
- BL separation caused by adverse pressure gradient

It works!

H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.



More complicated flows III

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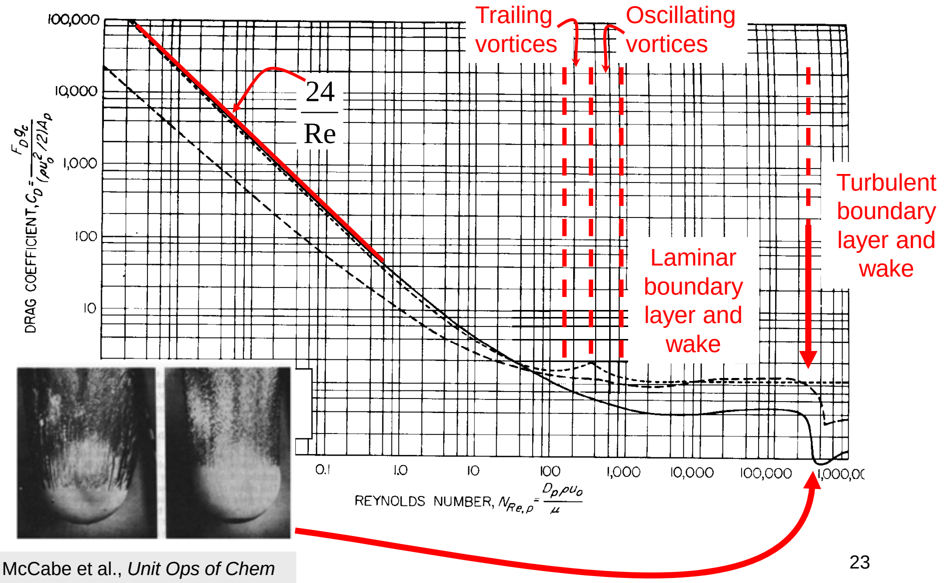
The pressure distribution is like a storage mechanism for momentum in the flow; as other momentum sources die out, the pressure drives the flow.

graphical correlation

Steady flow of an incompressible, Newtonian fluid around a sphere



McCabe et al., *Unit Ops of Chem Eng*, 5th edition, p147

What did we do?

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Solve Real Problems.

Powerful.

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See text

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MichiganTech

Miscellaneous Topics
Fluidized Beds

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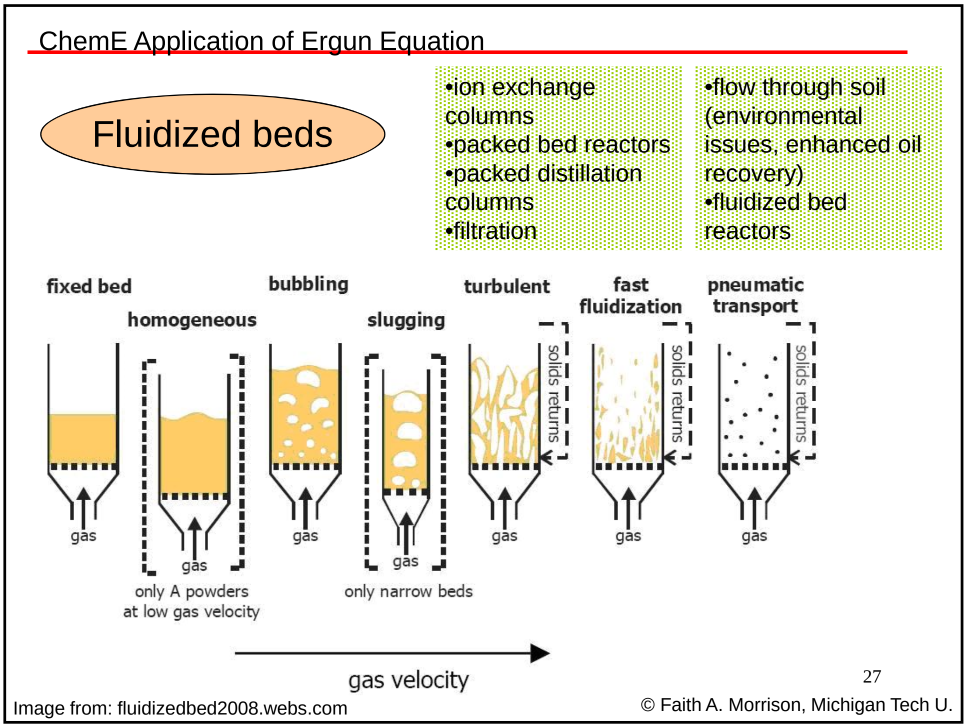
Department of Chemical Engineering
Michigan Technological University



ChemE Application of Ergun Equation

Fluidized beds

- ion exchange columns
- packed bed reactors
- packed distillation columns
- filtration

- flow through soil (environmental issues, enhanced oil recovery)
- fluidized bed reactors

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Image from: fluidizedbed2008.webs.com

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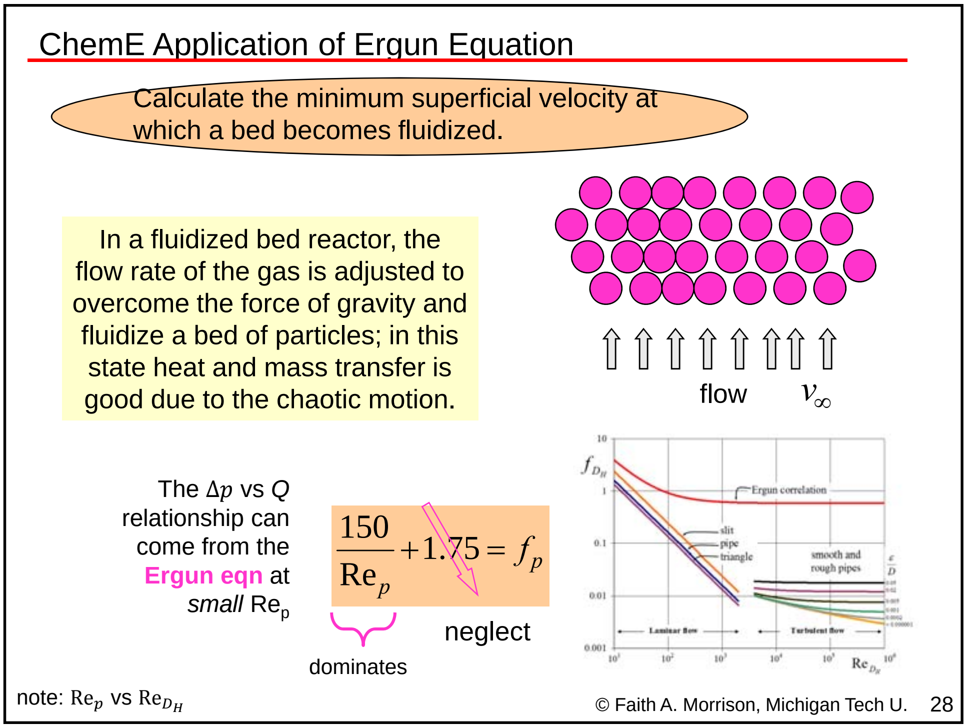

ChemE Application of Ergun Equation

Calculate the minimum superficial velocity at which a bed becomes fluidized.

In a fluidized bed reactor, the flow rate of the gas is adjusted to overcome the force of gravity and fluidize a bed of particles; in this state heat and mass transfer is good due to the chaotic motion.

The Δp vs Q relationship can come from the **Ergun eqn** at *small* Re_p

$$\underbrace{\frac{150}{\mathrm{Re}_p}}_{\text{dominates}} + \underbrace{1.75}_{\text{neglect}} = f_p$$

note: Re_p vs Re_{D_H}

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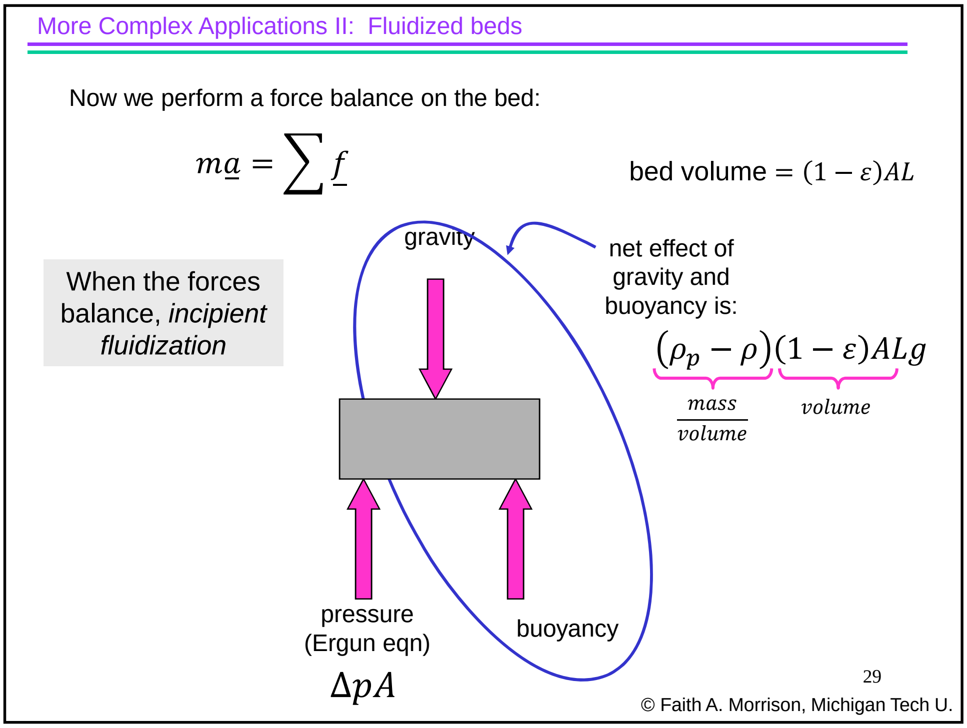


More Complex Applications II: Fluidized beds

Now we perform a force balance on the bed:

$$m\underline{a} = \sum \underline{f}$$

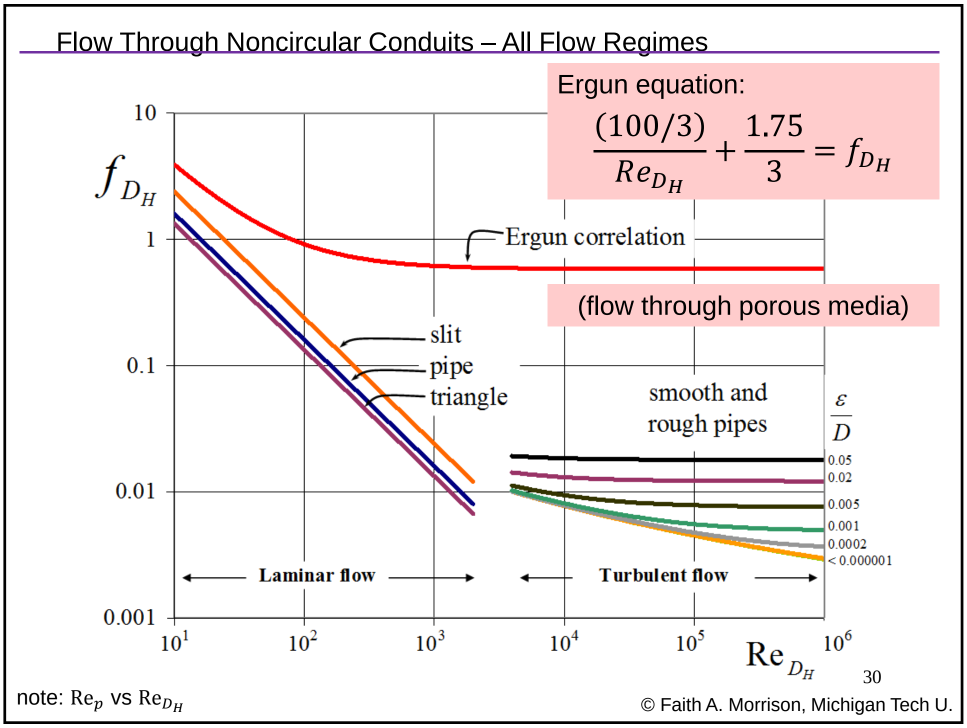
bed volume = $(1-\varepsilon)AL$

When the forces balance, *incipient fluidization*

ΔpA

net effect of gravity and buoyancy is:

$$\underbrace{\left(\rho_p-\rho\right)}_{\frac{mass}{volume}}\underbrace{(1-\varepsilon)AL}_{volume}g$$



Flow Through Noncircular Conduits – All Flow Regimes

Ergun equation:

$$\frac{(100/3)}{Re_{D_H}} + \frac{1.75}{3} = f_{D_H}$$

(flow through porous media)

note: Re_p vs Re_{D_H}

More Complex Applications II: Fluidized beds

When the forces balance, *incipient fluidization*

eliminate Δp;
solve for v_0

$$\left\{\begin{array}{l} \Delta p A = \left(\rho_p - \rho\right)(1-\varepsilon)ALg \\ \\ \dfrac{150}{Re_p} = f_p \end{array}\right.$$

note: Re_p vs Re_{D_H}

$$v_0 = \frac{\left(\rho_p - \rho\right) g D_p^2 \varepsilon^3}{150\mu(1-\varepsilon)}$$

velocity at the point of
incipient fluidization

Complete solution steps in Denn, Process Fluid Mechanics (Prentice Hall, 1980)

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Miscellaneous Topics
Compressible Flow

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Compressible Fluids

- most fluids are somewhat compressible
- in chemical-engineering processes, compressibility is unimportant at most operating pressures
- even gases may be modeled as incompressible if $\Delta p < p_{mean}$

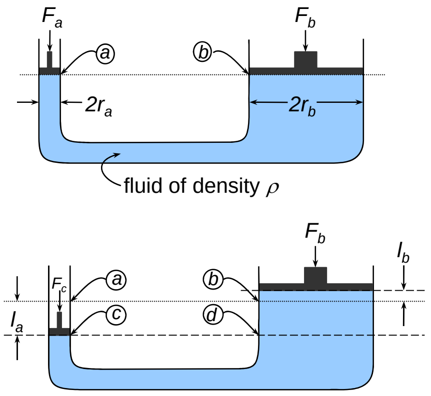
EXCEPT:
When the fluid velocity approaches the speed of sound

Compressible Fluids

How is pressure information transmitted in liquids and gases?

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.



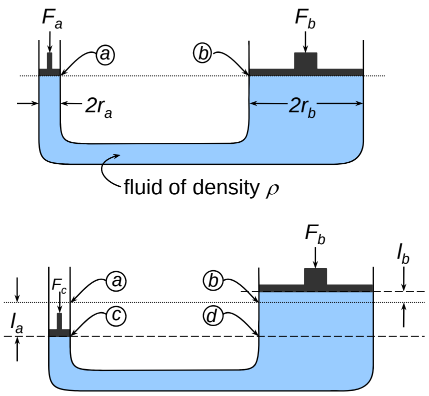
Compressible Fluids

For static incompressible liquids,

The Hydraulic Lift operates on Pascal's principle

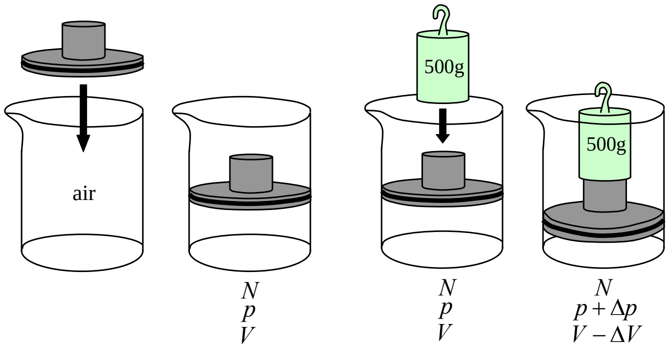
Pressure exerted on an enclosed liquid ***is transmitted equally*** *to every part of the liquid and to the walls of the container.*

and essentially, instantaneously



Compressible Fluids

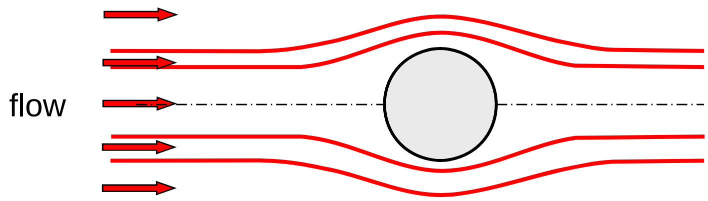
For static compressible fluids (gases), pressure causes volume change.



Compressible Fluids

For moving incompressible liquids and gases,

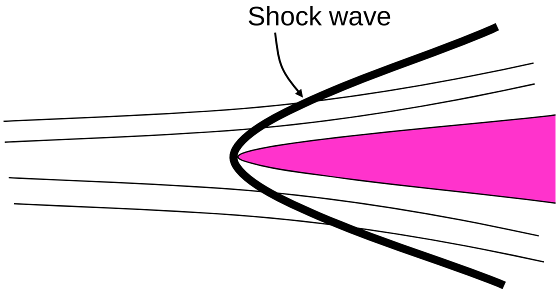
The presence of the obstacle is felt by the upstream fluid (pressure) and that information is transmitted very rapidly throughout the fluid.



The streamlines adjust according to momentum conservation.

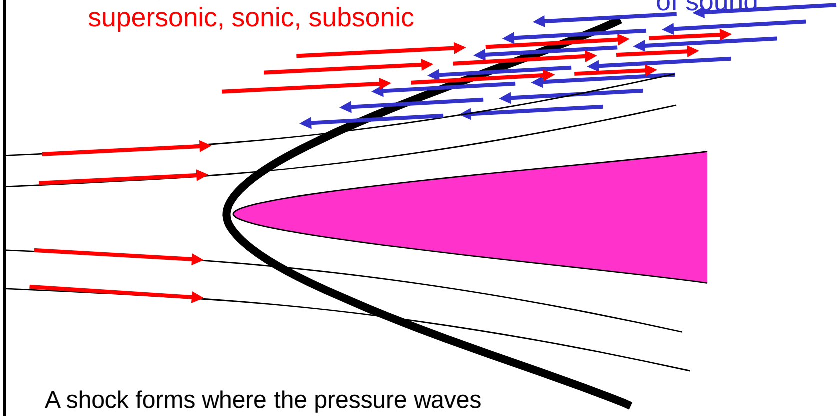
Compressible Fluids

For compressible fluids moving near sonic speeds, information (pressure) and the gas itself are moving at comparable speeds.



Pressure piles up at the shock wave

Compressible Fluids



Velocity of a fluid = variable = supersonic, sonic, subsonic

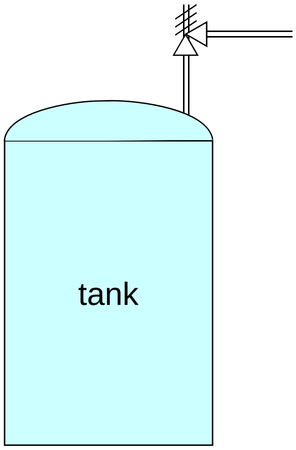
Velocity of a pressure wave = constant = speed of sound

A shock forms where the pressure waves from the obstacle stack up, and the speed of the pressure wave traveling upstream equals the speed of the fluid traveling downstream.

Compressible Fluids

Relief Valves (Safety Valves)

The rapid flows in relief valves can become sonic.



For supersonic flow, the flow rate is constant no matter what the pressure drop is.

(pressure waves pile up)

Choked Flow

Choked flow can be understood from basic equations of compressible fluid mechanics.

Compressible Fluids

Momentum and Energy in Compressible Fluids

incompressible

$$\underline{\underline{\tilde{\tau}}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T)$$

Microscopic momentum balance:

$$\boxed{\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \nabla\cdot\underline{\underline{\tilde{\tau}}} + \rho\underline{g}}$$

Mechanical energy balance:

incompressible

$$\boxed{\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}}$$

compressible?

Compressible Fluids

Momentum and Energy in Compressible Fluids

incompressible

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$$\underline{\underline{\tilde{\tau}}} = -\mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right) + \left(\frac{2}{3}\mu - \kappa\right)\nabla\cdot\underline{v}$$

compressible

κ = bulk viscosity

Mechanical energy balance:

incompressible

$$\boxed{\frac{\Delta p}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\delta z + F = \frac{W_{s,on}}{\dot{m}}}$$

MEB for compressible?

Compressible Fluids

Mechanical energy balance (compressible)

Back up one step in the derivation and reintegrate without constant ρ assumption.

$$\frac{dp}{\rho} + VdV + gdz + dF = \frac{dW_{s,on}}{\dot{m}}$$

Assume:
- constant cross section
- constant mass flow $\rho VA = GA$
- neglect gravity
- no shaft work

$$G \equiv \rho V = mass \;\; velocity$$

Compressible Fluids

Mechanical energy balance (compressible)

Ideal Gas Law

$$pV = NRT$$

$$\frac{V}{N} = \frac{RT}{p}$$

$$\frac{V}{MN} = \frac{RT}{pM}$$

$$\boxed{\frac{1}{\rho} = \frac{RT}{pM}}$$

For isothermal flow:

$$p_1V_1 = NRT$$

$$p_2V_2 = NRT$$

$$\boxed{\frac{p_1}{p_2} = \frac{V_2}{V_1}}$$

Also,

$$\frac{\rho_{av}}{p_{av}} = \frac{M}{RT}$$

$$\boxed{\frac{2\rho_{av}}{p_1 + p_2} = \frac{M}{RT}}$$

Compressible Fluids

Mechanical energy balance (compressible)

$$G \equiv \rho V = mass \;\; velocity$$

$$\boxed{\left(p_2 - p_1\right) + \frac{G^2}{\rho_{av}} \ln \frac{p_1}{p_2} + \frac{2fG^2}{\rho_{av} D}\left(L_2 - L_1\right) = 0}$$

The compressible MEB predicts that there is a maximum velocity at

(see book)

$$\boxed{V_{\max} = \sqrt{\frac{p_2}{\rho_2}} = \sqrt{\frac{RT}{M}} = \text{isothermal speed of sound}}$$

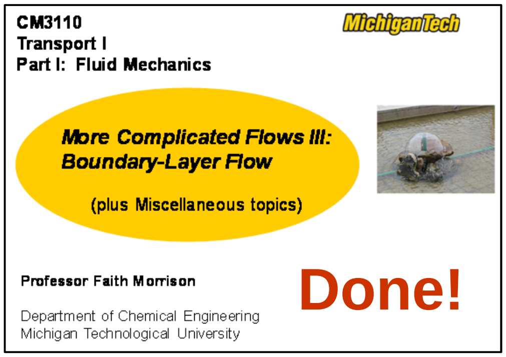
Compressible Fluids

A better assumption than isothermal flow is adiabatic flow (no heat transferred). For this case,

$$V_{\max} = \sqrt{\frac{\gamma p_2}{\rho_2}} = \sqrt{\frac{\gamma RT}{M}} = \text{adiabatic speed of sound}$$

$$\gamma = \frac{C_p}{C_v}$$

(see book)



CM3110
Transport I
Part I: Fluid Mechanics

More Complicated Flows III: Boundary-Layer Flow

(plus Miscellaneous topics)

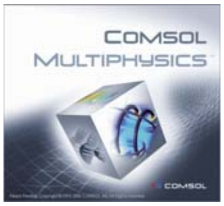
Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

Done!

Just one more thing

Numerical PDE Solving with Comsol 5.1



www.comsol.com

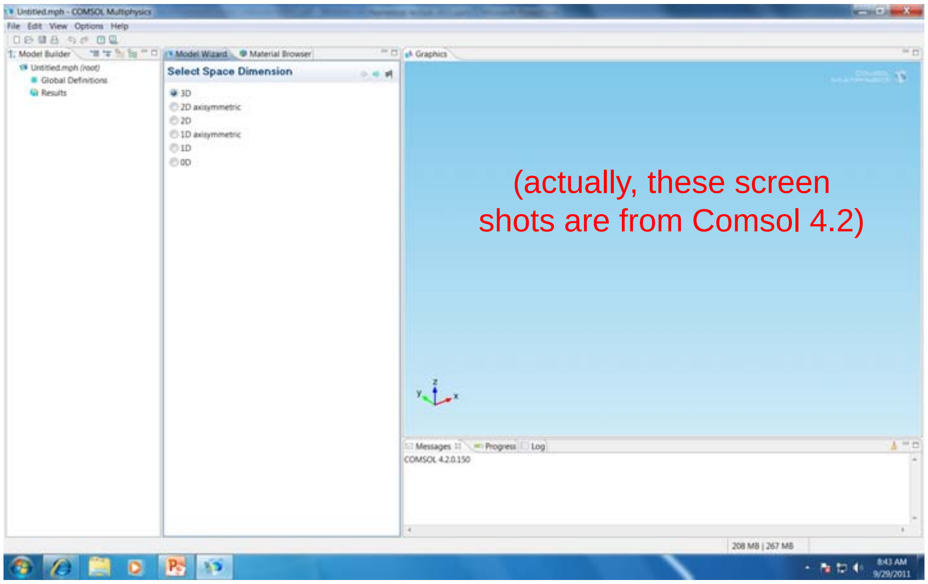
Finite-element numerical differential equation solver. Applications include fluid mechanics and heat transfer.

1. Choose the physics (2D, 2D axisymmetric, laminar, steady/unsteady, etc.)
2. Choose flow geometry and fluid (shape of the flow domain)
3. Define boundary conditions
4. Design and generate mesh
5. Solve the problem
6. Calculate and plot engineering quantities of interest.

Comsol Multiphysics 5.1

Launch the program

0



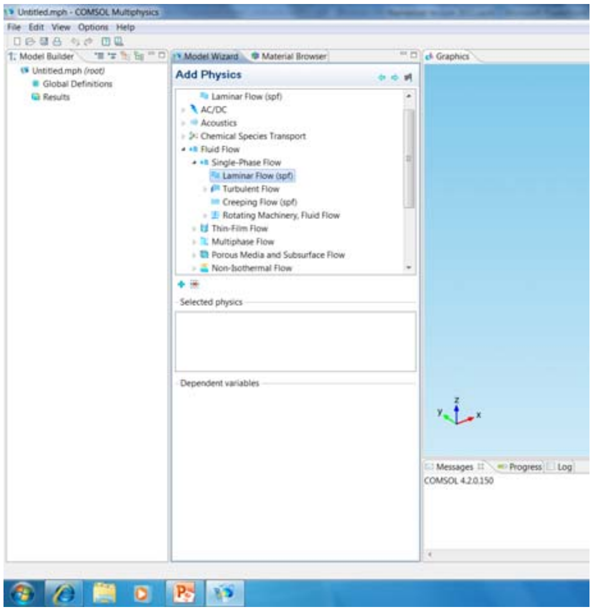
COMSOL 4.2.0.150

(actually, these screen shots are from Comsol 4.2)

Comsol Multiphysics 5.1

Choose the physics

1

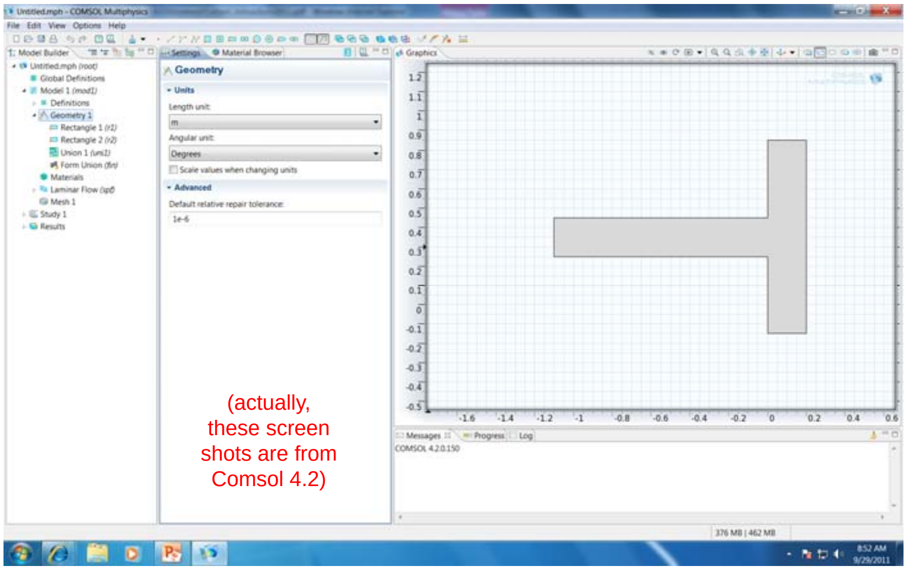


COMSOL 4.2.0.150

(actually, these screen shots are from Comsol 4.2)

Comsol Multiphysics 5.1

Choose flow geometry and fluid **2**



Geometry

Units

Length unit: m

Angular unit: Degrees

☐ Scale values when changing units

Advanced

Default relative repair tolerance: 1e-6

COMSOL 4.2.0.150

(actually, these screen shots are from Comsol 4.2)

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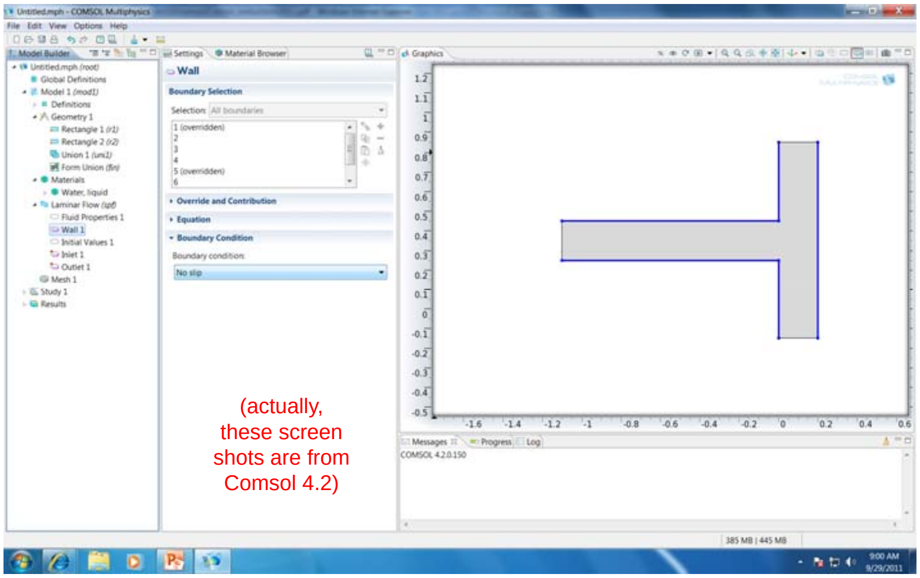
Comsol Multiphysics 5.1

Define boundary conditions **3**



Wall

Boundary Selection

Selection: All boundaries

1 (overridden)
2
3
4
5 (overridden)
6

Override and Contribution

Equation

Boundary Condition

Boundary condition: No slip

COMSOL 4.2.0.150

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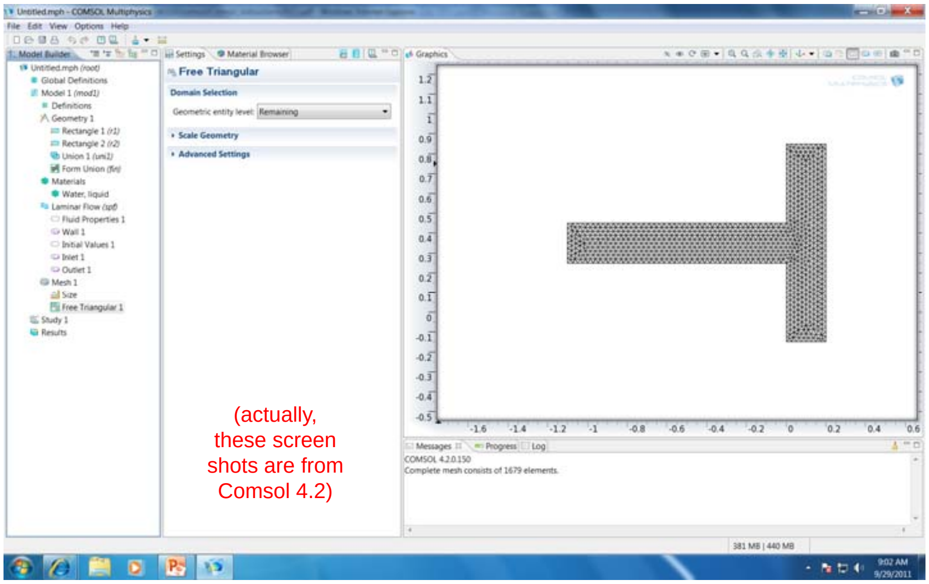
Comsol Multiphysics 5.1

Design and generate mesh **4**



Free Triangular

Domain Selection

Geometric entity level: Remaining

- Scale Geometry
- Advanced Settings

(actually, these screen shots are from Comsol 4.2)

```
COMSOL 4.2.0.150
Complete mesh consists of 1679 elements.
```

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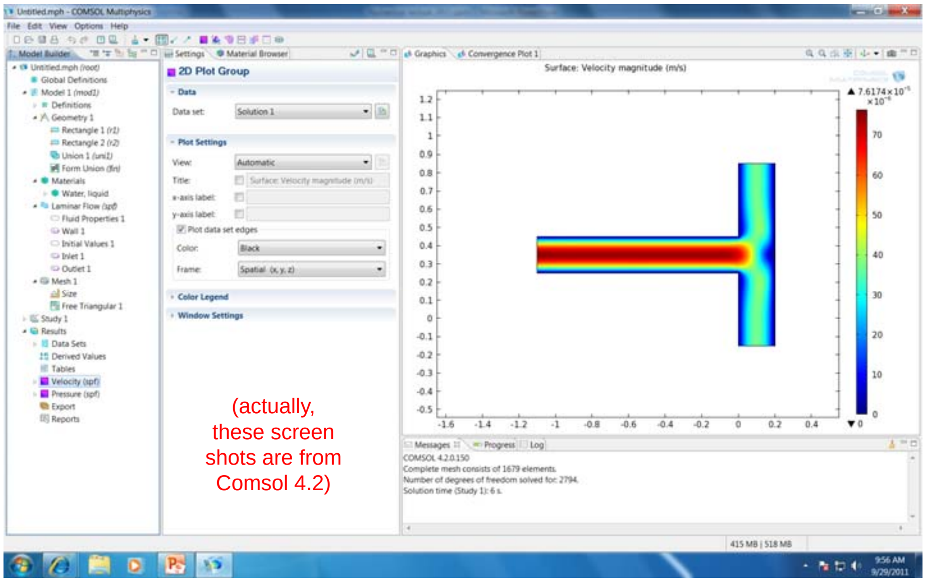
Comsol Multiphysics 5.1

Solve the problem **5**



2D Plot Group

Data

Data set: Solution 1

Plot Settings

View: Automatic

Title: ☐ Surface: Velocity magnitude (m/s)

x-axis label: ☐

y-axis label: ☐

☑ Plot data set edges

Color: Black

Frame: Spatial (x, y, z)

- Color Legend
- Window Settings

(actually, these screen shots are from Comsol 4.2)

```
COMSOL 4.2.0.150
Complete mesh consists of 1679 elements.
Number of degrees of freedom solved for: 2794.
Solution time (Study 1): 6 s.
```

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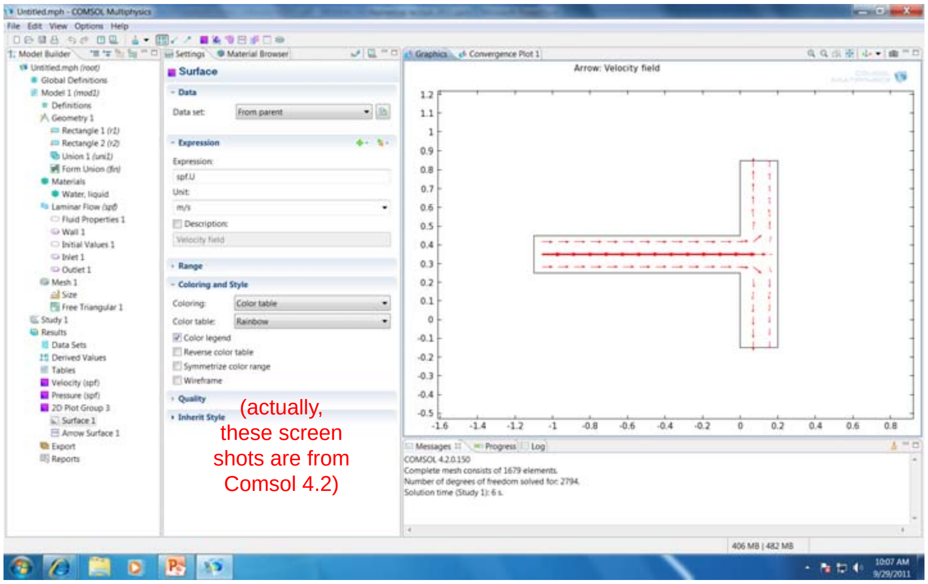
Comsol Multiphysics 5.1

View the solution

5



Surface

- Data

Data set: From parent

- Expression

Expression: spf.U

Unit: m/s

☐ Description: Velocity field

- Range
- Coloring and Style

Coloring: Color table

Color table: Rainbow

☑ Color legend

☐ Reverse color table

☐ Symmetrize color range

☐ Wireframe

- Quality
- Inherit Style

```
COMSOL 4.2.0.150
Complete mesh consists of 1679 elements.
Number of degrees of freedom solved for: 2794.
Solution time (Study 1): 6 s.
```

(actually, these screen shots are from Comsol 4.2)

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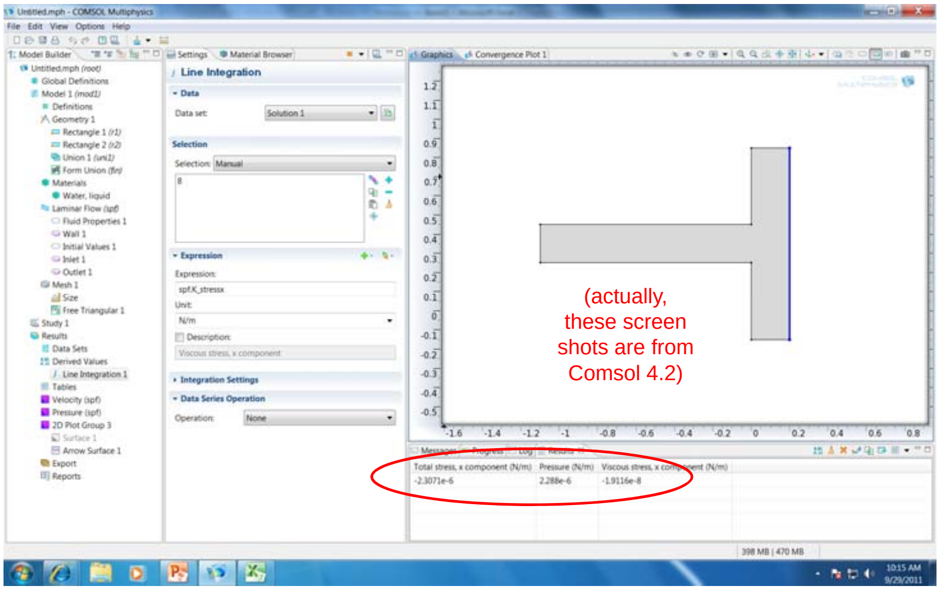
Comsol Multiphysics 5.1

Calculate engineering problems of interest

6



Line Integration

- Data

Data set: Solution 1

Selection

Selection: Manual

8

- Expression

Expression: spf.K_stressx

Unit: N/m

☐ Description: Viscous stress, x component

- Integration Settings
- Data Series Operation

Operation: None

Total stress, x component (N/m)	Pressure (N/m)	Viscous stress, x component (N/m)
-2.3071e-6	2.288e-6	-1.9116e-8

(actually, these screen shots are from Comsol 4.2)

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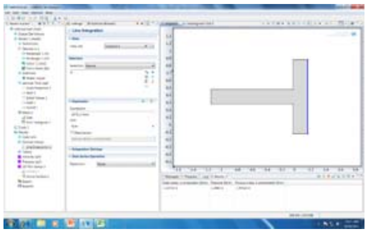
Comsol Multiphysics



Comsol project:

- Due last day of classes
- Individual work
- 2 points for part 1 (instructions given)
- 3 points for part 2 (no instructions)
- Coming soon

Adds on top
of your
course grade

CM3110
Transport I
Part II: Heat Transfer

MichiganTech

Introduction to Heat Transfer

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University