MTH123: Algebra I

Students develop algebraic fluency by learning the skills needed to solve equations and perform manipulations with numbers, variables, equations, and inequalities. They also learn concepts central to the abstraction and generalization that algebra makes possible. Students learn to use number properties to simplify expressions or justify statements; describe sets with set notation and find the union and intersection of sets; simplify and evaluate expressions involving variables, fractions, exponents, and radicals; work with integers, rational numbers, and irrational numbers; and graph and solve equations, inequalities, and systems of equations. They learn to determine whether a relation is a function and how to describe its domain and range; use factoring, formulas, and other techniques to solve quadratic and other polynomial equations; formulate and evaluate valid mathematical arguments using various types of reasoning; and translate word problems into mathematical equations and then use the equations to solve the original problems.

Compared to MTH122, this course has a more rigorous pace as well as more challenging assignments and assessments. It also covers additional topics such as number, cost, and mixture problems. Also included are translating functions, higher degree roots (such as cube roots and fourth roots), and using more difficult factoring techniques that are not covered in MTH122.

COURSE LENGTH: Two semesters

MATERIALS: *Algebra I: A Reference Guide and Problem Sets*

PREREQUISITES: K[12] Pre-Algebra B, MTH113: Pre-Algebra, or equivalent

NOTE: Students who have already succeeded in K[12] middle school Algebra 1 should not enroll in this course.

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SEMESTER ONE

Unit 1: Algebra Basics

The English word algebra and the Spanish word *algebrista* both come from the Arabic word *al-jabr*, which means "restoration." A barber in medieval times often called himself an *algebrista*. The *algebrista* also was a bonesetter who restored or fixed bones. Mathematicians today use algebra to solve problems. Algebra can find solutions and "fix" certain problems that you encounter.

- Semester Introduction
- Expressions
- Variables
- Translating Words into Variable Expressions
- Equations
- Translating Words into Equations
- Replacement Sets
- Problem Solving

Unit 2: Properties of Real Numbers

There are many different kinds of numbers. Negative numbers, positive numbers, integers, fractions, and decimals are just a few of the many groups of numbers. What do these varieties of numbers have in common? They all obey the rules of arithmetic. They can be added, subtracted, multiplied, and divided.

- Number Lines
- Sets
- Comparing Expressions
- Number Properties

- Distributive Property
- Algebraic Proof
- Opposites and Absolute Value

Unit 3: Operations with Real Numbers

There are many different kinds of numbers. Negative numbers, positive numbers, integers, fractions, and decimals are just a few of the many groups of numbers. What do these varieties of numbers have in common? They all obey the rules of arithmetic. They can be added, subtracted, multiplied, and divided.

- Addition
- Subtraction
- Multiplication
- Reciprocals and Division
- Applications: Number Problems

Unit 4: Solving Equations

The Greek mathematician Diophantus is often called "the father of algebra." His book *Arithmetica* described the solutions to 130 problems. He did not discover all of these solutions himself, but he did collect many solutions that had been found by Greeks, Egyptians, and Babylonians before him. Some people of long ago obviously enjoyed doing algebra. It also helped them—and can help you—solve many real-world problems.

- Addition and Subtraction Equations
- Multiplication and Division Equations
- Multiple Transformations
- Variables on Both Sides of an Equation
- Transforming Formulas
- Estimating Solutions
- Cost Problems

Unit 5: Solving Inequalities

Every mathematician knows that 5 is less than 7, but when is $y < x$? An inequality symbol can be used to describe how one number compares to another. It can also indicate a relationship between values.

- Inequalities
- Solving Inequalities
- Combined Inequalities
- Absolute Value Equations and Inequalities
- Applications: Inequalities

Unit 6: Applying Fractions

What do a scale drawing, a bicycle's gears, and a sale at the local store all have in common? They all present problems that can be solved using equations with fractions.

- Ratios
- Proportions
- Percents
- Applications: Percents
- Applications: Mixture Problems

Unit 7: Linear Equations and Inequalities

You've probably heard the phrase, "That's where I draw the line!" In algebra, you can take this expression literally. Linear functions and their graphs play an important role in the never-ending quest to model the real world.

- Equations in Two Variables
- Graphs
- Lines and Intercepts
- Slope
- Slope-Intercept Form
- Point-Slope Form
- Parallel and Perpendicular Lines
- Equations from Graphs
- Applications: Linear Models
- Graphing Linear Inequalities
- Inequalities from Graphs

Unit 8: Systems of Equations

When two people meet, they often shake hands or say "hello" to each other. Once they start talking to each other, they can find out what they have in common. What happens when two lines meet? Do they say anything? Probably not, but whenever two lines meet, you know they have at least one point in common. Finding the point at which they meet can help you solve problems in the real world.

- Systems of Equations
- Substitution Method
- Linear Combination
- Linear Combination with Multiplication
- Applications: Systems of Linear Equations
- Systems of Linear Inequalities

Unit 9: Semester Review and Test

- Semester Review
- Semester Test

SEMESTER TWO

Unit 1: Relations and Functions

A solar cell is a little machine that takes in solar energy and puts out electricity. A mathematical function is a machine that takes in a number as an input and produces another number as an output. There are many kinds of functions. Some have graphs that look like lines, while others have graphs that curve like a parabola. Functions can take other forms as well. Not every function has a graph that looks like a line or a parabola. Not every function has an equation. The important thing to remember is that if you put any valid input into a function, you will get a single result out of it.

- Semester Introduction
- Relations
- Functions
- Function Equations
- Absolute Value Functions
- Direct Linear Variation

- Quadratic Variation
- Inverse Variation
- Translating Functions

Unit 2: Rationals, Irrationals, and Radicals

Are rational numbers very levelheaded? Are irrational numbers hard to reason with? Not really, but rational and irrational numbers have things in common and things that make them different.

- Rational Numbers
- Terminating and Repeating Numbers
- Square Roots
- Irrational Numbers
- Evaluating and Estimating Square Roots
- Radicals with Variables
- Roots of Equations
- The Pythagorean Theorem
- Higher Roots

Unit 3: Working with Polynomials

Just as a train is built from linking railcars together, a polynomial is built by bringing terms together and linking them with plus or minus signs. You can perform basic operations on polynomials in the same way that you add, subtract, multiply, and divide numbers.

- Overview of Polynomials
- Adding and Subtracting Polynomials
- Multiplying Monomials
- Multiplying Polynomials by Monomials
- Multiplying Polynomials
- The FOIL Method

Unit 4: Factoring Polynomials

A polynomial is an expression that has variables that represent numbers. A number can be factored, so you should be able to factor a polynomial, right? Sometimes you can and sometimes you can't. Finding ways to write a polynomial as a product of factors can be quite useful.

- Factoring Integers
- Dividing Monomials
- Factoring Common Factors
- Dividing Polynomials by Monomials
- Factoring Perfect Squares
- Factoring Differences of Squares
- Factoring Quadratic Trinomials
- Factoring Quadratic Trinomials, $a \neq 1$
- Factoring Completely
- Finding Roots of Equations

Unit 5: Quadratic Equations

Solving equations can help you find answers to many kinds of problems in your daily life. Linear equations usually have one solution, but what about quadratic equations? How can you solve them and what do the solutions look like?

- Solving Perfect Square Equations
- Completing the Square
- The Quadratic Formula
- Solving Quadratic Equations
- Equations and Graphs: Roots and Intercepts
- Applications: Area Problems
- Applications: Projectile Motion

Unit 6: Rational Expressions

A fraction always has a number in the numerator and in the denominator. However, those numbers can actually be expressions that represent numbers, which means you can do all sorts of interesting things with fractions. Fractions with variable expressions in the numerator and denominator can help you solve many kinds of problems.

- Simplifying Rational Expressions
- Multiplying Rational Expressions
- Dividing Rational Expressions
- Like Denominators
- Adding and Subtracting Rational Expressions

Unit 7: Logic and Reasoning

Professionals use logical reasoning in a variety of ways. Just as lawyers use logical reasoning to formulate convincing arguments, mathematicians use logical reasoning to formulate and prove theorems. Once you have mastered the uses of inductive and deductive reasoning, you will be able to make and understand arguments in many areas.

- Reasoning and Arguments
- Hypothesis and Conclusion
- Forms of Conditional Statements
- Inductive and Deductive Reasoning
- Analyzing and Writing Proofs
- Counterexample

Unit 8: Semester Review and Test

- Semester Review
- Semester Test