XLR8 – Accelerating Mathematics Learning

XLR8 Unit 03

Multiplicative change of quantities

03

2016

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ACKNOWLEDGEMENTS

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods.

“YuMi” is a Torres Strait Islander Creole word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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XLR8 Program: Scope and Sequence

	2 year program	3 year program
Unit 01: Comparing, counting and representing quantity Students study countable attributes of their immediate environment, including attributes of the group of students (e.g., more boys than girls, less students with blue eyes than brown eyes) in the classroom and attributes of the school (e.g., quantity of windows in a classroom, quantity of ceiling tiles, length of classroom in steps). This context is limited to those attributes which can be described and represented using whole numbers.	1	1
Unit 02: Additive change of quantities Students extend their investigations of numbers from features of their immediate environment, to features of larger populations in their state, country or world. This context is limited to those features which can be counted using whole numbers and which can be used in additive number stories (for which the total or one of the parts is unknown).	1	1
Unit 03: Multiplicative change of quantities Students explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.	1	1
Unit 04: Investigating, measuring and changing shapes Students explore 3D objects, their 2D surfaces and the 1D attributes of point, line and angle. This context includes measurement of the attribute of turn (angle) and mathematical transformations of 2D shapes and 3D objects including reflection, rotation and translation of shapes and how these may be combined with tessellation to generate and describe designs.	1	1
Unit 05: Dealing with remainders Students extend their investigations of partitioning and quotitioning features of their immediate environment and features of larger populations in their state, country or world to include situations that result in a remainder. Students will explore partitioning whole items into fractions, quotitioning into smaller units, and sharing of remainders of collections. Continuous measures such as length provide useful contexts for partitioning and quotitioning.	1	1
Unit 06: Operations with fractions and decimals Students connect the common fraction representations of tenths, hundredths and thousandths to their decimal fraction representations in contexts common to students' immediate environments including money, measurement and parts of discrete wholes. Students will develop strategies to calculate additive and multiplicative changes involving fractional amounts represented as both common fractions and decimals.	1	2
Unit 07: Percentages Students extend their representations of fractions to include percentage. Percentage is used to compare values multiplicatively and to describe quantity comparisons, recommended daily intake of nutrients, discounts, markups, tax and simple interest. Students will be encouraged to work flexibly between common fractions, decimal fractions and percentages.	1	2

	2 year program	3 year program
Unit 08: Calculating coverage Students extend their investigations of attribute measurement from one-dimensional length measures to two-dimensional measures of coverage or area. This idea starts with coverage which can be counted using whole numbers before extending to fractional measures. Area measurement and calculation provides an opportunity for consolidation of multiplication and division with larger numbers, and consolidation of multiplication and division of fractional quantities expressed as common fractions, mixed numbers or decimal numbers.	2	2
Unit 09: Measuring and maintaining ratios of quantities Students develop their ability to measure duration, convert between units of measure and describe proportional relationships between quantities of discrete items or measurements using ratio notation. Students will also explore changing overall quantities while maintaining consistent proportions between the parts.	2	2
Unit 10: Summarising data with statistics Students develop their ability to gather, organise and represent data from primary and secondary sources. Ideas of sample, population and inference will be used to inform decision making from the gathered data. Students will also develop their ability to analyse measures of central tendency and variation within data sets and learn to represent and interpret these aspects on graphical representations (stem and leaf plots and box and whiskers graphs). Further analysis of the misrepresentation of data will conclude this unit's development of ideas surrounding critical analysis and interpretation of data and statistics.	2	2
Unit 11: Describing location and movement Students develop their ability to describe location and movement along a 1D line and in 2D space with respect to an origin and extending from internal to external frames of reference. Generating 2D representations of location and movement on scale maps and grids using alphanumeric coordinates and compass bearings and distance will be extended to include geometric location of points and collections of points on the Cartesian plane. Students will explore Pythagoras' theorem to find diagonal distances travelled.	2	3
Unit 12: Enlarging maps and plans Students develop their ability to describe proportional relationships between quantities of measurements using ratio notation. Ratio will also be used to describe enlargement and reduction transformations to create similar shapes, scale maps and grids, representations of shapes and paths on the Cartesian plane, and plan drawings. Explorations can be extended to trigonometric ratios between similar figures and the application of scale factor to area of similar figures.	2	3
Unit 13: Modelling with linear relationships Students explore parallels between ratio and rate in the context of relationships between measured attributes. These understandings will be extended to algebraic equations which can also be represented on the Cartesian plane to assist with visualisation of relationships and use of equations and algebraic calculations for finding gradient and distances between points on a line.	2	3
Unit 14: Volume of 3D objects Students explore relationships between measurements of solid objects that lead to calculations of formulae, relationships between solid volume and surface area and investigations of contexts that require calculation of solid volume of composite objects.	2	3
Unit 15: Extended probability Students extend upon their ability to determine theoretical probability and make inferences based upon likelihood of an event. Students will explore and compare theoretical and experimental probabilities, recognise when events are mutually inclusive, mutually exclusive or complementary and determine the probability of single-step and multi-step events.	2	3

Overview

Context

In this unit, students will explore multiplicative relationships and changes using real-world situations that involve discrete items. This context is limited to those features which can be counted using whole numbers, which can be used in multiplicative number stories (for which the product or one of the factors is unknown), and for which divisions also result in whole numbers.

Scope

This unit is based upon the **number-as-count** meaning of **cardinal** number. Once a collection is counted, the collection can be arranged using simple **multiplicative relationships** that describe the collection in terms of the **product** of **factors**. These relationships can be referred to as **factor-factor-product**. **Multiplication strategies** can be used to **compute** the **product** if the **factors** are known. **Division strategies** can be used to **compute** the **unknown factor** if the other **factor** and the **product** are known.

These relationships can be applied to a range of contexts including scaling of axes on graphs using a multiplier, and partitioning measured lengths to find how many tiles are needed to span a specified length.

The organisation of these and other related concepts is shown in Figure 1, in which the scope of concepts to be developed in this unit is highlighted in blue, concepts that may be connected to and reinforced are highlighted in green and number and algebra concepts and processes that are reinforced and applied within this area are highlighted in black.

Assessment

This unit provides a variety of items that may be considered as evidence of students' demonstration of learning outcomes:

- *Diagnostic Worksheets:* The diagnostic worksheet should be completed before starting to teach each RAMR cycle. This may show what students already understand. Not all objectives are represented on diagnostic worksheets.
- *Anecdotal Evidence:* Some evidence of student understanding is best gathered through observation or questions. A checklist may be used to record these instances.
- *Summative Worksheet:* The summative worksheet should be completed at the end of teaching the unit. This may be compared with student achievement on the diagnostic worksheets to determine student improvement in understanding.
- *Portfolio task*: The portfolio task P3: The Big Party accompanying Unit 03 engages students with exploring multiplicative operations in the context of planning catering and seating for a large party.

  This task could be further extended by providing students with access to actual shopping catalogues and allowing them to make their own choices for party food and drinks although for this unit we have limited computation to whole number values only.

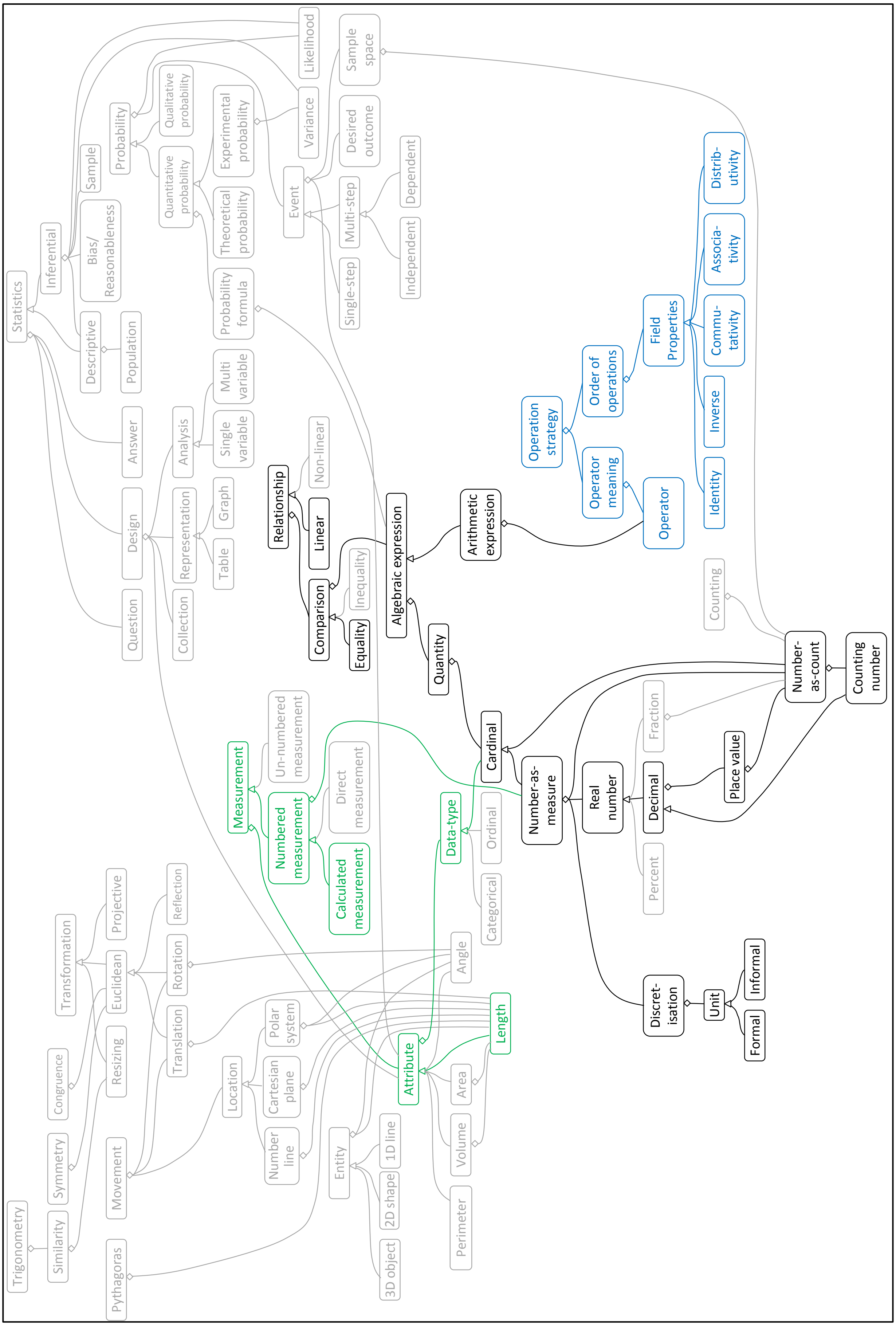


Figure 1 Scope of this unit

Cycle Sequence

In this unit, concepts identified in the preceding section are developed in the following suggested sequence:

Cycle 1: Repeated Additive Change

In this cycle, the simplest meanings for multiplication and division of repeated addition and repeated subtraction of multiple same-size sets are explored using a series of function machines with the same change on each machine. The function machine is also used to explore multiplicative inverse, backtracking and recording of multiplicative change. Equivalence between additive and multiplicative equations is explored using physical and drawn balances.

Cycle 2: Combining Equal Groups

In this cycle, activities extend the repeated addition meaning for multiplication to the combining equal groups meaning (product meaning), using set and array model representations. This cycle will also develop basic fact strategies built around field principles of commutativity, distributivity, associativity, identity and inverse.

Cycle 3: Sharing and Separating Collections

In this cycle, the partitioning meaning of division is developed and identified as an inverse of combining. This cycle will also develop the symbolic notation for division, basic fact strategies, and consolidate students' interpretation and construction of worded multiplicative problems.

Cycle 4: Factorisation

A facility to work flexibly with quantities is an essential component of mental and written computation strategies, and is also used to determine equivalence in fractions and ratios. Factorisation both relies on, and reinforces number facts as it focuses on equivalent number sentences. This idea extends from factorisation of numeric quantities to simplification and expansion of expressions containing variables. This cycle also provides an opportunity to explore and discuss odd, even, prime, composite and square numbers.

Cycle 5: Multiplicative Comparison

In this cycle, activities broaden the range of meanings for multiplicative relationships to include static comparison of groups where no change occurs (e.g., 3 times as many, 5 times fewer).

Cycle 6: Multiplicative Combinations

In this cycle, activities extend the meanings for multiplicative relationships to include multiplicative combinations (e.g., 3 bread types, 5 possible fillings, how many possible sandwiches on the menu).

Cycle 7: Multiplicative Strategies for Larger Numbers

Traditionally, calculation of multiples of greater than single digit numbers needed to be completed mentally or manually as did division calculations (now often completed with calculators). Arguably, strategies for written and mental calculation are important for instances where calculating devices are not available. However, mental and written calculation strategies also develop and demonstrate a facility to work flexibly with number, enhance logical thinking processes, promote problem solving strategies, build number sense and are generalisable to strategies that extend to algebraic understandings and field properties.

Notes on Cycle Sequence:

The proposed cycle sequence outlined should be completed sequentially as it is presented.

Literacy Development

Core to the development of number and operation concepts and their expression at varying levels of representational abstraction (from concrete-enactive through to symbolic) is the use of language that is consistent with the organisation of the mathematical concepts. In this unit the following key language should be explicitly developed with students, ensuring that students understand both the everyday and mathematical uses of each term and, where applicable, the differences and similarities between these.

Cycle 1: Repeated Additive Change

Repeated addition, groups of, lots of, same size groups, grouping, multiples, times, multiplication, repeated subtraction, sharing, shared, partitioned, division, divided, undoes, opposite, inverse, input, output, change, balance, same as, equivalence, equivalent

Cycle 2: Combining Equal Groups

Groups of, lots of, sets of, array, multiple, factor, product, multiply multiplication, combining equal groups, turnaround, identity, commutative law, associative law, distributive law, unknown, variable

Cycle 3: Sharing and Separating Collections

Division, shared, partitioned, divided, sets, groups, rows, columns, equal shares, inverse

Cycle 4: Factorisation

Groups, sets, collections, array, multiple, factor, product, multiply, multiplication, divide, division, compare, comparison, inverse, unknown, variable

Cycle 5: Multiplicative Comparison

Groups, sets, collections, array, multiple, factor, product, multiply, multiplication, divide, division, compare, comparison, inverse, unknown, variable

Cycle 6: Multiplicative Combinations

Multiple, factor, product, multiply, multiplication, divide, division, compare, combinations, options, possibilities, inverse, unknown, variable

Cycle 7: Multiplicative Strategies for Larger Numbers

Multiplication, division, array, area model, number fact strategies, factors, multiples, brackets, strategies, mental computation, separation, sequencing, compensation

Name:____________________________________ Date:________

Can you do this? #1

1. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) A relay team had 4 people running. Each person ran 3 laps of the oval. There were 12 laps run altogether.	4 × 3 = 12
	b) Three children had 3 ballons each. There were 9 balloons altogether.	6 × 2 = 12
	c) Twelve students entered the classroom in pairs. Six pairs entered the room.	3 × 3 = 9

Obj. 3.1.1
a)i□ii□
b)i□ii□
c)i□ii□

2. Fill in the blanks in the Input/Output Tables from a Function Machine.

a)

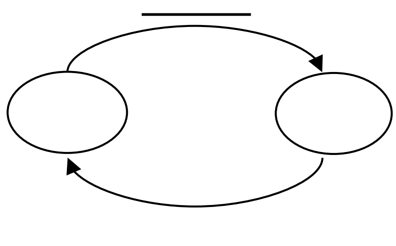
Change ______	
Input	Output
3	9
4	12
5	15
7	____
____	27

b)

Change ______	
Input	Output
6	36
7	42
9	54
11	____
____	24

Obj. 3.1.4
a)i. □
ii. □
iii. □
b)i. □
ii. □
iii. □

3. Draw a multiplication backtracking diagram for 4 + 4 + 4 = 12.



Obj. 3.1.5
i. □
ii. □
iii. □
Obj. 3.1.2
iv. □

4. Is the following equation true or false?

 × 1 =  T □ F □

Obj. 3.1.3
□

5. For each of the following stories:
   i. Underline the factors.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

(a) 21 students gave $6 to the class trip. How much money did the class have altogether? ______________________

____

(b) I bought 3 phones for $60 each. How much did I pay in total?

____

____

6. A paint store works out the price of its tins of paints using the equation: Q × $5 + $3 = C

(Q is the quantity of paint ordered in Litres; C is the final price)

If you order 8L of paint, what will be the price of the paint?

____

____

____

Obj. 3.1.6
a)i. □
b)i. □
Obj. 3.1.7
a)ii. □
b)ii. □
Obj. 3.1.8
a)iii. □
b)iii. □
Obj. 3.1.9
a)iv. □
b)iv. □

Obj. 3.1.10
i. □
ii. □
iii. □

Cycle 1: Repeated Additive Change

Overview

Big Idea

The simplest meanings for multiplication and division are repeated addition and repeated subtraction, where multiple, same-size sets are either added together (multiplication) or subtracted (division) a number of times. This idea can be explored with a series of function machines with the same change on each machine. As for addition and subtraction, the function machine can also be used to explore multiplicative inverse, backtracking and methods of recording multiplicative change as equations. It is also beneficial to reinforce the meaning of the equals sign as "same value as".

Objectives

By the end of this cycle, students should be able to:

3.1.1 Act out, interpret and represent repeated addition multiplication stories informally. [2NA031]

3.1.2 Explore multiplicative inverse for multiplication using function machines. [2NA032]

3.1.3 Identify identity element for multiplication. [5NA098]

3.1.4 Record multiplicative change and inverse using input-output tables. [4NA081]

3.1.5 Record multiplicative change and inverse using backtracking diagrams. [5NA121]

3.1.6 Identify factors within repeated addition multiplication stories as number of repeats and size of repeats. [5NA098]

3.1.7 Identify the product within repeated addition multiplication stories as the total to be found. [5NA098]

3.1.8 Represent repeated addition multiplication stories as equations using symbols for unknowns. [7NA175]

3.1.9 Solve repeated addition multiplication stories using equations with a symbol for the unknown. [4NA082]

3.1.10 Evaluate algebraic expressions by substituting a given value for each variable. [7NA176]

Conceptual Links

The conceptual understanding from this cycle is necessary for understanding and expressing multiplicative operations as written equations for the remaining cycles in this unit.

Materials

For Cycle 1 you may need:

- Variety of everyday items to use to demonstrate sameness or equivalence
- Simple pan balance
- Function machine (repeatable)
- Input, output and change cards

Key Language

Repeated addition, groups of, lots of, same-size groups, grouping, multiples, times, multiplication, repeated subtraction, sharing, shared, partitioned, division, divided, undoes, opposite, inverse, input, output, change, balance, same as, equivalence, equivalent

Definitions

Partition: separate into a number of same-size pieces or groups, usually by sharing.

Repeated addition: same number added a quantity of times. Most easily represented using set or length materials. For example, 4 × 2 → how many are 4 groups of 2 → 2 + 2 + 2 + 2 → 8.

Repeated subtraction: same number subtracted a quantity of times. Most easily represented using set or length materials. For example, 8 ÷ 2 is the same as how many times can you take a group of 2 from 8 → 8 – 2 – 2 – 2 – 2 → 4 times.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How can you find how many there are altogether?
- Is there a simpler way to say this instead of using addition?
- Are the groups you have all the same size?
- How many groups do you have?
- Can you write an equation for this story using multiplication?

Portfolio Task

The student portfolio task *P3: The Big Party* provides students with opportunities to practise multiplication as repeated addition.

RAMR Cycle

The focus of this cycle is to understand the conceptual difference between additive changes which take the form of part-part-total, and multiplicative changes which take the form of factor-factor-product. It is also important to consolidate equivalence and the role of equals in regards to the concept of multiplicative change. Using the meaning of multiplicative change as repeated additive change, students should be able to identify that 2 + 2 + 2 + 2 = 4 × 2.

Note: Consistency of language in regard to mathematical symbols is important. Consider the following story: *A small packet contains three biscuits. Each time I take a packet, I take three biscuits. If I take 5 packets, I have taken 15 biscuits.* Initially, at least, this story should be represented using the number-sentence 5 × 3 = 15, not the number sentence 3 × 5 = 15. The sentence 5 × 3 = 15 reads "5 packets of 3 biscuits equals 15 biscuits" or "5 lots of 3 equals 15". Whilst mathematically we know that 5 × 3 = 3 × 5, using the latter number sentence (3 × 5) we potentially confuse students since the order of what we say when we read the number sentence may not match the order of what students see or act-out. Keep in mind the convention mathematical convention multiplier (how many groups) × multiplicand (size of group) = product (total).

Reality

Use descriptive stories that involve additive repeats of the same amount (for example, buying five cans of soft drink through the self-serve at the supermarket). Students frequently experience this phenomenon lining up for class. If they line up and are only able to enter the room in pairs, the number of students could be skip counted as they step through the doorway (e.g., 2, 4, 6), or repeatedly added (e.g., 2 + 2 + 2). How else could we represent and record this action? (i.e., 3 × 2)

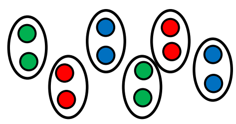
Abstraction

The abstraction sequence for this cycle takes students from an understanding of additive operations to repeated additive operations that can be expressed as multiplicative operations. Once this meaning of multiplication is established. Division will be introduced during the Mathematics phase of learning.

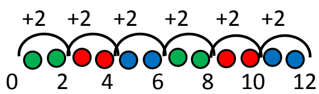
A suggested sequence of activities is presented in the following. Note that several different representations are used (set models, length models, back tracking diagrams, arrow-maths notation, tables of input-out data). There is an opportunity to use these various representations in parallel to one another, especially the tabular representation which could be used to link all representations together.

1. *Kinaesthetic activity.* Lining up and adding a pair of students to the classroom in sequence provides an opportunity for students to act out repeated addition. Repeated subtraction can also be acted out by removing students from the classroom a pair at a time. Refer back to the idea of subtraction as the inverse of addition.

2. *Model/Represent.* Encourage students to represent the actions in their own way. This could be modelled with drawn representations of students coming into the classroom in pairs. Focus students' attention on the count of each group entering the classroom as being of the same the same size (i.e., 2).



1. *Model/Represent.* Represent the actions of entering the room with counters arranged into a length model. Encourage students to move counters in pairs as they repeatedly add. Have students arrange counters in a line and bridge across pairs with 'jumps' (this will provide an effective base to link to the use of a number line representation.



2. *Connect language.* Count how many pairs of students entered the room or were added (e.g., 6 pairs). Connect the everyday language of *pairs* to *groups of 2* (e.g., 6 groups of 2). Discuss with students what they could use in place of the words *groups of*. Students should be familiar with the multiplication symbol (×). Ensure students understand that in multiplication each group has to be the same size.

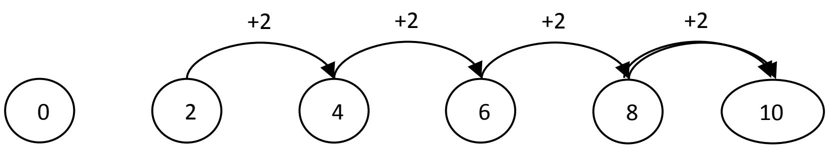
3. *Kinaesthetic activity – function machines.* To reinforce and practise the idea of repeating additive change of the same quantity, repeat the preceding sequence using a series of function machines. Ensure students walk from one side of each machine to the other to physically enact the changes and find a total after each machine (this links to the skip counting strategy for doubles, fives and tens facts). Ask students to identify how many times the same amount has been added.

   Note: To connect effectively to multiplication, it is necessary to start with zero as the first input to the function machine so that the number of additions matches the multiplication or to ensure that students recognise that the input and the change amounts are all the same size and the input must be counted along with the repeats to determine the multiplier.

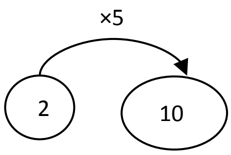
Resource Resource 3.1.1 Repeatable function machine (A3)

4. *Model/Represent – Backtracking diagrams.* When students are comfortable with the function machines for representing change, backtracking diagrams may be useful for more abstractly representing the repeated addition. Backtracking diagrams can be used for recording inputs, changes and outputs and, in the future, recording of inverse operations when outputs are known and changes or inputs are unknown.

Repeated addition Backtracking diagram



Multiplication Backtracking diagram



5. *Model/Represent – Input-Output tables.* It is common in mathematics to record inputs and outputs (or x, y values) in tables, which may help to facilitate students' identification of patterns. When students are exploring instances in which the change is unknown, recording a series of inputs and associated outputs can be a useful representation. Tables can be vertical or horizontal, although vertical tables may be helpful as the arrangement of symbols are more closely aligned to the Arrowmath and conventional number-sentence representations.

Input	1	2	3
Output	5	10	15
Change	___	___	___

Horizontal table

Change ___	
Input	**Output**
1	5
2	10
3	15

Vertical table

   To introduce tables of values to students, start with either function machines or backtracking diagrams. Connect the input, output and change of the function machine or backtracking diagram to the relevant part of the table. Then look for patterns in the numbers that point to the change. Ensure students understand that they are looking for what they do to the first pair of numbers that they can repeat for the other pairs of numbers so that the equations are satisfied. For example, 1 + 4 = 5, but this does not work for the next pair so '+4' is not the change. Conversely, 1 × 5 = 5, 2 × 5 = 10, 3 × 5 = 15 matches the pattern and so the change rule is '×5'. Make sure students practice examples with unknown values as the output, input (find using inverse operation to the change) and change (look for a pattern between the inputs and outputs that identifies the change. Ensure students understand that when they identify the change that they must identify both the operator and the magnitude.

6. *Model/*Represent - *Arrowmath.* Draw suitable pictures to illustrate the "changes" of the stories. Use Arrowmath notation to represent the changes additively and multiplicatively (e.g., 2 $\xrightarrow{\times 6}$ 12).
7. Model/*Represent – Number sentences.* Record the changes as multiplicative equations (e.g., 6 × 2 = 12). Highlight that the alternative way to represent the repeated addition is to use multiplication (i.e., 2+2+2+2+2=6x2)

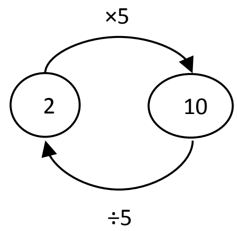
Mathematics

To reinforce and consolidate ideas from the Abstraction sequence, and to introduce division as the inverse of multiplication, the following activities may be beneficial.

Language/symbols and practice

Repeat abstraction steps 3 - 9 for division. For example, 12 students in the room walked out in pairs; how many pairs left the room? Connect *how many pairs* to the symbols ÷ 2.

To support the recognition of division as the 'undoing' of multiplication (i.e., inverse), reversed backtracking diagrams, as shown to the right, may also be constructed.



Ensure students are able to identify the inputs, changes and outputs. Where the input is unknown, students can record the inverse change below the backtracking diagram as on the right.

Connections

Equivalent Groupings

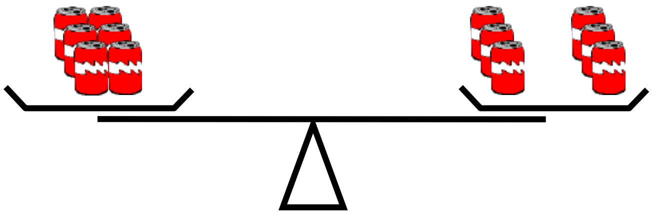
Students' experiences with equivalent equations should be revised and connected to multiplication. In this case the physical balance is used to connect equivalent additive equations to their multiplicative counterparts, which can be further extended to finding equivalent factor pairs. Use physical balances to explore repeated addition of same-size groups of items on one side of the basket that *balance* multipacks of the item on the other side of the balance. This will provide groundwork for number fact strategies in Cycles 2 and 3. For example, adding a four-pack of baked beans three times to one side of the scales is the same as putting three four-packs to the other (make the language connection between 4 + 4 + 4 as 3 lots of 4); four six-packs of soft drink is the same as a 24 can box; a three egg box of Kinder eggs is the same as three single Kinder eggs.

This is an informal opportunity to develop students' understanding of factorisation, which will be more formally addressed in a later cycle.

Commutativity

Extend students' thinking beyond simple equivalence leading to commutativity. For example, if four six-packs of soft drink balances a 24 can box, and four six-packs of soft drink is the same as two 12 can boxes, then two 12 can boxes are the same as a 24 can box.

See if students can come up with their own examples where multiple items provide equivalent outcomes.



A six-pack	balances	2 three-packs
A six-pack	balances	3 pairs
A six-pack	balances	6 single cans

Resource Resource 3.1.2 Multiplicative balances

Reflection

Check the idea

Ask students to create their own multiplicative change scenarios that they express using a diagram and (written) words or letter symbols featuring the use of Arrowmath or a backtracking diagram.

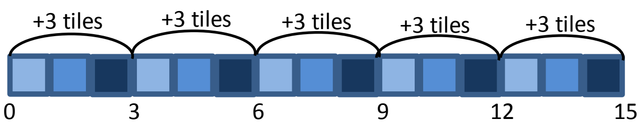
Thinkboards or concept maps may be used to demonstrate a repeated addition or subtraction problem and its representation with materials, language and symbols.

Engage students with devising as many stories and contexts as they can for a given multiplicative equation (e.g., 5 × 3 = 15 may be 5 groups of 3 students, 5 yards with 3 cars each, 5 buckets holding 3 fish each, 5 boxes of 3 Kinder eggs).

Apply the idea

Repeating patterns are used frequently in daily life, for example, songs, dance, jewellery, keys on the piano, heartbeat, cycles of the moon, fabric patterns, bead strings, borders around pages or frames, mosaic patterns and so on.

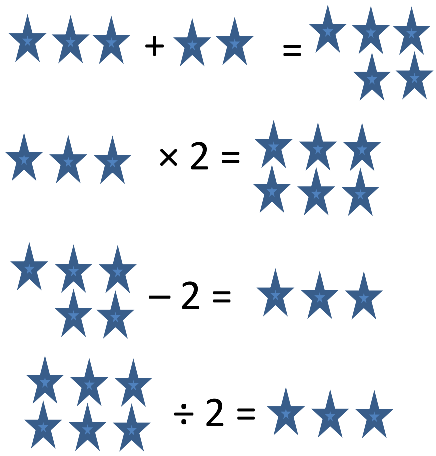
Multiplicative operations may be used to determine how many of each element in the pattern is needed to create a particular number of repeats of the pattern. For example, three colours of tiles are used in a repeating pattern along the edge of a border, if the set of three tiles is to be repeated 5 times, how many tiles will there be in total? If each tile is 5cm wide, how long will the border be? Multiplication as repeated addition is useful to solve this problem.



Extend the idea

Generalise

Engage students in comparison of additive and multiplicative problems. Ensure that students can generalise how additive and multiplicative operations are similar and different. For example, the following patterns or rules should be explored:



- Adding: group sizes may be the same or different values
- Multiplying: groups are all repeats of the same size (can be found using repeated addition or skip counting forwards)
- Subtracting: group sizes may be the same or different values
- Dividing: groups are all repeats of the same size (can be found using repeated subtraction or skip counting backwards)

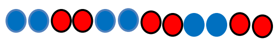
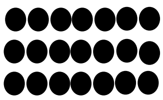
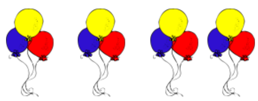
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #2

1. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) There were 3 classes with 7 students in each class. There were 21 students in total.	4 × 3 = 12
	b) Four children had 3 ballons each. There were 12 balloons altogether.	6 × 2 = 12
	c) Twelve students entered the classroom in pairs. Six pairs entered the room.	3 × 7 = 21

Obj. 3.2.1
a)i☐ii☐
b)i☐ii☐
c)i☐ii☐

2. Mark these equations right (✓) or wrong (×).

(a) 36 = 9 × 4 (b) 3 × 8 = 24 – 2

Obj. 3.2.4
a) ☐
b) ☐

3. Look at the story below.
   i. Underline the factors.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

   There were 5 children looking after 4 birds each. How many birds altogether? ____

   ____

Obj. 3.2.2
i. ☐
Obj. 3.2.3
ii. ☐
Obj. 3.2.5
iii. ☐
Obj. 3.2.6
iv. ☐

4. Think about this **3** × **7** = **21** equation:

   Tick (✓) the box next to the correct answer:

a) If the **3** is replaced with a bigger number and the **7** stays the same, what will happen to the **21**?

☐ The **21** will be replaced with a bigger number

Or ☐ The **21** will be replaced with a smaller number

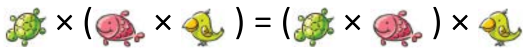
b) If the **3** is replaced with a smaller number and the **21** stays the same, what happens to the **7**?

☐ The **7** will be replaced with a bigger number

Or ☐ The **7** will be replaced with a smaller number

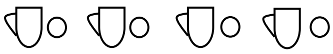
Obj. 3.2.4
a) ☐
b) ☐

5. Tick true or false for each equation.

(a)		T ☐	F ☐
(b)		T ☐	F ☐
(c)		T ☐	F ☐
(d)		T ☐	F ☐

Obj. 3.2.8
a) ☐
Obj. 3.2.9
b) ☐
c) ☐
Obj. 3.2.7
d) ☐

6. Write an expression to represent the drawing of cups and counters.

 ______________________________

Obj. 3.2.5
i. ☐
ii. ☐

7. Draw cups and counters to represent 4y+1

Obj. 3.2.5
i. ☐
ii. ☐

8. Write a story for the equation: 5 × c = 125

____

____

Obj. 3.2.5
i. ☐
ii. ☐
iii. ☐
iv. ☐

9. Four students bought lunch. Each student bought a burger for $4 and a can of soft drink. How much did they spend on lunch altogether?

(a) Write an equation with variables to represent the story.

____

(b) If soft drink cans were $1 each, how much did the students spend on lunch altogether? ______________________________

____

Obj. 3.2.5
a)i. ☐
ii. ☐
iii. ☐
iv. ☐
Obj. 3.2.6
b)i. ☐
b)ii. ☐

Cycle 2: Combining Equal Groups

Overview

Big Idea

For students to succeed at interpreting multiplication problems, they need to understand that the written equations they see represent a broad range of real world situations that involve multiplicative change or relationships, which can be modelled using materials or drawings. However, many students recognise only the repeated addition meaning of multiplication in problems and use skip counting as their only multiplication strategy.

In this cycle, activities extend the repeated addition meaning for multiplication to the combining equal groups meaning (product meaning), using set and array model representations. This cycle will also develop basic fact strategies built around field principles of commutativity, distributivity, associativity, identity and inverse. Related meanings of division will be developed in Cycle 3. Additional meanings of multiplication (i.e., comparison and combinations) are addressed in Cycles 5 and 6.

Objectives

By the end of this cycle, students should be able to:

3.2.1 Act out, interpret and represent combining equal groups multiplication stories informally. [2NA031]

3.2.2 Identify factors within combining equal groups multiplication stories as number of groups and size of groups. [5NA098]

3.2.3 Identify the product within combining equal groups multiplication stories as the total to be found. [5NA098]

3.2.4 Apply number fact strategies for multiplication. [4NA075]

3.2.5 Solve combining equal groups multiplication stories using equations with a symbol for the unknown. [7NA176]

3.2.6 Represent combining equal groups multiplication stories as equations using symbols for unknowns. [7NA175]

3.2.7 Apply commutative property for multiplication. [7NA151]

3.2.8 Apply associative property for multiplication. [7NA151]

3.2.9 Apply distributive property for multiplication. [7NA151]

3.2.10 Apply laws and properties of arithmetic to algebraic terms and expressions. [7NA177]

Conceptual Links

This cycle introduces a range of strategies for performing whole number multiplication and, through practice, aims to build a degree of recall-based answering of simple multiplication problems since this will assist more complex calculations.

Materials

For Cycle 2 you may need:

- Counters
- Calculators
- Grids for quickly structuring arrays
- Unifix cubes
- Multiplication representation match cards
- Foam cups

Key Language

Groups of, lots of, sets of, array, multiple, factor, product, multiply multiplication, combining equal groups, turnaround, identity, commutative law, associative law, distributive law, unknown, variable

Definitions

Associative law/Associativity: multiple operators within a calculation problem can be grouped in any order (for multiplication and division). For example, $8 \times 6 \div 3 = (8 \times 6) \div 3 = 8 \times (6 \div 3)$

$= 48 \div 3 = 8 \times 2 = 16$

Commutative law/Commutativity: factors may be multiplied together in any order to find the product (most simply written algebraically: $a \times b = b \times a$). Applies to addition and multiplication operations (known as turnaround when applied to number facts). Does not apply to subtraction or division operations.

Distributive law/Distributivity: multiplying a group of numbers added together is the same as doing each multiplication separately. For example, $3 \times (2 + 4) = 3 \times 2 + 3 \times 4 = 6 + 12 = 18$

Identity: multiplier that does not change the initial quantity. For multiplicative operations the identity value is 1.

Inverse: opposite operation. For multiplication, inverse operation is division and vice versa.

Patterning: Collection of number fact strategies that rely on known patterns to assist with mental calculation. Examples include multiplication by 0, 1, 2, 5, 9, 10.

Turnarounds: strategy for reducing the quantity of number facts to learn. Based on commutative law.

Use known facts: Collection of number fact strategies that rely on known number facts and conceptual understanding to assist with mental calculation of other facts. Examples include use doubles, build up and build down.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What numbers are you multiplying?
- What strategy can you use?
- What would that division be as multiplication?

Portfolio Task

The student portfolio task *P3: The Big Party* engages students with finding quantities and total costs of items to purchase for the party.

RAMR Cycle

The main focus of learning in this cycle is to extend the concept of multiplication to include the combining equal groups (or product) meaning of multiplication using set and array models and to develop multiplication number fact strategies.

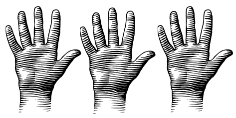
Reality

Find situations in the local area or around the school where objects are multiplied or combined in equal groups of discrete objects. Some examples may include: lolly bags (same quantity in each bag); bags of marbles; small groups of students. Ensure students can identify the number in each set and the number of sets (multiplier).

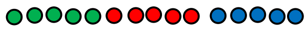
Abstraction

In this abstraction sequence the focus is on consolidating the concept of multiplication as combining equal groups of items and connecting the concept to the array model. A suggested abstraction sequence is as follows:

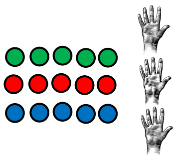
1. *Kinaesthetic activity.* Choose a multiplication problem that can be acted out. For example, 3 students hold up 5 fingers each. Have students draw the problem in their books. Reinforce the language of 3 groups of 5 fingers.



2. *Model/Represent.* Use three colours of counters or unifix cubes to model 3 groups of 5 on the desk in sets. Reorganise the groups so that the 3 groups are end to end. Discuss how students see the problem (5 + 5 + 5 or 3 × 5).



3. Reorganise the counters into an array. Discuss how students see the problem now. Connect each row of counters to the quantity of fingers held up, and the quantity of rows of counters to the quantity of students with their hands up. Students' hands could also be rearranged to an array to reinforce the connection.



   Note: This sequence could be acted out with students standing in random groups, standing in a line where each group can be identified, and standing in an array.

4. *Recording.* Connect the language used to describe 3 groups of 5 and 3 rows of 5 to factor-factor-product as an equation. Students should recognise that 3 and 5 are the factors that are multiplied together to reach the product of 15 (3 × 5 = 15).
5. Using the previous example as a base, compare situations in the local area or around the school where objects are arranged in an array which can be seen as rows and columns. Some examples may include: light panels on ceilings; rows or groups of desks in classrooms; rooms in buildings (if buildings have consistent numbers of rooms per floor or windows in rooms of buildings when seen from the outside); number of houses down each side of a street (could be simple double but is an example); paved/tiled areas; and cars parked in the parking lot in several parallel rows.
6. Pose problems that involve combining equal groups to find a product. Provide materials for students to model problems if needed. As appropriate, use function-machine, backtracking, Arrowmath and table representations to draw links between the physical situation and symbolic equations. Ensure students can write equations for each story using the format of factor-factor-product (or, initially, number of groups × group-size = total). Encourage students to use a symbol for the unknown (question mark, box or letter). Thinkboards may be used to collate ideas.

Resource Resource 3.2.1 Multiplication thinkboard or concept map

Mathematics

Once students are able to connect contexts to language and symbols and understand the concept of multiplication as combining equal size groups and the array representation, it is important to consolidate these connections. Also, once the symbolic representation of multiplication has been achieved, number fact strategies should be explored and practised with a view to freeing up working memory that is needed for wider applications of operations in finding solutions to complex, real-world problems.

Language/symbols and practice

Number Facts

Once the meaning of multiplication as combining equal groups to create a product has been established and connected to symbolic representations, it is possible to work within the realm of mathematics to establish strategies for the basic number facts. Materials should still be provided to scaffold student visualisation of strategies and properties of quantities to generate memorable mental images.

There are three main strategies that can be used to learn single digit multiplication:

- Patterning
- Turnarounds
- "Use known facts"

When viewed together, these strategies cover (sometimes with repetition or in combination with one-another) all of the basic multiplication facts (i.e., 0×0 … 9×9). To develop students' proficiency in using the various facts, instruction can begin with concrete materials (e.g., counters) paralleled with symbolic representations and then progress to 'in the mind' use of the strategy.

Students may be generally weak in multiplication facts or may be proficient with some and not with others. It is useful to check if there are general gaps which may be remediated with the focused teaching of a specific strategy. *Resource 3.2.2 Multiplication Number Fact Diagnosis* will be useful for teachers and students to identify the number facts they need to focus on for teaching and learning.

Resource Resource 3.2.2 Multiplication Number Fact Diagnosis

Using strategies to teach basic facts

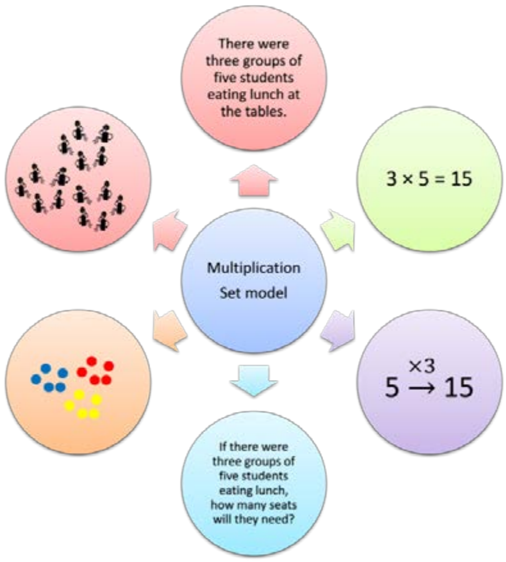
Strategies are used differently for multiplication and division than they are for addition and subtraction. In addition and subtraction, strategies covered a variety of facts. In multiplication and division, there is more focus on specific sets of basic facts (e.g., 4× and 7× basic facts). Basic fact strategies make use of ***identity, inverse,*** and the ***commutative, associative*** and ***distributive laws.*** These principles underpin useful strategies to use when solving mathematical problems and should be highlighted and connected to where they arise.

Resource Resource 3.2.3 Multiplication Number Fact Strategies

Reflection

Check the idea

As an assessment or evidence of learning artefact it is possible to give students the idea and a story (or equation or model …) and have them fill out as many other representations as they can. This will demonstrate the breadth of students' connections within a topic. An example for the set model with many connections is shown on right.



Students could work in pairs or threes to come up with a multiplication story, swap maps with another group to draw a picture, swap again to draw models, and swap again to write equations.

Apply the idea

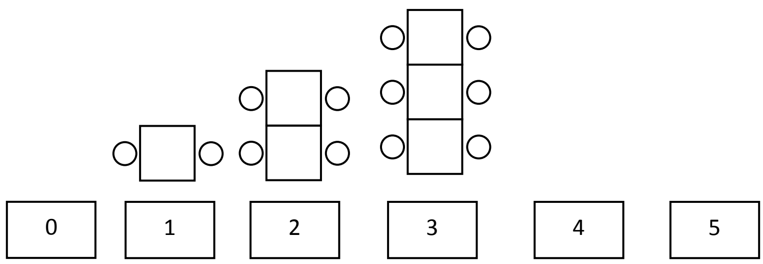
Resource 3.2.4 Challenge Investigations allows students to explore the relationship between the factors and the product, and what happens to the quantities when the factors and product change.

Resource Resource 3.2.4 Challenge investigations

Growing patterns are similar to repeating patterns in that there is a pattern of change that may be identified. These types of patterns are often found in growth of plants and physical arrangements of resources such as tables and chairs, tiling patterns for incrementally increasing sizes of floor and stacking of resources (such as cans), and so on.

Tables of values may be used, but instead of recording inputs and outputs it is necessary to label the terms in the growing pattern and the number of elements in the pattern. For example:

> A table in a restaurant seats 2, two tables together seats 4, three tables together seats 6, and so on. How many tables will 15 tables pushed together seat? How many tables are needed for 48 people?



Encourage students to draw the next two terms to establish they understand the pattern. Students may be able to identify the pattern as 2 x number of tables = number of seats. Connect to a table of values as follows (in this case the number of tables = term no.):

Tables	0	1	2	3	4	5		15		
Seats	0	2	4	6	8	10				48
Change		×2	×2	×2	×2	×2				

Factor sets should also be continued to be developed (i.e., building upon Cycle 1 experiences). For example, 12 = 12 × 1 = 6 × 2 = 4 × 3 = 3 × 4 = 2 × 6 = 1 × 12 = 2 × 2 × 3 = 2 × 3 × 2; counters, unifix cubes or grid paper may be useful to assist students in modelling these.

Extend the idea

Maintaining Equivalence

Consolidate and generalise understanding of the multiplication concept and number facts by exploring the effect increasing the size of a factor has on the product (remind students of work in *Resource 3.2.4: Challenge Investigations*). Also explore what happens to the other factor when one factor is changed and the product remains the same. Connect these ideas to factor sets (i.e., double the factor, double the product).

Multi-term multiplication

Extend students multiplication ability to include expressions with many terms. For example, boxes may be arranged on a pallet in layers, where each layer is comprised of several rows of boxes. The total number of boxes is therefore the number of layers x number of rows x number of boxes in each row. This can also include simple powers (squared, cubed etc.), in particular base 10 and making links to the decimal place-value system.

To support this thinking, backtracking diagrams and the like maybe also be used.

Powers of 10

Connect powers of 10 (10^1, 10^2, 10^3, 10^4) to their expanded multiplication forms (10, 10×10, 10×10×10, 10×10×10×10). Connect these to place values (tens, hundreds, thousands, ten thousands).

Introducing variable using cups and counters

An effective method for introducing understanding of operations with variables is to use cups to represent an unknown but repeatable quantity of counters as in *Resource 3.2.5 Introducing variables*. The idea is to extend students' experiences with materials (particularly the set model) for representing operations with discrete amounts to representing operations with variable amounts. In this case, the multiplicand (group size) is the variable quantity represented by the cup and the multiplier is the number of cups. Constants are represented by discrete materials such as counters.

Resource Resource 3.2.5 Introducing variables

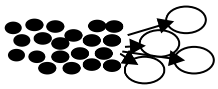
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #3

1. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) 20 fish were caught and put evenly into 4 buckets. Each bucket had 5 fish.	10 ÷ 5 = 2
	b) 5 girls shared a ribbon that was 10m long and was cut into 2m lengths.	7 ÷ 7 = 1
	c) Seven children shared a bunch of 7 balloons. Each child received 1 balloon.	20 ÷ 4 = 5

Obj. 3.3.1 a)i□ii□ c)i□ii□ Obj. 3.3.2 b)i□ii□

2. Mark these equations right (✓) or wrong (×).

(a) 16 – 2 = 14 ÷ 2 (b) 27 ÷ 3 = 9

Obj. 3.3.5 a) □ b) □

3. Look at the stories below. For each story:
   i. Underline the <u>factors</u>.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

(a) Each student gave $6 to the class trip. The class had $126 altogether. How many students paid for the trip?

____

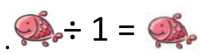
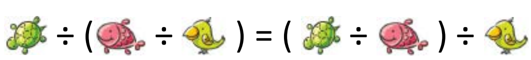
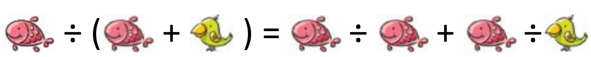
____

(b) I spent $120 on 3 phones. If the phones were all the same, how much was each phone? ____________________________

____

Obj. 3.3.6 a)i. □ b)i. □ Obj. 3.3.7 a)ii. □ b)ii. □ Obj. 3.3.8 a)iii. □ b)iii. □ Obj. 3.3.9 a)iv. □ b)iv. □

4. Tick true or false for each equation.

(a)		T □	F □
(b)		T □	F □
(c)		T □	F □
(d)		T □	F □

Obj. 3.3.10 a) □ Obj. 3.3.4 b) □ Obj. 3.3.10 c) □ d) □

5. Think about this equation: $12 \div 2 = 6$

Tick (✓) the box next to the correct answer:

(a) If the 12 is replaced with a smaller number and the 2 stays the same, what will happen to the 6?

☐ The 6 will be replaced with a bigger number

Or ☐ The 6 will be replaced with a smaller number

(b) If the 12 is replaced with a bigger number and the 2 is replaced with a smaller number, what happens to the 6?

☐ The 6 will be replaced with a bigger number

Or ☐ The 6 will be replaced with a smaller number

Obj. 3.3.5
a) ☐
b) ☐

Pretend to use the calculator.

Which operation key (× or ÷) would you use to solve each of these problems?

6. Circle the correct button beside each question.

(a) I bought 4 packets of chips at the shop, each one cost $2. How much did I spend?	× ÷
(b) I cut a hose into five pieces the same length. The hose was 20 m long to begin with. How long was each piece?	× ÷

Obj. 3.3.8
a) ☐
b) ☐

7. Write a story for the equation: $125 \div c = 5$

____

____

Obj. 3.3.8
i. ☐
ii. ☐
iii. ☐
iv. ☐

8. Four students bought lunch. Each student bought a burger for $5 and a can of soft drink. How much did they spend on lunch altogether?

(a) Write an equation with variables to represent the story.

____

(b) If the students spent $24 altogether, what did a can of soft drink cost? __________________________________

____

Obj. 3.3.8
a)i. ☐
ii. ☐
iii. ☐
iv. ☐
Obj. 3.3.9
b)i. ☐
b)ii. ☐

Cycle 3: Sharing and Separating Collections

Overview

Big Idea

Whole number division appears difficult for students even though it is naturally experienced and applied from an early age when sharing. Symbols in division represent real-world situations that involve multiplicative change and relationships where a whole amount is distributed into equal shares. This distribution can happen in one of two ways: the share size is known but the number of shares is unknown (repeated subtraction, as developed in Cycle 1); or the number of shares is known, but the size of each share is unknown (partitioning or sharing). Part of the difficulty lies in the use of language to describe divided by and divided into.

In this cycle, the partitioning meaning of division is developed and identified as the inverse of combining. This cycle will also develop the symbolic notation for division, basic fact strategies, and consolidate students' interpretation and construction of worded multiplicative problems.

Objectives

By the end of this cycle, students should be able to:

3.3.1 Act out, interpret and represent partitive (number of groups is known, size of group is unknown) division stories informally. [2NA032]

3.3.2 Act out, interpret and represent quotitive (size of group is known, number of groups is unknown) division stories informally. [2NA032]

3.3.3 Explore multiplicative inverse for division using function machines. [2NA032]

3.3.4 Identify identity element for division. [5NA098]

3.3.5 Apply number fact strategies for division. [4NA075]

3.3.6 Identify factors within division stories (divisor, dividend). [5NA098]

3.3.7 Identify the quotient within division stories as the total before sharing or separating. [5NA098]

3.3.8 Represent division stories as equations using symbols for unknowns. [7NA175]

3.3.9 Solve sharing and separating stories using equations with a symbol for the unknown. [7NA176]

3.3.10Test field properties for division. [7NA151]

Conceptual Links

This cycle relies on previously developed understanding of additive operations and multiplication meanings of repeated addition and combining equal groups.

This cycle introduces strategies for performing whole number division and aims to build a degree of recall-based answering of simple division problems to assist more complex calculations.

Materials

For Cycle 3 you may need:

- Counters
- Thinkboard or concept map
- Unifix cubes
- Physical combination resources

Key Language

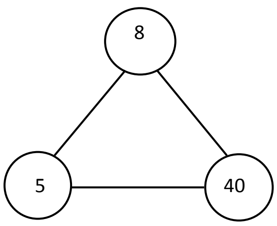
Division, shared, partitioned, divided, sets, groups, rows, columns, equal shares, inverse

Definitions

Partitive division/partitioning: Division problems in which the product is known, the number of groups is known, and the size of each group is unknown. Usually, sharing is initially computed by allocating items to each group one at a time to find how many in each group.

Quotitive division/quotitioning: Division problems in which the size of the groups is known, the number of groups is unknown, and sharing takes place by the repeated removal of known-size parts to determine how many groups result.

Triadic relationships: Depicted as a triangle with a quantity at each corner. Quantities are related such that two points of the triad multiply to the third quantity. Fact families may be generated by exploring relationships between the values around the triad.



Fact families:

8 × 5 = 40	***Note:*** also	40 = 8 × 5
5 × 8 = 40		40 = 5 × 8
40 ÷ 8 = 5		5 = 40 ÷ 8
40 ÷ 5 = 8		8 = 40 ÷ 5

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Can you write an equation for this question?
- What is the question asking you to find?
- Do you know the number of groups?
- Do you know the size of the groups?
- Do you know the product?
- What would the equation look like as a multiplication?
- Can you rearrange this now to be a division?
- What is the inverse operation?

Portfolio Task

Portfolio task *P3: The Big Party* engages students with finding quantities and total costs of items to purchase for the party.

RAMR Cycle

Division problems can be classified into two types:

- ***Quotitive*** division problems in which the size of the groups is known, the number of groups is unknown, and sharing takes place by the repeated removal of known-size parts to determine how many groups result; and
- ***Partitive*** division problems in which the product is known, the number of groups is known, and the size of each group is unknown. Usually, sharing is initially computed by allocating items to each group one at a time to find how many in each group.

The main focus of learning in this cycle is to extend the concept of division to include distributing a set into a known number of groups (partitive division) or into a number of groups of a known size (quotitive division) using set and array models and to explore number fact strategies.

Reality

Find situations in the local area or around the school where objects are shared or partitioned into equal groups of discrete objects. Some examples may include: spreading the class out into four equal groups of students in the classroom or evenly distributing animals among yards or cages. Students should be encouraged to come up with their own examples of larger collections that are shared or partitioned into smaller groups of the same size.

Focus upon situations in the local area or around the school where objects are arranged in arrays which can be seen as rows and columns. Some examples may include: light panels on ceilings; rows or groups of desks in classrooms; rooms in buildings (if buildings have consistent numbers of rooms per floor or windows in rooms of buildings when seen from the outside); banks of lockers; number of houses down each side of a street (could be simple double but is an example); paved/tiled areas; fruit trees or vegetable rows with the same number of plants in each row. Such situations should be familiar from the previous cycles' exploration of multiplication.

Abstraction

In this abstraction sequence the focus is on the quotitive meaning; the partitive meaning will be developed in the Mathematics phase.

1. *Kinaesthetic activity.* Choose a division problem that can be acted out. For example, 15 students make groups of 5 around the room. How many groups of students? Have students separate 5 at a time and count the groups. Students should draw the problem in their books.
2. *Model/Represent.* Use three colours of counters or unifix cubes to model the groups of 5 on the desk in sets. Reorganise the groups so that the 3 groups are end to end. Discuss how students see the problem (15 – 5 – 5 – 5 or 15 broken into groups of 5 or 5 groups of 3). Connect 5 × 3 to the inverse of 15 ÷ 5.

3. *Recording.* Connect the language used to describe the collection and the size of the group to the factor-factor-product equation. Students should recognise that 15 is the product, and the size of the group (5) is one of the factors. The other factor is unknown and may be represented in a variety of ways, including using a pronumeral.
4. Pose a range of quotitive division problems. Allow students to model these with counters or other materials as needed. Ensure that students can write an equation for each division story using the format factor-factor-product. The teaching activity Clumps can be useful for generating division problems quickly from groups of students.

Teaching Activity: Clumps

The game of "clumps" is useful here to quickly build a number of examples of partitioning the class group and can lead to discussion of remainders where there are incomplete groups for some divisions. To play, a teacher or student calls out number that the students are required to 'clump' into. This number choice could be a randomised number website, for example: http://www.mathgoodies.com/calculators/random_no_custom.html)

When interpreting worded problems, students may write the equation as a multiplication, then rearrange to a division equation using inverse. For example, $5 \times ? = 15$ and so $? = 15 \div 5$.

Thinkboards can be useful for collating the various representations of problems separating collections into same-size groups.

Resource Resource 3.3.1 Division thinkboard or concept map

Mathematics

Language/symbols and practice

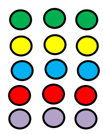
Partitive Division

The following sequence of activities is similar to that used in the Abstraction sequence for Quotitive Division.

1. *Kinaesthetic activity.* Choose a division problem that can be acted out. For example, 15 students make 5 groups around the room. How many students in each group? Have students move one at a time to designated areas. When all students are shared into groups, count up the size of a group. Students should draw the problem in their books. Reinforce the language of 15 students shared into 5 groups of students is 3 students per group.

2. *Model/Represent.* Use circles on a page and counters or unifix cubes to model sharing into 5 (different coloured) groups. Reorganise the groups so that the 5 groups are end to end. Discuss how students see the problem.

3. Reorganise the counters into an array. Discuss how students see the problem now. Connect the array to the whole group of students, each row of counters to the quantity of students in each group, and the quantity of rows of counters to the number of groups.



4. *Recording*. Connect the language used to describe the whole collection and the size of the group to factor-factor-product as an equation. Students should recognise that 15 is the product, and the size of the group (3) is one of the factors. The other factor is unknown and may be represented with a pronumeral.

5. Pose a range of partitive multiplication problems. Allow students to model these with counters or other materials as needed. Ensure that students can write an equation for each division story using the format factor-factor-product. As in the Abstraction sequence, the *Resource 3.3.1: Division thinkboard or concept map* can be useful for collating the various representations of partitive division problems.

Connections

Multiplicative Inverse

Reconsider the activities used in the development of quotitive and partitive division, in particular the re-arrangement of counters from set, length and array forms. Stress the inverse meanings of repeated addition multiplication with quotitive (repeated subtraction) division and combining-equal-groups (product) multiplication with partitive division.

Ensure students can interpret and pose a range of multiplicative problem types. Draw students' attention to multiplication and division. Ensure that students can determine when they need to use multiplication or division, and where distributivity, associativity and commutativity can and cannot be applied to multiplication and division.

As appropriate, continue to use a range of materials such as counter arranged in arrays to reinforce multiplicative concepts, including communitivity.

Resource Resource 3.3.2 Interpreting and constructing multiplicative problems
Resource 3.3.3 Choosing appropriate operations

Language/symbols and practice

Division Fact Strategies

Once the meaning of division as distributing into same-sized groups to find a quantity per share or quantity of shares has been established and connected to symbolic representations, it is possible to work within mathematics to reinforce strategies for basic number facts.

The major strategies for single digit division are *think multiplication* and the use of *fact families* (triadic relationships). The basic facts are also extended to *multiples of tens facts*. These strategies rely on understanding that division is the inverse operation for multiplication. This is an important connection to reinforce when introducing the think multiplication strategy for basic facts.

Think multiplication

This strategy is for all division facts. The division facts are reversed in thinking to multiplication form, for example, 36 ÷ 9 is rethought as "what times 9 equals 36".

Fact Families (Triadic relationships)

This strategy reinforces 'think multiplication' and relates multiplication to division. For each multiplication/division fact, there are 4 members of the family. For example, 5 × 3; 3 × 5; 15 ÷ 5; and 15 ÷ 3 are all members of the same fact family that relates Factor (5) – Factor (3) – Product (15).

Extended multiples of tens facts

Once basic facts are known, it is important to extend them to multiples of ten facts.

Further detail about each of these strategies and associated examples of problems can be found in *Resource 3.3.4 Division number fact strategies.*

Resource Resource 3.3.4 Division Number Fact Strategies

Reflection

Check the idea

Give students a story (or equation or model …) and have them fill out as many other representations as they can on a concept map or Thinkboard. This will demonstrate the breadth of students' connections within a topic. Practice division facts using multiplication and division games with number cards and dice that involve mix and match activities as well as cover the board style games.

Apply the idea

Resource 3.3.5 Challenge Investigations allows students to explore the relationship between the factors and the product, and what happens to the quantities when the factors and product change.

Resource Resource 3.3.5 Challenge investigations Division

Extend the idea

Remainders

Extend students' ability to consider problems in which the divisor is not a factor of the product (e.g., 27 ÷ 4 = 7 remainder 3). The remainder only needs to the identified, as the concept of creating fractions as a result of division (i.e., sharing the remainder) will be dealt with in a future cycle.

Discuss with students when remainders make sense. If the discrete objects in the set or array are living things, what will leftovers or incomplete groups mean for providing a suitable answer in context? For example, when planning seating around tables, will another table be needed to accommodate attendees or do number of attendees need to be restricted; if people are travelling, will a further vehicle be needed or do some people not go; ordering food items in multi-packs will mean there is food left over or someone will miss out. Students should identify examples from their own experiences. Explore a variety of problems, translate to equations, and then practise adjusting the solution to the equation according to the context of the question asked.

Maintaining Equivalence

Consolidate and generalise students' understanding of the division concept and number facts by exploring the effect on the product of increasing the size of a factor (link to *Resource 3.3.5 Challenge Investigations Division*).

Repeated Division

Extend the idea of repeated multiplication to include repeated division. Function machines may be used that repeat a multiplicative change (start with an input of 1 not 0). Explore a series of ÷2 changes and record on backtracking diagrams. Discuss other notations used to represent these ($\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$). In the case of squaring and square-rooting, explicitly link the language to the construction of squares using counters. Similarly, cube and cube-root can be linked to the construction of cubes using unifix cubes. Where appropriate, use calculators to find powers and roots of large numbers.

Consolidating variable

Reinforce students' confidence with unknowns and symbols by extending variable work (started in Cycle 2) to division stories. Cups and counters may still be used to model if needed.

Resource Resource 3.2.5 Introducing variables

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:____ Date:________

Can you do this? #4

1. (a) Circle the multiples of 3.

(i) 18 (ii) 127 (iii) 123 (iv) 53 (v) 42

(b) Circle the multiples of 8.

(i) 28 (ii) 128 (iii) 74 (iv) 56 (v) 42

2. (a) What number has factors of 3, 5? ____________________

(b) What number has factors of 2, 3, 5, 7? ____________________

3. Write down as many sets of factors for 12 as you can.

(Sets of factors can have more than 2 factors in them.)

____

____

4. Write down the set of prime factors for 18. (A factor tree will help.)

____

____

____

(a) Write down the factors of 32. ____________________

____

(b) Write down the factors of 20. ____________________

____

(c) Write down the common factors of 20 and 32. ____________

(d) Write down the highest common factor of 20 and 32. ________

(e) Write down the lowest common factor of 20 and 32. ________

5. (a) A fence uses panels of length P. Write the expression for the length of a fence that is 16 panels long. ____________________

(b) Simplify the expression: (6a + 3b). ____________________

Obj. 3.4.1
a)i☐ii☐
iii☐iv☐
v☐
b)i☐ii☐
iii☐iv☐
v☐
Obj. 3.4.2
a) ☐
b) ☐
Obj. 3.4.2
☐☐☐☐
☐☐☐☐
☐
Obj. 3.4.3
i. ☐
ii. ☐
iii. ☐
Obj. 3.4.2
a)☐☐☐
☐☐☐
Obj. 3.4.3
b)☐☐☐
☐☐☐
Obj. 3.4.4
c)☐☐☐
Obj. 3.4.5
d)☐
Obj. 3.4.6
e)☐
Obj. 3.4.7
a)☐
Obj. 3.4.8
b)☐

Cycle 4: Factorisation

Overview

Big Idea

The facility to work flexibly with quantities is an essential component of mental and written computation strategies, and is also used to determine equivalence in fractions and ratios. Factorisation both relies on, and reinforces, number facts, as it focuses on equivalent number sentences. This idea extends from factorisation of numeric quantities to simplification and expansion of expressions containing variables. This cycle also provides an opportunity to explore and discuss odd, even, prime, composite and square numbers. The array model (discrete objects) is most appropriate for early factorisation activities and may be extended to an area model (continuous quantities), particularly when addressing factorisation and expansion of algebraic expressions.

Objectives

By the end of this cycle, students should be able to:

3.4.1 Identify multiples of a numeric quantity. [5NA098]

3.4.2 Separate a numeric quantity into factors. [5NA098]

3.4.3 Identify prime factors of a numeric quantity. [5NA098]

3.4.4 Identify common factors for a pair of numeric quantities. [5NA098]

3.4.5 Identify lowest common factors for a pair of numeric quantities. [5NA098]

3.4.6 Identify highest common factors for a pair of numeric quantities. [5NA098]

3.4.7 Identify multiples of expressions containing variables. [7NA177]

3.4.8 Identify common factors and simplify expressions containing variables. [8NA192]

Conceptual Links

This cycle reinforces multiplicative number fact strategies and focuses students' attention on the relationships between factors that combine multiplicatively to create a product.

Factorisation skills will be further connected a range of strategies for performing whole number multiplication, determination of equivalence in fractions, and ratio and proportion.

Materials

For Cycle 4 you may need:

- Counters
- Calculators
- Grids for quickly structuring arrays
- Unifix cubes
- Thinkboards or concept maps

Key Language

Groups, sets, collections, array, multiple, factor, product, multiply, multiplication, divide, division, compare, comparison, inverse, unknown, variable, highest common factor, lowest common factor

Definitions

Factor: number that can be multiplied to reach another quantity. Many pairs of factors or sets of factors exist for large factors. Prime numbers have only one pair of factors (prime number, 1)

Factor tree: way of arranging equivalent multiplications so that the factors may be determined. A useful diagrammatic method for finding prime factors.

Highest common factor: highest possible whole number that each of a given set of quantities can all be divided by to reach a whole number answer.

Lowest common factor: lowest possible whole number (greater than 1) that each of a given set of quantities can all be divided by to reach a whole number answer. If 1 is the only factor in common then there are no common factors.

Prime factors: set of prime numbers that give a particular quantity when multiplied together.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- What numbers can you multiply together to reach this quantity?
- Are there any more sets of numbers you can multiply together to reach the same quantity?
- Are these the smallest possible numbers you can multiply together to reach that quantity?
- Will a factor tree help you to work that out?
- If you list all the factors of each number, what is the highest factor they have the same?
- If you list all the factors of each number, what is the lowest factor they all have the same?

Portfolio Task

Portfolio task *P3: The Big Party* engages students with finding multiples of numeric quantities.

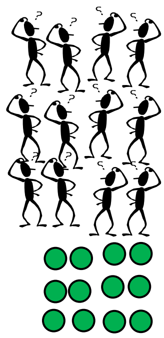
RAMR Cycle

Reality

Find situations in the local area or around the school where quantities may be separated multiplicatively in varying configurations. Some examples may include: seating of students in groups within the classroom (12 students may be sat in 1 group of 12, 2 groups of 6, 3 groups of 4 and so on); arrangements of bags on shelves; seating areas for house groups on sports day (it may only make sense to have 1 row of 4, but it is also possible to make 2 rows of 2). Consider also seating arrangements on trains, in stadiums, theatres and halls. If a set number of seats is required, how many different ways might these be arranged and which is best for a given space. Also, does it make sense for trains to have half a carriage with only a single row of seats along each side? Would more people be seated in carriages if the whole carriage has two rows of double seating?

Abstraction

In this abstraction sequence the focus is on connecting number facts to the relationships between factors that combine to make a product and identifying number facts with equivalent outcomes. A suggested abstraction sequence is as follows:



1. *Kinaesthetic activity.* Choose an arrangement problem that can be acted out. For example, have twelve students arrange themselves in different group sizes. Ensure students can use language to identify the configurations (e.g., twelve can be made up of three groups of four students).
2. *Model/Represent.* Use counters or unifix cubes to model the configurations on the desk in sets or arrays.
3. *Represent with drawings.* Use squared paper and shade arrays to represent factor sets that make 12.
4. *Connect to language and symbols.* Record the number sentence or equation for each configuration (e.g., 12 × 1, 6 × 2, and so on).
5. *Connect to inverse.* Ensure students understand that to find factors, they start with the product and separate into groups of a given size or a given number of groups. Connect this to the division facts (e.g., 12 ÷ 4 = 3).

Mathematics

Once students are able to connect contexts to language and symbols for finding factors, it is important to practice these connections.

Language/symbols and practice

Exploring multiples of a quantity

Increasing an initial quantity by progressively larger amounts generates multiples of the quantity. For example, starting with two, multiply by 1, 2, 3 and so on to find multiples of 2. Ensure that students also understand that each of these values has 2 as a factor. Repeat for other examples.

Finding all possible factor pairs for a quantity

Students should practise finding factor sets for a range of quantities. These could be explored in the context of possible seating arrangements for events.

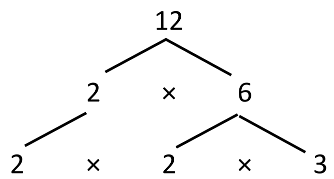
Factor trees to find smallest possible whole number (prime) factors

On occasion, it is useful to reduce a quantity to its prime number factors. This is an opportune time to further explore prime, composite and square numbers and associativity.

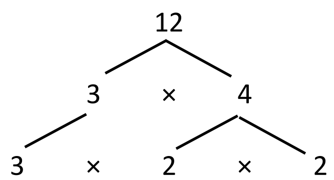
Connect to the ideas from Cycle 2 of this unit and explore the possible arrays of a given quantity. Connect the shapes of these arrays to prime, composite and square numbers.

Resource Resource 3.4.1 Prime, composite and square numbers

These equivalent equations or factor sets for numbers can be explored symbolically using factor trees. For example, the product of 12 items can be distributed or separated in a variety of ways. Explore the use of factor trees to find prime factors and equivalent multiplication expressions. For example, using a factor tree, 12 can be broken into the following expressions:



Factor trees can be used to break a number into its factors. Once a row of prime factors have been reached, it is possible to explore further relationships to determine whether a number is squared, or cubed, or has a square or cubed number as one of its factors. For example, 12 is not a square number, but the prime factor of 2 appears twice so 2^2 is a factor of 12. Thus, the following is also a valid factor tree of 12:



Resource Resource 3.4.2 Factor trees
Resource 3.4.3 Associativity and Brackets

Finding common factors

Frequently it is desirable in mathematics to find the highest or lowest common factor between quantities. Engage students with practice finding possible factors of pairs of quantities and then identifying the common factors, highest common factors and lowest common factors. As appropriate, the common factors can also be represented using array models as well as symbols. These ideas will later contribute to finding equivalent fractions and working with ratio and proportion.

Resource Resource 3.4.4 Finding common factors

Extend arithmetic to algebra

Expressions containing variables may also be used as a base to explore multiples and common factors. This idea reinforces and extends on the use of brackets and order of operations. Finding multiples of algebraic expressions is an opportune time to explore distributivity applied to multiplicative relationships. The simplest everyday example of this is finding perimeter of rectangles.

Resource Resource 3.4.5 Distributivity and algebraic expressions

Reflection

Check the idea

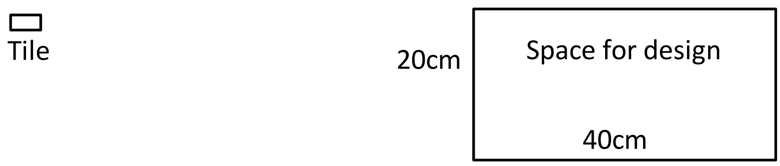
Effective contexts for finding multiples and/or factorisation can be increasing or decreasing a recipe or altering the scale on a drawing, map or plan. While these contexts are also explored in ratio and proportion, the process is fundamentally one of using multiplication and division. For example:

- Provide students with a simple recipe to serve four people and ask them to increase the ingredients to serve eight, twelve, sixteen or twenty-four. Conversely, give a recipe that serves 24 and determine quantities for 3, 4, 6, 8 or 12 guests. Consider marking out a running track for 60m, and then work out how far a person runs if they complete the track in multiples of 60m.
- How far a sports person travels on the field as play moves back and forth.
- Suppose a basketball court needs marking out in squares of whole metres for a scaling exercise. Find the common factors between the length of the court and the width of the court that will result in squares (e.g., squares may be 1m, 2m, 3m … and so on depending on the dimensions of the court).

Apply the idea

Tessellations

Tessellations of 2D shapes are frequently used in tiling and mosaic designs. If the dimensions of the shape or tessellating design are known, it is possible to determine how many will fit within a given space using multiplicative understandings. For example, if tiles within a tessellating design are 2cm across and 4cm long, and the place where the design is to be created is 20cm across and 40cm long, it is possible to use division to work out how many tiles across and how many tiles long the design will be. Using the combining or product meaning of multiplication and an array model, it is also possible to ascertain how many tiles will be needed to create the tessellating design.



If the tile is placed so that it has the same orientation as the space for the design, then

20cm ÷ 2cm = 10 tiles across; 40cm ÷ 4cm = 10 tiles along; 10 tiles across × 10 tiles along = 100 tiles.

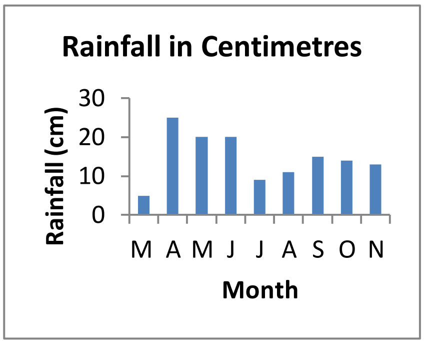
Explore other examples including designs that are not square.

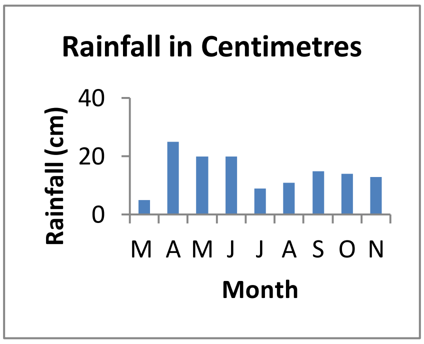
Note: The focus here is not on Area calculation but on determining how many will fit across and along the shape to be covered. The important ideas here are finding how many by breaking side lengths into pieces as long or as wide as the tessellating shape.

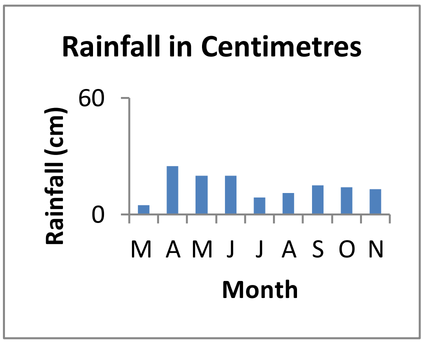
Graphing

Engage students with data gathering activities to generate one-to-many picture graphs and column graphs where the scale works in multiples. Generate some graphs that use the same information but compress the scale differently (e.g., a graph that is scaled in 10s compared to a graph of the same information scaled in 5s).

Use the graphs of same information but different scales to discuss the effect of condensing or stretching scales. How does this affect an ‘at a glance’ view of the data? Discuss how these ideas are used to inform or misinform. For example:







Resource

Resource 3.4.6 One-to-many picture graphs
Resource 3.4.7 From one-to-many picture graphs to column graphs
Resource 3.4.8 Create a scaled column graph

Extend the idea

Explore divisibility rules

Discuss and generalise shortcut methods for divisibility. For example, all multiples of 5 end in 5 or 0; all multiples of 3 result in numbers where the digits add to 3, 6, or 9; all even numbers are multiples of 2.

Explore patterns with square numbers

For example, find successive square numbers (1, 4, 9, 16, 25). Use a calculator to find the next few or complete these from number facts. Explore the differences between the square numbers (1 + 3 = 4, 4 + 5 = 9 and so on). This can be explored further by considering the factors of the square number (1 × 1 + 1 + 2 = 4, 2 × 2 + 2 + 3 = 9, 3 × 3 + 3 + 4 = 16, and so on). Squared paper is useful for modelling this exploration to help with the pattern. Following this pattern, if a square number is known, the next square number is found by adding double the square-root of this square number plus 1 to the square number (e.g., 12 × 12 + 12 + 12 + 1 = 13 × 13). This particular example can be illustrated using an array model and connected to the distribution rule i.e., 13x13 = (12+1)x(12+1) = 12x12 + 12x1 + 1x12 + 1x1 = 12x12 + 12 +12 +1

Extend to algebraic expressions

Extend students’ work with multiples and factors to simple algebraic expressions. For example, fences need fence panels and posts, there will be the same number of posts as fence panels plus an extra post (2m panel + post) + post. How many panels and posts for a 100m fence? 50(2m panel + post) + post = 50 panels + 50 posts + 1 post = 50 panels + 51 posts.

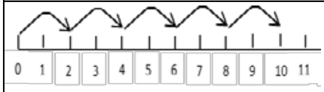
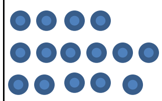
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________ Date:________

Can you do this? #5

1. Match the stories to the pictures and symbols that represent them.

Pictures		Stories	Symbols
		a) I caught 3 fish. You caught 5 times as many fish as me. You caught 15 fish.	9 ÷ 3 = 3
I: 	You: 	b) Fred has 9 stickers. You have 3 times fewer stickers than Fred. You have 3 stickers.	5 × 2 = 10
Fred: 	You: 	c) Tom walked 2km. Frances walked 5 times Tom's disance. Frances walked 10km.	5 × 3 = 15

Obj. 3.5.1
a)i□ii□
b)i□ii□
c)i□ii□

2. Look at the stories below. For each story:
   i. Underline the factors.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

   (a) Logan had 20mm of rain. Brisbane had 5 times as much rain as Logan. How much rain did Brisbane have?

   ____

   ____

   (b) Jeans at Just Jeans were $125. Big W's jeans were 5 times cheaper. What was the cost of jeans at Big W?

   ____

   ____

Obj. 3.5.2
a)i. □
b)i. □
Obj. 3.5.3
a)ii. □
b)ii. □
Obj. 3.5.4
a)iii. □
b)iii. □
Obj. 3.5.5
a)iv. □
b)iv. □

3. Fred and George bought lunch. Fred spent twice as much as George. How much did Fred spend on lunch?

   (a) Write an equation with variables to represent the story.

   ____

   (b) If George spent $6, how much did Fred spend on lunch?

   ____

   ____

Obj. 3.5.4
a)i. □
ii. □
iii. □
iv. □
Obj. 3.5.5
b)i. □
b)ii. □

Pretend to use the calculator.

Which operation key (× or ÷) would you use to solve each of these problems?

4. Circle the correct button beside each question.

(a) A roll of rope is 50m long. Another roll of rope is 5 times shorter. How long is the second roll of rope?	× ÷
(b) Mark has 12 lollies which is 4 times as many as Colin. How many lollies does Colin have?	× ÷

Obj. 3.5.4
a) ☐
b) ☐

5. Write a comparison story for the equation: $5 \times c = 125$

____

____

Obj. 3.5.4
i. ☐
ii. ☐
iii. ☐
iv. ☐

Cycle 5: Multiplicative Comparison

Overview

Big Idea

For students to succeed at interpreting multiplicative problems, they need to understand that the written equations they see represent a broad range of real world situations that involve multiplicative change or relationships, which can be modelled using materials or drawings. However, many students recognise only multiplicative change. Students also need to experience, recognise and construct multiplicative relationships in which two distinct groups are compared statically and are not changed (e.g., 3 times as many boys as girls, 5 times fewer buses as people). In this cycle, activities extend the known meanings for multiplicative relationships to include such multiplicative comparison.

Objectives

By the end of this cycle, students should be able to:

3.5.1 Act out, interpret and represent multiplicative comparison stories informally. [2NA031]

3.5.2 Identify factors within multiplicative comparison stories as multiplier and part of the whole group size. [5NA098]

3.5.3 Identify the product within multiplicative comparison stories as the whole comparative group size. [5NA098]

3.5.4 Represent multiplicative comparison stories as equations using symbols for unknowns. [7NA175]

3.5.5 Solve for unknown values in multiplicative comparison problems. [7NA176]

Conceptual Links

This cycle introduces a range of strategies for performing whole number multiplication and, through practice, aims to build a degree of recall-based answering of simple multiplication problems since this will assist more complex calculations. Multiplicative comparison problems form a basis for exploring ratio in later cycles.

Materials

For Cycle 5 you may need:

- Counters
- Calculators
- Grids for quickly structuring arrays
- Unifix cubes
- Thinkboards or concept maps
- Multiplication representation match cards

Key Language

Groups, sets, collections, array, multiple, factor, product, multiply, multiplication, divide, division, compare, comparison, inverse, unknown, variable

Definitions

N times as many/N times as much/N times fewer/N times shorter: multiplier or divisor that is needed to describe the relationship between two quantities. Note that it is important to not add confusing or ambiguous language to questions, for example, *4 times more than* is easily misinterpreted and confusing.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Which values in the problem are factors?
- What is the question asking you to find?
- Which operation will you use?
- Are you combining groups or comparing groups?
- Which is the larger group?
- What is the relationship between the quantities in the groups?

Portfolio Task

The student portfolio task *P3: The Big Party* does not involve multiplicative comparison although students could be extended to consider what the total cost would be if some items were twice the price or if alternative items were located that were three times cheaper.

RAMR Cycle

The main focus of learning in this cycle is to explore multiplicative relationships where one quantity is compared to another. Acting out these situations involves statically comparing the two groups to determine how many times more or less one group is than the other. The set, length and, to a lesser degree, array models are useful for representing multiplicative comparison problems.

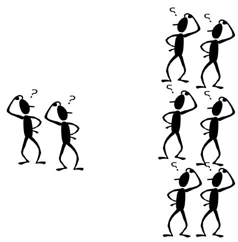
Reality

Find situations in the local area or around the school where objects may be compared multiplicatively. Some examples may include: numbers of boys compared to numbers of girls in class, number of people with brown eyes compared with number of people with blue eyes or green eyes, healthy compared to unhealthy menu choices at the tuckshop, number of trees near a waterway compared to further away, different flavour varieties of soft drink brands, costs of branded items compared with costs generically branded items, or comparing numbers of friends on Facebook.

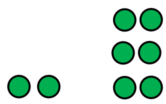
Abstraction

In this abstraction sequence the focus is on developing the meaning of multiplicative comparison. A suggested abstraction sequence is as follows:

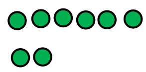
1. *Kinaesthetic activity.* Choose a multiplicative comparison problem that can be acted out. For example, have two students stand on the left side of the room and have six students stand on the right side of the room. Ask students to compare how many students are standing on each side of the room. How many more students are standing on the right? (4 more is an additive answer). Look for a multiple of how many times (three times as many).



2. *Model/Represent.* Use counters or unifix cubes to model the two collections on the desk in sets or arrays. Ensure that students can see that one group is 3 times higher than the other group (6 = 3 × 2).



3. *Connect to language and symbols.* Discuss ways of seeing this relationship. Another way of seeing this image is one group is 3 times shorter than the other group (6 ÷ 3 = 2), one group needs to be repeated 3 times to make the other group (3 × 2 = 6), or one group fits into the other 3 times (6 ÷2 = 3).
4. Reorganise the groups so that the two collections are compared horizontally. Discuss how students see the problem (one group is 3 times the length of the other group, 6 = 3 × 2 or 6 ÷ 3 = 2).



5. *Recording.* Ensure students can identify the number in each collection to be compared and the number of times greater or fewer (multiplier or divisor). Factor-factor-product may still be used as an organiser but students need to recognise that in this instance one of the collections is the product, the other collection is a factor and the other factor is the multiplier (e.g., There are 2 students standing on the left side of the classroom and 3 times as many students standing on the right. 2 students on the left is a factor (multiplicand), 3 times as many is the multiplier, and the number of students on the right is the product (i.e., 3 × 2 = 6).
6. *Connect multiplicative inverse.* Ensure students can recognise the multiplicative inverse in comparison problems. For example, there are 6 students standing on the right side of the room and 2 students standing on the left. How many times as many students are standing on the right side of the room as the left? Factor = 2; Factor = ? Product = 6; ? × 2 = 6 or? = 6 ÷ 2.

Mathematics

Once students are able to connect contexts to language and symbols and understand the concept of multiplicative comparison, it is important to practice these connections.

Language/symbols and practice

Multiplicative Comparison Problems - Multiplication

Multiplicative comparison describes the static relationship between quantities in each group, and identifies relative size of one quantity compared to the other. Language of multiplicative comparison problems includes *times as many, times as much, times as long, times fewer.* Examples of multiplicative comparison problems where the factors are known values and the products are unknown values are included in *Resource 3.5.1 Comparison problems – Multiplication.*

Resource Resource 3.5.1 Comparison Problems – Multiplication

Multiplicative Comparison Problems - Division

Multiplicative comparison problems in which one of the factors is unknown and the product and the other factor are known use division (or think multiplication) for their solution. Examples of multiplicative comparison problems that may be solved using division are included in *Resource 3.5.2 Comparison Problems – Division.*

Resource Resource 3.5.2 Comparison Problems – Division

Reflection

Check the idea

As evidence of learning give students the idea and a story (or equation or model) and have them determine as many other representations as they can. This will demonstrate the breadth of students' connections within the topic. Alternatively, students could work in pairs or threes to come up with a multiplicative story and then swap with another group who then draw pictures and models and write equations.

Apply the idea

Return to some of the graphs created previously with students (i.e., in Cycle 1). Determine if there are multiplicative comparison relationships between some of the categories depicted in the graphs.

Extend the idea

Maintaining the relationship

Explore the effect of changing the size of one quantity upon the other quantity if the multiplicative relationship is to be maintained. These explorations could be scaffolded using a suitably selected length arrangement of materials. For example, in a bowl of punch 1 litre of lemonade and 3 litres of juice is used. If 4 litres of lemonade was used, how much juice is needed? These quantities could be represented using a length model, mimicking the markings on a (large) measuring jug.

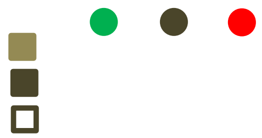
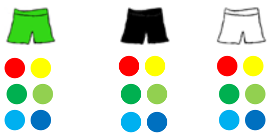
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:_______________________________________ Date:________

Can you do this? #6

1. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) You have 6 shirts and 3 pairs of shorts. You can make 18 different outfits.	3 × 2 = 6
	b) Fred makes 6 different desserts from 3 flavours of ice cream and 2 types of cone.	9 ÷ 3 = 3
	c) Tom makes 9 different types of sandwiches to sell. He has brown, white and gluten free bread. He needs 3 different fillings.	6 × 3 = 18

2. Look at the stories below. For each story:
    i. Underline the <u>factors</u>.
    ii. Circle the product.
    iii. Write an equation for the story with a symbol for the unknown.
    iv. Write down the answer

(a) If an ice cream stall has 6 flavours of ice cream for sale; how many different toppings will they need for 18 people to have different ice cream and topping combinations?

____

____

(b) A travel agent has 3 discount holiday destinations with 2 different ways of getting there. How many different discount holiday deals can they advertise?

____

____

Obj. 3.6.1
a)i□ii□
b)i□ii□
c)i□ii□

Obj. 3.6.2
a)i. □
b)i. □
Obj. 3.6.3
a)ii. □
b)ii. □
Obj. 3.6.4
a)iii. □
b)iii. □
Obj. 3.6.5
a)iv. □
b)iv. □

3. Fred has three models of car for sale. Each car can be painted different colours. How many people can buy different looking cars from Fred?

   (a) Write an equation with variables to represent the story.

   ____

   (b) If there are 6 different colours of paint, how many different looking cars can Fred sell?

   ____

   ____

Obj. 3.6.4
a)i. □
ii. □
iii. □
iv. □
Obj. 3.6.5
b)i. □
b)ii. □

4. Write a combinations story for the equation: 18 ÷ p = 3

   ____

   ____

Obj. 3.6.4
i. □
ii. □
iii. □
iv. □

Cycle 6: Multiplicative Combinations

Overview

Big Idea

Students should experience, recognise and construct multiplicative relationships that involve possible combinations (e.g., 3 jeans, 5 shirts, how many possible outfits). These relationships are neither change nor comparison. In this cycle, activities extend the meanings for multiplicative relationships to include multiplicative combinations.

Objectives

By the end of this cycle, students should be able to:

3.6.1 Act out, interpret and represent multiplicative combinations stories informally. [2NA031]

3.6.2 Identify factors within multiplicative combinations stories as quantity of options of first item to be combined with quantity of options of subsequent items to generate all possible outcomes. [5NA098]

3.6.3 Identify the product within multiplicative combinations stories as the whole selection of combinations. [5NA098]

3.6.4 Represent multiplicative combinations stories as equations using symbols for unknowns. [7NA175]

3.6.5 Solve for unknown values in multiplicative combinations problems. [7NA176]

Conceptual Links

This cycle introduces a range of strategies for performing whole number multiplication and, through practice, aims to build a degree of recall-based answering of simple multiplication problems since this will assist more complex calculations. Combination representations are useful connections to future Units which explore probability.

Materials

For Cycle 6 you may need:

- Counters
- Calculators
- Grids for quickly structuring arrays
- Unifix cubes
- Thinkboards or concept maps
- Multiplication representation match cards

Key Language

Multiple, factor, product, multiply, multiplication, divide, division, compare, combinations, options, possibilities, inverse, unknown, variable

Definitions

Combinations: number of different arrangements or outcomes that can be generated from the combination of a selection of inputs. For example, 3 pairs of shorts and 5 t-shirts can be paired in 15 different combinations of outfits; 3 bread types and 5 sandwich fillings can be paired to create 15 different sandwiches.

Combination table: a table may be used to identify all possible combinations of separate items (particularly useful for representing menu possibilities). May also be used in later exploration of Probability outcomes in Unit 15 (has applications in Biology when considering genetics).

Tree diagram: a method for diagrammatically representing all possible combinations of separate items. Also useful in later exploration of Probability outcomes in Unit 15.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- How many of each item do you have?
- How many combinations will you have with just this first item and all the others?
- Can you repeat that for the other items?
- How many is that added all together?
- Is there another way to think of these instead of adding them (repeated addition idea)?
- Can you just multiply this amount by that amount?
- Will that work every time without drawing pictures?

Portfolio Task

The student portfolio task *P3: The Big Party* engages students with finding possible combinations of items to purchase for the party.

RAMR Cycle

The main focus of learning in this cycle is to explore multiplicative relationships that involve possible combinations (e.g., 3 jeans, 5 shirts, how many possible outfits). Acting out these situations involves determining the range of possible options when two or more elements are combined in alternative configurations. These problems are modelled using tree models (a form of set arrangement) or tables (similar to array models).

Using a table, elements to be used in the combination can be identified in heading row and column and possible combinations enumerated within the body of the table. This model connects quite easily to the product meaning of multiplication due to its resemblance to the array model.

Using a tree model, an item can be systematically matched with other possible elements to generate a number of options. Some form of recording is needed since there are seldom enough items to create all the possible options from materials. Acting out is initially beneficial, but options are easier to model using drawings or sketches.

Reality

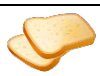
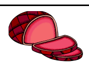
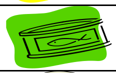
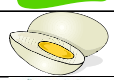
Find situations in which entities are combined into a set of possibilities. Some examples may include: menu items at a sandwich bar with optional breads and fillings, ice cream flavours and toppings, snack and drink combinations on an excursion, all possible combinations of tops and bottoms (complete range of options without considering taste or fashion sense).

Abstraction

In this abstraction sequence the focus is on developing the meaning of identifying possible combinations. Two sequences are presented: one using table representations and one using tree-diagram representations. Either sequence can be used initially to develop the combinations meaning, then the other can be used to consolidate the meaning.

Tables

1. *Kinaesthetic activity and Model/Represent.* Choose a combinations problem that can be acted out. For example, possible menu items for a sandwich bar can be acted out on a maths mat with options drawn on paper plates (or a cheap vinyl tablecloth marked into a grid with options drawn on paper plates for sandwich fillings and bread types). Place the elements in heading row and column and have students identify the options for the menu. Students should also complete their own tables while determining choices. For example:

		Bread Type		
				
Filling Type		Ham sandwich	Ham roll	Ham wrap
		Chicken sandwich	Chicken roll	Chicken wrap
		Tuna sandwich	Tuna roll	Tuna wrap
		Egg sandwich	Egg roll	Egg wrap
		Salad sandwich	Salad roll	Salad wrap

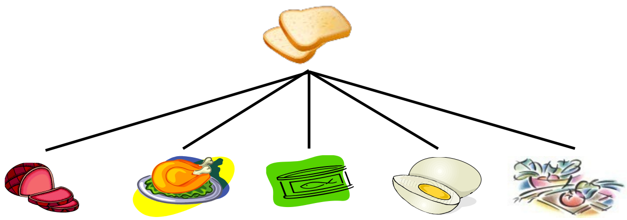
2. *Connect to language and symbols.* Discuss ways of seeing the possible combinations as number of possible bread choices by the number of possible fillings. This could be written as No. Option 1 choices x No. Option 2 choices = Total no. of Combinations, or No. Breas x No. Fillings = No. combinations. Connect this language to the symbolic representation of 3 × 5 = 15.

3. *Connect* to the idea of *turnarounds (commutativity)* for multiplication. Discuss whether it makes any difference if the possible breads are listed down the side of the table and the possible fillings across the top. Or if you choose the filling type first then the bread type (i.e., 5x3=15). Test this out to be sure.

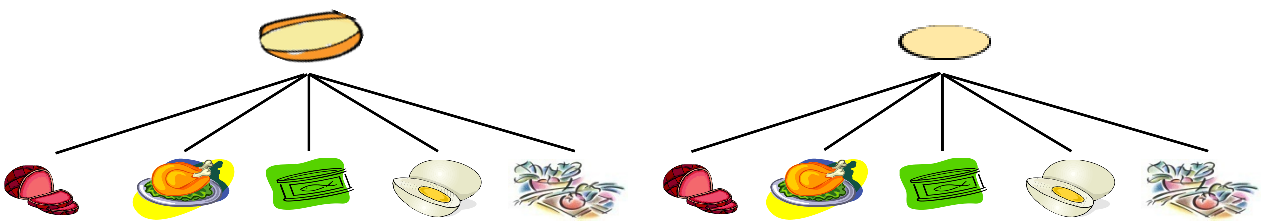
4. *Connect* to the idea of *multiplicative inverse.* Discuss with students what will happen if the sandwich bar decides it needs to be able to offer 30 different menu items for its clientele. If they have only three bread options, how many different fillings will they need to provide? Ensure students can connect this problem to the language of 3 by ? gives 30 options and the symbolic representation of 3 × ? = 30 and then 30 ÷ 3 = ? if choosing buttons on a calculator.

Tree Diagrams

1. *Kinaesthetic activity and Model/Represent.* Choose a combinations problem that can be acted out. For example, possible menu items for a sandwich bar can be acted out with options drawn on paper plates. Choose the type of bread to use and identify all the possible sandwiches that could be made by drawing these on paper plates. Arrange these under the type of bread plate. Draw these on the whiteboard and encourage students to draw their options in their books. For example,



2. Repeat the action for the other bread types. For example,



3. *Connect to language and symbols.* Discuss ways of seeing the possible combinations as number of possible bread choices by the number of possible fillings. Connect this language to the symbolic representation of 3 × 5 = 15.

4. *Connect* to the idea of *turnarounds (commutativity)* for multiplication. Discuss whether it makes any difference if the problem is drawn as three types of bread with five possible fillings or five possible fillings with three possible breads. Test this out to be sure if necessary.

Mathematics

Once students are able to connect contexts to language and symbols and understand the concept of identifying all possible combinations problems, it is important to practice these connections.

Language/symbols and practice

Identifying all possible combinations problems - Multiplication

Combinations problems do not involve generating a product or partitioning a whole. Instead, these problems determine the number of different ways components may be put together. Acting out or representing these problems with materials uses tree diagrams (often used in probability) or the creation of a table of possibilities with rows and columns (similar to the array model). In each case, the number of possible combinations may be counted or the quantity of each piece of the proposed combination can be multiplied together to find the corresponding product. *Resource 3.6.1 Combinations Problems – Multiplication* has some suggestions for exploring combinations problems.

Resource Resource 3.6.1 Combinations Problems - Multiplication

Identifying all possible combinations problems – Division and Multiplicative Inverse

Consider situations in the local area or around the school where objects need to be combined with other objects to create a specific number of alternative combinations. Examples from multiplication such as sandwich types and fillings or outfits from jeans and shirts are possibilities. For example:

> Discuss with students what will happen if the sandwich bar decides it needs to be able to offer 30 different menu items for its clientele. If they have only three bread options, how many different fillings will they need to provide? Ensure students can connect this problem to the language of 3 by ? gives 30 options and the symbolic representation of 3 × ? = 30 and then 30 ÷ 3 = ? if choosing buttons on a calculator.

More examples of Combinations problems that may be solved using division are included in *Resource 3.6.2 Combinations Problems – Division*.

Resource Resource 3.6.2 Combinations Problems - Division

Reflection

Check the idea

As evidence of learning give students the idea and a story (or equation or model …) and have them fill out as many other representations as they can. This will demonstrate the breadth of students' connections within a topic. Alternatively, students could work in pairs or threes to come up with a multiplicative story, swap maps with another group to draw picture, models, and write equations.

Apply the idea

Combinations are often used to randomise outcomes in games (e.g., Twister), and can be used to identify possible locations of pieces (e.g., chess, battleship). Encourage students to identify how many possible moves, locations, and outcomes for games. For example, if chess spaces are labelled A to H one way and 1 to 8 the other way, how many possible locations are there for a piece (same idea for battleship)? If Twister has four colours and four hands/feet, how many possible configurations are there? If you are devising a game like Twister and want only 12 possible configurations what do you need to change? What if you want more options, what could you change?

Note: These ideas do lead onto and are forerunners to location and direction and probability activities. The idea is not to go into these aspects of these activities, simply to use multiplication to identify possible combinations or outcomes.

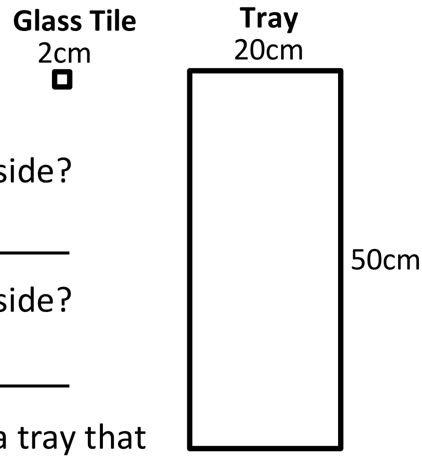
Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Name:______________________________________ Date:________

Can you do this? #7

1. You want to cover a pot stand with square glass tiles that are 2cm across.



(a) How many tiles will fit along the 50cm side?

____

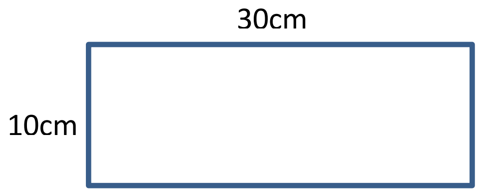
(b) How many tiles will fit along the 20cm side?

____

(c) How many tiles will you need to cover a tray that is 20cm wide and 50cm long?

____

2. Calculate the distance around this rectangle.



3. Solve the following questions in the space provided.

Write down all the steps you take to reach your answer.

(a) 26 × 5	(b) 21 ×23
(c) 84 ÷ 6	(d) 612 ÷ 12

Obj. 3.7.2
a) i. □
a) ii. □
a) iii. □
b) i. □
b) ii. □
b) iii. □

Obj. 3.7.1
c) i. □
c) ii. □
c) iii. □

Obj. 3.7.1
i. □
ii. □
iii. □

Obj. 3.7.1
a) □
b) □

Obj. 3.7.2
c) □
d) □

Cycle 7: Multiplicative Strategies for Larger Numbers

Overview

Big Idea

Traditionally, calculation of multiples of greater than single digit numbers needed to be completed mentally or manually as did division calculations (now often completed with calculators). Arguably, strategies for written and mental calculation are important for instances where calculating devices are not available. However, mental and written calculation strategies also develop and demonstrate a facility to work flexibly with number, enhance logical thinking processes, promote problem solving strategies, build number sense and are generalisable to strategies that extend to algebraic understandings and field properties. This cycle presents abstraction sequences to develop mental and written computation strategies of separation (traditional algorithm), sequencing and compensation for multiplication and division computation.

It is suggested that each of the strategies be developed separately before encouraging students to move flexibly between them when solving problems and thus lead to the comparison of the strategies' similarities and differences in the reflection phase. The order of introduction of these strategies is not critical. Choose the order based on what will suit your students best. Be aware, that if students have frequently failed with the traditional separation algorithm, they may need to start with an alternative strategy to experience success.

With regard to student recording of computation using any of the strategies, there are three ways this could be done:

(a) answer only – when full mental methods used or when calculators are used;
(b) informal writing or doodling – numbers and drawings that assist the mental processes, mostly idiosyncratic to the learner; and
(c) pen-paper recordings that imitate the material manipulation – ways of recording that lead on from materials and can replace the material thinking.

Objectives

By the end of this cycle, students should be able to:

3.7.1 Solve for unknown values in multiplication stories using mental and written strategies where quantities extend beyond number facts. [5NA100]
3.7.2 Solve for unknown values in division stories using mental and written strategies where quantities extend beyond number facts and leave no remainders. [4NA076]

Conceptual Links

This cycle connects to previous cycles' exploration of multiplicative relationships and number fact strategies.

This cycle introduces a range of strategies for performing multiplicative computation with greater than single-digit whole numbers.

Materials

For Cycle 7 you may need:

- Counters
- Number lines
- Calculators
- 99 boards
- Unifix cubes
- Grids with large squares for quickly structuring arrays of materials
- PVC (hundred, tens, ones) and MAB

Key Language

Multiplication, division, array, area model, number fact strategies, factors, multiples, brackets, strategies, mental computation, separation, sequencing, compensation

Definitions

Algorithm: written procedure to perform calculation

Separation strategy: Based on the traditional algorithm; separate into place-value components, calculate separately, and combine. Also of value when calculating mentally is the ability to split numbers in other ways that simplify mental calculation (e.g., 35 may be split into 25 and 10). Separation methods are best taught by set or array/area models. Set models use place value charts (PVCs) and size materials such as bundling sticks, MAB and money placed on top of these PVCs (separation by place value). The area model is best taught with pictures, although earlier work with counters and dot/graph paper can be used. The area model used with the separation strategy (by place value or other splits) generalises to the distributive law, algebraic thinking and equations.

Sequencing strategy: Involves one number being left as is and the other number being separated, so that parts of it are operated on in sequence. The second number may be split additively or multiplicatively. Additive splits are based on the distributive law and can occur in one of two ways: into 'tens' and 'ones' (using place value); or, in a way that provides simpler operations to be completed (e.g., 35 may be broken to multiply by 30 and by 5 or to multiply by 25 and by 10). Multiplicative splits make use of the associative law and rely on splitting one of the numbers into factors that are then successively multiplied by the other number.

Compensation strategy: Leaves both numbers unseparated, but changes the problem to an easier one and then compensates for the change. Like the sequencing strategy, the compensation strategy is best taught using the area model. Sound number sense and reasoning is required for this method.

Assessment

Anecdotal Evidence

Some possible prompting questions:

- Are the numbers easy to work with (number facts you know)?
- Can you split/change one number to make the multiplication easier?

Portfolio Task

The student portfolio task *P3: The Big Party* engages students with finding quantities and total costs of items to purchase for the party that may involve larger numbers.

RAMR Cycle

The focus of this RAMR cycle is the development of techniques to calculate two and three digit multiplication problems. For larger operands estimation and calculators are recommended. In this cycle, both set and array/area models are used to represent the operands and operations.

Mental and written strategies for multiplicative computation with larger numbers can be grouped into three strategy types: separation, sequencing and compensation. Explanations of these strategies are included in the Definitions section of the Overview for this cycle.

Reality

Consider real world applications of two-digit multiplication and division relevant to students. For example, set models may include payment for multiple items like meals, sporting fees, clothing or luxury items. Area model problems are common in construction or decorating contexts such as concreting, paving, tiling and painting or in horticultural contexts such as spraying pesticides on areas of vegetation, laying turf, and any other context that involves calculation of area. A selection of menus, shopping catalogues, floor plans or garden plans may be useful stimuli.

Abstraction

Students may or may not be proficient at large number computation using a traditional algorithm or mental computation strategies. If students have an effective method of calculating and recording that works, it is not necessary to overtly teach another method but can be interesting to extend their range of mental computation strategies and increase number and operation sense. The abstraction phase for this cycle is general with more details about modelling each strategy with materials available in *Resource 3.7.1 Computation strategies: Large number multiplication* and *Resource 3.7.2 Computation strategies: Large number division*. The strategies may be taught in any order as suits concept development for your class. Some mathematics practice activities between each will be beneficial to ensure that students are confident with a strategy before moving on. It is recommended to complete each example through from Kinaesthetic activity to symbols in a coherent set to keep problem, model, language and symbolic representations together and connected.

Developing Computation Strategies for Multiplication of Larger Numbers
(Separation strategy, Sequencing strategy, Compensation strategy)

1. *Kinaesthetic activity.* Act out real world problems with simple props. For example (set model problem), if it costs $37 for a meal, how much will we pay for 4 meals? Act this out with money and food (imaginary or drawn on paper plates). Ensure students can identify the operation they need for each problem solution. Write each problem in symbols. Link to language in previously built word banks. Identify language already known or add new words and meanings. Once students can recognise that they have to use multiplication to solve the problems, the algorithm for solutions can be taught.

   Other suitable problems might be:

   - *Area model:* How many pavers are needed for a pathway 37 pavers long and 4 pavers wide? This can be acted out using squares of paper if needed.
   - *Area model:* The floor to be tiled was 45m wide and 63m long. What is the total area to be tiled?
   - *Set model:* Balloons were inflated for fundraising. If we used 15 canisters of balloon gas and each canister of balloon gas inflates 62 balloons, how many balloons do we have for sale?

2. *Model/represent with materials*. Determine whether you wish to use set or array/area model with students. These have been detailed in *Resource 3.7.1 Computation strategies: Large number multiplication*.
3. *Connect to language and symbols*. Connect the model/representation with drawings and materials to the steps/stages in the strategy and record using the algorithm.
4. *Replace materials with sketches*. Note that it is not necessary to remove students' reliance on sketches if they help them to make sure that all steps of the calculation are completed.
5. *Practice*. Complete further examples to practise computation strategies. Engage students with devising or finding their own real world problems that involve large numbers and multiplication.

Resource Resource 3.7.1 Computation strategies: Large number multiplication

Developing Computation Strategies for Division of Larger Numbers
(Separation strategy, Sequencing strategy, Compensation strategy)

1. *Kinaesthetic activity.* Act out real world problems with simple props. For example (set model problem), *4 people shared in a lotto win of $92, how much money did each person win?* Act this out with money (use nine $10 notes and have available twelve $1 coins) and people. Ensure students can identify the operation they need for each problem solution. Write each problem in symbols. Link to language in previously built word banks. Identify language already known or add new words and meanings. Once students can recognise that they have to use division to solve the problems, the algorithm for solutions can be taught.

   Other suitable problems might be:

   - *Area model:* We have 136 tiles to build a paved area that is 4 tiles wide. How many rows of pavers will there be? This can be acted out using squares of paper if needed.
   - *Area model:* We have 936 tiles to run 4 rows of tile horizontally along a section of wall. How many vertical rows will there be?
   - *Set model:* We have $2580 to give prizes of $12 each, how many prizes can we make up?
2. *Model/represent with materials*. Determine whether you wish to use set or array/area model with students. These have been detailed in *Resource 3.7.1 Computation strategies: Large number multiplication*.
3. *Connect to language and symbols*. Connect the model/representation with drawings and materials to the steps/stages in the strategy and record using the algorithm.
4. *Replace materials with sketches*. Note that it is not necessary to remove students' reliance on sketches if they help them to make sure that all steps of the calculation are completed.
5. *Practice*. Complete further examples to practise computation strategies. Engage students with devising or finding their own real world problems that involve large numbers and multiplication.

Resource Resource 3.7.2 Computation strategies: Large number division

Mathematics

Language/symbols and practice

Engage students with additional practice solving and posing multiplicative problems that involve large numbers. Ensure that students are able to identify and generate a suitable equation from worded problems, identify the appropriate operator, then solve to find a solution for the problem.

Multi-step problems

Extend students' problem solving and interpretation skills by introducing multi-step problems. Ensure students are able to identify the parts of the problem, decide what operations to do in what order, and then work through the combined steps to find a solution to the problem.

Some suitable problems to engage students with include:

- *Resource 3.7.3 Aquarium activity*
- *Resource 3.7.4 Assorted worksheet resources*

Resource Resource 3.7.3 Aquarium activity
Resource 3.7.4 Assorted worksheet resources

Reflection

Check and Apply the idea

Engage students with generating and solving multiplicative problems. Ensure that a wide range of multiplicative style problems are addressed including repeated addition, combining, comparison, and combinations.

Provide students with a variety of solutions or final products (these might be areas, shopping bill totals, menu selection totals) and ask students what possible questions may be asked. For example, I spent $200 at the shop, what multiple items might I have bought? We have enough tiles to cover 480 m^2, what possible room dimensions can we tile?

Teacher Reflective Notes

This page is provided for you to record any notes with respect to resources you found useful, additional resources, activities and/or models that worked well/not so well.

Unit 03 Portfolio Task – Teacher Guide

The Big Party

Content Strand/s: Number and Algebra

Resources Supplied:

- Task sheet
- Table template

Other Resources Needed:

- Rulers
- Scissors

Summary:

Students use their operation knowledge to plan a party for two year 8 classes. The students need to work through the activity sequentially

Variations:

- Allow students to provide more detail about the party (e.g., decorations, live music, etc).

ACARA Proficiencies Addressed:

Understanding
Fluency
Problem Solving
Reasoning

Content Strands:

Number and Algebra

3.2.1 Act out, interpret and represent combining equal groups multiplication stories informally. [2NA031]

3.2.5 Solve combining equal groups multiplication stories using equations with a symbol for the unknown. [7NA176]

3.3.9 Solve sharing and separating stories using equations with a symbol for the unknown. [7NA176]

3.4.1 Identify multiples of a numeric quantity. [5NA098]

3.5.4 Represent multiplicative comparison stories as equations using symbols for unknowns. [7NA175]

3.6.2 Identify factors within multiplicative combinations stories as quantity of options of first item to be combined with quantity of options of subsequent items to generate all possible outcomes. [5NA098]

3.6.5 Solve for unknown values in multiplicative combinations problems. [7NA176]

3.7.1 Solve for unknown values in multiplication stories using mental and written strategies where quantities extend beyond number facts. [5NA100]

The Big Party

Name	
Teacher	
Class	

Your Task:

You have been given the job of planning a party for two classes in your year level. The party will take place in a room that you choose in the school.

You will be:

- determining the number of people attending your party,
- calculating quantities and costs for food and drinks,
- planning the layout of tables and chairs in the room.

Within Portfolio Task 3, you demonstrated the following characteristics:

			A	B	C	D	E
Understanding and Fluency	Conceptual understanding	3.2.5 Solve combining equal groups multiplication stories using equations with a symbol for the unknown. 3.4.1 Identify multiples of a numeric quantity. 3.5.4 Represent multiplicative comparison stories as equations using symbols for unknowns.	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
Problem Solving and Reasoning	Problem solving approaches	33.3.9 Solve sharing and separating stories using equations with a symbol for the unknown. 3.6.2 Identify factors within multiplicative combinations stories as quantity of options of first item to be combined with quantity of options of subsequent items to generate all possible outcomes. 3.6.5 Solve for unknown values in multiplicative combinations problems. 3.7.1 Solve for unknown values in multiplication stories using mental and written strategies where quantities extend beyond number facts.	Systematic application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches
	Mathematical modelling	3.2.1 Act out, interpret and represent combining equal groups multiplication stories informally.	Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar	Development of mathematical models and representations in complex familiar or simple unfamiliar situations	Development of mathematical models and representations in simple familiar situations	Statements about simple mathematical models and representations	Isolated statements about given mathematical models and representations

Comments:

Food and Drinks

1. You are having a party for two year eight classes. How many students are going to be at your party? ____

2. You need three bottles of drink per student. What is the total number of drinks required? Show your working.

3. The catering company provides party food for $6 per person. What is the total cost of buying the food? Show your working.

4. The catering company is providing platters of sausage rolls, mini meat pies and spring rolls. 3 sauce types are also provided: barbeque sauce, tomato sauce and sweet chili sauce.
   How many combinations of food and sauce can be made?

5. You can afford two types of drink for the party. Cola comes in single bottles, and ginger beer comes in packs of 4. You must have some of each type of drink.

   (a) How many bottles of coke and 4-packs of ginger beer will you have at your party?

   Remember, the total number of bottles here must match your answer in question 1.

   Number of bottles of cola: _________ Number of 4-pack of ginger beer: _________.

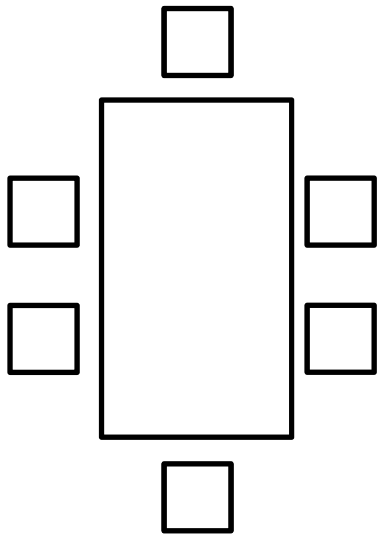
6. Using the costs below, work out the costs of your drinks:
   Single bottle of cola = $2 4-pack of ginger beer = $5

   Total drink cost: _______
7. What is the total cost of food and drink for the party?

8. How much do you need to charge students to cover party costs?
   Make it a whole number.

Charge per student: ____________

Room Set Up



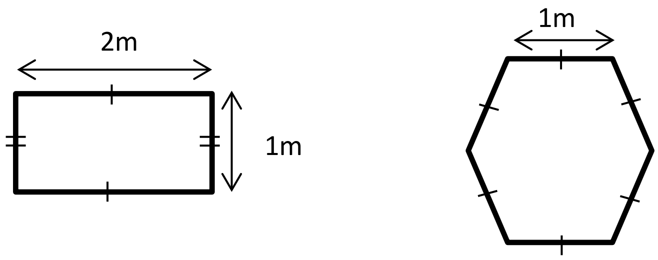
9. Organise the tables and chairs for your guests.
   If each table is large enough to seat six students, how many tables and chairs are needed for your party?

   Number of tables: _____ Number of chairs:_____

10. Measure to the nearest metre the dimensions of the room in the school that you think would be suitable for the party.

    Length of room: _____ Width of room: _____

11. There are two types of tables available:.



Create two different plans for your party room. On each of the following grid pages, draw the walls of the room and mark any doorways. Create two different designs by cutting out the templates of each shape and placing them in your rooms.

Remember that space needs to be left for chairs, and you may like to leave a space for activities such as games.

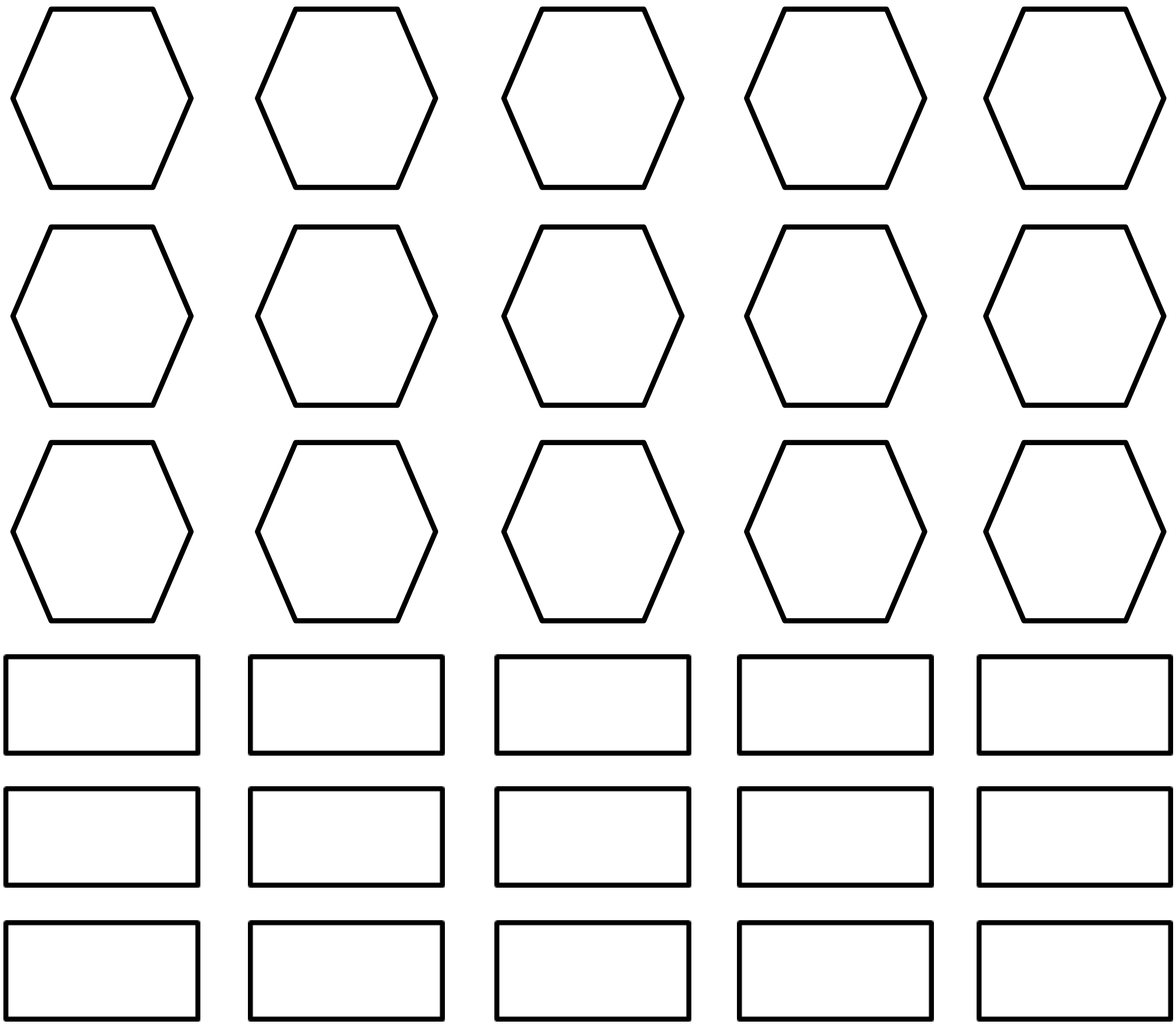
Design 1

Design 2

12. Which room would you choose for your party and why?

The Big Party – Table Template

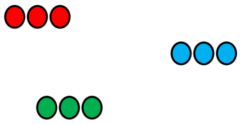
Cut out the tables to glue onto your table design.



Name:______________________________________ Date:________

Can you do this now? Unit 03

1. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) A relay team had 4 people running. Each person ran 3 laps of the oval. There were 12 laps run altogether.	12 = 4 × 3
	b) Three children had 3 ballons each. There were 9 balloons altogether.	6 × 2 = 12
	c) Twelve students entered the classroom in pairs. Six pairs entered the room.	3 × 3 = 9

Obj. 3.1.1
a)i☐ii☐
b)i☐ii☐
c)i☐ii☐

2. Fill in the blanks in the Input/Output Tables from a Function Machine.

a)

Change ________	
Input	Output
4	16
5	20
6	24
8	____
____	8

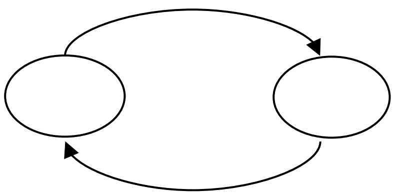
b)

Change ________	
Input	Output
4	28
5	35
6	42
9	____
____	21

Obj. 3.1.4
a)i. ☐
ii. ☐
iii. ☐
b)i. ☐
ii. ☐
iii. ☐

3. Draw a multiplication backtracking diagram for 8 + 8 + 8 = 24.

____



____

Obj. 3.1.5
i. ☐
ii. ☐
iii. ☐
Obj. 3.1.2
iv. ☐

4. Is the following equation true or false?

.  × 1 =    T ☐   F ☐

Obj. 3.1.3
☐

5. For each of the following stories:
   i. Underline the factors.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

   (a) 12 students paid $5 for lunch. How much money did the students pay altogether for lunch? ______________________________

   ____

   (b) I bought 3 phones for $29 each. How much did I pay in total?

   ____

   ____

Obj. 3.1.6
a)i. □
b)i. □
Obj. 3.1.7
a)ii. □
b)ii. □
Obj. 3.1.8
a)iii. □
b)iii. □
Obj. 3.1.9
a)iv. □
b)iv. □

6. A paint store works out the price of its tins of paints using the equation: Q × $9 + $5 = C

   (Q is the quantity of paint ordered in Litres; C is the final price)

   If you order 10L of paint, what will be the price of the paint?

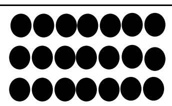
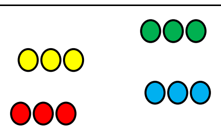
   ____

   ____

   ____

Obj. 3.1.10
i. □
ii. □
iii. □

7. Match the stories to the pictures and symbols that represent them.

Pictures	**Stories**	**Symbols**
	a) There were 3 classes with 7 students in each class. There were 21 students in total.	4 × 3 = 12
	b) Four children had 3 ballons each. There were 12 balloons altogether.	6 × 2 = 12
	c) Twelve students entered the classroom in pairs. Six pairs entered the room.	3 × 7 = 21

Obj. 3.2.1
a)i□ii□
b)i□ii□
c)i□ii□

8. Mark these equations right (✓) or wrong (×).

   (a) 72 = 9 × 8 (b) 4 × 8 = 34 – 2

Obj. 3.2.4
a) □
b) □

9. Look at the story below.
   i. Underline the factors.
   ii. Circle the product.
   iii. Write an equation for the story with a symbol for the unknown.
   iv. Write down the answer

   There were 5 children looking after 4 birds each. How many birds altogether? ______________________________

   ____

Obj. 3.2.2 i. ☐
Obj. 3.2.3 ii. ☐
Obj. 3.2.5 iii. ☐
Obj. 3.2.6 iv. ☐

10. Think about this $3 \times 8 = 24$ equation:

Tick (✓) the box next to the correct answer:

a) If the 3 is replaced with a bigger number and the 8 stays the same, what will happen to the 24?

☐ The 24 will be replaced with a bigger number

Or ☐ The 24 will be replaced with a smaller number

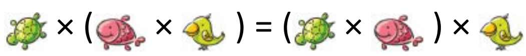
b) If the 3 is replaced with a smaller number and the 24 stays the same, what happens to the 8?

☐ The 8 will be replaced with a bigger number

Or ☐ The 8 will be replaced with a smaller number

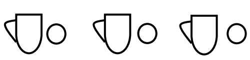
Obj. 3.2.4 a) ☐ b) ☐

11. Tick true or false for each equation.

(a)		T ☐	F ☐
(b)		T ☐	F ☐
(c)		T ☐	F ☐
(d)		T ☐	F ☐

Obj. 3.2.8 a) ☐
Obj. 3.2.9 b) ☐ c) ☐
Obj. 3.2.7 d) ☐

12. Write an expression to represent the drawing of cups and counters.

 ______________________________

Obj. 3.2.5 i. ☐ ii. ☐

13. Draw cups and counters to represent 3y+2

Obj. 3.2.5 i. ☐ ii. ☐

14. Write a story for the equation: $6 \times c = 126$

____

____

Obj. 3.2.5
i. □
ii. □
iii. □
iv. □

15. Three students bought lunch. Each student bought a burger for $5 and a can of soft drink. How much did they spend on lunch altogether?

(a) Write an equation with variables to represent the story.

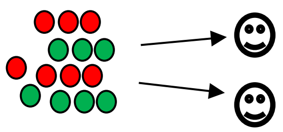
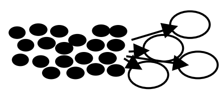
____

(b) If soft drink cans were $2 each, how much did the students spend on lunch altogether? ______________________

____

Obj. 3.2.5
a)i. □
ii. □
iii. □
iv. □
Obj. 3.2.6
b)i. □
b)ii. □

16. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) 20 fish were caught and put evenly into 4 buckets. Each bucket had 5 fish.	$10 \div 5 = 2$
	b) 5 girls shared a ribbon that was 10m long and was cut into 2m lengths.	$14 \div 2 = 7$
	c) Two children shared a bunch of 14 balloons. Each child received 7 balloons.	$20 \div 4 = 5$

Obj. 3.3.1
a)i□ii□
c)i□ii□
Obj. 3.3.2
b)i□ii□

17. Mark these equations right (✓) or wrong (×).

(a) $16 - 9 = 14 \div 2$

(b) $27 \div 4 = 9$

Obj. 3.3.5
a) □
b) □

18. Look at the stories below. For each story:

i. Underline the factors.

ii. Circle the product.

iii. Write an equation for the story with a symbol for the unknown.

iv. Write down the answer

(a) Each student paid $8 for the class party. The class had $128 altogether. How many students paid for the party?

____

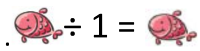
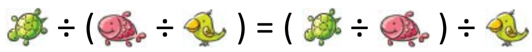
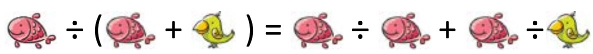
____

(b) I spent $120 on 4 books. If the books were all the same price, how much was each book? __________________________

____

Obj. 3.3.6 a)i. ☐ b)i. ☐
Obj. 3.3.7 a)ii. ☐ b)ii. ☐
Obj. 3.3.8 a)iii. ☐ b)iii. ☐
Obj. 3.3.9 a)iv. ☐ b)iv. ☐

19. Tick true or false for each equation.

(a)		T ☐	F ☐
(b)		T ☐	F ☐
(c)		T ☐	F ☐
(d)		T ☐	F ☐

Obj. 3.3.10 a) ☐
Obj. 3.3.4 b) ☐
Obj. 3.3.10 c) ☐ d) ☐

20. Think about this equation: **18 ÷ 3 = 6**

Tick (✓) the box next to the correct answer:

(a) If the **18** is replaced with a smaller number and the **3** stays the same, what will happen to the **6**?

☐ The **6** will be replaced with a bigger number

Or ☐ The **6** will be replaced with a smaller number

(b) If the **18** is replaced with a bigger number and the **3** is replaced with a smaller number, what happens to the **6**?

☐ The **6** will be replaced with a bigger number

Or ☐ The **6** will be replaced with a smaller number

Obj. 3.3.5 a) ☐ b) ☐

Pretend to use the calculator.

Which operation key (× or ÷) would you use to solve each of these problems?

21. Circle the correct button beside each question.

(a) I bought 6 packets of chips at the shop, each one cost $2. How much did I spend?	× ÷
(b) I cut a hose into five pieces the same length. The hose was 30 m long to begin with. How long was each piece?	× ÷

Obj. 3.3.8
a) □
b) □

22. Write a story for the equation: 126 ÷ c = 61

____

____

Obj. 3.3.8
i. □
ii. □
iii. □
iv. □

23. Four students bought lunch. Each student bought a burger for $4 and a can of soft drink. How much did they spend on lunch altogether?

(a) Write an equation with variables to represent the story.

____

(b) If the students spent $20 altogether, what did a can of soft drink cost? ________________________________

____

Obj. 3.3.8
a)i. □
ii. □
iii. □
iv. □
Obj. 3.3.9
b)i. □
b)ii. □

24. (a) Circle the multiples of 6.

(i) 18 (ii) 127 (iii) 123 (iv) 53 (v) 42

(b) Circle the multiples of 4.

(i) 28 (ii) 128 (iii) 74 (iv) 56 (v) 42

Obj. 3.4.1
a)i□ii□
iii□iv□
v□
b)i□ii□
iii□iv□
v□

25. (a) What number has factors of 7, 9? ____________________

(b) What number has factors of 3, 5, 6, 7? ____________________

Obj. 3.4.2
a) □
b) □

26. Write down as many sets of factors for 20 as you can.

(Sets of factors can have more than 2 factors in them.)

____

____

Obj. 3.4.2 □□□□ □□□□ □

27. Write down the set of prime factors for 12. (A factor tree will help.)

____

____

____

Obj. 3.4.3 i. □ ii. □ iii. □

28. (a) Write down the factors of 28. ______________________

____

(b) Write down the factors of 32. ______________________

____

(c) Write down the common factors of 28 and 32. __________

(d) Write down the highest common factor of 28 and 32. ________

(e) Write down the lowest common factor of 28 and 32. ________

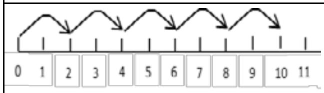
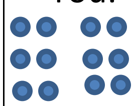
Obj. 3.4.2 a)□□□ □□□
Obj. 3.4.3 b)□□□ □□□
Obj. 3.4.4 c)□□□
Obj. 3.4.5 d)□
Obj. 3.4.6 e)□

29. (a) A fence uses panels of length P. Write the expression for the length of a fence that is 12 panels long. ______________________

(b) Simplify the expression: (4a + 2b). ______________________

Obj. 3.4.7 a)□
Obj. 3.4.8 b)□

30. Match the stories to the pictures and symbols that represent them.

Pictures		Stories	Symbols
		a) I caught 2 fish. You caught 6 times as many fish as me. You caught 12 fish.	6 ÷ 3 = 2
I: 	You: 	b) Fred has 6 stickers. You have 3 times fewer stickers than Fred. You have 2 stickers.	5 × 2 = 10
Fred: 	You: 	c) Tom walked 2km. Frances walked 5 times Tom's disance. Frances walked 10km.	6 × 2 = 12

Obj. 3.5.1 a)i□ii□ b)i□ii□ c)i□ii□

31. Look at the stories below. For each story:

i. Underline the factors.

ii. Circle the product.

iii. Write an equation for the story with a symbol for the unknown.

iv. Write down the answer

(a) Kingston had 20mm of rain. Brisbane had 6 times as much rain as Kingston. How much rain did Brisbane have?

____

____

(b) Jeans at Jeans West were $90. Target's jeans were 3 times cheaper. What was the cost of jeans at Target?

____

____

Obj. 3.5.2 a)i. □ b)i. □
Obj. 3.5.3 a)ii. □ b)ii. □
Obj. 3.5.4 a)iii. □ b)iii. □
Obj. 3.5.5 a)iv. □ b)iv. □

32. Fred and George bought lunch. Fred spent 3 times as much as George. How much did Fred spend on lunch?

(a) Write an equation with variables to represent the story.

____

(b) If George spent $5, how much did Fred spend on lunch?

____

____

Obj. 3.5.4 a)i. □ ii. □ iii. □ iv. □
Obj. 3.5.5 b)i. □ b)ii. □

Pretend to use the calculator.

Which operation key (× or ÷) would you use to solve each of these problems?

33. Circle the correct button beside each question.

(a) A roll of rope is 50m long. Another roll of rope is 5 times shorter. How long is the second roll of rope?	× ÷
(b) Mark has 12 lollies which is 4 times as many as Colin. How many lollies does Colin have?	× ÷

Obj. 3.5.4 a) □ b) □

34. Write a comparison story for the equation: $12 \times c = 120$

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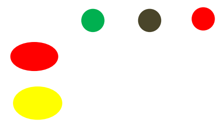
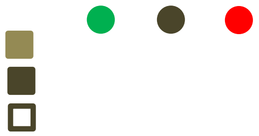
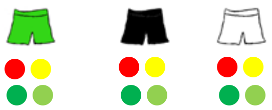
Obj. 3.5.4
i. □
ii. □
iii. □
iv. □

35. Match the stories to the pictures and symbols that represent them.

Pictures	Stories	Symbols
	a) You have 4 shirts and 3 pairs of shorts. You can make 12 different outfits.	$6 = 3 \times 2$
	b) Fred makes 6 different desserts from 3 flavours of ice cream and 2 types of fruit.	$9 \div 3 = 3$
	c) Tom makes 9 different types of sandwiches to sell. He has brown, white and gluten free bread. He needs 3 different fillings.	$4 \times 3 = 12$

Obj. 3.6.1
a)i□ii□
b)i□ii□
c)i□ii□

36. Look at the stories below. For each story:

i. Underline the factors.
ii. Circle the product.
iii. Write an equation for the story with a symbol for the unknown.
iv. Write down the answer

(a) If an ice cream stall has 8 flavours of ice cream for sale; how many different toppings will they need for 24 people to have different ice cream and topping combinations?

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(b) A travel agent has 6 discount holiday destinations with 3 different accommodation packages. How many different discount holiday deals can they advertise?

____

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Obj. 3.6.2
a)i. □
b)i. □
Obj. 3.6.3
a)ii. □
b)ii. □
Obj. 3.6.4
a)iii. □
b)iii. □
Obj. 3.6.5
a)iv. □
b)iv. □

37. Fred has three different shirt patterns. Each shirt can be made in different sizes. How many different shirts can Fred make to sell?

(a) Write an equation with variables to represent the story.

____

(b) If there are 6 different sizes for each shirt, how many different shirts can Fred make to sell?

____

____

Obj. 3.6.4
a)i. ☐
ii. ☐
iii. ☐
iv. ☐
Obj. 3.6.5
b)i. ☐
b)ii. ☐

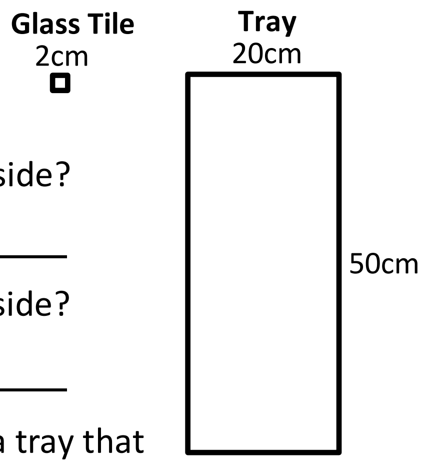
38. Write a combinations story for the equation: 12 ÷ p = 3

____

____

Obj. 3.6.4
i. ☐
ii. ☐
iii. ☐
iv. ☐

39. You want to cover a pot stand with square glass tiles that are 2cm across.



(a) How many tiles will fit along the 50cm side?

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(b) How many tiles will fit along the 20cm side?

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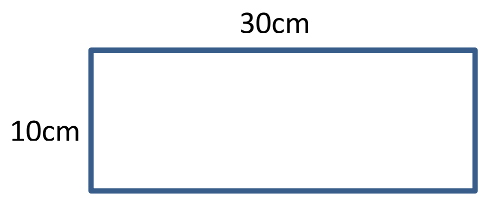
(c) How many tiles will you need to cover a tray that is 20cm wide and 50cm long?

____

Obj. 3.7.2
a) i. ☐
a) ii. ☐
a) iii. ☐
b) i. ☐
b) ii. ☐
b) iii. ☐

Obj. 3.7.1
c) i. ☐
c) ii. ☐
c) iii. ☐

40. Calculate the distance around this rectangle.



Obj. 3.7.1
i. ☐
ii. ☐
iii. ☐

41. Solve the following questions in the space provided.

Write down all the steps you take to reach your answer.

(a) 23 × 5	(b) 22 ×16
(c) 72 ÷ 6	(d) 852 ÷ 12

Obj. 3.7.1
a) □
b) □

Obj. 3.7.2
c) □
d) □

XLR8 – Accelerating Mathematics Learning

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