CEE 271: Applied Mechanics II, Dynamics – Lecture 4: Ch.12, Sec.6–7 –

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Date: ____________________

MOTION OF A PROJECTILE (12.6)

Today's objectives: Students will be able to

1. Analyze the ***free-flight*** motion of a projectile.

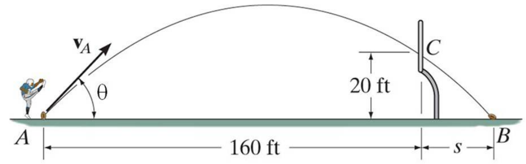
In-class activities:

- Reading Quiz
- Applications
- ***Kinematic*** Equations for
- ***Projectile*** Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

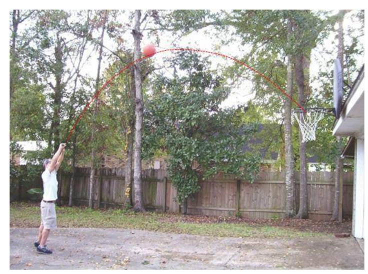
1. The downward acceleration of an object in free-flight motion is
   (a) zero.
   (b) increasing with time.
   (c) 9.81 m/s^2.
   (d) 9.81 ft/s^2.
   ANS: __
2. The horizontal component of velocity remains ________ during a free-flight motion.
   (a) zero
   (b) constant
   (c) at 9.81 m/s^2
   (d) at 32.2 ft/s^2
   ANS: __

APPLICATIONS



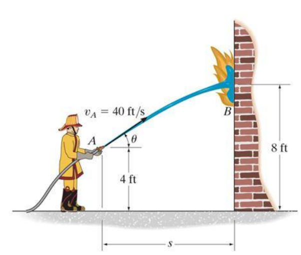
- A good kicker *instinctively* knows at what ***angle***, θ, and ***initial velocity***, v_A, he must kick the ball to make a field goal.
- For a given kick 'strength', at what angle should the ball be kicked to get the ***maximum distance***?

APPLICATIONS(continued)



- A basketball is shot at a certain angle. What ________ should the shooter consider in order for the basketball to pass through the basket?
- Distance, speed, the basket location, . . . anything else ?

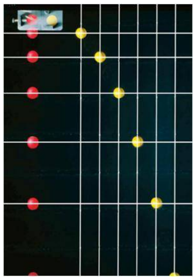
APPLICATIONS (continued)



- A firefighter needs to know the maximum height on the wall he/she can project water from the hose. What parameters would you program into a wrist computer to find the _______, that he/she should use to hold the hose?

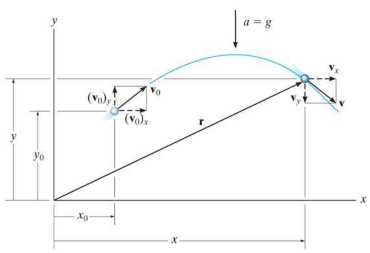
MOTION OF A PROJECTILE (Section 12.6)

Projectile motion can be treated as two rectilinear motions, one in the ________ direction experiencing ___ acceleration and the other in the ______ direction experiencing ______ acceleration (i.e., from _____).



For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture ***in this sequence*** is taken after the same time interval. Notice both balls are subjected to the ***same*** downward acceleration since they remain at the ***same*** elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is ***constant*** since the velocity in the horizontal direction is ***constant***.

KINEMATIC EQUATIONS: HORIZONTAL MOTION



- Since _____, the velocity in the horizontal direction remains ***constant*** ($v_x = v_{0x}$) and the position in the x-direction can be determined by:
- Why is $a_x = 0$ (assuming movement through the air)?

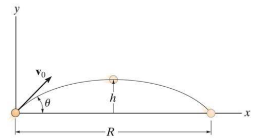
KINEMATIC EQUATIONS: VERTICAL MOTION

- Since the positive y-axis is directed *upward*, .
- Application of the ______ acceleration equations yields:

$$= v_{0y} - gt \quad (1)$$
$$= y_0 + (v_{0y})t - \tfrac{1}{2}gt^2 \quad (2)$$
$$= v_{0y}^2 - 2g(y - y_0) \quad (3)$$

- For any given problem, only two of these three equations can be used. Why?

EXAMPLE I



- Given: v_0 and θ
- Find: The equation that defines y as a function of x.
- Plan: *Eliminate* time from the kinematic equations.
- Solution : Using $v_x = v_0 \cos\theta$ and $v_y = v_0 \sin\theta$ ______
- We can write: $x = (v_0 \cos\theta)t$ or $t = x/(v_0 \cos\theta)$
- By substituting for t in:

$$y = (v_0 \sin\theta)t - \tfrac{1}{2}g(t)^2 \quad (4)$$
$$y = (v_0 \sin\theta) \times \frac{x}{v_0 \cos\theta} - \tfrac{1}{2}g \times \left(\frac{x}{v_0 \cos\theta}\right)^2 \quad (5)$$

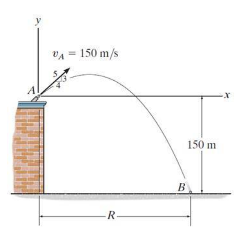
EXAMPLE I (continued)

- Simplifying the last equation, we get:

$$y =$$

- The above equation is called the ___________ which describes the path of a particle in projectile motion.
- The equation shows that the path is _______.

EXAMPLE II



- Given : Projectile is fired with $v_A = 150\,\text{m/s}$ at point A.
- Find: The horizontal distance it travels (R) and the time in the air.
- Plan:
  1. Establish a fixed x, y coordinate system (in this solution, the origin of the coordinate system is placed at A).
  2. Apply the ***kinematic*** relations in x- and y-directions

EXAMPLE II (continued)

- Solution:
  1. Place the coordinate system at point A. Then, write the equation for ***horizontal*** motion.

$$x_B = x_A + v_{Ax} t_{AB}$$

  where $x_B = R$, $x_A = 0$, $v_{Ax} = 150(\frac{4}{5})$ m/s. Range R will be $R = 120 t_{AB}$.
  2. Now write a ***vertical*** motion equation. Use the distance equation.

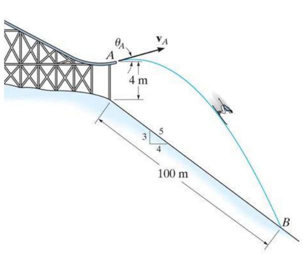
$$y_B = y_A + v_{Ay} t_{AB} - 0.5 g t_{AB}^2$$

  where ________, _____, and ______________. We get the following equation: $-150 = 90 t_{AB} + 0.5(-9.81) t_{AB}^2$
  3. Solving for t_{AB} first, __________. Then, $R = 120\, t_{AB} = 120\,(19.89) = 2387\,$m

CONCEPT QUIZ

1. In a projectile motion problem, what is the maximum number of unknowns that can be solved?
   (a) 1
   (b) 2
   (c) 3
   (d) 4
   ANS: __
2. The time of flight of a projectile, fired over level ground, with initial velocity v_0 at angle θ, is equal to?
   (a) $(v_0 \sin\theta)/g$
   (b) $(2v_0 \sin\theta)/g$
   (c) $(v_0 \cos\theta)/g$
   (d) $(2v_0 \cos\theta)/g$
   ANS: __

GROUP PROBLEM SOLVING



- Given: A skier leaves the ski jump ramp at $\theta_A = 25^o$ and hits the slope at B.
- Find: The skier's initial speed v_A.
- Plan: Establish a ___ x,y coordinate system (in this solution, the origin of the coordinate system is placed at A). Apply the _______ relations in x- and y-directions.

GROUP PROBLEM SOLVING (continued)

Solution:

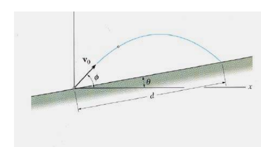
- Motion in x-direction:

$$x_B = x_A + v_{0x}(t_{AB}) \quad (6)$$
$$(4/5)100 = \quad (7)$$
$$t_{AB} = \frac{80}{v_A(\cos 25°)} = \quad (8)$$

- Motion in y-direction:

$$y_B = y_A + v_{0y}(t_{AB}) - \tfrac{1}{2} g (t_{AB})^2 \quad (9)$$
$$-64 = \quad (10)$$
$$v_A = \quad \text{m/s} \quad (11)$$

ATTENTION QUIZ



1. A projectile is given an initial velocity v_0 at an angle ϕ above the horizontal. The velocity of the projectile when it hits the slope is ________ the initial velocity v_0.
   (a) less than
   (b) equal to
   (c) greater than
   (d) None of the above.
   ANS: __
2. A particle has an initial velocity v_0 at angle θ with respect to the horizontal. The maximum height it can reach is when
   (a) $\theta = 30°$
   (b) $\theta = 45°$
   (c) $\theta = 60°$
   (d) $\theta = 90°$
   ANS: __

Note

CURVILINEAR MOTION: NORMAL AND TANGENTIAL COMPONENTS (12.7)

Today's objectives: Students will be able to

1. Determine the ***normal*** and ***tangential*** components of velocity and acceleration of a particle traveling along a ***curved*** path.

In-class activities:

- Reading Quiz
- Applications
- Normal and Tangential Components of Velocity and Acceleration
- Special Cases of Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

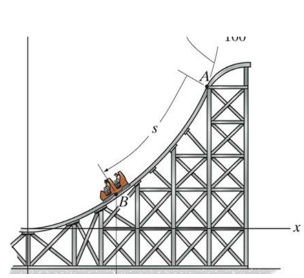
READING QUIZ

1. If a particle moves along a curve with a constant speed, then its tangential component of acceleration is
   (a) positive.
   (b) negative.
   (c) zero.
   (d) constant.
   ANS: __
2. The normal component of acceleration represents
   (a) the time rate of change in the magnitude of the velocity.
   (b) the time rate of change in the direction of the velocity.
   (c) magnitude of the velocity.
   (d) direction of the total acceleration.
   ANS: __

APPLICATIONS

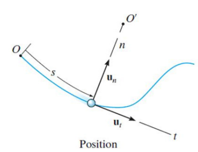
- Cars traveling along a clover-leaf interchange experience an ***acceleration*** due to a change in velocity as well as due to a change in _______ of the ***velocity***.
- If the car's speed is increasing at a known rate as it travels along a curve, how can we determine the ***magnitude*** and ***direction*** of its total acceleration?
- Why would you care about the ***total acceleration*** of the car?

APPLICATIONS(continued)



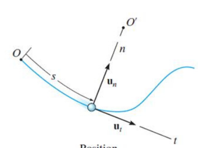
- A roller coaster travels down a hill for which the path can be approximated by a function
- The roller coaster starts from rest and increases its speed at a _______ rate.
- How can we determine its velocity and acceleration at the bottom?
- Why would we want to know these values?

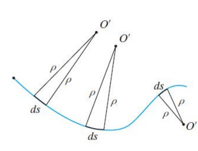
NORMAL AND TANGENTIAL COMPONENTS (Section 12.7)



- When a particle moves along a ______ path, it is sometimes convenient to describe its motion using coordinates ***other than*** Cartesian. When the path of motion is known, _________ and __________ coordinates are often used.
- In the $n - t$ coordinate system, the origin is located ***on the particle*** (the origin ***moves*** with the particle).
- The t-axis is ***tangent*** to the path (curve) at the instant considered, ***positive*** in the direction of the particle's ***motion***.
- The n-axis is ***perpendicular*** to the t-axis with the ***positive*** direction ***toward the center*** of curvature of the curve.

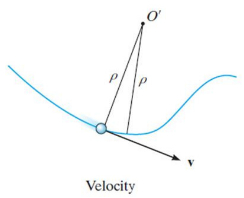
NORMAL AND TANGENTIAL COMPONENTS (continued)





- The positive n and t directions are defined by the unit vectors $\hat{u}_n$ and $\hat{u}_t$, respectively.
- The center of curvature, O', always lies on the ***concave*** side of the curve.
- The _______________, ρ, is defined as the ***perpendicular distance*** from the curve to the center of curvature at that point.
- The ***position*** of the particle at any instant is defined by the distance, s, along the curve from a fixed reference point.

VELOCITY IN THE $n - t$ COORDINATE SYSTEM

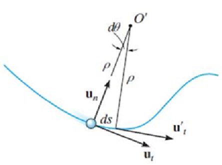


Velocity

- The velocity vector is always ______ to the path of motion (t-direction)..

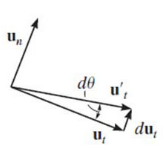
- The magnitude is determined by taking the ***time derivative*** of the path function, $s(t)$: $\boldsymbol{v} = v\hat{u}_t$ where $v = \dot{s} = ds/dt$
- Here v defines the ***magnitude*** of the velocity (speed) and $\hat{u}_t$ defines the ***direction*** of the velocity vector.

ACCELERATION IN THE $n - t$ COOR. SYSTEM



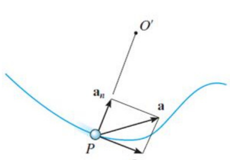
- Acceleration is the time rate of change of velocity:

- Here $\dot{v}$ represents the change in the ***magnitude*** of velocity and $\dot{\hat{u}}_t$ represents the rate of change in the ***direction*** of $\hat{u}_t$.
- After mathematical manipulation, the acceleration vector can be expressed as:



$$\boldsymbol{a} = \dot{v}\hat{u}_t + \quad \hat{u}_n = a_t\hat{u}_t + a_n\hat{u}_n$$

ACCELERATION IN THE $n - t$ COOR. SYSTEM (continued)



Acceleration

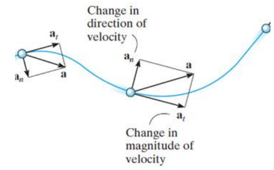
- So, there are ***two*** components to the acceleration vector: $a = a_t\hat{u}_t + a_n\hat{u}_n$
- The ***tangential*** component is tangent to the curve and in the direction of increasing or decreasing velocity.

  or

- The ***normal*** or ***centripetal*** component is always directed ______________ of curvature of the curve:
- The ***magnitude*** of the acceleration vector is

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$

SPECIAL CASES OF MOTION



There are some special cases of motion to consider.

1. The particle moves along a straight line:

   The ***tangential*** component represents the *time rate* of change in the ***magnitude*** of the velocity.
2. The particle moves along a ***curve*** at ***constant*** speed:

   The ***normal*** component represents the time rate of change in the ***direction*** of the velocity.

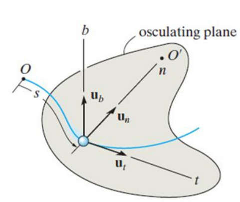
SPECIAL CASES OF MOTION (continued)

1. The tangential component of acceleration is constant, $a_t = (a_t)_c$. In this case,

$$s = s_o + v_o t + (\tfrac{1}{2})(a_t)_c t^2 \quad (12)$$
$$v = v_o + (a_t)_c t \quad (13)$$
$$v^2 = (v_o)^2 + 2(a_t)_c(s - s_o) \quad (14)$$

   As before, s_o and v_o are the ***initial position*** and ***velocity*** of the particle at $t = 0$. How are these equations related to ________ motion equations? Why?
2. The particle moves along a path expressed as $y = f(x)$. The ***radius of curvature, ρ***, at any point on the path can be calculated from

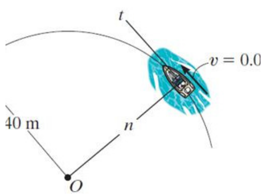
THREE-DIMENSIONAL MOTION



- If a particle moves along a ***space curve***, the n and t axes are defined as before. At any point, the t-axis is ***tangent*** to the path and the n-axis points toward the center of curvature. The plane containing the n and t axes is called the ***osculating*** plane.

- A third axis can be defined, called the ***binomial axis, b***. The ***binomial unit vector, $\hat{u}_b$***, is directed perpendicular to the ***osculating*** plane, and its sense is defined by the ***cross*** product $\hat{u}_b = \hat{u}_t \times \hat{u}_n$.
- There is __ motion, thus __ velocity or acceleration, in the binomial direction.

EXAMPLE



- Given: A boat travels around a circular path, ρ = 40 m, at a speed that increases with time, $v = (0.0625t^2)$ m/s.
- Find: The magnitudes of the boat's velocity and acceleration at the instant $t = 10$ s.
- Plan:
  The boat starts from rest ($v = 0$ when $t = 0$).
  1. Calculate the **speed** at $t = 10$ s using $v(t)$.
  2. Calculate the tangential and normal components of acceleration and then the magnitude of the acceleration vector.

EXAMPLE (continued)

Solution:

1. The velocity vector is $\boldsymbol{v} = v\hat{u}_t$, where the magnitude is given by $v = 0.0625t^2$ m/s.
   At t = 10s: $v = 0.0625t^2 = 0.0625(10)^2 =$ m/s
2. The acceleration vector is
   $\boldsymbol{a} = a_t\hat{u}_t + a_n\hat{u}_n =$.

a. Tangential component:
   $a_t = \dot{v} = d(.0625t^2)/dt =$ m/s^2
   At $t = 10$ s: $a_t = 0.125t = 0.125(10) = 1.25$ m/s^2

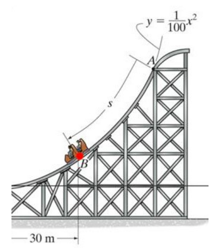
b. Normal component: $a_n =$ m/s^2
   At $t = 10$ s: $a_n = (6.25)^2/(40) = 0.9766$ m/s^2

c. The magnitude of the acceleration is
   $a = \left[(a_t)^2 + (a_n)^2\right]^{0.5} = [(1.25)^2 + (0.9766)^2]^{0.5} =$ m/s^2

CHECK YOUR UNDERSTANDING QUIZ

1. A particle traveling in a circular path of radius 300 m has an instantaneous velocity of 30 m/s and its velocity is increasing at a constant rate of 4 m/s^2. What is the magnitude of its total acceleration at this instant?
   (a) 3 m/s^2
   (b) 4 m/s^2
   (c) 5 m/s^2
   (d) -5 m/s^2
   ANS: __
2. If a particle moving in a circular path of radius 5 m has a velocity function $v = 4t^2$ m/s, what is the magnitude of its total acceleration at $t = 1$ s?
   (a) 8 m/s^2
   (b) 8.6 m/s^2
   (c) 3.2 m/s^2
   (d) 11.2 m/s^2
   ANS: __

GROUP PROBLEM SOLVING



- Given: A roller coaster travels along a vertical parabolic path defined by the equation $y = 0.01x^2$. At point B, it has a speed of 25 m/s, which is increasing at the rate of 3 m/s^2.
- Find: The magnitude of the roller coaster's acceleration when it is at point B.
- Plan:

1. The change in the speed of the car (3 m/s^2) is the tangential component of the total acceleration.
2. Calculate the radius of curvature of the path at B.
3. Calculate the normal component of acceleration.
4. Determine the magnitude of the acceleration vector.

GROUP PROBLEM SOLVING (continued)

Solution:

1. The tangential component of acceleration is the rate of increase of the roller coaster's speed, so $a_t = \dot{v} =$ m/s^2.
2. Determine the radius of curvature at point B (x = 30 m):
   $\frac{dy}{dx} = \frac{d}{dx}(0.01x^2) =$, $\frac{d^2y}{dx^2} = \frac{d}{dx}(0.02x) =$
   At $x = 30$ m, $\frac{dy}{dx} = 0.02(30) = 0.6$, $\frac{d^2y}{dx^2} = 0.02$
   $\rho =$ $= \frac{\left[1+(0.6)^2\right]^{3/2}}{|0.02|} =$ m
3. The normal component of acceleration is
   $a_n = v^2/\rho = (25)^2/(79.3) =$ m/s^2
4. The magnitude of the acceleration vector is
   $a = [(a_t)^2 + (a_n)^2]^{0.5} = [(3)^2 + (7.881)^2]^{0.5} =$ m/s^2

ATTENTION QUIZ

1. The magnitude of the normal acceleration is
   (a) proportional to radius of curvature.
   (b) inversely proportional to radius of curvature.
   (c) sometimes negative.
   (d) zero when velocity is constant.
   ANS: __
2. The directions of the tangential acceleration and velocity are always
   (a) perpendicular to each other.
   (b) collinear.
   (c) in the same direction.
   (d) in opposite directions.
   ANS: __