Multiloop and Multivariable Control

- Process Interactions and Control Loop Interactions
- Pairing of Controlled and Manipulated Variables
- Singular Value Analysis
- Tuning of Multiloop PID Control Systems
- Decoupling and Multivariable Control Strategies

Control of Multivariable Processes

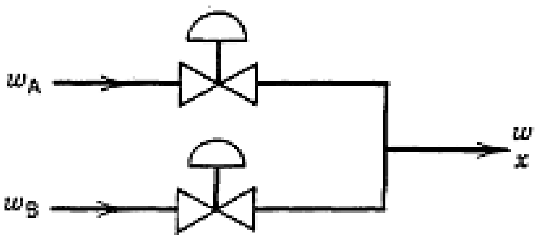
- Control systems that have only one controlled variable and one manipulated variable.
  - ➢ Single-input, single-output (**SISO**) control system
  - ➢ Single-loop control system

- In practical control problems there typically are a number of process variables which must be controlled and a number of variables which can be manipulated.
  - ➢ Multi-input, multi-output (**MIMO**) control system

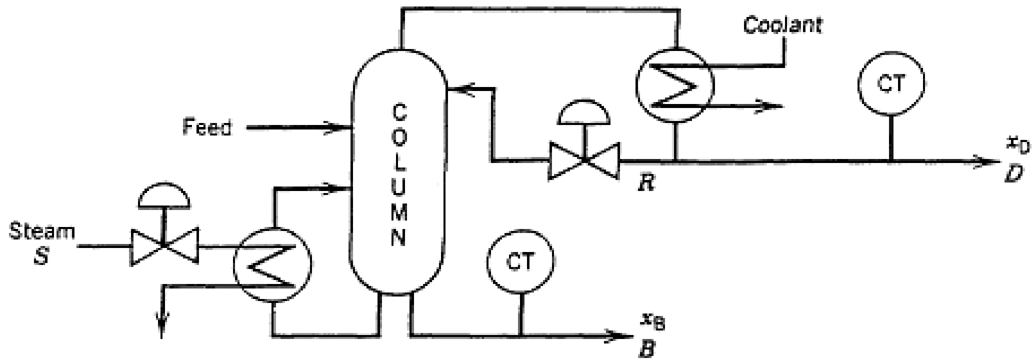
Example: product *quality* and *throughput* must usually be controlled.

Note the "**process interactions**" between controlled and manipulated variables.

Several simple physical examples



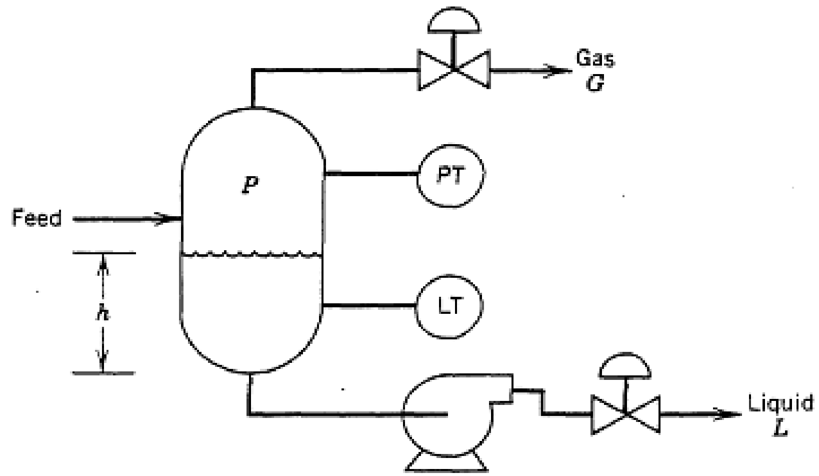
(a) In-line blending system



(b) Distillation column

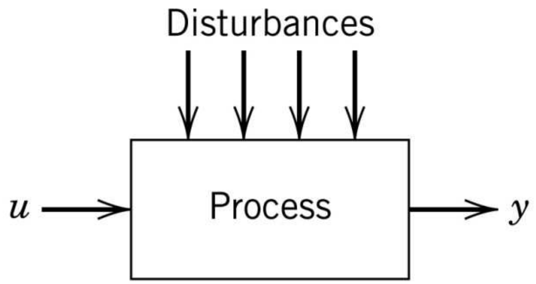
Process interactions :

Each manipulated variable can affect ***both*** controlled variables



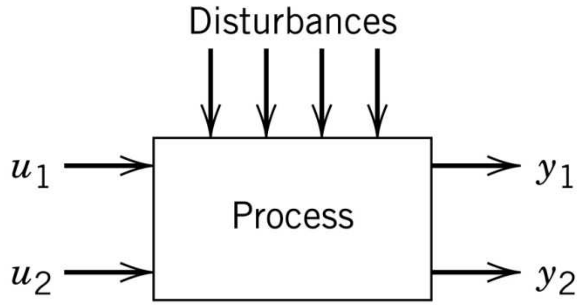
(c) Gas–liquid separator

SISO

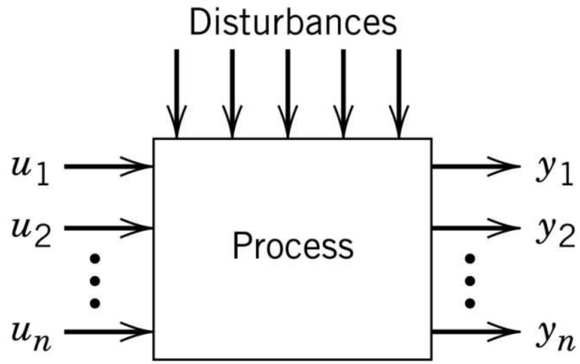


(a) Single-input, single-output proce
with multiple disturbances

MIMO



(b) Multiple-input, multiple-output process (2×2)



(c) Multiple-input, multiple-output process $(n \times n)$

- In this chapter we will be concerned with characterizing process interactions and selecting an appropriate multiloop control configuration.
- If process interactions are significant, even the best multiloop control system may not provide satisfactory control.
- In these situations there are incentives for considering multivariable control strategies.

Definitions:

- **Multiloop control:** Each manipulated variable depends on only a single controlled variable, i.e., a set of conventional feedback controllers.
- **Multivariable Control:** Each manipulated variable can depend on two or more of the controlled variables.

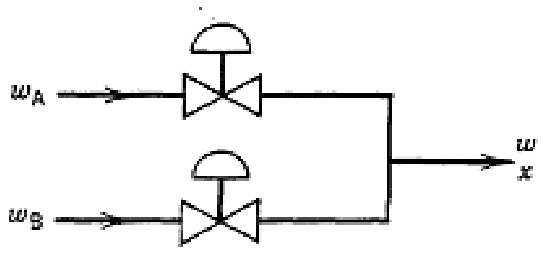
***Examples**:* decoupling control, model predictive control

Multiloop Control Strategy

- Typical industrial approach
- Consists of using several standard FB controllers (e.g., PID), one for each controlled variable.

- **Control system design**
  1. Select controlled and manipulated variables.
  2. Select pairing of controlled and manipulated variables.
  3. Specify types of FB controllers.

Example: 2 x 2 system



(a) In-line blending system

Two possible controller pairings:

U_1 *with* Y_1, U_2 *with* Y_2 (1-1/2-2 pairing)

or

U_1 *with* Y_2, U_2 *with* Y_1 (1-2/2-1 pairing)

Note: For n x n system, $n!$ possible pairing configurations.

Process Interactions

Transfer Function Model (2 x 2 system)

- Two controlled variables and two manipulated variables (4 transfer functions required)

$$\frac{Y_1(s)}{U_1(s)} = G_{p11}(s), \quad \frac{Y_1(s)}{U_2(s)} = G_{p12}(s)$$
$$\frac{Y_2(s)}{U_1(s)} = G_{p21}(s), \quad \frac{Y_2(s)}{U_2(s)} = G_{p22}(s) \qquad (18-1)$$

- Thus, the input-output relations for the process can be written as:

$$Y_1(s) = G_{P11}(s)U_1(s) + G_{P12}(s)U_2(s) \qquad (18-2)$$
$$Y_2(s) = G_{P21}(s)U_1(s) + G_{P22}(s)U_2(s) \qquad (18-3)$$

Multiloop and Multivariable Control

In vector-matrix notation as

$$\boldsymbol{Y}(s)=\boldsymbol{G}_p(s)\boldsymbol{U}(s) \qquad (18-4)$$

where $\boldsymbol{Y}$(s) and $\boldsymbol{U}$(s) are vectors

$$\boldsymbol{Y}(s)=\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} \quad \boldsymbol{U}(s)=\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \qquad (18-5)$$

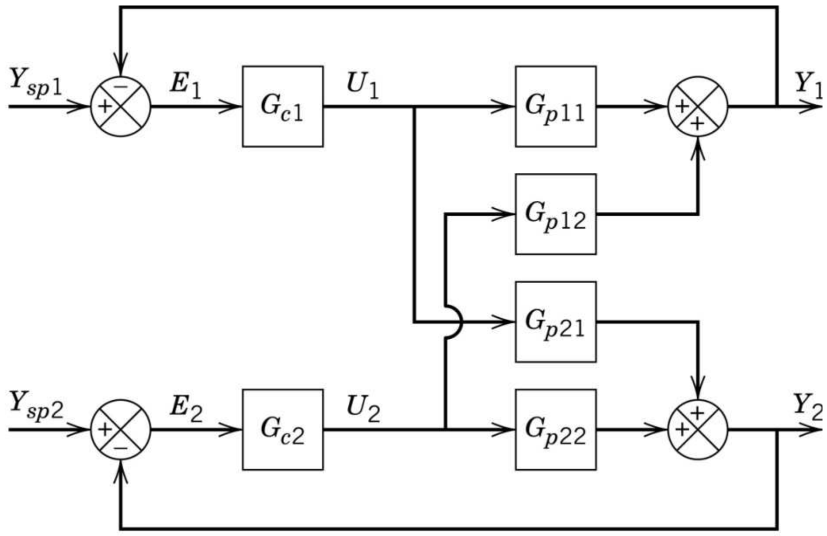
And $\boldsymbol{G}_p(s)$ is the transfer function matrix for the process

$$\boldsymbol{G}_p(s)=\begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix} \qquad (18-6)$$

The steady-state process transfer matrix (s=0) is called the process *gain matrix* ***K***

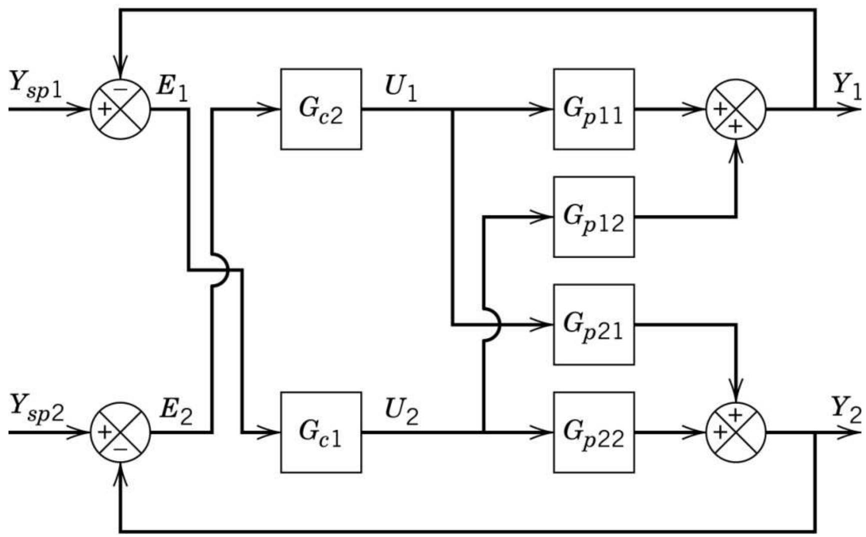
$$\boldsymbol{K}=\begin{bmatrix} G_{p11}(0) & G_{p12}(0) \\ G_{p21}(0) & G_{p22}(0) \end{bmatrix}=\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Block Diagram for 2x2 Multiloop Control



(a) 1–1/2–2 controller pairing

1-1/2-2 control scheme



(b) 1–2/2–1 controller pairing

1-2/2-1 control scheme

Multiloop and Multivariable Control

Control-loop Interactions

- Process interactions may induce undesirable interactions between two or more control loops.

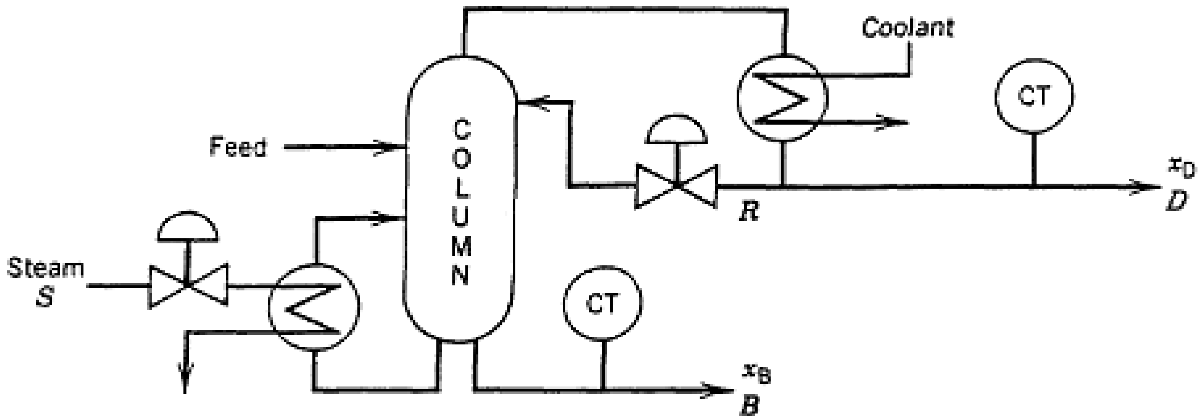
Example: 2 x 2 system

Change in U_1 has two effects on Y_1

(1) direct effect : $U_1 \rightarrow G_{p11} \rightarrow Y_1$

(2) indirect effect :

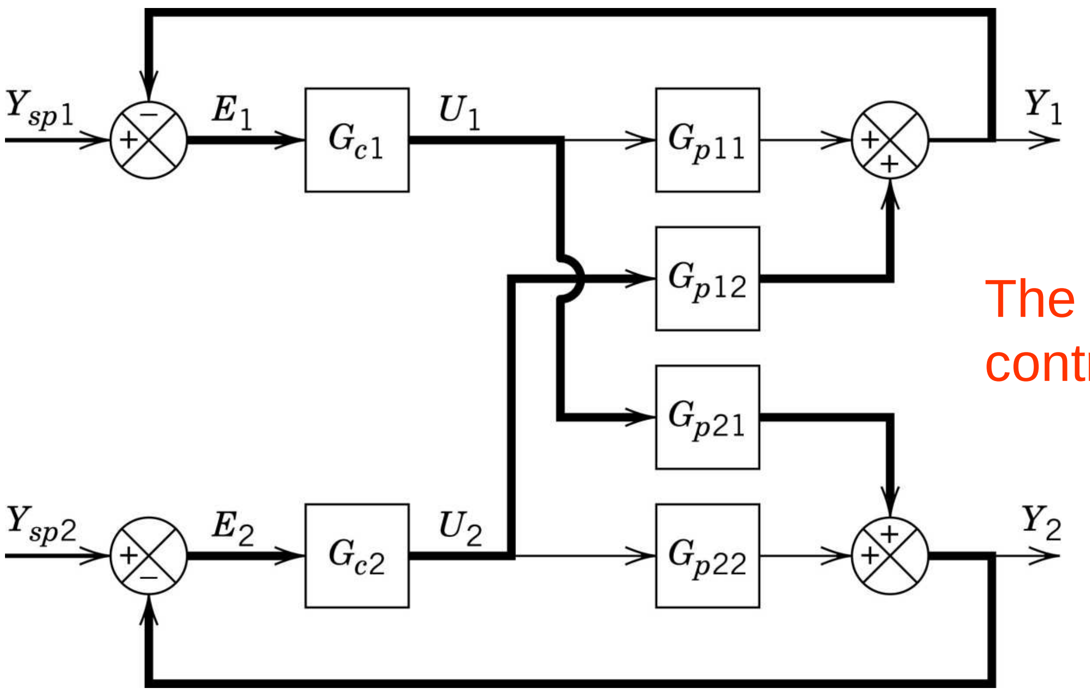
$$U_1 \rightarrow G_{p21} \rightarrow Y_2 \rightarrow G_{c2} \rightarrow U_2 \rightarrow G_{p12} \rightarrow Y_1$$



(b) Distillation column

- Control loop interactions are due to the presence of a third feedback loop.

Example: 1-1/2-2 pairing



The hidden feedback control loop (in dark lines)

- Problems arising from control loop interactions
  i. Closed-loop system may become destabilized.
  ii. Controller tuning becomes more difficult.

Block Diagram Analysis

For the multiloop control configuration, the transfer function between a controlled and a manipulated variable depends on whether the other feedback control loops are open or closed.

Example: 2 x 2 system, 1-1/2 -2 pairing

From block diagram algebra we can show

$$\frac{Y_1(s)}{U_1(s)} = G_{p11}(s) \qquad \text{(second loop open)}$$

$$\frac{Y_1(s)}{U_1(s)} = G_{p11} - \frac{G_{p12}G_{p21}G_{c2}}{1+G_{c2}G_{p22}} \qquad \text{(second loop closed)}$$

Note that the last expression contains G_{C2}.

➔ The two controllers should not be tuned independently

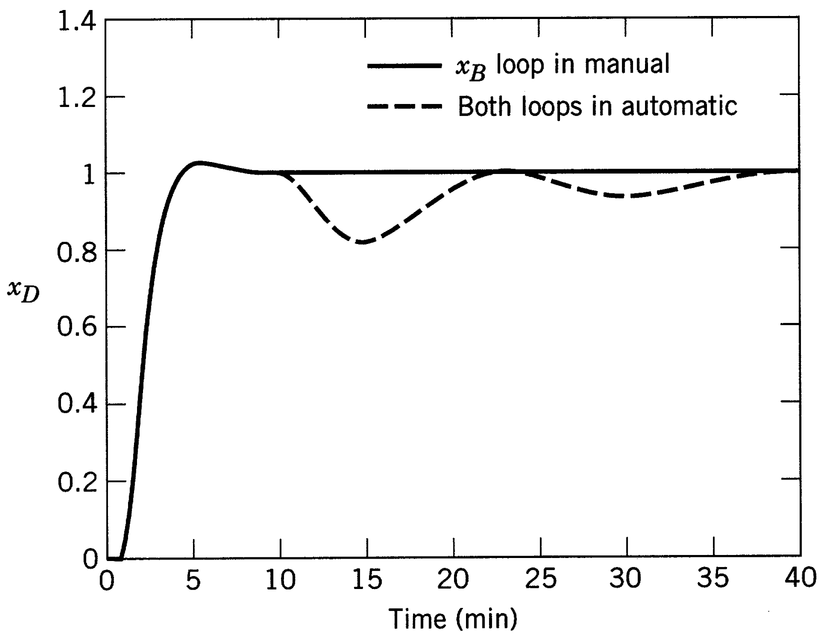
Example: Empirical model of a distillation column

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

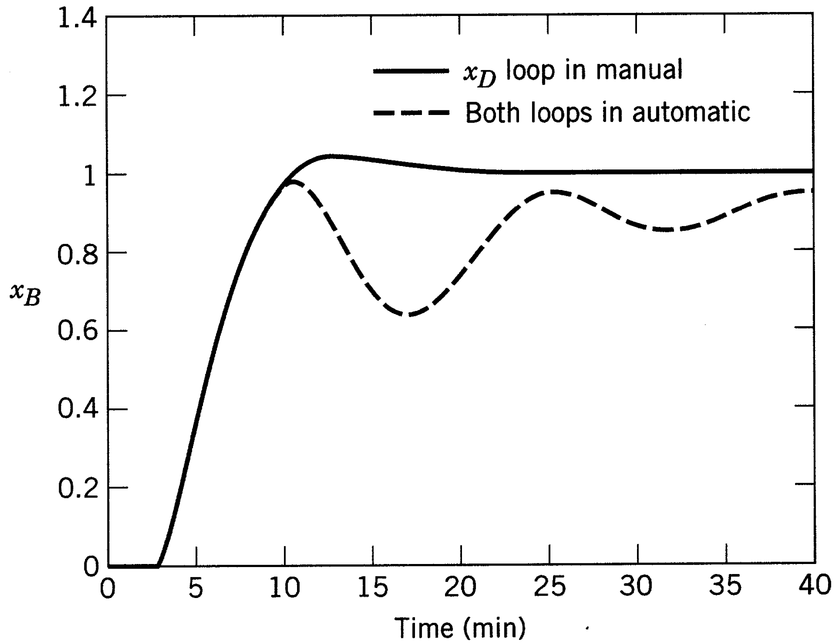
Single-loop ITAE tuning

Pairing	Kc	τ_I
x_D - R	0.604	16.37
x_B - S	-0.127	14.46

x_D set-point response



x_B set-point response



Closed-Loop Stability

- Relation between controlled variables and set-points

$$Y_1 = \Gamma_{11} Y_{sp1} + \Gamma_{12} Y_{sp2}$$

$$Y_2 = \Gamma_{21} Y_{sp1} + \Gamma_{22} Y_{sp2}$$

- Closed-loop transfer functions

$$\Gamma_{11} = \frac{G_{c1}G_{p11} + G_{c1}G_{c2}(G_{p11}G_{p22} - G_{p12}G_{p21})}{\Delta(s)}$$

$$\Gamma_{12} = \frac{G_{c2}G_{p12}}{\Delta(s)}$$

$$\Gamma_{21} = \frac{G_{c1}G_{p21}}{\Delta(s)}$$

$$\Gamma_{22} = \frac{G_{c2}G_{p22} + G_{c1}G_{c2}(G_{p11}G_{p22} - G_{p12}G_{p21})}{\Delta(s)}$$

where $\Delta(s) = (1 + G_{c1}G_{p11})(1 + G_{c2}G_{p22}) - G_{c1}G_{c2}G_{p12}G_{p21}$

- Characteristic equation

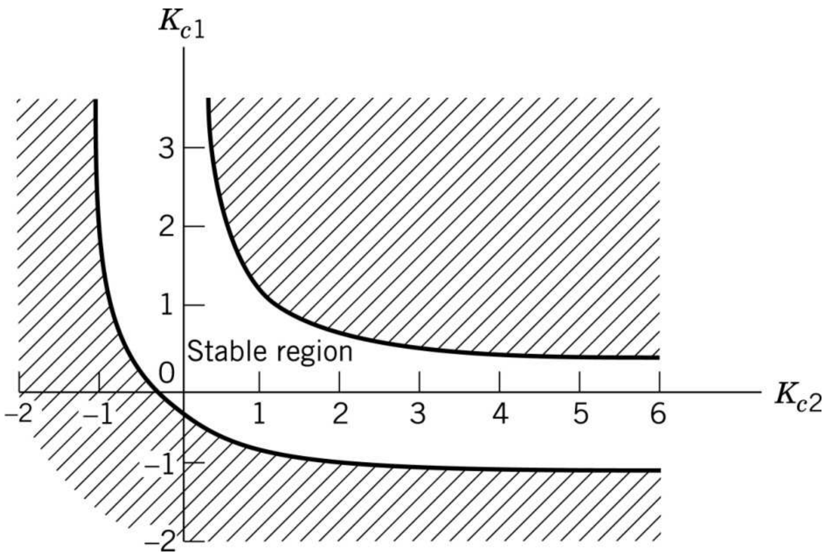
$$(1 + G_{c1}G_{p11})(1 + G_{c2}G_{p22}) - G_{c1}G_{c2}G_{p12}G_{p21} = 0$$

Example: Two P controllers are used to control the process

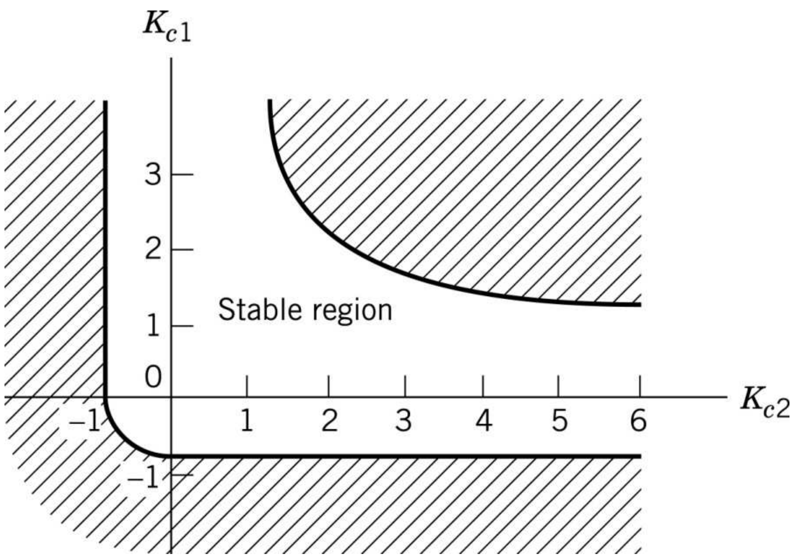
$$G_p(s) = \begin{bmatrix} \dfrac{2}{10s+1} & \dfrac{1.5}{s+1} \\ \dfrac{1.5}{s+1} & \dfrac{2}{10s+1} \end{bmatrix}$$

Stable region for K_{c1} and K_{c2}

1-1/2-2 pairing



1-2/2-1 pairing

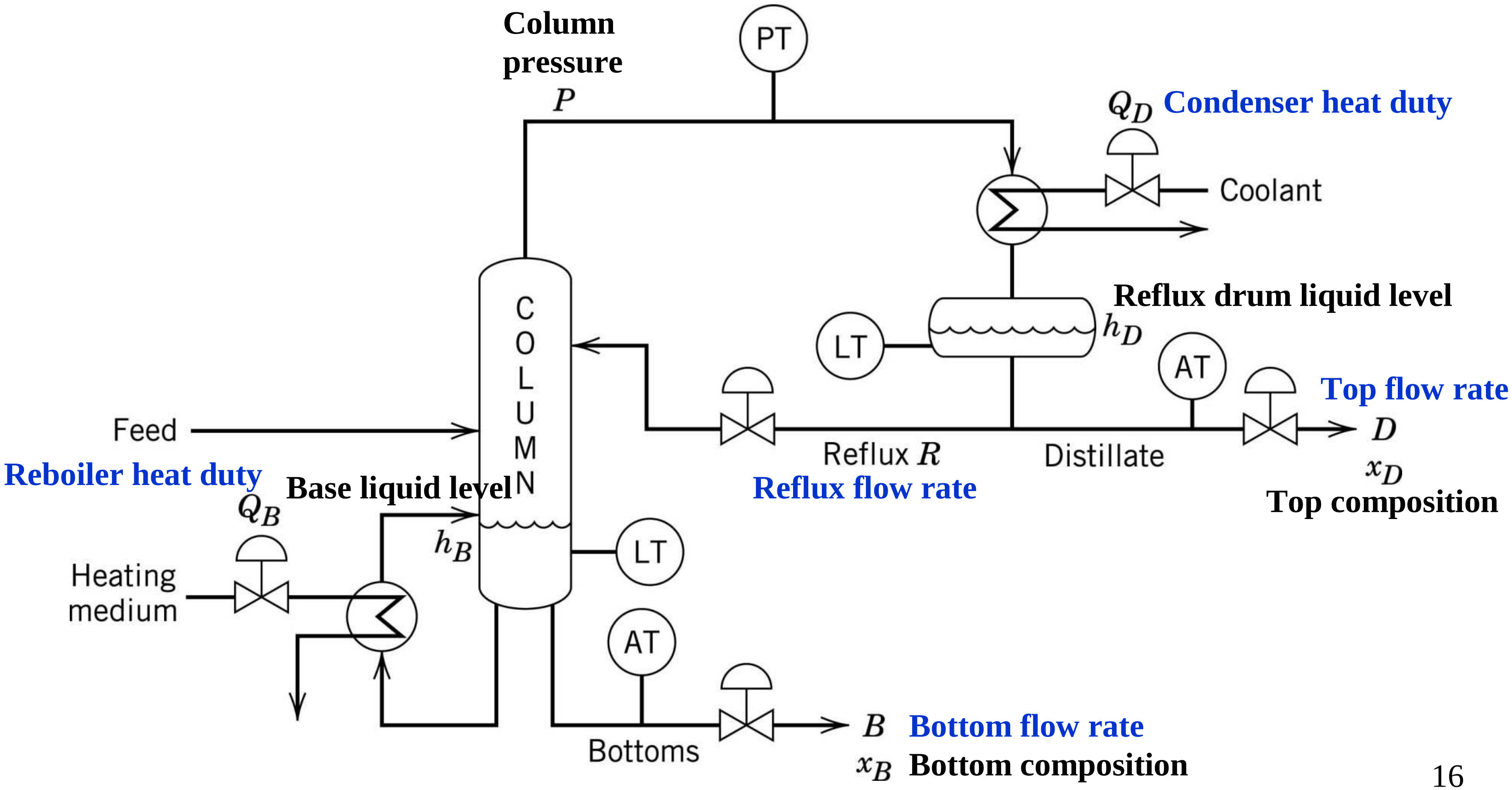


Pairing of Controlled and Manipulated Variables

- **Control of distillation column**
  - Controlled variables: x_D, x_B, P, h_D, h_B
  - Manipulated variables: D, B, R, Q_D, Q_B

Possible multiloop control strategies

= 5! = 120



- **One of the practical pairing**

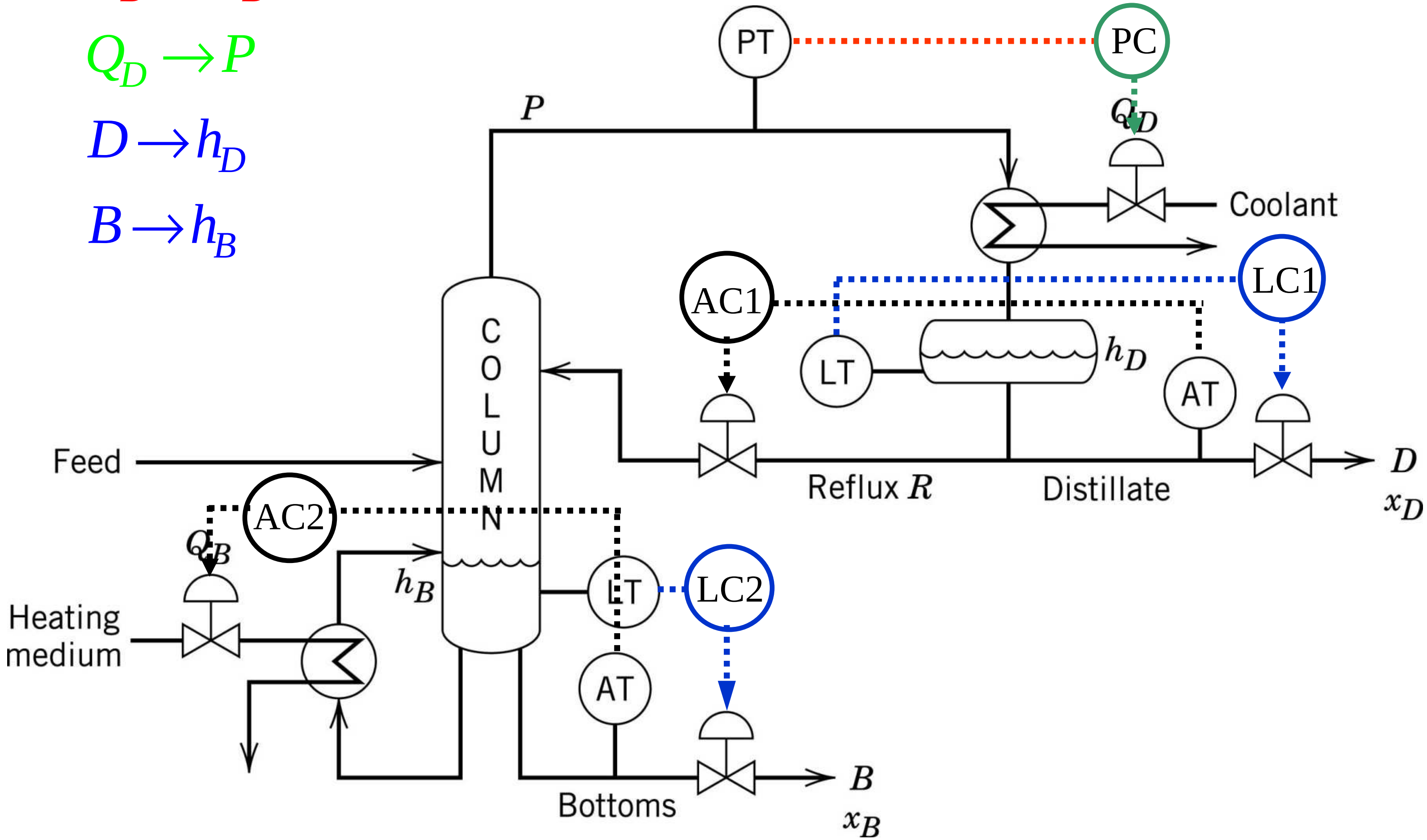
$R \rightarrow x_D$

$Q_B \rightarrow x_B$

$Q_D \rightarrow P$

$D \rightarrow h_D$

$B \rightarrow h_B$



Relative Gain Array (RGA)
(Bristol, 1966)

- **Provides two types of useful information:**
  1. Measure of process interactions
  2. Recommendation about best pairing of controlled and manipulated variables.

- **Requires knowledge of steady-state gains but not process dynamics.**

Multiloop and Multivariable Control

Example of RGA Analysis: 2 x 2 system

- Steady-state process model

$$y_1 = K_{11}u_1 + K_{12}u_2$$
$$y_2 = K_{21}u_1 + K_{22}u_2$$

or $\boldsymbol{y} = \boldsymbol{K}\boldsymbol{u}$

The RGA, $\boldsymbol{\Lambda}$, is defined as:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

where the relative gain, λ_{ij}, relates the i^{th} controlled variable and the j^{th} manipulated variable

$$\lambda_{ij} \triangleq \frac{\left(\partial y_i / \partial u_j\right)_u}{\left(\partial y_i / \partial u_j\right)_y} = \frac{\textbf{open-loop gain}}{\textbf{closed-loop gain}}$$

$\left(\partial y_i / \partial u_j\right)_u$: partial derivative evaluated with all of the manipulated variables except u_j held constant (K_{ij})

$\left(\partial y_i / \partial u_j\right)_y$: partial derivative evaluated with all of the controlled variables except y_i held constant

Scaling Properties:

i. λ_{ij} is dimensionless

ii. $\sum_i \lambda_{ij} = \sum_j \lambda_{ij} = 1$

For a 2 x 2 system,

$$\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}, \qquad \lambda_{12} = 1 - \lambda_{11} = \lambda_{21}$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \qquad (\lambda = \lambda_{11})$$

Recommended Controller Pairing

It corresponds to the λ_{ij} which have the largest **positive** values that are **closest to one**.

In general:

1. Pairings which correspond to negative pairings should not be selected.
2. Otherwise, choose the pairing which has λ_{ij} closest to one.

Examples:

Process Gain Matrix, $\boldsymbol{K}$:		Relative Gain Array, $\boldsymbol{\Lambda}$:
$\begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix}$	$\Rightarrow$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\begin{bmatrix} 0 & K_{12} \\ K_{21} & 0 \end{bmatrix}$	$\Rightarrow$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\begin{bmatrix} K_{11} & K_{12} \\ 0 & K_{22} \end{bmatrix}$	$\Rightarrow$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\begin{bmatrix} K_{11} & 0 \\ K_{21} & K_{22} \end{bmatrix}$	$\Rightarrow$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

For 2 x 2 systems:

$$y_1 = K_{11}u_1 + K_{12}u_2$$
$$y_2 = K_{21}u_1 + K_{22}u_2$$

$$\lambda_{11} = \frac{1}{1 - \dfrac{K_{12}K_{21}}{K_{11}K_{22}}}, \qquad \lambda_{12} = 1 - \lambda_{11} = \lambda_{21}$$

Example 1:

$$\boldsymbol{K} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1.5 \\ 1.5 & 2 \end{bmatrix}$$

$$\therefore \quad \Lambda = \begin{bmatrix} 2.29 & -1.29 \\ -1.29 & 2.29 \end{bmatrix}$$

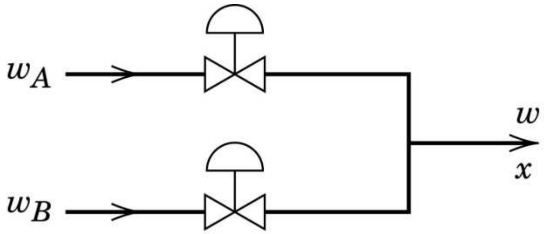
$\therefore$ Recommended pairing is Y_1 and U_1, Y_2 and U_2.

Example 2:

$$\boldsymbol{K} = \begin{bmatrix} -2 & 1.5 \\ 1.5 & 2 \end{bmatrix} \Rightarrow \quad \Lambda = \begin{bmatrix} 0.64 & 0.36 \\ 0.36 & 0.64 \end{bmatrix}$$

$\therefore$ Recommended pairing is Y_1 with U_1 and Y_2 with U_2.

EXAMPLE: Blending System



Controlled variables: w and x
Manipulated variables: w_A and w_B

Steady-state model:

$$w = w_A + w_B$$

$$xw = w_A \quad \Rightarrow \quad x = \frac{w_A}{w_A + w_B}$$

Steady-state gain matrix:

$$\boldsymbol{K} = \begin{bmatrix} 1 & 1 \\ \dfrac{1-x}{w} & \dfrac{-x}{w} \end{bmatrix}$$

The RGA is:

$$\boldsymbol{\Lambda} = \begin{array}{cc} & \begin{array}{cc} w_A & \quad w_B \end{array} \\ \begin{array}{c} w \\ x \end{array} & \begin{bmatrix} x & 1-x \\ 1-x & x \end{bmatrix} \end{array}$$

Note that each relative gain is between 0 and 1. The recommended controller pairing depends on the desired product composition *x*.

For $x = 0.4$, w-w_B / x-w_A (large interactions)
For x = 0.9, w-w_A / x-w_B (small interactions)

Multiloop and Multivariable Control

RGA for Higher-Order Systems

For a *n x n* system,

$$\Lambda = \begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n1} & \cdots & \lambda_{nn} \end{bmatrix} \end{array} \qquad (18-25)$$

Each λ_{ij} can be calculated from the relation,

$$\lambda_{ij} = K_{ij} H_{ij} \qquad (18-37)$$

where K_{ij} is the *(i,j)* -element of the steady-state gain $\boldsymbol{K}$ matrix,

H_{ij} is the *(i,j)* -element of the $\boldsymbol{H} = \left(\boldsymbol{K}^{-1}\right)^T$.

In matrix form, $\boldsymbol{\Lambda} = \boldsymbol{K} \otimes \boldsymbol{H}$

$\otimes$: **Schur product**
(element by element multiplication)

Note : $\boldsymbol{\Lambda} \neq \boldsymbol{KH}$

Example: Hydrocracker

The RGA for a hydrocracker has been reported as,

$$\Lambda = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} & \begin{bmatrix} 0.931 & 0.150 & 0.080 & -0.164 \\ -0.011 & -0.429 & 0.286 & 1.154 \\ -0.135 & 3.314 & -0.270 & -1.910 \\ 0.215 & -2.030 & 0.900 & 1.919 \end{bmatrix} \end{array}$$

Recommended controller pairing?

Dynamic Consideration

An important disadvantage of RGA approach is that it ignores process dynamics

Example:

$$G_p(s) = \begin{bmatrix} \dfrac{-2e^{-s}}{10s+1} & \dfrac{1.5e^{-s}}{s+1} \\ \dfrac{1.5e^{-s}}{s+1} & \dfrac{2}{10s+1}e^{-s} \end{bmatrix}$$

$$\lambda_{11} = 0.64$$

Recommended controller pairing?

Singular Value Analysis

- *Any real m x n matrix can be factored as,*

$$\boldsymbol{K} = \boldsymbol{W}\,\boldsymbol{\Sigma}\boldsymbol{V}^T$$

- *Matrix* $\boldsymbol{\Sigma}$ *is a diagonal matrix of singular values:*

$$\boldsymbol{\Sigma} = \mathrm{diag}\,(\sigma_1, \sigma_2, \ldots, \sigma_r)$$

- The singular values are the positive square roots of the eigenvalues of $\boldsymbol{K}^T\boldsymbol{K}$ (r = the rank of $\boldsymbol{K}^T\boldsymbol{K}$).
- The columns of matrices $\boldsymbol{W}$ and $\boldsymbol{V}$ are *orthonormal*. Thus,

$$\boldsymbol{W}\boldsymbol{W}^T = \boldsymbol{I} \quad \text{and} \quad \boldsymbol{V}\boldsymbol{V}^T = \boldsymbol{I}$$

- Can calculate $\boldsymbol{\Sigma}$, $\boldsymbol{W}$, and $\boldsymbol{V}$ using MATLAB command, *svd*.
- *Condition number* (CN) is defined to be the ratio of the largest to the smallest singular value,

$$CN \triangleq \frac{\sigma_1}{\sigma_r}$$

- A large value of *CN* indicates that $\boldsymbol{K}$ is ill-conditioned.

Condition Number

- CN is a measure of sensitivity of the matrix properties to changes in individual elements.
- Consider the RGA for a 2x2 process,

$$\boldsymbol{K} = \begin{bmatrix} 1 & 0 \\ 10 & 1 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{\Lambda} = \boldsymbol{I}$$

- If K_{12} changes from 0 to 0.1, then $\boldsymbol{K}$ becomes a singular matrix, which corresponds to a process that is difficult to control.
- RGA and SVA used together can indicate whether a process is easy (or difficult) to control.

$$\Sigma(\boldsymbol{K}) = \begin{bmatrix} 10.1 & 0 \\ 0 & 0.1 \end{bmatrix} \qquad \text{CN} = 101$$

- $\boldsymbol{K}$ is poorly conditioned when CN is a large number (e.g., > 10). Thus small changes in the model for this process can make it very difficult to control.

Selection of Inputs and Outputs

- Arrange the singular values in order of largest to smallest and look for any $\sigma_i/\sigma_{i-1} > 10$; then one or more inputs (or outputs) can be deleted.
- Delete one row and one column of $\boldsymbol{K}$ at a time and evaluate the properties of the reduced gain matrix.

- **Example:**

$$\boldsymbol{K} = \begin{bmatrix} 0.48 & 0.90 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{bmatrix}$$

$$\boldsymbol{W} = \begin{bmatrix} 0.5714 & 0.3766 & 0.7292 \\ 0.6035 & 0.4093 & -0.6843 \\ -0.5561 & 0.8311 & 0.0066 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1.618 & 0 & 0 \\ 0 & 1.143 & 0 \\ 0 & 0 & 0.0097 \end{bmatrix} \qquad \boldsymbol{V} = \begin{bmatrix} 0.0541 & 0.9984 & 0.0151 \\ 0.9985 & -0.0540 & -0.0068 \\ -0.0060 & 0.0154 & -0.9999 \end{bmatrix}$$

- CN = 166.5 (σ_1/σ_3)

The RGA is:

$$\boldsymbol{\Lambda} = \begin{bmatrix} -2.4376 & 3.0241 & 0.4135 \\ 1.2211 & -0.7617 & 0.5407 \\ 2.2165 & -1.2623 & 0.0458 \end{bmatrix}$$

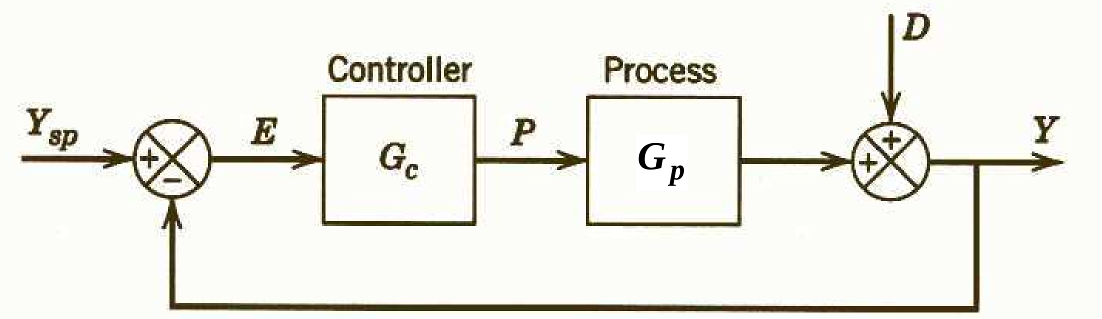
Preliminary pairing: y_1-u_2, y_2-u_3, y_3-u_1.

CN suggests only two output variables can be controlled. Eliminate one input and one output (3x3$\rightarrow$2x2).

Table 18.3 CN and λ for Different 2×2 Pairings, Example 18.7

Pairing Number	Controlled Variables	Manipulated Variables	CN	λ
1	y_1, y_2	u_1, u_2	184	39.0
2	y_1, y_2	u_1, u_3	72.0	0.552
3	y_1, y_2	u_2, u_3	133	0.558
4	y_1, y_3	u_2, u_1	1.51	0.640
5	y_1, y_3	u_1, u_3	69.4	0.640
6	y_1, y_3	u_2, u_3	139	1.463
7	y_2, y_3	u_2, u_1	1.45	0.634
8	y_2, y_3	u_1, u_3	338	3.25
9	y_2, y_3	u_2, u_3	67.9	0.714

Matrix Notation for Multiloop Control Systems



	Single loop	Multi-loop
CLTF	$Y = \dfrac{G_p G_c}{1+G_p G_c} Y_{sp}$	$\boldsymbol{Y} = \left(\boldsymbol{I} + \boldsymbol{G_p G_c}\right)^{-1} \boldsymbol{G_p G_c Y_{sp}}$
Characteristic equation	$1 + G_p G_c = 0$	$\det\left(\boldsymbol{I} + \boldsymbol{G_p G_c}\right) = 0$

$\boldsymbol{Y}$: (n x 1) vector of control variables
$\boldsymbol{Y_{sp}}$: (n x 1) vector of set-points
$\boldsymbol{G_p}$: (n x n) matrix of process transfer functions
$\boldsymbol{G_c}$: (n x n) diagonal matrix of controller transfer functions

Tuning of Multiloop PID Control Systems

- **Detuning method**
  - Each controller is first designed, ignoring process interactions
  - Then interactions are taken into account by detuning each controller
    - More conservative controller settings (decrease controller gain, increase integral time)

  - **Tyreus-Luyben (TL) tuning**

Ziegler-Nichols	K_c	τ_I	τ_D
P	$0.5K_{cu}$	—	—
PI	$0.45K_{cu}$	$P_u/1.2$	—
PID	$0.6K_{cu}$	$P_u/2$	$P_u/8$
Tyreus-Luyben†	**K_c**	**τ_I**	**τ_D**
PI	$0.31K_{cu}$	$2.2P_u$	—
PID	$0.45K_{cu}$	$2.2P_u$	$P_u/6.3$

† Luyben and Luyben (1997).

Biggest log-modulus tuning (BLT) method
(Luyben, 1986)

- **Log-modulus :** a robustness measure of control systems
  - ***Single loop***

$$L_c = 20\log\left|\frac{G_p G_c}{1+G_p G_c}\right| = 20\log\left|\frac{G}{1+G}\right|$$

$$L_c^{\max} = \max_{\omega} L_c = \max_{\omega}\left\{20\log\left|\frac{G}{1+G}\right|\right\}$$

A specification of $L_c^{\max} = 2\ \text{dB}$ has been suggested.

  - ***Multi-loop***

Define $W = -1 + \det\left(\boldsymbol{I} + \boldsymbol{G_p G_c}\right)$

$$L_c = 20\log\left|\frac{W}{1+W}\right|$$

Luyben suggest that $L_c^{\max} = \max_{\omega} L_c = 2n$

where n is the dimension of the multivariable system.

Tuning Procedure of BLT method

1. Calculate Z-N PI controller settings for each control loop

$$K_{c,ZN} = 0.45 K_{cu}, \quad \tau_{I,ZN} = P_u / 1.2$$

2. Assume a factor ***F***; typical values between 2 and 5
3. Calculate new values of controller parameters by

$$K_{ci} = \frac{K_{ci,ZN}}{F}, \quad \tau_{Ii} = F\,\tau_{Ii,ZN}\ ; \quad i = 1, 2, \cdots, n$$

(detuning)

4. Compute $W = -1 + \det\left(\boldsymbol{I} + \boldsymbol{G_p G_c}\right)$ for $0 \le \omega < \infty$

   for example, 2x2 system

$$\det\left(\boldsymbol{I} + \boldsymbol{G_p G_c}\right) = 1 + G_{c1}G_{p11} + G_{c2}G_{p22} + G_{c1}G_{c2}\left(G_{p11}G_{p22} - G_{p12}G_{p21}\right)$$

5. Determine $L_c^{\max} = \max_{\omega}\left\{20\log\left|\frac{W}{1+W}\right|\right\}$

6. If $L_c^{\max} \neq 2n$, select a new value of F and return to step 2 until $L_c^{\max} = 2n$

Multiloop IMC Controller

- Design IMC controller based in diagonal process transfer functions

$$\boldsymbol{G_p} = \begin{bmatrix} G_{p11} & \cdots & G_{p1n} \\ \vdots & \ddots & \vdots \\ G_{pn1} & \cdots & G_{pnn} \end{bmatrix}$$

- The IMC controller is designed as

$$\boldsymbol{G_c} = \mathrm{diag}\left[G_{c1} \quad G_{c2} \quad \vdots \quad G_{cn}\right]$$

with $$G_{ci} = G_{pii-}^{-1} f_i \qquad i = 1, 2, \cdots, n$$

- Since the off-diagonal terms of $\boldsymbol{G_p}$ have been dropped, modeling error are always present.

Alternative Strategies for Dealing with Undesirable Control Loop Interactions

1. "Detune" one or more FB controllers.
2. Select different manipulated or controlled variables.
   e.g., nonlinear functions of original variables
3. Use a decoupling control scheme.
4. Use some other type of multivariable control scheme.

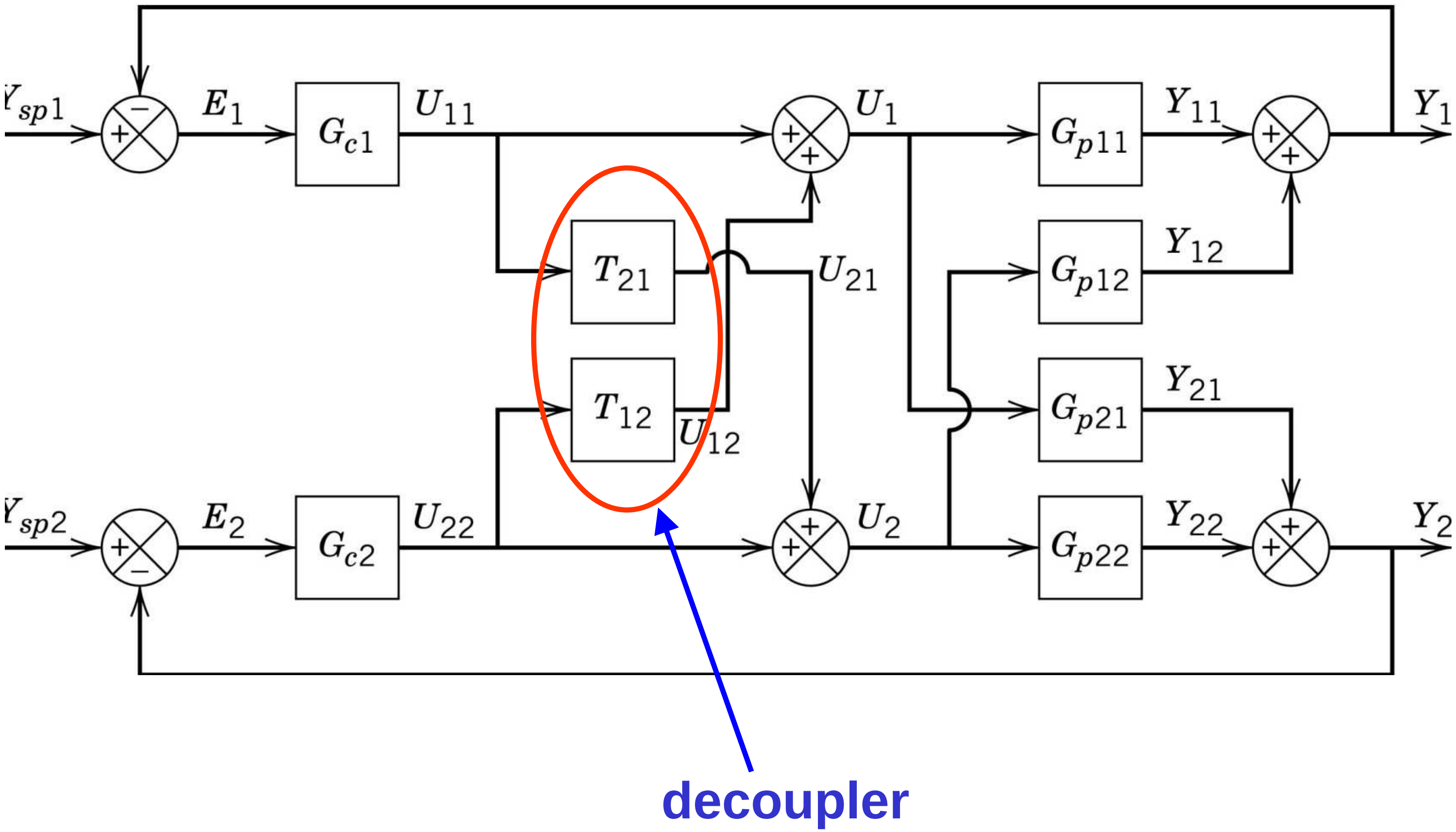
Decoupling Control Systems

- **Basic Idea**: Use additional controllers (decoupler) to compensate for process interactions and thus reduce control loop interactions

- Ideally, decoupling control allows setpoint changes to affect only the desired controlled variables.

- Typically, decoupling controllers are designed using a simple process model (e.g., a steady-state model or transfer function model)

A Decoupling Control System



Decoupler Design Equations

Multiloop and Multivariable Control

We want cross-controller, T_{12}, to cancel the effect of U_2 on Y_1. Thus, we would like $G_{p11}U_{12} + G_{p12}U_{22} = 0$

or $$G_{p11}T_{12}U_{22} + G_{p12}U_{22} = 0$$

Because $U_{22} \neq 0$ in general, then

$$T_{12} = -\frac{G_{p12}}{G_{p11}}$$

Similarly, we want T_{12} to cancel the effect of U_1 on Y_2. Thus, we require that,

$$G_{p22}T_{21}U_{11} + G_{p21}U_{11} = 0$$

$$\therefore \; T_{21} = -\frac{G_{p21}}{G_{p22}}$$

Compare with the design equations for feedforward control based on block diagram analysis

Variations on a Theme

1. Partial Decoupling:

Use only one “cross-controller.”

2. Static Decoupling:

Design to eliminate Steady-State interactions

Ideal decouplers are merely gains:

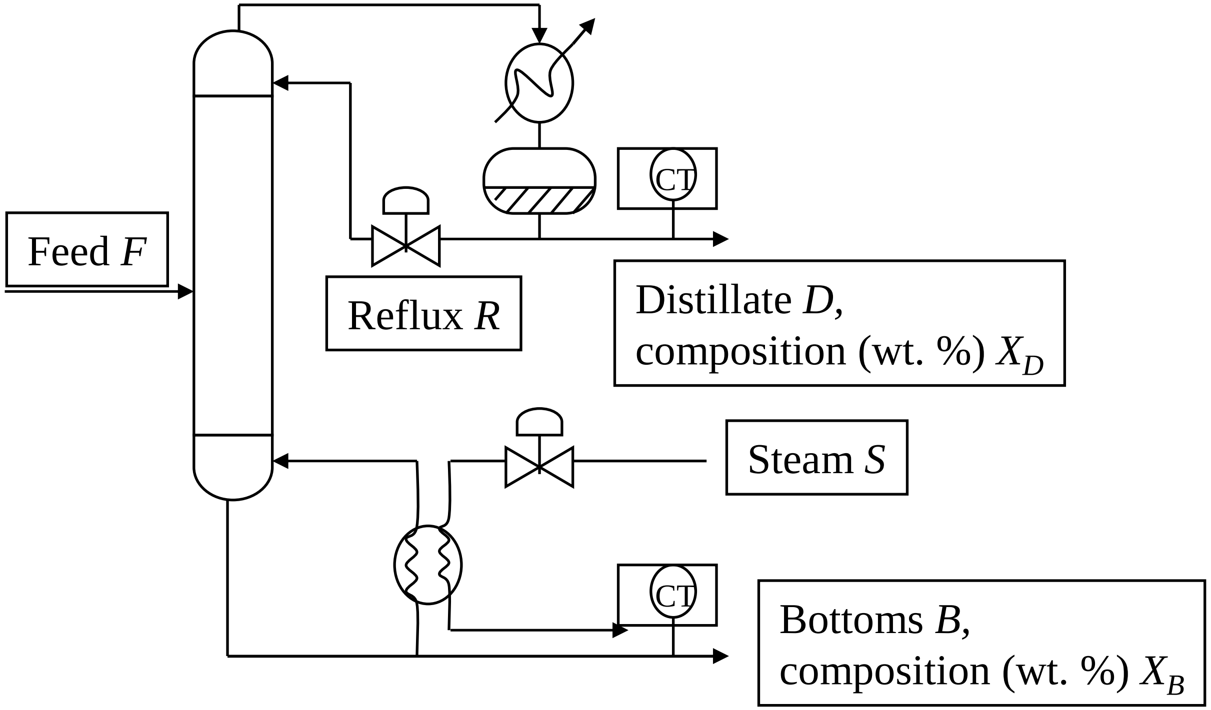
$$T_{12} = -\frac{K_{p12}}{K_{p11}}$$

$$T_{21} = -\frac{K_{p21}}{K_{p22}}$$

3. Nonlinear Decoupling

Appropriate for nonlinear processes.

Wood-Berry Distillation Column Model (methanol-water separation)



Wood-Berry Distillation Column Model

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \qquad (18-12)$$

where:

$y_1 = x_D =$ distillate composition, %MeOH

$y_2 = x_B =$ bottoms composition, %MeOH

$u_1 = R =$ reflux flow rate, lb/min

$u_1 = S =$ reflux flow rate, lb/min

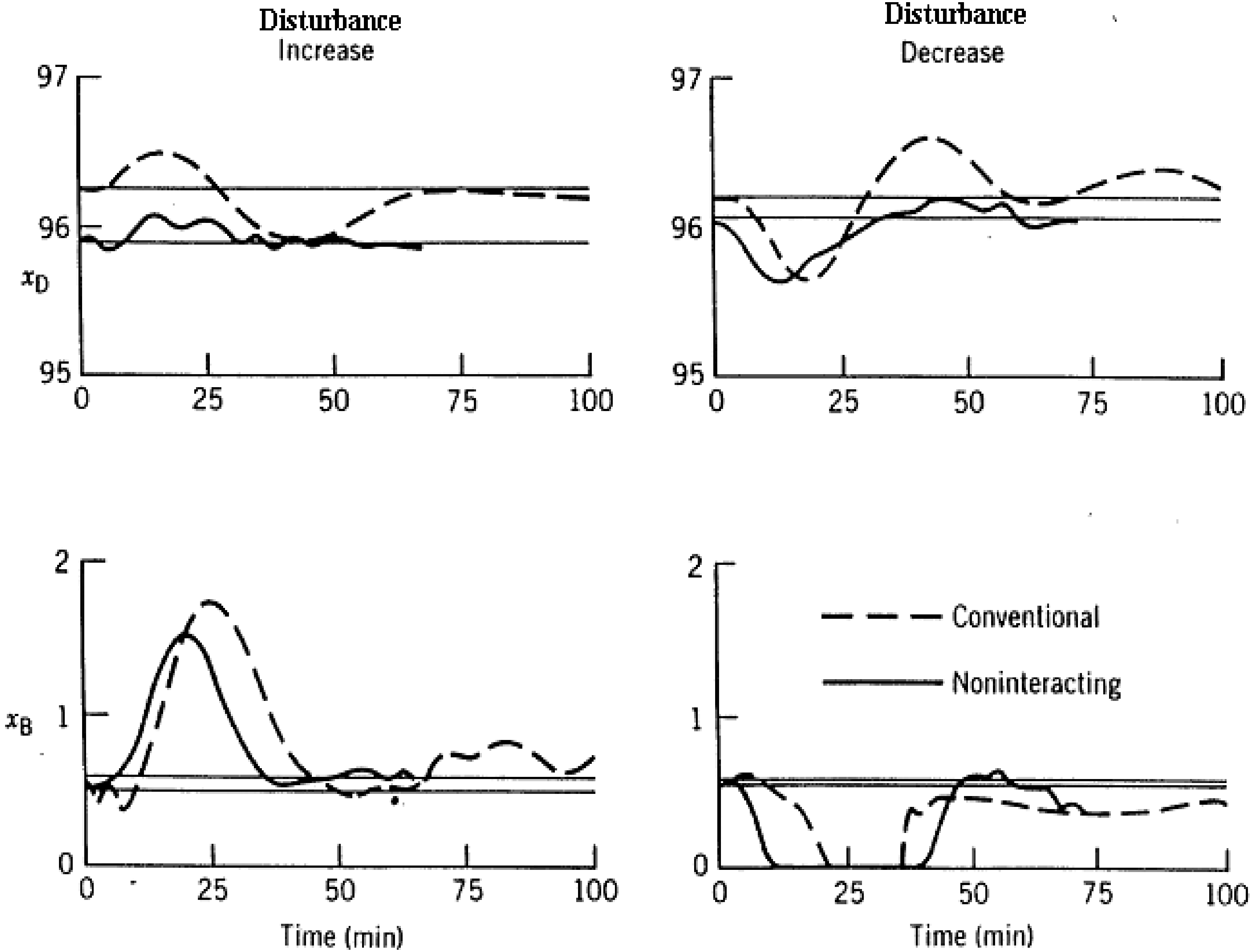


Figure 19.13. An experimental application of decoupling (noninteracting) control to a distillation column [3].