Integrated Circuits for Communication Berkeley

MOS Wideband Noise and Distortion Amplifier Examples

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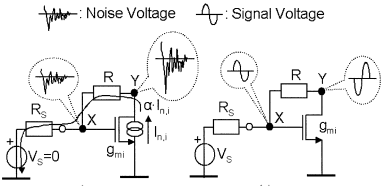
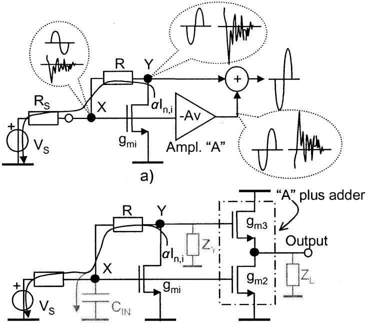
Wideband Noise Cancellation



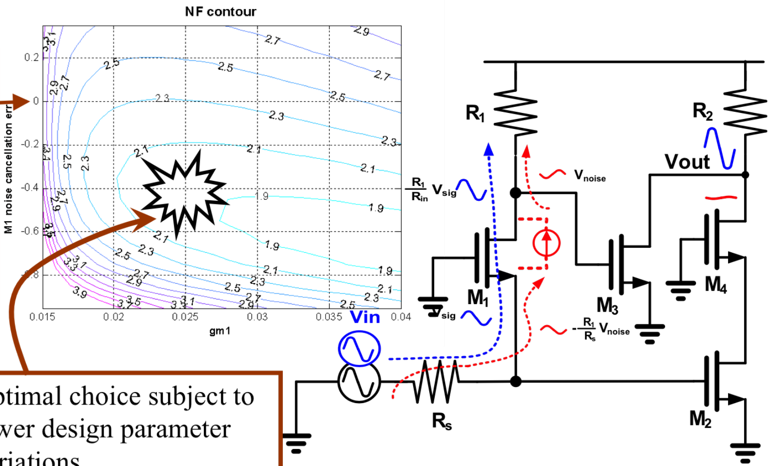


- Take advantage of amplifier topologies where the output thermal noise flows into the input (CG amplifier, shunt feedback amplifer, etc).
- Cancel thermal noise using a second feedforward path. Can we also cancel the distortion?
- Source: F. Bruccoleri, E. A. M. Klumperink, B. Nauta, "Wide-Band CMOS Low-Noise Amplifier Exploiting Thermal Noise Canceling," *JSSC*, vol. 39, Feb. 2004.

Noise Cancellation LNA

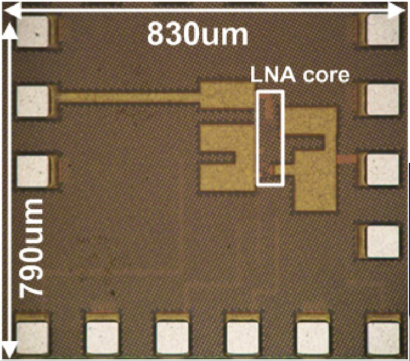
- Motivated by [Bruccoleri, *et al.*, ISSCC 2002]

M1 noise full cancellation



Optimal choice subject to fewer design parameter variations

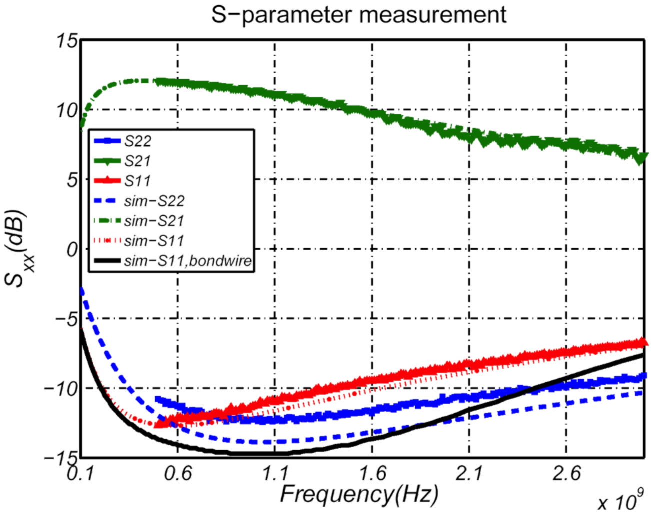
130nm LNA Prototype



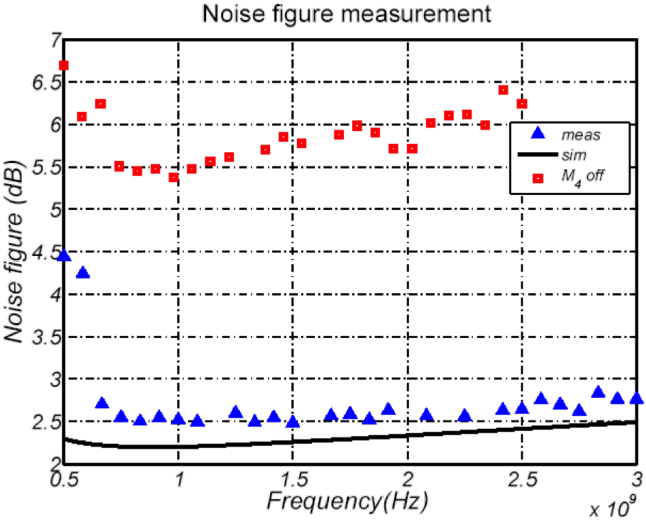
- 130nm CMOS, 1.5V, 12mA
- Employ only thin oxide transistors

- W.-H. Chen, G. Liu, Z. Boos, A. M. Niknejad, "A Highly Linear Broadband CMOS LNA Employing Noise and Distortion Cancellation," *IEEE Journal of Solid-State Circuits*, vol. 43, pp. 1164-1176, May 2008.

Measured LNA S-Parameters

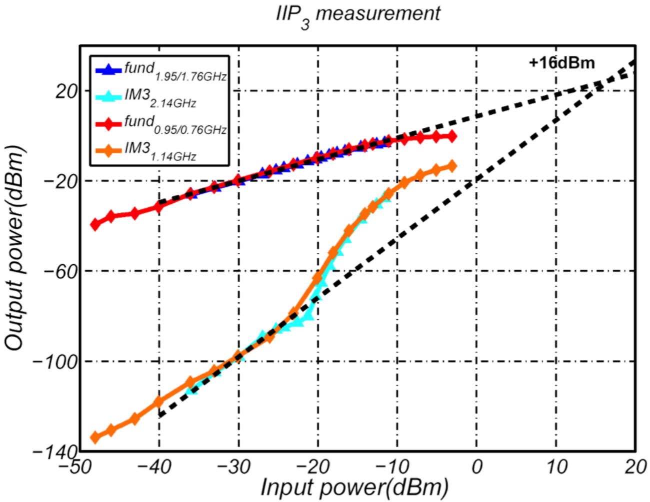


- Matches simulations well. Very broadband performance.



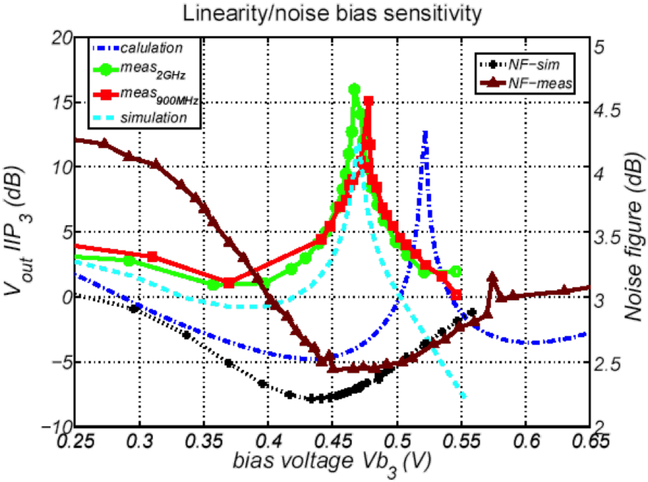
- Noise cancellation is clearly visible. This is also a “knob” for dynamic operation to save current.

Measured Linearity



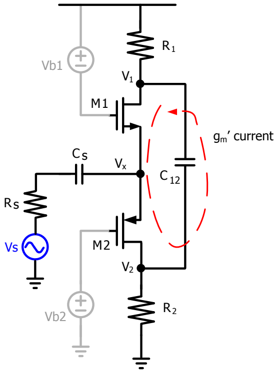
- Record linearity of +16 dBm for out of band blockers (at the time of publication).
- Linearity works over entire LNA band.

Linearity Bias Dependence



- As we vary the bias of key transistor, we simulate the effects of process/temp variation. There is a 50 mV window where the performance is still acceptable.

LNA Distortion Analysis



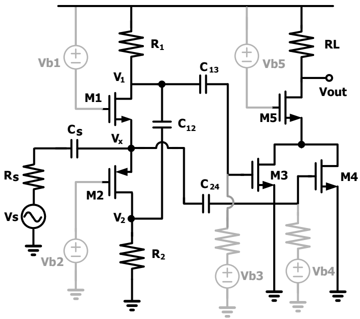
- Let's assume that the drain current is a power-series of the V_{gs} voltage:

$$i_d = g_{m1} V_{gs} + \frac{g_{m2}}{2!} V_{gs}^2 + \frac{g_{m3}}{3!} V_{gs}^3$$

- The purpose of the "differential" input is to cancel the 2nd order distortion of the first stage (to minimize 2nd order interaction):

$$i_{out} = i_{ds,n} + i_{ds,p} = (g_{mn} + g_{mp})\, v_{in} + \frac{g'_{mn} - g'_{mp}}{2} v_{in}^2 + \frac{g''_{mn} + g''_{mp}}{6} v_{in}^3$$

Distortion Equivalent Circuit



- Assume $R1/R2$ and RL are small so that r_o non-linearity is ignored.

$$V_x = A_1(s_1) \circ V_s + A_2(s_1, s_2) \circ V_s^2 + A_3(s_1, s_2, s_3) \circ V_s^3$$

$$V_1 = B_1(s_1) \circ V_s + B_2(s_1, s_2) \circ V_s^2 + B_3(s_1, s_2, s_3) \circ V_s^3$$

$$V_2 = C_1(s_1) \circ V_s + C_2(s_1, s_2) \circ V_s^2 + C_3(s_1, s_2, s_3) \circ V_s^3$$

$$i_{m1} + \frac{V_x - V_1}{r_{o1}} + i_{m2} + \frac{V_x - V_2}{r_{o2}} + \frac{V_x}{Z_x(s)} = \frac{V_s - V_x}{Z_s(s)}$$

$$i_{m1} + \frac{V_x - V_1}{r_{o1}} = \frac{V_1}{Z_1(s)} + \frac{V_1 - V_2}{Z_{12}(s)}$$

$$i_{m2} + \frac{V_x - V_2}{r_{o2}} = \frac{V_2}{Z_2(s)} + \frac{V_2 - V_1}{Z_{12}(s)}$$

$$Z_1 = R_1 || \frac{1}{sC_1}$$

$$Z_s = R_s + \frac{1}{sC_s}$$

$$Z_x = \frac{1}{sC_x}$$

Drain Current Non-Linearity

- Assuming that the gates of the input transistors are grounded at RF:

$$i_{m1} = -\left(g_{m1}(-V_x) + \frac{g'_{m1}}{2}(-V_x)^2 + \frac{g''_{m1}}{6}(-V_x)^3\right)$$

$$= g_{m1}V_x - \frac{g'_{m1}}{2}V_x^2 + \frac{g''_{m1}}{6}V_x^3$$

$$i_{m2} = g_{m2}V_x + \frac{g'_{m2}}{2}V_x^2 + \frac{g''_{m2}}{6}V_x^3$$

- Solve for first order Kernels from:

$$g_{m1}A_1(s) + \frac{A_1(s) - B_1(s)}{r_{o1}} + g_{m2}A_1(s) + \frac{A_1(s) - C_1(s)}{r_{o2}} + \frac{A_1(s)}{Z_x(s)} = \frac{1 - A_1(s)}{Z_s(s)}$$

$$g_{m1}A_1(s) + \frac{A_1(s) - B_1(s)}{r_{o1}} = \frac{B_1(s)}{Z_1(s)} + \frac{B_1(s) - C_1(s)}{Z_{12}(s)}$$

$$g_{m2}A_1(s) + \frac{A_1(s) - C_1(s)}{r_{o2}} = \frac{C_1(s)}{Z_2(s)} + \frac{C_1(s) - B_1(s)}{Z_{12}(s)}$$

Simplified First-Order

- At the frequency of interest, $Z_{12} \sim 0$ and $B_1 \sim C_1$

$$A_1(s) = \frac{(Z_1(s) \parallel Z_2(s)) + (r_{o1} \parallel r_{o2})}{H(s)}$$

$$B_1(s) = \frac{Z_1(s) \parallel Z_2(s)}{\left(\frac{Z_1(s) \parallel Z_2(s) + (r_{o1} \parallel r_{o2})}{1+(g_{m1}+g_{m2})(r_{o1} \parallel r_{o2})}\right)} A_1(s)$$

$$H(s) = Z_s(s)\Big(1 + (g_{m1} + g_{m2})(r_{o1} \parallel r_{o2})\Big) + \Big((Z_1(s) \parallel Z_2(s)) + (r_{o1} \parallel r_{o2})\Big)\left(1 + \frac{Z_s(s)}{Z_x(s)}\right)$$

Second-Order Terms

- Retaining only 2nd order terms in the KCL equations:

$$g_{m1}A_2(s_1,s_2) - \frac{g'_{m1}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2) - B_2(s_1,s_2)}{r_{o1}} +$$
$$g_{m2}A_2(s_1,s_2) + \frac{g'_{m2}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2) - C_2(s_1,s_2)}{r_{o2}} + \frac{A_2(s_1,s_2)}{Z_x(s_1+s_2)} = \frac{-A_2(s_1,s_2)}{Z_s(s_1+s_2)}$$

$$g_{m1}A_2(s_1,s_2) - \frac{g'_{m1}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2) - B_2(s_1,s_2)}{r_{o1}} = \frac{B_2(s_1,s_2)}{Z_1(s_1+s_2)} + \frac{B_2(s_1,s_2) - C_2(s_1,s_2)}{Z_{12}(s_1+s_2)}$$
$$g_{m2}A_2(s_1,s_2) + \frac{g'_{m2}}{2}A_1(s_1)A_1(s_2) + \frac{A_2(s_1,s_2) - C_2(s_1,s_2)}{r_{o2}} = \frac{C_2(s_1,s_2)}{Z_2(s_1+s_2)} + \frac{C_2(s_1,s_2) - B_2(s_1,s_2)}{Z_{12}(s_1+s_2)}$$

Second-Order Kernels

- Solving above equations we arrive at:

$$A_2(s_1,s_2) = \frac{\frac{1}{2}(g'_{m1} - g'_{m2})(r_{o1} \parallel r_{o1}) Z_s(s_1+s_2) A_1(s_1) A_1(s_2) + \triangle A_2(s_1,s_2)}{H(s_1+s_2) + \triangle H(s_1,s_2)}$$

$$B_2(s_1,s_2) = \frac{-\frac{Z_1(s_1+s_2)\parallel Z_2(s_1+s_2)}{Z_x(s_1+s_2)\parallel Z_s(s_1+s_2)}\left(\frac{1}{2}(g'_{m1} - g'_{m2})(r_{o1} \parallel r_{o1}) Z_s(s_1+s_2) A_1(s_1) A_1(s_2)\right) + \triangle B_2(s_1,s_2)}{H(s_1+s_2) + \triangle H(s_1,s_2)}$$

$$\triangle A_2(s_1,s_2) = \frac{1}{2} Z_{12}(s_1+s_2) A_1(s_1) A_1(s_2) \frac{Z_s(s_1+s_2)}{Z_1(s_1+s_2) + Z_2(s_1+s_2)} \times \left((g'_{m1} - g'_{m2})(r_{o1} \parallel r_{o2}) + \frac{g'_{m1} r_{o1} Z_2(s_1+s_2) - g'_{m2} r_{o2} Z_1(s_1+s_2)}{r_{o1} + r_{o2}}\right)$$

$$\triangle B_2(s_1,s_2) = -\frac{1}{2} Z_{12}(s_1+s_2) A_1(s_1) A_1(s_2) \frac{Z_1(s_1+s_2)}{Z_1(s_1+s_2) + Z_2(s_1+s_2)} \frac{1}{r_{o1} + r_{o2}} \times \left(g'_{m1} r_{o1}\left(Z_2(s_1+s_2) + r_{o2}\right)\left(1 + \frac{Z_s(s_1+s_2)}{Z_x(s_1+s_2)}\right) + Z_s(s_1+s_2)\left(g'_{m2} r_{o2}(1 + g_{m1} r_{o1}) + g'_{m1} r_{o1}(1 + g_{m2} r_{o2})\right)\right)$$

$$\triangle H(s_1,s_2) = Z_{12}(s_1+s_2) \frac{Z_s(s_1,s_2)}{Z_1(s_1,s_2) + Z_2(s_1,s_2)} \frac{1}{r_{o1} + r_{o2}} \times \left(\frac{(r_{o1} + Z_1(s_1+s_2))(r_{o2} + Z_2(s_1+s_2))}{Z_x(s_1+s_2) \parallel Z_s(s_1+s_2)} + \left((1 + g_{m1} r_{o1})(r_{o2} + Z_2(s_1+s_2)) + (1 + g_{m2} r_{o2})(r_{o1} + Z_1(s_1+s_2))\right)\right)$$

Third-Order Terms

$$g_{m1}A_3(s_1,s_2,s_3)+\frac{g''_{m1}}{6}A_1(s_1)A_1(s_2)A_1(s_3)-g'_{m1}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{r_{o1}}$$

$$+g_{m2}A_3(s_1,s_2,s_3)+\frac{g''_{m2}}{6}A_1(s_1)A_1(s_2)A_1(s_3)+g'_{m2}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{r_{o2}}$$

$$=-\frac{A_3(s_1,s_2,s_3)}{Z_s(s_1+s_2+s_3)}$$

$$g_{m1}A_3(s_1,s_2,s_3)+\frac{g''_{m1}}{6}A_1(s_1)A_1(s_2)A_1(s_3)-g'_{m1}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{r_{o1}}$$

$$=\frac{B_3(s_1,s_2,s_3)}{Z_1(s_1+s_2+s_3)}+\frac{B_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{Z_{12}(s_1+s_2+s_3)}$$

$$g_{m2}A_3(s_1,s_2,s_3)+\frac{g''_{m2}}{6}A_1(s_1)A_1(s_2)A_1(s_3)+g'_{m2}\overline{A_1(s_1)A_2(s_2,s_3)}+\frac{A_3(s_1,s_2,s_3)-C_3(s_1,s_2,s_3)}{r_{o1}}$$

$$=\frac{C_3(s_1,s_2,s_3)}{Z_2(s_1+s_2+s_3)}+\frac{C_3(s_1,s_2,s_3)-B_3(s_1,s_2,s_3)}{Z_{12}(s_1+s_2+s_3)}$$

Third-Order Kernel

- Assuming $Z_{12} \sim 0$ (at $s_1 + s_2 + s_3$):

$$A_3(s_1,s_2,s_3) = \frac{-Z_s(r_{o1} \parallel r_{o2})\Big(-(g'_{m1}+g'_{m2})\overline{A_1(s_1)A_2(s_2,s_3)} + \frac{1}{6}(g''_{m1}+g''_{m2})A_1(s_1)A_1(s_2)A_1}{H(s_1+s_2+s_3)}$$

$$B_3(s_1,s_2,s_3) = \frac{-Z_1(s_1+s_2+s_3)}{Z_x(s_1+s_2+s_3) \parallel Z_s(s_1+s_2+s_3)} A_3(s_1,s_2,s_3)$$

Output Voltage

- The output voltage is given by a new Volterra series. Assume for simplicity the following:

$$V_{out} = \left(g_{m3} V_1 + g_{m4} V_x + \frac{g'_{m3}}{2} V_1^2 + \frac{g'_{m4}}{2} V_x^2 + \frac{g''_{m3}}{6} V_1^3 + \frac{g''_{m4}}{6} V_x^3 \right) \times Z_L$$

- The fundamental and third-order output are therefore:

$$V_{out,fund} = \left((A_1(s) \circ V_s) \times g_{m4} + (B_1(s) \circ V_s) \times g_{m3} \right) \times Z_L$$

$$\begin{aligned} V_{out,3^{rd}} = \Big(& ((A_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m4} + (B_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m3}) \\ & + ((A_1(s) \circ V_s)^3 \times \frac{g''_{m4}}{6} + (B_1(s) \circ V_s)^3 \times \frac{g''_{m3}}{6}) \\ & + ((\overline{A_1(s_1) A_2(s_2,s_3)} \circ V_s^3) \times g'_{m4} + (\overline{B_1(s_1) B_2(s_2,s_3)} \circ V_s^3) \times g'_{m3}) \Big) \times Z_L \end{aligned}$$

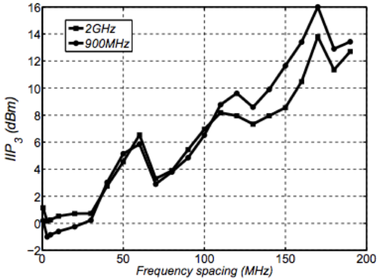
Focus on Third-Order Output

- At low frequencies:
  - $A_1/B_1 \sim R_{in}/R_1$
  - $A_2/B_2 \sim -R_s/R_1$
  - $A_3/B_3 \sim -R_s/R_1$

$$V_{out,3^{rd}} = \Big(((A_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m4} + (B_3(s_1,s_2,s_3) \circ V_s^3) \times g_{m3})$$
$$+ ((A_1(s) \circ V_s)^3 \times \frac{g''_{m4}}{6} + (B_1(s) \circ V_s)^3 \times \frac{g''_{m3}}{6})$$
$$+ ((\overline{A_1(s_1)A_2(s_2,s_3)} \circ V_s^3) \times g'_{m4} + (\overline{B_1(s_1)B_2(s_2,s_3)} \circ V_s^3) \times g'_{m3}) \Big) \times Z_L$$

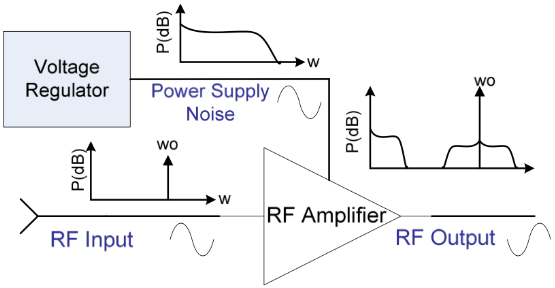
- First term cancels like thermal noise
- Second term: New distortion generated at output. Cancel with MGTR.
- Thrid term: Due to second-order interaction: Must use $g' = 0$

Two-Tone Spacing Dependence



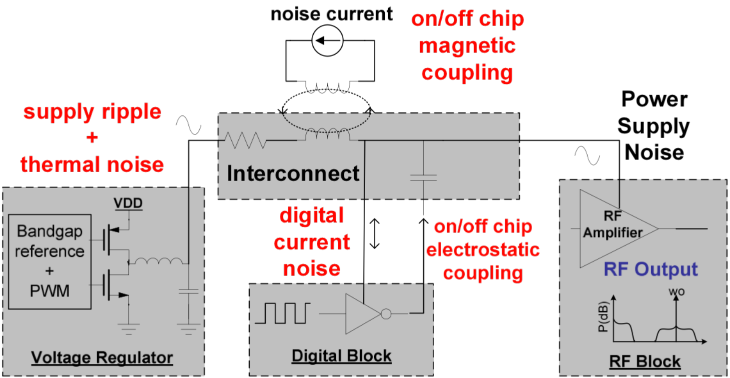
- Because 2nd order interaction is minimized by using a PMOS and NMOS in parallel, the capacitor C_{12} plays an important role.
- When second order distortion is generated at low frequencies, $f_1 - f_2$, the capacitor C_{12} has a high reactance and distortion cancellation does not take place.
- There is therefore a dependency to the two-tone spacing.

Power Supply Ripple



- In RF systems, the supply ripple can non-linearity transfer noise modulation on the supply to the output.
- This problem was analyzed by Jason Stauth: *Energy Efficient Wireless Transmitters: Polar and Direct-Digital Modulation Architectures*, Ph.D. Dissertation, U.C. Berkeley.

Supply Noise Sources



Multi-Port Memoryless Non-linearity

- The output voltage is a non-linear function of both the supply voltage and the input voltage. A two-variable Taylor series expansion can be used if the system is memory-less:

$$S_{out}(S_{in}, S_{vdd}) = a_{10} S_{in} + a_{20} S_{in}^2 + \mathrm{a}_{30} \mathrm{S}_{in}^3 + \cdots$$
$$+ a_{11} S_{in} S_{vdd} + a_{21} S_{in}^2 S_{vdd} + \cdots$$
$$+ a_{01} S_{vdd} + a_{02} S_{vdd}^2 + a_{03} S_{vdd}^3 + \cdots$$

Supply Noise Sideband

- Assume the input is at RF and the supply noise is a tone. Then the output signal will contain a noise sideband given by:

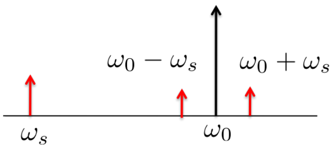
$$S_{in} = v_i \cos(\omega_0 t)$$

$$S_{vdd} = v_s \cos(\omega_s t)$$

$$v_{out}(\omega_0 \pm \omega_s) = \frac{1}{2} a_{11} \mathrm{v}_i v_s$$

$$\mathrm{Sideband(dBc)} = dB\left(\frac{2a_{10}}{a_{11}} \cdot \frac{1}{v_s}\right)$$

$$PSSR = dB\left(\frac{2a_{10}}{a_{11}}\right)$$



Multi-Port Volterra Series

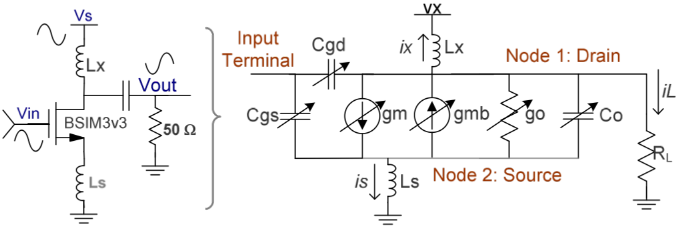
- Extending the concept of a Volterra Series to a two input-port system, we have

$$v_{out}(t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}(v_1(t), v_2(t))$$

$$F_{mn}(v_1(t), v_2(t)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{mn}(\tau_1, \cdots, \tau_{m+n})$$
$$v_1(t-\tau_1)\cdots v_1(\tau-\tau_m) v_2(t-\tau_{m+1})\cdots v_2(\tau-\tau_{m+n})$$
$$\times d\tau_1 \cdots d\tau_{m+n}$$

$$\begin{aligned} S_{out} = {} & A_{10}(j\omega_a)^\circ S_1 + A_{20}(j\omega_a, j\omega_b)^\circ S_1^2 + A_{30}(j\omega_a, j\omega_b, j\omega_c)^\circ S_1^3 + \cdots \\ & + A_{01}(j\omega_a)^\circ S_2 + A_{02}(j\omega_a, j\omega_b)^\circ S_2^2 + A_{03}(j\omega_a, j\omega_b, j\omega_c)^\circ S_2^3 + \cdots \\ & + A_{11}(j\omega_a, j\omega_b)^\circ S_1 S_2 + A_{21}(j\omega_a, j\omega_b, j\omega_c)^\circ S_1^2 S_2 + A_{12}(j\omega_a, j\omega_b, j\omega_c)^\circ S_1 S_2^2 + \cdots \end{aligned}$$

Example



$$
\begin{aligned}
i_d &= g_{m1} v_{gs} + g_{m2} v_{gs}^2 + g_{m3} v_{gs}^3 + \cdots \\
&\quad - g_{mb1} v_{sb} - g_{mb2} v_{sb}^2 - g_{mb3} v_{sb}^3 + \cdots \\
&\quad + g_{mo11} v_{ds} \cdot v_{gs} + g_{mo12} v_{ds} \cdot v_{gs}^2 + g_{mo21} v_{ds}^2 v_{gs} + \cdots \\
&\quad + C_1 \frac{d}{dt}(v_{db}) + \frac{C_2}{2}\frac{d}{dt}(v_{db}^2) + \frac{C_3}{3}\frac{d}{dt}(v_{db}^3) + \cdots
\end{aligned}
$$

- Several important terms:
  - g_m, and g_o non-linearity is usual transconductance and output resistance terms
  - g_{mo} is the interaction between the input/output
  - C_j is the output voltage non-linear capacitance

First Order Terms

$$y_1(j\omega) = go_1 + j\omega C$$
$$y_x(j\omega) = (j\omega L_C)^{-1}$$
$$y_s(j\omega) = (j\omega L_s)^{-1}$$

- First-order transfer function: (RF Node Transfer)

$$A^1_{10}(j\omega) = -y_s(j\omega)\frac{g_{m1}}{K_0(j\omega)}$$

$$K_0(j\omega) = (gm_1 + gmb_1 + y_1)(y_x + y_L) + y_s(y_x + y_1 + y_L)$$

- Supply Node Transfer: (superscripts are node numbers)

$$A^1_{01}(j\omega) = \frac{g_{m1}(y_x + y_L)}{K_0(j\omega)}$$

$$A^2_{10}(j\omega) = \frac{y_x(gm_1 + y_1 + gmb_1 + y_s)}{K_0(j\omega)}$$

$$A^2_{01}(j\omega) = \frac{y_x y_L}{K_0(j\omega)}$$

Mixing Product

- The most important term for now is the supply-noise mixing term:

$$v_{out}(\omega_0 \pm \omega_s) = A_{11}^1(j\omega_0, j\omega_s)^\circ[V_i(\omega_0), V_s(\omega_s)]$$

$$A_{11}^1(j\omega_a, j\omega_b) = y_s \frac{gmo_{11}K_1 + 2y_2K_2 + 2gm_2K_3 - 2gmb_2K_4}{K_0}$$

$$K_1(j\omega_a, j\omega_b) = A_{01}^2[1 + A_{10}^1(j\omega_a) - 2A_{10}^2(j\omega_a)] - A_{01}^1(j\omega_b)[1 - A_{10}^2(j\omega_a)]$$

$$K_2(j\omega_a, j\omega_b) = A_{01}^2(j\omega_B)[A_{10}^1(j\omega_a) - A_{10}^2(j\omega_a)] + A_{01}^1(j\omega_b)[A_{10}^2(j\omega_a) - A_{10}^1(j\omega_a)]$$

$$K_3(j\omega_a, j\omega_b) = A_{01}^2(j\omega_b)[1 - A_{10}^1(j\omega_a)]$$

$$K_4(j\omega_a, j\omega_b) = A_{10}^2(j\omega_a)A_{01}^2(j\omega_b)$$

PSSR Reduction

$$PSRR = dB[\frac{gm_1}{gmo_{11}K_1 + 2y_2K_2 + 2gm_2K_3 - 2gmb_2K_4}]$$

- Increase gm_1
- Reduce second order conductive non-linearity at drain (go_2)
- Reduce the non-linear junction capacitance at drain
- Reduce cross-coupling term by shielding the device drain from supply noise (cascode)

Output Conductance Non-Linearity

- For short-channel devices, due to DIBL, the output has a strong influence on the drain current. A complete description of the drain current is therefore a function of $f(v_{ds}, v_{gs})$.
- This is especially true if the device is run close to triode region (large swing or equivalently high output impedance):

$$\begin{aligned} i_{ds}(v_{gs}, v_{ds}) = g_{m1} v_{gs} + g_{ds1} v_{ds} + g_{m2} v_{gs}^2 + g_{ds2} v_{ds}^2 + \\ x_{11} v_{gs} v_{ds} + g_{m3} v_{gs}^3 + g_{ds3} v_{ds}^3 + \\ x_{12} v_{gs} v_{ds}^2 + x_{21} v_{gs}^2 v_{ds} + \cdots \end{aligned}$$

$$g_{mk} = \frac{1}{k!} \frac{\partial^k I_{ds}}{\partial V_{gs}^k}; g_{dsk} = \frac{1}{k!} \frac{\partial^k I_{ds}}{\partial V_{ds}^k}; x_{pq} = \frac{1}{p!q!} \frac{\partial^{p+q} I_{ds}}{\partial V_{gs}^p \partial V_{gs}^q}$$

Total Distortion

- Including the output conductance non-linearity modifies the distortion as follows

$$
\begin{aligned}
v_{ds} &= c_1 v_{gs} + c_2 v_{gs}^2 + c_3 v_{gs}^3 + \cdots \\
c_1 &= -g_{m1}(R_{CS}||g_{ds1}^{-1}) \\
c_2 &= -(g_{m2} + g_{ds2}c_1^2 + x_{11}c_1)\cdot(R_{CS}||g_{ds1}^{-1}) \\
c_3 &= -(g_{m3} + g_{ds3}c_1^3 + 2g_{ds2}c_1c_2 + x_{11}c_2 + x_{12}c_1^2 + x_{21}c_1)\cdot(R_{CS}||g
\end{aligned}
$$

- Source: S. C. Blaakmeer, E. A. M. Klumperink, D. M. W. Leenaerts,, B. Nauta, "Wideband Balun-LNA With Simultaneous Output Balancing, Noise-Canceling and Distortion-Canceling," *JSSC*, vol. 43, Jun. 2008.

Example *IIP* Simulation

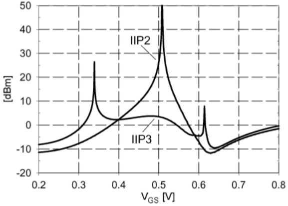


Fig. 6. Simulated IIP2 and IIP3 of a resistively loaded CS-stage.

$$\mathrm{IIP2_{dBm}} = 20 \cdot \log_{10}\left(\left|\frac{c_1}{c_2}\right|\right) + 10\ \mathrm{dB}$$

$$\mathrm{IIP3_{dBm}} = 20 \cdot \log_{10}\left(\sqrt{\left|\frac{4}{3}\frac{c_1}{c_3}\right|}\right) + 10\ \mathrm{dB}$$

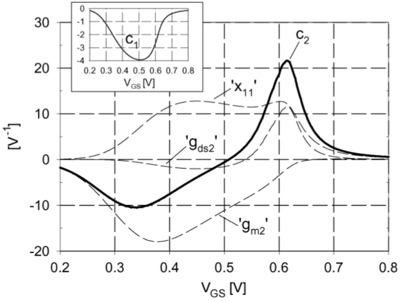


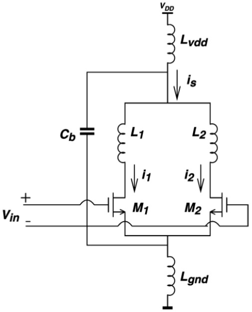
Fig. 5. Simulated second-order nonlinearity coefficient (c_2) and individual contributions due to the transistor coefficients (g_{m2}, g_{ds2} and x_{11}). Inset: linear gain coefficient (c_1) of the CS-stage.

- Contributions to c_2 are shown above.
- For low bias, g_{ds2} contributes very little but x_{11} and g_{m2} are significant. They also have opposite sign.

PA Power Supply Modulation

- When we apply a 1-tone to a class AB PA, the current drawn from the supply is constant.
- For when we apply 2-tones, there is a low-frequency component to the input:

$$V_{in} = A\sin(\omega_1 t) + A\sin(\omega_2 t)$$
$$= 2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$$
$$= 2A\cos(\omega_m t)\sin(\omega_c t)$$



- This causes a low frequency current to be drawn from the supply as well, even for a balanced circuit.
  - P. Haldi, D. Chowdhury, P. Reynaert, G. Liu, A. M Niknejad, "A 5.8 GHz 1 V Linear Power Amplifier Using a Novel On-Chip Transformer Power Combiner in Standard 90 nm CMOS," *IEEE Journal of Solid-State Circuits*, vol. 43, pp.1054-1063, May 2008.

Supply Current

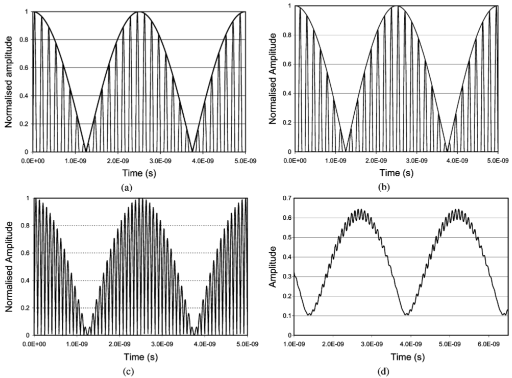


Fig. 9. (a) Drain current waveform of M1, operating in Class B, under two-tone excitation. (b) Drain current waveform of M2. (c) Sum of drain currents of M1 and M2. (d) Supply current after on-chip bypassing.

- The supply current is a full-wave rectified sine.

Fourier Components of Supply Current

- Substitute the Fourier series for the sine and cosine.
- Note that an on-chip bypass can usually absorb the higher frequencies ($2f_c$) but not the low frequency beat (f_s and harmonics)

$$i_s = k\left(\frac{2}{\pi} + \frac{4}{\pi}\frac{\cos(2\omega_m t)}{3} - \cdots\right) \times \left(\frac{2}{\pi} - \frac{4}{\pi}\frac{\cos(2\omega_c t)}{3} - \cdots\right)$$

$$= k\left(\cdots - \frac{8}{\pi^2}\frac{\cos(2\omega_c t)}{3} + \frac{8}{\pi^2}\frac{\cos(2\omega_m t)}{3} - \cdots\right)$$

$$= k\left(\cdots - \frac{8}{\pi^2}\frac{\cos(2\omega_c t)}{3} + \frac{8}{\pi^2}\frac{\cos(\omega_s t)}{3} - \cdots\right)$$

Supply Ripple Voltage

- The finite impedance of the supply means that the supply ripple has the following form.

$$V_{dd} = V_{DD} + A_2 \cdot \cos(\omega_s t) + \cdots$$

- Assuming a multi-port Volterra description for the transistor results in:

$$\begin{aligned} S_o = F_1(\omega_a)^\circ S_1 + F_2(\omega_a, \omega_b)^\circ S_1^2 + F_3(\omega_a, \omega_b, \omega_c)^\circ S_1^3 + \cdots \\ G_1(\omega_a)^\circ S_2 + G_2(\omega_a, \omega_b)^\circ S_2^2 + G_3(\omega_a, \omega_b, \omega_c)^\circ S_3^3 + \cdots \\ H_{11}(\omega_a, \omega_b)^\circ (S_1 \cdot S_2) + H_{12}(\omega_a, \omega_b, \omega_c)^\circ (S_1 \cdot S_2^2) + \\ H_{21}(\omega_a, \omega_b, \omega_c)^\circ (S_1^2 \cdot S_2) + \cdots \end{aligned}$$

$$S(\omega_1 \pm \omega_s) = H_{11}{}^\circ (S_1 \cdot S_2)$$

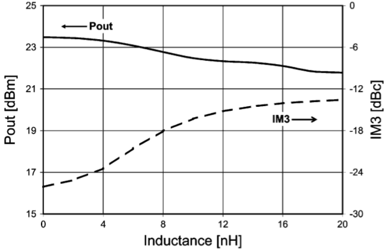


Fig. 14. Degradation in IM3 with increased supply inductance.

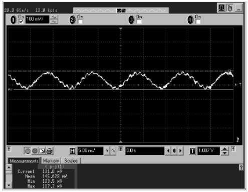
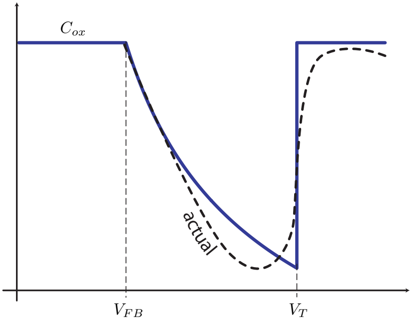


Fig. 15. Measured supply voltage ripple in a two-tone test with 100 MHz tone spacing.

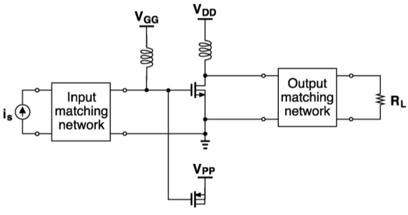
- Even though the PA is fully balanced, the supply inductance impacts the linearity.
- Measurements confirm the source of the IM_3 at low offsets arising from supply modulation.

MOS CV Non-Linearity

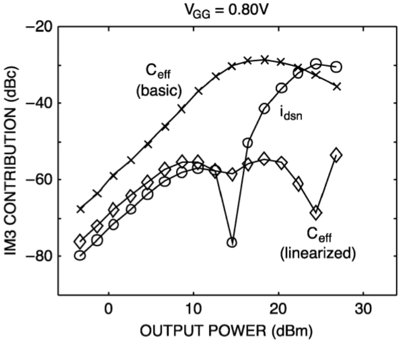


- C_{gs}, C_{μ} and C_{db} all contribute to the non-linearity.
- As expected, the contribution is frequency dependent and very much a strong function of the swing (drain, gate).
- Gate cap is particular non-linear around the threshold of the device.

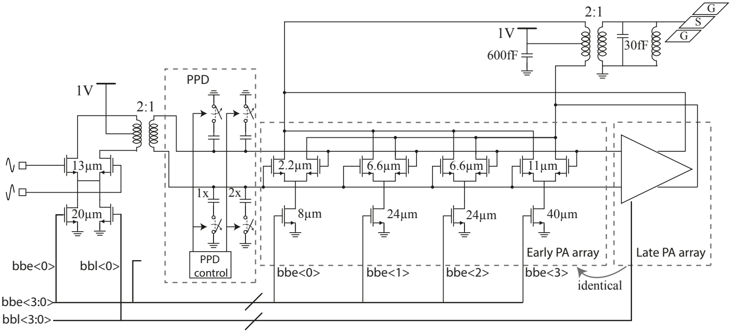
PMOS Compensation Technique



(b)



- Make an overall flat *C*-*V* curve by adding an appropriately sized PMOS device.
- Source: C. Wang, M. Vaidyanathan, L. Larson, "A Capacitance-Compensation Technique for Improved Linearity in CMOS Class-AB Power Ampliers," *JSSC*, vol. 39, Nov. 2004.



- "Digital" PA cell at 60 GHz uses non-binary weighted elements for RF-DAC to compensate for amplitude compression.
- Phase AM-to-PM distortion is compensated by switching in capacitance versus codeword.

- Source: Jiashu Chen, Lu Ye, D. Titz, F. Gianesello, R. Pilard, A. Cathelin, F. Ferrero, C. Luxey, A. Niknejad, "A digitally modulated mm-Wave cartesian beamforming transmitter with quadrature spatial combining," *IEEE International Solid-State Circuits Conference Digest of Technical Papers (ISSCC)*, 17-21 Feb. 2013, pp. 232-233.