Fox School of Business
TEMPLE UNIVERSITY®

MS/AcSci

Probability Self-Assessment

Read the following **INSTRUCTIONS** carefully before you begin the assessment.

- The assessment consists of 20 questions, each worth 1 point.
- The questions resemble those on actuarial *Exam P*, in both topics and style. Therefore, we believe that they are suitable to assess your probability background as it relates to passing the corresponding actuarial exam. If you have already passed Exam P, you can use the assessment to refresh your working knowledge of probability.
- There is no formal time restriction. For reference, on Exam P you would have about 2 hours to complete the 20 questions. We suggest that you give yourself 4 hours to work on the assessment, but feel free to take more time if you need it.
- Only the following resources are permitted during this self-assessment:
  - An SOA-approved calculator.
  - A standard normal distribution table.

  In particular, you should NOT consult: textbooks, study manuals, personal notes, solutions, internet, people, graphing calculators, etc.

  Keep in mind that this is a *self*-assessment; if you violate these guidelines, you are only cheating yourself.
- Recall the following information:
  - The p.m.f. of the *Poisson* distribution with mean λ is $f(k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$.
  - The p.d.f. of the *Exponential* distribution with mean μ is $f(x) = \dfrac{1}{\mu} e^{-x/\mu}$.
- Answers are provided on the final page of this document. Do not check them until you have completed the entire assessment.
- **Self-evaluation**: Your probability background is "very strong" if you attain 15 or more points; "strong" if you score between 12 and 14 points, and "insufficient" if your score is 11 points or less.

Good Skill & Good Luck!

Question 1

An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto policy and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto policy and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

Question 2

An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident. Calculate the probability that the driver was age 16-20.

Question 3

A company establishes a fund of 120 from which it wants to pay an amount, C, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year. The events of different employees achieving a high performance level during the coming year are mutually independent.

Calculate the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

Question 4

Ten cards from a deck of playing cards are in a box: two diamonds, three spades, and five hearts. Two cards are randomly selected without replacement.

Calculate the variance of the number of diamonds selected, given that no spade is selected.

Question 5

The number of days an employee is sick each month is modeled by a Poisson distribution with mean 1. The numbers of sick days in different months are mutually independent.

Calculate the probability that an employee is sick more than two days in a three-month period.

Question 6

Let N_1 and N_2 represent the numbers of claims submitted to a life insurance company in April and May, respectively. The joint probability function of N_1 and N_2 is

$$p(n_1, n_2) = \begin{cases} \frac{3}{4}\left(\frac{1}{4}\right)^{n_1-1} e^{-n_1}\left(1-e^{-n_1}\right)^{n_2-1}, & n_1 = 1, 2, 3, ..., \; n_2 = 1, 2, 3, ... \\ 0, & \text{otherwise} \end{cases}$$

Calculate the expected number of claims that will be submitted to the company in May, given that exactly 2 claims were submitted in April.

Question 7

The number of tornadoes in a given year follows a Poisson distribution with mean 3.

Calculate the variance of the number of tornadoes in a year given that at least one tornado occurs.

Question 8

For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000.

For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000.

The total claim amounts of the two companies are independent.

Calculate the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount.

Question 9

A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$.

Calculate $E[X]$.

Question 10

A company insures homes in three cities, J, K, and L. Since sufficient distance separates the cities, it is reasonable to assume that the losses occurring in these cities are mutually independent. The moment generating functions for the loss distributions of the cities are:

$$M_J(t) = (1-2t)^{-3}, \quad M_K(t) = (1-2t)^{-2.5}, \quad M_L(t) = (1-2t)^{-4.5}.$$

Let X represent the combined losses from the three cities. Calculate $E\left[X^3\right]$.

Question 11

An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x > 200 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

Question 12

In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be mutually independent, exponentially distributed random variables with respective means 1.0, 1.5, and 2.4.

Calculate the probability that the maximum of these losses exceeds 3.

Question 13

Let T_1 be the time between a car accident and reporting a claim to the insurance company. Let T_2 be the time between the report of the claim and payment of the claim. The joint density function of T_1 and T_2, $f(t_1, t_2)$, is constant over the region $\mathbb{C} = \{(y_1, y_2) : 0 < t_1 < 6,\ 0 < t_2 < 6,\ 0 < t_1 + t_2 < 10\}$, and zero otherwise.

Calculate $E[T_1 + T_2]$, the expected time between a car accident and payment of the claim.

Question 14

A device containing two key components fails when, and only when, both components fail. The lifetimes, T_1 and T_2 of these components are independent with common density function

$$f(t) = \begin{cases} e^{-t}, & t > 0, \\ 0, & \text{otherwise} \end{cases}$$

The cost, X, of operating the device until failure is $2T_1 + T_2$. Let g be the density function for X. Determine $g(x)$, for $x > 0$.

Question 15

The joint probability density for X and Y is

$$f(x,y) = \begin{cases} 2e^{-(x+2y)}, & x > 0,\ y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of Y given that $X > 3$ and $Y > 3$.

Question 16

A motorist just had an accident. The accident is minor with probability 0.75 and is otherwise major. Let b be a positive constant. If the accident is minor, then the loss amount follows a uniform distribution on the interval $[0, b]$. If the accident is major, then the loss amount follows a uniform distribution on the interval $[b, 3b]$.

The median loss amount due to this accident is 672. Calculate the mean loss amount due to this accident.

Question 17

Claim amounts at an insurance company are independent of one another. In year one, claim amounts are modeled by a normal random variable X with mean 100 and standard deviation 25. In year two, claim amounts are modeled by the random variable $Y = 1.04X + 5$.

Calculate the probability that a random sample of 25 claim amounts in year two average between 100 and 110.

Question 18

A motorist makes three driving errors, each independently resulting in an accident with probability 0.25. Each accident results in a loss that is exponentially distributed with mean 0.80. Losses are mutually independent and independent of the number of accidents.

The motorist's insurer reimburses 70% of each loss due to an accident. Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these driving errors.

Question 19

The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \dfrac{x+y}{8}, & 0 < x < 2 \text{ and } 0 < y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of $(X + Y)/2$.

Question 20

A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

(i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

(ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25.

(iii) The events of different new hires reaching retirement and the events of different new hires being married at retirement are all mutually independent events.

Calculate the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

End of Assessment.

See following page for answers.

Answers

Assessment Question	Answer	SOA Exam P Sample Question*
1	0.53	7
2	0.1584	19
3	60	31
4	50/147	295
5	0.577	212
6	e^2	131
7	2.6609	290
8	0.223	42
9	$2 + 3\,e^{-2/3}$	46
10	10,560	58
11	93.06	59
12	0.414	103
13	5.72	94
14	$e^{-x/2} - e^{-x}$	108
15	0.25	124
16	882	282
17	0.5335	223
18	0.0756	149
19	10/72	162
20	0.9887	86

(*) Check this document for the official solutions.