Multiple-Choice Answer Key

The following contains the answers to the multiple-choice questions in this exam.

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: D

Question 2: B

Question 3: B

Question 4: E

Question 5: D

Question 6: B

Question 7: C

Question 8: C

Question 9: D

Question 10: A

Question 11: D

Question 12: E

Question 13: E

Question 14: A

Question 15: C

Question 16: D

Question 17: B

Question 18: B

Question 19: B

Question 20: D

Question 21: A

Question 22: E

Question 23: B

Question 24: A

Question 25: C

Question 26: E

Question 27: D

Question 28: B

Question 76: D

Question 77: E

Question 78: A

Question 79: C

Question 80: E

Question 81: A

Question 82: E

Question 83: B

Question 84: D

Question 85: C

Question 86: E

Question 87: E

Question 88: D

Question 89: E

Question 90: A

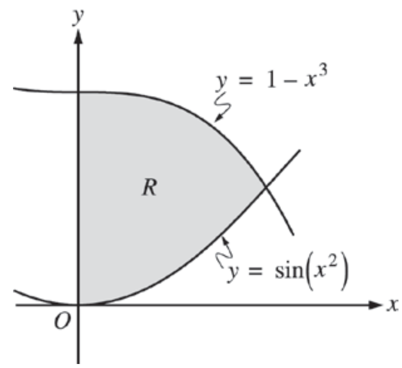
Question 91: B

Question 92: A

Free-Response Scoring Guidelines

The following contains the scoring guidelines
for the free-response questions in this exam.

Question 1



Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$, as shown in the figure above.

(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

The graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.

1 : correct limits in an integral in (a), (b), or (c)

(a) $$\text{Area} = \int_0^A \left(1 - x^3 - \sin\left(x^2\right)\right) dx = 0.533 \text{ (or } 0.534)$$

2 : { 1 : integrand; 1 : answer }

(b) $k = B = 0.552352$

$$\int_0^A \left(1 - x^3 - k\right) dx = 0.257 \text{ (or } 0.256)$$

$$\int_0^A \left(k - \sin\left(x^2\right)\right) dx = 0.277 \text{ (or } 0.276)$$

The two regions have unequal areas.

3 : { 1 : integral(s) with k value; 1 : value(s) of integral(s); 1 : conclusion tied to part (a) or comparison of two integrals }

Note: Stating k value only does not earn a point.

(c) $$\text{Volume} = \pi \int_0^A \left(\left(1 - x^3 + 3\right)^2 - \left(\sin\left(x^2\right) + 3\right)^2\right) dx = 11.841 \text{ (or } 11.840)$$

3 : { 2 : integrand; 1 : answer }

Question 2

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

(a) $P'(0) = B(0) - D(0) = 1000 - 250 = 750$ penguins per year	1 : answer
(b) $P(40) = 100000 + \int_0^{40} (B(t) - D(t))\,dt$ $= 100000 + 33057.56459$ There are 133,058 penguins on the island.	3 : { 1 : limits 1 : integrand 1 : answer
(c) $\frac{1}{40}\int_0^{40} (B(t) - D(t))\,dt = 826.439$ OR $\frac{P(40) - P(0)}{40 - 0} = \frac{133058 - 100000}{40} = 826.45$ The average rate of change is 826 penguins per year.	1 : answer
(d) $B(t) - D(t) = 0$ $1000e^{0.06t} = 250e^{0.1t} \Rightarrow t = A = \frac{\ln 4}{0.04} = 34.657359$ The absolute minimum and absolute maximum occur at a critical point or at an endpoint. $P(0) = 100000$ $P(A) = 100000 + \int_0^{A} (B(t) - D(t))\,dt = 139166.667$ $P(40) = 133058$ The minimum population is 100,000 and the maximum population is 139,167 penguins.	4 : { 1 : $B(t) - D(t) = 0$ 1 : solves for t 1 : minimum value 1 : maximum value

Question 3

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

Solution	Points
(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$. $W(32) \approx 0.5(32 - 30) + 125 = 126$	1 : answer
(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$ $W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$	3 : { 1 : left Riemann sum 1 : approximation 1 : answer
(c) W' is differentiable $\Rightarrow$ W' is continuous. $W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$ By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.	2 : explanation
(d) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3} \cdot \frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}} \cdot \frac{dW}{dt}$ $\left.\frac{dA}{dt}\right\rvert_{t=30} = \frac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$	3 : { 2 : $\frac{dA}{dt}$ 1 : answer

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Question 4

Let f be the function given by $f(x) = \left(x^2 - 2x - 1\right)e^x$.

(a) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

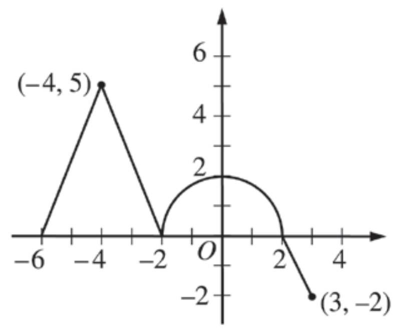
(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

(a) $\lim_{x\to\infty} f(x) = \infty$ or does not exist $\lim_{x\to-\infty} f(x) = 0$	1 : answers
(b) $f'(x) = (2x-2)e^x + \left(x^2 - 2x - 1\right)e^x$ $= \left(x^2 - 3\right)e^x$ $f'(x) = 0$ when $x = -\sqrt{3},\ x = \sqrt{3}$ $f'(x) > 0$ for $-\infty < x < -\sqrt{3}$ and $\sqrt{3} < x < \infty$. f is increasing on the intervals $-\infty < x \le -\sqrt{3}$ and $\sqrt{3} \le x < \infty$.	$4 : \begin{cases} 2 : f'(x) \\ 1 : \text{analysis} \\ 1 : \text{intervals} \end{cases}$
(c) $f''(x) = 2xe^x + \left(x^2 - 3\right)e^x$ $= \left(x^2 + 2x - 3\right)e^x = (x+3)(x-1)e^x$ $f''(x) < 0$ for $-3 < x < 1$ The graph of f is concave down on the interval $-3 < x < 1$.	$4 : \begin{cases} 2 : f''(x) \\ 1 : \text{analysis} \\ 1 : \text{interval} \end{cases}$

Question 5

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(-6) = \int_{-2}^{-6} f(t)\,dt = -\int_{-6}^{-2} f(t)\,dt = -\frac{1}{2}\cdot 4\cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\,dt = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

2 : { 1 : $g(-6)$
1 : $g(3)$ }

(b) $g'(0) = f(0) = 2$

1 : $g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

3 : { 1 : horizontal tangent at $x = -2$ and $x = 2$
2 : answers with justifications }

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

3 : { 2 : values of x
1 : explanation }

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Question 6

Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\dfrac{3x^2}{y}$.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

(b) Find an expression for $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{y}$ with the initial condition $f(2) = -8$.

(a) Slope $= \dfrac{(3)(4)}{-8} = -\dfrac{3}{2}$

An equation for the tangent line is $y = -\dfrac{3}{2}(x - 2) - 8$.

$f(1.8) \approx -\dfrac{3}{2}(1.8 - 2) - 8 = -7.7$

$3 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $\int y\,dy = \int 3x^2\,dx$

$\dfrac{1}{2}y^2 = x^3 + C$

$\dfrac{1}{2}(-8)^2 = 2^3 + C \Rightarrow C = 24$

$y^2 = 2\left(x^3 + 24\right) = 2x^3 + 48$

$y = -\sqrt{2x^3 + 48}$

Note: This solution is valid for $x > -\sqrt[3]{24}$.

$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables