

5

MULTIPLICATION AND DIVISION WITH RATIONAL NUMBERS



Kitty Hawk, North Carolina, is famous for being the place where the first airplane flight took place. The brothers who flew these first flights grew up in Ohio, but they chose Kitty Hawk for its steady winds, soft landings, and privacy.



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Chapter 5 Overview

This chapter use models to develop a conceptual understanding of multiplication and division with respect to the set of integers. These strategies are formalized, and then extended to operations with respect to the set of rational numbers.

Lessons		CCSS	Pacing	Highlights	Models	Worked Examples	Peer Analysis	Talk the Talk	Technology
5.1	Multiplying and Dividing Integers	7.NS.2.a 7.NS.2.b 7.NS.2.c	1	<p>This lesson develops the conceptual understanding of multiplication and division of signed numbers using two-color counters and number lines.</p> <p>Questions encourage students to draw the models, look for patterns, and write their own rules for multiplying and dividing signed numbers.</p>	X			X	
5.2	Multiplying and Dividing Rational Numbers	7.NS.2.a 7.NS.2.b 7.NS.2.c	1	This lesson extends the rules for multiplying and dividing integers to multiplying and dividing rational numbers.				X	
5.3	Simplifying Arithmetic Expressions with Rational Numbers	7.NS.1.d 7.NS.2.a 7.NS.2.c	1	This lesson requires the use of properties to justify the simplification process for various expressions involving rational numbers.					
5.4	Evaluating Expressions with Rational Numbers	7.NS.1.d 7.NS.2.a 7.NS.2.b 7.NS.2.c 7.NS.3	1	This lesson presents real-world situations that involve the four operations with rational numbers.					
5.5	Exact Decimal Representations of Fractions	7.NS.1.d 7.NS.2.d	1	<p>This lesson explores the relationship between mixed numbers and their decimal equivalents.</p> <p>Questions lead students to conclude that the decimal form of a rational number can either terminate or repeat. The lesson provides a graphic organizer for students to summarize the four types of decimals: terminating, non-terminating, repeating, and non-repeating.</p>				X	

Skills Practice Correlation for Chapter 5

Lessons		Problem Set	Objective(s)
5.1	Multiplying and Dividing Integers	1 - 6	Draw two-color counter models to determine products
		7 - 14	Use number lines to determine products
		15 - 20	Write expressions as repeated addition sentences
		21 - 24	Determine products to extend patterns
		25 - 28	Determine products
		29 - 34	Determine integers to make number sentences true
		35 - 40	Write fact families of integers
		41 - 48	Determine unknown integers to make number sentences true
5.2	Multiplying and Dividing Rational Numbers	1 - 10	Calculate products of rational numbers
		11 - 20	Calculate quotients of rational numbers
5.3	Simplifying Arithmetic Expressions with Rational Numbers	1 - 8	Identify operations or properties used to simplify expressions
		9 - 16	Determine the steps to simplify an expression using the operations or properties provided
		17 - 22	Simplify expressions by listing the properties or operations at each step
5.4	Evaluating Expressions with Rational Numbers	1 - 10	Solve word problems with rational numbers
		11 - 20	Evaluate expressions for given values
5.5	Exact Decimal Representations of Fractions		Vocabulary
		1 - 10	Convert fractions to decimals and classify as terminating, non-terminating, repeating, or non-repeating
		11 - 20	Convert fractions and mixed numbers to decimals and evaluate expressions

5.1

Equal Groups

Multiplying and Dividing Integers

Learning Goals

In this lesson, you will:

- ▶ Multiply integers.
- ▶ Divide integers.

Essential Ideas

- Multiplication can be thought of as repeated addition.
- Multiplication of integers can be modeled using two-color counters that represent positive charges (yellow counters) and negative charges (red counters).
- Multiplication of integers can be modeled using a number line.
- The product that results from multiplying two positive integers is always positive.
- The product that results from multiplying two negative integers is always positive.
- The product that results from multiplying a negative integer and a positive is always negative.
- The product that results from multiplying an odd number of negative integers is always negative.
- The product that results from multiplying an even number of negative integers is always positive.
- Division and multiplication are inverse operations.
- The algorithms for determining the sign of the quotient when performing division are the same as the algorithms for determining the sign of the product when performing multiplication.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
 - c. Apply properties of operations as strategies to multiply and divide rational numbers.

Overview

Two-color counters and number lines are used to model the product and quotient of two integers. Through a series of activities, students develop rules to determine the sign of a product and quotient of two integers. They will conclude that multiplying or dividing two positive integers or two negative integers, always results in a positive product or quotient and that multiplying or dividing a positive integer by a negative integer always results in a negative product or quotient.

The first set of activities demonstrates the use of two-color counters and number lines to perform multiplication. Examples of modeling the product of two integers with opposite signs using two-color counters and a number line are provided. Students will determine products using one of these two methods and describe the expression in words. They then complete a table in which number sentences are given by writing the expressions using words, writing an addition sentence, and computing the product. Students will notice patterns and describe an algorithm that will help them multiply any two integers. Questions focus students on the sign of a product resulting from the multiplication of two positive integers, two negative integers, and one positive and one negative integer.

Students then explore the quotient of two integers. Students will conclude that the algorithms to determine the sign of a quotient when performing division are the same as the algorithms to determine the sign of the product when performing multiplication.

Warm Up

1. Rewrite this addition problem as a multiplication problem.

$$(-10) + (-10) + (-10) + (-10)$$

$$(4)(-10)$$

2. Is the product of the answer to Question 1 positive or negative? Explain your reasoning.

$$(4)(-10) = -40$$

The product is negative because $(-10) + (-10) + (-10) + (-10) = -40$.

3. Rewrite this addition problem as a multiplication problem.

$$(-8) + (-8) + (-8) + (-8) + (-8) + (-8) + (-8)$$

$$(7)(-8)$$

4. Is the product of the answer to Question 3 positive or negative? Explain your reasoning.

$$(7)(-8) = -56$$

The product is negative because $(-8) + (-8) + (-8) + (-8) + (-8) + (-8) + (-8) = -56$.

5.1

EQUAL GROUPS

Multiplying and Dividing Integers

Learning Goals

In this lesson, you will:

- ▶ Multiply integers.
- ▶ Divide integers.

Pick any positive integer. If the integer is even, divide it by 2. If it is odd, multiply it by 3 and then add 1. Repeat this process with your result.

No matter what number you start with, eventually you will have a result of 1. This is known as the Collatz Conjecture—a conjecture in mathematics that no one has yet proven or disproven. How do you think it works?

Problem 1

Multiplication is described as repeated addition. Examples of the multiplication of two integers using the two-color counter method and the number line method are provided. Students are given number sentences and will use one of the methods to compute each product, and then describe each expression using words. A table that contains number sentences is given and students rewrite the number sentence using words, as an addition sentence, and as a product. They will notice patterns that result from multiplication problems and use these patterns to create an algorithm to multiply any two integers. Questions focus students on the sign of the product that results from multiplying two integers.

5

Grouping

Ask the students to read the worked examples on their own. Then discuss the information as a class.

Problem 1 Multiply Integers

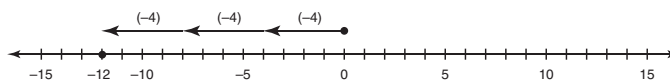
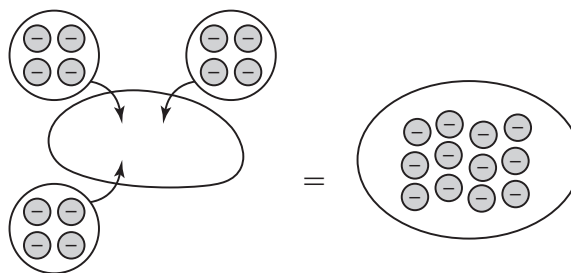


When you multiply integers, you can think of multiplication as repeated addition.

Consider the expression $3 \times (-4)$.

As repeated addition, it means $(-4) + (-4) + (-4) = -12$.

You can think of $3 \times (-4)$ as three groups of (-4) .

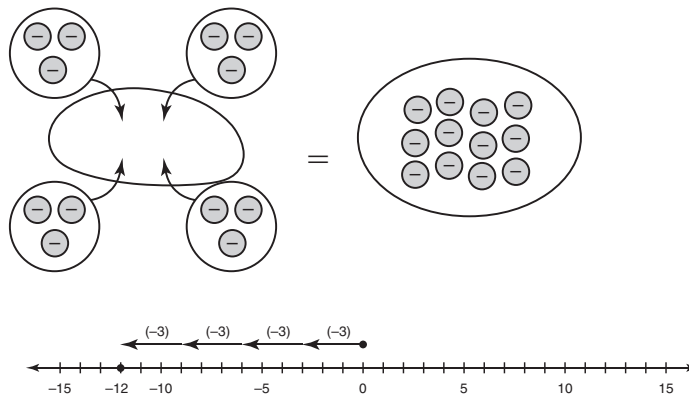


Grouping

Ask the students to read both worked examples on their own. Then discuss the information as a class.

Here is another example: $4 \times (-3)$.

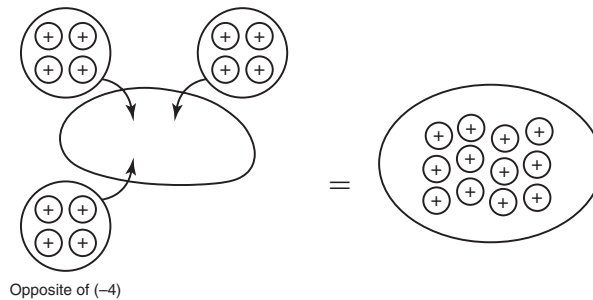
You can think of this as four sets of (-3) , or $(-3) + (-3) + (-3) + (-3) = -12$.



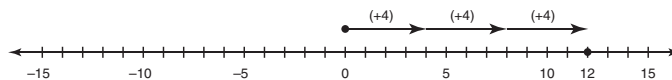
And here is a third example: $(-3) \times (-4)$.

You know that $3 \times (-4)$ means "three groups of (-4) " and that -3 means "the opposite of 3." So, $(-3) \times (-4)$ means "the opposite of 3 groups of (-4) ."

Opposite of (-4) Opposite of (-4)



Opposite of (-4)



Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- Is the sign of the product of the two integers positive or negative?
- Can you glance at the number sentence and know the sign of the product?
- Which number sentences resulted in the same product?
- Do you need to use the two-counter method or the number line method to compute the product of the two integers?

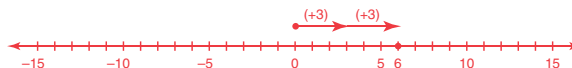


1. Draw either a number line representation or a two-color counter model to determine each product. Describe the expression in words.

The answers provided use a number line representation. Your students may also draw two-color counters.

a. 2×3

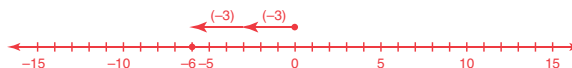
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The expression 2×3 means "two groups of 3."

b. $2 \times (-3)$

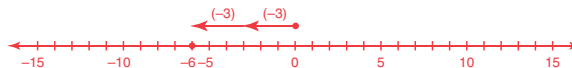
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The expression $2 \times (-3)$ means "two groups of -3."

c. $(-2) \times 3$

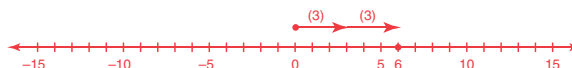
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The expression $(-2) \times 3$ means "the opposite of two groups of 3."

d. $(-2) \times (-3)$

6



The expression $(-2) \times (-3)$ means "the opposite of two groups of (-3)."

Use the examples if you need help.



Grouping

Have students complete Questions 2 through 4 with a partner. Then share the responses as a class.

Share Phase, Questions 2 through 4

- Can every multiplication problem be rewritten as an addition problem?
- How does rewriting a multiplication problem as an addition problem help you to determine the product?



2. Complete the table.

Expression	Description	Addition Sentence	Product
3×5	Three groups of 5	$5 + 5 + 5 = 15$	15
$(-3) \times 5$	The opposite of three groups of 5	$-(5 + 5 + 5) = -(15)$	-15
$3 \times (-5)$	Three groups of (-5)	$(-5) + (-5) + (-5) = -15$	-15
$(-3) \times (-5)$	The opposite of three groups of (-5)	$-((-5) + (-5) + (-5)) = -(-15)$	15

3. Analyze each number sentence.

$4 \times 5 = 20$
 $4 \times 4 = 16$
 $4 \times 3 = 12$
 $4 \times 2 = 8$
 $4 \times 1 = 4$
 $4 \times 0 = 0$

What pattern do you notice in the products as the numbers multiplied by 4 decrease?

The products are decreasing by 4 each time.

4. Determine each product. Describe the pattern.

- a. $4 \times (-1) =$ -4
b. $4 \times (-2) =$ -8
c. $4 \times (-3) =$ -12

The pattern continues. Each product is 4 less than the previous product.



Grouping

Have students complete Questions 5 through 10 with a partner. Then share the responses as a class.

Share Phase, Questions 5 through 10

- Explain why the product of a negative integer and a positive integer is always a negative integer.
- How do these patterns help you to write an algorithm?
- What algorithm would work for the multiplication of more than two integers?



5. Write the next three number sentences that extend this pattern.

$$-5 \times 5 = -25$$

$$-5 \times 4 = -20$$

$$-5 \times 3 = -15$$

$$-5 \times 2 = -10$$

$$-5 \times 1 = -5$$

$$-5 \times 0 = 0$$

$$-5 \times (-1) = 5$$

$$-5 \times (-2) = 10$$

$$-5 \times (-3) = 15$$

6. How do these products change as the numbers multiplied by -5 decrease?

The products increase by 5 each time as the numbers multiplied by -5 decrease.

7. Determine each product.

a. $-5 \times (-1) =$ 5

b. $-5 \times (-2) =$ 10

c. $-5 \times (-3) =$ 15

d. $-5 \times (-4) =$ 20

- e. Write the next three number sentences that extend this pattern.

$$-5 \times (-5) = 25$$

$$-5 \times (-6) = 30$$

$$-5 \times (-7) = 35$$

When you multiply by the opposite, you go in the opposite direction!



8. What is the sign of the product of two integers when:

- a. they are both positive?

The product is positive.

- b. they are both negative?

The product is positive.

- c. one is positive and one is negative?

The product is negative.

- d. one is zero?

The product is zero.

Grouping

Have students complete Questions 11 through 15 with a partner. Then share the responses as a class.

9. If you know that the product of two integers is negative, what can you say about the two integers? Give examples.

If the product of two integers is negative, then one of the integers has to be negative and the other positive.

Examples: $-7 \times 6 = -42$; $3 \times -3 = -9$



10. Describe an algorithm that will help you multiply any two integers.

I can ignore the signs and multiply as I would with positive whole numbers to calculate the product. If I multiply two positive integers, then the result is a positive integer. If I multiply two negative integers, then the result is also a positive integer. If I multiply a positive integer and a negative integer, then the result is a negative integer.



11. Use your algorithm to simplify these expressions.

a. 6×5	30	b. -8×7	-56
$6 \times (-5)$	-30	$-8 \times (-7)$	56
-6×5	-30	$8 \times (-7)$	-56
$-6 \times (-5)$	30	8×7	56

c. $-3 \times 2 \times (-4)$ 24

$-3 \times (-2) \times (-4)$ -24

$3 \times (-2) \times 4$ -24

$-3 \times (-2) \times 4$ 24

$3 \times 2 \times (-4)$ -24

$-3 \times 2 \times 4$ -24

12. Determine the single-digit integers that make each number sentence true.

a. _____ \times _____ = -42 7, -6 or -7, 6

b. _____ \times _____ = 56 7, 8 or -7, -8

c. _____ \times (-9) = 63 -7

d. _____ \times _____ = -48 6, -8 or -6, 8

Share Phase, Questions 11 through 15

- If you know that the product of three numbers is negative, what can you say about the three integers?
- If you know that the product of three numbers is positive, what can you say about the three integers?
- If you know that the product of four numbers is negative, what can you say about the four integers?
- If you know that the product of four numbers is positive, what can you say about the four integers?
- If you know that the product of five numbers is negative, what can you say about the five integers?
- If you know that the product of five numbers is positive, what can you say about the five integers?
- If you know that the product of an odd number of integers is positive, what can you say about the integers?
- If you know that the product of an even number of integers is negative, what can you say about the integers?

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13. Describe the sign of each product and how you know.

a. the product of three negative integers

The product of three negative integers is negative because there is an odd number of negative integers.

b. the product of four negative integers

The product of four negative integers is positive because there is an even number of negative integers.

c. the product of seven negative integers

The product of seven negative integers is negative because there is an odd number of negative integers.

d. the product of ten negative integers

The product of ten negative integers is positive because there is an even number of negative integers.

14. What is the sign of the product of any odd number of negative integers?

Explain your reasoning.

The product of any odd number of negative integers is negative.



15. What is the sign of the product of three positive integers and five negative integers?

Explain your reasoning.

The product is negative because there is an odd number of negative integers.

Problem 2

A fact family for integer multiplication and division is provided. Patterns are noted and students create their own fact family. Students conclude the algorithms used to determine the sign of the product of two integers are the same as the algorithms used to determine the sign of the quotient of two integers.

Grouping

- Ask a student to read the information and example of a fact family before Question 1 aloud. Then discuss the worked example as a class.
- Have students complete Questions 1 through 3 with a partner. Then share the responses as a class.

Problem 2 Division of Integers



When you studied division in elementary school, you learned that multiplication and division were inverse operations. For every multiplication fact, you can write a corresponding division fact.

The example shown is a fact family for 4, 5, and 20.

Fact Family

$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

$$20 \div 4 = 5$$

$$20 \div 5 = 4$$

Similarly, you can write fact families for integer multiplication and division.

Examples:

$$-7 \times 3 = -21$$

$$-8 \times (-4) = 32$$

$$3 \times (-7) = -21$$

$$-4 \times (-8) = 32$$

$$-21 \div (-7) = 3$$

$$32 \div (-8) = -4$$

$$-21 \div 3 = -7$$

$$32 \div (-4) = -8$$



1. What pattern(s) do you notice in each fact family?

If the product is negative, then just one of the factors is negative. If both factors are negative, then the product is positive. When you are writing the fact families, you must pay attention to the sign of the numbers.

2. Write a fact family for -6 , 8 , and -48 .

$$-6 \times 8 = -48$$

$$8 \times (-6) = -48$$

$$-48 \div (-6) = 8$$

$$-48 \div 8 = -6$$

Share Phase, Question 3

- Will more than one number make this number sentence true?
- How do the algorithms for multiplication compare to the algorithms for division?
- How does the sign of the quotient rules compare to the sign of the product rules?

Talk the Talk

Students summarize the rules to determine the appropriate sign of the quotient given certain conditions.

Grouping

Have students complete Questions 1 and 2 on their own. Then share the responses as a class.

5

3. Fill in the unknown numbers to make each number sentence true.

a. $56 \div (-8) = \underline{-7}$

b. $28 \div (-4) = \underline{-7}$

c. $-63 \div \underline{9} = -7$

d. $24 \div \underline{-3} = -8$

e. $\underline{32} \div (-8) = -4$

f. $-105 \div \underline{21} = -5$

g. $\underline{0} \div (-8) = 0$

h. $-26 \div \underline{26} = -1$



Talk the Talk



1. What is the sign of the quotient of two integers when

a. both integers are positive?

The quotient is positive.

b. one integer is positive and one integer is negative?

The quotient is negative.

c. both integers are negative?

The quotient is positive.

d. the dividend is zero?

The quotient is zero.

2. How do the answers to Question 1 compare to the answers to the same questions about the multiplication of two integers? Explain your reasoning.

The rules for determining the sign of a product are the same for determining the sign of a quotient.



Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 5.1 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 5.1 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 5.

Check for Students’ Understanding

1. Complete the table by writing the sign (+, −, or +/−) to describe each sum, difference, product, or quotient.

Description of Integers	Addition (Sum)	Subtraction (Difference)	Multiplication (Product)	Division (Quotient)
Two positive integers	+	+/−	+	+
Two negative integers	−	+/−	+	+
One positive and one negative integer	+/−	+/−	−	−

2. Which situations in Question 1 could produce both a positive and negative result?

Subtracting two positive integers
Subtracting two negative integers
Adding one negative and one positive integer
Subtracting one negative and one positive integer

3. Create a problem that results in a positive sum.

$4 + 6 = 10$

4. Create a problem that results in a negative sum.

$4 + (-6) = -2$

5. Create a problem that results in a positive difference.

$14 - 6 = 8$

6. Create a problem that results in a negative difference.

$-14 - 6 = -20$

7. Create a problem that results in a positive product.

$$4 (6) = 24$$

8. Create a problem that results in a negative product.

$$4 (-6) = -24$$

9. Create a problem that results in a positive quotient.

$$12 \div 6 = 2$$

10. Create a problem that results in a negative quotient.

$$12 \div (-6) = -2$$

5.2

What's My Product or Quotient?

Multiplying and Dividing Rational Numbers

Learning Goals

In this lesson, you will:

- ▶ Multiply rational numbers.
- ▶ Divide rational numbers.

Essential Idea

- The rules for multiplying and dividing integers also apply to multiplying and dividing rational numbers.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
 - c. Apply properties of operations as strategies to multiply and divide rational numbers.

Overview

Students apply their knowledge of multiplying and dividing positive and negative integers to the set of rational numbers.

5.2

WHAT'S MY PRODUCT OR QUOTIENT?

Multiplying and Dividing Rational Numbers

Learning Goals

In this lesson, you will:

- ▶ Multiply rational numbers.
- ▶ Divide rational numbers.

Look at these models. The top model shows $\frac{6}{8}$, and the bottom model shows $\frac{3}{8}$.



To determine $\frac{6}{8} \div \frac{3}{8}$, you can ask, "How many $\frac{3}{8}$ go into $\frac{6}{8}$?" You can see that the answer, or quotient, is just $6 \div 3$, or 2.

So, if you are dividing two fractions with the same denominators, can you always just divide the numerators to determine the quotient?

Try it out and see!

Problem 1

Students use the rule for multiplying positive and negative integers to determine the product of an expression with rational numbers. They will then solve six additional problems which involve computing the product of mixed numbers and decimals.

Grouping

- Ask a student to read Question 1 aloud. Discuss the information and complete Question 1 as a class.
- Have students complete Question 2 with a partner. Then share the responses as a class.

Discuss Phase, Question 1

- Can the rules you learned for multiplying integers be applied to multiplying mixed numbers? How?
- To multiply two mixed numbers, do you need a common denominator? Explain.
- What is an improper fraction?
- To multiply two mixed numbers, do you need to change them into improper fractions?
- How do you change a mixed number into an improper fraction?

Problem 1 From Integer to Rational



In this lesson, you will apply what you learned about multiplying and dividing with integers to multiply and divide with rational numbers.

1. Consider this multiplication sentence:

$$-2\frac{1}{2} \times 3\frac{1}{5} = ?$$

- a. What is the rule for multiplying signed numbers?

If I multiply two positive integers, then the result is a positive integer. If I multiply two negative integers, then the result is also a positive integer. If I multiply a positive integer and a negative integer, then the result is a negative integer.

- b. Use the rule to calculate the product. Show your work.

$$-2\frac{1}{2} \times 3\frac{1}{5} = -\frac{5}{2} \times \frac{16}{5} = -8$$

When you convert a mixed number to an improper fraction, ignore the sign at first. Put it back in when you have finished converting.



2. Calculate each product and show your work.

- a. $-5\frac{1}{3} \times -4\frac{1}{4} =$

$$-5\frac{1}{3} \times -4\frac{1}{4} = -\frac{16}{3} \times -\frac{17}{4} = \frac{68}{3} = 22\frac{2}{3}$$

- c. $2\frac{1}{6} \times -7\frac{1}{5} =$

$$2\frac{1}{6} \times -7\frac{1}{5} = \frac{13}{6} \times -\frac{36}{5} = -\frac{78}{5} = -15\frac{3}{5}$$

- b. $5.02 \times -3.1 =$

$$\begin{array}{r} 5.02 \times -3.1 = -15.562 \\ 5.02 \\ \times 3.1 \\ \hline 502 \\ 15060 \\ \hline 15.562 \end{array}$$

- d. $-20.1 \times -19.02 =$

$$\begin{array}{r} -20.1 \times -19.02 = 382.302 \\ 19.02 \\ \times 20.1 \\ \hline 1902 \\ 380400 \\ \hline 382302 \end{array}$$

It doesn't matter what numbers I have. The rules for the signs are the same.



Share Phase, Question 2, part (a)

- How is $-5\frac{1}{3}$ written as an improper fraction?
- How is $-4\frac{1}{4}$ written as an improper fraction?
- Before computing the product, how can you reduce $-\frac{16}{3} \times \frac{17}{4}$?
- Can the answer of $\frac{68}{3}$ be reduced even further?



$$e. -4\frac{1}{2} \times -3\frac{2}{3} =$$

$$-4\frac{1}{2} \times -3\frac{2}{3} = -\frac{9}{2} \times -\frac{11}{3} = \frac{33}{2} = 16\frac{1}{2}$$

$$f. -2\frac{1}{2} \times 3\frac{1}{5} \times -1\frac{2}{3} =$$

$$-2\frac{1}{2} \times 3\frac{1}{5} \times -1\frac{2}{3} = -\frac{5}{2} \times \frac{16}{5} \times -\frac{5}{3} = \frac{40}{3} = 13\frac{1}{3}$$

Problem 2

Students use the rule for dividing positive and negative integers to determine the quotient of an expression with rational numbers. They will then solve six additional problems which involve computing the quotient of mixed numbers and decimals.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- Can the rules you learned for dividing integers be applied to dividing mixed numbers? How?
- To divide two mixed numbers, do you need a common denominator? Explain.
- To divide two mixed numbers, do you need to change them into improper fractions?
- What is the rule for dividing two fractions?
- What is a reciprocal?

Problem 2 And On to Dividing



1. Consider this division sentence:

$$-3\frac{1}{3} \div 2\frac{1}{2} = ?$$

- a. What is the rule for dividing signed numbers?

The rules for determining the sign of a quotient are the same for determining the sign of a product.

- b. Use the rule to calculate the quotient. Show your work.

$$-3\frac{1}{3} \div 2\frac{1}{2} = -\frac{10}{3} \times \frac{2}{5} = -\frac{4}{3} = -1\frac{1}{3}$$

2. Calculate each quotient and show your work.

a. $-2\frac{1}{8} \div -4\frac{1}{4} =$

$$-2\frac{1}{8} \div -4\frac{1}{4} = -\frac{17}{8} \times -\frac{4}{17} = \frac{1}{2}$$

b. $4.03 \div -3.1 =$

$$4.03 \div -3.1 = -1.3$$

$$\begin{array}{r} 1.3 \\ 3.1 \overline{)4.03} \\ \underline{31} \\ 93 \\ \underline{93} \\ 0 \end{array}$$

c. $-\frac{5}{6} \div -2\frac{1}{7} =$

$$-\frac{5}{6} \div -2\frac{1}{7} = -\frac{5}{6} \times -\frac{7}{15} = \frac{7}{18}$$

d. $-20.582 \div -4.1 =$

$$-20.582 \div -4.1 = 5.02$$

$$\begin{array}{r} 5.02 \\ 4.1 \overline{)20.582} \\ \underline{205} \\ 82 \\ \underline{82} \\ 0 \end{array}$$

- When using a fraction, how do you write the reciprocal of the fraction?
- How do you divide two mixed numbers?
- When dividing two mixed numbers, is the operation of division actually used? Explain.
- Why is multiplying by the reciprocal of the fraction equivalent to dividing by the fraction?
- How can the problem be rewritten to perform the division?

Share Phase, Question 2

Before computing the quotient, how is the problem rewritten as a multiplication problem?



e. $-11 \div \frac{2}{3} =$

$$-11 \div \frac{2}{3} = -\frac{11}{1} \times \frac{3}{2} = -\frac{33}{2} = -16\frac{1}{2}$$

f. $\left(-5\frac{1}{2} \div \frac{1}{5}\right) \div -1\frac{2}{3} =$

$$\left(-5\frac{1}{2} \div \frac{1}{5}\right) \div -1\frac{2}{3}$$

$$\left(-\frac{11}{2} \times \frac{5}{1}\right) \div -1\frac{2}{3}$$

$$-\frac{55}{2} \div -1\frac{2}{3}$$

$$-\frac{55}{2} \times -\frac{3}{2} = \frac{33}{2} = 16\frac{1}{2}$$

Talk the Talk

Students determine the product or quotient of several problems using the algorithms they wrote in the previous problems.

Grouping

Have students complete Questions 1 through 8 with a partner. Then share the responses as a class.

Talk the Talk



Determine each product or quotient.

1. $\frac{2}{5} \times -\frac{4}{7}$
 $-\frac{8}{35}$

2. $\frac{-72}{-24}$
 3

3. $-9 \div 81$
 $-\frac{1}{9}$

4. $-3.3 \times 2\frac{2}{3}$
 -8.8

5. $10.8 \div -2.4$
 -4.5

6. $-\frac{3}{8} \times \frac{8}{3}$
 -1

7. $-\frac{66}{33}$
 -2

8. $-\frac{4}{5} \div -\frac{1}{4}$
 $-\frac{4}{5} \left(-\frac{4}{1}\right) = \frac{16}{5}$
 $= 3\frac{1}{5}$



Be prepared to share your solutions and methods.

Share Phase, Questions 1 through 8

- What algorithm did you use to determine the product?
- What algorithm did you use to determine the quotient?
- How are the algorithms used to multiply or divide two integers used to multiply or divide two fractions?
- How are the algorithms used to multiply or divide two integers used to multiply or divide two decimals?

Follow Up

Assignment

Use the Assignment for Lesson 5.2 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 5.2 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 5.

Check for Students' Understanding

Is the quotient of each of the following positive or negative?

1. $\frac{1}{6} \div -\frac{1}{3}$

The quotient is negative.

2. $-\frac{2}{7} \div -\frac{2}{5}$

The quotient is positive.

3. $-\frac{1}{2} \div \frac{3}{5}$

The quotient is negative.

4. $\frac{1}{3} \div \frac{4}{5}$

The quotient is positive.

5. Compute the answers to Questions 1 through 4.

Question 1:

$$\frac{1}{6} \div -\frac{1}{3} = \frac{1}{6} \times -\frac{3}{1} = -\frac{3}{6} = -\frac{1}{2}$$

Question 2:

$$-\frac{2}{7} \div -\frac{2}{5} = -\frac{2}{7} \times -\frac{5}{2} = \frac{10}{14} = \frac{5}{7}$$

Question 3:

$$-\frac{1}{2} \div \frac{3}{5} = -\frac{1}{2} \times \frac{5}{3} = -\frac{5}{6}$$

Question 4:

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{5}{12}$$

5.3

Properties Schmoperties

Simplifying Arithmetic Expressions with Rational Numbers

Learning Goal

In this lesson, you will:

- ▶ Simply arithmetic expressions using the number properties and the order of operations.

Essential Ideas

- The Order of Operations can be used to simplify arithmetic expressions.
- Number properties are used to simplify arithmetic expressions.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
 - d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - c. Apply properties of operations as strategies to multiply and divide rational numbers.

Overview

Problems involving mixed numbers and decimals are solved step by step. Each step is justified using reasons such as, addition, subtraction, multiplication, division, Commutative Property of Addition, Commutative Property of Multiplication, Associative Property of Addition, Associative Property of Multiplication, Distributive Property of Multiplication over Addition, Distributive Property of Division over Subtraction, and the Distributive Property of Division over Addition.

Warm Up

Match each example with the appropriate operation or property.

Example	Operation/Property
1. $-5\frac{1}{6} \div -7\frac{2}{3} = \frac{31}{46}$ (G)	A. Commutative Property of Addition
2. $-5\frac{1}{6} \times \left(-7\frac{2}{3} \times -7\frac{2}{3}\right) = \left(-5\frac{1}{6} \times -7\frac{2}{3}\right) \times -7\frac{2}{3}$ (J)	B. Associative Property of Addition
3. $5\frac{1}{6} + 7\frac{2}{3} = 13$ (F)	C. Subtraction
4. $\frac{5\frac{1}{6} - 7\frac{2}{3}}{2} = \frac{5\frac{1}{6}}{2} - \frac{7\frac{2}{3}}{2}$ (I)	D. Distributive Property of Multiplication over Addition
5. $-5\frac{1}{6} \times -7\frac{2}{3} = 39\frac{11}{18}$ (K)	E. Distributive Property of Division over Addition
6. $-5\frac{1}{6} \times -7\frac{2}{3} = -7\frac{2}{3} \times -5\frac{1}{6}$ (H)	F. Addition
7. $5\frac{1}{6} + 7\frac{2}{3} = 7\frac{2}{3} + 5\frac{1}{6}$ (A)	G. Division
8. $5\frac{1}{6} \left(7\frac{2}{3} + 7\frac{2}{3}\right) = \left(7\frac{2}{3}\right) 5\frac{1}{6} + \left(7\frac{2}{3}\right) 5\frac{1}{6}$ (D)	H. Commutative Property of Multiplication
9. $-5\frac{1}{6} - 7\frac{2}{3} = -13$ (C)	I. Distributive Property of Division over Subtraction
10. $-5\frac{1}{6} + \left(-7\frac{2}{3} + -7\frac{2}{3}\right) = \left(-5\frac{1}{6} + -7\frac{2}{3}\right) + -7\frac{2}{3}$ (B)	J. Associative Property of Multiplication
11. $\frac{5\frac{1}{6} - 7\frac{2}{3}}{2} = \frac{5\frac{1}{6}}{2} + -\frac{7\frac{2}{3}}{2}$ (E)	K. Multiplication

5.3

PROPERTIES SCHMOPERTIES

Simplifying Arithmetic Expressions with Rational Numbers

Learning Goal

In this lesson, you will:

- ▶ Simply arithmetic expressions using the number properties and the order of operations.

Suppose you didn't know that a negative times a negative is equal to a positive. How could you prove it? One way is to use properties—in this case, the Zero Property and the Distributive Property.

The Zero Property tells us that any number times 0 is equal to 0, and the Distributive Property tells us that something like $4 \times (2 + 3)$ is equal to $(4 \times 2) + (4 \times 3)$. We want these properties to be true for negative numbers too.

So, start with this:

$$-5 \times 0 = 0$$

That's the Zero Property. We want that to be true. Now, let's replace the first 0 with an expression that equals 0:

$$-5 \times (5 + -5) = 0$$

Using the Distributive Property, we can rewrite that as

$$(-5 \times 5) + (-5 \times -5) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ -25 & + & ? = 0 \end{array}$$

For the properties to be true, -5×-5 has to equal *positive* 25!

What other number properties do you remember learning about?

Hey, did you
hear what Zero
said to Eight?
"Nice belt."



5

Problem 1

Students identify the usage of the number properties and operations in several equations. Equations are simplified step by step and students will determine the appropriate number property or operation associated with each step in the solution process. In the next activity, students write the steps to solve an equation while the number property or operation associated with each step is given.

Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

Share Phase, Question 1

- What are the four arithmetic operations?
- What properties are considered Commutative properties?
- What properties are considered Associative properties?
- What properties are considered Distributive properties?
- Which properties always involve using parenthesis?
- How is the Commutative property of Addition different from the Associative property of Addition?
- How is the Commutative property of Multiplication different from the Associative property of Multiplication?

Problem 1 Properties and Operations



1. For each equation, identify the number property or operation used.

Equation	Number Property/Operation
a. $3\frac{1}{2} + 2\frac{1}{4} = 5\frac{3}{4}$	Addition
b. $-3\frac{1}{2} + 2\frac{1}{4} = 2\frac{1}{4} + (-3\frac{1}{2})$	Commutative Property of Addition
c. $(3\frac{1}{2} \times 2\frac{1}{4}) \times 5\frac{3}{4} = 3\frac{1}{2} \times (2\frac{1}{4} \times 5\frac{3}{4})$	Associative Property of Multiplication
d. $-3\frac{1}{2} \div 2\frac{1}{4} = -1\frac{5}{9}$	Division
e. $-3\frac{1}{2} + (-2\frac{1}{4} + 5\frac{3}{4}) = (-3\frac{1}{2} + (-2\frac{1}{4})) + 5\frac{3}{4}$	Associative Property of Addition
f. $2\frac{1}{4} \times 5\frac{3}{5} = 12\frac{3}{5}$	Multiplication
g. $-3\frac{1}{2} - 2\frac{1}{4} = -5\frac{3}{4}$	Subtraction
h. $(-3\frac{1}{2} + 2\frac{1}{4})1\frac{5}{9} = (-3\frac{1}{2})1\frac{5}{9} + (2\frac{1}{4})1\frac{5}{9}$	Distributive Property of Multiplication over Addition
i. $\frac{-3\frac{1}{2} - 2\frac{1}{4}}{4} = \frac{-3\frac{1}{2}}{4} - \frac{2\frac{1}{4}}{4}$	Distributive Property of Division over Subtraction
j. $(-7.02)(-3.42) = (-3.42)(-7.02)$	Commutative Property of Multiplication



Grouping

Have students complete Questions 2 and 3 with a partner. Then share the responses as a class.

Share Phase, Question 2

- What is the difference between an arithmetic operation and a number property?
- How do you know when to justify a step using an operation?
- How do you know when to justify a step using a number property?
- How can you recognize the use of an Associative property?
- How can you recognize the use of a Distributive property?



2. For each step of the simplification of the expression, identify the operation or property applied.

Number Property/Operation

a. $3\frac{1}{2} + 5\frac{3}{4} + 2\frac{1}{2} =$

$$3\frac{1}{2} + 2\frac{1}{2} + 5\frac{3}{4} =$$

$$6 + 5\frac{3}{4} =$$

$$11\frac{3}{4}$$

Commutative Property of Addition

Addition

Addition

b. $\left(-3\frac{1}{5} + 5\frac{3}{4}\right) + 3\frac{1}{4} =$

$$-3\frac{1}{5} + \left(5\frac{3}{4} + 3\frac{1}{4}\right) =$$

$$-3\frac{1}{5} + 9 =$$

$$5\frac{4}{5}$$

Associative Property of Addition

Addition

Addition

c. $-3\frac{1}{3}\left(-3\frac{1}{5} + 5\frac{3}{5}\right) =$

$$-3\frac{1}{3}\left(2\frac{2}{5}\right) =$$

$$-8$$

Addition

Multiplication

d. $-3\frac{1}{3}\left(-3\frac{1}{5} + 5\frac{3}{5}\right) =$

$$\left(-3\frac{1}{3}\right)\left(-3\frac{1}{5}\right) + \left(-3\frac{1}{3}\right)\left(5\frac{3}{5}\right) =$$

$$\frac{32}{3} + -\frac{56}{3} =$$

$$-8$$

Distributive Property of Multiplication over Addition

Multiplication

Addition

3. Supply the next step in each simplification using the operation or property provided.

	Number Property/Operation
a. $3\frac{3}{4} + \left(-5\frac{2}{5}\right) + 7\frac{1}{4} =$	
$3\frac{3}{4} + 7\frac{1}{4} + \left(-5\frac{2}{5}\right) =$ or $\left(-5\frac{2}{5}\right) + 3\frac{3}{4} + 7\frac{1}{4} =$	Commutative Property of Addition
$11 + \left(-5\frac{2}{5}\right) =$ or $\left(-5\frac{2}{5}\right) + 11 =$	Addition
$5\frac{3}{5}$	Addition
b. $\left(5\frac{1}{6} + -3\frac{3}{4}\right) + -3\frac{1}{4} =$	
$5\frac{1}{6} + \left(-3\frac{3}{4} + -3\frac{1}{4}\right) =$	Associative Property of Addition
$5\frac{1}{6} + (-7) =$	Addition
$-1\frac{5}{6}$	Addition
c. $-5.2(-93.7 + 3.7) =$	
$-5.2(-90) =$	Addition
468	Multiplication

Problem 2

Equations are given and students provide both the steps in the solution process and the number property or operation associated with each step in the solution.

Grouping

Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.



Problem 2 On Your Own



Simplify each expression step by step, listing the property or operation(s) used.

Possible solutions are shown.

$$1. \ 5\left(-3\frac{1}{4}\right) + 5\left(-6\frac{3}{4}\right) =$$

Number Property/Operation

$$5\left(-3\frac{1}{4} + -6\frac{3}{4}\right) =$$

Distributive Property of Multiplication over Addition

$$5(-10) =$$

Addition

$$-50$$

Multiplication

d. $-5.1(70 + 3) =$	
$(-5.1)(70) + (-5.1)(3) =$	Distributive Property of Multiplication over Addition
$-357 + -15.3 =$	Multiplication
-372.3	Addition
e. $\left(-3\frac{1}{4}\right)\left(5\frac{1}{6}\right) + \left(-3\frac{1}{4}\right)\left(2\frac{5}{6}\right) =$	
$\left(-3\frac{1}{4}\right)\left(5\frac{1}{6} + 2\frac{5}{6}\right) =$	Distributive Property of Multiplication over Addition
$\left(-3\frac{1}{4}\right)(8) =$	Addition
-26	Multiplication

Share Phase, Question 1

- How is the first step an example of using the Distributive Property of Multiplication over Addition?
- What operation was used to go from the second step to the third step?
- What operation was used to go from the third step to the last step?

Share Phase, Question 2

- How is the first step an example of using the Associative Property of Addition?
- What operation was used to go from the second step to the third step?
- What operation was used to go from the third step to the last step?

Share Phase, Question 3

- How is the first step an example of using the Commutative Property of Multiplication over Addition?
- What operation was used to go from the second step to the third step?
- What operation was used to go from the third step to the last step?

Share Phase, Question 4

- How is the first step an example of using the Distributive Property of Division over Addition?
- What operation was used to go from the second step to the third step?
- What operation was used to go from the third step to the last step?

Share Phase, Question 5

What two operations were used to solve this problem?

$$2. \left(-3\frac{1}{4} - 2\frac{1}{5}\right) + \left(-6\frac{3}{5}\right) =$$

$$-3\frac{1}{4} + \left(-2\frac{1}{5} + \left(-6\frac{3}{5}\right)\right) =$$

$$-3\frac{1}{4} + \left(-8\frac{4}{5}\right) =$$

$$-12\frac{1}{20}$$

Number Property/Operation
Associative Property of Addition

Addition

Addition

$$3. \frac{7}{8} \times \left(-\frac{4}{5}\right) \times \left(-\frac{8}{7}\right) =$$

$$\left(-\frac{4}{5}\right) \times \frac{7}{8} \times \left(-\frac{8}{7}\right) =$$

$$\left(-\frac{4}{5}\right) \times (-1) =$$

$$\frac{4}{5}$$

Number Property/Operation
Commutative Property of Multiplication

Multiplication

Multiplication

$$4. \frac{\frac{8}{9} + \frac{4}{5}}{4} =$$

$$\frac{8}{9} \div 4 + \left(-\frac{4}{5}\right) \div 4 =$$

$$\frac{2}{9} + \left(-\frac{1}{5}\right) =$$

$$\frac{1}{45}$$

Number Property/Operation
Distributive Property of Division over Addition

Division

Addition

$$5. -3.1(90.7 - (-4.3)) =$$

$$-3.1(95) =$$

$$-294.5$$

Number Property/Operation

Addition

Division

$$6. (-11.4)(6.4) + (-11.4)(-12.4) =$$

$$(-11.4)(6.4 + -12.4) =$$

$$(-11.4)(-6) =$$

$$68.4$$

Number Property/Operation
Distributive Property of Multiplication over Addition
Addition
Multiplication



Be prepared to share your solutions and methods.

Share Phase, Question 6

How many properties and operations were used to solve this problem?

Follow Up

Assignment

Use the Assignment for Lesson 5.3 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 5.3 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 5.

Check for Students’ Understanding

Complete the table by writing an example, property, or operation.

Example	Operation/Property
1. $-5\frac{1}{6} \div -7\frac{2}{3} = \frac{31}{46}$ Answers will vary.	Division
2. $-5\frac{1}{6} \times (-7\frac{2}{3} \times -7\frac{2}{3}) = (-5\frac{1}{6} \times -7\frac{2}{3}) \times -7\frac{2}{3}$	Associative Property of Multiplication
3. $5\frac{1}{6} + 7\frac{2}{3} = 13$	Addition
4. $\frac{5\frac{1}{6} - 7\frac{2}{3}}{2} = \frac{5\frac{1}{6}}{2} - \frac{7\frac{2}{3}}{2}$ Answers will vary.	Distributive Property of Division over Subtraction
5. $-5\frac{1}{6} \times -7\frac{2}{3} = 39\frac{11}{18}$	Multiplication
6. $-5\frac{1}{6} \times -7\frac{2}{3} = -7\frac{2}{3} \times -5\frac{1}{6}$	Commutative Property of Multiplication
7. $5\frac{1}{6} + 7\frac{2}{3} = 7\frac{2}{3} + 5\frac{1}{6}$	Commutative Property of Addition
8. $5\frac{1}{6} (7\frac{2}{3} + 7\frac{2}{3}) = (7\frac{2}{3}) 5\frac{1}{6} + (7\frac{2}{3}) 5\frac{1}{6}$ Answers will vary.	Distributive Property of Multiplication over Addition
9. $-5\frac{1}{6} - 7\frac{2}{3} = -13$	Subtraction
10. $-5\frac{1}{6} + (-7\frac{2}{3} + -7\frac{2}{3}) = (-5\frac{1}{6} + -7\frac{2}{3}) + -7\frac{2}{3}$ Answers will vary.	Associative Property of Addition
11. $\frac{5\frac{1}{6} - 7\frac{2}{3}}{2} = \frac{5\frac{1}{6}}{2} + \frac{-7\frac{2}{3}}{2}$ Answers will vary.	Distributive Property of Division over Addition

5.4

Building a Wright Brothers' Flyer

Evaluating Expressions with Rational Numbers

Learning Goals

In this lesson, you will:

- ▶ Model a situation with an expression using rational numbers.
- ▶ Evaluate rational expressions.

Essential Idea

- Expressions and equations composed of rational numbers are used to solve real world problems.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
- c. Apply properties of operations as strategies to multiply and divide rational numbers.
3. Solve real-world and mathematical problems involving the four operations with rational numbers.

5

Overview

Arithmetic operations are performed on expressions and equations composed of mixed numbers to solve for unknown quantities. The problems used are written within the context of a real world situation. Students will define variables for quantities that change, write an equation that represents the situation, and use the equation to solve for unknown quantities.

Warm Up

Use the indicated operation to solve each problem.

1. $20 \div \left(2\left(7\frac{2}{3}\right)\right)$

$$20 \div 2\left(\frac{23}{3}\right) = 20 \div \frac{46}{3} = \frac{20}{1} \div \frac{46}{3} = \frac{20}{1} \times \frac{3}{46} = \frac{30}{23} = 1\frac{7}{23}$$

2. $20 - 2\left(7\frac{2}{3}\right)$

$$20 - \frac{46}{3} = \frac{60}{3} - \frac{46}{3} = \frac{14}{3} = 4\frac{2}{3}$$

3. $\frac{3}{4}\left(7\frac{2}{3}\right)$

$$\frac{3}{4}\left(7\frac{2}{3}\right) = \frac{3}{4}\left(\frac{23}{3}\right) = \frac{69}{12} = 5\frac{3}{4}$$

4. $-16\frac{1}{4} - \left(5\frac{2}{3}\right)6$

$$-16\frac{1}{4} - \left(5\frac{2}{3}\right)6 = -\frac{65}{4} - \left(\frac{17}{3}\right)6 = -\frac{65}{4} - \left(\frac{102}{3}\right) = -\frac{195}{12} - \left(\frac{408}{12}\right) = -\frac{603}{12} = -50\frac{1}{4}$$

5.4

BUILDING A WRIGHT BROTHERS' FLYER

Evaluating Expressions with Rational Numbers

Learning Goals

In this lesson, you will:

- ▶ Model a situation with an expression using rational numbers.
- ▶ Evaluate rational expressions.

On December 17, 1903, two brothers—Orville and Wilbur Wright—became the first two people to make a controlled flight in a powered plane. They made four flights that day, the longest covering only 852 feet and lasting just 59 seconds.

Human flight progressed amazingly quickly after those first flights. In the year before Orville died, Chuck Yeager had already piloted the first flight that broke the sound barrier!

Problem 1

A balsa wood model of the Wright Brother's plane provides a real-world situation. Students divide rational numbers to solve for the number of stays that can be cut from 10 inch and 12 inch wooden spindles. They will use subtraction to determine the length of wood left over.

Grouping

Have students complete Questions 1 through 4 with a partner. Then share responses as a class.

Share Phase, Questions 1 and 2

- How do you divide a whole number by a mixed number?
- How do you write a whole number as a fraction?
- How do you write a mixed number as a fraction?
- How do you determine how many times $3\frac{1}{4}$ divides into 10?
- How do you determine how much wood is left over?

Problem 1 Building a Wright Brothers' Flyer

In order to build a balsa wood model of the Wright brothers' plane, you would need to cut long lengths of wood spindles into shorter lengths for the wing stays, the vertical poles that support and connect the two wings. Each stay for the main wings of the model needs to be cut $3\frac{1}{4}$ inches long.

Show your work and explain your reasoning.



1. If the wood spindles are each 10 inches long, how many stays could you cut from one spindle?

$$\begin{aligned} 10 \div 3\frac{1}{4} &= \frac{10}{1} \times \frac{4}{13} \\ &= \frac{40}{13} \\ &= 3\frac{1}{13} \end{aligned}$$

I could cut three stays because 10 divided by $3\frac{1}{4}$ is $3\frac{1}{13}$, so there are 3 full pieces and $\frac{1}{13}$ of a stay left over.

2. How many inches of the spindle would be left over?

$$\begin{aligned} 10 - 3\left(3\frac{1}{4}\right) &= 10 - \frac{39}{4} \\ &= \frac{40}{4} - \frac{39}{4} \\ &= \frac{1}{4} \end{aligned}$$

or

$$\frac{1}{13} \left(3\frac{1}{4} \right) = \frac{1}{13} \left(\frac{13}{4} \right) = \frac{1}{4}$$

There would be $\frac{1}{4}$ of an inch left over.

Share Phase, Questions 3 and 4

- How do you determine how many times $3\frac{1}{4}$ divides into 12?
- How do you determine how much wood is left over?

Grouping

Have students complete Questions 5 through 9 with a partner. Then share the responses as a class.

Share Phase, Questions 5 and 6

- How do you determine how many times $1\frac{5}{8}$ divides into 10?
- How do you determine how much wood is left over?

3. If the wood spindles are each 12 inches long, how many stays could you cut from one spindle?

$$\begin{aligned} 12 \div 3\frac{1}{4} &= \frac{12}{1} \times \frac{4}{13} \\ &= \frac{48}{13} \\ &= 3\frac{9}{13} \end{aligned}$$

I could still only cut three stays because 12 divided by $3\frac{1}{4}$ is $3\frac{9}{13}$, so there are 3 full pieces and $\frac{9}{13}$ of a stay left over.

4. How many inches of the spindle would be left over?

$$\begin{aligned} 12 - 3\left(3\frac{1}{4}\right) &= 12 - \frac{39}{4} \\ &= \frac{48}{4} - \frac{39}{4} \\ &= \frac{9}{4} \\ &= 2\frac{1}{4} \end{aligned}$$

or

$$\frac{9}{13} \left(3\frac{1}{4}\right) = \frac{9}{13} \left(\frac{13}{4}\right) = \frac{9}{4} = 2\frac{1}{4}$$

There would be $2\frac{1}{4}$ inches left over.



You also need to cut vertical stays for the smaller wing that are each $1\frac{5}{8}$ inches long.

5. If the wood spindles are each 10 inches long, how many of these stays could you cut from one spindle?

$$\begin{aligned} 10 \div 1\frac{5}{8} &= \frac{10}{1} \times \frac{8}{13} \\ &= \frac{80}{13} \\ &= 6\frac{2}{13} \end{aligned}$$

I could cut six stays because 10 divided by $1\frac{5}{8}$ is $6\frac{2}{13}$, so there are 6 full pieces and $\frac{2}{13}$ of a stay left over.

Share Phase, Questions 7 and 8

- How do you determine how many times $1\frac{5}{8}$ divides into 12?
- How do you determine how much wood is left over?

6. How many inches of the spindle would be left over?

$$\begin{aligned} 10 - 6\left(1\frac{5}{8}\right) &= 10 - \frac{78}{8} \\ &= \frac{80}{8} - \frac{78}{8} \\ &= \frac{2}{8} \\ &= \frac{1}{4} \end{aligned}$$

or

$$\frac{2}{13} \left(1\frac{5}{8}\right) = \frac{2}{13} \left(\frac{13}{8}\right) = \frac{1}{4}$$

There would be $\frac{1}{4}$ of an inch left over.

7. If the wood spindles are each 12 inches long, how many stays could you cut from one spindle?

$$\begin{aligned} 12 \div 1\frac{5}{8} &= \frac{12}{1} \times \frac{8}{13} \\ &= \frac{96}{13} \\ &= 7\frac{5}{13} \end{aligned}$$

I could still only cut seven stays because 12 divided by $1\frac{5}{8}$ is $7\frac{5}{13}$, so there are 7 full pieces and $\frac{5}{13}$ of a stay left over.

8. How many inches of the spindle would be left over?

$$\begin{aligned}12 - 7\left(1\frac{5}{8}\right) &= 12 - \frac{91}{8} \\&= \frac{96}{8} - \frac{91}{8} \\&= \frac{5}{8}\end{aligned}$$

or

$$\frac{5}{13}\left(1\frac{5}{8}\right) = \frac{5}{13}\left(\frac{13}{8}\right) = \frac{5}{8}$$

There would be $\frac{5}{8}$ of an inch left over.



9. Which length of spindle should be used to cut each of the different stays so that there is the least amount wasted?

If either stay is cut from the 10-inch spindle, there will be just $\frac{1}{4}$ inch of waste. However, if you have some 12-inch-long spindles, the shorter stays should be cut from these with only $\frac{5}{8}$ of an inch left over.

Problem 2

Students continue to divide rational numbers to solve for the number of stays that can be cut from a 36 inch wooden spindle and use subtraction to determine the length of wood left over. They will then define variables for the number of $3\frac{1}{4}$ inch stays and the amount of the 36 inch spindle left over. The variables are used to write an equation, and the equation is used to solve for unknown quantities.

Grouping

Have students complete Questions 1 through 6 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3

- How is this problem different from the last problem?
- How is this problem similar to the last problem?
- How do you determine how many times $3\frac{1}{4}$ divides into 36?
- How many times does $3\frac{1}{4}$ divide into 36?
- How do you determine how much wood is left over?
- What are the two quantities that change in the situation?

Problem 2 Building a Wright Brothers' Flyer Redux



Remember, a stay is $3\frac{1}{4}$ inch.



There are longer spindles that measure 36 inches.

1. How much of a 36-inch-long spindle would be left over if you cut one of the stays from it?

$$36 - 3\frac{1}{4} = 32\frac{3}{4}$$

$$36 = 35\frac{4}{4}$$

$$-3\frac{1}{4} = \underline{3\frac{1}{4}}$$

$$32\frac{3}{4}$$

There would be $32\frac{3}{4}$ inches left over.

2. How much of this spindle would be left over if you cut two of the stays from it?

$$36 - 2(3\frac{1}{4}) = 29\frac{1}{2}$$

$$36 = 35\frac{2}{2}$$

$$-6\frac{1}{2} = \underline{6\frac{1}{2}}$$

$$29\frac{1}{2}$$

There will be $29\frac{1}{2}$ inches left over.

3. Define variables for the number of $3\frac{1}{4}$ inch stays and the amount of the 36-inch-long spindle that is left over.

Let x be equal to the number of stays and let y be equal to the amount of the spindle left over.

Show your work and explain your reasoning.



Share Phase, Questions 4 through 6

- Is there more than one way to write the equation?
- Which variable does 10 replace in the equation?
- Which variable does 13 replace in the equation?

4. Write an equation for the relationship between these variables.

$$y = 36 - \left(3\frac{1}{4}\right)x$$

5. Use your equation to calculate the amount of the spindle left over after cutting 10 stays.

$$\begin{aligned} y &= 36 - \left(3\frac{1}{4}\right)10 \\ &= 36 - \frac{13}{4} \left(\frac{10}{1}\right) \\ &= 36 - 32\frac{1}{2} \\ &= 3\frac{1}{2} \end{aligned}$$

There would be $3\frac{1}{2}$ inches left over.

6. Use your equation to calculate the amount of the spindle left over after cutting 13 stays.

$$\begin{aligned} y &= 36 - \left(3\frac{1}{4}\right)13 \\ &= 36 - \frac{13}{4} \left(\frac{13}{1}\right) \\ &= 36 - 42\frac{1}{4} \\ &= -6\frac{1}{4} \end{aligned}$$

Since the number is negative you cannot cut 13 stays.



Problem 3

Students evaluate expressions for a given variable.

Grouping

Have students complete Questions 1 and 2 with a partner. Then share the responses as a class.

Share Phase, Question 1

- After replacing the variable with the value -5 , what is the first step in evaluating the expression?
- After writing the mixed number as a fraction, what is the second step in evaluating the expression?
- What is the third step?
- Can these steps be done in a different order? Explain.
- Which steps can be done differently? Explain.

Problem 3 Evaluating Expressions



1. Evaluate the expression $-12\frac{1}{2} - \left(3\frac{1}{3}\right)v$ for:

a. $v = -5$

$$\begin{aligned} -12\frac{1}{2} - \left(3\frac{1}{3}\right)(-5) &= -12\frac{1}{2} - \left(\frac{10}{3}\right)(-5) \\ &= -12\frac{1}{2} - \left(-\frac{50}{3}\right) \\ &= -12\frac{1}{2} + 16\frac{2}{3} \\ &= 4\frac{1}{6} \end{aligned}$$

b. $v = 3$

$$\begin{aligned} -12\frac{1}{2} - \left(3\frac{1}{3}\right)(3) &= -12\frac{1}{2} - \left(\frac{10}{3}\right)(3) \\ &= -12\frac{1}{2} - (10) \\ &= -12\frac{1}{2} + 10 \\ &= -22\frac{1}{2} \end{aligned}$$

c. $v = -\frac{6}{7}$

$$\begin{aligned} -12\frac{1}{2} - \left(3\frac{1}{3}\right)\left(-\frac{6}{7}\right) &= -12\frac{1}{2} - \left(\frac{10}{3}\right)\left(-\frac{6}{7}\right) \\ &= -12\frac{1}{2} - \left(-\frac{20}{7}\right) \\ &= -12\frac{1}{2} + 2\frac{6}{7} \\ &= -12\frac{7}{14} + 2\frac{12}{14} \\ &= -9\frac{9}{14} \end{aligned}$$

d. $v = 2\frac{2}{5}$

$$\begin{aligned} -12\frac{1}{2} - \left(3\frac{1}{3}\right)\left(2\frac{2}{5}\right) &= -12\frac{1}{2} - \left(\frac{10}{3}\right)\left(\frac{12}{5}\right) \\ &= -12\frac{1}{2} - 8 \\ &= -20\frac{1}{2} \end{aligned}$$

To evaluate an expression, substitute the values for the variables and then perform the operations.



2. Evaluate the expression $\left(-1\frac{1}{4}\right)x - 8\frac{7}{8}$ for:

a. $x = -\frac{2}{5}$

b. $x = -2$

$$\begin{aligned} \left(-1\frac{1}{4}\right)\left(-\frac{2}{5}\right) - 8\frac{7}{8} &= \left(\frac{-5}{4}\right)\left(\frac{-2}{5}\right) - 8\frac{7}{8} & \left(-1\frac{1}{4}\right)(-2) - 8\frac{7}{8} &= \left(\frac{-5}{4}\right)\left(\frac{-2}{1}\right) - 8\frac{7}{8} \\ &= \frac{1}{2} - 8\frac{7}{8} & &= 2\frac{1}{2} - 8\frac{7}{8} \\ &= \frac{4}{8} + -8\frac{7}{8} & &= 2\frac{4}{8} + -8\frac{7}{8} \\ &= -8\frac{3}{8} & &= -6\frac{3}{8} \end{aligned}$$



Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 5.4 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 5.4 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 5.

Check for Students' Understanding

Evaluate the expression $9\frac{4}{5} - \left(2\frac{1}{8}\right)b$ for :

1. $b = -2$

$$9\frac{4}{5} - \left(2\frac{1}{8}\right)(-2)$$

$$\frac{49}{5} - \left(-\frac{34}{8}\right) = \frac{49}{5} + \frac{34}{8} = \frac{392}{40} + \frac{170}{40} = \frac{562}{40} = 14\frac{1}{20}$$

2. $b = \frac{1}{2}$

$$9\frac{4}{5} - \left(2\frac{1}{8}\right)\left(\frac{1}{2}\right)$$

$$\frac{49}{5} - \left(\frac{17}{16}\right) = \frac{49}{5} - \frac{17}{16} = \frac{784}{80} - \frac{85}{80} = \frac{699}{80} = 8\frac{59}{80}$$

5.5

Repeat or Not? That Is the Question!

Exact Decimal Representations of Fractions

Learning Goals

In this lesson, you will:

- ▶ Use decimals and fractions to evaluate arithmetic expressions.
- ▶ Convert fractions to decimals.
- ▶ Represent fractions as repeating decimals.

Key Terms

- ▶ terminating decimals
- ▶ non-terminating decimals
- ▶ repeating decimals
- ▶ non-repeating decimals
- ▶ bar notation

Essential Ideas

- Decimals are classified as terminating, non-terminating, repeating, and non-repeating.
- Bar notation is used when writing repeating decimals.

Common Core State Standards for Mathematics

7.NS The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
 - d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

5

Overview

Students solve problems as fractions and as equivalent decimals. Division is used to convert fractions into decimals. The terms terminating decimal, non-terminating decimal, repeating decimal, non-repeating decimal, and bar notation are introduced. Students will complete a graphic organizer with examples for each type of decimal.

Warm Up

Change each fraction to a decimal and determine what all of the decimals have in common.

1. $\frac{2}{3}$

$$\frac{2}{3} = 0.666666 \dots$$

2. $\frac{5}{22}$

$$\frac{5}{22} = 0.2272727 \dots$$

3. $\frac{1}{9}$

$$\frac{1}{9} = 0.1111111 \dots$$

4. $\frac{13}{15}$

$$\frac{13}{15} = 0.8666666 \dots$$

All of the decimals do not end.

5.5

REPEAT OR NOT? THAT
IS THE QUESTION!Exact Decimal Representations
of Fractions

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- ▶ non-repeating decimals
- ▶ bar notation

Sometimes calculating an exact answer is very important. For example, making sure that all the parts of an airplane fit exactly is very important to keep the plane in the air. Can you think of other examples where very exact answers are necessary?

Problem 1

Students compare the answers to a problem using mixed numbers and using the decimal equivalents of the mixed numbers. They will conclude the answers are the same when the decimal terminates.

Grouping

- Ask a student to read the introduction aloud. Discuss the context as a class.
- Have students complete Questions 1 through 5 with a partner. Then share the responses as a class.

Share Phase, Questions 1 through 3

- What least common denominator should Jayme use?
- Does using Jayme's method result in an exact answer? Explain.
- How do you determine the decimal equivalents for each fraction?

Problem 1 Not More Homework!

Jayme was complaining to her brother about having to do homework problems with fractions like this:

$$2\frac{1}{2} + \left(-3\frac{3}{4}\right) + 5\frac{2}{5} = ?$$

Jayme said, "I have to find the least common denominator, convert the fractions to equivalent fractions with the least common denominator, and then calculate the answer!"

Her brother said, "Whoa! Why don't you just use decimals?"



1. Calculate the answer using Jayme's method.

$$\begin{aligned} 2\frac{1}{2} + \left(-3\frac{3}{4}\right) + 5\frac{2}{5} &= 2\frac{10}{20} + \left(-3\frac{15}{20}\right) + 5\frac{8}{20} \\ &= 4\frac{3}{20} \end{aligned}$$

2. Convert each mixed number to a decimal and calculate the sum.

$$\begin{aligned} 2\frac{1}{2} + \left(-3\frac{3}{4}\right) + 5\frac{2}{5} &= 2.5 + (-3.75) + 5.4 = 4.15 \\ &= 4\frac{15}{100} \\ &= 4\frac{3}{20} \end{aligned}$$

3. In this case, which method do you think works best?

Answers will vary, but most students will like using the decimals.

Jayme said: "That's okay for that problem, but what about this next one?"

$$5\frac{1}{3} + \left(-4\frac{1}{6}\right) + \left(-2\frac{1}{2}\right) =$$

Share Phase, Questions 4 and 5

- Does using Jayme's brother's method result in an exact answer? Explain.
- What is the decimal equivalent of $-1\frac{1}{3}$?
- Is this an exact answer? Why or why not?
- What do you consider to be an exact answer?
- What do you consider to be an answer that is not exact?

Problem 2

Students convert fractions to decimals. All of the decimals are classified as terminating or non-terminating decimals and repeating or non-repeating decimals. Bar notation is introduced to express repeating decimals. Fractions and decimals that either terminate or repeat are referred to as rational numbers.

Grouping

Have students complete Question 1 with a partner. Then share the responses as a class.

4. Calculate the answer using Jayme's method.

$$\begin{aligned} 5\frac{1}{3} + \left(-4\frac{1}{6}\right) + \left(-2\frac{1}{2}\right) &= 5\frac{2}{6} + \left(-4\frac{1}{6}\right) + \left(-2\frac{3}{6}\right) \\ &= 5\frac{2}{6} + \left(-6\frac{4}{6}\right) \\ &= -1\frac{2}{6} \\ &= -1\frac{1}{3} \end{aligned}$$



5. Will Jayme's brother's method work for the second problem? Why or why not?

Answers will vary. Most students should understand that for thirds and sixths the decimal representations are decimals that do not end.

Problem 2 Analyzing Decimals



1. Convert each fraction to a decimal.

a. $\frac{11}{25}$

The decimal equivalent is 0.44.
terminating

b. $\frac{1}{6}$

The decimal equivalent is 0.1666...
repeating: 0.16

c. $\frac{27}{50}$

The decimal equivalent is 0.54.
terminating

d. $\frac{15}{64}$

The decimal equivalent is 0.234375.
terminating

Note

Reinforce the importance of not dividing by zero.

Share Phase, Question 1

- How do you convert a fraction to a decimal?
- What operation is used to convert a fraction to a decimal?
- Does the decimal end or does it seem to go on forever?
- Can you tell before you divide if the decimal will end or go on forever? Explain.

Grouping

Ask a student to read the information aloud. Discuss the classifications of decimals, and bar notations used with repeating decimals as a class.

5

Discuss Phase, Decimal Classification

- Can a decimal be both terminating and repeating?
- Can a decimal be both non-terminating and non-repeating?
- How many decimal places are needed when expressing a repeating decimal?
- Can bar notation be used to express all repeating decimals?

e. $\frac{7}{9}$

The decimal equivalent is 0.777...
repeating: $0.\overline{7}$

f. $\frac{5}{11}$

The decimal equivalent is 0.4545...
repeating: $0.\overline{45}$

g. $\frac{7}{22}$

The decimal equivalent is
0.3181818... repeating: $0.\overline{318}$

h. $\frac{5}{8}$

The decimal equivalent is 0.625.
terminating



i. $\frac{3}{7}$

The decimal equivalent is
0.428571428571... repeating:
 $0.\overline{428571}$

j. $\frac{39}{60}$

The decimal equivalent is 0.65.
terminating



Decimals can be classified in four different ways:

- terminating,
- non-terminating,
- repeating,
- or non-repeating.

A **terminating decimal** has a finite number of digits, meaning that the decimal will end, or terminate.

A **non-terminating** decimal is a decimal that continues without end.

A **repeating decimal** is a decimal in which a digit, or a group of digits, repeat(s) without end.

A **non-repeating decimal** neither terminates nor repeats.

Bar notation is used for repeating decimals. Consider the example shown. The sequence 142857 repeats. The numbers that lie underneath the bar are those numbers that repeat.

$$\frac{1}{7} = 0.142857142857... = 0.\overline{142857}$$

The bar is called a vinculum.



- How do you know where to place the bar when expressing a repeating decimal?
- Is 0.33 a repeating decimal? Why or why not?

Grouping

Have students complete Questions 2 through 4 with a partner. Then share the responses as a class.



2. Classify each decimal in Question 1, parts (a) through (j) as terminating, non-terminating, repeating, or non-repeating. If the decimal repeats, rewrite it using bar notation.
3. Can all fractions be represented as either terminating or repeating decimals? Write some examples to explain your answer.

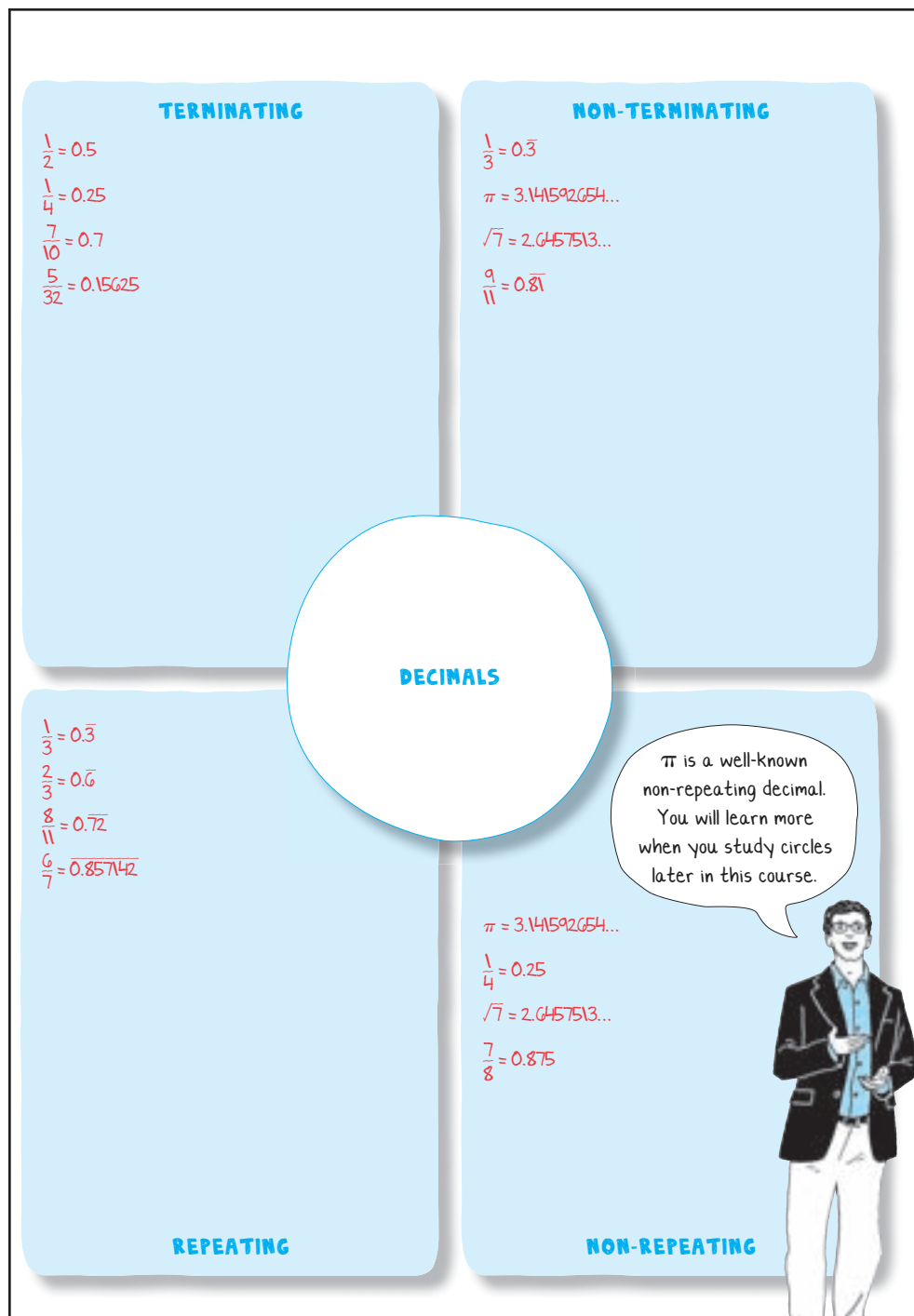
No. Some decimals do not repeat and do not terminate.

Examples may vary.

4. Complete the graphic organizer.
 - Describe each decimal in words.
 - Show examples.



Be prepared to share your solutions and methods.



Follow Up

Assignment

Use the Assignment for Lesson 5.5 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 5.5 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 5.

Check for Students' Understanding

Decide which answer(s) is most exact and explain your reasoning.

$$\frac{2}{3}$$

0.666666 . . .

0.667

$0.\overline{6}$

Both answers $\frac{2}{3}$ and $0.\overline{6}$ are the most exact answers.

The answer 0.666666 . . . is not complete.

The answer 0.667 has been truncated or rounded off.

Chapter 5 Summary

Key Terms

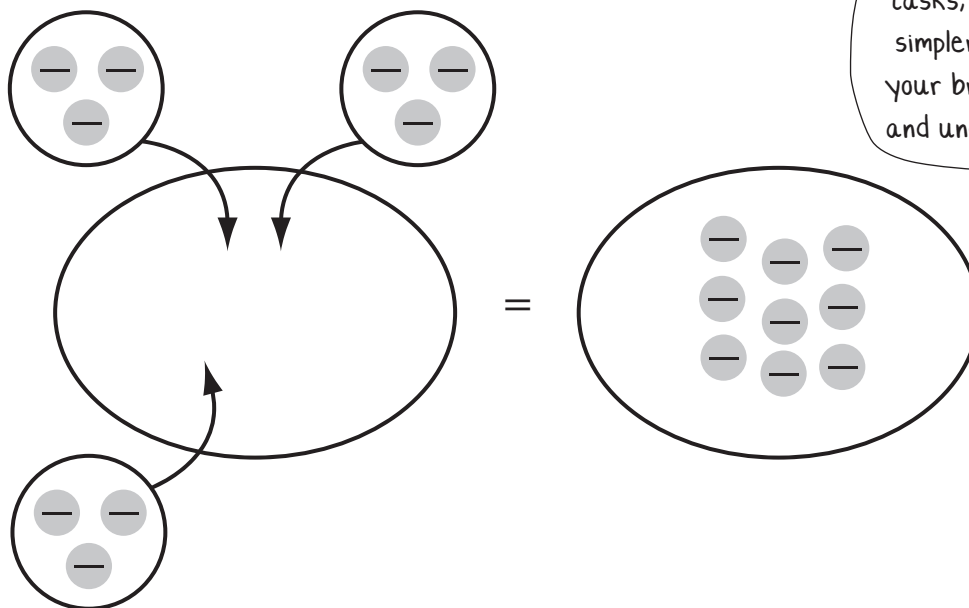
- ▶ terminating decimals (5.5)
- ▶ non-terminating decimals (5.5)
- ▶ repeating decimals (5.5)
- ▶ non-repeating decimals (5.5)
- ▶ bar notation (5.5)

5.1 Multiplying Integers

When multiplying integers, multiplication can be thought of as repeated addition. Two-color counter models and number lines can be used to represent multiplication of integers.

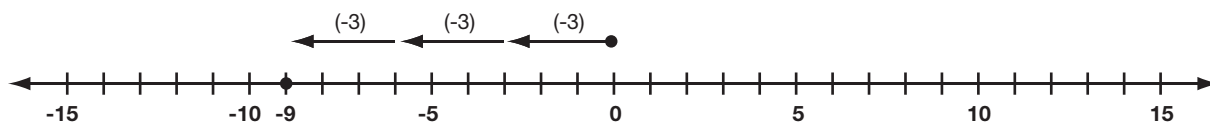
Example

Consider the expression $3 \times (-3)$. As repeated addition, it means $(-3) + (-3) + (-3) = -9$. The expression $3 \times (-3)$ can be thought of as three groups of (-3) .



Dividing complicated tasks, or problems, into simpler pieces can help your brain better learn and understand. Try it!

5



5.1

Dividing Integers

Multiplication and division are inverse operations. For every multiplication fact, there is a corresponding division fact.

Example

This is a fact family for 3, 8, and 24.

$$3 \times 8 = 24$$

$$8 \times 3 = 24$$

$$24 \div 8 = 3$$

$$24 \div 3 = 8$$

5.1

Determining the Sign of a Product or Quotient

The sign of a product or quotient of two integers depends on the signs of the two integers being multiplied or divided. The product or quotient will be positive when both integers have the same sign. The product or quotient will be negative when one integer is positive and the other is negative.

Example

Notice the sign of each product or quotient.

$$5 \times 7 = 35$$

$$35 \div 5 = 7$$

$$-5 \times 7 = -35$$

$$-35 \div 5 = -7$$

$$-5 \times -7 = 35$$

$$35 \div -5 = -7$$

$$-5 \times -7 = 35$$

$$-35 \div -5 = 7$$

he

5

5.2

Multiplying and Dividing Rational Numbers

The rules used to determine the sign of a product or quotient of two integers also apply when multiplying and dividing rational numbers.

Example

The product or quotient of each are shown following the rules for determining the sign for each.

$$3\frac{1}{4} \times 5\frac{1}{3} = \frac{13}{4} \times \frac{16}{3} = \frac{13}{1} \times \frac{4}{3} = \frac{52}{3} = 17\frac{1}{3}$$

$$12.1 \times -5.6 = -67.76$$

$$-6\frac{3}{4} \div 1\frac{7}{8} = -\frac{27}{4} \div \frac{15}{8} = -\frac{27}{4} \times \frac{8}{15} = -\frac{9}{1} \times \frac{2}{5} = -\frac{18}{5} = -3\frac{3}{5}$$

$$-58.75 \div -6.25 = 9.40$$

5.3

Simplifying Expressions with Rational Numbers

When simplifying arithmetic expressions involving rational numbers, it is often helpful to identify and use the number properties or operations that make the simplification easier.

Example

The steps for simplifying the expression are shown.

$$\begin{aligned}
 2\frac{3}{4}\left(5\frac{1}{2}\right) + 2\frac{3}{4}\left(2\frac{1}{2}\right) &= && \text{Number Property/Operation} \\
 2\frac{3}{4}\left(5\frac{1}{2} + 2\frac{1}{2}\right) &= && \text{Distributive Property of Multiplication over Addition} \\
 2\frac{3}{4}(8) &= && \text{Addition} \\
 22 & && \text{Multiplication}
 \end{aligned}$$

5.4

Evaluating Expressions with Rational Numbers

To evaluate an expression containing variables, substitute the values for the variables and then perform the necessary operations.

Example

The evaluation of the expression $8\frac{3}{4}\left(m - 6\frac{1}{5}\right)$ when $m = 7$ is shown.

$$\begin{aligned}
 8\frac{3}{4}\left(7 - 6\frac{1}{5}\right) &= 8\frac{3}{4}\left(\frac{4}{5}\right) \\
 &= \frac{35}{4}\left(\frac{4}{5}\right) \\
 &= \frac{7}{1}\left(\frac{1}{1}\right) \\
 &= 7
 \end{aligned}$$

5.5

Representing Fractions as Decimals

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator.

A terminating decimal has a finite number of digits, meaning that the decimal will end or terminate. A non-terminating decimal is a decimal that continues without end. A repeating decimal is a decimal in which a digit, or a group of digits, repeats without end. When writing a repeating decimal, bar notation is used to indicate the digits that repeat.

A non-repeating decimal neither terminates nor repeats.

Example

The fraction $\frac{3}{4}$ is a terminating decimal. The decimal equivalent of $\frac{3}{4}$ is 0.75.

The fraction $\frac{2}{11}$ is a non-terminating, repeating decimal. The decimal equivalent of $\frac{2}{11}$ is 0.181818... Using bar notation, $\frac{2}{11}$ is written as $0.\overline{18}$.