# Chapter 3

# Moving Reference Frame Kinematics Homework

# Homework H.3.A

**Given:** The disk shown is rotating about its center with a constant rotation rate of  $\Omega$ . Four slots have been cut into the disk, and a particle in each slot has a constant speed of  $v_{rel}$  relative to the disk. Each particle is at a distance of R from the center of the disk.

**Find:** Determine the velocity and acceleration of each particle in the disk. Write your answers as vectors.



## Homework H.3.B

**Given:** An L-shaped telescoping arm is pinned to ground at point O. End A of the arm is constrained to move within a stationary circular slot with a constant speed of  $v_A$ . At the instant shown, the arm is oriented horizontally with end A located immediately to the left of the center C of the circular slot. A set of xyz axes are attached to section OB of the telescoping arm, with an observer also attached to this section of the arm.

**Find:** Determine the angular velocity and angular acceleration of the arm. Write your answers as vectors.



# Homework H.3.C

**Given:** Arm AD is made from a quarter circular arc bar of radius R and is pinned to fixed ground at end A. Slotted arm BE is pinned to fixed ground at end B with pin B located directly below pin A, as shown. A pin at end D of the curved arm is allowed to slide within the slot of arm BE. At the position shown, arm BE is horizontal, and arm AD is rotating CW with a constant rate of  $\omega_{AD}$ .

Find: For this position,

- (a) Determine the angular velocity of arm BE and the value of  $\dot{d}$ .
- (b) Determine the angular acceleration of arm BE and the value of  $\ddot{d}$ .

HINT: Use an observer attached to the slotted arm BE, and relate the kinematics of points B and D through the moving reference frame kinematics equations.



#### Homework H.3.D

**Given:** A guide rod is able to control the angular position of the L-shaped arm BOE through its end A moving vertically and through the slot cut into section OE of the arm. In turn, arm BOE controls the horizontal position of slider C through end B being constrained to move within the slot in slider C. For a particular task of this mechanism, end A of the guide rod is moving upward with a constant speed  $v_A$ . Our goal is to determine the velocity and acceleration of slider C.

**Find:** For this problem:

- (a) Determine the angular velocity and angular acceleration of arm BOE.
- (b) Determine the velocity and acceleration of pin B on BOE.
- (c) Determine the velocity and acceleration of slider C.

HINT: Consider using an observer attached to the slotted arm BOE.



Use the following parameters in your analysis:  $v_A = 20$  ft/s,  $\theta = 30^{\circ}$ , L = 2 ft and d = 1.5 ft.

#### Homework H.3.E

**Given:** A shaft rotates about a fixed vertical axis at a constant rate of  $\Omega$ , as shown below. A straight bar OA, having a length of L, is pinned to point O on the shaft, with O being on the rotation axis of the shaft. At the instant when  $\theta = 0^{\circ}$ , bar OA is being raised at a rate of  $\dot{\theta}$  from the horizontal plane, with this rate changing at a rate of  $\ddot{\theta}$ . A set of xyz coordinate axes is attached to bar OA with its origin at O. A second set of coordinate axes, XYZ, are fixed to ground. At the instant when  $\theta = 0^{\circ}$ , the xyz and XYZ axes are aligned with each other.

Find: For the instant when  $\theta = 0^{\circ}$ , determine the angular velocity and angular acceleration of bar OA.



Use the following parameters in your analysis:  $\Omega = 5 \text{ rad/s}$ ,  $\dot{\theta} = 4 \text{ rad/s}$ ,  $\ddot{\theta} = -3 \text{ rad/s}^2$  and L = 2 m.

# Homework H.3.F

**Given:** A shaft is rotating at a constant rate of  $\Omega$  about a fixed axis. A disk of radius R is able to roll without slipping in a slot that is cut longitudinally into the shaft. The position of the disk's center A is controlled by an hydraulic cylinder that is extending at a constant rate of  $\dot{d}$ . Consider a set of coordinate axes xyz that are attached to the disk, and a set of coordinate axes XYZ that are fixed in space.

Find: For this problem,

- (a) determine the angular velocity and angular acceleration of the disk.
- (b) determine the acceleration of point B on the perimeter of the disk at a time when B is immediately to the right of A, as shown in the figure.



## Homework H.3.G

**Given:** The yoke shown below rotates about a fixed axis with a constant rate of  $\omega_0$ . A disk, of radius r, rotates about its center C at a constant rate of  $\omega_{disk}$  relative to yoke. The XYZ coordinate system is fixed with the X-axis aligned with the fixed rotation axis of the yoke. The xyz coordinate system is attached to the disk with the z-axis aligned with the rotation axis of the disk for all time. For the position shown below, the xyz axes are aligned with the XYZ axes.

Find: For the position shown:

- (a) Determine the angular velocity and angular acceleration of the disk. Write your answers as vectors in terms of their xyz components.
- (b) Determine the acceleration of point B of the disk. Write your answer as a vector in terms of its xyz components.



Use the following parameters in your analysis:  $\omega_0 = 3 \text{ rad/s}$ ,  $\omega_{disk} = 2 \text{ rad/s}$ , d = 0.5 m, b = 1.5 m and r = 0.25 m.

#### Homework H.3.H

**Given:** A caster wheel is supported by an L-shaped bracket. The bracket is rotating about a fixed vertical axis with a constant rate of  $\omega_1$ . The wheel rotates with respect to the bracket with a constant rate of  $\omega_2$ .

Find: For this problem, determine:

- 1. The angular velocity and angular acceleration of the wheel. Write your answers as vectors.
- 2. The acceleration of point A on the wheel at the instant shown when A is immediately to the right of the center O of the wheel.
- 3. The acceleration of point B on the wheel at the instant shown when B is immediately above the center O of the wheel.



Use the following parameters in your analysis:  $\omega_1 = 2 \text{ rad/s}$ ,  $\omega_2 = 5 \text{ rad/s}$ , r = 200 mm and d = 100 mm.

#### Homework H.3.I

**Given:** Particle P travels in a tube with  $\dot{R} = constant$ . The tube is being raised at a constant rate of  $\dot{\theta}$ . In addition, the tube is attached to a vertical shaft which is rotating about the fixed Y axis with a constant rate of  $\omega$ . An observer is attached to the tube with the xyz axes also attached to the tube with its origin at point O.

Find: For the position shown, determine:

- (a) The angular velocity vector of the observer.
- (b) The angular acceleration vector of the observer.
- (c) The velocity of point P as seen by the observer.
- (d) The acceleration of point P as seen by the observer.
- (e) The acceleration of point P using the above results.



Use the following parameters in your analysis: R = 5 ft,  $\theta = 36.87^{\circ}$ ,  $\dot{R} = 6$  ft/s,  $\dot{\theta} = 3$  rad/s and  $\omega = 4$  rad/s.

#### Homework H.3.J

**Given:** A shaft is rotating about the fixed X-axis at a constant rate of  $\Omega$ . A square plate is pinned at its center O to the centerline of the shaft and is rotating relative to the shaft about O at a constant rate of  $\dot{\theta}$ . A set of xyz axes are attached to the plate with its origin at O. An insect on the plate is walking along the y-axis with a constant speed of  $v_{rel}$  relative to the plate.

Find: Determine the velocity and acceleration of the insect when the insect has reached the edge of the plate. The insect reaches the edge of the plate when  $\theta = 0^{\circ}$ .



Use the following parameters in your analysis: b = 6 in,  $v_{rel} = 12$  in/s,  $\Omega = 3$  rad/s and  $\dot{\theta} = 5$  rad/s.