Written as per the revised 'G' Scheme syllabus prescribed by the Maharashtra State Board of Technical Education (MSBTE) w.e.f. academic year 2012-2013

ENGINEERING MATHEMATICS

COMMON TO ALL BRANCHES

FIRST YEAR DIPLOMA

SEMESTER - II

First Edition: December 2015

Salient Features

- Concise content with complete coverage of revised G-scheme syllabus.
- Simple & lucid language.
- Illustrative examples showing detailed solution of problems.
- MSBTE Questions from Summer-2007 to Winter-2015.
- MSBTE Question Papers of Summer, Winter - 2014 and 2015.
- Three Model Question Papers for practice.
- List of formulae for quick reference.

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PREFACE

Target's "Engineering Mathematics" is a complete and thorough book critically analysed and extensively drafted to boost the students' confidence. The book is prepared as per the revised scheme [G-scheme] of MSBTE curriculum effective from June 2012.

Each unit from the syllabus is divided into chapters bearing 'specific objectives' in mind. The sub-topic wise classification of this book helps the students in easy comprehension.

Each chapter includes the following features:

Theory of each mathematical concept is explained with appropriate references. Diagrams and illustrations are provided wherever necessary. *Italicized definitions* are mentioned for important topics.

Illustrative Examples are provided in order to explain the method of solving the problems. The detailed step-by-step solution in these problems helps students to understand and remember each minute step with proper justification of the same.

Solved Problems covering every type of MSBTE question gives students the confidence to attempt all the questions in the examination.

Exercise *(With final answers)* covers a variety of questions from simple to complex to help the students gain thorough revision in solving various types of problems.

MSBTE Problems covering questions from year 2006 to 2015 are solved exactly as they are expected to be solved by the students in the examination.

Formulae section is provided for quick recap and last minute revision of all the formulae at one glance. **MSBTE Question Papers** of year 2014 and 2015 are added at the end to make students familiar with the MSBTE examination pattern. A set of **three Model Question Papers** are designed as per MSBTE Pattern for thorough revision and to prepare the students for the final examination.

The journey to create a complete book is strewn with triumphs, failures and near misses. If you think we've nearly missed something or want to applaud us for our triumphs, we'd love to hear from you.

Please write to us on : mail@targetpublications.org

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

From,
Publisher

SYLLABUS

Topic and Contents	Hours	Marks
Topic 1 - Complex Number		
1.1 Complex Number: [14] **Specific objectives:** ➢ Find roots of algebraic equations which are not in real. • Definition of complex number, Cartesian, polar and exponential forms of complex number. • Algebra of complex number such as equality, addition, subtraction, multiplication and division. • De - Moivre's theorem with simple examples. • Euler's form of circular functions, hyperbolic functions and relation between circular and hyperbolic functions.	**08**	**14**
Topic 2 - Differential Calculus		
2.1 Function: [14] **Specific objectives:** ➢ Identify the function and find the value of function. • Definition of function, range and domain of function. • Value of function at a point. • Types of functions and examples.	**08**	**58**
2.2 Limits: [20] **Specific objectives:** ➢ To evaluate limit of function. • Concept and definition of limit. • Limits of algebraic, trigonometric, logarithmic and exponential functions with examples.	**08**	
2.3 Derivatives: [24] **Specific objectives:** ➢ Find the derivatives by first principle. ➢ Solve problems using rules and methods of derivatives • Definition of derivatives, notation, derivatives of standard function using first principle. • Rules of differentiation such as, derivatives of sum or difference, product, and quotient with proofs. • Derivative of composite function with proof (Chain rule) • Derivatives of inverse trigonometric functions using substitution. • Derivatives of inverse function. • Derivatives of implicit function. • Derivatives of parametric function. • Derivatives of one function w.r.t another function. • Logarithmic differentiation. • Second order differentiation.	**12**	

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<tr><td>3.1 Solution of algebraic equation [14]
Specific objectives:
➢ Find the approximate root of algebraic equation.
• Bisection method
• Regula falsi method
• Newton Rapshon method</td><td>06</td><td rowspan="2">28</td></tr>
<tr><td>3.2 Numerical solution of simultaneous equations [14]
➢ Solve the system of equations in three unknowns.
• Gauss elimination method
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UNIT I

Complex Number

Chapter - 1 Complex Number

01 Complex Number

1.1 Introduction

A linear equation in x of the form ax + b = 0 has a real root. Also in case of a quadratic equation the solution is obtained by factorization.

However every quadratic equation is not factorizable. For example $x^2 + 3 = 0$ has no factors in the set of real numbers. Also $x^2 = -3$ is not possible in the set of real numbers, as squares of real numbers are non-negative.

Inspite of the facts mentioned, the solution set of equation $x^2 + 3 = 0$ is

$$x^2 = -3$$

$\therefore \quad x = \pm\sqrt{-3}$

$\therefore \quad x = \pm\sqrt{3\times -1}$

$\therefore \quad x = \pm\sqrt{3}\times\sqrt{-1}$

where $\sqrt{-1}$ is called imaginary unit and it is denoted by i.

i.e., $\quad \mathrm{i} = \sqrt{-1}$

$\therefore \quad \mathrm{i}^2 = -1$

In general, $x = \pm\sqrt{a}\,\mathrm{i}$ is the solution of equation $x^2 + a = 0$, where a is a positive real number.

Thus i is an imaginary number.

Now, consider the equation $x^2 - 6x + 13 = 0$

$\therefore \quad x^2 - 6x + 9 = -4$

$\therefore \quad (x-3)^2 = 4\mathrm{i}^2$

$\therefore \quad x - 3 = \pm 2\mathrm{i}$

$\therefore \quad x = 3 \pm 2\mathrm{i}$

$\therefore \quad x = 3 + 2\mathrm{i}$ or $x = 3 - 2\mathrm{i}$

Hence the equation $x^2 - 6x + 13 = 0$ has two solutions 3 + 2i and 3 – 2i, which are not real numbers. These numbers are called complex numbers.

Integral Powers of i:

$$\mathrm{i} = \sqrt{-1}$$

$\therefore \quad \mathrm{i}^2 = \left(\sqrt{-1}\right)^2$

$\therefore \quad \mathrm{i}^2 = -1$

Similarly, we have the following results

$\mathrm{i}^3 = \mathrm{i}^2\,\mathrm{i} = (-1)\,\mathrm{i} = -\mathrm{i}$

$\mathrm{i}^4 = (\mathrm{i}^2)^2 = (-1)^2 = 1$

$\mathrm{i}^5 = \mathrm{i}^4.\,\mathrm{i} = (1)^4 \times \mathrm{i} = \mathrm{i}$

$\mathrm{i}^6 = (\mathrm{i}^2)^3 = (-1)^3 = -1$ and so on

Also, $\dfrac{1}{\mathrm{i}} = \dfrac{1}{\mathrm{i}}\times\dfrac{\mathrm{i}}{\mathrm{i}} = \dfrac{\mathrm{i}}{\mathrm{i}^2} = \dfrac{\mathrm{i}}{-1}$

$\therefore \quad \dfrac{1}{\mathrm{i}} = -\mathrm{i}$

1.2 Definition of a Complex Number

A number of the type $x + iy$ *or* $x + yi$*, where* x *and* y *are real numbers and* $i = \sqrt{-1}$ *is called a* ***complex number****.*
The complex number is denoted by z.
$\therefore \quad z = x + iy$
In a complex number $x + iy$, x is called the real part and is denoted by Re(z) and y is called the imaginary part and is denoted by Im(z).
$\therefore \quad \text{Re}(z) = x$ and $\text{Im}(z) = y$.

Example:
If $z = 2 + 3i$ is a complex number, then Re(z) = 2 and Im(z) = 3

Note:
i. A complex number whose real part is 0 is called a **purely imaginary number.**
ii. A complex number whose imaginary part is 0 is a purely real number. Thus, every real number can be considered as a complex number whose imaginary part is zero.
iii. A complex number whose both real and imaginary parts are zero is a zero complex number.
iv. The real part and imaginary part of a complex number are real numbers.

1.3 Conjugate of a Complex Number

If $z = x + iy$ is a complex number, then its conjugate complex number is $x - iy$ and is denoted by $\bar{z}$.
$\therefore \quad \bar{z} = x - iy$
Examples:

Complex numbers	Conjugate complex numbers
$3 + 2i$	$3 - 2i$
$4 - \sqrt{5}\,i$	$4 + \sqrt{5}\,i$
$2i - 3$	$-3 - 2i$
$\cos\theta + i\sin\theta$	$\cos\theta - i\sin\theta$

1.4 Algebra of Complex Numbers

1.4.(a) Equality of complex numbers:

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $x_1 = x_2$ and $y_1 = y_2$.
i.e., $z_1 = z_2$ if $x_1 = x_2$ and $y_1 = y_2$.
Examples:
i. If $x_1 + iy_1 = 3 + 2i$, then by equality of complex numbers we have,
$x_1 = 3$ and $y_1 = 2$.
ii. If $\sqrt{3} - 4i = x_2 + iy_2$, then by equality of complex numbers we have,
$x_2 = \sqrt{3}$ and $y_2 = -4$.

1.4.(b) Addition of complex numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then their sum is $z_1 + z_2$ and is defined as $z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$

$= (x_1 + x_2) + i(y_1 + y_2)$

$\therefore$ $\text{Re}(z_1 + z_2) = \text{Re}(z_1) + \text{Re}(z_2)$ and $\text{Im}(z_1 + z_2) = \text{Im}(z_1) + \text{Im}(z_2)$

Thus, $z_1 + z_2$ is a complex number.

Illustrative Example

If $z_1 = 3 - 7i$ and $z_2 = 5 + 3i$, find $z_1 + z_2$.

***Solution*:**

Given, $z_1 = 3 - 7i$ and $z_2 = 5 + 3i$

$\therefore$ $z_1 + z_2 = (3 - 7i) + (5 + 3i)$

$= 3 - 7i + 5 + 3i$

$= (3 + 5) + (-7 + 3)i$

$\therefore$ $\mathbf{z_1 + z_2 = 8 - 4i}$

Properties of addition:

If z_1, z_2, z_3 are complex numbers, then

i. $z_1 + z_2 = z_2 + z_1$

ii. $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

iii. $z_1 + 0 = 0 + z_1 = z_1$

1.4.(c) Subtraction of complex numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then their subtraction is $z_1 - z_2$ and is defined as

$z_1 - z_2 = (x_1 + iy_1) - (x_2 + iy_2) = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore$ $\text{Re}(z_1 - z_2) = \text{Re}(z_1) - \text{Re}(z_2)$ and $\text{Im}(z_1 - z_2) = \text{Im}(z_1) - \text{Im}(z_2)$

Thus, $z_1 - z_2$ is a complex number.

Illustrative Example

If $z_1 = 5 + 13i$ and $z_2 = 4 + 7i$, find $z_1 - z_2$.

***Solution*:**

Given, $z_1 = 5 + 13i$ and $z_2 = 4 + 7i$

$\therefore$ $z_1 - z_2 = (5 + 13i) - (4 + 7i)$

$= 5 + 13i - 4 - 7i = (5 - 4) + (13 - 7)\,i$

$\therefore$ $\mathbf{z_1 - z_2 = 1 + 6i}$

1.4.(d) Multiplication of complex numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex numbers, then their product is $z_1.z_2$ and is defined as

$z_1.z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$= x_1x_2 + i(x_1y_2) + i(y_1x_2) + i^2(y_1y_2)$

$= x_1x_2 + i(x_1y_2 + y_1x_2) - y_1y_2$ ....[$\because i^2 = -1$]

$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

Thus, $z_1.z_2$ is a complex number.

Illustrative Example

If $z_1 = 1 + i$ and $z_2 = 2 - 3i$, find z_1z_2.

***Solution*:**

Given, $z_1 = 1 + i$ and $z_2 = 2 - 3i$

$\therefore$ $z_1z_2 = (1 + i) . (2 - 3i)$

$= 1(2 - 3i) + i (2 - 3i)$

$= (2 - 3i) + (2i - 3i^2)$

$= 2 - 3i + 2i - 3(-1)$ ....$[\because i^2 = -1]$

$= 2 - 3i + 2i + 3$

$= (2 + 3) + (-3 + 2)i$

$\therefore$ $\mathbf{z_1z_2 = 5 - i}$

<u>Properties of multiplication:</u>

If z_1, z_2, z_3 are complex numbers, then

i. $z_1.z_2 = z_2.z_1$

ii. $(z_1. z_2).z_3 = z_1.(z_2.z_3)$

iii. $z_1.1 = 1.z_1 = z_1$

iv. $z_1. \bar{z}_1$ is purely a real number.

Note: If $z = x + iy$ and $\bar{z} = x - iy$, then

$z. \bar{z} = (x + iy)(x - iy) = x^2 - i^2y^2 = x^2 + y^2$ ....$[\because i^2 = -1]$

Thus, the product of two complex conjugates is a positive real number.

1.4.(e) Division of complex numbers:

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are the two complex numbers, then their division $\frac{z_1}{z_2}$ is defined as

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$= \frac{(x_1 + iy_1)}{(x_2 + iy_2)} \times \frac{(x_2 - iy_2)}{(x_2 - iy_2)}$$

$$= \frac{x_1x_2 - ix_1y_2 + iy_1x_2 - i^2y_1y_2}{x_2^2 - i^2y_2^2}$$

$$= \frac{x_1x_2 + i(y_1x_2 - x_1y_2) + y_1y_2}{x_2^2 + y_2^2} \quad ....[\because i^2 = -1]$$

$$\therefore \quad \frac{z_1}{z_2} = \left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}\right) + i\left(\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}\right)$$

Thus, $\frac{z_1}{z_2}$ is a complex number.