Mrs. Ambre's Math Notebook

Almost everything you need to know for 7th grade math

Plus a little about 6th grade math

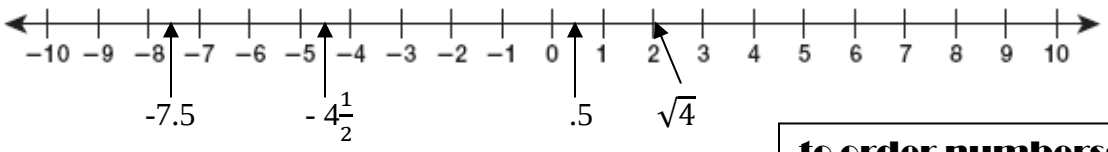
And a little about 8th grade math

Table of Contents by Outcome

My Multiplication Chart 1-12

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Place Value and Number Lines



Millions	Hundred Thousands	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

to order numbers:
Line up the decimals then compare the number in each place value (if a number has no decimal it is at the end) If the numbers are fractions, change them to decimals! (see fraction notes)

To Round:

- Find the place value you're supposed to round to and underline the digit.
- Copy all the numbers to the left of the underlined digit (those don't change)
- Look to the right of the underlined digit. If it's 5 or higher, add one to the underlined digit, if not, keep the underlined digit the same.
- Write zeroes for all the remaining digits to the right of the underlined digit.
- Your rounded number should have the same number of digits that you started with!

Ex: Round **568.24** to the nearest tenth

568.24

568.

4 < 5 so it stays the same 568.2

568.20

Divisibility Rules

A number is divisible by:
10 – if it ends in 0
5 – if it ends in 0 or 5
2 – if it ends in 0,2,4,6,8 (even numbers)
3 – if all the digits are added and their total is a multiple of 3 (3,6,9,12,15,18…)

Prime Numbers are not divisible by anything (except 1 & itself) Ex:2, 3, 5, 7, 11, 13, 17, 19…
Composite Numbers are composed of other numbers and have many Factors

Order of Operations
Please Excuse My Dear Aunt Sally

P – Parenthesis ( )
E – Exponents ^{3}
M/D – Multiply or Divide **Left to Right**
A/S – Subtract or Add **Left to Right**

Prime Factors: Numbers 1 through 100.

2:	2	36:	2x2x3x3	70:	2x5x7
3:	3	37:	37	71:	71
4:	2x2	38:	2x19	72:	2x2x2x3x3
5:	5	39:	3x13	73:	73
6:	2x3	40:	2x2x2x5	74:	2x37
7:	7	41:	41	75:	3x5x5
8:	2x2x2	42:	2x3x7	76:	2x2x19
9:	3x3	43:	43	77:	7x11
10:	2x5	44:	2x2x11	78:	2x3x13
11:	11	45:	3x3x5	79:	79
12:	2x2x3	46:	2x23	80:	2x2x2x2x5
13:	13	47:	47	81:	3x3x3x3
14:	2x7	48:	2x2x2x2x3	82:	2x41
15:	3x5	49:	7x7	83:	83
16:	2x2x2x2	50:	2x5x5	84:	2x2x3x7
17:	17	51:	3x17	85:	5x17
18:	2x3x3	52:	2x2x13	86:	2x43
19:	19	53:	53	87:	3x29
20:	2x2x5	54:	2x3x3x3	88:	2x2x2x11
21:	3x7	55:	5x11	89:	89
22:	2x11	56:	2x2x2x7	90:	2x3x3x5
23:	23	57:	3x19	91:	7x13
24:	2x2x2x3	58:	2x29	92:	2x2x23
25:	5x5	59:	59	93:	3x31
26:	2x13	60:	2x2x3x5	94:	2x47
27:	3x3x3	61:	61	95:	5x19
28:	2x2x7	62:	2x31	96:	2x2x2x2x2x3
29:	29	63:	3x3x7	97:	97
30:	2x3x5	64:	2x2x2x2x2x2	98:	2x7x7
31:	31	65:	5x13	99:	3x3x11
32:	2x2x2x2x2	66:	2x3x11	100:	2x2x5x5
33:	3x11	67:	67		
34:	2x17	68:	2x2x17		
35:	5x7	69:	3x23		

Properties & Identities

Commutative Property		
Words	**Numbers**	**Algebra**
You can add numbers in any order and multiply numbers in any order.	$3 + 8 = 8 + 3$ $5 \cdot 7 = 7 \cdot 5$	$a + b = b + a$ $ab = ba$

The signs are all the same – the problem was just written with a different order

Associative Property		
Words	**Numbers**	**Algebra**
When you add or multiply, you can group the numbers together in any combination.	$(4 + 5) + 1 = 4 + (5 + 1)$ $(9 \cdot 2) \cdot 6 = 9 \cdot (2 \cdot 6)$	$(a + b) + c = a + (b + c)$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

The original signs are all multiply or all add. The parenthesis just moved

Distributive Property		
Numbers	$6 \cdot (9 + 14) = 6 \cdot 9 + 6 \cdot 14$	$8 \cdot (5 - 2) = 8 \cdot 5 - 8 \cdot 2$
Algebra	$a \cdot (b + c) = ab + ac$	$a \cdot (b - c) = ab - ac$

The problem is totally rewritten. A number is written more than once on the other side

Zero Property		
Words	**Numbers**	**Algebra**
The product of 0 and any number is 0.	$\mathbf{4 \cdot 0 = 0}$	$\mathbf{a \cdot 0 = 0}$

Multiply by zero

Identity Property		
Words	**Numbers**	**Algebra**
The sum of 0 and any number is the number. The product of 1 and any number is the number.	$4 + 0 = 4$ $8 \cdot 1 = 8$	$a + 0 = a$ $a \cdot 1 = a$

Tables & Plots

Frequency Table

- Tells the number of times something occurs

Favorite Food	Tally	Frequency
Taco	𝍸 ||	7
Burger	𝍸 ||||	9

Stem & Leaf Plot

Adding zero or multiplying by 1

Number of Card Tower Levels

Stems	Leaves
1	2 3 4 5 5 7 8 9
2	0 1 3 3 3 5
3	1 5
4	2 4
5	0 1

Key: 1\|5 means 15

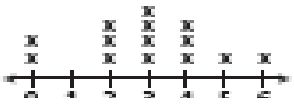
Arranges data by place value. The stems represent values of 10. The leaves are ones. Don't forget

Line Plot

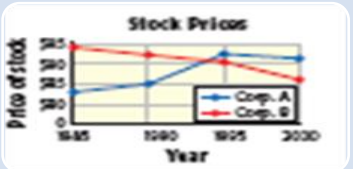
Uses a number line & X's or other symbols to show the frequencies of values.

Number of Items Purchased

4 1 5 4 3 3 1 2 4 3 5



Graphs



Bar Graph

To display countable data grouped in categories. Bars can't touch

Line Graph

To show change over time

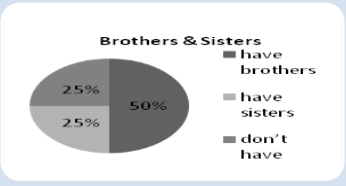
Plot each dot then connect the dots

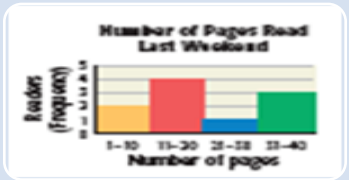
Double Line Graph

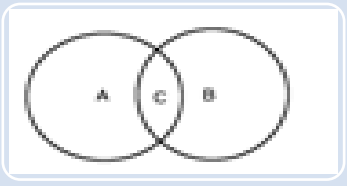
Use double lines or bars to compare similar data

All graphs must include:

*A title *A scale that starts at 0 (use the lightning bolt symbol to jump up)
*A label for the y axis (left side) what do those numbers stand for?
*A label for the x axis (bottom) & labels for each point or bar







Pie Chart

To compare parts to a whole

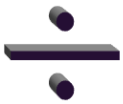
A complete circle is 100%

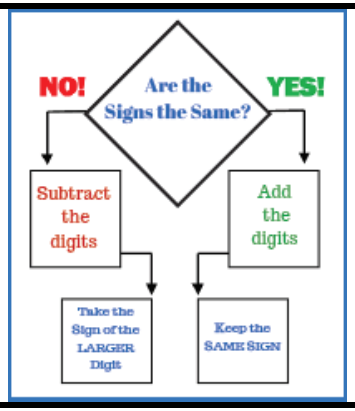
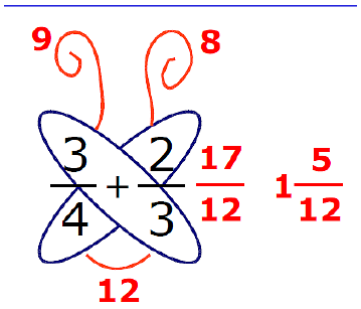
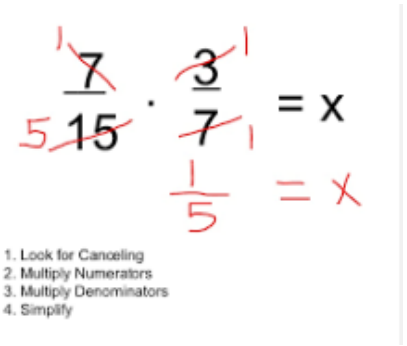
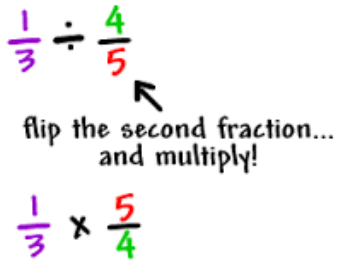
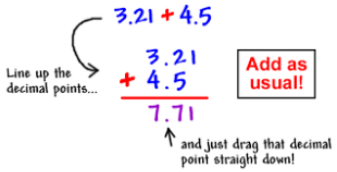
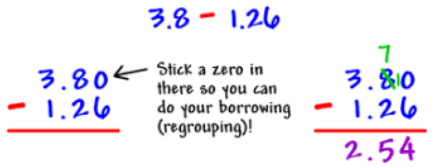
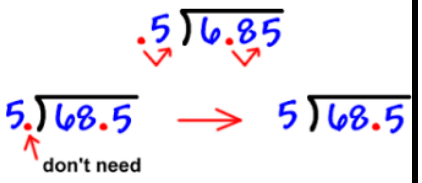
Histogram

A bar graph that shows frequency of occurance of each interval. The bars touch each other and the y axis.

Venn Diagram

Used to compare two sets of data. A is one thing. B is another. C is what they have in common.

Word Problem Clues	Expression	Word Phrases
Put parts together • What is the **sum**? • How many **in all**? • How many / how much **altogether**? • What is the **total**? • If its repeated addition it would be better to multiply	n + 5 	• **Add** 5 to a number • **Sum** of a number and 5 • 5 **more than** a number • The **total** of 5 and a number • A number **increased by** 5 • A number **plus** 5
How much more or less • How many **more**? **Less**? • Find the **difference**. • **Compare** two numbers: • How much bigg**er**? • How much tall**er**? • Heavi**er**? Old**er**?	n-11 	• **Subtract** 11 from a number • **Difference** of a number and 11 • 11 **less than** a number • A number **decreased by** 11 • A number **minus** 11 • **Take away** 11 • a number **less** 11
Put equal parts together • How many **in all?** • What is 12% **of** 3? • This much **per**…how many / how much **altogether?** • Find the **product.** • **Each** one is… total is? • He did it 3 **times**… total is?	3n 	• 3 **multiplied** by a number • **Product** of 3 and a number • a number **times** 3
Separate into equal parts • Find the **quotient.** • What would **one unit be**? • If **shared?** If **divided?** • How much **Each? Per?** • Find the **average / mean** (add numbers then divide by how many numbers were given)	$\frac{a}{7}$ or a ÷ 7 	• 7 **divided** by a number • **Quotient** of a number and 7 • 7 **divided into** a number

	Add	Subtract	Multiply	Divide
Integers		- 3 – 2 ADD the OPPOSITE –3 + (-2) = -5	⊕ x ⊕ = ⊕ ⊖ x ⊖ = ⊕ ⊕ x ⊖ = ⊖ ⊖ x ⊕ = ⊖	⊕ ÷ ⊕ = ⊕ ⊖ ÷ ⊖ = ⊕ ⊕ ÷ ⊖ = ⊖ ⊖ ÷ ⊕ = ⊖
Fractions		Use the Butterfly Method OR Find Common Denominators		
Decimals				

How do I do it?

Changing Fractions to Decimals

Example: $\frac{3}{4} \rightarrow 3 \div 4$

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ -28\downarrow \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

$$\frac{3}{4} = 0.75$$

Change Decimals to Fractions

Example:1

Write the decimal 0.35 as a fraction

$0.35 = \frac{35}{100}$ Express in lowest terms

Example2 Write 0.7 as a fraction

$0.7 = \frac{7}{10}$

Mixed Numbers to Improper Fractions

Then add.

$4\frac{1}{3} = \frac{13}{3}$

Multiply.

Improper Fractions to Mixed Numbers

improper fraction **into** mixed number

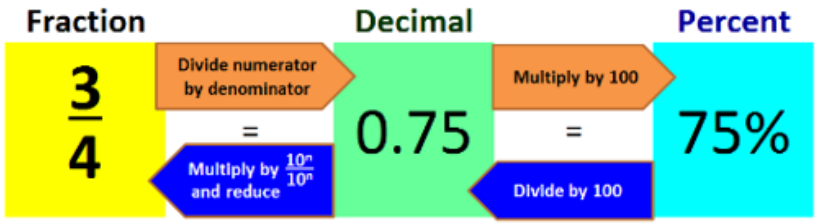
$\frac{25}{7}$ $\quad 7\overline{)25}$ $\quad 3\frac{4}{7}$

-21

4

keep the same denominator

Fractions Decimals and Percents



Benchmark Percents

$^{1}/_{20}$	.05
$^{1}/_{10}$	.1 or .10
$^{1}/_{8}$	.125
$^{1}/_{5}$	.2 or .20
$^{1}/_{4}$	.25
$^{1}/_{3}$	.333...
$^{1}/_{2}$	.5 or .50

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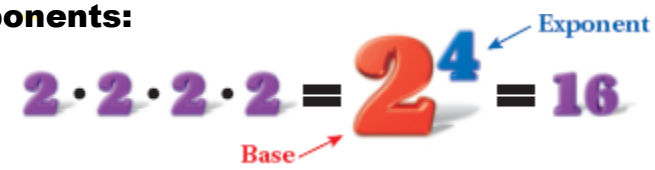
EXPONENTS & SCIENTIFIC NOTATION

Exponents:



Scientific Notation:

4.123×10^5

The first part is a number that is greater than 1 and less than 10.

The second part is a power of 10.

Write 8,296,000 in scientific notation.

8,296,000

8,296,000 — *Move the decimal point 6 places left. The power of 10 is 6.*

$8{,}296{,}000 = 8.296 \times 10^6$

Write 3.2×10^7 in standard form.

3.2×10^7 — *The power of 10 is 7.*

3.2000000 — *Move the decimal point 7 places right. Use zeros as placeholders.*

$3.2 \times 10^7 = 32{,}000{,}000$

1.35×10^{-4}

1.35×10^{-4} — $10^{-4} = \frac{1}{10{,}000}$

$1.35 \times \frac{1}{10{,}000}$ — *Divide by the reciprocal.*

$1.35 \div 10{,}000$ — *Think: Move the decimal left 4 places.*

0.000135

Scientific notation using a calculator: Enter the given number. Press the 2nd key then the DRG key. Use the arrows to move to SCI. Hit Enter twice. (To clear memory press the on and clear button at the same)

Standard notation using a calculator: Enter the given number in scientific notation. 3.2 x10 ^7 then hit Enter. The number will appear in standard notation.

Positive Exponent – move to the Right
Negative Exponent – move to the Left

The Ten Trick

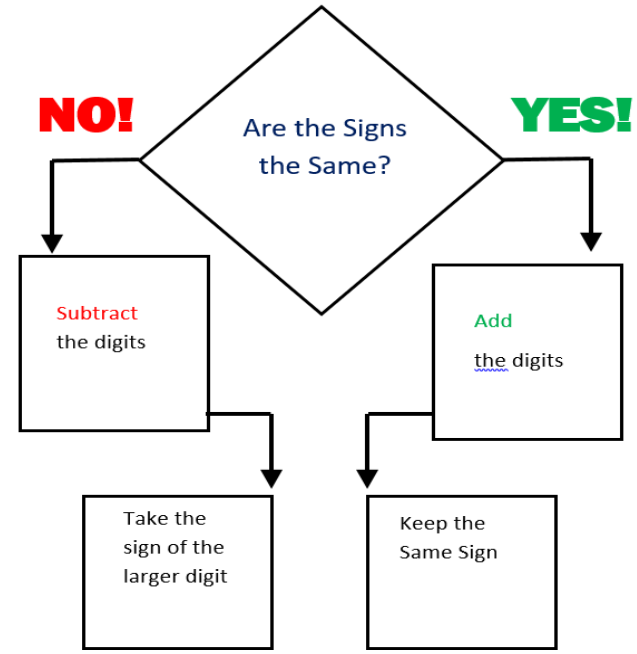
To multiply any number by ten or multiple of 10: multiply the front digits then add on the total number of zeroes to the end

2 x 10 *Think 2x1=2 then add 1 zero* = 20

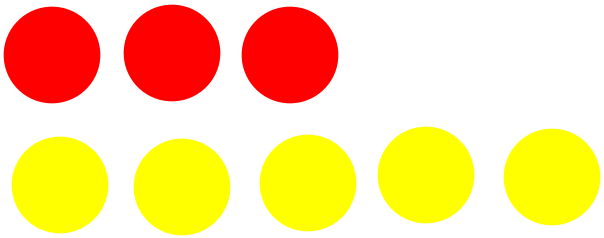
30 x 10 *Think 3x1=3 then add 2 zeroes* = 30

15 x 50 *Think 15x5=75 then add 1 zero* =750

Adding Integers



$$(-3) + 5 = 2$$



When you subtract integers:

ADD

THE

OPPOSITE

Subtracting Integers

Subtracting Integers it is no fuss...
Just change the Minus Sign
into a Plus.

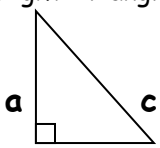
Don't forget the next digit's sign
Change that that too & you'll be fine.

It's now time to add,
you're good to go.
Just flip over the page and
go with the flow!

Pythagorean Theorem: $a^2 + b^2 = c^2$

(Used to find missing sides in a Right Triangle)

Example: 9, 15, ?



$a^2+b^2 = c^2$

$9^2 + b^2 = 15^2$

$81 + b^2 = 225$

$-81 \quad -81$

$b^2 = 144$

$b = \sqrt{144}$

$b = 12$

The sides that create the square are always the legs **a** & **b**.

The hypotenuse or slanty side across from the square is always **c**. **c** is always the largest number of the three sides.

Squares & Square Roots:

To find **Squares** enter the number then press the x^2 key. To find **Square Roots** Press the Blue 2nd key then the x^2 key to get the √ sign then enter the number you want to find the square root of.

Perfect Squares

$1\ 1^2$
$4\ 2^2$
$9\ 3^2$
$16\ 4^2$
$25\ 5^2$
$36\ 6^2$
$49\ 7^2$
$64\ 8^2$
$81\ 9^2$
$100\ 10^2$
$121\ 11^2$

Cube Roots

1x1x1 = 1
2x2x2 = 8
3x3x3 = 27

In your calculator use the exponent key (^) to find the cube. To find the cube root enter 3 press the 2nd key then the (^) key and the number you want to find the cube root of.

Integers (the set of positive & negative numbers)

Its best just to enter in your calculator & check twice!

Multiplying / Dividing: If the signs are the **same: Positive.** If the signs are **different: Negative**

Adding: Your answer will always be the sign of the "bigger" number (not looking at positive or negative – just the number)

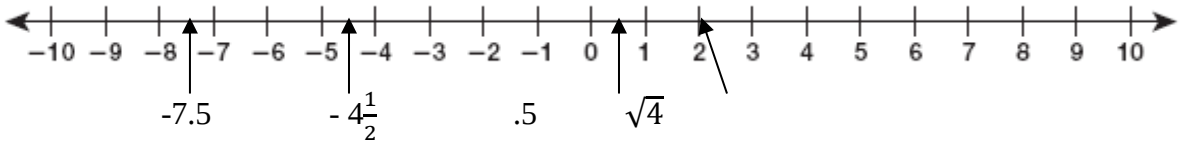
−10 + 5 = -5 - + + find the difference between the two numbers
10 + -5 = 5 + + - find the difference between the two numbers
-10 + -5 = -15 - + - add the two numbers
10 + 5 = 15 + + + add the two numbers

Subtracting: Add the opposite of the second number

-10 - -5 change to -10 + 5 = -5

10 - -5 change to 10 + 5 = 15

-10 - 5 change to -10 + -5 = -15



Absolute Value: the distance from 0 on the number line

|-5| the absolute value of -5 is 5

|6| the absolute value of 6 is 6

|-5 + 3| find the value of what's inside then take its absolute value of -5 +3 is 2

Place Value Chart

Thousands	Hundred	Tens	Ones	DECIMAL	Tenths	Hundreths	Thousandths	Ten Thousandths
				.				

There are three kinds of Estimation:

Front End, Rounding, and Compatible Numbers.

Front End Estimation: use only the whole number to estimate.

Rounding: Look at the tenths place to round. If it is a 0, 1, 2, 3, or 4 you should leave the number the same. If it is 5, 6, 7, 8, or 9 you should round up .

Compatible Numbers: Make the problems easier by finding close numbers that are easy to add, subtract, multiply or divide.

Adding and Subtracting Decimals

1. **Line Up Your Decimals**
2. **Like The Buttons On A Shirt.**
3. **Even out the Place Values with Zeros**
4. **Bring the Decimal Straight Down**
5. **Don't forget when to carry (adding)**
6. **And when to borrow (subtracting)**

3.02 + .4

$$\begin{array}{r} 3.02 \\ +\ .40 \\ \hline 3.42 \end{array}$$

12.03 - 4.8

$$\begin{array}{r} 12.03 \\ -\ 4.80 \\ \hline 7.23 \end{array}$$

FRACTIONS, LCM, GCF

WITH FRACTIONS WHATEVER YOU DO TO THE TOP – YOU MUST DO TO THE BOTTOM

Numerator	How many pieces you have	Part
Denominator	What size they are cut into	Whole
	(the number you need to make a whole)	

FRACTIONS TO DECIMALS

The fraction bar ALWAYS means divide

3/8 = 0.375

Using a calculator: enter fraction using $A^b/_c$ key, press 2nd key then PRB key & Enter

TO ENTER FRACTIONS INTO YOUR CALCULATOR

Use the $A^b/_c$ key

Ex: $\frac{1}{4}$ is entered as 1 $A^b/_c$ 4

$1\frac{2}{3}$ is entered as 1 $A^b/_c$ 2 $A^b/_c$ 3

To **Reduce** or **Simplify** just enter the fraction then the Enter key

DECIMALS TO FRACTIONS

0.27 = 27/100

Using a calculator: enter decimal, press 2nd key then PRB key & Enter

MIXED NUMBER TO IMPROPER FRACTION

Multiply the whole number by the denominator then add the numerator. Place that number over the original denominator.

Your calculator won't do this for you!

$1\frac{2}{3} = \frac{5}{3}$

"5 pieces of something cut into thirds" (3).

TO COMPARE FRACTIONS

	Ex: 3/5	1/4
•convert them to decimals	0.6	0.25
•Line up decimals then compare by place value adding zeroes if needed	0.60	0.25
•or find the cross products (see proportions)	0.6 > 0.25	
	3/5 > 1/4	

IMPROPER FRACTION TO A MIXED NUMBER

Divide the numerator by the denominator. The answer is your whole number. Any remainder should be written over the original denominator.

Using a calculator: Divide the numerator by the denominator then press the 2nd key then the PRB key.

PRIME FACTORIZATION

Break the number Down into prime numbers.

(2, 3, 5, 7, 11, 13…)

$36 = 3 \cdot 2 \cdot 2 \cdot 3$

EQUIVALENT FRACTIONS

Fractions that are equal

You can multiply or divide by a form of (the same number top and bottom) to create equivalent fractions.

Ex: $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ are all equivalent

RECIPROCALS

"Flip the fraction"

The reciprocal of $\frac{3}{4}$ *is* $\frac{4}{3}$

The reciprocal of 3 *is* $\frac{1}{3}$

Every whole number has 1 as a denominator

LCM (LEAST COMMON MULTIPLE)

smallest big number

12, 10, and 15

$12 = 2^2 \cdot 3$

$10 = 2 \cdot 5$

$15 = 3 \cdot 5$

$2^2 \cdot 3 \cdot 5$

$2^2 \cdot 3 \cdot 5 = 60$

LCM: 60

Write the factorization of each number in columns using exponents. Bring down the number with the biggest exponent from each column. Multiply to get the LCM.

GCF (GREATEST COMMON FACTOR)

biggest little number

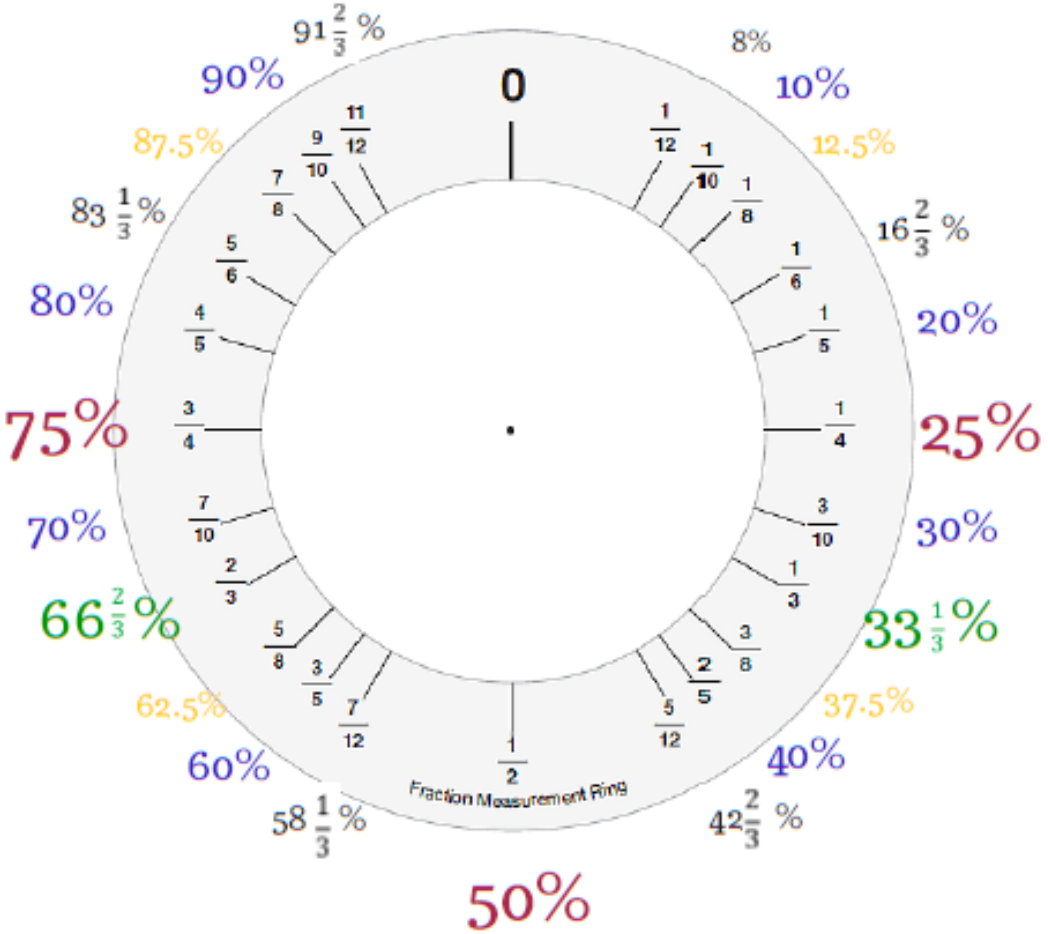
Find the prime factorization of each number. Line the factors up to see what they have in common. Multiply the common numbers to get the GCF.

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

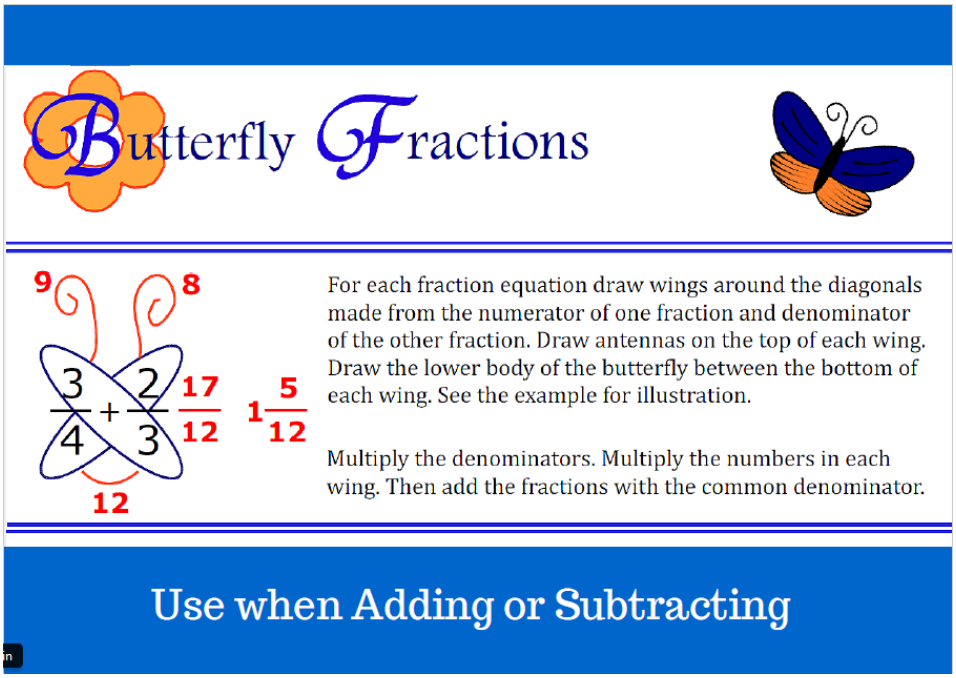
$2 \cdot 2 \cdot 3 = 12$

Fraction Ring

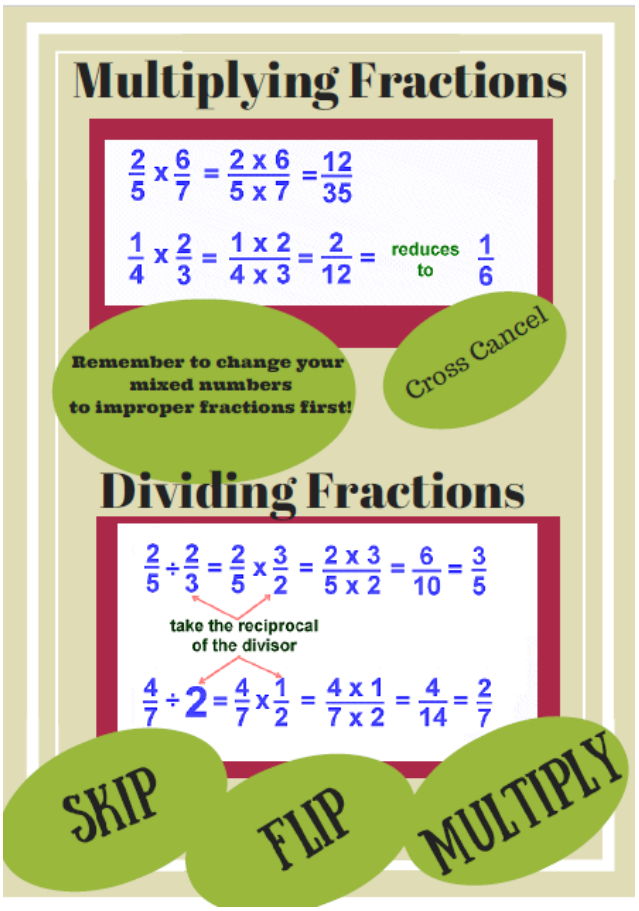


All Fractions and Percents are Part of a Whole

Adding and Subtracting Fractions



Multiplying and Dividing Fractions



Percents as Fractions

45%

$= \frac{45}{100}$ or $\frac{9}{20}$ Write the percent over 100,

Use $A^b/_c$ button to simplify

(If number on top is bigger – temporarily flip the numbers to simplify then flip them back)

Ratios (fractions) as Percents

At the Pet Expo, 35 out of every 50 dogs were Yellow Labs.

35 out of 50

$\frac{35}{50}$ Divide 35 by 50

= .7 move the decimal 2 places to the right

.7= 70% of the dogs were yellow labs

Decimals as Percents

0.3 = 30% Move the decimal 2 spaces to the right and add percent symbol.

Percents as Decimals

55% = 0.55

Divide percent by 100 and remove percent symbol (or just move the decimal 2 places to the left)

Find the Percent

18 is what percent of 120?

$\frac{is}{of} = \frac{\%}{100} \rightarrow \frac{18}{120} = \frac{x}{100}$

Replace *is* with 18

Replace *of* with 120.

Cross multiply to solve

Find the Part

18 is 15% of 120.

What number is 70% of 300?

$\frac{is}{of} = \frac{\%}{100} \rightarrow \frac{x}{300} = \frac{70}{100}$ 210 is 70% of 300.

Percent of Change

The Math Club had 20 members. Now it has 30 members. Find the percent of increase.

Step 1 Subtract to find the amount of change.

30 – 20 = 10

Step 2 Write a ratio that compares the amount of change to the original number of members. Write the ratio as a percent.

percent of change = $\frac{\text{amount of change}}{\text{original amount}}$

$= \frac{10}{20}$ The amount of change is 10

The original amount is 20.

= 0.5 or 50% Divide then write as a Percent

The percent of increase is 50%.

Sale Price So-Fro Fabrics is having a sale. All of their fabric is 25% off. Find the sale price of fabric originally priced $10 a yard.

Find the amount of the discount.

Find 25% of $10. of means multipy

0.25 · 10 Change the percent to a decimal and multiply = 2.5

Subtract the amount of the discount from the original price. $10 − $2.50 = $7.50

The discounted price is $7.50 a yard

Ratios: Compare two numbers

Ex: In basketball practice, I made 17 of the 25 shots I attempted.

17 to 25 17:25 17/25

The number stated first in the text is the number that goes first in the

Rates: Compares two numbers that have different labels (units)

Ex: Denise ran 5 miles in 40 minutes.

$\frac{5 \text{ miles}}{40 \text{ mins}}$ or $\frac{40 \text{ mins}}{5 \text{ miles}}$

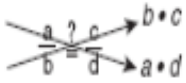
Unit Rates: Always have a denominator of 1. The fraction bar may be read as *per* in a rate.

Ex: Denise ran 5 miles in 40 minutes. How many minutes did it take her to run 1 mile?

Rate: $\frac{40 \text{ mins}}{5 \text{ miles}}$ Unit Rate: $\frac{8 \text{ mins}}{1 \text{ mile}}$

40 divided by 5 = 8 Or 8 minutes per mile

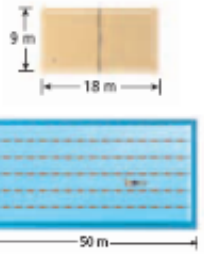
Cross Multiply



If $a \cdot d = b \cdot c$, then $\frac{a}{b} = \frac{c}{d}$.

Using proportions to solve problems

A volleyball court is a rectangle that is similar in shape to an Olympic-sized pool. Find the width of the pool.



	Court		Pool
Short side	9m	=	x
Long side	18m		50m

9x50 = 18x

450 = 18x

$\frac{450}{18} = x$ x = 25 The width of the pool is 25m.

To use proportions:

Joe ran the 500 yard dash for fun. How many feet does he run?

What you know What you want to find out

$\frac{1 \text{ yard}}{3 \text{ feet}} = \frac{500 \text{ yards}}{X \text{ feet}}$

What do you want to convert? Find the Fact from the table.

Information from given in the problem. This side contains the X.

Cross multiply to solve:

1 yard (x) = 3 feet (500 yards)

X = 1500 feet

Scale

- A model boat is 4 inches long. The scale factor is $\frac{1}{24}$. How long is the actual boat?

	Fact		Info from problem
model / actual	1	=	4 / x
	24		

1x = 24 x 4

x = 96 The actual boat is 96 in long.

Map measurement between points *A* and *B*: 4.5 in. Map scale: 1 in. = 50 mi

	Scale (fact)		Info from problem
map / actual	1in / 50mi	=	4.5 / x

1x = 50 x 4.5

x = 225

The actual distance between points A and B is 225 mi.

Scale Factor

	Race Car	Model
Length (in.)	132	11
Height (in.)	66	5.5

You can use the lengths *or* heights to find the scale factor.

$\frac{\text{model length}}{\text{race car length}} = \frac{11}{132} = \frac{1}{12}$

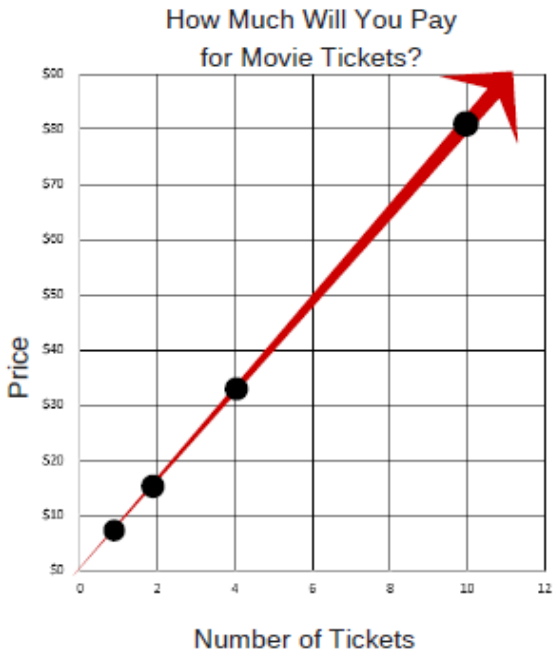
$\frac{\text{model height}}{\text{race car height}} = \frac{5.5}{66} = \frac{1}{12}$

Write a ratio then simplify. The scale factor is 1/12.

Proportional Relationship

Jessica is going to the movies with her friends. How much will she have to pay to bring X number of friends if tickets are $6 each.

Number of Movie Tickets (x)	Price (y)
1	$8
2	$16
4	$32
10	$80

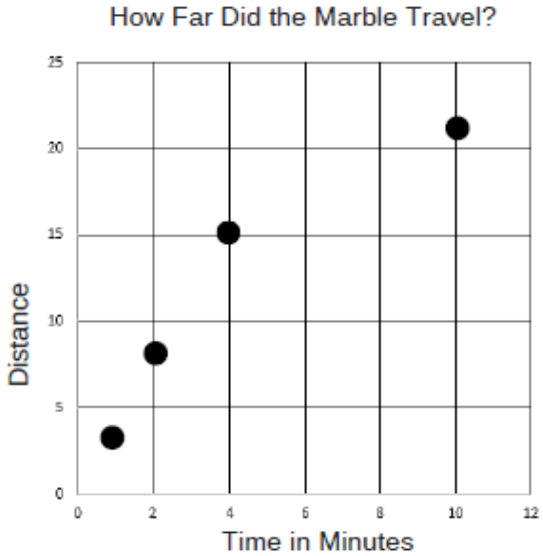


The Constant of Proportionality is $8. y = 8x

Non-Proportional Relationship

In a science experiment Logan had to roll a marble and chart the distance the marble traveled over 10 minutes.

Time in Minutes (x)	Distance in Feet(y)
1	3
2	8
4	15
10	21



There is no Constant of Proportionality

Customary Conversions

Measurement

Customary

	Customary Unit	Benchmark
Length	Inch (in)	Length of a small paperclip
	Foot (ft)	Length of a sheet of paper
	Mile (mi)	Length of a 18 football fields
Weight	Ounce (oz)	Weight of a slice of bread
	Pound (lb)	Weight of 3 apples
	Ton	Weight of a buffalo
Capacity	Fluid Ounce (fl oz)	About 2 tablespoons
	Cup (c)	Capacity of a standard mug
	Gallon (gal)	Capacity of a milk jug

TO USE PROPORTIONS:

Joe ran the 500 yard dash for fun. How many feet does he run?

What you know — What you want to find out

$$\frac{1 \text{ yard}}{3 \text{ feet}} = \frac{500 \text{ yards}}{X \text{ feet}}$$

What do you want to convert? Find the Fact from the table below.

Insert the info from the given problem. **Be sure to keep the units in the same row.**

Cross multiply to solve:

1 yard (x) = 3 feet (500 yards)

X = 1500 feet

Length	Weight	Capacity
12 inches (in.) = 1 foot (ft) 3 feet = 1 yard (yd) 5,280 feet = 1 mile (mi)	16 ounces (oz) = 1 pound (lb) 2,000 pounds = 1 ton	8 fluid ounces (fl oz) = 1 cup (c) 2 cups = 1 pint (pt) 2 pints = 1 quart (qt) 4 quarts = 1 gallon (gal)

Metric

Length	Mass	Capacity
1 cm = 10 mm 1 m = 100 cm = 1,000 mm 1 km = 1,000 m	1 g = 1,000 mg 1 kg = 1,000 g	1 L = 1,000 mL 1 kL = 1,000 L

	Metric Unit	Benchmark
Length	Millimeter (mm)	Thickness of a dime
	Centimeter (cm)	Width of your littlr finger
	Meter (m)	Width of a doorway
	Kilometer (km)	Length of 10 football fields
Weight	Milligram (mg)	Mass of a grain of sand
	Gram (g)	Mass of a small paperclip
	Kilogram (kg)	Mass of a textbook
Capacity	Milliliter (ml)	Amount of liquid in an eyedropper
	Liter (l)	Amount of liquid in large soda bottle
	Kiloliter (kl)	Capacity of 2 large refrigerators

To change smaller units to larger units, divide.

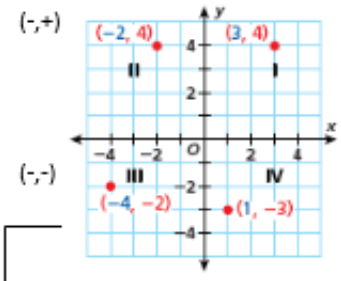
1 m = 100 cm
Centimeters are smaller than meters, so divide by 100.
Move the decimal point 2 places to the left: 046

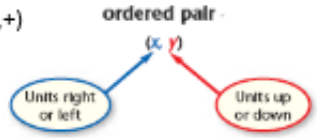
To change larger units to smaller units, multiply.

1 L = 1,000 mL
Liters are larger than milliliters, so multiply by 1,000.
Move the decimal point 3 places to the right: 5.300

1,000	100	10	1	0.1	0.01	0.001
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
Kilo-	*Hecto-*	*Deca-*	Base unit	*Deci-*	*Centi-*	*Milli-*

Coordinate Plane, Graphing, Slope & Rate of Change





Coordinate Plane

Finding Rate of Change

Pick 2 points on the graph that land perfectly on an intersection (nothing where you have to guess the #)

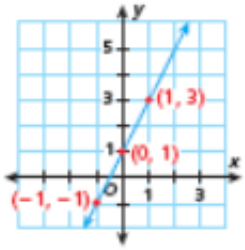
Formula = $\frac{y_2 - y_1}{x_2 - x_1}$

Enter the two points into the formula subtract then simplify leaving your answer as a fraction.
Do not divide or turn into a mixed number!

Graphing Functions

Input	Rule	Output	Ordered Pair
x	$2x + 1$	y	(x, y)
−1	$2(-1) + 1$	−1	(−1, −1)
0	$2(0) + 1$	1	(0, 1)
1	$2(1) + 1$	3	(1, 3)

Place each ordered pair on the coordinate grid. Then connect the points to form a line.



Creating a line given Slope

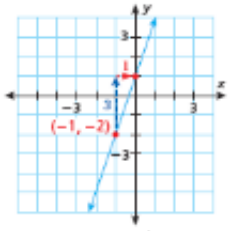
3; (−1, −2)

$3 = \frac{3}{1}$ *Write the slope as a fraction.*

slope = $\frac{\text{rise}}{\text{run}} = \frac{3}{1}$

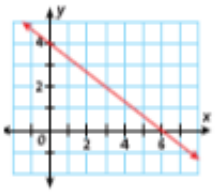
From point (−1, −2), move 3 units up and 1 unit right. Mark the point where you end up, and draw a line through the two points.

slope = $\frac{\text{rise}}{\text{run}}$

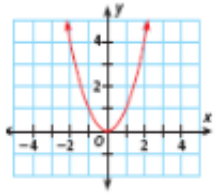


If a line rises from left to right, its slope is positive. If a line falls from left to right, its slope is negative.

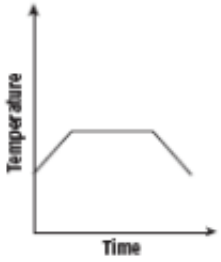
Rates of Change



The graph is a line, so the rate of change is constant.

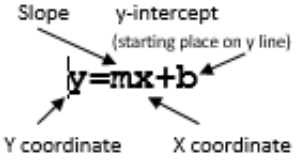


The graph is not a line, so the rate of change is variable.



Interpreting Graphs

The heat was turned on in the morning.
The temperature **increased** until it reached a comfortable level.
The temperature stayed **constant** throughout the workday.
Finally, the heat was turned off and the temperature **dropped.**



To solve an EQUATION you need to get the variable (letter) alone on one side of the equal sign.

$x+5 = -3$ right now x is being increased by 5 so we do the **opposite** to both sides of the equal sign to get it alone - 5

$x+5 = -3$

$-5 \quad -5$ +5 and -5 cancel each other out so all that's left on that side is x

$x = -8$ -3 -5 = -8 so that is what is on the right side of the equation

$-8+5 = -3$ substitute your answer for x to check if you are correct!

$x-3 = 7$ (x is being decreased by 3 so the opposite is to add 3)

$+3 \quad +3$

$x = 10$

$5x\frac{x}{5} = 15\ x5$ (x is being divided by 5 so the opposite is to multiply by 5)

$x = 75$

$2x = 10$ (x is being multiplied by 2 so the opposite is to divide by 2)

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

If an equation has **2 steps**

- cancel the addition (+) or subtraction (-) First! Then Cancel the (x) or (÷)

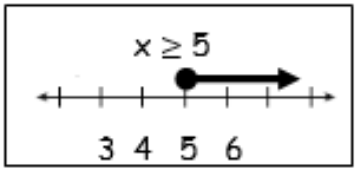
$2x - 5 = 1$

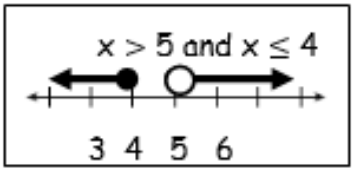
$+5 \quad +5$

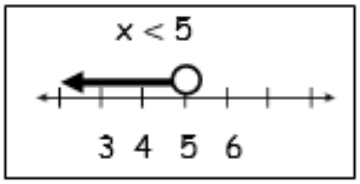
$\frac{2x}{2} = \frac{6}{2}$

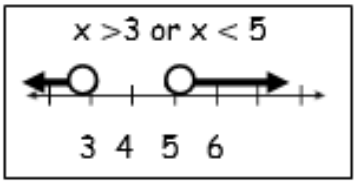
$x = 3$

Watch your signs! It is very easy to make mistakes when working with positive and negative numbers. Use your calculator and double check!









Remember to reverse the inequality sign when multiplying or dividing by a negative

$2x<14$ ÷ by positive

$\frac{2}{2} \quad \frac{2}{2}$

$x < 7$

$-2x<14$ ÷ by negative

$\frac{-2}{-2} \quad \frac{}{}$ reverse the inequality sign

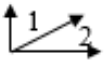
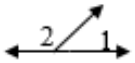
$x > -7$

Inequalities

Sign	*Meaning*	*Dot on graph*
$>$	*Greater than*	○
$\geq$	*Greater than or equal to*	●
$<$	*Less than*	○
$\leq$	*Less than or equal to*	●
$=$	*equal*	●

LINES & ANGLES & TRANSFORMATIONS

Point	Line	Segment	Ray	Plane	Parallel	Perpendicular
• A	⟷	———	——→	▱	⟺	✛
An exact location	Straight path that extends in both directions forever A line is 180°	A part of a line that has a beginning & end point	A part of a line that extends from a beginning point to forever	A flat surface that can be named by 3 points that don't all connect & extends forever	2 lines that do not ever intersect	2 lines that intersect at a right angle

Right Angle	Acute Angle	Obtuse Angle	Complementary Angles	Supplementary Angles	Vertical Angles
					
An angle that measures 90°	An angle that measures less than 90°	An angle that measures more than 90°	2 angles that add up to 90°	2 angles that add up to 180°	2 equal angles that are formed by intersecting lines.

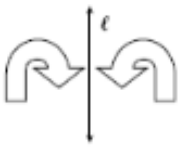
$m\angle 1 \cong m\angle 2 \cong m\angle 7 \cong m\angle 8$
$m\angle 3 \cong m\angle 4 \cong m\angle 5 \cong m\angle 6$

$\cong$ means congruent or equal

$m\angle 1$ & $m\angle 2$ are vertical angles

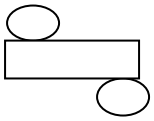
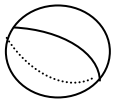
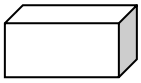
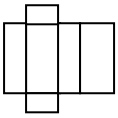
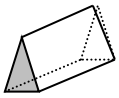
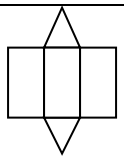
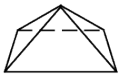
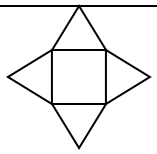
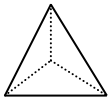
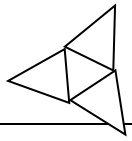
alternate **interior** angles are $m\angle 4$ & $m\angle 5$
alternate **exterior** angles are m $m\angle 3$ & $m\angle 6$

Transformations

Reflection	Rotation	Translation
		
A mirror image or FLIP	A TURN or rotation around a given point. Can rotate 90° or 180° clockwise (to the right) or counter clockwise (to the left)	A SLIDE can be to the side or down or both

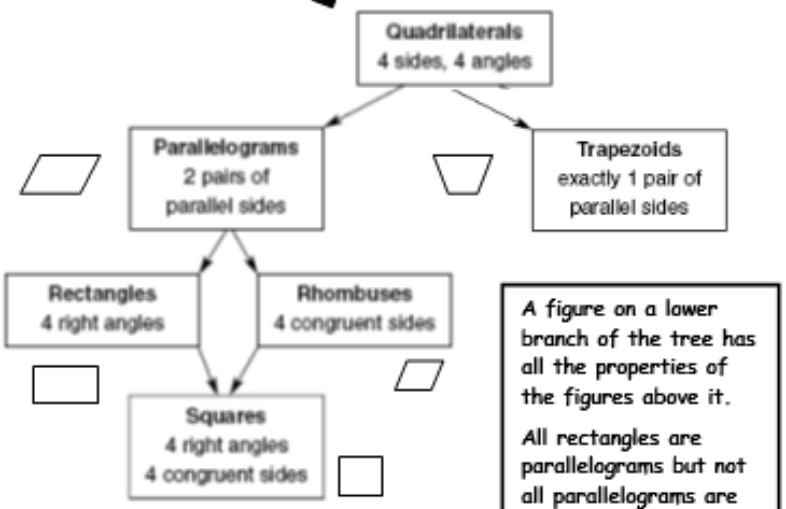
To tessalate a figure on a cooridinate grid you need to figure if you are going over a horizontal (x axis) or vertical (y axis) line, then the opposite x or y coordinate changes.
Ex 1: To reflect over the y axis; point (3,2) becomes (-3,2) the x coordinate changed to the opposite.
Ex 2: To slide 2 points up and one point to the left : point (3,2) becomes (2,4) as 3 -1 (to the left on a

3 D SHAPES AND NETS

	Picture	Net
Cylinder		
Sphere		
Cone		
Square / Rectangular Prism		
Triangular Prism		
Square Pyramid		
Triangular Pyramid		

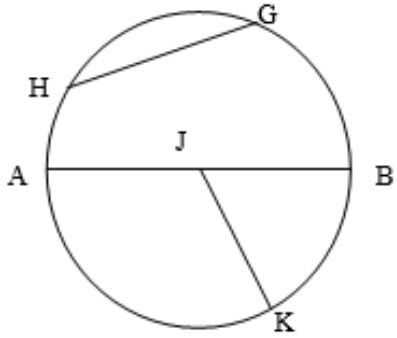
Polygons

Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Octagon	n-gon
Sides & Angles	3	4	5	6	8	any (n)
Regular: all sides & angles the same						Can find the measure of the angle by dividing the #of degrees in the shape (below) by the # of angles
Irregular: all sides & angles different						Can find the measure of missing angle by adding given angles and subtracting from #of degrees in the shape (found below)
# of degrees	180°	360°	540°	720°	1080°	n-2 x180°



A figure on a lower branch of the tree has all the properties of the figures above it.

All rectangles are parallelograms but not all parallelograms are rectangles



$\overline{AB}$ is the Diameter
$\overline{JK}$ is the Radius
$\overline{HG}$ is the chord
J is the center point

Isosceles	Right	Scalene	Obtuse	Equilateral
				
2 sides & 2 angles the same size (can be an acute triangle if each angle measures less than 90°)	Contains a 90° angle (also can be scalene if all sides are different lengths)	All 3 sides and angles are different	Has at least 1 obtuse angle (an obtuse angle is larger than 90°)	All 3 sides & angles are the same (always an acute triangle)

Triangle Sum Theorem: the measures of all 3 angles in a triangle add up to 180°

Answer (no exponent)

Perimeter: the length around the outside

P= Add up all the sides

C= 2πr

ex: **How much fence is needed?**

The formulas all ask for **r** (radius)
remember to find **r** if given diameter (d)
diameter÷ 2 = radius

d = diameter **r** = radius

Answer ²

Area: the amount of squares inside
(used on flat shapes 2D)

A = L x W

A= ½ L x W

A= πr^2

A= ½ h (b_1+b_2)

Never use a slanty side!
Height is always a straight line.

$\pi = 3.14$

It is only used with circles!

Answer ³

Volume: the amount INSIDE
(Used with 3D shapes)

ex: **How much water will it hold?**

V = L x W x H

V = ½ L x W x H

V = $\pi r^2 h$

Answer ²

Surface Area: covers the shape
(Used with 3D shapes)

ex: **How much wrapping paper is needed?**

S = Front L x W=
Side W x H=
Top H x L = ____ +

x 2

S = $2\pi r^2 + 2\pi rh$

Face : flat surface
Edge: where 2 flat surface meet
Vetex: where 3 or more edges meet (a 3D corner)

Analyzing Data

Example Data: 10, 20, 20, 10, 20, 20, 34, 10

Mean (Average)
Add all the numbers together then divide that answer by how many pieces of data you had

Ex:
10+20+20+10+20+20+34+10=144
144÷8=18
The mean of this data is 18

Median
Put the numbers in order from least to greatest and find the middle number (average the two numbers if it falls in the middle)

Ex: 10, 10, 10, 20, 20, 20, 20, 34 the middle number is 20
The median of this data is 20

Mode
The number that appears most (it can be more than one number if there is a tie or there might not be one if all numbers appear the same amount of times)

Ex: there are 3 10's, 4 20's and 1 34
The mode of this data is 20

The mean, median & mode MUST fall within the range of the data. It can't be higher or lower than the numbers you already have!

To find Scale & Interval
Find the range of the data. This sample is 24 so I can use a scale of 0-25 and use an interval of 5. My scale would be 0, 5, 10, 15, 20, 25.
If I had a data set that ranged from 150 to 578. The range would be 428. So I could use an interval of 100. My scale would be 0, lightning bolt, 100, 200, 300, 400, 500, 600.

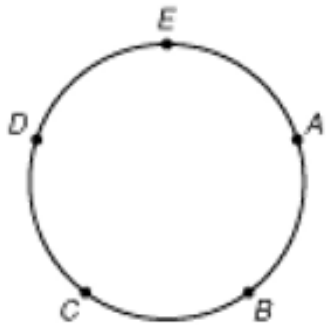
Outlier
The number that is much higher or lower than the rest of the data
The outlier of this data is 34

Scale & Interval
The numbers on the side of the graph. Scale always starts at 0. It can jump up using a lightning bolt symbol. The interval is the space between the numbers. It must be equal. Common intervals are 1,2,5,10, 50 & so on …

Range
The difference between the highest number and the lowest number in the data. It is used to determine the scale & interval to create a graph.
Ex: 34 – 10= 24 **The range of this data is 24**

COMBINATIONS

GEOMETRY:
Five points are located on a circle. How many line segments can be drawn with these points as endpoints?



Find the number of permutations of 5 points taken 2
at a time.
$P(5, 2) = 5 \cdot 4$ or 20

Since order is not important, divide the number of permutations by the number of ways 2 things can be arranged.
$\frac{20}{2!} = \frac{20}{2 \cdot 1}$ or 10

There are 10 segments that can be drawn.

Combination Notation

Find $C(7, 3)$.

$C(7, 3) = \frac{P(7, 3)}{3!}$ Definition of $C(7, 3)$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}$ or 35 $P(7, 3) = 7 \cdot 6 \cdot 5$ and $3! = 3 \cdot 2 \cdot 1$

Permutations

Permutation

A pizza place offers 12 different toppings. Jack wants to buy a three-topping pizza. How many pizzas can he buy if order is important?

number of possible toppings available	×	number of possible toppings available after the first topping is chosen	×	number of possible toppings available after the second topping is chosen	=	total number of possible pizzas with 3 toppings
12	×	11	×	10	=	1,320

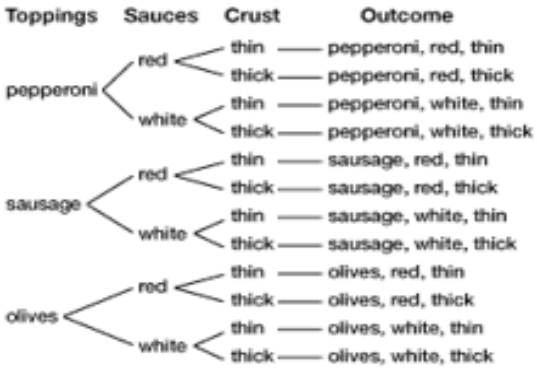
There are 1,320 different pizzas Jack can order.

Permutation Notation

Find $P(7, 3)$.
$P(7, 3) = 7 \cdot 6 \cdot 5$ or 210 7 things taken 3 at a time.

Tree Diagram

Classic Pizza bakes their pizzas in red or white sauce on a thin or thick crust. They offer pepperoni, sausage, or olives as their toppings. Draw a tree diagram to determine the number of different pizzas they offer with one topping.



There are 12 different pizzas.

Method 2:
3 toppings x 2 sauces x 2 crusts
3 x 2 x 2 = 12

Probability

A box contains 3 blue marbles, 6 red marbles, 4 black marbles, and 7 white marbles. A marble is picked at random. What is the probability the marble is blue?

There are 3 + 6 + 4 + 7 or 20 marbles in the box.

$$P(\text{blue}) = \frac{\text{blue marbles}}{\text{total number of marbles}}$$ Definition of probability

$$= \frac{3}{20}$$ There are 3 blue marbles out of 20 marbles.

The probability the marble is blue is $\frac{3}{20}$. The probability can also be written as 0.15 or 15%.

Probability of independent Events

The two spinners are spun. What is the probability that both spinners will show an odd number?

$P(\text{first spinner is odd}) = \frac{1}{2}$

$P(\text{second spinner is odd}) = \frac{4}{7}$

$P(\text{both spinners are odd}) = \frac{1}{2} \cdot \frac{4}{7}$ or $\frac{2}{7}$

Probability of Dependent Events

There are 3 red, 6 blue, and 11 green marbles in a bag. Once a marble is selected, it is not replaced. Find the probability that 2 red marbles are chosen.

Since the first marble is not replaced, the first event affects the second event. These are dependent events.

$P(\text{first marble is red}) = \frac{3}{20}$ Number of red marbles divided by the total number of marbles.

$P(\text{second marble is red}) = \frac{2}{19}$ Number of red marbles after one red marble is removed divided by the number of marbles after one red marble is removed.

$P(\text{two red marbles}) = \frac{3}{\cancel{20}\,10} \cdot \frac{\cancel{2}\,1}{19}$ or $\frac{3}{190}$

Theoretical Probability

What is the theoretical probability of getting heads on a coin and a 4 on a dice?

The theoretical probability is $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$.

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