## Chapter 3

## Moving Reference Frame Kinematics Homework

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## Homework H.3.A

Given: The disk shown is rotating about its center with a constant rotation rate of $\Omega$. Four slots have been cut into the disk, and a particle in each slot has a constant speed of $v_{\text {rel }}$ relative to the disk. Each particle is at a distance of $R$ from the center of the disk.

Find: Determine the velocity and acceleration of each particle in the disk. Write your answers as vectors.


## Homework H.3.B

Given: An L-shaped telescoping arm is pinned to ground at point O. End A of the arm is constrained to move within a stationary circular slot with a constant speed of $v_{A}$. At the instant shown, the arm is oriented horizontally with end A located immediately to the left of the center C of the circular slot. A set of $x y z$ axes are attached to section OB of the telescoping arm, with an observer also attached to this section of the arm.

Find: Determine the angular velocity and angular acceleration of the arm. Write your answers as vectors.


## Homework H.3.C

Given: Arm AD is made from a quarter circular arc bar of radius $R$ and is pinned to fixed ground at end A. Slotted arm BE is pinned to fixed ground at end B with pin B located directly below pin A, as shown. A pin at end D of the curved arm is allowed to slide within the slot of arm BE. At the position shown, arm BE is horizontal, and arm AD is rotating CW with a constant rate of $\omega_{A D}$.

Find: For this position,
(a) Determine the angular velocity of arm BE and the value of $\dot{d}$.
(b) Determine the angular acceleration of arm BE and the value of $\ddot{d}$.

HINT: Use an observer attached to the slotted arm BE, and relate the kinematics of points B and D through the moving reference frame kinematics equations.


## Homework H.3.D

Given: A guide rod is able to control the angular position of the L-shaped arm BOE through its end A moving vertically and through the slot cut into section OE of the arm. In turn, arm BOE controls the horizontal position of slider C through end B being constrained to move within the slot in slider C. For a particular task of this mechanism, end A of the guide rod is moving upward with a constant speed $v_{A}$. Our goal is to determine the velocity and acceleration of slider C.

Find: For this problem:
(a) Determine the angular velocity and angular acceleration of arm BOE.
(b) Determine the velocity and acceleration of pin B on BOE.
(c) Determine the velocity and acceleration of slider C.

HINT: Consider using an observer attached to the slotted arm BOE.


Use the following parameters in your analysis: $v_{A}=20 \mathrm{ft} / \mathrm{s}, \theta=30^{\circ}, L=2 \mathrm{ft}$ and $d=1.5 \mathrm{ft}$.

## Homework H.3.E

Given: A shaft rotates about a fixed vertical axis at a constant rate of $\Omega$, as shown below. A straight bar OA, having a length of $L$, is pinned to point O on the shaft, with O being on the rotation axis of the shaft. At the instant when $\theta=0^{\circ}$, bar OA is being raised at a rate of $\dot{\theta}$ from the horizontal plane, with this rate changing at a rate of $\ddot{\theta}$. A set of $x y z$ coordinate axes is attached to bar OA with its origin at O . A second set of coordinate axes, $X Y Z$, are fixed to ground. At the instant when $\theta=0^{\circ}$, the $x y z$ and $X Y Z$ axes are aligned with each other.

Find: For the instant when $\theta=0^{\circ}$, determine the angular velocity and angular acceleration of bar OA.


Use the following parameters in your analysis: $\Omega=5 \mathrm{rad} / \mathrm{s}, \dot{\theta}=4 \mathrm{rad} / \mathrm{s}, \ddot{\theta}=-3 \mathrm{rad} / \mathrm{s}^{2}$ and $L=2$ m.

## Homework H.3.F

Given: A shaft is rotating at a constant rate of $\Omega$ about a fixed axis. A disk of radius $R$ is able to roll without slipping in a slot that is cut longitudinally into the shaft. The position of the disk's center A is controlled by an hydraulic cylinder that is extending at a constant rate of $\dot{d}$. Consider a set of coordinate axes $x y z$ that are attached to the disk, and a set of coordinate axes $X Y Z$ that are fixed in space.

Find: For this problem,
(a) determine the angular velocity and angular acceleration of the disk.
(b) determine the acceleration of point $B$ on the perimeter of the disk at a time when $B$ is immediately to the right of A , as shown in the figure.


## Homework H.3.G

Given: The yoke shown below rotates about a fixed axis with a constant rate of $\omega_{0}$. A disk, of radius $r$, rotates about its center C at a constant rate of $\omega_{\text {disk }}$ relative to yoke. The XYZ coordinate system is fixed with the $X$-axis aligned with the fixed rotation axis of the yoke. The $x y z$ coordinate system is attached to the disk with the $z$-axis aligned with the rotation axis of the disk for all time. For the position shown below, the $x y z$ axes are aligned with the $X Y Z$ axes.

Find: For the position shown:
(a) Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors in terms of their $x y z$ components.
(b) Determine the acceleration of point $B$ of the disk. Write your answer as a vector in terms of its $x y z$ components.

front view

Use the following parameters in your analysis: $\omega_{0}=3 \mathrm{rad} / \mathrm{s}, \omega_{\text {disk }}=2 \mathrm{rad} / \mathrm{s}, d=0.5 \mathrm{~m}, b=1.5$ m and $r=0.25 \mathrm{~m}$.

## Homework H.3.H

Given: A caster wheel is supported by an L-shaped bracket. The bracket is rotating about a fixed vertical axis with a constant rate of $\omega_{1}$. The wheel rotates with respect to the bracket with a constant rate of $\omega_{2}$.

Find: For this problem, determine:

1. The angular velocity and angular acceleration of the wheel. Write your answers as vectors.
2. The acceleration of point A on the wheel at the instant shown when A is immediately to the right of the center O of the wheel.
3. The acceleration of point B on the wheel at the instant shown when B is immediately above the center O of the wheel.


Use the following parameters in your analysis: $\omega_{1}=2 \mathrm{rad} / \mathrm{s}, \omega_{2}=5 \mathrm{rad} / \mathrm{s}, r=200 \mathrm{~mm}$ and $d=$ 100 mm .

## Homework H.3.I

Given: Particle P travels in a tube with $\dot{R}=$ constant. The tube is being raised at a constant rate of $\dot{\theta}$. In addition, the tube is attached to a vertical shaft which is rotating about the fixed $Y$ axis with a constant rate of $\omega$. An observer is attached to the tube with the $x y z$ axes also attached to the tube with its origin at point O .

Find: For the position shown, determine:
(a) The angular velocity vector of the observer.
(b) The angular acceleration vector of the observer.
(c) The velocity of point P as seen by the observer.
(d) The acceleration of point P as seen by the observer.
(e) The acceleration of point P using the above results.


Use the following parameters in your analysis: $R=5 \mathrm{ft}, \theta=36.87^{\circ}, \dot{R}=6 \mathrm{ft} / \mathrm{s}, \dot{\theta}=3 \mathrm{rad} / \mathrm{s}$ and $\omega=4 \mathrm{rad} / \mathrm{s}$.

## Homework H.3.J

Given: A shaft is rotating about the fixed $X$-axis at a constant rate of $\Omega$. A square plate is pinned at its center O to the centerline of the shaft and is rotating relative to the shaft about O at a constant rate of $\dot{\theta}$. A set of $x y z$ axes are attached to the plate with its origin at O . An insect on the plate is walking along the $y$-axis with a constant speed of $v_{r e l}$ relative to the plate.

Find: Determine the velocity and acceleration of the insect when the insect has reached the edge of the plate. The insect reaches the edge of the plate when $\theta=0^{\circ}$.


Use the following parameters in your analysis: $b=6 \mathrm{in}$, $v_{r e l}=12 \mathrm{in} / \mathrm{s}, \Omega=3 \mathrm{rad} / \mathrm{s}$ and $\dot{\theta}=5$ rad/s.

