MTH 111

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Mathematics Department

Missouri State University-West Plains

MTH 111 Unit Objectives

Unit 1:

1. Simplify numeric expressions using the Order of Operations.
2. Simplify variable expressions by combining like terms and using the Distributive Property.
3. Evaluate variable expressions for given values.
4. Solve linear equations, including those containing decimals and fractions. Identify contradictions and identities.
5. Develop a variety of word problem strategies and use to solve application problems.
6. Solve linear, compound and three-part inequalities. Graph solution sets on a number line and write in interval notation.
7. Solve absolute value equations and inequalities. Graph solution sets on a number line and write in interval notation.
8. Understand and utilize the proper vocabulary associated with the concepts presented in this unit; develop critical thinking skills.

Unit 2:

1. Be familiar with radical expressions; simplify radicals.
2. Be familiar with imaginary numbers and complex numbers.
3. Add and subtract common radicals and complex numbers.
4. Solve radical equations; watch for extraneous roots.
5. Use the Pythagorean Theorem to find missing sides of a right triangle and solve basic application problems involving right triangles.
6. Solve rational equations; watch for restrictions.
7. Solve application problems involving rational equations.
8. Evaluate various expressions for given values.
9. Understand and utilize the proper vocabulary associated with the concepts presented in this unit; develop critical thinking skills.

Unit 3:

1. Understand the relationship between the input value and output value concerning various expressions.
2. Utilize the coordinate plane to plot points and further analyze the relationship between input values (x) and output values (y) concerning various expressions.
3. Graph linear equations in two variables, including horizontal and vertical lines.
4. Find the slope of a line and be familiar with various notes concerning slope.
5. Solve application problems involving linear equations in two variables.
6. Understand and utilize the proper vocabulary associated with the concepts presented in this unit; develop critical thinking skills.

MTH 111, Unit One

I. Order of Operations (PEMDAS)

a. Perform any work in parenthesis first
b. Convert/simplify terms with exponents
c. Perform all Multiplications/Divisions next (going left to right)
d. Perform all Additions/Subtractions (going left to right)

II. Example Problems:

Simplify the following numeric expressions using the Order of Operations:

1. $4 + 3(2 + 4) - 6$
2. $5^2 + 2^3 + 3^2$
3. $2 - (6 + 1)^2$
4. $(4 + 3)(5 - 2)$
5. $\dfrac{5x3+5}{(4-2)^2+1}$

Simplify the following variable expressions by combining like terms:

1. $4x + 3x + 5x$
2. $4x - 7x + 2x$
3. $4x + 3y + 2x + 6y$
4. $4x + 3y + 5 + 2x - 6y - 8$
5. $5x + 4x^2 + 3x + 2x^2 + 2$

Evaluate the last problem above for $x = 3$.

III. Distributive Property: $a(b + c) = ab + ac$; $a(b - c) = ab - ac$

Use the <u>distributive property</u> to simplify the following variable expressions:

1. $5(x - 3)$
2. $-(x^2 + 5x - 6)$
3. $3(x + 2) + 4(x - 3)$
4. $2(x + 4) - 2(x + 5)$

<u>Homework A - Simplify the following expressions:</u>

1. $4 + 3(2) + 5^2$
2. $3 - 2(5 - 1)$
3. $4 + 2[5 + 4(4 + 1)]$
4. $2^2 + 3^3 + 4$
5. $\sqrt{25} + \sqrt{16} - \sqrt{9}$
6. $(7 - 2)(2 - 7)$
7. $(10 - 3)^2$
8. $\dfrac{2 + 4x5}{(3-2)^2 + 1}$
9. $5x + 3x + 2x$
10. $10x - x$
11. $7x + 5y + 6 + 2x - 6y + 4$
12. $2(3x + 4)$
13. $-4(3x - 9)$
14. $-(x + 6)$
15. $8x + 3(4x - 5) - 8x$
16. $2(x - 1) + 3(x - 3)$
17. $3(x + 6) - 2(x + 4)$
18. $2(3x - 4) + 3(5x - 2) - (2x + 10)$
19. Evaluate $2x^2 + 5x - 3$ for $x = 2$.
20. Evaluate $3x - 4y + 3$ for $x = -1$ and $y = -2$

IV. When solving a **linear equation**, we tend to use PEMDAS in reverse - Use inverse (opposite) operations to isolate the variable.

a. Don't forget you can check your answer.
b. Keep up with the negative signs.
c. If you get $-x = a$, then $x = -a$.
d. Simplify each side of the equation before solving – use distribution if necessary and combine like terms.
e. An entire term can be "moved" from one side of an equation to another using inverse operations.
f. Zero can be an answer!
g. Watch for **contradictions** and **identities**.

V. **Solve the following linear equations**:

1. $x - 7 = -5$

2. $3p = 129$

3. $-x = 7$

4. $p - 8 = -15$

5. $c/3 = 14$

6. $0.2x + 0.4(x - 1) = 1.2$

7. $3x + 5 = 4 - 5x$

8. $3x - 4 = 3x + 7$

9. $3x - 4 = 3x - 4$

10. $6 - 4x = 6(x - 1) + 2(3x + 4)$

Homework B - Solve the following equations:

1. $4x + 6 = 18$
2. $10 - 2x = 5$
3. $3x + 6 = 2x + 10$
4. $2(x + 3) - 5x = 12$
5. $4(x + 1) = 3(x - 6)$
6. $x/5 + 2 = 6$
7. $\frac{1}{2}x + \frac{1}{3}x = 10$
8. $3x + 2(x - 1) = 4(x + 3)$
9. $3x + 2(x - 1) = 5(x + 3)$
10. $5x - 2(x - 4) = 10$
11. $5(x + 3) = 2x + 3(x + 5)$
12. $0.4x + 2 = 1.2$
13. $0.4x + 0.2(x + 1) = 0.8$
14. Four more than three times a number is 19. What is the number?

Word Problem Strategies
(Polya's 4 Step Problem Solving Process)

1. Read the problem carefully – make sure you understand what is being asked.

2. Devise a strategy
    a. Find a pattern
    b. Use mental arithmetic skills
    c. Make a table
    d. Work backwards
    e. Guess and check
    f. Draw a picture
    g. Use a Formula/Make an equation

3. Carry out the strategy!

4. Check your solution – make sure it is complete and make sure it makes sense.

Some Important Notes from Geometry:

- The sum of the angle measurements of the three angles in a triangle will be 180 degrees.
- A right angle has a measure of 90 degrees.
- If two angles form a right angle, then the sum of their angle measurements will be 90 degrees.
- Complementary angles have a sum of 90 degrees.
- Supplementary angles have a sum of 180 degrees.
- Perimeter is the distance around an object.

Examples:

1. $(2x + 4)$ and $(3x + 16)$ represent the degree measurements of two complementary angles. Find the actual measure of each angle.

2. Let ***x*** be the degree measure of the smallest angle in a triangle. If one of the other angles measures twice this angle and if the third angle measures three times the smallest angle, find the measure of all three angles.

3. How much is 12 pennies, 7 nickels, 5 dimes, and 3 quarters worth?

4. I have 50 coins consisting of dimes and quarters. If the total value of my money is $10.70, how many of each coin do I have?

5. There are 100 senators. If there are 10 more Republicans then Democrats, how many Republican Senators are there?

6. How much is 40% of 200?

7. How many gallons of a 10% bleach solution must be mixed with 20 gallons of a 40% bleach solution to get a mixture that is 30% bleach?

8. I have 10,000 to invest in two accounts, one earning 5% and the other earning 8%. How much should I put in each account in order to earn $740 in interest?

Homework C - Solve the following application problems:

1. If ***x*** represents the measure of an angle, then ______________ represents the measure of its supplement.

2. If I have 60 coins consisting of dimes and quarters, and if I have ***x*** dimes, how many quarters do I have?

3. The degree measurements of the three angles of a triangle are represented by the expressions $5x$, $2x + 3$, and $3x - 3$. Find the measure of all three angles.

4. The measurements of the three sides of a triangle are represented by the expressions $2x$, $3x$ and $4x$. If the perimeter of this triangle is 18 feet find the measurement of all three sides.

5. A certain school has a total of 85 fifth graders. If there are 13 more girls than boys, how many of these fifth graders are boys?

6. I have 60 dimes and quarters. If the value of these coins is $8.25, how many of each coin do I have?

7. Adult tickets to a movie cost $12 while children tickets cost $5. If a total of 500 tickets were sold and if $3,550 was collected, how many of each kind of ticket was sold?

8. I have $5,000 to invest in two accounts, one paying 4% and the other paying 6%. If I would like to make $270 in interest, how much should I invest in each account?

9. How much of a 10% solution should be mixed with 30 liters of a 50% solution in order to make a 40% solution?

10. Using the formula $V = lwh$, find w when $l = 10$, $h = 20$ and $V = 2000$.

VI. Solve **<u>inequalities</u>** like equations – except, graph answer(s) on a number line and shade the region that "works".

- When multiplying or dividing both sides of an inequality by a negative, reverse the inequality sign.
- A compound inequality using the word "and" must have an intersection, otherwise the answer is "no solution".
- A compound inequality using the word "or" generally has a left and right region for its solution set. However, the solution set could end up being the entire number line.
- A 3-part inequality needs to be "rearranged" so that the variable is isolated in the "middle" of the inequality.
- Solution sets can be written in Interval Notation

VII. Graph solution sets to each of the following on a number line. Also, give answers in interval notation.

1. $x + 5 \leq 4$

2. $2x - 7 > 5$

3. $-3x > 6$

4. $6x - 4 \geq 2(x - 5)$

5. $2 \leq x < 5$

6. $2 < 2x + 3 \leq 7$

7. $5(x + 3) > 10$ and $-2x + 16 > 2$

8. $0.5x + 3 > 4$ or $\frac{1}{4}x - 2 < -6$

Homework D - Solve the following inequalities and graph the solutions on number lines/write in interval notation:

1. $2x + 4 \geq 12$
2. $0.5x < 1.5$
3. $-2x < -20$
4. $-4x + 6 \leq 2$
5. $5 - 2x < 11$
6. $4(x + 2) \leq x + 11$
7. $\frac{2}{5}x > 10$
8. $5 < x + 2 < 20$
9. $-13 < 2x + 5 < 21$
10. $-4 \leq 2x - 6 \leq 12$
11. $2x + 4 < 10$ and $x - 3 \geq -11$
12. $5 + 2x > -11$ and $3(x + 4) < 12$
13. $x < -6$ and $x > 4$
14. $2x \leq -10$ or $5x + 1 \geq 11$
15. $x < 4$ or $x > -10$

VIII. To solve **absolute value equations**, use this note:

If $|x| = b$, then $x = b$ and $x = -b$.

- If $|x| = -b$, then there is no solution.
- The solution to an absolute value inequality will typically resemble the solution to a compound inequality.

IX. Solve each of the following absolute value equations/inequalities:

1. $|x| = 5$

2. $|2x + 3| = 9$

3. $|x + 5| = 11$

4. $|x - 3| = 0$

5. $|x| + 5 = 11$

6. $|2x - 5| + 4 = 6$

7. $|x| = -3$

X. Graph the solutions to each of the following Absolute Value Inequalities on a number line and give answers in interval notation.

1. $|x| \geq 2$

2. $|x| < 2$

3. $|3x - 1| \geq 5$

4. $|-x + 4| < 9$

5. $|x| > -2$

6. $|x + 4| + 2 < 10$

7. $|x + 4| + 10 < 2$

8. $|x + 2| \geq -4$

<u>Homework E - Solve the following absolute value equations and equalities; graph the solutions sets of the inequalities on number lines or write in interval notation:</u>

1. $|x| = 12$
2. $|x + 4| = 10$
3. $|x - 3| = 6$
4. $|2x + 5| = 0$
5. $|x + 4| + 5 = 10$
6. $|2x - 3| + 7 = 12$
7. $|5x + 10| = -11$
8. $|x| < 4$
9. $|2x + 6| > 10$
10. $|x + 4| - 3 \leq 6$
11. $|x - 10| \geq -3$
12. $|x| + 6 < 4$

MTH 111, Unit Two

I. <u>Radicals</u>

1. 16 and 25 are examples of perfect squares – these numbers have perfect square roots.

2. However, there are several numbers that don't have perfect square roots – these numbers are called **<u>irrational</u>**. $\sqrt{2}$ and $\sqrt{3}$ are two examples.

3. Notice that some non-perfect square roots can be factored: $\sqrt{50} = \sqrt{25} \times \sqrt{2}$

4. Simplifying we get $\sqrt{50} = 5\sqrt{2}$ (verbally we say 5 radical 2) – simplify the following:

   a. $\sqrt{32} =$

   b. $\sqrt{20} =$

   c. $\sqrt{8} =$

   d. $4\sqrt{27} =$

   e. $\sqrt[3]{27} =$

5. $\sqrt{a} + \sqrt{b} \neq \sqrt{(a + b)}$ – think about adding like terms.

6. However, $x\sqrt{a} + y\sqrt{a} = (x + y)\sqrt{a}$

7. So, $\sqrt{50} + \sqrt{32} = 5\sqrt{2} + 4\sqrt{2} = 9\sqrt{2}$

8. Perform the following operations:

   a. $\sqrt{36} + \sqrt{49} =$

   b. $4\sqrt{3} + 5\sqrt{3} =$

   c. $\sqrt{80} + \sqrt{20} =$

9. The number under the square root symbol is called the "radicand". If the radicand is negative, then we will always be able to factor out a $\sqrt{-1}$.

10. So, $\sqrt{-16} = \sqrt{16} \times \sqrt{-1}$ and $\sqrt{-50} = \sqrt{25} \times \sqrt{2} \times \sqrt{-1}$

11. Centuries ago It was decided that the square root of a negative number was not a real number – thus these numbers were called **<u>imaginary</u>**. This led to the following: $\boldsymbol{\sqrt{-1} = i}$.

12. Thus, $\sqrt{-16} = 4i$ and $\sqrt{-50} = 5i\sqrt{2}$

13. Simplify the following:

    a. $\sqrt{-25} =$

    b. $\sqrt{-32} =$

    c. $4\sqrt{-75}$

14. Also, a real number and an imaginary number cannot be added together. But two imaginary numbers can be added (or subtracted).

15. $5i + 12i = 17i$.

16. $(4 + 3i)$ is an example of a **complex number**. (This expression can't be simplified.)

17. $(4 + 3i) + (5 - 2i) = 9 + 1i$. (Combine like terms)

18. $(10 + 2i) - (6 - 3i) = 4 + 5i$ (Be careful with the subtraction!) Perform the following operations:

    a. $2i + 3i + 5i =$

    b. $6 + 2i - 5i =$

    c. $(6 + 2i) + (5 - 6i) =$

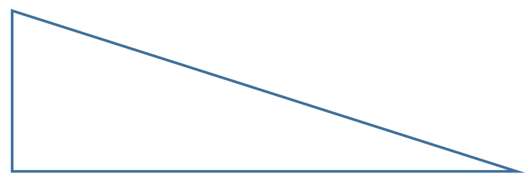
    d. $(3 + 4i) - (2 - 3i) =$

<u>Homework A – Simplify the following radicals and radical expressions:</u>

1. $\sqrt{25}$
2. $\sqrt{27}$
3. $\sqrt{72}$
4. $3\sqrt{20}$
5. $\sqrt[3]{8}$
6. $3\sqrt[3]{64}$
7. $\sqrt{-49}$
8. $\sqrt{-125}$
9. $-4\sqrt{-20}$
10. $\sqrt{\dfrac{16}{25}}$
11. $\sqrt{\dfrac{50}{9}}$
12. $\sqrt{25}+\sqrt{100}$
13. $3\sqrt{2}+4\sqrt{2}$
14. $3\sqrt{2}+4\sqrt{3}$
15. $\sqrt{8}-\sqrt{32}$
16. $\sqrt{-25}-\sqrt{-36}$
17. 5i + 3i + 2i
18. (4 + 3i) + (6 – 2i)
19. (5 – 2i) – (4 – 7i)
20. 2(5 + 3i) + 4(2 + 7i)

II. <u>The Pythagorean Theorem</u>

1. In a right triangle, the two sides that form the right angle are called "legs" and the long side opposite of the right angle is called the "hypotenuse". Often the two legs are labeled "a" and "b" and the hypotenuse is labeled "c".



2. The Pythagorean Theorem states that in a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse. In other words, $a^2 + b^2 = c^2$.

3. The most famous right triangle is the one with legs measuring 3 and 4 inches and the hypotenuse measuring 5 inches. Notice that $3^2 + 4^2 = 5^2$. Anytime three integers "work" in the Pythagorean Theorem they are called a <u>Pythagorean Triple</u>.

4. Is 6, 7 and 8 a Pythagorean Triple?

5. If two legs of a right triangle measure 5 and 12 inches, what would be the length of the hypotenuse?

6. If the legs of a right isosceles triangle measure 10 inches, what would be the length of the hypotenuse?

7. If the hypotenuse of a right triangle measured 20 cm and one of the legs measured 12 cm, what would be the **<u>perimeter</u>** of this triangle?

8. If the diagonal of a square measured 30 cm, what would be the length of one side of this square?

9. If a 20-foot ladder was placed 6 feet from the base of a wall and then the top of the ladder came to rest on the wall, how far up the wall would the top of the ladder reach?

Homework B – Solve the following problems involving the Pythagorean Theorem:

1. Are 6, 8 and 10 the sides of a right triangle?

2. If two legs of a right triangle measure 8 and 15 inches, what would be the length of the hypotenuse?

3. If one leg of a right triangle measures 4 cm and the hypotenuse measures 6 cm, what would be the length of the other leg?

4. A rectangle is 4 feet wide and 10 feet long. What is the length of its diagonal?

5. A guide wire is attached to the top of a 50-foot tower and anchored 10 feet away from the base. What is the length of this guide wire?

6. If the hypotenuse of an isosceles right triangle measures 10 inches, how long is a leg?

III. Radical Equations

1. To solve a **radical equation**:

   i. Isolate the radical
   ii. Square both sides (or use some other power as appropriate)
   iii. Solve the resulting equation
   iv. Check for extraneous roots – watch for "no solutions"

2. Solve the following radical equations:

   a. $\sqrt{x+2}=6$

   b. $\sqrt{x+2}+3=10$

   c. $\sqrt{x+2}+8=2$

   d. $\sqrt[3]{x+3}=4$

Homework C – Solve the following radical equations:

1. $\sqrt{x}=6$
2. $\sqrt{x+2}=5$
3. $\sqrt{x-7}+3=11$
4. $\sqrt{3x+1}-7=3$
5. $\sqrt{x+2}=-6$
6. $\sqrt{x+5}+10=3$
7. $3\sqrt{x-4}+1=7$
8. $\sqrt[3]{x+30}=5$

IV. Rational Equations

1. A rational equation typically has a variable in a denominator.

2. $\frac{10}{x} = 2$ is an example.

3. To solve rational equations:

   a. Use inverse operations to isolate the variable; or
   b. Use **proportion** concepts (cross products are equal); or
   c. Multiply the equation by the common denominator and then solve resulting equation.
   d. Don't forget: the denominator of a fraction cannot equal zero.

4. $\frac{3}{x+2} = \frac{2}{x}$

5. $\frac{1}{x} + \frac{2}{x} = 6$

6. $\frac{1}{x} + \frac{1}{3} = \frac{1}{5}$

Homework D – Solve the following rational equations:

1. $\frac{1}{x} = 3$
2. $\frac{30}{x} = 15$
3. $\frac{1}{x} = \frac{1}{10}$
4. $\frac{3}{5} = \frac{4}{x}$

5. $\frac{2}{x}-\frac{1}{2}=3$
6. $\frac{2}{x}+\frac{1}{5}=\frac{1}{2}$
7. $\frac{1}{2}x+\frac{1}{3}x=10$
8. $\frac{2}{x}-\frac{11}{x}=3$
9. $\frac{2}{x+1}=\frac{3}{x-4}$

V. <u>Applications</u>

Here are two new important concepts:

1. **D = RT**
   a. This is one of two "distance formulas" in mathematics.
   b. This is used to solve application problems involving distance, rate, and/or time.
   c. These problems often can be solved using a diagram. Harder problems that contain a lot of information might need a chart to keep track of everything and to help "develop" an equation that should help solve the problem.
   d. Don't forget to check to make sure that the problem is finished.
   e. Here is an example:
      i. Two cities are 500 miles apart. Steve leaves City A driving at 40 miles per hour heading toward City B. Mary leaves City B (at the same time Steve left City A) driving at 60 miles per hour towards City A. How long will it take for Steve and Mary to meet?

2. **Rate = ?/Time (Work Problems)**
   a. Rate isn't always expressed in miles per hour.
   b. If you stuff 100 envelopes in 5 minutes, then your "rate" is:
   c. If you take 3 hours to mow your lawn, then your "rate" is:
   d. If you can mow your lawn in 3 hours but I can mow it in 2 hours, would it take us 5 hours to mow your lawn if we "worked" together? How long would it take?
   e. So, in application problems involving "work", we need to use the idea that $\mathbf{R_1 + R_2 = R_T}$.

Solve the following examples:

A. I can buy 4 pounds of candy for \$1.50. How many pounds can I get for \$20?

B. In a certain forest 75 deer were captured and 15 of them were found to have a certain disease. If it is estimated that this forest contains 25,000 deer, how many of them probably have this disease?

C. I can mow a lawn in 6 hours. My son can mow the lawn in 4 hours. How long will it take us if we both mow at the same time?

D. A hose can fill a large vat in 20 minutes. If a second hose is used at the same time, the vat can be filled in 15 minutes. How long would it take this second hose to fill the vat by itself?

E. A supervisor does a certain job 3 times faster than his helper. If they work together they complete the job in 2 hours. How long would it take the helper working alone to get this job completed?

F. Two people leave a city at the same time heading in the same direction. If the first person drives 50 miles per hour and the second person drives 60 miles per hour, how long will it be before they are 100 miles apart?

G. Sue can drive 600 miles in the same amount of time as Sam can drive 400 miles. If Sue drives 10 miles per hour faster than Sam, what is the rate of each driver?

<u>Homework E – Solve the following application problems:</u>

1. I can buy 10 gallons of paint for $55. How many gallons can I get for $30?

2. It is found that 5 out of every 8 college students like algebra. If a certain college has 4,000 students, how many of them like algebra?

3. For every 15 minutes of exercise, you burn off 70 calories. How long would it take you to burn off 250 calories?

4. An experienced maid can clean a room in 30 minutes. A new maid can clean the same room in an hour (60 minutes). How long will it take to clean this room if both maids work together?

5. It takes me 5 hours to do a certain task. If my son helps we get done in 3 hours. How long would it take my son working by himself?

6. If I drive 70 miles per hour for eight and a half hours, how far will I have driven?

7. If I average 65 miles per hour, how long will it take me to drive 1,000 miles?

8. Two people leave a town at the same time traveling in opposite directions. If one person travels at 50 miles per hour and the other travels at 60 miles per hour, how long will it take for them to be 550 miles apart?

9. If the two people in the previous problem were heading in the same direction, how long would it take for them to be 550 miles apart?

10. I am heading north at 40 miles an hour. ½ hour later my wife leaves traveling on the same route at 50 miles per hour. How long will it take her to catch me?

VI. <u>Expressions & Equations Summary</u>

1. Equations are solved. Expressions are either simplified or evaluated.

2. $x^2 + 5x - 6$ is called a variable or **<u>polynomial</u>** expression.

3. $|3x + 5|$ is called an **Absolute Value** expression.

4. $\sqrt{10-3x}$ is called a **Radical** expression.

5. $\frac{3x-5}{2x+1}$ is called a **Rational** expression. The <u>restriction</u> of this rational expression is -½; said another way, $x \neq -\frac{1}{2}$.

6. There are many other kinds of expressions.

7. <u>Evaluate</u> each of the expressions in parts 2, 3, 4 & 5 for x = 0, x = -2, and x = 4.

8. $2x - 7 = 12$ is called a Linear Equation.

9. $|3x + 5| = 6$ is called an Absolute Value Equation.

10. $\sqrt{10-3x} = 5$ is called a Radical Equation.

11. $\frac{3x-5}{2x+1} = 2$ is called a Rational Equation.

12. There are many other kinds of equations. Don't forget, equations can have no solutions or infinite solutions.

13. <u>Solve</u> the equations in parts 8, 9, 10 & 11.

<u>Homework F – Complete the following:</u>

1. Evaluate $x^2 + 5x - 4$ for x = -2, x = 3 and x = 0.

2. Evaluate $|2x - 4|$ for x = 5 and x = -4.

3. What value of *x* would make the expression above <u>produce</u> 6?

4. Evaluate $\sqrt{3x+4}$ for x = -2, x = 0 and x = 20.

5. What value of *x* would make the expression above produce 5?

6. What value of *x* would make the expression above produce -5?

7. Evaluate $\dfrac{2x-3}{x+2}$ for x = 0 and for x = 1,000.

8. What is the "restriction" of the expression above?

9. What value of *x* would make the expression above produce 0?

MTH 111 – Unit 3
Coordinate Geometry

I. The Cartesian Coordinate System consists of two perpendicular number lines intersecting at the 0's. This point of intersection is called the origin. The horizontal number line is called the x-axis and the vertical number line is called the y-axis.

II. Four quadrants are formed by these perpendicular number lines: The first quadrant is in the upper right; the other three are labeled consecutively as we move counter-clockwise.

III. One purpose of the coordinate plane is to plot points. When certain points are plotted and then connected, various shapes (lines, triangles, squares, etc.…) can be formed.

Homework A – Complete the following charts/Plot points on a coordinate plane:

1. Plot the following points: (-6,0); (6,0); (6,4) & (-6,4). What shape could we get if we connected these points?

2. Complete the following chart:

$y = \lvert x + 3\rvert$	x	y
	-7	
	-5	
	-3	
	-1	
	1	

3. Complete the following chart:

$y = x^2 - 2$	x	y
	-3	
	-2	
	-1	
	0	
	1	
	2	

4. Complete the following chart:

$y = \sqrt{x+4}$	x	y
	-5	
	-4	
	0	
	5	
	12	

5. Plot the "answers" in these charts as points on the coordinate plane and then connect to show the "shape" of each of these equations.

IV. Linear equations in two variables (like *2x + y = 10*) have solutions that are listed as ordered pairs. These ordered pairs can then be plotted as points on the coordinate plane.

V. One method for finding solutions to linear equations in two variables is to form a table of values. Any number can be substituted in for x (or y) – then once the resulting equation is solved, the answer completes the ordered pair. For example if we put 4 in place of x in the equation *2x + y = 10* and then solve the resulting equation we get y = 2. Said another way, when x = 4, then y = 2. Thus (4,2) is a solution to this equation <u>and</u> can be plotted as a point on the coordinate plane.

VI. When several solutions to a linear equation in two variables are plotted and connected, a line will always result.

VII. Completing (0,_) and (_,0) finds two special points called **intercepts**. The point that lies on the x-axis is called the x-intercept and the point that lies on the y-axis is called the y-intercept. For the linear equation *2x + y = 10*, the intercepts are (5,0) and (0,10). Plotting these two points and connecting them allows us to see the line that represents <u>all</u> the solutions to this equation.

VIII. There are two special lines: horizontal and vertical. Plot the points (1,2), (2,2,) and (3,2). When connected we get a horizontal line. Notice the y- coordinate of all three points was 2. So, the equation of this line is ***y = 2***. Vertical lines will have the same x-coordinate of all the points they pass through.

IX. **<u>Examples</u>** (find the intercepts and/or use a table of values to graph the following):

a. x + y = 6

b. 2x – y = 12

c. 3x + 5y = 20

d. y = 3

e. x = -2

<u>Homework B – Graph the following linear equations on a coordinate plane:</u>

1. x + y = 10
2. 2x + 3y = 12
3. y = -2x + 4
4. y = 5 – x
5. y = x
6. y = 2x
7. y = 2
8. x = -3

X. Lines have a distinct property called a constant **slope**. The slope of a line is a number (often a fraction) that describes its "slant" in terms of a vertical rise over a horizontal run.

XI. To find the slope of a line when given two points that the line passes through use this formula: $\boldsymbol{m = (y_2 - y_1)/(x_2 - x_1)}$ where y_2 and x_2 come from one point and y_1 and x_1 come from the other point. This formula can be paraphrased as "the subtraction of the *y's* over the subtraction of the *x's*"; however, you must do your subtractions in the same order.

XII. If a line slants up (going left to right) then it will have a positive slope; if it slants down then it will have a negative slope.

XIII. The slope of a horizontal line is always zero.

XIV. The slope of a vertical line is always undefined (doesn't exist).

XV. The slopes of parallel lines are the same.

XVI. The slopes of perpendicular lines are negative reciprocals. For example, if the slope of a line was 2/3, the slope of any line perpendicular to it would be -3/2.

XVII. <u>Examples:</u>

a. Find the slope of the line passing through (2,3) & (1,10).

b. Find the slope of the line passing through (4,-1) and (-2,7).
c. Find the slope of the line passing through (3,5) & (-1,5).

d. Find the slope of the line passing through (2,2) and (2,-10).

e. Find the slope of a line perpendicular to a line with a slope of 3.

Homework C – Find the slope of the line passing through the following points:

1. Find the slope of the line that passes through (2,3) and (1,-3).
2. Find the slope of the line that passes through (-4,0) and (2,-2).
3. Find the slope of the line that passes through (-2,-5) and (1,0).
4. Find the slope of the line that passes through (2,5) and (4,5).
5. What is the slope of a horizontal line?
6. What is the slope of the line represented by $y = 3x - 4$?
7. What is the slope of the line perpendicular to the one given above?
8. What is the slope of the line represented by $2x + 3y = 12$?

XVIII. The slope of a line and one additional point is all that is needed to sketch the graph of a line.

XIX. A linear equation written in the form Ax + By = C (where A, B, and C are integers) is called Standard Form.

XX. A linear equation written in the form $\boldsymbol{y = mx + b}$ is called slope-intercept form because the value of m is the slope and the value of b is called the y-intercept. For example, if $y = 2x + 3$, then the slope of the line will be 2 and the y-intercept will be 3 [formally written as (0,3)] and this is all that is needed in order to sketch the graph of this equation.

XXI. **Examples: Graph the following linear equations**:

a. y = 2x + 3

b. $y = -x + 2$

c. $y = \frac{1}{4}x - 2$

d. $y = -3x$

e. $y = 6$

f. $x = 4$

g. $2x + 3y = 12$

h. $2x + 3y = 3$

Homework D – Graph Linear Equations; Applications:

1. Sketch the graph of $y = 3x - 1$
2. Sketch the graph of $y = -2x + 4$
3. Sketch the graph of $y = 2x$
4. Sketch the graph of $x = -2$
5. Sketch the graph of $3x - 2y = 6$
6. A salesman makes $30 per hour plus $500 when he sells a certain item. If it takes this salesman 20 hours to complete a sale, how much will he make?

Glossary/Addendum

Unit One

1. **Associative Property:** a + (b + c) = (a + b) + c; a(bc) = (ab)c

2. **Coefficient** – the numeric multiplier of an algebraic term; usually written before the variable. Should the coefficient be zero, then the entire term becomes zero because anything times zero equals zero.

3. **Commutative Property:** a + b = b + a; ab = ba

4. **Contradiction** – an equation that has no solution; an equation where no value for a variable satisfies the given equation.

5. **Evaluate** – the process of replacing variables with supplied values and then using the Order of Operations to find an "output" value of a variable expression.

6. **Identity** – an equation with infinite solutions; an equation where any number satisfies the equation.

7. **Interval Notation** – a way of writing infinite solutions in set notation. $(-\infty,\infty)$ is interval notation for "all solutions" (the answer to an identity). (3,8) is interval notation for all numbers between 3 and 8 (but doesn't include 3 and 8). [-4,10] is interval notation for all numbers from -4 to 10, including -4 and 10. $(-\infty,-2) \cup (4,\infty)$ is interval notation for all numbers less than -2 <u>union</u> all numbers greater than 4, excluding -2 and 4.

8. **Like Terms** – have the same variable raised to the same exponent (power). To add or subtract like terms you only add or subtract the coefficients, leaving the common variable and exponent alone. (Example: $5x^4 + 10x^4 = 15x^4$)

9. **Linear Equation** – an equation where the highest exponent on a variable is one. Does not contain square roots, absolute values, or denominators that have variables.

10. **Simplify** – the process of combining like terms (might have to use the distributive property first).

11. **Solve** – the process of finding a value for a variable that produces a given value; the process of finding a value for a variable that "satisfies" the given equation.

12. **Terms** – an algebraic term usually consists of three parts: a coefficient, a variable (sometimes called the base) and an exponent. Sometimes the coefficient and exponent are both "1" – when this happens we often do not write the "1". Should the exponent be zero, then we also do not write the variable because of the following note: $x^0 = 1$. Thus, a whole number like 5 is considered a term.

13. **Variable Expression** – a collection of unlike algebraic terms; should be written in simplified form – if not then it could be simplified before doing anything else.

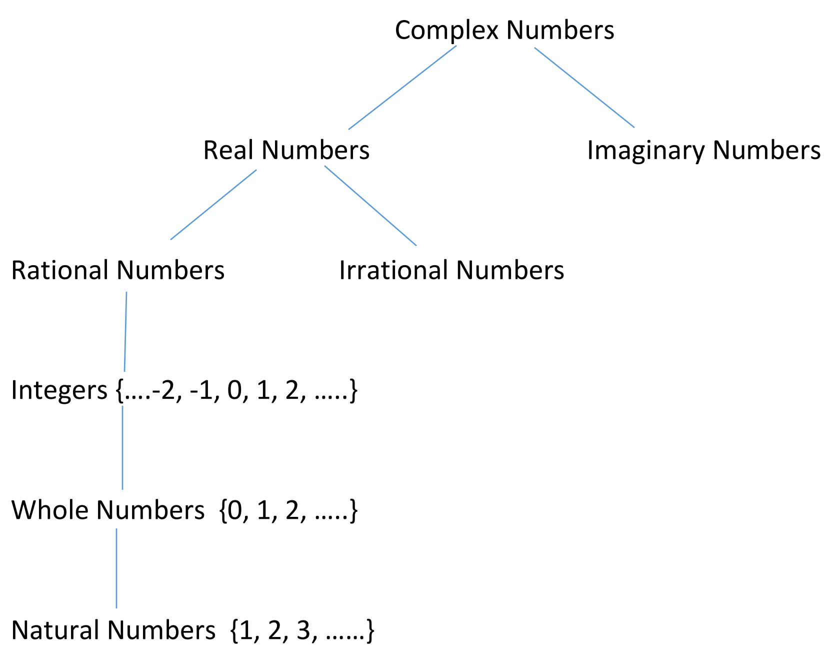
Unit Two

1. **Complex Numbers** – the set of numbers produced when combining the sets of Real and Imaginary numbers.

2. **Irrational Numbers (Radicals)** – numbers that do not have a terminating or repeating decimal representation; typically the square root of non-perfect squares. However, π is also an example.

3. **Imaginary Numbers** – typically the square root of a negative number.

4. **Number Tree:**



5. **Polynomials** – a variable expression that has more than one unlike terms (but uses only one variable). Example: $5x^3 + 6x^2 - 4x - 3$.

6. **Proportions** – two equal ratios. An important property of proportions is that the cross products are equal.

7. **Rational Numbers** – any number that can be written as a fraction (ratio).

8. **Ratios** – a fraction that represents a real-world occurrence. If a classroom has 20 kids and 12 of them are boys, the 12/20 is a ratio that represents the fraction of boys in this classroom. Often ratios are reduced (or simplified); they also can be converted to decimals or percentages.

9. **Real Numbers** – the set of numbers produced when combining the sets of Irrational and Rational numbers. Real Numbers are often represented by the number line.

Unit Three

1. **Horizontal** – usually referring to a line that when graphed goes "flat" to the left and right; a line without a positive or negative slope – the slope would be zero.

2. **Ordered Pairs (Coordinates)** – a notation describing a location (point) on the coordinate plane. In this context the ordered pair (3,8) means something completely different than what it meant when written in Interval Notation. On the coordinate plane, the first value of an ordered pair corresponds to the x-axis and the second value corresponds to the y-axis.

3. **Parallel** – lines that have the same slope. Lines that do not cross.

4. **Perpendicular** – lines that intersect to form right angles.

5. **Vertical** – usually referring to a line that when graphed goes straight up and down; a line that is not horizontal but still has no slant. Because of this we say the slope of a vertical line is "undefined".