Multiplier product matrix analysis for interregional input-output systems: an application to the Brazilian economy

Guilhoto, Joaquim José Martins and Sonis, Michael and Hewings, Geoffrey J.D.

Unviersidade de São Paulo, University of Illinois

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Multiplier Product Matrix Analysis for Interregional Input-Output Systems: An Application to the Brazilian Economy

Joaquim J.M. Guilhoto,[1] Michael Sonis[2], and Geoffrey J.D. Hewings[3]

Abstract

In this paper, using a set of interregional input-output tables built by the authors for the year of 1992 for 2 Brazilian regions (Northeast and rest of the economy), attention is focused on a new approach to the interpretation of Miyazawa's concepts of left and right multipliers in the decomposition of interregional input-output systems. Using the technique of the multiplier product matrix (Sonis *et al.*, 1997) and Sonis and Hewings (1999), the hierarchical decomposition proposed exploits the insights offered by the fields of influence theory and provides a way of interpreting Miyazawa's left and right multipliers in terms of interregional feedback loops. When this technique is applied to an interregional system for 2 Brazilian regions (Northeast and the Rest of Brazil) the results shows that: (a) the Rest of Brazil region seems to be more developed and has a more complex productive structure than the Northeast region; (b) the inputs that the Northeast region buys from the Rest of Brazil region practically make no contribution to the **total** linkages in either of the regions; and (c) when isolated from the whole economy system, there is little contribution from the Northeast region to the linkages of the Rest of Brazil region, and vice-versa. Extension of this work could be considered in two directions, namely, providing sectoral detail and extending this approach to the *n* region case.

1. Introduction

In recent years, several new perspectives on economic structure and structural change have been derived from those originally proposed by Miyazawa (1966, 1971). In this paper, an attempt is made to link these approaches in a way that provides a clear path from one to the other, thereby making the different insights generated by each component more directly comparable or complementary to the others. The paper begins with a presentation of the multiplier product matrix (MPM) and its associated economic landscapes; from here, the notions of interdependence, especially the identification of internal and external multipliers, originally

[1] University of São Paulo (USP), Brazil and University of Illinois. E-mail: guilhoto@usp.br.
[2] Bar Ilan University, Israel, and University of Illinois. E-mail: sonism@mail.bin.ac.il.
[3] University of Illinois. E-mail: hewings@uiuc.edu.

proposed by Miyazawa, can be generated and reinterpreted with the MPM structure. The methodology developed in this paper is then applied to a two-region (Northeast, Rest of Brazil) interregional input-output system constructed for the Brazilian economy for the year of 1992 (see Guilhoto, 1998).

2. Economic Cross-Structure Landscapes of MPM and the Rank-Size Hierarchies of Backward and Forward Linkages[4]

This section introduces the notion of *artificial economic landscapes* and the corresponding multiplier product matrices representing the essence of key sector analysis. The definition of the multiplier product matrix is as follows: let $A = \|a_{ij}\|$ be a matrix of direct inputs in the usual input-output system, and $B = (I - A)^{-1} = \|b_{ij}\|$ the associated Leontief inverse matrix and let $B_{\bullet j}$ and $B_{i\bullet}$ be the column and row multipliers of this Leontief inverse. These are defined as:

$$B_{\bullet j} = \sum_{i=1}^{n} b_{ij}, \quad B_{i\bullet} = \sum_{j=1}^{n} b_{ij} \quad j = 1, 2, ..., n \tag{1}$$

The row and column vectors of column and row multipliers take the following form:

$$M_c(B) = \left[B_{\bullet 1} B_{\bullet 2} ... B_{\bullet p} \right], \quad M_r(B) = \begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{bmatrix} \tag{2}$$

Let V be the global intensity of the Leontief inverse matrix:

$$V = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \tag{3}$$

Then, the input-output multiplier product matrix (MPM) is defined as:

[4] The first part of this section draws on Sonis e*t al*., (1997)

$$M = \frac{1}{V}\left\| B_{i\bullet} B_{\bullet j} \right\| = \frac{1}{V}\begin{pmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{pmatrix} \begin{matrix} B_{\bullet 1} & B_{\bullet 2} & \cdots & B_{\bullet n} \end{matrix} = \left\| m_{ij} \right\|$$

$$V = \sum_{i=1}^{n} B_{i\bullet} = \sum_{j=1}^{n} B_{\bullet j} = \begin{matrix} B_{\bullet 1} & B_{\bullet 2} & \cdots & B_{\bullet n} \end{matrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = 11\ldots1 \begin{pmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{pmatrix} \tag{4}$$

or, in vector notation:

$$M = \frac{1}{V} M_r(B) M_c(B);\;\; V = M_c(B) \times i' = i \times M_r(B) \tag{5}$$

The properties of the MPM that will now be considered will focus on (1) the hierarchy of backward and forward linkages and their economic landscape associated with the cross-structure of the MPM, and (2) the interpretation of MPM as a matrix of first order intensities of the fields of influence of individual changes in direct inputs.

The concept of key sectors is based on the notion of backward and forward linkages and has been associated with the work of both Rasmussen (1956) and Hirschman (1958). The major thrust of the analytical techniques, and subsequent modifications and extensions, has been towards the identification of sectors whose linkage structures are such that they create an above-average impact on the rest of the economy when they expand or in response to changes elsewhere in the system. Rasmussen (1956) proposed two types of indices drawing on entries in the Leontief inverse:

1. Power of dispersion for the backward linkages, BL_j, as follows:

$$\begin{aligned} BL_j &= \frac{1}{n}\sum_{i=1}^{n} b_{ij} \Big/ \frac{1}{n^2}\sum_{i,j=1}^{n} b_{ij} = \\ &= \frac{1}{n} B_{\bullet j} \Big/ \frac{1}{n^2} V = B_{\bullet j} \Big/ \frac{1}{n} V \end{aligned} \tag{6}$$

and

2. The indices of the sensitivity of dispersion for forward linkages, FL_i, as follows:

$$FL_i = \frac{1}{n}\sum_{j=1}^{n} b_{ij} \bigg/ \frac{1}{n^2}\sum_{i,j=1}^{n} b_{ij} = \\ = \frac{1}{n} B_{i\bullet} \bigg/ \frac{1}{n^2} V = B_{i\bullet} \bigg/ \frac{1}{n} V \tag{7}$$

The usual interpretation is to propose that $BL_j > 1$ indicates that a unit change in final demand in sector *j* will create an above average increase in activity in the economy; similarly, for $FL_i > 1$, it is asserted that a unit change in all sectors' final demand would create an above average increase in sector *i*. A key sector is usually defined as one in which both indices are greater than 1.

The definitions of backward and forward linkages provided by (6) and (7) imply that the rank-size hierarchies (rank-size ordering) of these indices coincide with the rank-size hierarchies of the column and row multipliers. It is important to underline, in this connection, that the column and row multipliers for MPM are the same as those for the Leontief inverse matrix. Thus, the structure of the MPM is essentially connected with the properties of sectoral backward and forward linkages.

The structure of the matrix, *M*, can be ascertained in the following fashion: consider the largest column multiplier, $B_{\bullet j}$, and the largest row multiplier, $B_{i\bullet}$, of the Leontief inverse, with the element, $m_{i_0 j_0} = \frac{1}{V} B_{i_0\bullet} B_{\bullet j_0}$, located in the place i_0, j_0 of the matrix, *M*. Moreover, all rows of the matrix, *M*, are proportional to the i_0^{th} row, and the elements of this row are larger than the corresponding elements of all other rows. The same property applies to the j_0^{th} column of the same matrix. Hence, the element located in i_0, j_0 defines the center of the largest *cross* within the matrix, *M*. If this cross is excluded from *M*, then the second largest cross can be identified and so on. Thus, the matrix, *M*, contains the rank-size sequence of crosses. One can reorganize the locations of rows and columns of *M* in such a way that the centers of the corresponding crosses appear on the main diagonal. In this fashion, the matrix will be reorganized so that a descending *economic landscape* will be apparent.

This rearrangement also reveals the descending rank-size hierarchies of the Rasmussen-Hirschman indices for forward and backward linkages. Inspection of that part of the landscape with indices > 1 (the usual criterion for specification of key sectors) will enable the identification of the key sectors. However, it is important to stress that the construction of the economic landscape for different regions or for the same region at different points in time would create the possibility for the establishment of a taxonomy of these economies.

3. Hierarchical Inclusion of Economic Landscapes[5]

In this section, attention will be directed to a description of multiple shifts in intraregional backward and forward linkages and the associated changes in the positions of key sectors under the influence of interaction between the region and the rest of economy. The approach creates the possibility to evaluate immediately when economic sectors became more important for the regional economy under the influence of synergetic interactions with the rest of economy.

The main analytical tool of the hierarchical inclusion of the economic landscapes will now be revealed. Consider the product, $B = B^{/} B^{//}$, of two matrices, $B^{/}$ and $B^{//}$, of the respective sizes $n \times m,\ \ m \times p$. Let

$$\begin{aligned}
B_{\bullet j} &= \sum_{i=1}^{n} b_{ij}; \quad B_{i\bullet} = \sum_{j=1}^{n} b_{ij} \\
B^{/}_{\bullet j} &= \sum_{i=1}^{n} b^{/}_{ij}; \quad B^{/}_{i\bullet} = \sum_{j=1}^{n} b^{/}_{ij} \\
B^{//}_{\bullet j} &= \sum_{i=1}^{n} b^{//}_{ij}; \quad B^{//}_{i\bullet} = \sum_{j=1}^{n} b^{//}_{ij}
\end{aligned} \tag{8}$$

be the column and row multipliers of these matrices. Using the definition of *V*, the global intensity of the matrix *B* from (3), the following multiplicative connections between the vectors of column and row multipliers of these matrices exist:

[5] This section draws on Sonis and Hewings (1999).

$$\left[B_{\bullet 1}B_{\bullet 2}...B_{\bullet p}\right]=\left[B^{'}_{\bullet 1}B^{'}_{\bullet 2}...B^{'}_{\bullet m}\right]\times B^{"};\quad \begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ : \\ B_{n\bullet} \end{bmatrix}=B^{'}\times\begin{bmatrix} B^{"}_{1\bullet} \\ B^{"}_{2\bullet} \\ : \\ B^{"}_{m\bullet} \end{bmatrix};$$

$$V=\left[B^{'}_{\bullet 1}B^{'}_{\bullet 2}...B^{'}_{\bullet m}\right]\times\begin{bmatrix} B^{"}_{1\bullet} \\ B^{"}_{2\bullet} \\ : \\ B^{"}_{m\bullet} \end{bmatrix} \tag{9}$$

These expressions can be checked by direct calculations of the components of the corresponding vectors and matrices.

Further, specify the following vectors:

$$\begin{aligned} M_c(B) &= \left[B_{\bullet 1}B_{\bullet 2}...B_{\bullet p}\right] \\ M_c(B') &= \left[B^{'}_{\bullet 1}B^{'}_{\bullet 2}...B^{'}_{\bullet m}\right] \\ M_c(B^{"}) &= \left[B^{"}_{\bullet 1}B^{"}_{\bullet 2}...B^{"}_{\bullet m}\right] \end{aligned} \tag{10}$$

$$M_r(B)=\begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ : \\ B_{n\bullet} \end{bmatrix}, M_r(B^{'})=\begin{bmatrix} B^{'}_{1\bullet} \\ B^{'}_{2\bullet} \\ : \\ B^{'}_{m\bullet} \end{bmatrix}, M_r(B^{"})=\begin{bmatrix} B^{"}_{1\bullet} \\ B^{"}_{2\bullet} \\ : \\ B^{"}_{m\bullet} \end{bmatrix}$$

as the row vectors and column vectors with components that are the column and row multipliers of the matrices, $B, B^{'}, B^{"}$. Using this notation, equation (9) may be presented in the following form:

$$\begin{aligned} M_c(B) &= M_c(B')B''; \\ M_r(B) &= B'M_r(B''); \\ V &= M_c(B')M_r(B'') \end{aligned} \tag{11}$$

Consider the economic system that is comprised of a region *r* and the rest of economy, *R*. The corresponding input-output system can be represented by the block matrix

$$A=\begin{pmatrix} A_{rr} & A_{rR} \\ A_{Rr} & A_{RR} \end{pmatrix} \tag{12}$$

Assume that the intra-regional matrix, A_{rr}, of the region r has the following incremental change E_{rr}, and A_{rR}, A_{Rr} are the inter-regional matrices representing direct input connections between region and the rest of the economy, while the matrix A_{RR} represents the intra-regional inputs within the rest of the economy.

The Leontief inverse $B = (I - A)^{-1}$ can be formally presented in the following block:

$$B = \begin{bmatrix} B_{rr} & B_{rR} \\ B_{Rr} & B_{RR} \end{bmatrix} \tag{13}$$

and this can be further elaborated with the help of the Schur-Banachiewicz formula (Schur, 1917; Banachiewicz, 1937; Miyazawa, 1966; Sonis and Hewings, 1993):

$$B = \begin{bmatrix} B_{rr} & B_{rr} A_{rR} B_R \\ B_R A_{Rr} B_r & B_{RR} \end{bmatrix} = \begin{bmatrix} B_{rr} & B_r A_{rR} B_{RR} \\ B_{RR} A_{Rr} B_{rr} & B_{RR} \end{bmatrix} \tag{14}$$

where the matrices $B_r = (I - A_{rr})^{-1}$ and $B_R = (I - A_{RR})^{-1}$ represent the Miyazawa internal matrix multipliers for the region r and the rest of economy (revealing the interindustry propagation effects within the isolated region and isolated rest of economy) while the matrices $A_{Rr}B_r, B_r A_{rR}, A_{rR} B_R$, and $B_R A_{Rr}$ show the induced effects on output or input between the two parts of input-output system (Miyazawa, 1966).

Further:

$$\begin{aligned} B_{rr} &= (I - A_{rr} - A_{rR} B_R A_{Rr})^{-1} \\ B_{RR} &= (I - A_{RR} - A_{Rr} B_r A_{rR})^{-1} \end{aligned} \tag{15}$$

are the extended Leontief multipliers for the region r and the rest of economy. The connections between these extended Leontief multipliers are:

$$\begin{aligned} B_{rr} &= B_r + B_r A_{rR} B_{RR} A_{Rr} B_r \\ B_{RR} &= B_R + B_R A_{Rr} B_{rr} A_{rR} B_R \end{aligned} \tag{16}$$

By using the Miyazawa decomposition, the extended Leontief inverses can be decomposed into the products of internal and external multipliers describing direct and induced self-influences (Miyazawa, 1966, 1976):

$$\begin{aligned} B_{rr} &= B_r B_{rr}^R = B_{rr}^L B_r \\ B_{RR} &= B_R B_{RR}^R = B_{RR}^L B_R \end{aligned} \tag{17}$$

where

$$\begin{aligned} B_{rr}^L &= \left(I - B_r A_{rR} B_R A_{Rr}\right); \quad B_{rr}^R = \left(I - A_{rR} B_R A_{Rr} B_r\right) \\ B_{RR}^L &= \left(I - B_R A_{Rr} B_r A_{rR}\right); \quad B_{RR}^R = \left(I - A_{Rr} B_r A_{rR} B_R\right) \end{aligned} \tag{18}$$

are the left and right Miyazawa external multipliers for the region r and the rest of economy.

It is easy to see that for the block Leontief inverse (13), the row vector $M_c(B)$ of the column multipliers has the following block form:

$$M_c(B) = \left[M_c(B_{rr}) + M_c(B_{Rr}) \quad M_c(B_{rR}) + M_c(B_{RR}) \right] \tag{19}$$

Using (14), one obtains:

$$\begin{aligned} M_c(B) &= \left[M_c(B_{rr}) + M_c(B_{RR}) A_{Rr} B_r \quad M_c(B_{rr}) A_{rR} B_R + M_c(B_{RR}) \right] = \\ &= M_c(B_{rr}) \left[I \quad A_{rR} B_R \right] + M_c(B_{RR}) \left[A_{Rr} B_r \quad I \right] \end{aligned} \tag{20}$$

Analogously, the column block vector of the row multipliers of the Leontief inverse B can be presented in the form:

$$\begin{aligned} M_r(B) &= \begin{bmatrix} M_r(B_{rr}) + M_r(B_{rR}) \\ M_r(B_{Rr}) + M_r(B_{RR}) \end{bmatrix} = \\ &= \begin{bmatrix} M_r(B_{rr}) + B_r A_{rR} M_r(B_{RR}) \\ B_R A_{Rr} M_r(B_{rr}) + M_r(B_{RR}) \end{bmatrix} = \\ &= \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) + \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) \end{aligned} \tag{21}$$

Therefore, the expressions (5) and (4) yield the following form of the multiplier product matrix for the block matrix A of the multiregional input-output system and its Leontief inverse:

$$
\begin{aligned}
&M(B) = \frac{1}{V(B)} M_r(B) M_c(B) = \\
&= \frac{1}{V(B)} \left\{ \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) + \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) \right\} \; M_c(B_{rr}) \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + M_c(B_{RR}) \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} \; = \\
&= \frac{1}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) M_c(B_{rr}) \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{1}{V(B)} \begin{bmatrix} I \\ B_R A_{Rr} \end{bmatrix} M_r(B_{rr}) M_c(B_{rr}) \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix} + \\
&+ \frac{1}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) M_c(B_{rr}) \begin{bmatrix} I & A_{rR} B_R \end{bmatrix} + \frac{1}{V(B)} \begin{bmatrix} B_r A_{rR} \\ I \end{bmatrix} M_r(B_{RR}) M_c(B_{rr}) \begin{bmatrix} A_{Rr} B_r & I \end{bmatrix}
\end{aligned}
\tag{22}
$$

It is important to underline that the application of equations (4) and (5) to the extended Leontief inverses, B_{rr}, B_{RR}, will provide the following extended intraregional multiplier product matrices for the region r and the rest of economy:

$$
\begin{aligned}
M_{rr} &= M(B_{rr}) = \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{rr}) \\
M_{RR} &= M(B_{RR}) = \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{RR})
\end{aligned}
\tag{23}
$$

By analogy it is possible to define the interregional extended multiplier product matrices:

$$
\begin{aligned}
M_{rR} &= \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{RR}) \\
M_{Rr} &= \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{rr})
\end{aligned}
\tag{24}
$$

By analogy it is possible to define the interregional extended multiplier product matrices:

$$
\begin{aligned}
M_{rR} &= \frac{1}{V(B_{rr})} M_r(B_{rr}) M_c(B_{RR}) \\
M_{Rr} &= \frac{1}{V(B_{RR})} M_r(B_{RR}) M_c(B_{rr})
\end{aligned}
\tag{24}
$$

Therefore, the multiplier product matrix $M(B)$ for the block matrix A of the multiregional input-output system reveals the following structure:

$$M(B)=\frac{V(B_{rr})}{V(B)}\begin{bmatrix} I \\ B_R A_{Rr}\end{bmatrix} M_{rr}\begin{bmatrix} I & A_{rR}B_R\end{bmatrix}+\frac{V(B_{rr})}{V(B)}\begin{bmatrix} I \\ B_R A_{Rr}\end{bmatrix} M_{rR}\begin{bmatrix} A_{Rr}B_r & I\end{bmatrix}+$$
$$+\frac{V(B_{RR}}{V(B)})\begin{bmatrix} B_r A_{rR} \\ I\end{bmatrix} M_{Rr}\begin{bmatrix} I & A_{rR}B_R\end{bmatrix}+\frac{V(B_{RR})}{V(B)}\begin{bmatrix} B_r A_{rR} \\ I\end{bmatrix} M_{RR}\begin{bmatrix} A_{Rr}B_r & I\end{bmatrix}$$

(25)

Denote the four components of the decomposition (25) as: $M(B)\ rr\ ; M(B)\ rR\ ; M(B)\ Rr\ ; M(B)\ RR$. Then:

$$M(B)=M(B)\ rr\ +M(B)\ rR\ +M(B)\ Rr\ +M(B)\ RR \tag{26}$$

where

$$M(B)\ rr\ =\frac{V(B_{rr})}{V(B)}\begin{bmatrix} I \\ B_R A_{Rr}\end{bmatrix} M_{rr}\begin{bmatrix} I & A_{rR}B_R\end{bmatrix}=\frac{V(B_{rr})}{V(B)}\begin{bmatrix} M_{rr} & M_{rr}A_{rR}B_R \\ B_R A_{Rr}M_{rr} & B_R A_{Rr}M_{rr}A_{rR}B_R\end{bmatrix} \tag{27}$$

$$M(B)\ rR\ =\frac{V(B_{rr})}{V(B)}\begin{bmatrix} I \\ B_R A_{Rr}\end{bmatrix} M_{rR}\begin{bmatrix} A_{Rr}B_r & I\end{bmatrix}=\frac{V(B_{rr})}{V(B)}\begin{bmatrix} M_{rR}A_{Rr}B_r & M_{rR} \\ B_R A_{Rr}M_{rR}A_{Rr}B_r & B_R A_{Rr}M_{rR}\end{bmatrix} \tag{28}$$

$$M(B)\ Rr\ =\frac{V(B_{RR})}{V(B)}\begin{bmatrix} B_r A_{rR} \\ I\end{bmatrix} M_{Rr}\begin{bmatrix} I & A_{rR}B_R\end{bmatrix}=\frac{V(B_{RR})}{V(B)}\begin{bmatrix} B_r A_{rR}M_{Rr} & B_r A_{rR}M_{Rr}A_{rR}B_R \\ M_{Rr} & M_{Rr}A_{rR}B_R\end{bmatrix} \tag{29}$$

$$M(B)\ RR\ =\frac{V(B_{RR})}{V(B)}\begin{bmatrix} B_r A_{rR} \\ I\end{bmatrix} M_{RR}\begin{bmatrix} A_{Rr}B_r & I\end{bmatrix}=\frac{V(B_{RR})}{V(B)}\begin{bmatrix} B_r A_{rR}M_{RR}A_{Rr}B_r & B_r A_{rR}M_{RR} \\ M_{RR}A_{Rr}B_r & M_{RR}\end{bmatrix} \tag{30}$$

Using the block structure of the components $M(B)\ rr\ ; M(B)\ rR\ ; M(B)\ Rr\ ; M(B)\ RR$, one can construct the block structure of the multiplier product matrix as:

$$M(B)=\begin{bmatrix} M(B)_{\ rr} & M(B)_{\ rR} \\ M(B)_{\ Rr} & M(B)_{\ RR}\end{bmatrix} \tag{31}$$

by summing the corresponding blocks from (27) - (30);

$$
\begin{aligned}
M(B)_{rr} &= \frac{V(B_{rr})}{V(B)} M_{rr} + \frac{V(B_{rr})}{V(B)} M_{rR} A_{Rr} B_r + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr} + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} A_{Rr} B_r; \\
M(B)_{rR} &= \frac{V(B_{rr})}{V(B)} M_{rR} + \frac{V(B_{rr})}{V(B)} M_{rr} A_{rR} B_R + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr} A_{rR} B_R; \\
M(B)_{Rr} &= \frac{V(B_{RR})}{V(B)} M_{Rr} + \frac{V(B_{RR})}{V(B)} M_{RR} A_{Rr} B_r + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rr} + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rR} A_{Rr} B_r; \\
[M(B)]_{RR} &= \frac{V(B_{RR})}{V(B)} M_{RR} + \frac{V(B_{RR})}{V(B)} M_{Rr} A_{rR} B_R + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rR} + \frac{V(B_{rr})}{V(B)} B_R A_{Rr} M_{rr} A_{rR} B_R
\end{aligned}
\tag{32}
$$

Here, a modification of an earlier approach to the region versus the rest of the economy is provided that extends the interpretation to a broader context (see Sonis *et al*., 1996). If attention was directed only to the regional part, *M(B)[rr],* of the economic landscape, *M(B),* then (32) may be shown as:

$$
M(B)_{rr} = \frac{V(B_{rr})}{V(B)} M_{rr} + \frac{V(B_{rr})}{V(B)} M_{rR} A_{Rr} B_r + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr} + \frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} A_{Rr} B_r
$$

This part of (32) describes the spread of changes within the region *r* caused by (1) the changes in direct inputs within the region, $\frac{V(B_{rr})}{V(B)} M_{rr}$; (2) changes in regional forward linkages, $\frac{V(B_{rr})}{V(B)} M_{rR} A_{Rr} B_r$; (3) changes of the regional backward linkages, $\frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{Rr}$ and, finally, (4) changes in the direct inputs within the isolated rest of economy, $\frac{V(B_{RR})}{V(B)} B_r A_{rR} M_{RR} A_{Rr} B_r$. This decomposition provides a summary of the changes differentiated into internal, forward, backward and external linkages.

4. An Application to the Brazilian Economy

The methodology presented in the previous section is now applied to the Brazilian economy, more specifically to an interregional input-output matrix constructed for the Brazilian economy for the year of 1992 for two regions – Northeast and Rest of Brazil (Guilhoto, 1998) at the level of 14 sectors (Table 1).

<<<< Insert Table 1 Here >>>>

When compared to Brazil, the Northeast region: a) occupies 18.28% of its territory; b) has 28.50% of its population; and c) has a share of around 14.00% of the Brazilian GDP.

The results for the Hirschman/Rasmussen Indexes (equations 6 and 7) are presented in Figures 1 to 3. From the analysis of these figures one can see that the Rest of Brazil region records higher values than the Northeast region, which is an indication of a higher level of the development in the Rest of Brazil region. How have the transactions between these two regions contributed to the value of these indices? An explanation can be derived through the use of the methodology presented in the previous section of this paper.

<<<< Insert Figures 1 Through 3 Here >>>>

Applying equation (32), which describes the spread of changes within a given region, it was possible to estimates the results presented in figures 4 through 8 for the Northeast region and in figures 9 through 13 for the rest of Brazil region. In each one of the figures 4 to 13, two schematic representations of the interregional system for the two-region case are shown. In the first scheme, the region of focus is identified (region 1 is the Northeast region and region 2 is the Rest of Brazil region). The second scheme shows which cell is contributing to the landscape of the region of focus; thus, in figures 4 and 9, all the cells in the second scheme are marked.

<<<< Insert Figures 4 Through 13 Here >>>>

The analysis of figures 4 to 13 shows that, in general, the Northeast region has a greater dependence on the Rest of Brazil economy than the latter has on the Northeast region; at the same time, the Rest of Brazil seems to be more developed as it reveals a more complex productive structure than the Northeast region, by virtue of the fact that the MPM matrix for the Rest of Brazil region shows higher values than the ones observed for the Northeast region. These results are directly related with the results of the Hirschman/Rasmussen indexes. Note that the inputs that the Northeast region buys from the Rest of Brazil region have practically no contribution to the total linkages in either one of the regions (figures 7 and 11). Also, when isolated from the system, practically there is no contribution of the Northeast region to the linkages of the Rest of Brazil region, and vice-versa.

5. Conclusions

While many decomposition techniques for interpreting structure and structural change have been proposed and subsequently modified, there have been few attempts to explore the links between sets of methodologies. This paper has tried to provide a mapping between several alternative, yet complementary approaches and, in the previous section, provide a summary interpretation of the insights that they offer. Particular attention is paid to the distinction, first articulated by Miyazawa (1966), between internal and external effects. Dominating this distinction is the strong presence of hierarchical influences and the superposition of different intersectoral and spatial mechanisms creating change.

The methodology presented here is a first approach to better understand how the transaction take place among regions; however more work needs to be done to estimate the linkages not only in terms of the productive relation among the sectors and regions but also in terms of value of production. A first attempt using the value of production of the sectors was done by Guilhoto, *et al*. (1999). However in this analysis, a different methodology was used and there needs to be some evaluation of the linkages between these two methodologies. Also, the methodology presented here focused on a two region (region versus the rest of the economy) context; extensions to the *n*-region case would afford the opportunity to explain the paths (directions) of change (see Guilhoto 1999).

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Table 1

Sectors Defined in the Interregional Model

Sector	Description
1	Agriculture
2	Mining
3	Metallurgy
4	Machinery
5	Electrical Equipment
6	Transport Equipment
7	Wood, Wood Products, Paper Products, and Printing
8	Chemicals
9	Textiles, Clothing and Footwear
10	Food Products
11	Miscellaneous Industries
12	Public Utilities
13	Construction
14	Services

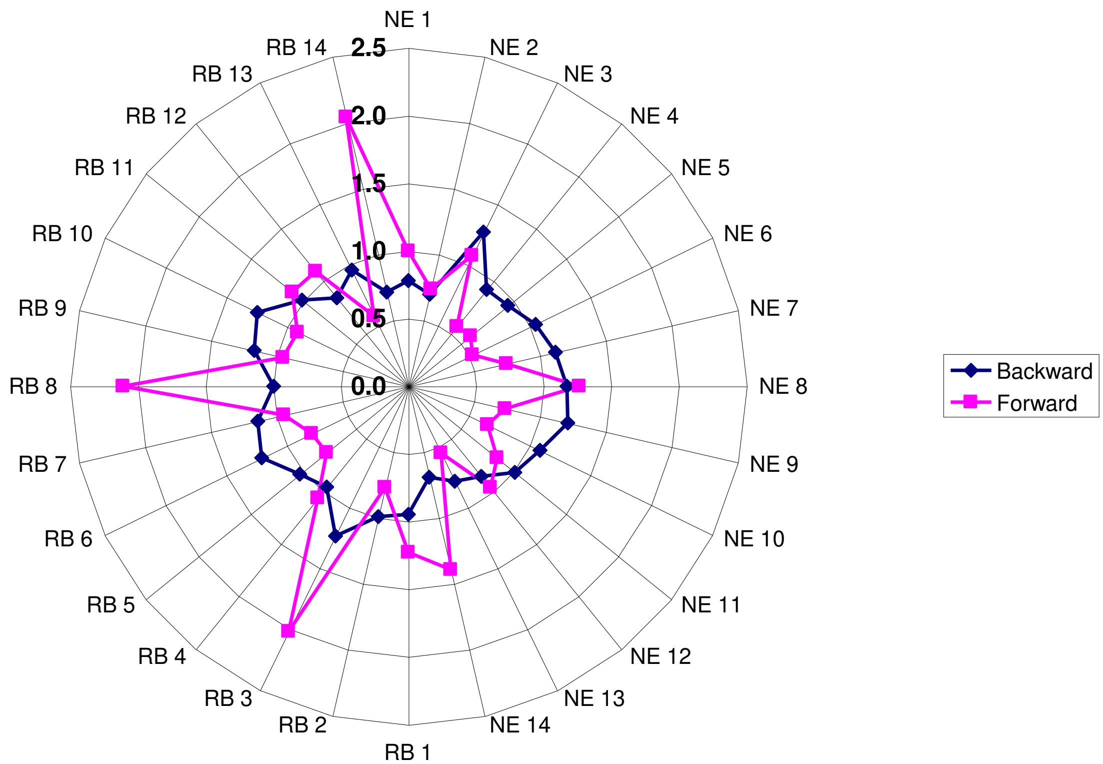


Figure 1

Hirschman/Rasmussen Backward and Forward Linkages for the Interregional System
Northeast and Rest of Brazil Regions – 1992

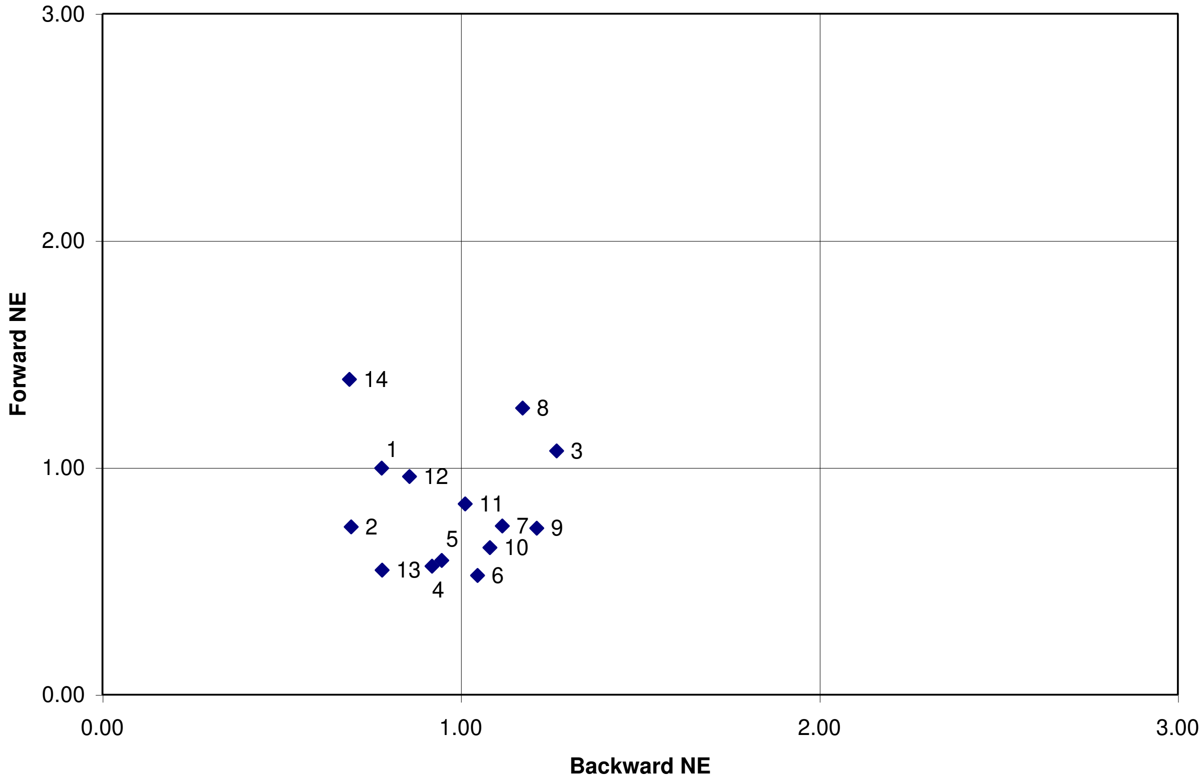


Figure 2

Hirschman/Rasmussen Backward and Forward Linkages for the Interregional System
Northeast Region – 1992

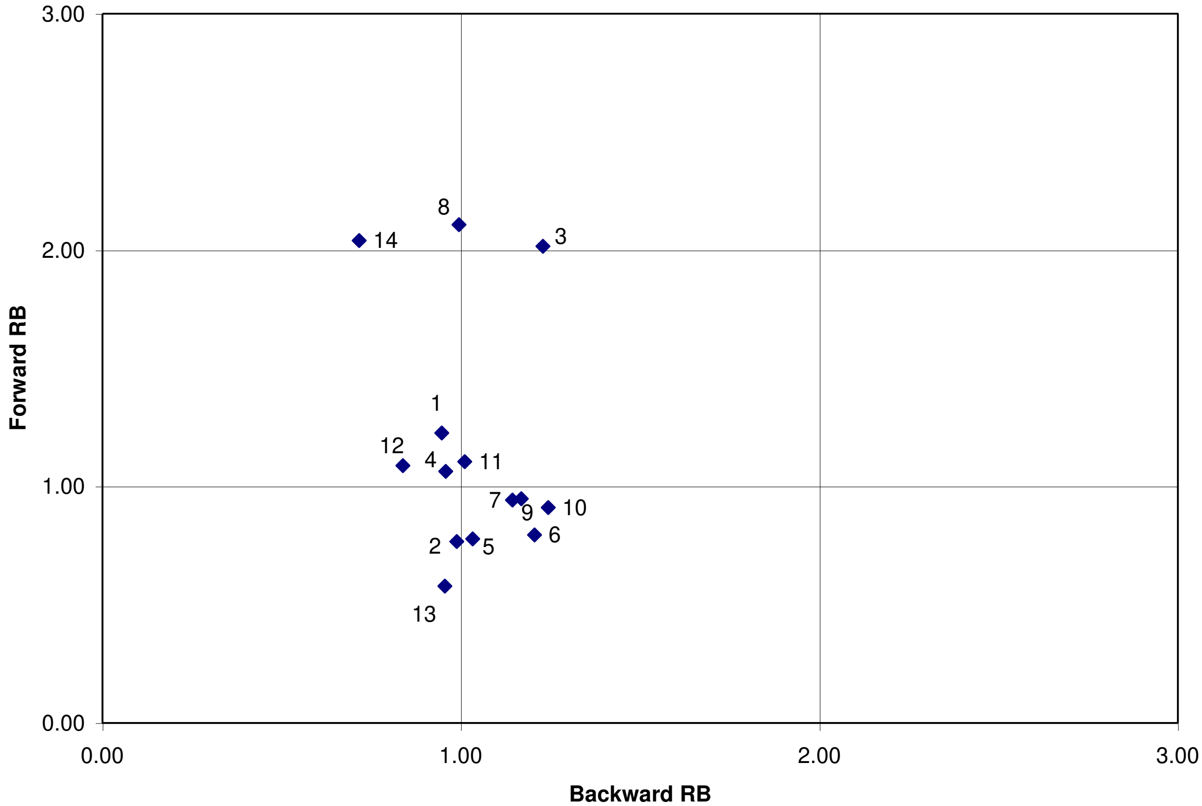
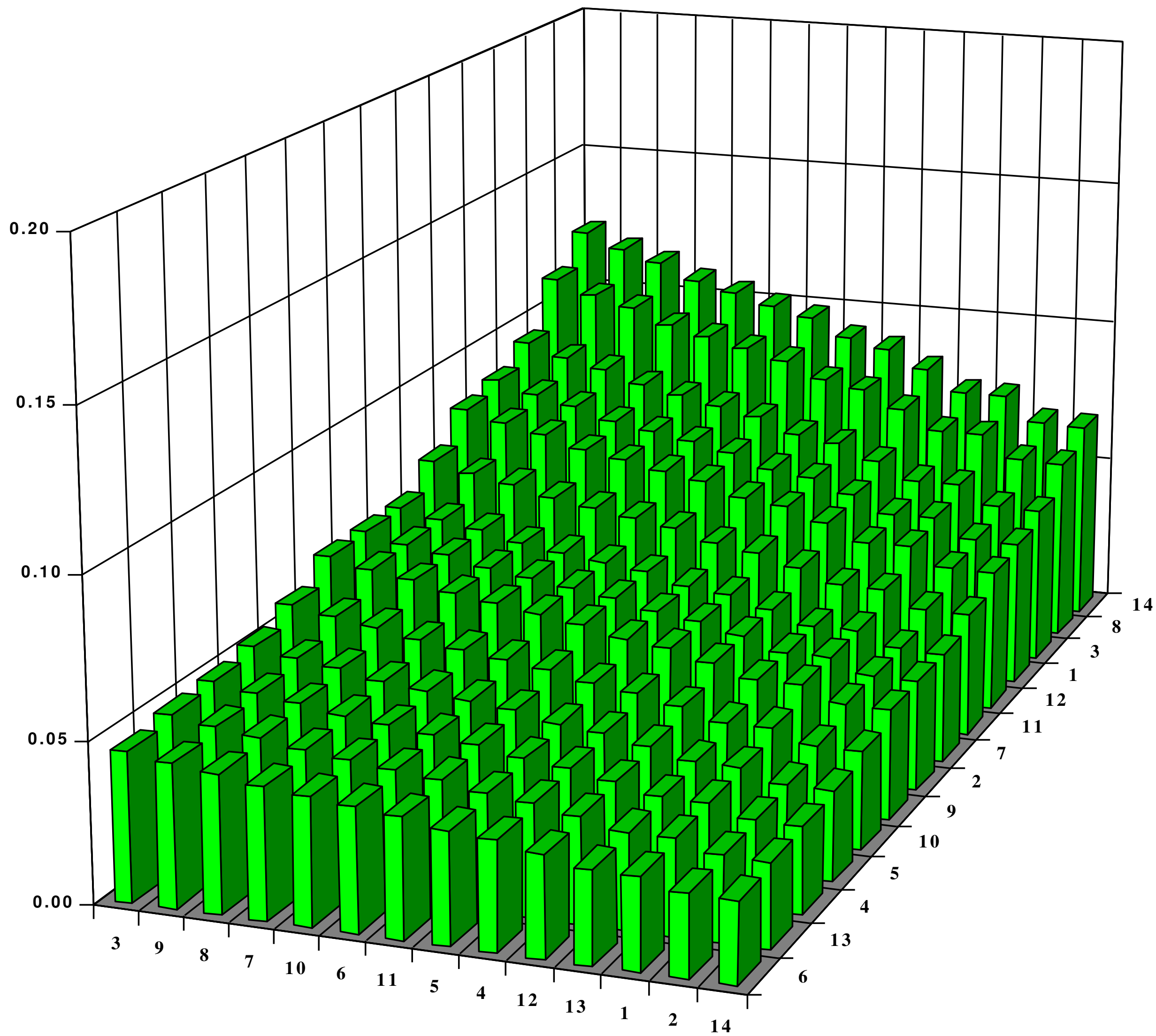
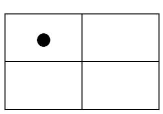


Figure 3

Hirschman/Rasmussen Backward and Forward Linkages for the Interregional System
Rest of Brazil Region – 1992

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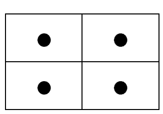
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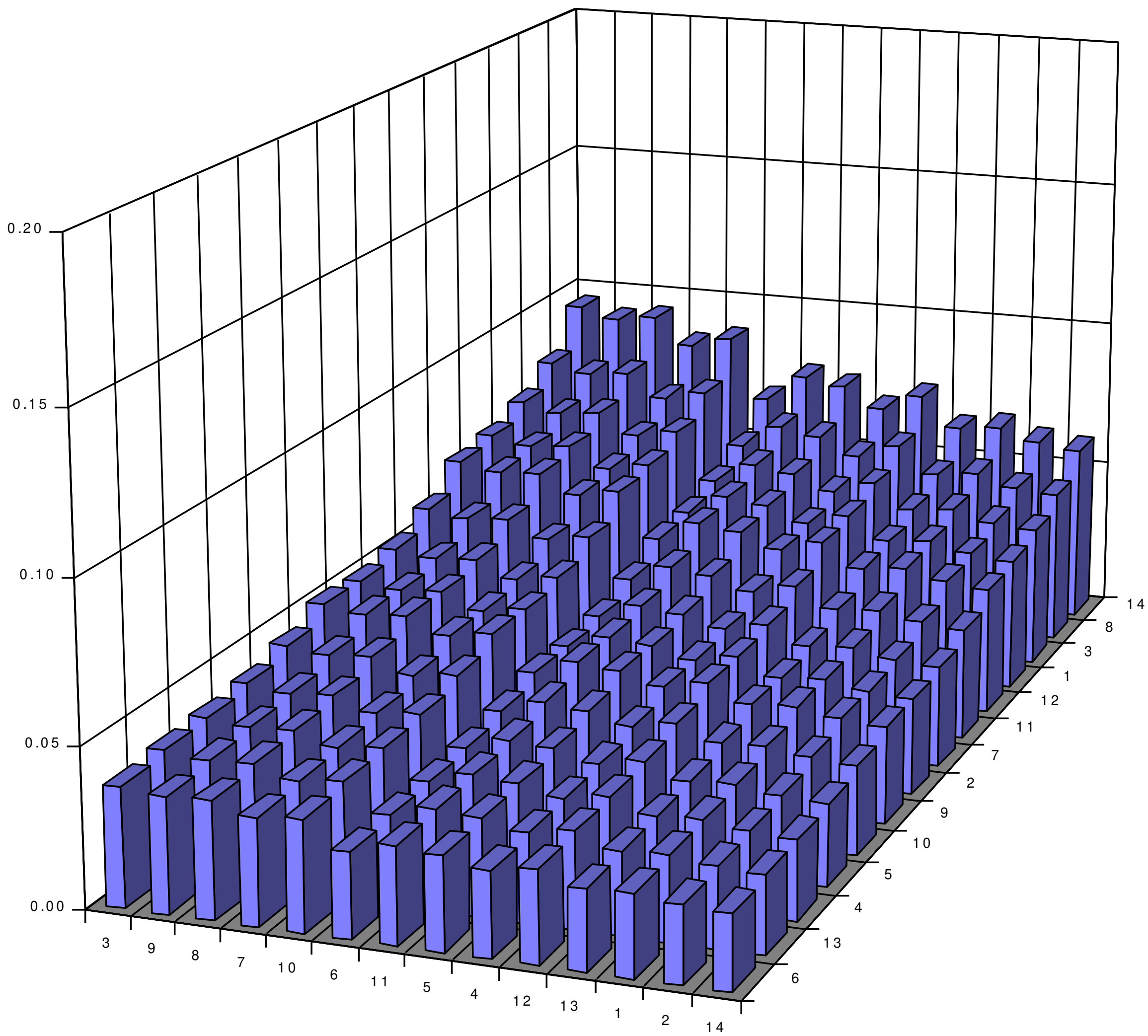
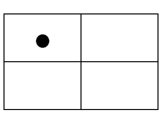
Figure 4

Landscape of the Northeast Region





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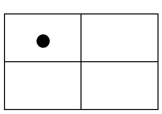

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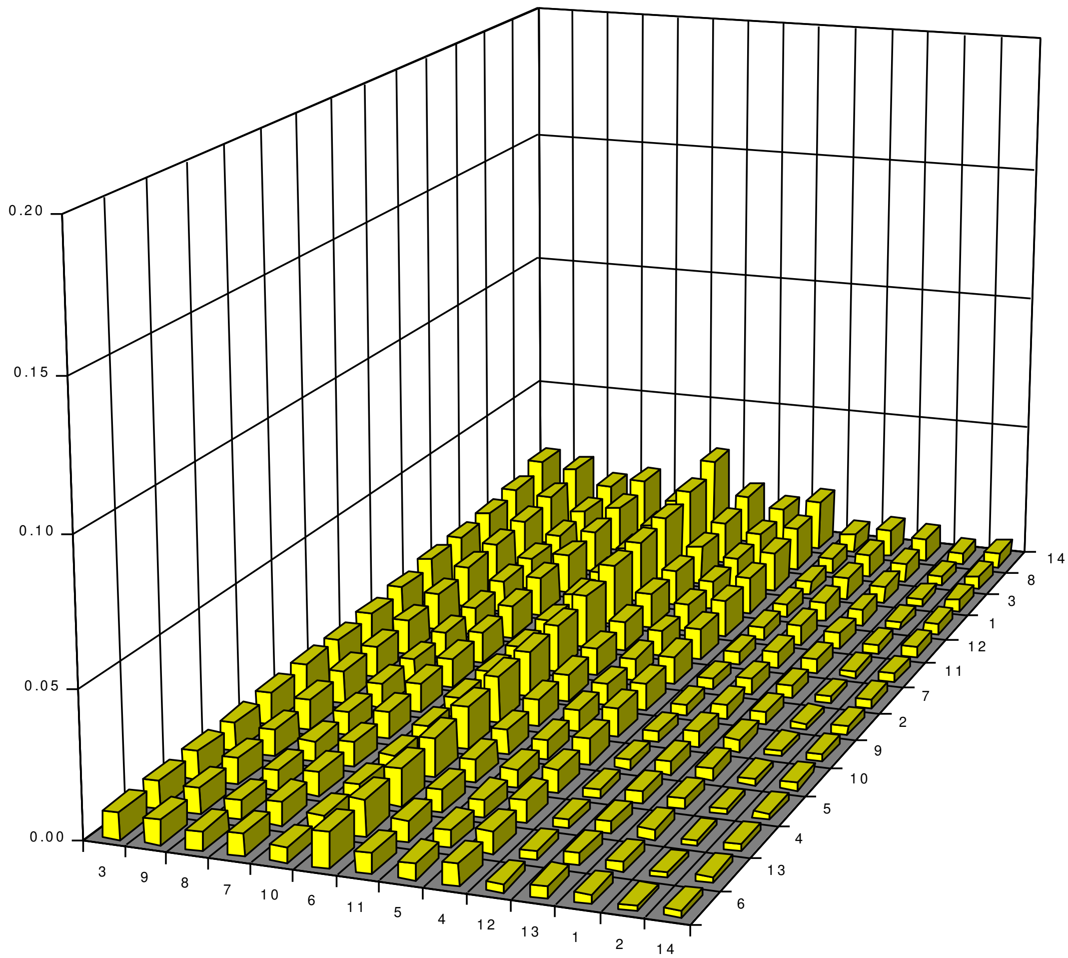
Figure 5

Contribution of Inputs Within the Northeast Region
to the Northeast Region Landscape



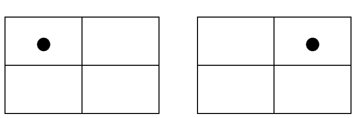


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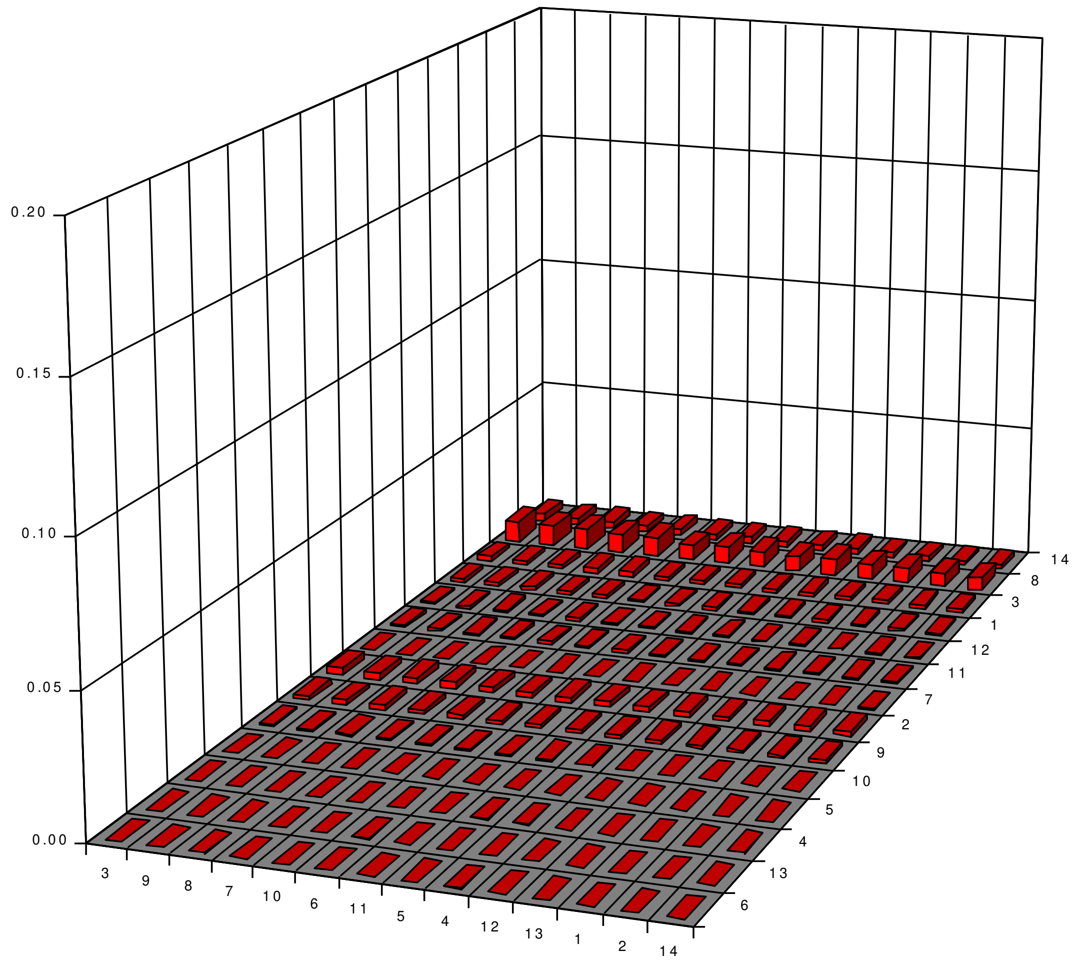
Figure 6

Contribution of Regional Forward Linkages
to the Northeast Region Landscape



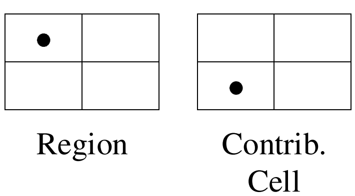
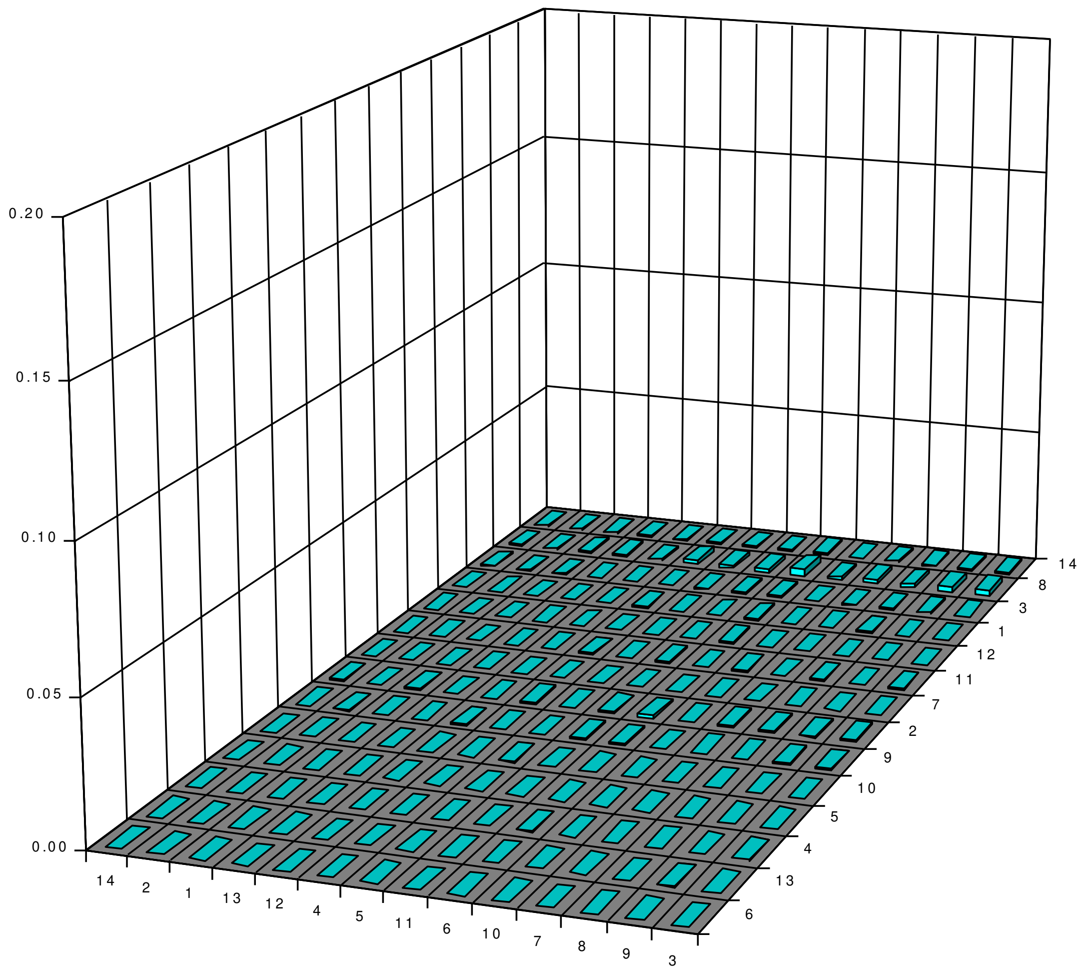


Figure 7

Contribution of Regional Backward Linkages
to the Northeast Region Landscape



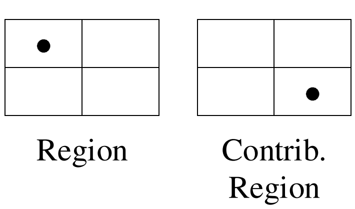
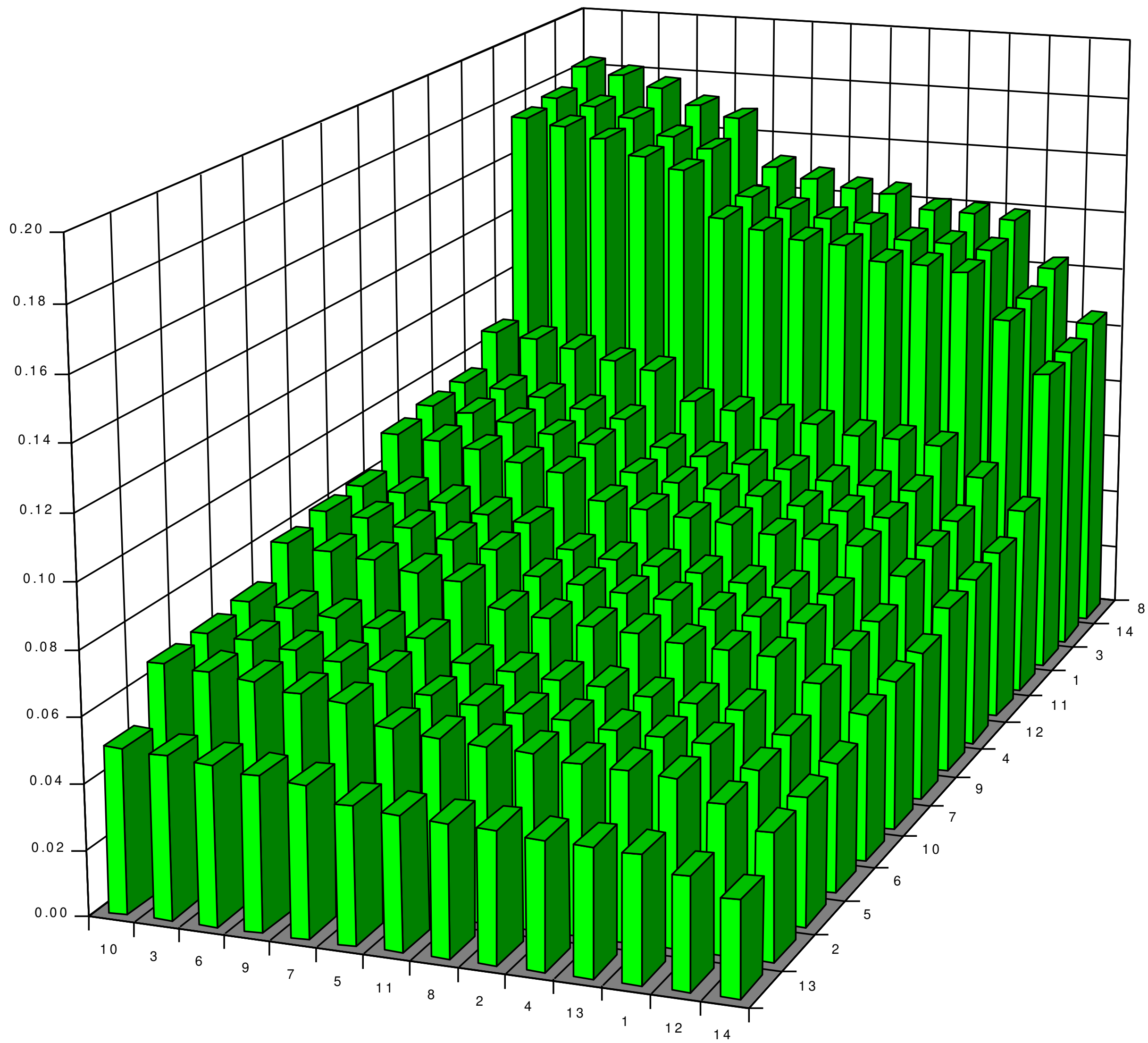


Figure 8

Contribution of Inputs Within the Rest of Brazil Region to the Northeast Region Landscape



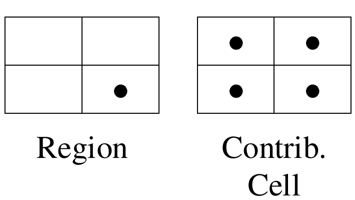
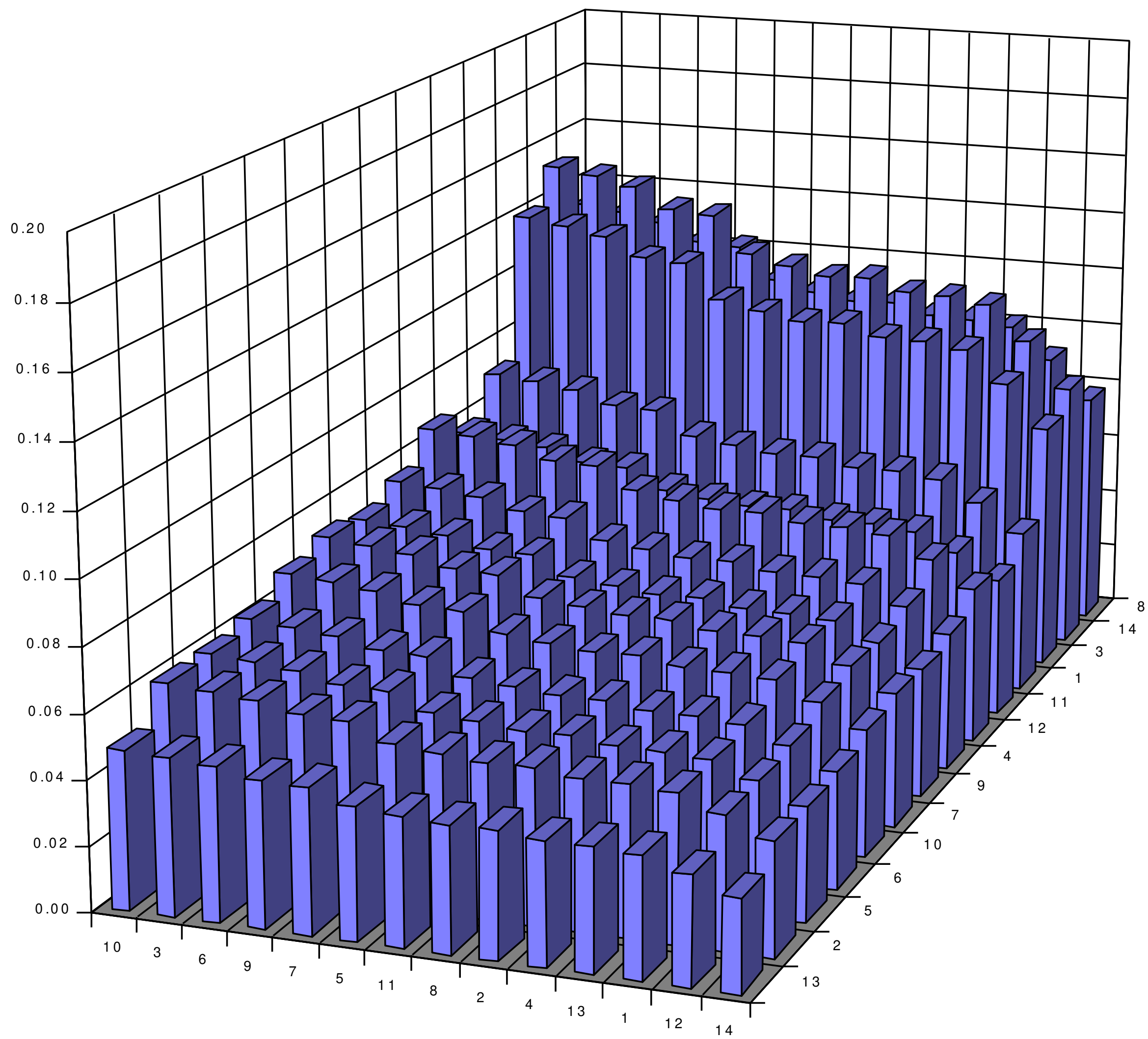


Figure 9

Landscape of the Rest of Brazil Region



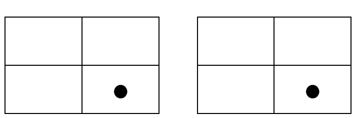


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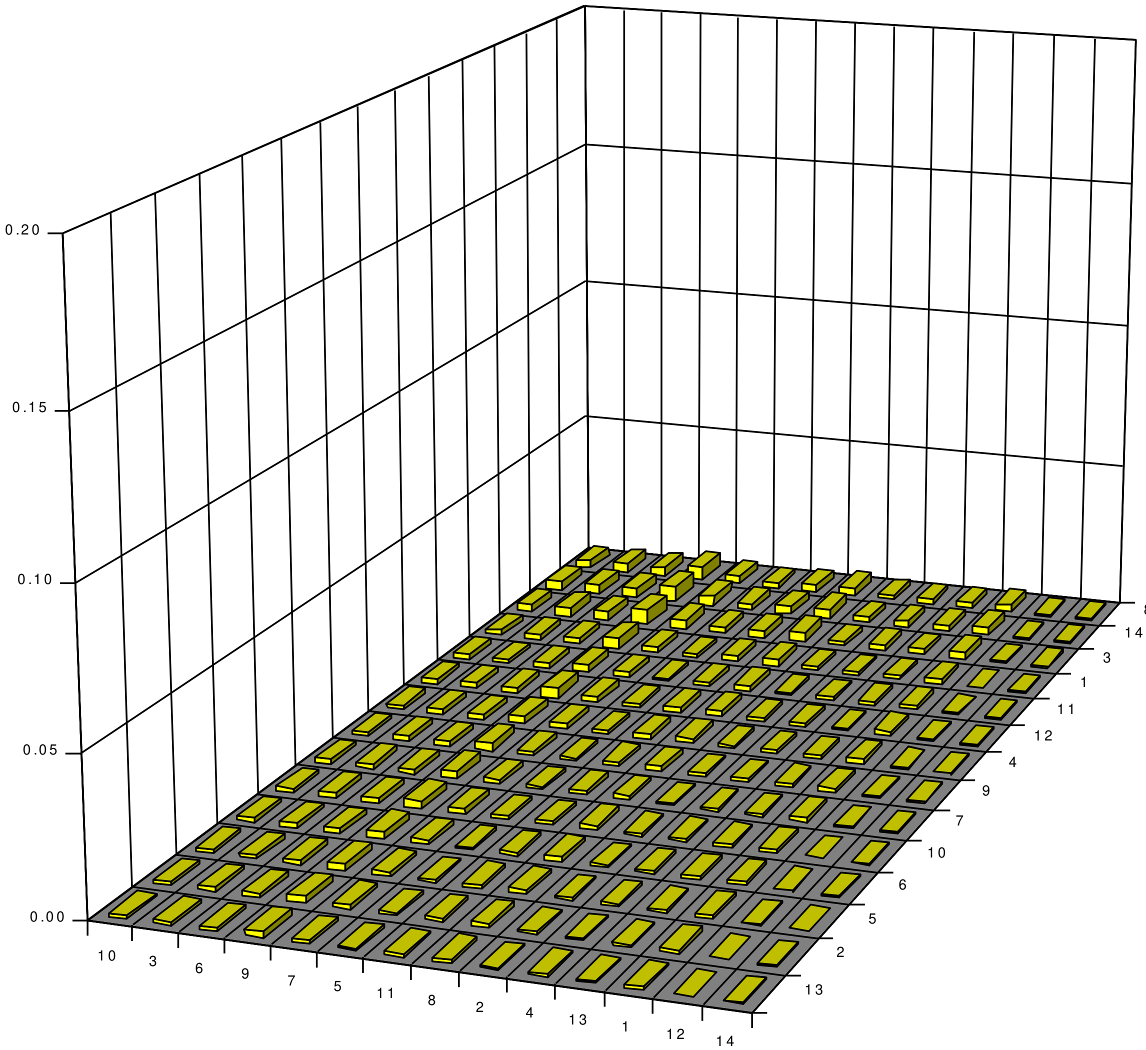
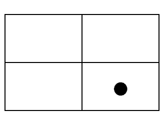
Figure 10

Contribution of Inputs Within the Rest of Brazil Region to the Rest of Brazil Region Landscape





Region

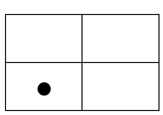


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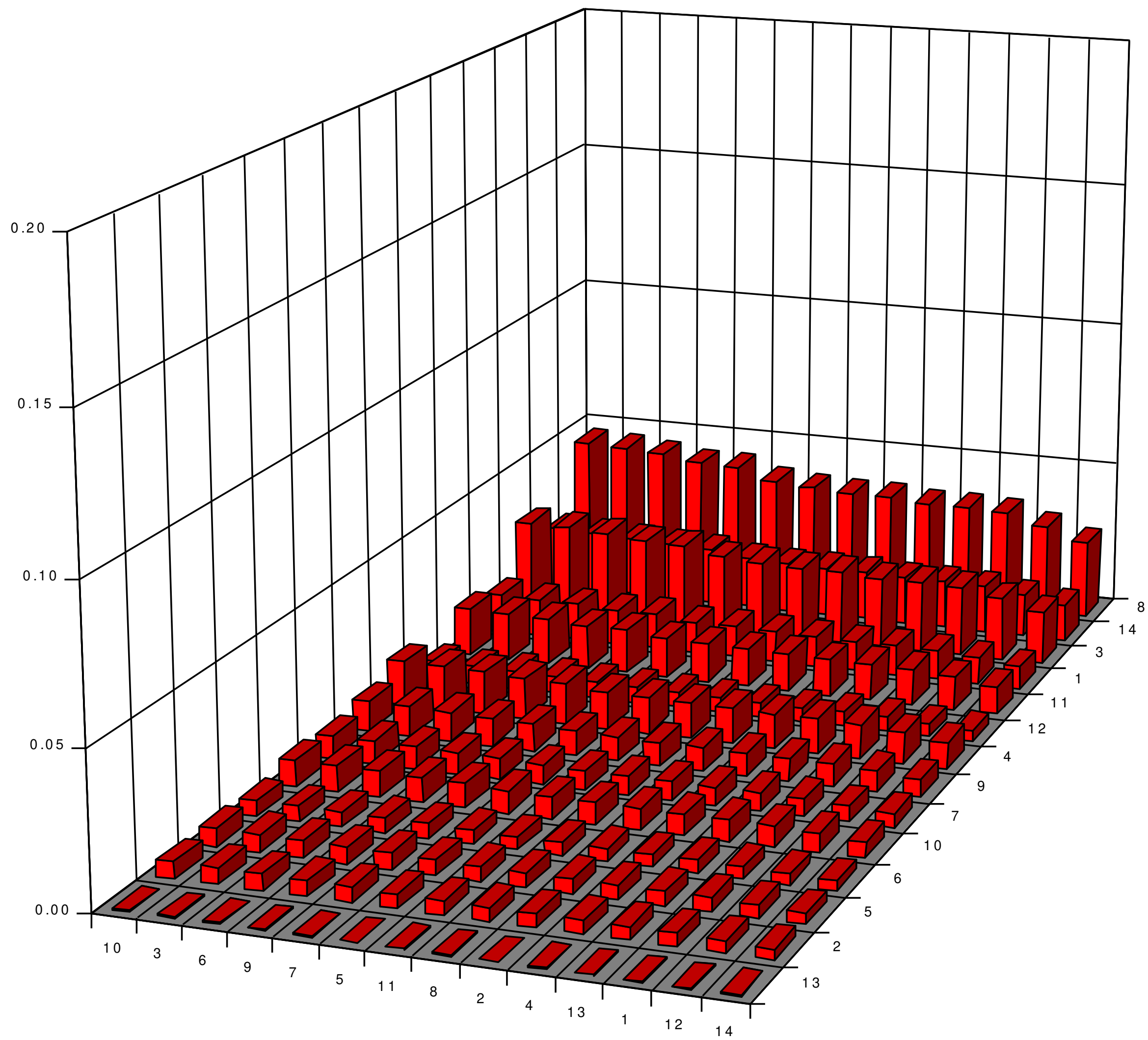
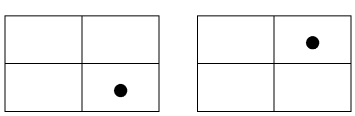
Figure 11

Contribution of Regional Forward Linkages
to the Rest of Brazil Region Landscape





Region

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Figure 12

Contribution of Regional Backward Linkages to the Rest of Brazil Region Landscape

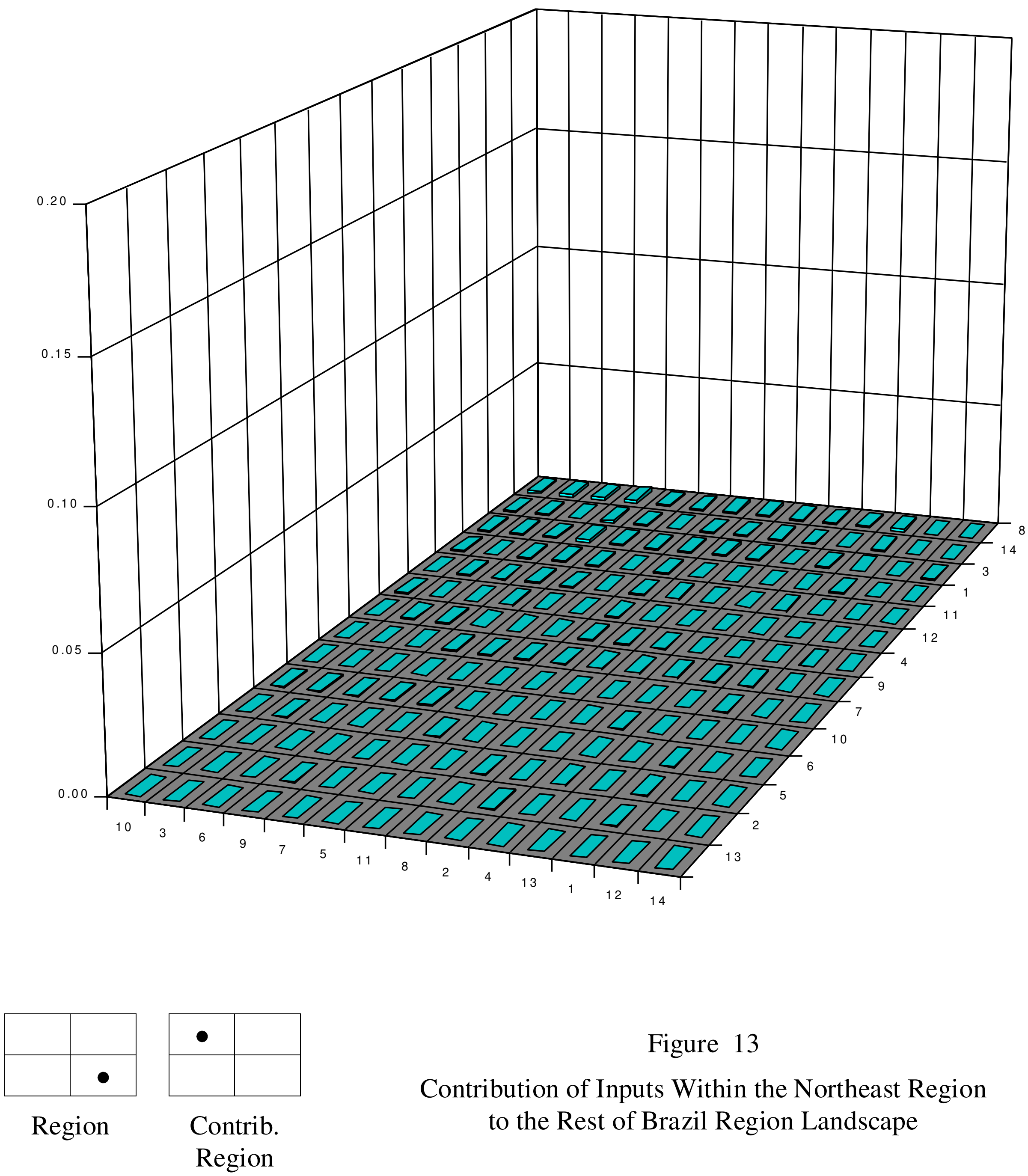

Figure 13

Contribution of Inputs Within the Northeast Region to the Rest of Brazil Region Landscape