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Annual Summary Report
for
PHASE V: STRESS ANALYSIS OF A DOUBLY-CURVED SKIN WITH A FLARED NOZZLE PORT

Contract NAS 8-2698
(RAC 1452-6)

Submitted to
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
George C. Marshall Space Flight Center
Huntsville, Alabama

REPUBLIC AVIATION CORPORATION
Farmingdale, L. I., N. Y. 11735

FOREWORD

This report was prepared by Drs. I. U. Ojalvo, M. Newman, M. Goldberg and Mr. N. Levine of Republic Aviation Corporation, Farmingdale, New York, under Contract NAS 8-2698, "Stress Analysis of a Doubly-Curved Skin with a Flared Nozzle Port."

The work was administered under the direction of Mr. David Hoppers of the Manufacturing Engineering Laboratory through Mr. Norman Schlemmer of the Propulsion and Vehicle Engineering Laboratory of the George C. Marshall Space Flight Center.

The Republic Program Manager is Dr. R. S. Levy.

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SUMMARY

26047

An analysis to determine the stresses and deflections caused by comparatively small flared nozzle openings in large pressure vessels and an associated digital computer program are described. The structural problem is idealized as a shell of revolution with axis normal to a thin, shallow, parent shell which is not axisymmetrical. The midsurfaces of the two shells are assumed to mate at a common intersecting circle and the entire configuration is subjected to internal pressurization and membrane edge forces.

Author

TABLE OF CONTENTS

SECTION I
INTRODUCTION

The evaluation of the magnitude of stresses in the neighborhood of nozzle ports is a technically important problem in the design of pressurized fuel tanks. Concurrent with stress concentration effects, resulting from geometric discontinuities, the nozzles serve as localized loading points which can transmit both thrusts and bending moments to the primary shell. Problems of this nature occur in the elliptical end closures of large boosters of the Saturn type, as well as with pressure vessels in the form of, spheres, cylinders, and tori (see Figure 1).

Recognizing a need for efficient nozzle fabrication techniques, Republic Aviation Corporation has recently developed a method of drawing flared nozzles from curved thin shells Reference [1]. Using the developed technique, the specific location of the nozzle drawing axis and the angle made with the local shell normal are quite arbitrary, thus enhancing the possibilities of the technique for general and, in particular, space vehicle applications. A significant advantage of the nozzle-forming procedure, is that it eliminates the need for complex weldments along awkward nozzle-shell junctions.

Although shell-nozzle interaction problems are of practical importance, comparatively little analytical work has been published in this area. Notable contributions have been made by Bijlaard [2, 3, 4] and Galletly [5] in the analysis of discontinuity stresses adjacent to central circular opening in spherical caps. In addition, Conway and Lissa [6] have considered the oblique intersection of a rigid cylinder with a shallow spherical shell. However, questions regarding the influence of unequal radii of curvature in the pressure vessel, at nozzle junctions, have remained unanswered.

This document describes a general approximate procedure for determining the stresses and deflections in flared nozzles with axes normal to comparatively large pressure vessels, as well as the stress distribution in the parent shell near the nozzle junction. Expressions for these discontinuity stress are derived

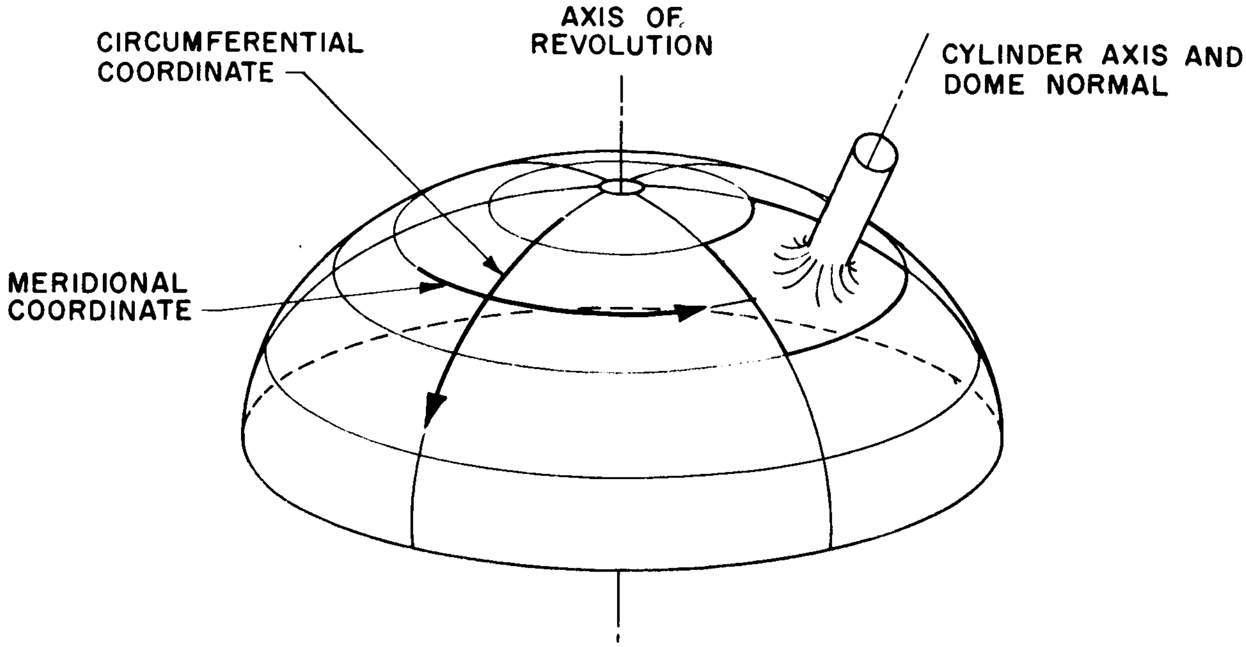
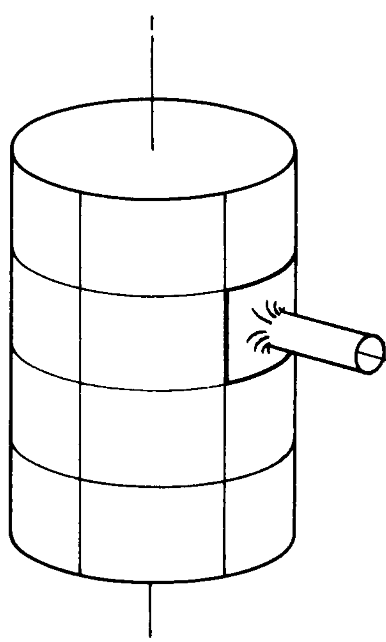


Figure 1a. Oblate Spheroid (Ellipsoid) Intersected by a Circular Cylinder With Axis Along a Surface Normal



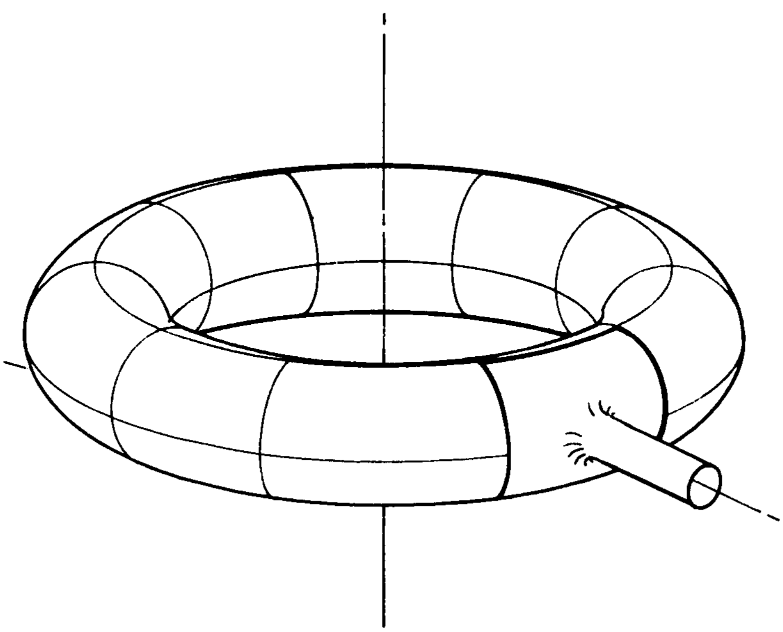


Figure 1b. Circular Cylinder Intersected Normally by a Second Circular Cylinder

Figure 1c. Circular Torus Intersected by a Circular Cylinder with Axis along a Surface Normal for an idealized model, which brings out the salient features of the problem.

It is assumed that the nozzle geometry, exclusive of the parent dome shell, can be approximated (see Figure 2) by a surface of revolution, consisting of a right circular cylinder joined to a bell-mouthed shell of variable thickness. The nozzle axis is taken normal to a thin, shallow, parent shell of double curvature which is idealized as a segment of an elliptic paraboloid. Because of the shallowness of the dome segment and the coincidence of nozzle axis and local shell normal, the curve describing the intersecting shells can be approximated as a circle. The entire system is subjected to internal pressure and it is assumed that membrane boundary conditions exist at the top of the cylinder and at the edges of the dome segment.

Sections II and III present the mathematical models for the cylinder-flare combination and dome, respectively. The analytical and numerical solution procedures associated with generating solution families for each are complete in their respective sections. Section IV deals with the joining of these two segments to form a composite shell by a point-matching [7, 8] scheme with least squares option.

The computer program, which numerically generates the appropriate solution families, joins the shell segments, and calculates the overall stresses and deformations, is described in Section V. A complete set of operating instructions is contained in the separate companion report entitled, "User's Manual for Stress Analysis of a Doubly-Curved Skin with a Flared Nozzle Port," Republic Aviation Corporation Report No. RAC 1452-7, dated 15 May 1965. Concurrent with the submission of this report, the computer program is being made available to the National Aeronautics and Space Adminstration, where it will be applied to the analysis of specific test problems.

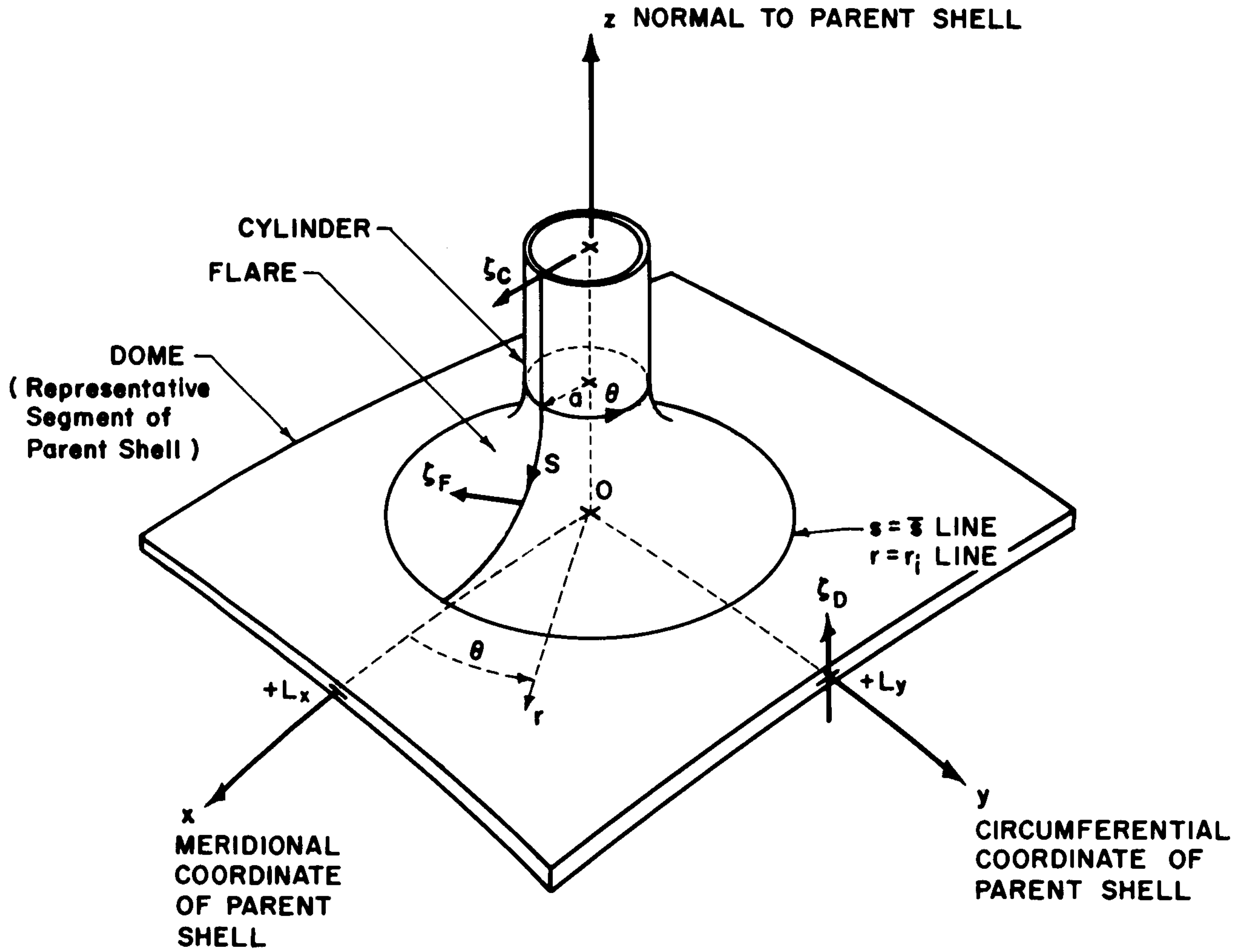


Figure 2. Idealization of a Flared-Nozzle as a Series of Shells

SECTION II
CYLINDER-FLARE ANALYSIS

A. INTRODUCTION

The following development describes the general numerical procedures, based on Flugge's linear elastic shell equations [9], for determining the stresses and deflections in the cylinder-flare portion of the composite shell. It is assumed that the nozzle geometry, exclusive of the parent dome shell, can be approximated by a surface of revolution, consisting of a circular cylinder joined to a bell mouthed shell of variable thickness. Because of the axisymmetric shape of the flare model, it is possible to expand all dependent variables into Fourier series in the circumferential direction such that the Fourier coefficients are functions only of the meridional coordinate. The shell equations then yield an uncoupled set of ordinary differential equations for the coefficients of each Fourier component. Finite difference forms of these equations are then solved by a modification of the matrix methods employed in Reference [10].

B. FORMULATION OF BASIC EQUATIONS

The flare consists of a shell of revolution (Figure 3) of variable wall thickness, t. The distance along the middle-surface meridional curve, measured from an edge of the shell is denoted by x; the normal distance from the axis to the curve by r(x). The functions $r_1(x)$ and $r_2(x)$ describe the meridional and hoop radii of curvature of the middle surface, respectively.

The exact equilibrium equations of a differential shell element in terms of force and moment resultants are given by [9]

$$(r N_\phi)^{\cdot} + r_1 N'_{\theta\phi} - r_1 N_\theta \cos\phi - rQ_\phi = - rr_1\, p_\phi \qquad (1a)$$

$$(rN_{\phi\theta})^{\cdot} + r_1 N'_\theta + r_1 N_{\theta\phi} \cos\phi - r_1 Q_\theta \sin\phi = - rr_1\, p_\theta \qquad (1b)$$

$$r_1\, N_\theta \sin\phi + r\, N_\phi + r_1\, Q'_\theta + (r\, Q_\phi)^{\cdot} = r\, r_1\, p \tag{1c}$$

$$(r\, M_\phi)^{\cdot} + r_1\, M'_{\theta\phi} - r_1\, M_\theta \cos\phi = r\, r_1\, Q_\phi \tag{1d}$$

$$(r\, M_{\phi\theta})^{\cdot} + r_1\, M'_\theta + r_1\, M_{\theta\phi} \cos\phi = r\, r_1\, Q_\theta \tag{1e}$$

$$\frac{M_{\phi\theta}}{r_1} - \frac{M_{\theta\phi}}{r_2} = N_{\phi\theta} - N_{\theta\phi} \ , \tag{1f}$$

where $\quad (\ )^{\cdot} = \frac{\partial}{\partial\phi}(\ )$

$\qquad\quad (\ )' = \frac{\partial}{\partial\theta}(\ )$,

p, p_ϕ and p_θ are the shell loads per unit of middle surface area in the ζ, ϕ and θ directions, respectively (Figure 3), and

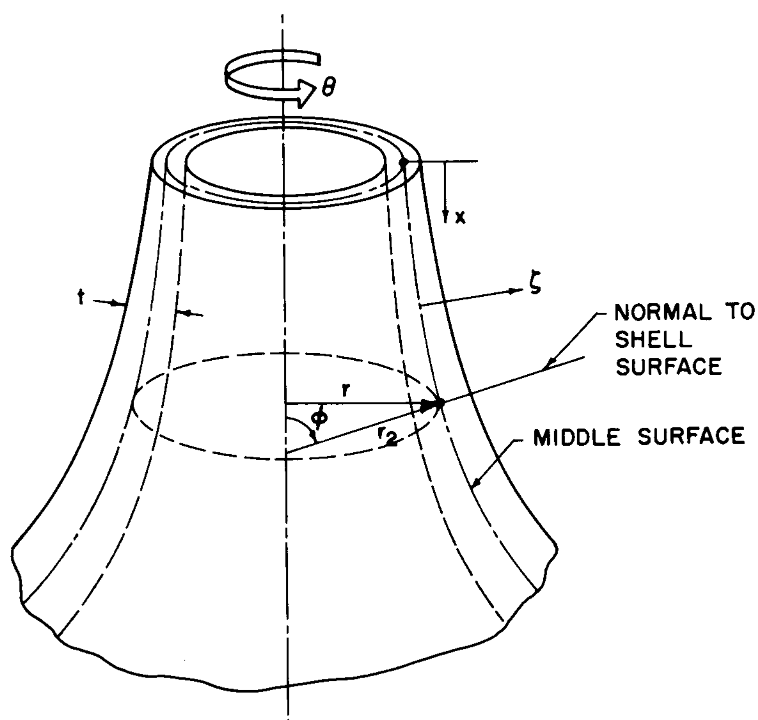


Figure 3. Flare Geometry

$$N_\phi = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\phi\phi} \frac{r_2 + \zeta}{r_2} \, d\zeta \tag{2a}$$

$$N_\theta = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\theta\theta} \frac{r_1 + \zeta}{r_1} \, d\zeta \tag{2b}$$

$$N_{\theta\phi} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\theta\phi} \frac{r_1 + \zeta}{r_1} \, d\zeta \tag{2c}$$

$$N_{\phi\theta} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\phi\theta} \frac{r_2 + \zeta}{r_2} \, d\zeta \tag{2d}$$

$$M_\phi = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\phi\phi} \frac{r_2 + \zeta}{r_2} \, \zeta \, d\zeta \tag{2e}$$

$$M_\theta = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\theta\theta} \frac{r_1 + \zeta}{r_1} \, \zeta \, d\zeta \tag{2f}$$

$$M_{\theta\phi} = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\theta\phi} \frac{r_1 + \zeta}{r_1} \, \zeta \, d\zeta \tag{2g}$$

$$M_{\phi\theta} = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\phi\theta} \frac{r_2 + \zeta}{r_2} \, \zeta \, d\zeta \tag{2h}$$

$$Q_\phi = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\phi\zeta} \frac{r_2 + \zeta}{r_2} \, d\zeta \tag{2i}$$

$$Q_\theta = - \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_{\theta\zeta} \frac{r_1 + \zeta}{r_1} \, d\zeta \tag{2j}$$

The stress components σ_{ij} $\begin{pmatrix} i = \phi, \theta \\ j = \phi, \theta, \zeta \end{pmatrix}$ are defined in accordance with conventional theory of elasticity notation. As a consequence, the force and moment resultants defined by equations (1) and (2) are positive when acting as shown in Figures 2c through 2e.

Equation (1f) is an identity which follows from the definitions of the force and moment resultants given by equations (2c), (2d), (2g), and (2h). Equations (1a) through (1e), therefore, constitute five independent equations involving ten resultants. The required connecting equations are provided by the constitutive relations [9]:

$$N_\phi = D\left[\frac{v^\cdot + w}{r_1} + \nu\,\frac{u' + v\cos\phi + w\sin\phi}{r}\right] + \frac{K}{r_1^2}\,\frac{r_2 - r_1}{r_2}\left[\frac{v - w^\cdot}{r_1}\,\frac{r_1^\cdot}{r_1} + \frac{w^{\cdot\cdot} + w}{r_1}\right], \tag{3a}$$

$$N_\theta = D\left[\frac{u' + v\cos\phi + w\sin\phi}{r} + \nu\,\frac{v^\cdot + w}{r_1}\right] - \frac{K}{rr_1}\,\frac{r_2 - r_1}{r_2}\left[-\frac{v}{r_1}\,\frac{r_2 - r_1}{r_2}\cos\phi + \frac{w\sin\phi}{r_2} + \frac{w''}{r} + \frac{w^\cdot\cos\phi}{r_1}\right], \tag{3b}$$

$$N_{\phi\theta} = D\,\frac{1-\nu}{2}\left[\frac{u^\cdot}{r_1} + \frac{v' - u\cos\phi}{r}\right] + \frac{K}{r_1^2}\,\frac{1-\nu}{2}\,\frac{r_2 - r_1}{r_2}\left[\frac{u^\cdot}{r_1}\,\frac{r_2 - r_1}{r_2} + \frac{u}{r_2}\,\frac{r_1 - r_2}{r_2}\cot\phi + \frac{w'^\cdot}{r} - \frac{w'}{r}\,\frac{r_1}{r}\cos\phi\right], \tag{3c}$$

$$N_{\theta\phi} = D\,\frac{1-\nu}{2}\left[\frac{u^\cdot}{r_1} + \frac{v' - u\cos\phi}{r}\right] + \frac{K}{rr_1}\,\frac{1-\nu}{2}\,\frac{r_2 - r_1}{r_2}\left[\frac{v'}{r_1}\,\frac{r_2 - r_1}{r_2} - \frac{w'^\cdot}{r_1} + \frac{w'\cos\phi}{r}\right], \tag{3d}$$

$$M_\phi = K\left[\frac{1}{r_1^2}\left(w^{\cdot\cdot} - w^\cdot\,\frac{r_1^\cdot}{r_1} - w\,\frac{r_1 - r_2}{r_2}\right) - \frac{v^\cdot}{r_1 r_2} + \frac{v}{r_1^2}\,\frac{r_1^\cdot}{r_1} + \nu\,\frac{w''}{r^2} + \nu\,\frac{w^\cdot\cos\phi}{rr_1} - \nu\,\frac{u'}{rr_2} - \nu\,\frac{v\cos\phi}{rr_1}\right], \tag{3e}$$

$$M_\theta = K\left[\frac{w''}{r^2} + \frac{w^\bullet \cos\phi}{r\,r_1} - \frac{w}{r_2^2}\,\frac{r_2 - r_1}{r_1} - \frac{u'}{r\,r_1} - \frac{v\cos\phi}{r\,r_1}\,\frac{2r_2 - r_1}{r_2}\right.$$

$$\left. + \frac{\nu}{r_1^2}\left(w^{\bullet\bullet} - w^\bullet \frac{r_1^\bullet}{r_1}\right) - \frac{\nu}{r_1^2}\left(v^\bullet - v\frac{r_1^\bullet}{r_1}\right)\right], \qquad \text{(3f)}$$

$$M_{\phi\theta} = K\,\frac{1-\nu}{2}\left[2\,\frac{w'^\bullet}{r\,r_1} - 2\,\frac{w'}{r^2}\cos\phi - \frac{u^\bullet}{r_1 r_2}\,\frac{2r_1 - r_2}{r_1}\right.$$

$$\left. + \frac{u}{r_2^2}\,\frac{2r_1 - r_2}{r_1}\cot\phi - \frac{v'}{r\,r_1}\right], \qquad \text{(3g)}$$

$$M_{\theta\phi} = K\,\frac{1-\nu}{2}\left[2\,\frac{w'^\bullet}{r\,r_1} - 2\,\frac{w'}{r^2}\cos\phi - \frac{u^\bullet}{r_1 r_2} + \frac{u}{r_2^2}\cot\phi - \frac{v'}{r\,r_1}\,\frac{2r_2 - r_1}{r_2}\right] \qquad \text{(3h)}$$

where u,v,w are the middle surface displacements in the circumferential, meridional and normal directions, respectively (see Figure 4a), and

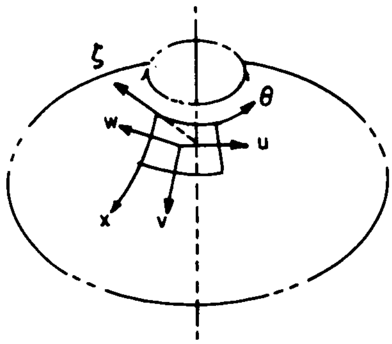
$$D = \frac{Et}{1-\nu^2} \qquad \text{(4a)}$$

$$K = \frac{Et^3}{12(1-\nu^2)} \qquad \text{(4b)}$$

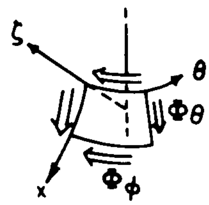
Equations (3) are based on the Kirchoff-Love hypothesis, which assumes the preservation of normals to the middle surface during deformation, and neglects direct stresses ($\sigma_{\zeta\zeta}$). Equations (3), however, are based upon the retention of thickness-curvature terms up to the order of $\left(\frac{t}{r}\right)^3$ and $\left(\frac{t}{r}\right)^3$ in the strain-displacement relations and stress resultant expressions, whereas, in the usual thin shell theory, founded on Love's first approximation [11], first powers of these quantities are neglected in comparison with unity.

The set of field equations given by (1a) through (1e) and (3a) through (3h) constitute thirteen independent relations in the thirteen field quantities N_ϕ, N_θ, $N_{\theta\phi}$, $N_{\phi\theta}$, M_ϕ, M_θ, $M_{\phi\theta}$, $M_{\theta\phi}$, Q_ϕ, Q_θ, u, v and w. These quantities and the surface loads are now expanded into Fourier series as follows:

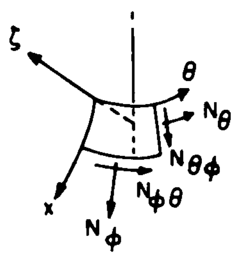
* NOTE:
USE RIGHT-HAND RULE FOR MOMENT AND ROTATION VECTORS



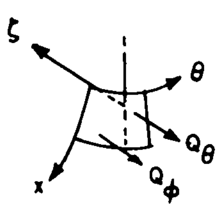
a. Displacements



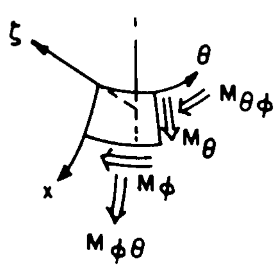
b. Rotations *



c. Tangential Force Resultants



d. Transverse Force Resultants



e. Moments *

Figure 4. Sign Conventions

$$\left\{\begin{matrix} N_\phi \\ N_\theta \end{matrix}\right\} = \sigma_c\, t_c \sum_{n=0}^{\infty} \left\{\begin{matrix} t_s^n \\ t_\theta^n \end{matrix}\right\} \cos n\theta$$

$$\left\{\begin{matrix} N_{\theta\phi} \\ N_{\phi\theta} \end{matrix}\right\} = \sigma_c\, t_c \sum_{n=1}^{\infty} \left\{\begin{matrix} t_{\theta s}^n \\ t_{s\theta}^n \end{matrix}\right\} \sin n\theta$$

$$\left\{\begin{matrix} M_\phi \\ M_\theta \end{matrix}\right\} = \frac{\sigma_c t_c^3}{r_c} \sum_{n=0}^{\infty} \left\{\begin{matrix} m_s^n \\ m_\theta^n \end{matrix}\right\} \cos n\theta$$

$$\left\{\begin{matrix} M_{\theta\phi} \\ M_{\phi\theta} \end{matrix}\right\} = \frac{\sigma_c t_c^3}{r_c} \sum_{n=1}^{\infty} \left\{\begin{matrix} m_{\theta s}^n \\ m_{s\theta}^n \end{matrix}\right\} \sin n\theta \qquad (5)$$

$$Q_\phi = \sigma_c\, t_c \sum_{n=0}^{\infty} f_s^n \cos n\theta$$

$$Q_\theta = \sigma_c t_c \sum_{n=1}^{\infty} f_\theta^n \sin n\theta$$

$$u = \frac{r_c \sigma_c}{E} \sum_{n=1}^{\infty} u_\theta^n \sin n\theta$$

$$\left\{\begin{matrix} v \\ w \end{matrix}\right\} = \frac{r_c \sigma_c}{E} \sum_{n=0}^{\infty} \left\{\begin{matrix} u_s^n \\ w^n \end{matrix}\right\} \cos n\theta$$

$$\left.\begin{matrix} \left\{\begin{matrix} p \\ p_\phi \end{matrix}\right\} = \dfrac{\sigma_c t_c}{r_c} \displaystyle\sum_{n=0}^{\infty} \left\{\begin{matrix} p^n \\ p_s^n \end{matrix}\right\} \cos n\theta \\ \\ p_\theta = \dfrac{\sigma_c t_c}{r_c} \displaystyle\sum_{n=1}^{\infty} p_\theta^n \sin n\theta \end{matrix}\right\} \qquad (6)$$

The quantities σ_c, t_c and r_c represent a reference stress level, thickness and radial dimension of the shell, respectively, which tend to normalize the dimensionless Fourier coefficients t_s^n, t_θ^n, ..., w^n. The expansions (5) and (6) and the development which follows are valid for loads and boundary conditions which produce symmetric deformations about the plane $\theta = 0$. Antisymmetric problems can be accommodated by interchanging the roles of sin nθ and cos nθ in Equations (5) and (6). Solutions for general asymmetric loading conditions can then be obtained by decomposing the complete problem into symmetric and antisymmetric components.

If Equations (5) and (6) are substituted into (1) and (3) the equations for the Fourier coefficients decouple into separate sets for each Fourier index n. Each set constitutes an eighth order system of ordinary differential equations with the meridional coordinate x as independent variable. To facilitate subsequent numerical computations, it is convenient to express the n^{th} general system as four second order differential equations in the Fourier coefficients u_s^n, u_θ^n, w^n and m_s^n. This is accomplished in the following manner: Solve Equations (1d) through (1f) for Q_ϕ, Q_θ and $N_{\theta\phi}$ and substitute the resulting expressions into Equations (1a) through (1c). Next, eliminate terms (3a) and (3f) by making use of (3e). Substituting (5) and (6) into the resulting equations and considering only normal pressure loading then yields the following for the n^{th} set of Fourier coefficients*:

$$\left.\begin{aligned}
&\rho\left[\gamma t_s + t_s' - \gamma t_\theta\right] + n\, t_{s\theta} - \lambda^2\left[\rho\,\omega_s\, m_s' \right.\\
&\quad + \rho\gamma\,\omega_s\,(m_s - m_\theta) + n\,(\omega_s - \omega_\theta)\, m_{\theta s}\\
&\quad \left. + n\omega_s\, m_{s\theta}\right] = 0\\
&\rho\left[\gamma t_{s\theta} + t_{s\theta}'\right] - n\, t_\theta - \lambda^2\left[\rho\gamma(\omega_s + \omega_\theta)\, m_{s\theta}\right.\\
&\quad \left. + \rho\,\omega_\theta\, m_{s\theta}' - n\omega_\theta\, m_\theta\right] = 0
\end{aligned}\right\} \qquad (7)$$

---

* For convenience the superscript n on the Fourier coefficients will be omitted in the following equations.

$$
\left.
\begin{aligned}
& - \omega_s t_s - \omega_\theta t_\theta - \lambda^2 \Big[m_s'' + 2\gamma m_s' - \omega_s \omega_\theta m_s \\
& \quad - \gamma m_\theta' + (\omega_s \omega_\theta - \frac{n^2}{\rho^2}) m_\theta + \frac{n}{\rho} (m_{s\theta}' + m_{\theta s}') \\
& \quad + \frac{n\gamma}{\rho} (m_{s\theta} + m_{\theta s}) \Big] = - p^n \\
& d \Big\{ w'' + \nu \gamma w' + \Big[\omega_s (\omega_s - \omega_\theta) - \frac{\nu n^2}{\rho^2} \Big] w - \omega_\theta u_s' \\
& \quad - (\omega_s' + \nu \gamma \omega_s) u_s - \frac{\nu n}{\rho} \omega_\theta u_\theta \Big\} - m_s = 0
\end{aligned}
\right\} \quad (7 \text{ cont'd})
$$

$$
\left.
\begin{aligned}
t_s &= b \Big[u_s' + (\omega_s + \nu \omega_\theta) w + \frac{\nu n}{\rho} u_\theta + \nu \gamma u_s \Big] + \lambda^2 (\omega_s - \omega_\theta) m_s \\
& \quad + d \lambda^2 (\omega_s - \omega_\theta) \Big[(\omega_s \omega_\theta + \frac{\nu n^2}{\rho^2}) w + \omega_\theta u_s' - \nu \gamma w' \\
& \qquad + \nu \omega_\theta \frac{n}{\rho} u_\theta + \nu \gamma \omega_s u_s \Big] \\
t_\theta &= b \Big[\frac{n}{\rho} u_\theta + \gamma u_s + \omega_\theta w + \nu u_s' + \nu \omega_s w \Big] \\
& \quad - d \lambda^2 (\omega_s - \omega_\theta) \Big[\gamma (\omega_\theta - \omega_s) u_s + (\omega_\theta^2 - \frac{n^2}{\rho^2}) w + \gamma w' \Big] \\
t_{s\theta} &= b \left(\frac{1-\nu}{2} \right) \Big[u_\theta' - \frac{n}{\rho} u_s - \gamma u_\theta \Big] \\
& \quad + d \lambda^2 \left(\frac{1-\nu}{2} \right) \left(\omega_s - \omega_\theta \right) \Big[(\omega_s - \omega_\theta) u_\theta' + \gamma (\omega_\theta - \omega_s) u_\theta \\
& \qquad - \frac{n}{\rho} w' + \frac{\gamma n}{\rho} w \Big] \\
m_\theta &= \nu m_s - d \Big\{ \Big[\omega_\theta (\omega_s - \omega_\theta) - \nu \omega_s (\omega_\theta - \omega_s) + (1-\nu^2) \frac{n^2}{\rho^2} \Big] w \\
& \qquad + \Big[\gamma (2\omega_s - \omega_\theta) - \nu^2 \gamma \omega_s \Big] u_s + \frac{n}{\rho} (\omega_s - \nu^2 \omega_\theta) u_\theta \\
& \qquad - \gamma (1-\nu^2) w' + \nu (\omega_s - \omega_\theta) u_s' \Big\}
\end{aligned}
\right\} \quad (8)
$$

$$\left.\begin{aligned} m_{\theta s} &= d\,\frac{(1-\nu)}{2}\Big[-\frac{2n}{\rho}\,w' + \frac{2n\gamma}{\rho}\,w - \omega_\theta u'_\theta + \gamma\,\omega_\theta\,u_\theta \\ &\qquad + \frac{n}{\rho}\,(2\omega_s - \omega_\theta)\;u_s\Big] \\ m_{s\theta} &= d\,\frac{(1-\nu)}{2}\Big[-\frac{2n}{\rho}\,w' + \frac{2n\gamma}{\rho}\,w - (2\omega_\theta - \omega_s)\,u'_\theta + \gamma\,(2\omega_\theta - \omega_s)\,u_\theta \\ &\qquad + \frac{n}{\rho}\,\omega_s\,u_s\Big] \end{aligned}\right\} \quad \text{(8 cont'd)}$$

The following dimensionless quantities and geometric identities have been employed in deriving Equations (7) and (8):

$$\begin{aligned} s &= \frac{x}{r_c} \\ \lambda &= \frac{t_c}{r_c} \\ d &= \frac{(t/t_c)^3}{12(1-\nu^2)} \\ b &= \frac{t/t_c}{1-\nu^2} \end{aligned} \qquad (9)$$

$$\begin{aligned} \omega_s &= \frac{r_c}{r_1} \\ \omega_\theta &= \frac{r_c}{r_2} \\ \rho &= \frac{r}{r_c} \end{aligned} \qquad (10)$$

$$\begin{aligned}
\sin\phi &= \rho\,\omega_\theta \\
\cos\phi &= \rho\gamma \\
\frac{\partial}{\partial\phi} &= \frac{1}{\omega_s}\frac{\partial}{\partial s} = \frac{1}{\omega_s}(\ )' \\
\frac{\partial}{\partial s} &= (\ )' \\
\omega_\theta' &= \gamma(\omega_s - \omega_\theta) \\
\rho' &= \rho\gamma \\
\gamma' &= -(\gamma^2 + \omega_s\,\omega_\theta)
\end{aligned} \tag{11}$$

The symbol $(\ )'$, which previously indicated differentiation with respect to θ is now employed to denote differentiation with respect to s in the Fourier component Equations (7) and (8). No confusion should arise since θ has been eliminated from the latter equations by Fourier decomposition.

When Equations (8) are substituted into (7), the resulting four second order differential equations are of the form:

$$\begin{aligned}
& a_1u_s'' + a_2u_s' + a_3u_s + a_4u_\theta' + a_5u_\theta \\
& + a_{5'}\,w'' + a_6\,w' + a_7\,w + a_8\,m_s' + a_9\,m_s = 0 \\[1ex]
& a_{10}u_s' + a_{11}u_s + a_{12}u_\theta'' + a_{13}u_\theta' + a_{14}u_\theta \\
& + a_{15}\,w'' + a_{16}\,w' + a_{17}\,w + a_{18}\,m_s = 0 \\[1ex]
& a_{19'}u_s'' + a_{19}u_s' + a_{20}u_s + a_{21}u_\theta'' + a_{22}u_\theta' \\
& + a_{23}u_\theta + a_{24}w'' + a_{25}w' + a_{26}w + a_{27}\,m_s'' \\
& + a_{28}m_s' + a_{29}\,m_s = -p \\[1ex]
& a_{30}u_s' + a_{31}u_s + a_{32}u_\theta + a_{33}\,w'' + a_{34}\,w' \\
& + a_{35}\,w + a_{36}\,m_s = 0
\end{aligned} \tag{12}$$

where the a's are listed in Appendix I.

Equations (12) can be conveniently written in the following matrix form:

$$E\,Z'' + F\,Z' + G\,Z = e \tag{13}$$

where Z is the solution vector

$$Z = \begin{Bmatrix} u_s \\ u_\theta \\ w \\ m_s \end{Bmatrix} \tag{14a}$$

and

$$E = \begin{bmatrix} a_1 & 0 & a_5' & 0 \\ 0 & a_{12} & a_{15} & 0 \\ a_{19}' & a_{21} & a_{24} & a_{27} \\ 0 & 0 & a_{33} & 0 \end{bmatrix} \tag{14b}$$

$$F = \begin{bmatrix} a_2 & a_4 & a_6 & a_8 \\ a_{10} & a_{13} & a_{16} & 0 \\ a_{19} & a_{22} & a_{25} & a_{28} \\ a_{30} & 0 & a_{34} & 0 \end{bmatrix} \tag{14c}$$

$$G = \begin{bmatrix} a_3 & a_5 & a_7 & a_9 \\ a_{11} & a_{14} & a_{17} & a_{18} \\ a_{20} & a_{23} & a_{26} & a_{29} \\ a_{31} & a_{32} & a_{35} & a_{36} \end{bmatrix} \tag{14d}$$

$$e = \begin{Bmatrix} 0 \\ 0 \\ -p \\ 0 \end{Bmatrix} \tag{14e}$$

C. BOUNDARY CONDITIONS

The eighth order system of equations describing deformations of the shell must be solved subject to four appropriately specified boundary conditions at the edges S = 0 and S = $\bar{S}$. In general, physically realizable edge conditions can be written in the following form:

$$K_{11}N_{\varphi} + K_{12}v = L_1 \tag{15a}$$

$$K_{21}\widehat{N}_{\varphi\theta} + K_{22}u = L_2 \tag{15b}$$

$$K_{31}\widehat{Q}_{\varphi} + K_{32}w = L_3 \tag{15c}$$

$$K_{41}\Phi_{\varphi} + K_{42}M_{\varphi} = L_4 \tag{15d}$$

Where $K_{11}, \ldots, K_{42}$ and $L_1, \ldots, L_4$ are specified constants and the quantities not previously defined are: (a) an effective membrane shear stress, resultant

$$\widehat{N}_{\varphi\theta} = N_{\varphi\theta} - \frac{M_{\varphi\theta}}{r_2}, \tag{16a}$$

(b) an effective transverse shear stress resultant

$$\widehat{Q}_{\varphi} = \frac{1}{rr_1}\left[\frac{\partial}{\partial\varphi}(rM_{\varphi}) + r_1\frac{\partial}{\partial\theta}(M_{\theta\varphi}) - r_1 M_{\theta}\cos\varphi\right] + \frac{1}{r}\frac{\partial M_{\varphi\theta}}{\partial\theta}, \tag{16b}$$

and

(c) the meridional slope change

$$\Phi_{\varphi} = \frac{1}{r_1}\left[\frac{\partial w}{\partial\varphi} - v\right], \tag{16c}$$

(see Figure 4b).

The Fourier coefficients corresponding to these additional quantities, i.e., $\hat{t}^{n}_{s\theta}$, $\hat{f}^{n}_{s}$ and $\bar{\varphi}^{n}_{s}$ are defined by the expansions

$$\begin{Bmatrix}\widehat{N}_{\varphi\theta}\\ \widehat{Q}_{\varphi}\end{Bmatrix} = \sigma_c t_c \sum_{n=0}^{\infty}\begin{Bmatrix}\hat{t}^{n}_{s\theta} & \sin n\theta\\ \hat{f}^{n}_{s} & \cos n\theta\end{Bmatrix}$$

$$\Phi_\varphi = \frac{\sigma_c}{E}\sum_{n=0}^{\infty} \bar{\varphi}_s^n \cos n\theta, \tag{17}$$

so that from (5) and (15) the boundary conditions for the n^{th} Fourier component can be written as

$$\Omega Y + \Lambda Z = \ell \tag{18}$$

where

$$\Omega = \begin{bmatrix} \lambda K_{11} & 0 & 0 & 0 \\ 0 & \lambda K_{21} & 0 & 0 \\ 0 & 0 & \lambda K_{31} & 0 \\ 0 & 0 & 0 & \dfrac{K_{41}}{E r_c} \end{bmatrix} \tag{19a}$$

$$\Lambda = \begin{bmatrix} \dfrac{K_{12}}{E} & 0 & 0 & 0 \\ 0 & \dfrac{K_{22}}{E} & 0 & 0 \\ 0 & 0 & \dfrac{K_{32}}{E} & 0 \\ 0 & 0 & 0 & \lambda^3 r_c K_{42} \end{bmatrix} \tag{19b}$$

$$\begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{Bmatrix} = \sigma_c r_c \sum_{n=0}^{\infty} \begin{Bmatrix} \ell_1^n \cos n\theta \\ \ell_2^n \sin n\theta \\ \ell_3^n \cos n\theta \\ \ell_4^n \cos n\theta \end{Bmatrix} \tag{19c}$$

and

$$Y = \begin{Bmatrix} t_s \\ \hat{t}_{s\theta} \\ \hat{f}_s \\ \bar{\sigma}_s \end{Bmatrix} \tag{20}$$

In order to express the boundary conditions entirely in terms of the solution vector z, Equations (16) are first written in Fourier component form and the resulting Fourier coefficients are then expressed in terms of u_s, u_θ, w and m_s by means of equations (8). The results of these calculations are given in matrix form by

$$Y = HZ' + JZ \tag{21}$$

where

$$H = \begin{bmatrix} b_1 & 0 & b_{4'} & 0 \\ 0 & b_6 & b_8 & 0 \\ b_{10'} & b_{11} & b_{13} & b_{15} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{22a}$$

$$J = \begin{bmatrix} b_2 & b_3 & b_4 & b_{4''} \\ b_5 & b_7 & b_9 & 0 \\ b_{10} & b_{12} & b_{14} & b_{16} \\ -\omega_s & 0 & 0 & 0 \end{bmatrix} \tag{22b}$$

and the b coefficients are given in Appendix II.

Substituting (21) into (18) yields

$$\Omega HZ' + (\Lambda + \Omega J)Z = \ell \tag{23}$$

Thus, the field equations (13) are to be solved subject to boundary conditions of the type (23) at the edges $S = 0$ and $S = \bar{S}$.

D. FINITE DIFFERENCE FORMULATION AND MATRIX SOLUTION OF THE DIFFERENCE EQUATIONS

The range of the independent variable $0 \leq S \leq \bar{S}$ is divided into N intervals by the grid points $i=0, 1, \ldots, N$. The i^{th} grid size is then defined by

$$\Delta_i = S_i - S_{i-1} \quad (i=1,2,\ldots,N) \tag{24}$$

The vector field Z is now replaced by the mesh vectors Z_i, defined only at the grid points. At all interior points, $1 \leq i \leq N-1$, the first two derivatives of Z are replaced by parabolic central difference expressions of the form

$$Z'_i = \frac{1}{(\Delta_{i+1} + \Delta_i)} \left[\frac{\Delta_i}{\Delta_{i+1}} Z_{i+1} - \left(\frac{\Delta_i}{\Delta_{i+1}} - \frac{\Delta_{i+1}}{\Delta_i} \right) Z_i - \frac{\Delta_{i+1}}{\Delta_i} Z_{i-1} \right] \tag{25a}$$

$$Z''_i = \frac{2}{(\Delta_{i+1} + \Delta_i)} \left[\frac{Z_{i+1}}{\Delta_{i+1}} - \left(\frac{1}{\Delta_{i+1}} + \frac{1}{\Delta_i} \right) Z_i + \frac{Z_{i-1}}{\Delta_i} \right] \tag{25b}$$

At the end points $i=0, N$ first derivatives of Z are approximated by forward and backward difference formulas, respectively, which involve the same truncation orders as for the central difference formulas (25).

$$Z'_o = - Z_2 \frac{\Delta_1}{\Delta_2 (\Delta_1 + \Delta_2)} + Z_1 \frac{\Delta_1 + \Delta_2}{\Delta_1 \Delta_2} - Z_o \left(\frac{\Delta_2}{\Delta_1 (\Delta_1 + \Delta_2)} + \frac{2}{\Delta_1 + \Delta_2} \right) \tag{26a}$$

$$Z'_N = Z_N \frac{\Delta_{N-1}}{\Delta_N (\Delta_N + \Delta_{N-1})} + \frac{2}{\Delta_N + \Delta_{N-1}} - Z_{N-1} \frac{\Delta_N + \Delta_{N-1}}{\Delta_N \Delta_{N-1}}$$

$$+ Z_{N-2} \frac{\Delta_N}{\Delta_{N-1} (\Delta \quad + \Delta_{N-1})} \tag{26b}$$

This, employing Equations (25) and (26) the boundary value problems given by Equations (13) and (23) are approximated by the matrix difference equations:

$$\bar{A}_o Z_1 + \bar{B}_o Z_o + \bar{C}_o Z_2 = g_o \tag{27a}$$

$$A_i Z_{i+1} + B_i Z_i + C_i Z_{i-1} = g_i \quad (i=1,2,\ldots,N-1) \tag{27b}$$

$$\bar{A}_N Z_{N-2} + \bar{B}_N Z_N + \bar{C}_N Z_{N-1} = \bar{g}_N \tag{27c}$$

where

$$\bar{A}_o = \frac{\Delta_1 + \Delta_2}{\Delta_1 \Delta_2} \Omega_o H_o$$

$$\bar{B}_o = \Lambda_o + \Omega_o \left[J_o - \frac{H_o}{\Delta_1} \left(\frac{\Delta_2 + 2\Delta_1}{\Delta_1 + \Delta_2} \right) \right] \tag{28a}$$

$$\bar{C}_o = - \frac{\Delta_1}{\Delta_2 (\Delta_1 + \Delta_2)} \Omega_o H_o$$

$$g_o = \ell_o$$

$$A_i = \frac{2E_i}{\Delta_{i+1}} + \frac{\Delta_i}{\Delta_{i+1}} F_i$$

$$B_i = -2E_i \left(\frac{1}{\Delta_{i+1}} + \frac{1}{\Delta_i} \right) - \left(\frac{\Delta_i}{\Delta_{i+1}} - \frac{\Delta_{i+1}}{\Delta_i} \right) F_i + (\Delta_{i+1} + \Delta_i) G_i$$

$$C_i = \frac{2E_i}{\Delta_i} - \frac{\Delta_{i+1}}{\Delta_i} F_i \tag{28b}$$

$$g_i = (\Delta_i + \Delta_{i+1}) e_i,$$

$$\bar{A}_N = \Omega_N H_N \frac{\Delta_N}{\Delta_{N-1}(\Delta_N + \Delta_{N-1})}$$

$$\bar{B}_N = \Lambda_N + \Omega_N \left[J_N + \frac{H_N}{\Delta_N} \left(\frac{\Delta_{N-1} + 2\Delta_N}{\Delta_N \Delta_{N-1}} \right) \right]$$

$$\bar{C}_N = - \Omega_N H_N \frac{\Delta_N + \Delta_{N-1}}{\Delta_N \Delta_{N-1}} \tag{28c}$$

$$g_N = \ell_N ,$$

and the subscripts zero and N refer to the grid values at i=0, N, respectively.

Equations (27) form a system of (N+1) linear algebraic equations in the N+1 unknown mesh vectors $(Z_i)_{i=0,1,\ldots,N}$. We assume that as $N \to \infty$ the solution of this algebraic system converges to the solution of the differential system.

Thus for a sufficiently small mesh size, the mesh vectors Z_i are expected to accurately approximate the vector field Z(s).

Matrix solutions of the difference equations are obtained by a modified Gaussian elimination scheme slightly different from the one developed in Reference [10]. The procedure is as follows:

Solve (27b) for Z_2 when $i = 1$,

$$Z_2 = A_1^{-1} (g_1 - B_1 Z_1 - C_1 Z_o) \tag{29}$$

and substitute (29) into (27a) to obtain

$$A_o Z_1 + B_o Z_o = g_o \tag{27c}$$

where

$$A_o = \bar{A}_o - \bar{C}_o A_1^{-1} B_1, \quad B_o = \bar{B}_o - \bar{C}_o A_1^{-1} C_1, \quad g_o = \bar{g}_o - \bar{C}_o A_1^{-1} g_1$$

Next solve (27b) for Z_{N-2} when $i = N-1$,

$$Z_{N-2} = C_{N-1}^{-1} \left(g_{N-1} - A_{N-1} Z_N - B_{N-1} Z_{N-1} \right)$$

and substitute into (27c) to obtain

$$B_N Z_N + A_N Z_{N-1} = g_N \tag{27e}$$

where

$$B_N = \bar{B}_N - \bar{A}_N C_{N-1}^{-1} A_{N-1}, \quad C_N = \bar{C}_N - \bar{A}_N C_{N-1}^{-1} B_{N-1},$$

$$g_N = \bar{g}_N - \bar{A}_{N-1} C_{N-1}^{-1} g_{N-1}$$

Next, Equations (27d) and (27b) (for $i = 1$) are solved simultaneously for

Z_o in terms of Z_1 and Z_2 and then for Z_1 in terms of Z_2:

$$Z_o = C_1^{-1} \left[g_1 - A_1 Z_2 - B_1 Z_1\right] \tag{30}$$

$$Z_1 = - \left[B_o C_1^{-1} B_1 - A_o\right]^{-1} \left[B_o C_1^{-1} A_1 Z_2 - B_o C_1^{-1} g_1 + g_o\right] \tag{31}$$

(This inversion procedure avoids singularities which can occur in the B_o matrix for certain types of boundary conditions). It is next assumed that the value of the mesh vector at a given grid point can be expressed in terms of the value at the following grid point in the form

$$Z_{i-1} = - P_{i-1} Z_i + X_{i-1} \quad (i=2,3,\ldots,N-1). \tag{32}$$

Recurrence relations for the P's and X's are established by substituting (32) into (27b);

$$P_i = \left[B_i - C_i P_{i-1}\right]^{-1} A_i$$

$$X_i = \left[B_i - C_i P_{i-1}\right]^{-1} \left[g_i - C_i X_{i-1}\right], \quad (i = 2,3,\ldots N-1). \tag{33}$$

However, from (31) the initial values are

$$P_1 = \left[B_o C_1^{-1} B_1 - A_o\right]^{-1} \left[B_o C_1^{-1} A_1\right] \tag{34a}$$

$$X_1 = \left[B_o C_1^{-1} B_1 - A_o\right]^{-1} \left[B_o C_1^{-1} g_1 - g_o\right], \tag{34b}$$

and therefore all the P's and X's up to P_{N-1} and X_{N-1} can be determined from (33) by recursions. The value of Z_N can now be determined by substituting

$$Z_{N-1} = -P_{N-1} Z_N + X_{N-1} \tag{35}$$

into (27e), yielding

$$Z_N = \left[B_N - C_N P_{N-1}\right]^{-1} \left[g_N - C_N X_{N-1}\right] \tag{36}$$

and therefore all the values of Z_i down to Z_2 can be determined by recursions of (32). Finally Z_1 and Z_0 are found in turn from (31) and (30). Thus, the solution for all the $Z_i\rceil_{i=0,1,\ldots,N}$ involves only the inversion of 4 x 4 matrices at each grid point, which can be rapidly accomplished on a digital computer.

After the Z_i vectors have been obtained in the above manner, numerical approxinations to the remaining field quantities can be calculated from obvious finite difference counterparts of equations (8).

The stresses σ_ϕ, σ_θ and $\sigma_{\varphi\theta}$ are obtained by first finding the total grid point displacements from the last two equations of (5). These are then substituted into finite difference equivalents of the strain-displacement relations:[9]

$$\begin{aligned}
\epsilon_\varphi &= \frac{v^\cdot}{r_1} - \frac{v}{r_1}\frac{r_i^\cdot}{r_1}\frac{\zeta}{r_1+\zeta} - \frac{w^{\cdot\cdot}}{r_1}\frac{\zeta}{r_1+\varphi} + \frac{w^\cdot}{r_1}\frac{r_i^\cdot}{r_1}\frac{\zeta}{r_1+\zeta} + \frac{w}{r_1+\zeta} \\
\epsilon_\theta &= \frac{u'}{r} + \frac{v}{r_1}\cot\varphi\,\frac{r_1+\zeta}{r_2+\zeta} - \frac{w''}{r\sin\varphi}\frac{\zeta}{r_2+\zeta} \\
&\quad - \frac{w^\cdot}{r_1}\cot\varphi\,\frac{\zeta}{r_2+\zeta} + \frac{w}{r_2+\zeta}\,, \\
\gamma_{\varphi\theta} &= \frac{u^\cdot}{r_2}\frac{r_2+\zeta}{r_1+\zeta} - \frac{ur_1}{r_2^2}\frac{r_2+\zeta}{r_1+\zeta}\cot\varphi + \frac{v'}{r_1\sin\varphi}\frac{r_1+\zeta}{r_2+\zeta} \\
&\quad - \frac{w'^\cdot\zeta}{r_1\sin\zeta}\left(\frac{1}{r_2+\zeta} + \frac{r_1}{r_2}\frac{1}{r_1+\zeta}\right) \\
&\quad + \frac{w'}{r_2}\frac{\cot\varphi}{\sin\varphi}\left(\frac{\zeta}{r_2+\zeta} + \frac{r_1}{r_2}\frac{\zeta}{r_1+\zeta}\right) .
\end{aligned} \tag{37}$$

The stresses are finally calculated by substituting the resulting strains into Hooke's law,

$$\sigma_{\varphi} = \frac{E}{1-\nu^2}(\epsilon_{\varphi} + \nu\epsilon_{\theta})$$

$$\sigma_{\theta} = \frac{E}{1-\nu^2}(\epsilon_{\theta} + \nu\epsilon_{\varphi}) \qquad (38)$$

$$\sigma_{\varphi\theta} = \sigma_{\theta\varphi} = \frac{E}{2(1+\nu)}\gamma_{\varphi\theta}$$

E. UNIT SOLUTIONS

Since the boundary conditions cannot be satisfied continuously at the flare-dome intersection, a solution procedure which matches conditions point-wise will be employed. Toward this end, we wish to develop unit solutions which satisfy the homogeneous flare equations and which, when multiplied by suitable constants, can be made to satisfy appropriate boundary conditions. To develop the unit solutions, we specialize the constants of Equations (15) so that the unit problem boundary conditions, satisfied for each n, are:

(1) $s = o$: $t_s = t_{s\theta} = f_s = m_s = o$
$s = \bar{s}$: $u_s = 1$, $u_\theta = w = m_s = o$

(2) $s = o$: same as for (1)
$s = \bar{s}$: $u_s = o$, $u_\theta = 1$, $w = m_s = 0$

(3) $s = o$: same as for (1)
$s = \bar{s}$: $u_s = u_\theta = o$, $w = 1$, $m_s = o$

(4) $s = o$: same as for (1)
$s = \bar{s}$: $u_s = u_\theta = w = o$, $m_s = 1$

In addition, for $n = o$, we require the particular solution corresponding to the internal pressure $= p$ (a constant) and boundary conditions, which are:

$s = o$: t_s^o = internal pressure membrane force

$t_{s\theta} = f_s = m_s = o$

$s = \bar{s}$: $u_s = u_\theta = w = m_s = o$

Multiplying the resulting unit solutions by constants and summing, we obtain, for eight of the more important derived quantities:

$$U_F(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, u_{s1k}^n(s) + u^0(s) \tag{39a}$$

$$V_F(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \sin n\theta \sum_{k=1}^{4} C_k^n \, u_{\theta k}^n(s) \tag{39b}$$

$$W_F(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, w_k^n(s) + w^0(s) \tag{39c}$$

$$M_S(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, m_k^n(s) + m^0(s) \tag{39d}$$

$$N_S(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, t_{s_k}^n(s) + t_s^0(s) \tag{39e}$$

$$N_{S\theta}(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \sin n\theta \sum_{k=1}^{4} C_k^n \, t_{s\theta_k}^n(s) \tag{39f}$$

$$\bar{Q}_S(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, f_{s_k}^n(s) + f_s^0(s) \tag{39g}$$

$$\Phi_S(S,\theta) = \sum_{n=0,2,\ldots}^{\bar{n}} \cos n\theta \sum_{k=1}^{4} C_k^n \, \bar{\varphi}_k^n(s) + \bar{\varphi}^0(s) \tag{39h}$$

where the subscript F refers to "Flare," the subscript S relates to the meridional coordinate and U_F and V_F are the meridional and circumferential deflections, respectively.

The remaining derived quantities may be formed in a similar manner where necessary.

Only the even integer (n) Fourier terms have been used, since the nozzle-dome geometry is assumed to be symmetric about the xz and yz planes. Writing the solutions in this form yields $4\,(\frac{\bar{n}}{2}+1)$ constants C_k^n to satisfy the nozzle-dome intersection compatibility and equilibrium conditions. However, because of the problem's symmetry it can be shown that $C_2^0 = 0$, thus reducing the number of unknowns to $2\bar{n} + 3$.

SECTION III

DOME ANALYSIS

A. INTRODUCTION

The governing equations for the bending and stretching of an arbitrarily shaped, thin, shallow shell were originally developed by Marguere [12]. For completeness, these equations are rederived (Appendix II) for the particular geometry of an elliptic-paraboloidal shell. The median surface of this shell is prescribed by (see Figure 5)

$$z = -\frac{1}{2}\left(\frac{x^2}{R_x} + \frac{y^2}{R_y}\right) \tag{1}$$

where, to the order of approximation in the analysis, R_x and R_y are the principal radii of curvature.

Marguerre's shallow shell theory is characterized by a linear system of eleven coupled equations for as many unknowns. In particular, this set consists of the equilibrium equations

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{2a}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{2b}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = Q_x \tag{2c}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y \tag{2d}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} = -p(x,y) \tag{2e}$$

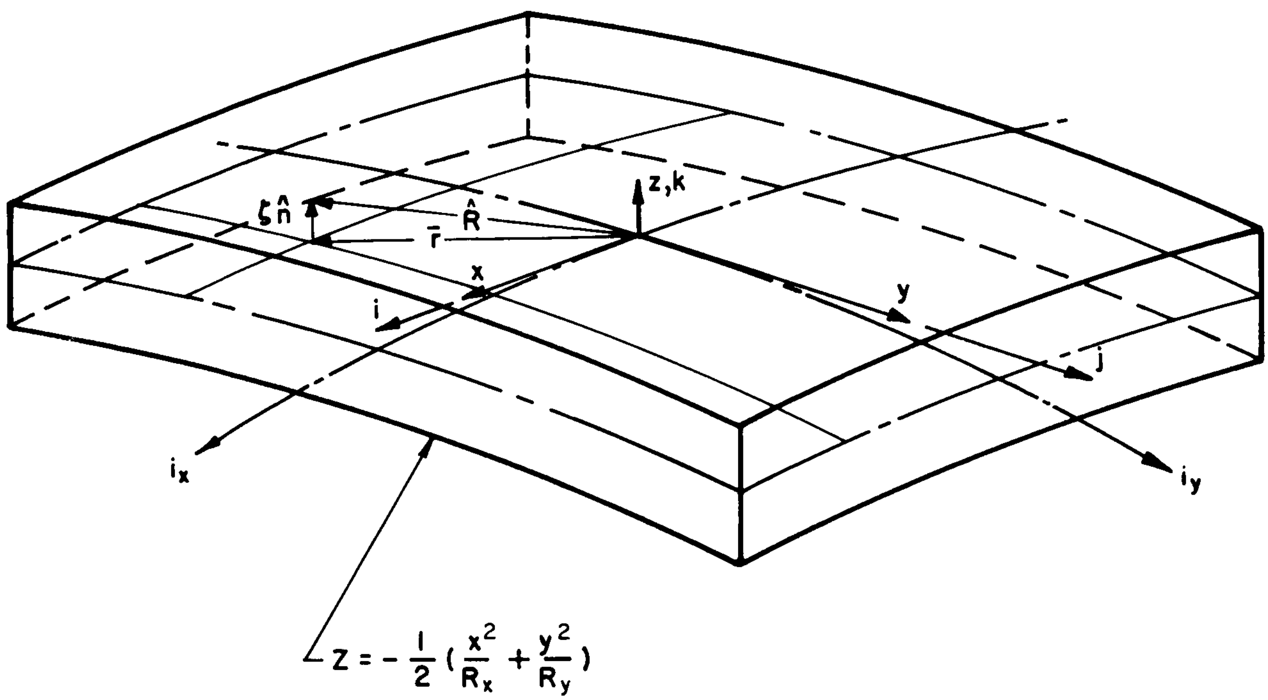


Figure 5. Coordinates and Notation for Shallow Elliptic-Paraboloidal Shell

and the stress resultant-displacement relations

$$N_x = A\left[\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} + \left(\frac{1}{R_x} + \frac{\nu}{R_y}\right) w\right] \tag{3a}$$

$$N_y = A\left[\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x} + \left(\frac{1}{R_y} + \frac{\nu}{R_x}\right) w\right] \tag{3b}$$

$$N_{xy} = \frac{(1-\nu)A}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \tag{3c}$$

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right] \tag{3d}$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right] \tag{3e}$$

$$M_{xy} = -(1-\nu)D\frac{\partial^2 w}{\partial x\,\partial y} \tag{3f}$$

The notation and sign convention is shown in Figure 6. Associated with Equations 2 and 3 are the admissible boundary conditions which specify either

$$u \quad \text{or} \quad \left[N_x \cos(\ell,x) + N_{xy}\cos(\ell,y)\right] \tag{4a}$$

$$v \quad \text{or} \quad \left[N_{xy} \cos(\ell,x) + N_y\cos(\ell,x)\right] \tag{4b}$$

$$w \quad \text{or} \quad \left[\left(Q_x + \frac{\partial M_{xy}}{\partial y}\right)\cos(\ell,x) + \left(Q_y + \frac{\partial M_{xy}}{\partial x}\right)\cos(\ell,y)\right] \tag{4c}$$

$$\frac{\partial w}{\partial x} \quad \text{or} \quad M_x \tag{4d}$$

$$\frac{\partial w}{\partial y} \quad \text{or} \quad M_y \tag{4e}$$

Equations 2 and 3 can be contracted to the single fourth order partial differential equation (Appendix III)

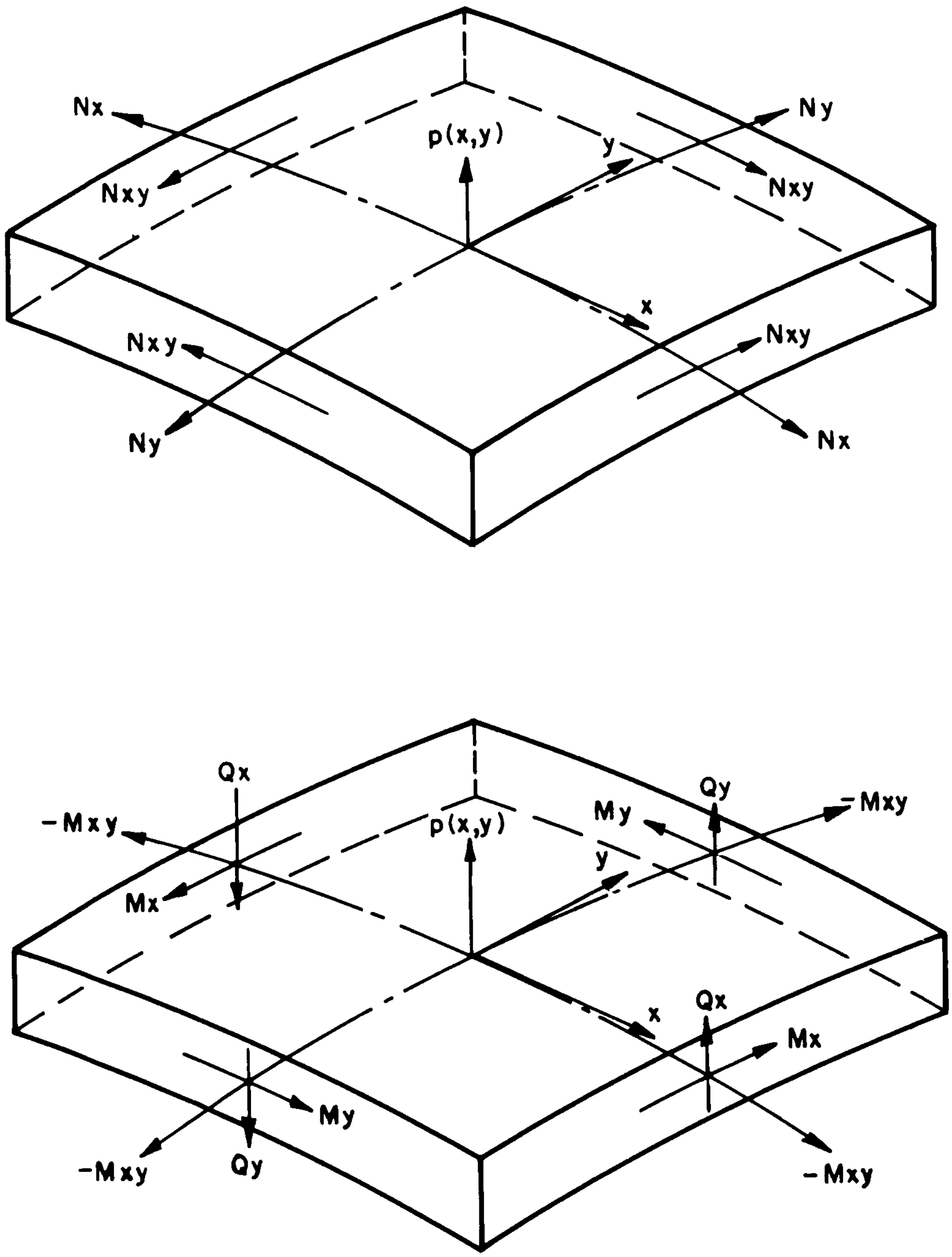


Figure 6. Sign Convention for the Stress Resultants

$$\nabla^4 \varphi - i\,\delta^2 \nabla_k^2 \varphi = \frac{p(x,y)}{D} \qquad (5a)$$

where φ, the complex deflection-stress function, is defined by

$$\varphi = w + \frac{i\,F}{[(1-\nu^2)\,AD]^{1/2}} \qquad (5b)$$

with

$$i = \sqrt{-1}$$

$$\delta^4 = \frac{(1-\nu^2)A}{D} \qquad (5c)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla_k^2 = \frac{1}{R_x}\frac{\partial^2}{\partial y^2} + \frac{1}{R_y}\frac{\partial^2}{\partial x^2}$$

and F is related to the stress resultants by

$$N_x = \frac{\partial^2 F}{\partial y^2}$$

$$N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \qquad (5d)$$

$$N_y = \frac{\partial^2 F}{\partial x^2}$$

Thus, Marguerre's shallow shell theory reduces to the solution of Equation 5a subject to the requisite boundary conditions selected from Equations 4.

B. BOUNDARY CONDITIONS

The controlling equation for the deformation of the elliptic-paraboloidal shell (Equation 5a) requires the stipulation of four boundary conditions at each edge to establish a unique solution. Four of these will be specified some distance away from the junction of the flare and dome.

In accordance with the local character of the discontinuity stress, the bending effects attenuate rapidly, and the stress field approaches the membrane field. Thus, away from the intersection, the following conditions are imposed,

at $x = L_x$ (Figure 7)

$$
\begin{aligned}
N_x &= \text{membrane force} \\
N_{xy} &= 0 \\
Q_x + \frac{\partial M_{xy}}{\partial y} &= 0 \\
M_x &= 0
\end{aligned}
\tag{6a}
$$

at $y = L_y$

$$
\begin{aligned}
N_y &= \text{membrane force} \\
N_{xy} &= 0 \\
Q_y + \frac{\partial M_{xy}}{\partial x} &= 0 \\
M_y &= 0
\end{aligned}
\tag{6b}
$$

The symmetry of the shell leads to the additional restrictions along $x = 0$

$$
\begin{aligned}
u &= 0 \\
N_{xy} &= 0 \\
\frac{\partial w}{\partial x} &= 0 \\
Q_x + \frac{\partial M_{xy}}{\partial y} &= 0
\end{aligned}
\tag{7a}
$$

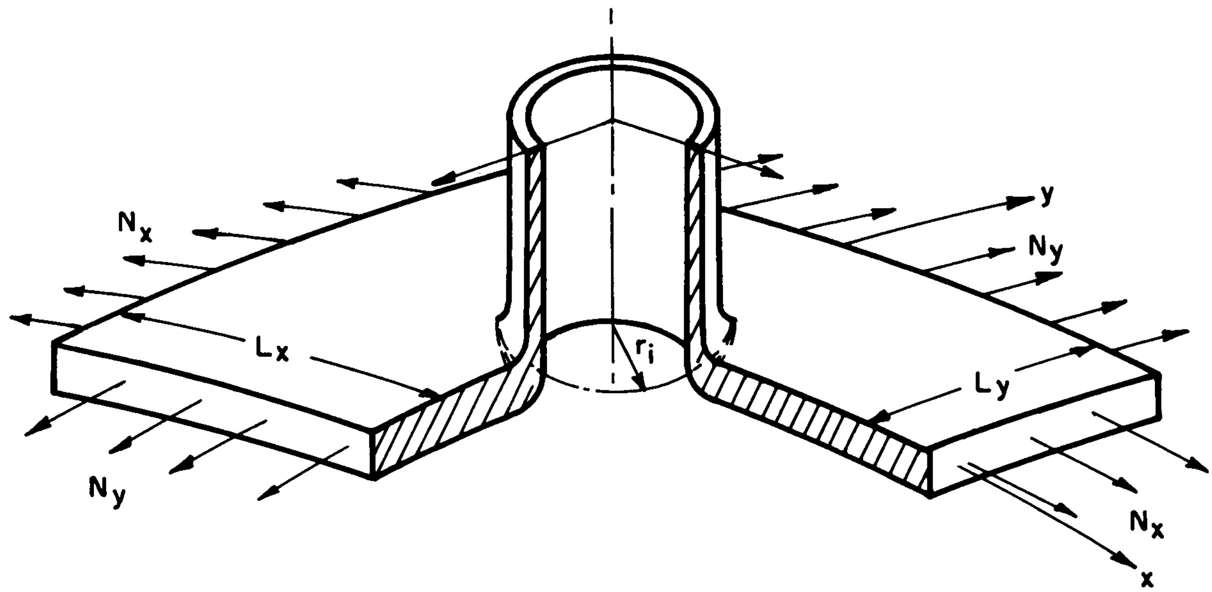


Figure 7. Intersection of the Flared Nozzle with the Elliptic-Paraboloidal Shell

and at $y = 0$

$$\begin{aligned} v &= 0 \\ N_{xy} &= 0 \\ \frac{\partial w}{\partial y} &= 0 \\ Q_y &= \frac{\partial M_{xy}}{\partial x} = 0 \end{aligned} \tag{7b}$$

Furthermore, the continuity conditions at the junction of the flare and the dome require that the displacements, normal slope, and stress resultants of each shell be equal. These compatibility relations will be described in detail in the section dealing with the "point-matching" of the flare to the dome. However, for subsequent use, the Cartesian components of the displacements and forces in the elliptic-paraboloidal shell are transformed to curvilinear components. To this end, the displacements are (Figure 8a)

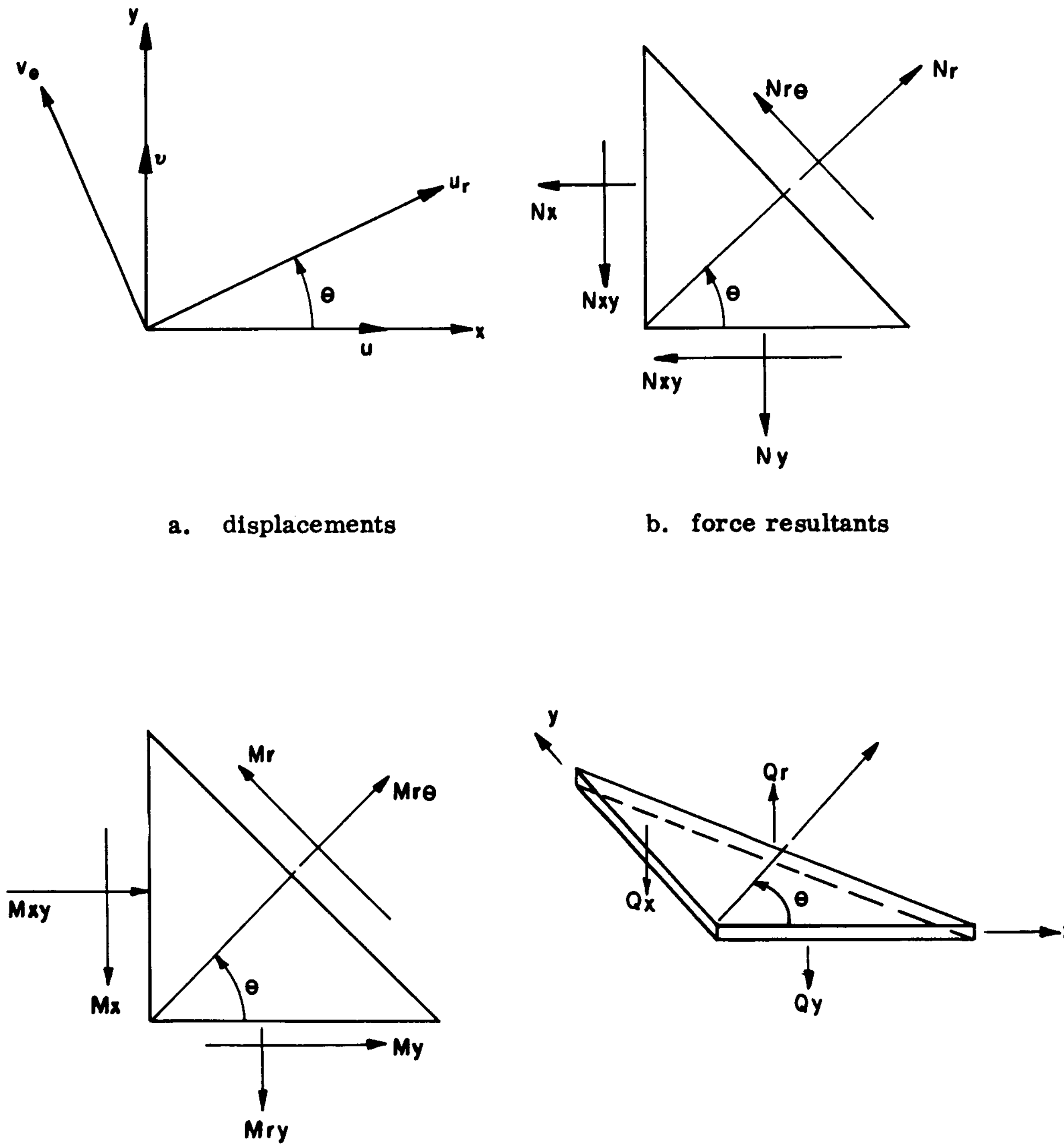
$$\begin{aligned} u_r &= u \cos\theta + v \sin\theta \\ v_\theta &= -u \sin\theta + v \cos\theta \end{aligned} \tag{8a}$$

and the in-plane force resultants are (Figure 8b)

$$\begin{aligned} N_r &= N_x \cos^2\theta + N_y \sin^2\theta + 2 N_{xy} \sin\theta \cos\theta \\ N_{r\theta} &= \left(N_y - N_x\right) \sin\theta \cos\theta + N_{xy} \left(\cos^2\theta - \sin^2\theta\right) \\ N_\theta &= N_x \sin^2\theta + N_y \cos^2\theta - 2 N_{xy} \sin\theta \cos\theta \end{aligned} \tag{8b}$$

and the moment resultants are (Figure 8c)

$$\begin{aligned} M_r &= M_x \cos^2\theta + M_y \sin^2\theta + 2 M_{xy} \sin\theta \cos\theta \\ M_{r\theta} &= \left(M_y - M_x\right) \sin\theta \cos\theta + M_{xy} \left(\cos^2\theta - \sin^2\theta\right) \\ M_\theta &= M_x \sin^2\theta + M_y \cos^2\theta - 2 M_{xy} \sin\theta \cos\theta \end{aligned} \tag{8c}$$



a. displacements

b. force resultants

c. moment resultants

d. transverse shear resultants

Figure 8. Cartesian and Polar Coordinate Representations of the Displacements and Stress Resultants

and the transverse shear resultants are (Figure 8d)

$$Q_r = Q_x \cos\theta + Q_y \sin\theta$$
$$Q_\theta = -Q_x \sin\theta + Q_y \cos\theta \tag{8d}$$

C. SOLUTION OF THE PROBLEM: ELLIPTIC-PARABOLOIDAL SHELL

1. Technical Approach

The mathematical difficulties associated with the simultaneous satisfaction of the field equation (Equation 5a) and the associated boundary conditions (Equations 6, 7, and the interaction conditions) are sizable. Consequently, an approximate solution technique known as "Least-Squares Point-Matching", will be employed, to effect a solution to the composite (interaction) problem.

To prepare the way for the application of the point-matching procedure, two families of functions which satisfy the homogeneous field Equation, 5a, are constructed. Each family is in the form of an infinite series with coefficients which must be determined from the boundary conditions. The solution (applicable to the elliptic-paraboloidal shell) is composed of three distinct parts. These are (1) a particular solution of Equation 5a (suitable for the description of the membrane field), (2) a solution to the homogeneous equation which decays exponentially in the y direction and has a trigonometric variation in the x direction, and (3) a companion solution to the homogeneous equation which decays exponentially in the x direction and has a trigonometric variation in the y direction.

The required solutions will be developed in subsection 2, and the boundary conditions satisfied by each of the solutions will be discussed in subsection 3.

2. Component Solutions

a. Particular Solution

A particular solution of Equation 5a, which satisfies the far field boundary conditions (Equation 6) as well as the symmetry requirements

(Equations 7), can be constructed from the membrane theory of shells. After neglecting the quantities multiplied by the flexural rigidity, D, Equation 5a reduces to

$$\nabla_k^2 F = p \tag{9}$$

A suitable choice of F (x, y) is

$$F(x,y) = p \frac{R_y}{4} \left[y^2 + \left(2 - \frac{R_y}{R_x} \right) x^2 \right] \tag{10}$$

and from 5d, the stress resultants are

$$\begin{aligned} N_x &= p \frac{R_y}{2} \\ N_{xy} &= 0 \\ N_y &= p \frac{R_y}{2} \left(2 - \frac{R_y}{R_x} \right) \end{aligned} \tag{11a}$$

Substituting Equation 11a into 3a, 3b, and 3c, arbitrarily setting the normal deflection, w, equal to zero, and then integrating, leads to the membrane displacements

$$\begin{aligned} u &= p \frac{R_y}{2Et} \left[1 - 2\nu + \nu \frac{R_y}{R_x} \right] x \\ v &= p \frac{R_y}{2Et} \left[2 - \nu - \frac{R_y}{R_x} \right] y \\ w &= 0 \end{aligned} \tag{11b}$$

b. Complementary Solution

A complementary solution to Equation 5a, which gives rise to stress resultants that decay exponentially as y increases, can be presented in the form

$$\varphi_n = C \, e^{-\beta y} \cos \alpha_n x \tag{12a}$$

where α_n is taken as

$$\alpha_n = \frac{n\pi}{2L_x} \quad ; \quad n = \text{integer} \tag{12b}$$

and β is a characteristic number to be determined and C is an arbitrary constant. The constant L_x is the projected semi-span in the x direction (Figure 7).

Substituting from Equation 12a into 5a leads to the characteristic equation

$$\left(\beta^2 - \alpha_n^2\right)^2 - \frac{i\,\delta^2}{R_x}\left(\beta^2 - \alpha_n^2\right) - i\,\delta^2\left(\frac{1}{R_x} - \frac{1}{R_y}\right)\alpha_n^2 = 0 \tag{13}$$

for the acceptable values of β. They are,

$$\begin{aligned} \beta_{1,2} &= \pm\left(p_1 + i\,q_1\right) \\ \beta_{3,4} &= \pm\left(p_2 + i\,q_2\right) \end{aligned} \tag{14}$$

with the definitions

$$p_1^2 = \frac{1}{2}\left(\alpha_n^2 - \frac{\delta^2 b}{2R_x}\right)\left(1 \pm \sqrt{1 + \left(\frac{(1+a)\,\delta^2}{2R_x\alpha_n^2 - \delta^2 b}\right)^2}\right) \tag{15a}$$

$$p_2^2 = \frac{1}{2}\left(\alpha_n^2 + \frac{\delta^2 b}{2R_x}\right)\left(1 \pm \sqrt{1 + \left(\frac{(1-a)\,\delta^2}{2R_x\alpha_n^2 + \delta^2 b}\right)^2}\right) \tag{15b}$$

$$q_1^2 = \frac{1}{2}\left(\alpha_n^2 - \frac{\delta^2 b}{2R_x}\right)\left(-1 \pm \sqrt{1 + \left(\frac{(1+a)\,\delta^2}{2R_x\alpha_n^2 - \delta^2 b}\right)^2}\right) \tag{15c}$$

$$q_2^2 = \frac{1}{2}\left(\alpha_n^2 + \frac{\delta^2 b}{2R_x}\right)\left(-1 \pm \sqrt{1 + \left(\frac{(1-a)\,\delta^2}{2R_x\alpha_n^2 + \delta^2 b}\right)^2}\right) \tag{15d}$$

$$a^2 = \frac{1}{2}\left(1 + \sqrt{1 + \frac{16R_x^4\alpha_n^4}{\delta^4}\left(\frac{1}{R_x} - \frac{1}{R_y}\right)^2}\right) \tag{15e}$$

$$b = - \frac{2 R_x^2 \alpha_n^2}{a \delta^2} \left(\frac{1}{R_x} - \frac{1}{R_y} \right) \tag{15f}$$

Since p_1, p_2, q_1, and q_2 are real numbers, the choice of signs preceding the radical is dependent upon the signs of the terms $\left(\alpha_n^2 - \frac{\delta^2 b}{2R_x}\right)$ and $\left(\alpha_n^2 + \frac{\delta^2 b}{2R_x}\right)$. For example, in computing p_1, if

$$\alpha_n^2 - \frac{\delta^2 b}{2R_x} > 0 \tag{16a}$$

the positive sign is required, and if

$$\alpha_n^2 - \frac{\delta^2 b}{2R_x} < 0 \tag{16b}$$

the negative sign is required. Furthermore, if

$$\alpha_n^2 - \frac{\delta^2 b}{2R_x} = 0 \tag{16c}$$

equation 15a reduces to

$$p_1^2 = \frac{(1+a)\,\delta^2}{4 R_x} \tag{16d}$$

The remaining quantities, q_1, p_2, and q_2 are obtained in the same manner.

Employing Equation 14, the expanded form of Equation 12a becomes,

$$\varphi_n = \cos \alpha_n x \left[C_1 e^{(p_1+iq_1)y} + C_2 e^{-(p_1+iq_1)y} + C_3 e^{(p_2+iq_2)y} + C_4 e^{-(p_2+iq_2)y} \right] \tag{17}$$

where C_1, C_2, C_3, and C_4 are complex constants. Since Equation 17 is valid for any n, the complete solution will be composed of the sum over all values on n. With this understanding, further discussions will be concerned only with a generic term of the series.

Retaining the terms which give rise to negative exponents in 17, since the stress resultants must decay as y grows large, and letting

$$C_2 = A_1 + i\, B_1$$

$$C_4 = A_2 + i\, B_2$$

Equations 17 becomes

$$\begin{aligned} \varphi_n = \cos \alpha_n x \Big[& \left\{ A_1 \cos q_1 y + B_1 \sin q_1 y \right\} e^{-p_1 y} \\ & + \left\{ A_2 \cos q_2 y + B_2 \sin q_2 y \right\} e^{-p_2 y} \\ & + i \left\{ B_1 \cos q_1 y - A_1 \sin q_1 y \right\} e^{-p_1 y} \\ & + i \left\{ B_2 \cos q_2 y - A_2 \sin q_2 y \right\} e^{-p_2 y} \Big] \end{aligned} \tag{18}$$

Equating real and imaginary parts, as prescribed by Equation 5b, leads to the deflection

$$\begin{aligned} w_n = \cos \alpha_n x \Big[& \left\{ A_1 \cos q_1 y + B_1 \sin q_1 y \right\} e^{-p_1 y} \\ & + \left\{ A_2 \cos q_2 y + B_2 \sin q_2 y \right\} e^{-p_2 y} \Big] \end{aligned} \tag{19a}$$

and the stress function

$$\begin{aligned} \frac{F_n}{[(1-\nu^2)AD]^{1/2}} = \cos \alpha_n x \Big[& \left\{ -A_1 \sin q_1 y + B_1 \cos q_1 y \right\} e^{-p_1 y} \\ & + \left\{ -A_2 \sin q_2 y + B_2 \cos q_2 y \right\} e^{-p_2 y} \Big] \end{aligned} \tag{19b}$$

The stress resultants associated with Equations 19 are obtained from the interconnecting relations (Equations 3d, 3e, 3f, and 5d) and are

$$N_x = [(1-\nu^2)AD]^{1/2} \cos \alpha_n x \Big[e^{-p_1 y} \Big\{ \Big[\left(q_1^2 - p_1^2\right) A_1 + 2 q_1 p_1 B_1 \Big] \sin q_1 y$$

$$+ \Big[2 q_1 p_1 A_1 - \left(q_1^2 - p_1^2\right) B_1 \Big] \cos q_1 y \Big\}$$

$$+ e^{-p_2 y} \Big\{ \Big[\left(q_2^2 - p_2^2\right) A_2 + 2 q_2 p_2 B_2 \Big] \sin q_2 y$$

$$+ \Big[2 q_2 p_2 A_2 - \left(q_2^2 - p_2^2\right) B_2 \Big] \cos q_2 y \Big\} \Big] \qquad (20a)$$

$$N_{xy} = \alpha_n [(1-\nu^2)AD]^{1/2} \sin \alpha_n x \Big[e^{-p_1 y} \Big\{ \left(p_1 A_1 - q_1 B_1\right) \sin q_1 y$$

$$- \left(q_1 A_1 + p_1 B_1\right) \cos q_1 y \Big\}$$

$$+ e^{-p_2 y} \Big\{ \left(p_2 A_2 - q_2 B_2\right) \sin q_2 y - \left(q_2 A_2 + p_2 B_2\right) \cos q_2 y \Big\} \Big] \qquad (20b)$$

$$N_y = -\alpha_n^2 [(1-\nu^2)AD]^{1/2} \cos \alpha_n x \Big[e^{-p_1 y} \Big\{ -A_1 \sin q_1 y + B_1 \cos q_1 y \Big\}$$

$$+ e^{-p_2 y} \Big\{ -A_2 \sin q_2 y + B_2 \cos q_2 y \Big\} \Big] \qquad (20c)$$

$$M_x = D \cos \alpha_n x \Big[e^{-p_1 y} \Big\{ \Big[\left(\alpha_n^2 - \nu \left[p_1^2 - q_1^2\right]\right) A_1 + 2 \nu p_1 q_1 B_1 \Big] \cos q_1 y +$$

$$+ \Big[-2\nu p_1 q_1 A_1 + \left(\alpha_n^2 - \nu \left[p_1^2 - q_1^2\right]\right) B_1 \Big] \sin q_1 y \Big\}$$

$$+ e^{-p_2 y} \Big\{ \Big[\left(\alpha_n^2 - \nu \left[p_2^2 - q_2^2\right]\right) A_2 + 2\nu p_2 q_2 B_2 \Big] \cos q_2 y$$

$$+ \Big[-2\nu p_2 q_2 A_2 + \left(\alpha_n^2 - \nu \left[p_2^2 - q_2^2\right]\right) B_2 \Big] \sin q_2 y \Big\} \Big] \qquad (20d)$$

$$M_y = -D\cos\alpha_n x\Bigg[e^{-p_1 y}\Big\{\Big[\Big(p_1^2 - q_1^2 - \nu\,\alpha_n^2\Big)A_1 - 2p_1q_1B_1\Big]\cos q_1 y +$$

$$\Big[2p_1q_1A_1 + \Big(p_1^2 - q_1^2 - \nu\,\alpha_n^2\Big)B_1\Big]\sin q_1 y\Big\} +$$

$$e^{-p_2 y}\Big\{\Big[\Big(p_2^2 - q_2^2 - \nu\alpha_n^2\Big)A_2 - 2p_2q_2B_2\Big]\cos q_2 y +$$

$$\Big[2\,p_2q_2A_2 + \Big(p_2^2 - q_2^2 - \nu\,\alpha_n^2\Big)B_2\Big]\sin q_2 y\Big\}\Bigg] \quad \textbf{(20e)}$$

$$M_{xy} = (1-\nu)\,D\alpha_n\sin\alpha_n x\Bigg[e^{-p_1 y}\Big\{\Big(-p_1A_1 + q_1B_1\Big)\cos q_1 y -$$

$$\Big(q_1A_1 + p_1B_1\Big)\sin q_1 y\Big\} +$$

$$e^{-p_2 y}\Big\{\Big(-p_2A_2 + q_2B_2\Big)\cos q_2 y - \Big(q_2A_2 + p_2B_2\Big)\sin q_2 y\Big\}\Bigg] \quad \textbf{(20f)}$$

The transverse shear resultants are, from Equations 2c and 2d,

$$Q_x = \alpha_n\,D\sin\alpha_n x\Bigg[e^{-p_1 y}\Big\{\Big(\Big[p_1^2 - q_1^2 - \alpha_n^2\Big]A_1 - 2\,p_1q_1B_1\Big)\cos q_1 y$$

$$+\Big(2\,p_1q_1A_1 + \Big[p_1^2 - q_1^2 - \alpha_n^2\Big]B_1\Big)\sin q_1 y\Big\} +$$

$$e^{-p_2 y}\Big\{\Big(\Big[p_2^2 - q_2^2 - \alpha_n^2\Big]A_2 - 2\,p_2q_2B_2\Big)\cos q_2 y$$

$$+\Big(2\,p_2q_2A_2 + \Big[p_2^2 - q_2^2 - \alpha_n^2\Big]B_2\Big)\sin q_2 y\Big\}\Bigg] \quad \textbf{(21a)}$$

$$Q_y = D\cos\alpha_n x\Big[e^{-p_1 y}\Big\{\Big(\big[p_1^3-3p_1q_1^2-p_1\alpha_n^2\big]A_1+\big[q_1^3-3p_1^2q_1+q_1\alpha_n^2\big]B_1\Big)\cos q_1y$$

$$+\Big(-\big[q_1^3-3p_1^2q_1+q_1\alpha_n^2\big]A_1+\big[p_1^3-3p_1q_1^2-p_1\alpha_n^2\big]B_1\Big)\sin q_1y\Big\}$$

$$+\,e^{-p_2y}\Big\{\Big(\big[p_2^3-3p_2q_2^2-p_2\alpha_n^2\big]A_2+\big[q_2^3-3p_2^2q_2+q_2\alpha_n^2\big]B_2\Big)\cos q_2y$$

$$+\Big(-\big[q_2^3-3p_2^2q_2+q_2\alpha_n^2\big]A_2+\big[p_2^3-3p_2q_2^2-p_2\alpha_n^2\big]B_2\Big)\sin q_2y\Big\}\Big] \quad (21b)$$

The displacements u and v are evaluated from Equations 3a, 3b, and 3c where N_x, N_{xy}, N_y, and w are now known functions. Solving the first order partial differential equations results in

$$u = \left[\frac{D}{(1-\nu^2)A}\right]^{1/2}\frac{\sin\alpha_n x}{\alpha_n}\Big[e^{-p_1y}\Big\{\Big[\big(q_1^2-p_1^2-\alpha_n^2\nu\big)A_1+\Big(2p_1q_1-\Big\{\frac{(1-\nu^2)A}{D R_x^2}\Big\}^{1/2}\Big)B_1\Big]\sin q_1y$$

$$+\Big[\Big(2q_1p_1-\Big\{\frac{(1-\nu^2)A}{D R_x^2}\Big\}^{1/2}\Big)A_1-\big(q_1^2-p_1^2-\alpha_n^2\nu\big)B_1\Big]\cos q_1y\Big\}+$$

$$e^{-p_2y}\Big\{\Big[\big(q_2^2-p_2^2-\alpha_n^2\nu\big)A_2+\Big(2p_2q_2-\Big\{\frac{(1-\nu^2)A}{DR_x^2}\Big\}^{1/2}\Big)B_2\Big]\sin q_2y$$

$$+\Big[\Big(2p_2q_2-\Big\{\frac{(1-\nu^2)A}{DR_x^2}\Big\}^{1/2}\Big)A_2-\big(q_2^2-p_2^2-\alpha_n^2\nu\big)B_2\Big]\cos q_2y\Big\}\Big] \quad (22a)$$

$$
\begin{aligned}
v = \left[\frac{D}{(1-\nu^2)A}\right]^{1/2} \frac{\cos \alpha_n x}{\alpha_n^2} \Bigg[e^{-p_1 y} \Bigg\{ \Bigg[\Bigg(& p_1^3 - 3p_1 q_1^2 + \alpha_n^2 p_1 \nu + q_1 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} - 2(1+\nu)\,\alpha_n^2 p_1 \Bigg) A_1 \\
& + \Bigg(q_1^3 - 3p_1^2 q_1 - q_1 \alpha_n^2 \nu + p_1 \left\{\frac{(1-\nu^2)A}{R_x^2 D}\right\}^{1/2} + 2(1+\nu)\,\alpha_n^2 q_1 \Bigg) B_1 \Bigg] \sin q_1 y \\
& + \Bigg[\Bigg(q_1^3 - 3q_1 p_1^2 - q_1 \alpha_n^2 \nu + p_1 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} + 2(1+\nu)\,\alpha_n^2 q_1 \Bigg) A_1 \\
& - \Bigg(p_1^3 - 3q_1^2 p_1 + p_1 \alpha_n^2 \nu + q_1 \left\{\frac{(1-\nu^2)A}{R_x^2 D}\right\}^{1/2} - 2(1+\nu)\,\alpha_n^2 p_1 \Bigg) B_1 \Bigg] \cos q_1 y \Bigg\} \\
& + e^{-p_2 y} \Bigg\{ \Bigg[\Bigg(p_2^3 - 3p_2 q_2^2 + \alpha_n^2 p_2 \nu + q_2 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} - 2(1+\nu)\,\alpha_n^2 p_2 \Bigg) A_2 \\
& + \Bigg(q_2^3 - 3p_2^2 q_2 - q_2 \alpha_n^2 \nu + p_2 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} + 2(1+\nu)\,\alpha_n^2 q_2 \Bigg) B_2 \Bigg] \sin q_2 y \\
& + \Bigg[\Bigg(q_2^3 - 3q_2 p_2^2 - q_2 \alpha_n^2 \nu + p_2 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} + 2(1+\nu)\,\alpha_n^2 q_2 \Bigg) A_2 \\
& - \Bigg(p_2^3 - 3q_2^2 p_2 + p_2 \alpha_n^2 \nu + q_2 \left\{\frac{(1-\nu^2)A}{D R_x^2}\right\}^{1/2} - 2(1+\nu)\,\alpha_n^2 p_2 \Bigg) B_2 \Bigg] \cos q_2 y \Bigg\} \Bigg]
\end{aligned}
\tag{22b}
$$

Thus, all the field quantities derivable from the assumed solution (Equation 12a) are now known for integral values of n. The special case of $n = 0$ must be examined independently. This case corresponds to a one dimensional problem in the coordinate y. Therefore, Equation (5a) reduces to (with $p = 0$)

$$\frac{d^4\varphi_o}{dy^4} - i\frac{\delta^2}{R_x}\frac{d^2\varphi_o}{dy^2} = 0 \tag{23a}$$

and φ_o is

$$\varphi_o = C_o' + C_1'y + C_2' e^{-\beta_o(1+i)y} + C_3' e^{\beta_o(1+i)y} \tag{23b}$$

where

$$\beta_o = \left(\frac{\delta^2}{2R_x}\right)^{1/2} \tag{23c}$$

and C_o', C_1', C_2' and C_3' are complex constants.

Again, since the field quantities are to decay exponentially, only the negative exponential is retained. Furthermore, the rigid body constant C_o' is also omitted. Therefore, analogous to the case $n \neq 0$, the displacements and stress resultants for $n = 0$ are (with $C_2' = A_o + i B_o$)

$$w_o = (A_o \cos\beta_o y + B_o \sin\beta_o y)\, e^{-\beta_o y} \tag{24a}$$

$$\frac{F_o}{[(1-\nu^2)AD]^{1/2}} = (-A_o \sin\beta_o y + B_o \cos\beta_o y)\, e^{-\beta_o y} \tag{24b}$$

$$N_x = [(1-\nu^2)AD]^{1/2}\, 2\beta_o^2\, e^{-\beta_o y}(A_o \cos\beta_o y + B_o \sin\beta_o y) \tag{24c}$$

$$N_{xy} = 0 \tag{24d}$$

$$N_y = 0 \tag{24e}$$

$$M_x = -2\nu D \beta_o^2 e^{-\beta_o y} (A_o \sin \beta_o y - B_o \cos \beta_o y) \tag{24f}$$

$$M_{xy} = 0 \tag{24g}$$

$$M_y = -2 D \beta_o^2 e^{-\beta_o y} (A_o \sin \beta_o y - B_o \cos \beta_o y) \tag{24h}$$

$$Q_x = 0 \tag{24i}$$

$$Q_y = -2 D \beta_o^3 e^{-\beta_o y} \left(- \left[A_o - B_o \right] \sin \beta_o y + \left[A_o + B_o \right] \cos \beta_o y \right) \tag{24j}$$

$$u_o = 0 \tag{24k}$$

$$v_o = \frac{e^{-\beta_o y}}{2\beta_o} \Big\{ (\sin \beta_o y - \cos \beta_o y) A_o - (\cos \beta_o y + \sin \beta_o y) B_o + \text{constant} \Big\} \tag{24ℓ}$$

All quantities related to the product type solution (Equation 12a) displaying exponentially decaying stress fields in the y direction have now been evaluated. The companion solution, i.e., the solution which gives rise to an exponentially decaying stress field in the x direction, can be developed in a similar manner. Here the stress function is taken as

$$\Phi = \sum_n e^{\bar{\beta} x} \cos \bar{\alpha}_m y \tag{25a}$$

where

$$\alpha_m = \frac{m\pi}{2L_y} \tag{25b}$$

Proceeding as before, Equations 12a to 24 would yield the similar relations (Equations 13, 15, and 17 through 24), with the following changes:

replace:	R_x	by	R_y
	R_y		R_x
	L_x		L_y
	L_y		L_x
	x		y
	y		x
	u_n		v_m
	v_n		u_m
	w_n		w_m
	M_x		M_y
	M_y		M_x
	M_{xy}		M_{xy}
	N_x		N_y
	N_y		N_x
	N_{xy}		N_{xy}
	p_1		p_3
	p_2		p_4
	q_1		q_3
	q_2		q_4
	α_n		$\overline{\alpha}_m$
	A_1		A_3
	A_2		A_4
	B_1		B_3
	B_2		B_4

(26)

The equations resulting from this cyclic interchange (Equation 26) will not be listed since they are employed symbolically in subsequent analysis. That is, the first solution (Equations 12a to 24) is tabulated and the companion solution can always be obtained by the cyclic interchange listed in Equation 26.

3. Satisfaction of the Boundary Conditions

It is not possible to satisfy all the boundary conditions continuously with the type solutions employed herein. Therefore, a solution procedure will be used, which satisfies the boundary conditions at discrete points. An efficient function to be employed in this procedure can be constructed from each of the component complementary solutions. To this end, each family (with unknown constants such as A_1, A_2, B_1, and B_2) is made to satisfy as many boundary conditions as possible while leaving one free constant. Thus, three of the four unknowns are evaluated from some of the boundary conditions, and the fourth unknown will be evaluated in accordance with the remaining conditions by the "Least-Squares Point-Matching" procedure.

The symmetry conditions with respect to the x axis, Equation 7a, are identically satisfied for all values of y and n (including n = 0) by virtue of the trigonometric variation. Similar requirements with respect to the y direction Equation 7b, lead to the relations, for all values of x and n ≥ 1,

$$\begin{aligned}
&\left[q_1^3 - 3q_1p_1^2 + (2+\nu)\,\alpha_n^2 q_1 + p_1\left\{\frac{(1-\nu^2)A}{D\,R_x^2}\right\}^{1/2}\right]A_1 \\
&- \left[p_1^3 - 3q_1^2p_1 - (2+\nu)\,\alpha_n^2 p_1 + q_1\left\{\frac{(1-\nu^2)A}{DR_x^2}\right\}^{1/2}\right]B_1 \\
&+ \left[q_2^3 - 3q_2p_2^2 + (2+\nu)\,\alpha_n^2 q_2 + p_2\left\{\frac{(1-\nu^2)A}{DR_x^2}\right\}^{1/2}\right]A_2 \\
&- \left[p_2^3 - 3q_2^2p_2 - (2+\nu)\,\alpha_n^2 p_2 + q_2\left\{\frac{(1-\nu^2)A}{DR_x^2}\right\}^{1/2}\right]B_2 = 0 \qquad (27a)
\end{aligned}$$

$$q_1 A_1 + p_1 B_1 + q_2 A_2 + p_2 B_2 = 0 \tag{27b}$$

$$-p_1 A_1 + q_1 B_1 - p_2 A_2 + q_2 B_2 = 0 \tag{27c}$$

$$\left[p_1^3 - 3p_1 q_1^2 - (2-\nu)\alpha_n^2 p_1\right] A_1 + \left[q_1^3 - 3p_1^2 q_1 + (2-\nu)\alpha_n^2 q_1\right] B_1$$

$$+ \left[p_2^3 - 3p_2 q_2^2 - (2-\nu)\alpha_n^2 p_2\right] A_2 + \left[q_2^3 - 3p_2^2 q_2 + (2-\nu)\alpha_n^2 q_2\right] B_2 = 0 \tag{27d}$$

Making use of Equations 15, 27b and 27c in Equations 27a and 27d and simplifying, results in the following compact set of equations

$$(q_1 b - p_1 a)A_1 + (p_1 b + q_1 a)B_1 - (q_2 b - p_2 a)A_2 - (p_2 b + q_2 a) B_2 = 0 \tag{28a}$$

$$q_1 A_1 + p_1 B_1 + q_2 A_2 + p_2 B_2 = 0 \tag{28b}$$

$$-p_1 A_1 + q_1 B_1 - p_2 A_2 + q_2 B_2 = 0 \tag{28c}$$

$$-(p_1 b + q_1 a)A_1 + (q_1 b - p_1 a)B_1 + (p_2 b + q_2 a)A_2 - (q_2 b - p_2 a)B_2 = 0 \tag{28d}$$

The above system of homogeneous equations is linearly independent and therefore has the trivial solution, $A_1 = A_2 = B_1 = B_2 = 0$. Thus, the form of the solution does not admit to termwise satisfaction of the symmetry conditions. However, in the solution procedure to be employed, all the boundary conditions which are not satisfied continuously will be satisfied at specified points. With this in mind, Equation 28d is arbitrarily omitted and the remaining three equations are solved in the terms of A_1. Consequently, all the field quantities will be given in terms of the undetermined constant A_1. This constant, A_1, will be evaluated by "Least-Squares Point-Matching". One of the conditions to be matched is the vanishing of the transverse shear force at $y = 0$, which is required to satisfy Equation 28d.

A similar argument for the companion solution (Equation 26) results in an additional constant A_3, which must be evaluated in the same manner as was employed for A_1.

For the special case $n = 0$, the symmetry conditions lead to the non-vanishing field equations

$$w = A_o (\cos \beta_o y + \sin \beta_o y)\, e^{-\beta_o y} \tag{29a}$$

$$N_x = [(1-\nu^2)AD]^{1/2}\, 2\beta_o^2 e^{-\beta_o y} A_o (\cos \beta_o y + \sin \beta_o y) \tag{29b}$$

$$M_x = -2\nu D\beta_o^2 e^{-\beta_o y} A_o (\sin \beta_o y - \cos \beta_o y) \tag{29c}$$

$$M_y = -2D\beta_o^2 e^{-\beta_o y} A_o (\sin \beta_o y - \cos \beta_o y) \tag{29d}$$

$$Q_y = -4D\beta_o^3 e^{-\beta_o y} A_o \cos \beta_o y \tag{29e}$$

$$v = \frac{e^{-\beta_o y}}{\beta_o} A_o [1 - \cos \beta_o y] \tag{29f}$$

where again, A_o is the undetermined coefficient and is to be evaluated by point-matching methods.

Thus, the dome solutions may be combined with undetermined parameters A_i and B_i as follows:

$$u_D = \sum_{i=0}^{I} A_i (\bar{u}_{1i}(x,y) \cos\theta + \bar{v}_{1i}(x,y) \sin\theta) + \sum_{j=0}^{J} B_j (\bar{u}_{2j} \cos\theta + \bar{v}_{2j} \sin\theta) + u_p \tag{30a}$$

$$v_D = \sum_{i=0}^{I} A_i (\bar{u}_{1i}(x,y) \sin\theta + \bar{v}_{1i} \cos\theta) + \sum_{j=0}^{J} B_j (-\bar{u}_{2j} \sin\theta + \bar{v}_{2j} \cos\theta) + v_p \tag{30b}$$

$$w_D = \sum_{i=0}^{I} A_i w_{1i}(x,y) + \sum_{j=0}^{J} B_j w_{2j}(x,y)$$

$$M_r = \sum_{i=0}^{I} A_i \left(m_{1ix} \frac{1+\cos 2\theta}{2} + m_{1iy} \frac{1-\cos 2\theta}{2} + m_{1ixy} \sin 2\theta\right)$$

$$+ \sum_{j=0}^{J} B_j \left(m_{2jx} \frac{1+\cos 2\theta}{2} + m_{2jy} \frac{1-\cos 2\theta}{2} + m_{2jxy} \sin 2\theta\right)$$

$$N_r = \sum_{i=0}^{I} A_i \left(n_{1ix} \frac{1+\cos 2\theta}{2} + n_{1iy} \frac{1-\cos 2\theta}{2} + n_{1ixy} \sin 2\theta\right)$$

$$+ \sum_{j=0}^{J} B_j \left(n_{2jx} \frac{1+\cos 2\theta}{2} + n_{2jy} \frac{1-\cos 2\theta}{2} + n_{2jxy} \sin 2\theta\right) + N_p(x, y)$$

$$\overline{N}_{r\theta} = \sum_{i=0}^{I} A_i \left(n_{1iy} - n_{1ix} \quad \frac{\sin 2\theta}{2} + n_{1ixy} \cos 2\theta\right)$$

$$+ \sum_{j=0}^{J} B_j \left(n_{2jy} - n_{2jx} \quad \frac{\sin 2\theta}{2} + n_{2jxy} \cos 2\theta\right) + \overline{N}_p \tag{30f}$$

$$\overline{Q}_r = \sum_{i=0}^{I} A_i \left(q_{1ix} \cos\theta + q_{1iy} \sin\theta \quad - 2m_{1ixy} \sin 2\theta + (m_{1ix} - m_{1iy}) \cos 2\theta\right)$$

$$+ \sum_{j=0}^{J} B_j \left(q_{2jx} \cos\theta + q_{2jy} \sin\theta \quad - 2m_{2jxy} \sin 2\theta + (m_{2jx} - m_{2jy}) \cos 2\theta\right) \tag{30g}$$

$$\Phi_r = \sum_{i=0}^{I} A_i \left(\varphi_{1ix} \cos\theta + \varphi_{1iy} \sin\theta\right) + \sum_{j=0}^{J} B_j \left(\varphi_{2jx} \cos\theta + \varphi_{2jy} \sin\theta\right) \tag{30h}$$

where the $\overline{u}_{1i}$, $\overline{u}_{2j}$, $\overline{v}_{1i}, \ldots, \varphi_{2jy}$ are the unit solutions, functions of x and y and y($\theta = \tan^{-1} \frac{y}{x}$), as developed in subsection C-2. For example, $\overline{u}_{1i}$ used in

Equation (30a) is the u of (22a), whereas the $\bar{u}_{2j}$ of (30a) is the u of (22a) after the replacements indicated by Equation (26) have been made.

Although the solution, characterized by negative exponentials in the y direction, gives rise to a decaying stress field, the companion solution will have a trigonometric variation in the same direction. Therefore, the far field boundary conditions, Equations 6, will be satisfied in a least-squares sense. For example, consider the first condition of 62. At $x = L_x$, the particular solution satisfies the boundary condition and the remaining two solutions must vanish. For n equal to an odd integer, N_x given by Equation 20a vanishes identically. However, this will not be the case for n equal to even integers. The companion solution will decay exponentially for all values of the index. The vanishing of the complementary solutions at $x = L_x$ must be accomplished pointwise.

The general description presented here is characteristic of all the far field boundary conditions (Equations 6). Satisfaction of these conditions as well as the shell joining will be described in Section IV.

SECTION IV
FLARE-DOME SHELL POINT-MATCHING

A. INTRODUCTION

As has been shown, solutions for the dome and flare shell equations of equilibrium and compatibility consist of truncated series [Section II Equations (39) and Section III Equations (30)], each term of which satisfies the homogeneous differential equations and is multiplied by an undetermined coefficient, and particular solutions which satisfy the nonhomogeneous shell equations.

The coefficients of the solutions to the homogeneous equations must now be determined to obtain expressions which satisfy the boundary conditions at the top edge of the cylinder-flare, the flare-dome junction, and the far field of the dome. In addition, the resulting solutions must satisfy conditions of symmetry about the x and y axes. The symmetry conditions follow from the assumption of constant (but different from one another) curvatures in the principal dome directions.

B. POINT-MATCHING

To determine the A_i and B_i of Equations (30) (Section II), boundary and symmetry conditions will be satisfied pointwise. For example, at the flare-dome intersection,

compatibility requires:

$$\begin{aligned} U_F &= U_D \\ V_F &= V_D \\ W_F &= W_D \\ \Phi_S &= \Phi_r \end{aligned} \tag{1}$$

and equilibrium requires:

$$\begin{aligned} N_S &= N_r \\ N_{S\theta} &= N_{r\theta} \\ \bar{Q}_S &= \bar{Q}_r \\ M_S &= -M_r \end{aligned} \tag{2}$$

Therefore, select one point, r_i, θ_t (see Figure 9) at the intersection curve of the two shells, and generate eight linear equations in the A, B, C for each point.

In a similar manner, point-matching the dome membrane conditions at $x = L_x$ (Figure 9) and various y's:

$$\begin{aligned} N_x &= \text{membrane force} \\ N_{xy} &= 0 \\ Q_x &= 0 \\ M_x &= 0 \end{aligned} \tag{3}$$

at $y = L_y$ (Figure 9) and various x's:

$$\begin{aligned} N_y &= \text{membrane force} \\ N_{xy} &= 0 \\ \bar{Q}_y &= 0 \\ M_y &= 0 \end{aligned} \tag{4}$$

leads to four simultaneous equations in the A_i and B_j for each point matched.

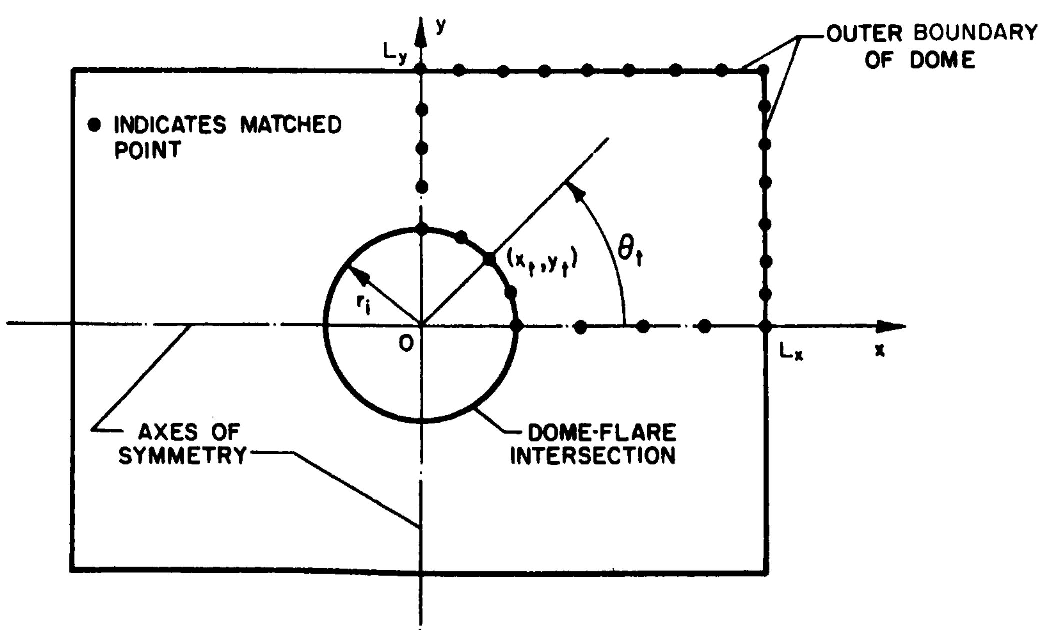


Figure 9. **Typical Array of Matched Points**

Symmetry conditions about the x and y axes, not automatically satisfied by the trial functions are point-matched:

$$\text{at } x = 0 \text{ and various y's, } \bar{Q}_x = 0 \tag{5}$$

while

$$\text{at } y = 0 \text{ and various x's, } \bar{Q}_y = 0 \tag{6}$$

Conditions at the cylinder edge need not be point-matched, since the M_S, N_S, $N_{S\theta}$, $\bar{Q}_S$ automatically satisfy membrane boundary conditions created by internal pressurization.

By judicious selection of the points matched, an equal number of unknowns (A_i, B_j and $C_k{}^n$) and linear point-matched algebraic equations may be obtained. This can be achieved by the appropriate choice of the maximum Fourier index $\bar{n}$ and the maximum dome summation indeces I and J. These are then solved simultaneously for the constants and reinserted in the series solutions of Sections II and III. However, if the number of unknowns is less than the number of equations, a solution may be obtained in the least-square sense. This is explained in the following subsection.

C. LEAST-SQUARES

Let the system of linear equations, generated as described above, be denoted as

$$\begin{aligned}
C_{11}\,x_1 + C_{12}\,x_2 + C_{13}\,x_3 + \dots + C_{1n}\,x_n - T_1 &= 0 \\
C_{21}\,x_1 + C_{22}\,x_2 + C_{23}\,x_3 + \dots + C_{2n}\,x_n - T_2 &= 0 \\
C_{31}\,x_1 + C_{32}\,x_2 + C_{33}\,x_3 + \dots + C_{3n}\,x_n - T_3 &= 0 \\
\cdot \qquad \cdot \qquad \cdot \qquad \cdot \qquad &\cdot \quad \cdot \\
C_{m1}\,x_1 + C_{m2}\,x_2 + C_{m3}\,x_3 + \dots + C_{mn}\,x_n - T_m &= 0
\end{aligned} \tag{7}$$

where $m > n$, and $x_1, x_2 \ldots x_n$ are the unknowns.

The equations (7) may be expressed in matrix form as

$$[C]\{x\} - \{T\} = 0 \tag{8}$$

where $[C]$ is the m x n matrix of the coefficients C_{11}, C_{12}, ... C_{mn}; $\{x\}$ is the column matrix $x_1, x_2, .. x_n$, and $\{T\}$ is the column matrix $T_1, T_2, .. T_m$. The well-known least squares procedure is to pre-multiply the equation (8) by the transpose of $[C]$, say $[C]'$, that is, form the equation

$$[C]'[C]\{x\} - [C]'\{T\} = 0 \tag{9}$$

The resulting system of equations (9) is n equations in n unknowns. Furthermore, if $[C]$ is of rank n and $[C]'[C]$ is of rank n, then the system of equations has a unique solution. It shall now be established that the above procedure minimizes the sum of the squares of the residuals, $\sum_{i=1}^{m} e_i^2$ where

$$e_i = C_{i1}x_1 + C_{i2}x_2 + \dots + C_{in}x_n - T_i\,, \tag{10}$$

thereby justifying the name "least-squares."

For the purposes of economy of notation, we rewrite (10) as

$$e_i = C_{ik}x_k - T_i \qquad \begin{matrix}(k = 1, 2, \dots n)\\(i = 1, 2, \dots m;\ m > n)\end{matrix} \tag{11}$$

where the repeated subscript indicates summation and the rank of the $[C]$ matrix is n.

Let y be defined by

$$y = e_i\, e_i \tag{12}$$

For stationary values,

$$\frac{\partial y}{\partial x_j} = 0 = 2e_i \frac{\partial e_i}{\partial x_j} = 2\, e_i\, C_{ij} \qquad (j = 1, 2, .. n) \tag{13}$$

From (11) and (13),

$$C_{ik}\, C_{ij}\, x_k - T_i\, C_{ij} = 0 \tag{14}$$

Now $C_{ik}\ C_{ij}$ is the kj element of $[C]'\ [C]$ which is taken to be of ran n, that is, it is nonsingular. Let x_k^o be the unique solution of (14).

Now consider $y = y\ (x_k)$. Then by Taylor's expansion about x_k^o,

$$y\ (x_k) = y\ (x_k^o) + \left(\frac{\partial y}{\partial x_k}\right)_o (x_k - x_k^o)$$
$$+ \left(\frac{\partial^2 y}{\partial x_j\,\partial x_k}\right)_o (x_j - x_j^o)\ (x_k - x_k^o) \tag{15}$$

The series terminates because all higher derivatives are zero.

Therefore, since the first partial derivatives are zero,

$$y\ (x_k) - y\ (x_k^o) = \left(\frac{\partial^2 y}{\partial x_j\,\partial x_k}\right)_o (x_j - x_j^o)\ (x_k - x_k^o)$$

$$y\ (x_k) - y\ (x_k^o) = 2\ C_{ik}\ C_{ij}\ \delta_j\ \delta_k \tag{16}$$

where $\delta_j = x_j - x_j^o$ (17)

$$y\ (x_k) - y\ (x_k^o) = 2\ (C_{ij}\ \delta_j)\ (C_{ik}\ \delta_k)$$
$$= \text{sum of squares} \geq 0$$

Thus, $y\ (x_k) - y\ (x_k^o) \geq 0$

But $C_{ik}\ \delta_k = 0$ implies that $\delta_k = 0$, since the rank of C is n. Therefore

$$y\ (x_k) - y\ (x_k^o) > 0 \quad \text{if} \quad x_k - x_k^o \neq 0$$

and consequently $y\ (x_k^o)$ is an absolute minimum.

SECTION V
COMPUTER PROGRAM

A. INTRODUCTION

A computer program which employs the analyses presented in Sections II through IV has been developed. It consists of six separate parts as described below:

Part 1: Computer unit solutions of the flare equations. Output from this portion of the program includes five tapes to be input in subsequent parts of the program.

Part 2: Obtains the point-matching matrix $[a_{ij}]$, and right-hand side $\{y_i\}$ for the system $[a_{ij}]\{x_i\} = \{y_i\}$, where the x_i are the unknowns A_i, B_j, and C_k^n. Output from this part is used as input to Part 3.

Part 3: Computes the least-square matrix if $[a_{ij}]$ is not a square matrix (if it is square, this step is omitted), normalizes the point-matching equations, orders them to facilitate more accurate solutions, and solves for the A_i, B_j, and C_k^n, using double precision. These are reproduced on tape which is used in Parts 4, 5, and 6.

Part 4: Determines the dome stresses and deflections using the A_i and B_j obtained in Part 3.

Part 5: Computes the flare stresses and deflections employing the C_k^n of Part 3, together with tapes 1 - 5 developed in Part 1.

Part 6: Checks the point-matching equations of Part 2, to determine how well they are satisfied, using the A_i, B_j and C_k^n of Part 3 and the $[a_{ij}]$ of Part 2.

It should be pointed out that although the program appears segmented, it is possible to run each part consecutively in a single machine pass; if the input cards for each section are properly interspersed between the program decks of successive parts.

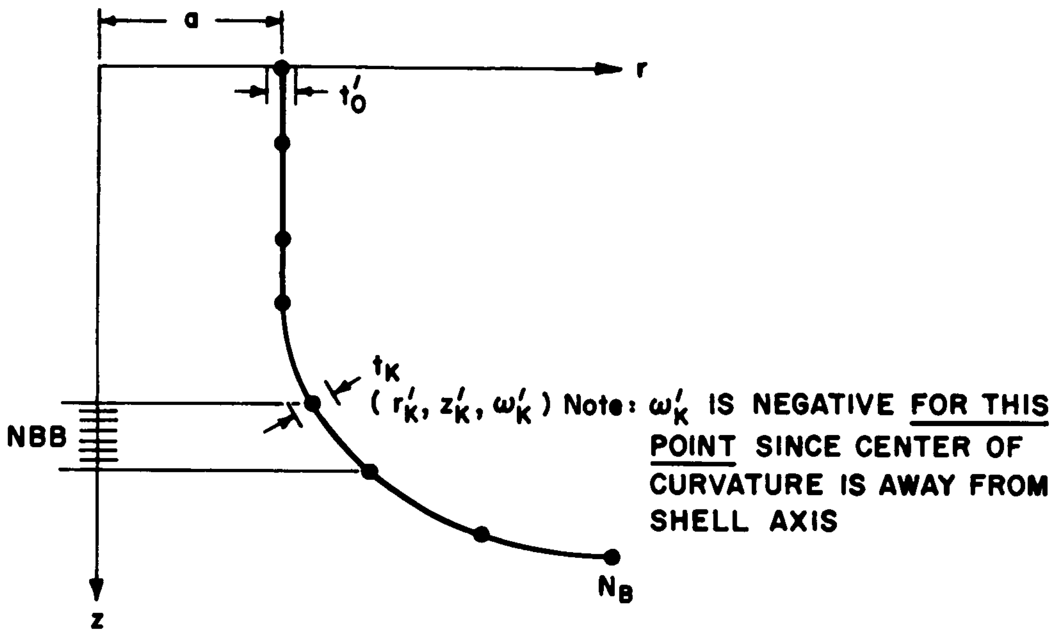
B. GEOMETRY OF FLARE

Application of the matrix solution of Section II to typical flared nozzle problems requires the specification of certain geometric quantities, from which the system-properties matrices A_i, B_i, C_i, and g_i, $i = 0, 1, 2, \ldots, N$, may be computed. Two input options, explained below, are available for this purpose.

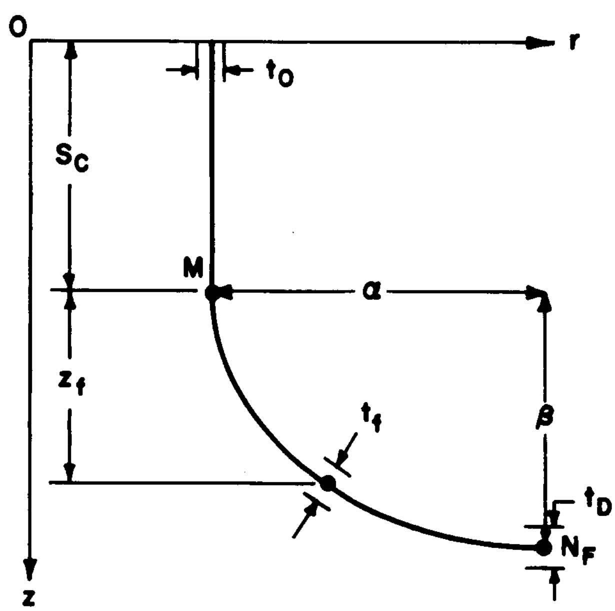
Option 1 inputs an array of axial and radial coordinates on the shell's meridional middle-surface, as well as the thickness and meridional curvature at these locations (see Figure 10a). A cubic curve fit is then made through the first three points with the additional condition that slopes of shell thickness, meridional curvature, and middle-surface, are zero with respect to the shell axis. This simplification is permissible, since the nozzle starts out as a circular cylinder of uniform wall thickness. The length of curve between the first two points is then divided into $\bar{\bar{N}}$ finite difference intervals, equally spaced with respect to the nozzle's axial coordinate. A cubic fit is then made, using the second, third, and fourth points, as well as the slope of the previous cubic, at the second input station. This back-stepping of the cubic fit is continued to insure a smooth flare contour. The interval between the second and third points is again subdivided into $\bar{\bar{N}}$ finite difference intervals, equally spaced with respect to the nozzle axis. This process is continued until finally, the last three input points $\bar{N}-2$, $\bar{N}-1$, and $\bar{N}$, as well as the slope of the previous cubic slope at the $\bar{N}-2$ station, are used to generate the last cubic, which is then subdivided into $2\bar{\bar{N}}$ equal intervals, between $\bar{N}-2$ and N, along the cylinder axis. Thus, the nozzle geometry is subdivided into $N = \bar{N} \times \bar{\bar{N}}$ finite difference intervals with $N + 1$ end points denoted by $i = 0, 1, 2, \ldots, N$. Next, dimensionless arc-lengths, Δ_i, are computed using the Pythagorean Theoreum and coordinates of adjacent points. Then, the following quantities, in the order indicated, are computed employing parabolic central finite-difference differentiations where appropriate:

$$\rho_i,\ \rho_i',\ \gamma_i,\ \omega_{\theta i} = \frac{\sqrt{1-(\rho_i')^2}}{\rho_i},\ \omega_s',$$

$$b_i = \frac{t_i/t_c}{1-\nu^2},\ b_i',\ d_i = \frac{(t_i/t_c)^3}{12(1-\nu^2)},\ d_i',\ \lambda = \frac{t_c}{r_c}$$



a. Input Option 1



b. Input Option 2

Figure 10. Cylinder-Flare Input Geometry

It is now possible to compute the a's and b's of Appendices 1 and 2, from which the matrices of Section II, Equations (14b, c, d, etc.) may be obtained.

Option 2 inputs the radius, a, of a circular cylinder and the semi-horizontal, α, and vertical, β, axes of a quarter of an elliptical torus which mates smoothly with the cylinder (see Figure 10b). The variables z_f, t_f, and t_D permit a quadratic thickness variation in the flare. The cylinder, of length S_c, is divided into M equally spaced difference intervals, while the total number of intervals for the combined shell is N_F. The elliptical dimensionless arc lengths, are given by

$$\Delta_i = \frac{\sqrt{(r_i - r_{i-1})^2 + (\bar{z}_i - \bar{z}_{i-1})^2}}{r_c}$$

where:

$$r_i = a + \alpha (1-\cos \varphi_i)$$

$$\bar{z}_i = \beta \sin \phi_i + S_c$$

and:

$$\varphi_i = \frac{(i-M)\,\pi}{2(N-M)}, \quad i = M + 1,\ M + 2,\ \ldots, N$$

The ω_{si} and ω'_{si} are computed from

$$\omega_{si} = - r_c \beta \alpha \left[\alpha^2 \sin^2 \varphi + \beta^2 \cos^2 \varphi_i\right]^{-3/2}$$

$$\omega'_{si} = 3 \left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right) \omega_s^2 \cos \varphi_i \sin \varphi_i$$

and the remaining quantities, ρ_i, ρ'_i, γ_i, b_i, b'_i, d_i, d'_i, and $\omega_{\theta i}$ are computed as for Option 1.

C. DOME GEOMETRY

For a complete description, the dome geometry simply requires the principal curvatures, the dome boundaries L_x, L_y, a constant thickness t_D, and boundary radius, r_i, at which the dome joins with the flare.

The outer boundaries of the dome, defined by L_x and L_y, remain somewhat arbitrary in that they are usually not uniquely defined, but must be chosen within certain bounds. These bounds are determined as follows:

(1) The minimum distance away from the flare intersection curve at which localized nozzle attachment effects must be essentially negligable is a lower bound for L_x and L_y and,

(2) L_x and L_y must be sufficiently small so as not to violate the dome assumptions such as constant principal radii of curvature and shallowness.

Based on experimental data, the minimum L_x and L_y should be approximately four times the flare-dome intersection radius. However, the only true method of establishing whether the magnitudes of L_x and L_y are adequate, is to rerun any given problem with different $L_{x's}$ and $L_{y's}$. This should be repeated until the stress and deflection results stabilize to the same value near the nozzle junction, while still decaying to the membrane stress field at L_x and L_y. These same comments are applicable for the cylinder length coordinate at $z = 0$, where it is assumed that membrane conditions prevail.

D. POINT-MATCHING

The points at which boundary conditions will be matched are indicated by dots in Figure 11.

NDY is the number of points along the y axis at which $\bar{Q}_x$ is set equal to zero. The YD are the ordinates of these points. In a similar manner, NDX and NDXP establish the points at which $\bar{Q}_y$ is set equal to zero.

NDXP is the number of points along $y = L_y$ at which membrane boundary conditions, of which there are four, are satisfied, and XPD are the specific points. Similarly, NDYP and YPD relate to membrane conditions along $x = L_x$. The membrane condition used are presented in Section IV, Equations (3) and (4).

NDTHP is the number of points along the flare-dome intersection at which compatibility and equilibrium are satisfied (Equations (1) and (2) of Section IV). The specific points are defined by the THDP.

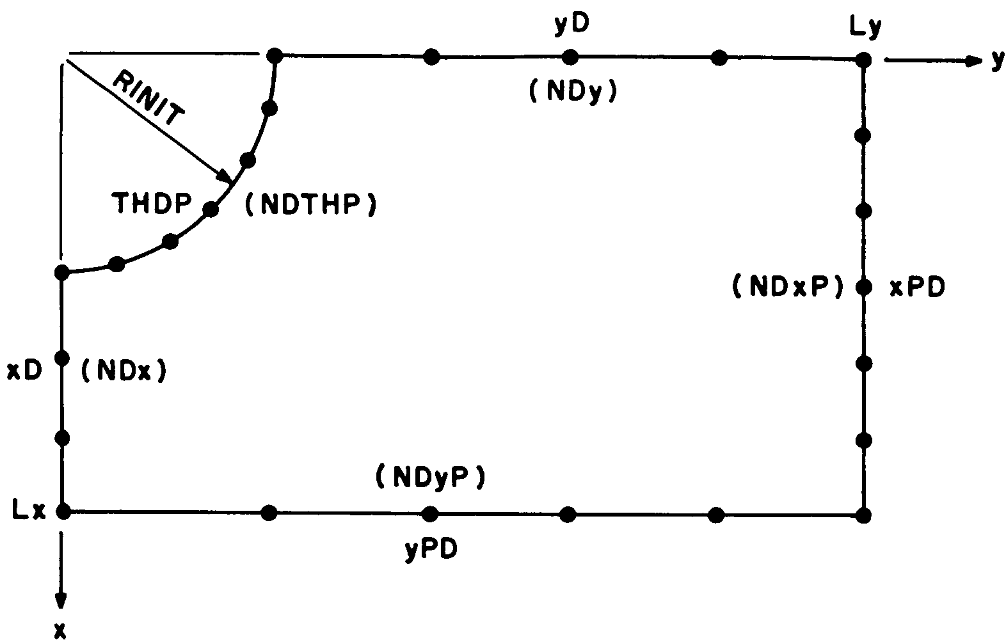


Figure 11. Points at Which Boundary Conditions are Matched

E. NOMENCLATURE AND PROGRAM FLOW CHARTS

The Input Nomenclature is presented in the following pages and Appendix IV contains the program Flow Charts. In addition, a separate simplified User's Manual, supplements this report.

INPUT NOMENCLATURE

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
E	psi	EF, ED, ELAS	Modulus of elasticity; assumed the same for dome-flare-cylinder configuration
γ		NU, NUD	Poisson's ratio
p	psi	PSI	Internal pressure
r_c	in.	RC	Characteristic radius
t_c	in.	TC	Characteristic radius

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
σ_c	psi	SIGC	Characteristic stress
$\bar{n}$		NBAR	Maximum Fourier index for flare

If NC option 2 = 1, use the following flare input geometry. (Reference Figure 9a)

$\bar{N}$		NB	Number of flare input coordinate points
$\bar{\bar{N}}$		NBB	Number of finite-difference intervals into which segment between successive input points is subdivided
N_F		NF	Number of flare finite difference intervals = $\bar{N} \cdot \bar{\bar{N}}$
a	in.	A, ACR	Radius of cylinder
r'_k	in.	RPK	Radii of cylinder-flare input points
z'_k	in.	ZPK	Vertical coordinate of cylinder-flare input points
t'_o	in.	TZP	Thickness at top of cylinder
ω'_o	$in.^{-1}$	OMZP	Curvature at top of cylinder
ω'_k	$in.^{-1}$	OMPK	Curvature at cylinder-flare input points

If NC Option 2 = 0, use the following flare input geometry. (Reference Figure 9b)

α	in.	ALPHA	Horizontal semi-axis (ellipse) for flare geometry
β	in.	BETA	Vertical semi-axis (ellipse) for flare geometry
z_F	in.	ZF	Flare thickness parameter; adjusted so quadratic flare thickness approximates desired thickness variation
S_c	in.	SC	Cylinder length
$\bar{t}_D$	in.	TTDD	Flare thickness parameter
L_x	in.	LX	Dome half-span in x direction
L_y	in.	LY	Dome half-span in y direction

Physical Symbols	Dimensional Units	Program Symbols	Physical Description
t_D	in.	TD	Dome thickness
$1/R_x$	in.$^{-1}$	RATIO X	Dome curvature in x direction
$1/R_y$	in.$^{-1}$	RATIO Y	Dome curvature in y direction
I		ID	Maximum index on series which decays exponentially in y direction
J		JD	Maximum index on series which decays exponentially in x direction
NDY		NDY	Number of points matched along y axis
NDX		NDX	Number of points matched along x axis
NDYP		NDYP	Number of points matched along $y = L_y$
NDXP		NDXP	Number of points matched along $x = L_x$
NDTHP		NDTHP	Number of points matched along $r = r_i$
r_i	in.	RINIT	Radius of flare-dome intersection
Δr (or ΔR)	in.	DELTAR	Radial increment for output stations for stress and deflection computation
x_o	in.	XO	Cartesian output stations
Δx	in.	DELTAX	Increment for cartesian output stations
Δy	in.	DELTAY	Increment for cartesian output stations
θ_o	degrees	THETAO	Polar output stations
$\Delta\theta$	degrees	DTHETA	Polar increment for output stations
θ_ℓ	degrees	THLAST	Final angle for polar output stations

SECTION VI

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APPENDIX I

FORMULAS FOR THE "a" COEFFICIENTS OF SECTION II

$$a_1 = \left[b + d\lambda^2 (\omega_s - \omega_\theta)\,\omega_\theta \right]$$

$$a_2 = \left[\gamma b + b' + \gamma d \lambda^2 (\omega_s - \omega_\theta)^2 + d' \lambda^2 \omega_\theta (\omega_s - \omega_\theta) + d\lambda^2 \omega_s' \omega_\theta \right]$$

$$\begin{aligned} a_3 = {} & \gamma(\nu b' - \gamma b) - \nu b \omega_s \omega_\theta - \frac{b(1-\nu)}{2}\left(\frac{n}{\rho}\right)^2 \\ & + \nu\lambda^2 \left[(\gamma d' \omega_s - \gamma^2 d \omega_s - d\omega_s^2 \omega_\theta)(\omega_s - \omega_\theta) \right. \\ & \left. + \gamma d \omega_s' (2\omega_s - \omega_\theta) + \nu d \gamma^2 \omega_s^2 \right] \\ & - \lambda^2 d \left(\gamma^2 + \frac{(1-\nu)}{2}\left(\frac{n}{\rho}\right)^2 \right)\left(3\omega_s^2 - 3\omega_\theta \omega_s + \omega_\theta^2 \right) \end{aligned}$$

$$a_4 = \frac{b(1+\nu)}{2}\,\frac{n}{\rho} + d\lambda^2 \frac{n}{\rho}\left[\frac{1+\nu}{2}\omega_s \omega_\theta - \nu \omega_\theta^2 \right]$$

$$\begin{aligned} a_5 = {} & \left[\nu b' - \frac{(3-\nu)}{2}\gamma b \right]\frac{n}{\rho} + \nu\lambda^2 \frac{n}{\rho}\left[d' \omega_\theta (\omega_s - \omega_\theta) \right. \\ & \left. + d\omega_\theta \omega_s' \right] + \lambda^2 \left(\frac{n}{\rho}\right)\gamma d \left[-(1-\nu)\omega_s^2 \right. \\ & \left. - \left(\frac{1+5\nu}{2} - \nu^2 \right)\omega_s \omega_\theta + 2\nu \omega_\theta^2 \right] \end{aligned}$$

$$a_5' = -\nu d \lambda^2 \gamma (\omega_s - \omega_\theta)$$

$$\begin{aligned} a_6 = {} & b(\omega_s + \nu\omega_\theta) - \nu\gamma\lambda^2 \left[d'(\omega_s - \omega_\theta) + d\omega_s' \right] \\ & + d\lambda^2 \left[(1+\nu)(\omega_s - \omega_\theta)(\omega_s \omega_\theta + \gamma^2) \right. \\ & \left. + (1-\nu^2)\gamma^2 \omega_s + \frac{(3-\nu)}{2}\left(\frac{n}{\rho}\right)^2 \omega_s - \frac{(1+\nu)}{2}\omega_\theta \left(\frac{n}{\rho}\right)^2 \right] \end{aligned}$$

$$\begin{aligned} a_7 = {} & b'(\omega_s + \nu\omega_\theta) + \lambda^2 \left(\omega_s \omega_\theta + \nu \frac{n^2}{\rho^2} \right)\left[d'(\omega_s - \omega_\theta) \right. \\ & \left. + d\omega_s' \right] + b\left[\gamma(\omega_s - \omega_\theta) + \omega_s' \right] + \text{(continued)} \end{aligned}$$

$$a_7 \text{ (continuation)} + d\lambda^2 \omega_\theta (\omega_s - \omega_\theta)\omega_s'$$
$$+ d\gamma\lambda^2 \Big[(\omega_s - \omega_\theta)^3 - \nu \omega_s^2 (\omega_s - \omega_\theta)$$
$$- \left(\frac{n}{\rho}\right)^2 \Big\{ (\omega_s - \omega_\theta) - \frac{(1-5\nu)}{2}(\omega_s - \omega_\theta)$$
$$+ (1-\nu^2)\omega_s + (1-\nu)(2\omega_s - \omega_\theta) \Big\} \Big]$$
$$a_8 = -\lambda^2 \omega_\theta$$
$$a_9 = \lambda^2 [\omega_s' - \gamma(1-\nu)\omega_s]$$
$$a_{10} = -\frac{b}{2}\left(\frac{n}{\rho}\right)(1+\nu) - \lambda^2 d\, \omega_\theta \left(\frac{n}{\rho}\right)\left[\frac{(1+\nu)}{2}\omega_s - \nu\omega_\theta\right]$$
$$a_{11} = -\left(\frac{n}{\rho}\right)\left[\frac{b'(1-\nu)}{2} + b\gamma\right] - \left(\frac{n}{\rho}\right)\lambda^2 \omega_\theta \frac{(1-\nu)}{2}(d'\omega_s + d\omega_s')$$
$$- \left(\frac{n}{\rho}\right)\lambda^2 \gamma d\, \omega_s \left[\frac{(3-\nu)}{2}\omega_s - \nu^2 \omega_\theta\right]$$
$$a_{12} = \frac{(1-\nu)}{2}\left[b + d\lambda^2(\omega_s^2 - 3\omega_\theta\omega_s + 3\omega_\theta^2)\right]$$
$$a_{13} = \frac{(1-\nu)}{2}\Big[b' + d'\lambda^2(\omega_s^2 - 3\omega_\theta\omega_s + 3\omega_\theta^2)$$
$$+ d\lambda^2 \omega_s'(2\omega_s - 3\omega_\theta) + d\lambda^2\gamma(2\omega_\theta - 3\omega_s)(\omega_s - 2\omega_\theta)\Big]$$
$$a_{14} = \frac{(1-\nu)}{2}\Big[b\omega_s\omega_\theta - b'\gamma - d'\lambda^2\gamma(\omega_s^2 - 3\omega_s\omega_\theta + 3\omega_\theta^2)$$
$$+ d\lambda^2\gamma\omega_s'(3\omega_\theta - 2\omega_s) + d\lambda^2\gamma^2(3\omega_s - 2\omega_\theta)(\omega_s - 2\omega_\theta)$$
$$+ d\lambda^2\omega_s\omega_\theta(\omega_s^2 - 3\omega_s\omega_\theta + 3\omega_\theta^2)\Big]$$
$$- \left(\frac{n}{\rho}\right)^2 \left[b + \lambda^2 d\omega_\theta(\omega_s - \nu^2\omega_\theta)\right]$$
$$a_{15} = \frac{(1-\nu)}{2}\left(\frac{n}{\rho}\right)\lambda^2 d\,[3\omega_\theta - \omega_s]$$

$$a_{16} = \frac{(1-\nu)}{2}\left(\frac{n}{\rho}\right)\lambda^2\left[d'(3w_\theta - w_s) - dw_s'\right]$$
$$+\left(\frac{n}{\rho}\right)\gamma d\lambda^2\left[(3w_s - 2w_\theta) - 2\nu(w_s - w_\theta) - \nu^2 w_\theta\right]$$
$$a_{17} = \frac{(1-\nu)}{2}\left(\frac{n}{\rho}\right)\gamma\lambda^2\left[d'(w_s - 3w_\theta) + dw_s'\right] - \frac{bn}{\rho}(\nu w_s + w_\theta)$$
$$-2(1-\nu)\left(\frac{n}{\rho}\right)d\gamma^2\lambda^2(w_s - w_\theta) - \left(\frac{n}{\rho}\right)^3 d\lambda^2(w_s - \nu^2 w_\theta)$$
$$+\frac{n}{2\rho}d\lambda^2 w_s w_\theta\left[(3w_\theta - w_s) - \nu(w_s + w_\theta)\right]$$
$$a_{18} = \nu\left(\frac{n}{\rho}\right)\lambda^2 w_\theta$$
$$a_{19}' = -\nu\lambda^2\gamma d(w_s - w_\theta)$$
$$a_{19} = -b(w_s + \nu w_\theta) - \nu\lambda^2\gamma d'(w_s - w_\theta) - (1-\nu)d\lambda^2 w_s w_\theta(w_s - w_\theta)$$
$$-\frac{d\lambda^2}{2}\left(\frac{n}{\rho}\right)^2\left[(3w_s - w_\theta) - \nu(w_s + w_\theta)\right]$$
$$-d\lambda^2\gamma^2\left[(2-\nu-\nu^2)w_s - (1-\nu)w_\theta\right] - \nu d\lambda^2\gamma w_s'$$
$$a_{20} = -b\gamma(\nu w_s + w_\theta) - \lambda^2 d\gamma\left[\nu w_s^3 - (3+\nu-2\nu^2)w_s^2 w_\theta + w_\theta^3\right]$$
$$-\lambda^2\gamma^2 d'\left[(2-\nu^2)w_s - w_\theta\right] - \frac{(1-\nu)}{2}\lambda^2\left(\frac{n}{\rho}\right)^2 d'(3w_s - w_\theta)$$
$$-\lambda^2\gamma^3 d\left[2w_\theta - (3-\nu^2)w_s\right] - \lambda^2 d w_s'\left[(2-\nu^2)\gamma^2\right.$$
$$\left.+\frac{3(1-\nu)}{2}\left(\frac{n}{\rho}\right)^2\right] - \lambda^2\gamma d\left(\frac{n}{\rho}\right)^2\left[\left(\frac{3}{2} + \frac{\nu}{2} - \nu^2\right)w_s\right.$$
$$\left.-\frac{(1+\nu)}{2}w_\theta\right]$$
$$a_{21} = \frac{(1-\nu)}{2}\lambda^2 d\left(\frac{n}{\rho}\right)(3w_\theta - w_s)$$
$$a_{22} = -\lambda^2\gamma d\left(\frac{n}{\rho}\right)\left[-\frac{(1-3\nu)}{2}w_s + \left(\frac{3}{2} - \frac{3}{2}\nu - \nu^2\right)w_\theta\right]$$
$$-\frac{(1-\nu)}{2}\lambda^2\left(\frac{n}{\rho}\right)\left[-d'(3w_\theta - w_s) + dw_s'\right]$$

$$a_{23} = -b\left(\frac{n}{\rho}\right)(\nu w_s + w_\theta) + \lambda^2 d\left(\frac{n}{\rho}\right) w_s w_\theta \left[\frac{(1-\nu)}{2} w_s + \left(\frac{3-\nu}{2} - \nu^2\right) w_\theta\right]$$
$$- \lambda^2 \gamma d\left(\frac{n}{\rho}\right)\left[\left(\frac{1+\nu}{2}\right) w_s + \left(\frac{3}{2} - \frac{3}{2}\nu - \nu^2\right) w_\theta\right]$$
$$- \lambda^2 \gamma^2 d\left(\frac{n}{\rho}\right)\left[\left(\frac{1-3\nu}{2} - \nu^2\right) w_s + \left(-\frac{3}{2} + \frac{3}{2}\nu + 2\nu^2\right) w_\theta\right]$$
$$- \lambda^2 \gamma d\left(\frac{n}{\rho}\right) w_s' \frac{(1+\nu)}{2} - \lambda^2 d\left(\frac{n}{\rho}\right)^3 (w_s - \nu^2 w_\theta)$$

$$a_{24} = \lambda^2 d\left[(1-\nu^2)\gamma^2 + 2(1-\nu)\left(\frac{n}{\rho}\right)^2\right]$$

$$a_{25} = -\lambda^2 \gamma d\left[(1-\nu^2)(\gamma^2 + 2 w_s w_\theta) + 2(1-\nu)\left(\frac{n}{\rho}\right)^2\right]$$
$$+ \lambda^2 d'\left[(1-\nu^2)\gamma^2 + 2(1-\nu)\left(\frac{n}{\rho}\right)^2\right]$$

$$a_{26} = -b(w_s^2 + 2\nu w_s w_\theta + w_\theta^2) - d\lambda^2 (w_s - w_\theta) w_\theta \left[(1-\nu) w_s^2\right.$$
$$\left. - w_s w_\theta - w_\theta^2\right] - d\lambda^2 \left(\frac{n}{\rho}\right)^2 \left[2(w_s - w_\theta)(\nu w_s + w_\theta)\right.$$
$$\left. - (3 - 2\nu - \nu^2) w_s w_\theta + (1-\nu^2)\left(\frac{n}{\rho}\right)^2\right]$$
$$- d'\lambda^2 \gamma\left[(w_s - w_\theta)(w_\theta + \nu w_s) + (3 - 2\nu - \nu^2)\left(\frac{n}{\rho}\right)^2\right]$$
$$- \lambda^2 \gamma d\left[\gamma (w_s - w_\theta)\{(1-\nu) w_s - 2 w_\theta\} + w_s'\{(1-\nu) w_\theta + 2\nu w_s\}\right.$$
$$\left. - 2(2 - \nu - \nu^2)\left(\frac{n}{\rho}\right)^2 \gamma\right]$$

$$a_{27} = -\lambda^2$$

$$a_{28} = -\gamma \lambda^2 (2-\nu)$$

$$a_{29} = -\lambda^2\left[w_s^2 - (2-\nu) w_s w_\theta - \nu\left(\frac{n}{\rho}\right)^2\right]$$

$$a_{30} = d w_\theta$$

$$a_{31} = d(w_s' + \nu \gamma w_s)$$

$$a_{32} = \nu d \left(\frac{n}{\rho}\right) w_\theta$$

$$a_{33} = -d$$

$$a_{34} = -\nu d \gamma$$

$$a_{35} = -d \left[\omega_s (\omega_s - \omega_\theta) - \nu \left(\frac{n}{\rho}\right)^2 \right]$$

$$a_{36} = 1$$

APPENDIX II

FORMULAS FOR THE "b" COEFFICIENTS OF SECTION II

$$b_1 = b + d\lambda^2(\omega_s - \omega_\theta)\omega_\theta$$

$$b_2 = \nu b \gamma + \nu \gamma d \lambda^2 \omega_s (\omega_s - \omega_\theta)$$

$$b_3 = \nu b\left(\frac{n}{\rho}\right) + \nu d \lambda^2\left(\frac{n}{\rho}\right)\omega_\theta(\omega_s - \omega_\theta)$$

$$b_4' = -\nu \gamma d \lambda^2(\omega_s - \omega_\theta)$$

$$b_4 = b(\omega_s + \nu\omega_\theta) + d\lambda^2(\omega_s - 2\omega_\theta)\left(\omega_s\omega_\theta + \nu\left(\frac{n}{\rho}\right)^2\right)$$

$$b_4'' = \lambda^2(\omega_s - \omega_\theta)$$

$$b_5 = -\frac{(1-\nu)}{2}\left(\frac{n}{\rho}\right)(b + \lambda^2 d\,\omega_s\omega_\theta)$$

$$b_6 = \frac{(1-\nu)}{2}\left[b + d\lambda^2\{(\omega_s - \omega_\theta)^2 + \omega_\theta(2\omega_\theta - \omega_s)\}\right]$$

$$b_7 = -\gamma b_6$$

$$b_8 = \frac{(1-\nu)}{2} d\lambda^2\left(\frac{n}{\rho}\right)(3\omega_\theta - \omega_s)$$

$$b_9 = -\gamma b_8$$

$$b_{10}' = \gamma \nu d \lambda^2(\omega_s - \omega_\theta)$$

$$b_{10} = d\lambda^2\gamma^2\left[(2\omega_s - \omega_\theta) - \nu^2\omega_s\right] + \frac{(1-\nu)}{2} d\lambda^2\left(\frac{n}{\rho}\right)^2(3\omega_s - \omega_\theta)$$

$$b_{11} = -\frac{(1-\nu)}{2} d\lambda^2\left(\frac{n}{\rho}\right)(3\omega_\theta - \omega_s)$$

$$b_{12} = d\lambda^2\gamma\left(\frac{n}{\rho}\right)\left[\omega_s - \nu^2\omega_\theta + \frac{(1-\nu)}{2}(3\omega_\theta - \omega_s)\right]$$

$$b_{13} = -2(1-\nu)\,d\lambda^2\left(\frac{n}{\rho}\right)^2 - d\lambda^2\gamma^2(1-\nu^2)$$

$$b_{14} = d\lambda^2\gamma\left[(\omega_\theta+\nu\omega_s)(\omega_s-\omega_\theta)+(3-2\nu-\nu^2)\left(\frac{n}{\rho}\right)^2\right]$$

$$b_{15} = \lambda^2$$

$$b_{16} = \lambda^2\gamma(1-\nu)$$

APPENDIX III

SHALLOW SHELL EQUATIONS

A. INTRODUCTION

A system of partial differential equations suitable for the analysis of thin, shallow shells was originally developed by Marguerre (Reference 12). The assumption of shallowness, which entails neglecting quantities such as $\left(\frac{x}{R_x}\right)^2$ in comparison to unity, represents an essential simplification in the analysis of arbitrarily shaped shells. Although the use of a shallow shell theory approach offers attractive possibilities for the solution of engineering problems, standard reference texts, such as Flügge (Reference 9) and Timoshenko and Woinowsky-Krieger (Reference 13) do not include the particulars of Marguerre's investigation. Therefore, to insure completeness of the present report, the governing equations for a shallow, elliptic-paraboloidal shell will be developed in detail.

It should be noted that the elliptic paraboloid is capable of representing a variety of shapes provided that

- The region represented is shallow
- The curvatures are essentially constant
- The Gaussian curvature is positive.

The results obtained herein also apply to shells of negative Gaussian curvature, which could be shown by following a similar development using an hyperbolic paraboloid.

The geometric assumptions (these form the basis for the shallow shell theory), the constituent stress-strain relations, and the associated equilibrium equations (compatible with the shallowness assumptions) will be discussed in Subsections B, C, and D, respectively. Reduction of the resulting system of fourteen equations, involving the same number of unknowns, to one fourth order partial differential equation for a complex deflection-stress function, will be presented in subsection E.

B. GEOMETRICAL CONSIDERATIONS

1. Shell Geometry

The geometry of a shell surface is completely determined by specifying the equation of the surface. For the elliptic-paraboloidal shell, with respect to a centrally located Cartesian coordinate system (Figure 5), this surface is represented by

$$z = -\frac{1}{2}\left(\frac{x^2}{R_x} + \frac{y^2}{R_y}\right) = f(x, y) \qquad \text{(A-1)}$$

where R_x and R_y are constants. An arbitrary point on the middle surface is given by the vector

$$\hat{r} = xi + yj + f(x, y)k \qquad \text{(A-2)}$$

where i, j, and k are unit vectors aligned in the x, y, and z directions, respectively. The locations of all other field points are measured along a normal ($\hat{n}$) to the median surface. Thus, a generic point in the shell is given by (Figure 5)

$$\hat{R}(x, y, \zeta) = \hat{r} + \zeta\hat{n} \qquad \text{(A-3)}$$

where ζ is measured along the local normal.

A preliminary step in developing the field equations is the construction of a curvilinear coordinate system on the reference surface of the shell. Since this surface is given in Monge's form (Equation A-1), then the Cartesian coordinates x and y are taken as parameters. Thus, the curvilinear coordinate system consists of the parametric curves formed by the intersection of the surface with the planes x = constant and y = constant. Unit vectors, tangent to the coordinate curves are denoted as i_x and i_y (Figure A-1) and are defined as

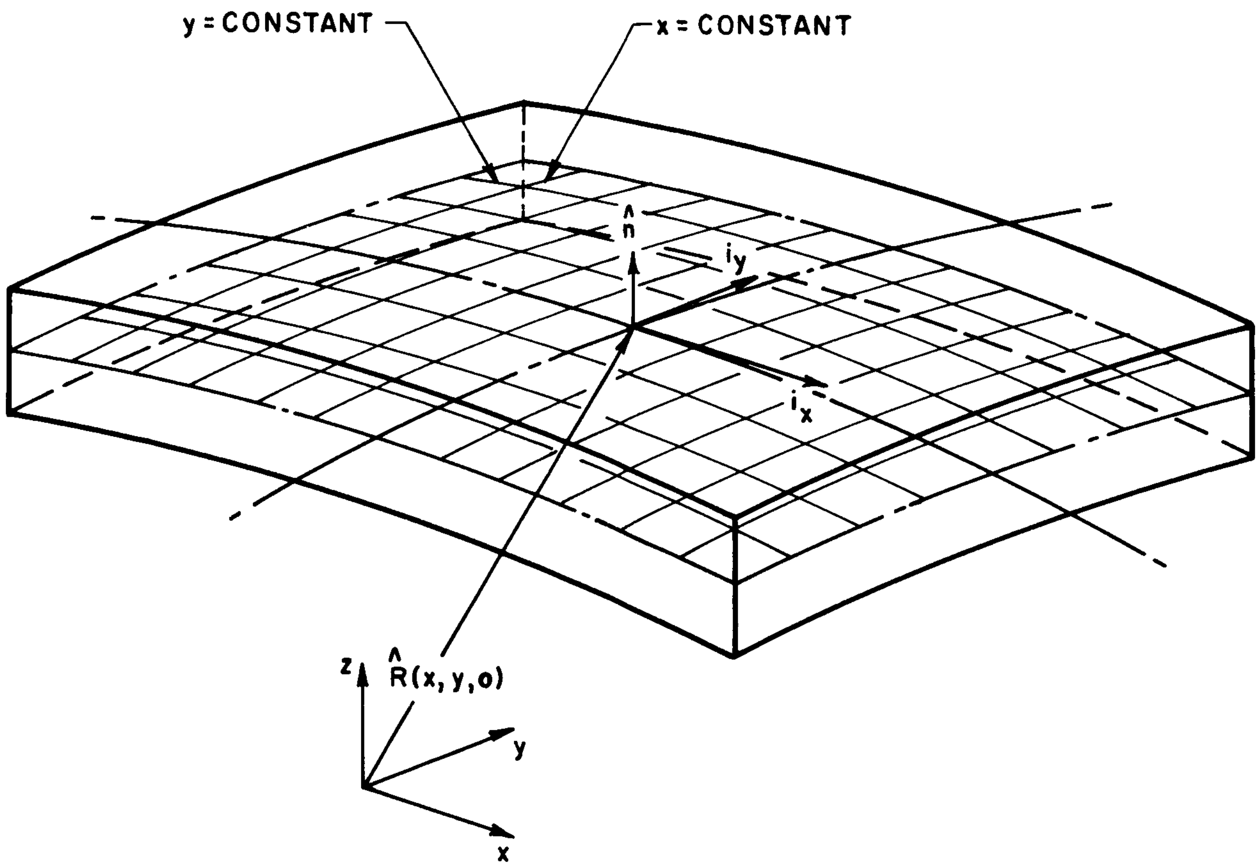


Figure A-1. Parametric Curves on an Elliptic-Paraboloidal Shell Surface

$$i_x = \frac{\dfrac{\partial \hat{R}}{\partial x}}{\left|\dfrac{\partial \hat{R}}{\partial x}\right|}$$

$$i_y = \frac{\dfrac{\partial \hat{R}}{\partial y}}{\left|\dfrac{\partial \hat{R}}{\partial y}\right|} \qquad \text{(A-4a)}$$

in addition the unit normal is

$$\hat{n} = \frac{\dfrac{\partial \hat{R}}{\partial x} \times \dfrac{\partial \hat{R}}{\partial y}}{\left|\dfrac{\partial \hat{R}}{\partial x} \times \dfrac{\partial \hat{R}}{\partial y}\right|} \qquad \text{(A-4b)}$$

Putting Equation A-3, with $\zeta = 0$, into Equations A-4 and making use of Equation A-1 results in

$$i_x = \frac{i - \dfrac{x}{R_x}\, k}{\left[1+\left(\dfrac{x}{R_x}\right)^2\right]^{\frac{1}{2}}}$$

$$i_y = \frac{j - \dfrac{y}{R_y}\, k}{\left[1+\left(\dfrac{y}{R_y}\right)^2\right]^{\frac{1}{2}}} \qquad \text{(A-5)}$$

$$\hat{n} = \frac{\dfrac{x}{R_x}\, i + \dfrac{y}{R_y}\, j + k}{\left[1+\left(\dfrac{x}{R_x}\right)^2 + \left(\dfrac{y}{R_y}\right)^2\right]^{\frac{1}{2}}}$$

Considering the shell to be shallow, then $\left(\frac{x}{R_x}\right)^2$ and $\left(\frac{y}{R_y}\right)^2$ will be negligible in comparison with unity and Equation A-5 becomes

$$i_x = i - \frac{x}{R_x} k$$

$$i_y = j - \frac{y}{R_y} k \qquad \text{(A-6)}$$

$$n = \frac{x}{R_x} i + \frac{y}{R_y} j + k$$

The differential length of arc between two neighboring points, with position vectors $\hat{R}(x, y, \zeta)$ and $\hat{R}+d\hat{R}$, respectively (Figure A-2) is

$$(ds)^2 = d\hat{R} \cdot d\hat{R} \qquad \text{(A-7a)}$$

where

$$d\hat{R} = \frac{\partial \hat{R}}{\partial x} dx + \frac{\partial \hat{R}}{\partial y} dy + \frac{\partial \hat{R}}{\partial \zeta} d\zeta \qquad \text{(A-7b)}$$

and to the order of approximation in the analysis

$$d\hat{R} = \left(i_x + \zeta \frac{\partial \hat{n}}{\partial x}\right) dx + \left(i_y + \zeta \frac{\partial \hat{n}}{\partial y}\right) dy + \hat{n}\, d\,\zeta \qquad \text{(A-7c)}$$

Putting Equation A-7c into A-7a, and carrying out the indicated operations leads to

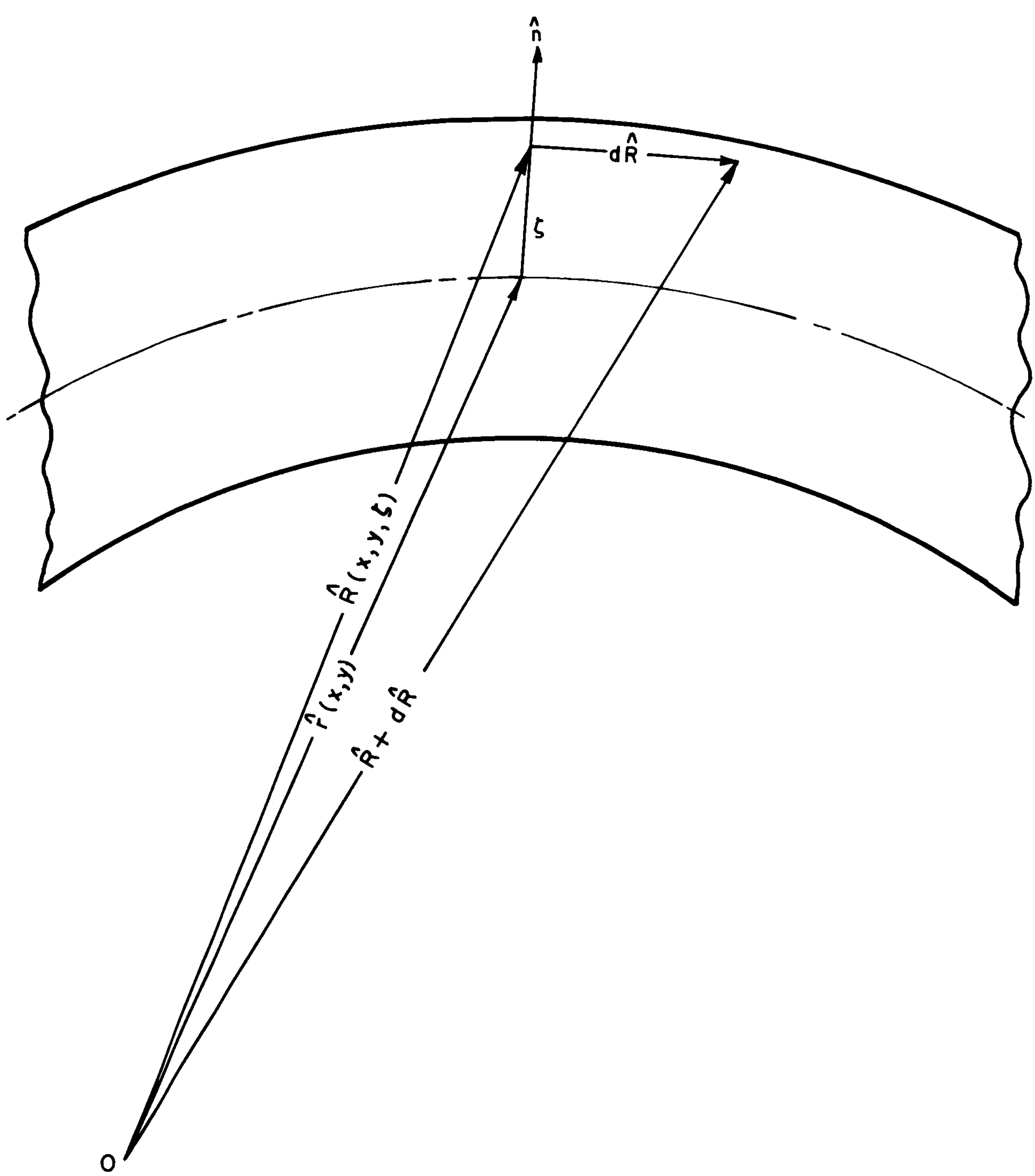


Figure A-2. Displacement of a Generic Point in the Shell

$$(ds)^2 = \left[\left(1+\frac{\zeta}{R_x}\right)^2 + \left(\frac{x}{R_x}\right)^2\right](dx)^2 + \left[\left(1+\frac{\zeta}{R_y}\right)^2 + \left(\frac{y}{R_y}\right)^2\right](dy)^2$$

$$+ \left[1+\left(\frac{x}{R_x}\right)^2 + \left(\frac{y}{R_y}\right)^2\right](d\zeta)^2 + \frac{2x\zeta}{R_x^2}\, dxd\zeta$$

$$+ \frac{2y\zeta}{R_y^2}\, dyd\zeta + \frac{2xy}{R_xR_y}\, dxdy \qquad \text{(A-7d)}$$

After neglecting terms of the type $\left(\frac{x}{R_x}\right)^2$, $\left(\frac{xy}{R_xR_y}\right)$, etc., when compared to unity, in accordance with the shallowness assumption, Equation A-7d reduces to

$$(ds)^2 = \left(1+\frac{\zeta}{R_x}\right)^2 (dx)^2 + \left(1+\frac{\zeta}{R_y}\right)^2 (dy)^2 + (d\zeta)^2 \qquad \text{(A-8a)}$$

which is the first fundamental form of the shallow elliptic-paraboloidal shell. In addition, the principal radii of curvature, R_1 and R_2, can be written as

$$\frac{1}{R_1} = \frac{1}{R_x\left(1+\left[\frac{x}{R_x}\right]^2\right)\sqrt{1+\left(\frac{x}{R_x}\right)^2+\left(\frac{y}{R_y}\right)^2}}$$

$$\text{(A-8b)}$$

$$\frac{1}{R_2} = \frac{1}{R_y\left(1+\left[\frac{y}{R_y}\right]^2\right)\sqrt{1+\left(\frac{x}{R_x}\right)^2+\left(\frac{y}{R_y}\right)^2}}$$

and, as a consequence of the shallowness assumption, R_x and R_y are approximately equal to the principal radii of curvature.

Equation A-8a is employed to determine the strain displacement components. The metric coefficients, i.e., $\left(1 + \frac{\zeta}{R_x}\right)$, $\left(1 + \frac{\zeta}{R_y}\right)$, and 1 are also required to define the stress and moment resultants.

2. Strain Displacement Relations

General expressions for the strain components in an arbitrarily shaped body have been derived by Wang (Reference 14). These expressions are referred to an orthogonal curvilinear coordinate system, where a differential length of arc is specified by

$$(ds)^2 = (A_1 d\xi_1)^2 + (A_2 d\xi_2)^2 + (A_3 d\xi_3)^2 \tag{A-9}$$

The strain-displacement relations are

$$\epsilon_1 = \frac{1}{A_1}\frac{\partial u_1}{\partial \xi_1} + \frac{u_2}{A_1 A_2}\frac{\partial A_1}{\partial \xi_2} + \frac{u_3}{A_1 A_3}\frac{\partial A_1}{\partial \xi_3} \tag{A-10a}$$

$$\epsilon_2 = \frac{1}{A_2}\frac{\partial u_2}{\partial \xi_2} + \frac{u_3}{A_2 A_3}\frac{\partial A_2}{\partial \xi_3} + \frac{u_1}{A_2 A_1}\frac{\partial A_2}{\partial \xi_1} \tag{A-10b}$$

$$\epsilon_3 = \frac{1}{A_3}\frac{\partial u_3}{\partial \xi_3} + \frac{u_1}{A_3 A_1}\frac{\partial A_3}{\partial \xi_1} + \frac{u_2}{A_3 A_2}\frac{\partial A_3}{\partial \xi_2} \tag{A-10c}$$

$$\gamma_{12} = \frac{A_2}{A_1}\frac{\partial}{\partial \xi_1}\left(\frac{u_2}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \xi_2}\left(\frac{u_1}{A_1}\right) \tag{A-10d}$$

$$\gamma_{13} = \frac{A_1}{A_3}\frac{\partial}{\partial \xi_3}\left(\frac{u_1}{A_1}\right) + \frac{A_3}{A_1}\frac{\partial}{\partial \xi_1}\left(\frac{u_3}{A_3}\right) \tag{A-10e}$$

$$\gamma_{23} = \frac{A_3}{A_2}\frac{\partial}{\partial \xi_2}\left(\frac{u_3}{A_3}\right) + \frac{A_2}{A_3}\frac{\partial}{\partial \xi_3}\left(\frac{u_2}{A_2}\right) \tag{A-10f}$$

where u_1, u_2, and u_3 are the displacements in the coordinate directions ξ_1, ξ_2, and ξ_3, respectively. The notation used for extensional strains (ϵ_1, ϵ_2, ϵ_3) and shear strains (γ_{12}, γ_{23}, γ_{31}) is in accordance with the usual convention of elasticity.

The strain displacement relations for a shallow elliptic paraboloidal shell can be obtained directly from Equations A-10 by formally identifying the metric coefficients in Equation A-8a with their counterparts in Equation A-9. Therefore, with

$$
\begin{aligned}
\xi_1 &= x \\
\xi_2 &= y \\
\xi_3 &= \zeta \\
A_1 &= 1 + \frac{\zeta}{R_x} \\
A_2 &= 1 + \frac{\zeta}{R_y} \\
A_3 &= 1 \\
u_1 &= U \\
u_2 &= V \\
u_3 &= w
\end{aligned}
\tag{A-11}
$$

Equations A-10 become

$$\epsilon_x = \frac{1}{\left(1 + \frac{\zeta}{R_x}\right)} \left[\frac{\partial U}{\partial x} + \frac{w}{R_x}\right] \tag{A-12a}$$

$$\epsilon_y = \frac{1}{\left(1 + \frac{\zeta}{R_y}\right)} \left[\frac{\partial V}{\partial y} + \frac{w}{R_y}\right] \tag{A-12b}$$

$$\epsilon_\zeta = \frac{\partial w}{\partial \zeta} \tag{A-12c}$$

$$\gamma_{xy} = \frac{1}{\left(1 + \frac{\zeta}{R_x}\right)} \frac{\partial V}{\partial x} + \frac{1}{\left(1 + \frac{\zeta}{R_y}\right)} \frac{\partial U}{\partial y} \tag{A-12d}$$

$$\gamma_{x\zeta} = \left(1 + \frac{\zeta}{R_x}\right) \frac{\partial}{\partial \zeta}\left(\frac{U}{1 + \frac{\zeta}{R_x}} \right) + \frac{1}{\left(1 + \frac{\zeta}{R_x}\right)} \frac{\partial w}{\partial x} \tag{A-12e}$$

$$\gamma_{y\zeta} = \frac{1}{\left(1 + \frac{\zeta}{R_y}\right)} \frac{\partial w}{\partial y} + \left(1 + \frac{\zeta}{R_y}\right) \frac{\partial}{\partial \zeta}\left(\frac{V}{1 + \frac{\zeta}{R_y}} \right) \tag{A-12f}$$

These relations can be simplified by assuming that normals to the undeformed middle surface remain normal to the deformed middle surface, and are unextended. Mathematically, these assumptions require

$$\gamma_{x\zeta} = \gamma_{y\zeta} = \epsilon_\zeta = 0 \tag{A-13}$$

which, in terms of the displacements (Equations A-12c, A-12e, and A-12f), lead to the constraints

$$\begin{aligned}
&\frac{\partial w}{\partial \zeta} = 0 \\
&\frac{\partial}{\partial \zeta}\left(\frac{U}{1 + \frac{\zeta}{R_x}} \right) + \frac{1}{\left(1 + \frac{\zeta}{R_x}\right)^2} \frac{\partial w}{\partial x} = 0 \\
&\frac{\partial}{\partial \zeta}\left(\frac{V}{1 + \frac{\zeta}{R_y}} \right) + \frac{1}{\left(1 + \frac{\zeta}{R_y}\right)^2} \frac{\partial w}{\partial y} = 0
\end{aligned} \tag{A-14}$$

The first of Equations A-14 specifies that

$$w = w(x,y) \tag{A-15a}$$

and after integrating the remaining two equations with respect to ζ, the displacements become

$$U(x,y,\zeta) = \left(1 + \frac{\zeta}{R_x}\right) u(x,y.0) - \zeta \frac{\partial w}{\partial x} \tag{A-15b}$$

$$V(x,y,\zeta) = \left(1 + \frac{\zeta}{R_y}\right) v(x,y,0) - \zeta \frac{\partial w}{\partial y} \tag{A-15c}$$

where $u(x,y,0)$ and $v(x,y,0)$ are the median surface displacements.

Substituting Equations A-15 into Equations A-12a, A-12b, and A-12d and assuming that the shell is sufficiently thin so that $\frac{\zeta}{R_x}$ and $\frac{\zeta}{R_y}$ are negligible in comparison to unity, yields the final form of the strain displacement relations for the elliptic-paraboloidal shell. These kinematic relations are

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} + \frac{w}{R_x} - \zeta \frac{\partial^2 w}{\partial x^2} \\ \epsilon_y &= \frac{\partial v}{\partial y} + \frac{w}{R_y} - \zeta \frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2\zeta \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{A-16}$$

C. STRESS-STRAIN RELATIONS

Stress-strain relations, suitable for thin shells, are

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) \tag{A-17}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \gamma_{xy}$$

where the normal stress, σ_ζ, has been omitted, since it does not contribute to the analysis. However, the integral of σ_ζ through the thickness of the shell is an essential feature of the problem, as it gives rise to the surface loading.

Substituting the strain displacement relations, Equations A-16, into Equations A-17 results in

$$\sigma_x = \left(\frac{E}{1-\nu^2}\right) \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} + \left(\frac{1}{R_x} + \frac{\nu}{R_y}\right) w - \zeta \quad \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}$$

$$\sigma_y = \left(\frac{E}{1-\nu^2}\right) \frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} + \left(\frac{1}{R_y} + \frac{\nu}{R_x}\right) w - \zeta \quad \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}$$

$$\tau_{xy} = \frac{E}{2(1+\nu)} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2\zeta \frac{\partial^2 w}{\partial x \partial y}\right] \tag{A-18}$$

which are the requisite stress displacement relations for a thin, shallow, elliptic paraboloidal shell.

In accordance with the thinness assumption, i.e., $\frac{\zeta}{R_x}$ and $\frac{\zeta}{R_y}$ are negligible in comparison with unity, the force and moment resultants are defined by

$$N_x = \int_{-t/2}^{t/2} \sigma_x \, d\zeta \qquad M_x = \int_{-t/2}^{t/2} \zeta \, \sigma_x \, d\zeta$$

$$N_y = \int_{-t/2}^{t/2} \sigma_y \, d\zeta \qquad M_y = \int_{-t/2}^{t/2} \zeta \, \sigma_y \, d\zeta \qquad \text{(A-19)}$$

$$N_{xy} = \int_{-t/2}^{t/2} \tau_{xy} \, d\zeta \qquad M_{xy} = \int_{-t/2}^{t/2} \zeta \, \tau_{xy} \, d\zeta$$

with the associated sign convention shown in Figure 6. The transverse shear resultants, Q_x and Q_y, and the surface load $p(x, y)$ are given by

$$Q_x = \int_{-t/2}^{t/2} \tau_{x\zeta} \, d\zeta$$

$$Q_y = \int_{-t/2}^{t/2} \tau_{y\zeta} \, d\zeta \qquad \text{(A-20)}$$

$$p\,(x, y) = \sigma_\zeta \Big|_{-t/2}^{t/2}$$

The force and moment resultants can be evaluated from Equations A-18, and are

$$N_x = A\left[\frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} + \left(\frac{1}{R_x} + \frac{\nu}{R_y}\right) w\right]$$

$$N_y = A\left[\frac{\partial v}{\partial y} + \nu \frac{\partial u}{\partial x} + \left(\frac{1}{R_y} + \frac{\nu}{R_x}\right) w\right]$$

$$N_{xy} = \frac{(1-\nu)}{2} A\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \qquad \text{(A-21)}$$

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right]$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right]$$

$$M_{xy} = -(1-\nu) D \frac{\partial^2 w}{\partial x \partial y}$$

where as usual $D = \dfrac{Et^3}{12(1-\nu^2)}$ and $A = \dfrac{Et}{1-\nu^2}$. Consequently, the stresses can now be expressed in terms of the resultants and Equations A-18 will then take the form

$$\sigma_x = \frac{N_x}{t} + \frac{12\zeta M_x}{t^3}$$

$$\sigma_y = \frac{N_y}{t} + \frac{12\zeta M_y}{t^3} \qquad \text{(A-22)}$$

$$\tau_{xy} = \frac{N_{xy}}{t} + \frac{12\zeta M_{xy}}{t^3}$$

D. EQUILIBRIUM EQUATIONS

A consistent set of equilibrium equations and associated boundary conditions, compatible with the assumed kinematic and constituent stress-strain relations, will be developed by employing the Theorem of Minimum Potential Energy. The potential energy (π) is given by

$$\pi = \pi_s - \pi_w \tag{A-23}$$

where π_s is the strain energy and π_w is the work done by the boundary forces. Setting the first variation of the potential energy equal to zero will lead to the equilibrium conditions.

The strain energy functional is taken as

$$\pi_s = \frac{1}{2} \iiint \left\{ \sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy} \right\} d_x d_y d\zeta \tag{A-24a}$$

and after putting Equations A-16 and A-18 into A-24a and integrating the resulting equation with respect to ζ the strain energy becomes, in terms of the displacements,

$$\pi_s = \frac{A}{2} \iint \left\{ \left(\frac{\partial u}{\partial x} + \frac{w}{R_x} \right)^2 + 2\nu \left(\frac{\partial u}{\partial x} + \frac{w}{R_x} \right)\left(\frac{\partial v}{\partial y} + \frac{w}{R_y} \right) + \left(\frac{\partial v}{\partial y} + \frac{w}{R_y} \right)^2 \right.$$

$$\left. + \frac{(1-\nu)}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} dxdy + \frac{D}{2} \iint \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right.$$

$$\left. + 2\nu \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2(1-\nu) \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dxdy \tag{A-24b}$$

The work done by the boundary forces (π_w) is given by

$$\pi_w = \iint_{\zeta\, s} \bar{F} \cdot \bar{Z} \, ds d\zeta + \iint pw \, dxdy \tag{A-25a}$$

where $\overline{F}$ is the force vector

$$\overline{F} = Xi_x + Yi_y + Z\hat{n} \tag{A-25b}$$

and the components are

$$\begin{aligned} X &= \sigma_x \cos(\ell, x) + \tau_{xy} \cos(\ell, y) \\ Y &= \tau_{xy} \cos(\ell, x) + \sigma_y \cos(\ell, y) \\ Z &= \tau_{x\zeta} \cos(\ell, x) + \tau_{y\zeta} \cos(\ell, y) \end{aligned} \tag{A-25c}$$

In the above equations ℓ is the outward drawn normal to the boundary curve in the reference surface of the shell. The displacement vector $\overline{Z}$ is

$$\overline{Z} = Ui_x + Vi_y + w\hat{n} \tag{A-25d}$$

or (from Equations A-15b and A-15c with $\frac{\zeta}{R_x}$; $\frac{\zeta}{R_y} << 1$)

$$\overline{Z} = \left(u - \zeta \frac{\partial w}{\partial x}\right) i_x + \left(v - \zeta \frac{\partial w}{\partial y}\right) i_y + w\, n \tag{A-25e}$$

Substituting Equations A-25b, A-25c, and A-25e into Equation A-25a and integrating through the thickness results in

$$\begin{aligned} \pi_w = \int_s \Big\{ & \Big[N_x \cos(\ell, x) + N_{xy} \cos(\ell, y) \Big] u \\ & + \Big[N_{xy} \cos(\ell, x) + N_y \cos(\ell, y) \Big] v + \Big[Q_x \cos(\ell, x) + Q_y \cos(\ell, y) \Big] w \\ & - \Big[M_x \cos(\ell, x) + M_{xy} \cos(\ell, y) \Big] \frac{\partial w}{\partial x} - \Big[M_{xy} \cos(\ell, x) \\ & + M_y \cos(\ell, y) \Big] \frac{\partial w}{\partial y} \Big\} ds + \iint p\, w\, dxdy \end{aligned} \tag{A-26}$$

Before proceeding with the minimization process a portion of the work integral (π_w) is recast in order to display the "Kirchhoff free edge condition." To this end, consider the identity

$$\int \Big\{ \Big[Q_x \cos(\ell, x) + Q_y \cos(\ell, y) \Big] w - \Big[M_x \cos(\ell, x) + M_{xy} \cos(\ell, y) \Big] \frac{dw}{dx}$$

$$- \Big[M_{xy} \cos(\ell, x) + M_y \cos(\ell, y) \Big] \frac{\partial w}{\partial y} \Big\} \, ds =$$

$$\int \Big\{ \Big[\Big(Q_x + \frac{\partial M_{xy}}{\partial y} \Big) \cos(\ell, x) + \Big(Q_y + \frac{\partial M_{xy}}{\partial x} \Big) \cos(\ell, y) \Big] w$$

$$- M_x \cos(\ell, x) \frac{\partial w}{\partial x} - M_y \cos(\ell, y) \frac{\partial w}{\partial y}$$

$$- \frac{\partial}{\partial x} (M_{xy} w) \cos(\ell, y) - \frac{\partial}{\partial y} (M_{xy} w) \cos(\ell, x) \Big\} \, ds \qquad \text{(A-27a)}$$

According to the sign convention shown in Figure 6 and the relations between dx, dy, and ds (Figure A-3), which are

$$\cos(\ell, x) = \frac{dy}{ds}$$

$$\cos(\ell, y) = -\frac{dx}{ds} \qquad \text{(A-27b)}$$

then

$$\int \Big\{ \frac{\partial}{\partial x} (M_{xy} w) \cos(\ell, y) + \frac{\partial}{\partial y} (M_{xy} w) \cos(\ell, x) \Big\} \, ds =$$

$$= \int \Big\{ - \frac{\partial (M_{xy} w)}{\partial x} \, dx + \frac{\partial (M_{xy} w)}{\partial y} \, dy \Big\} = 0 \qquad \text{(A-27c)}$$

The vanishing of this integral follows from the evaluation of the contour integral over a rectangular path with the sign convention given in Figure 6.

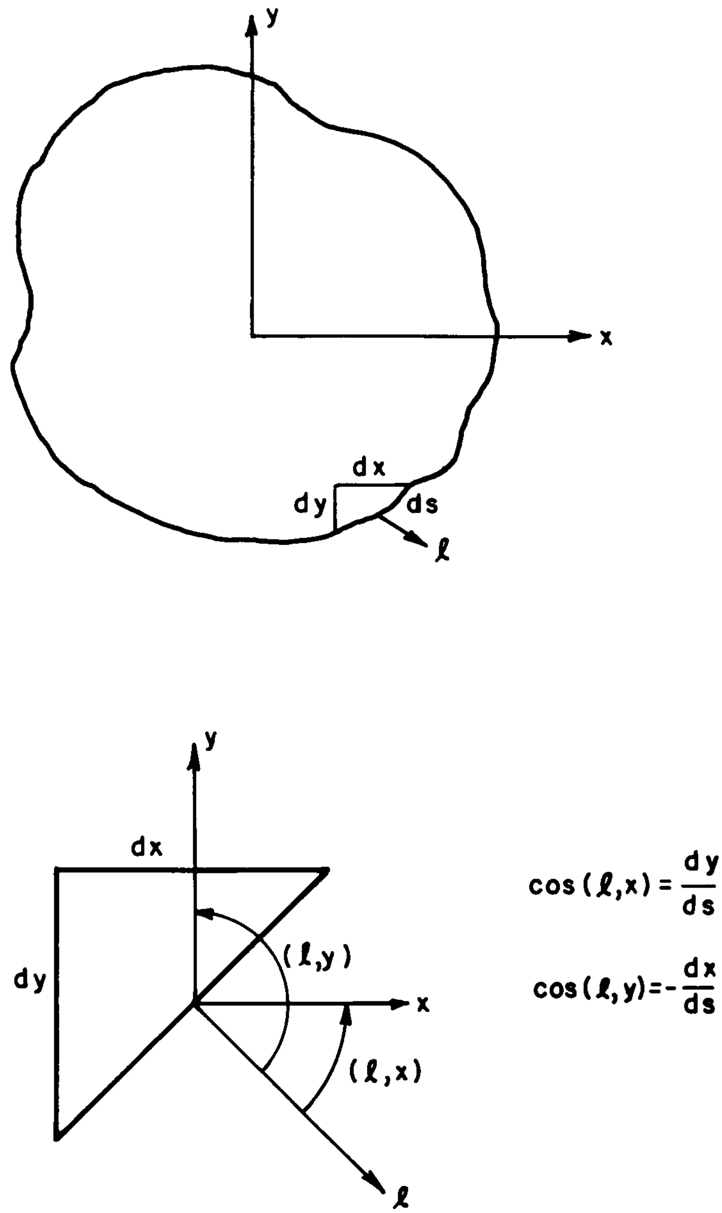


Figure A-3. Direction Cosines Along a Contour in the x, y Plane

Making use of the identity, Equation A-27a, with the associated simplification given by Equation A-27c, in A-26, the work integral reduces to

$$\pi_w = \int \Big\{ [N_x \cos(\ell, x) + N_{xy} \cos(\ell, y)]\, u + [N_{xy} \cos(\ell, x) + N_y \cos(\ell, y)]\, v$$

$$+ \left[\left(Q_x + \frac{\partial M_{xy}}{\partial y} \right) \cos(\ell, x) + \left(Q_y + \frac{\partial M_{xy}}{\partial x} \right) \cos(\ell, y) \right] w$$

$$- M_x \cos(\ell, x) \frac{\partial w}{\partial x} - M_y \cos(\ell, y) \frac{\partial w}{\partial y} \Big\}\, ds + \iint pw\, dxdy \qquad \text{(A-28)}$$

Thus, the potential energy is composed of the difference between the strain energy, given by A-24b, and the work done by the boundary forces, given by A-28.

Minimization of the potential energy follows by application of the usual procedures in the Calculus of Variations. That is, the displacements u, v, and w are replaced by the one parameter family of comparison functions $u+\eta\delta u$, $v+\eta\delta v$, and $w+\eta\delta w$ in Equation A-23. The potential energy is then expressed as a function of the parameter η, and the necessary condition for π to assume a stationary value is that $\frac{d\pi(\eta)}{d\eta} = 0$ as η approaches zero.

Making the appropriate substitution in Equations A-24b and A-28 and performing the indicated operations results in

$$A \iint \Big\{ \left(\frac{\partial u}{\partial x} + \frac{w}{R_x} \right) \left(\frac{\partial \delta u}{\partial x} + \frac{\delta w}{R_x} \right) + \nu \left(\frac{\partial u}{\partial x} + \frac{w}{R_x} \right) \left(\frac{\partial \delta v}{\partial y} + \frac{\delta w}{R_y} \right)$$

$$+ \nu \left(\frac{\partial \delta u}{\partial x} + \frac{\delta w}{R_x} \right) \left(\frac{\partial v}{\partial y} + \frac{w}{R_y} \right) + \left(\frac{\partial v}{\partial y} + \frac{w}{R_y} \right) \left(\frac{\partial \delta v}{\partial y} + \frac{\delta w}{R_y} \right)$$

$$+\frac{(1-\nu)}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\left(\frac{\partial \delta u}{\partial y}+\frac{\partial \delta v}{\delta x}\right)\Bigg\} \, dxdy + D\int\int\Bigg\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \delta w}{\partial x^2}$$

$$+\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \delta w}{\partial y^2}+\nu\frac{\partial^2 \delta w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \delta w}{\partial y^2}+2(1-\nu)\frac{\partial^2 w}{\partial xdy}\frac{\partial^2 \delta w}{\partial xdy}\Bigg\} \, dxdy$$

$$-\int_s\Bigg\{[\overline{N}_x \cos(\ell,x)+\overline{N}_{xy}\cos(\ell,y)]\,\delta u+[\overline{N}_{xy}\cos(\ell,x)+\overline{N}_y\cos(\ell,y)]\,\delta v$$

$$+\left[\left(\overline{Q}_x+\frac{\partial \overline{M}_{xy}}{\partial y}\right)\cos(\ell,x)+\left(\overline{Q}_y+\frac{\partial \overline{M}_{xy}}{\partial x}\right)\cos(\ell,y)\right]\delta w$$

$$-\overline{M}_x\cos(\ell,x)\frac{\partial \delta w}{\partial x}-\overline{M}_y\cos(\ell,y)\frac{\partial \delta w}{\partial y}\Bigg\}\,ds-\int\int p\,\delta w\,dxdy=0 \qquad \text{(A-29)}$$

where the bar notation on the force and moment resultants denotes prescribed edge quantities. Recalling the force and moment resultant-displacement relations (Equations A-21), permits Equation A-29 to be put into the form

$$\int\int\Bigg\{N_x\frac{\partial \delta u}{\partial x}+N_y\frac{\partial \delta v}{\partial y}+N_{xy}\left(\frac{\partial \delta u}{\partial y}+\frac{\partial \delta v}{\partial x}\right)+\left(\frac{N_x}{R_x}+\frac{N_y}{R_y}-p\right)\cdot\delta w$$

$$-M_x\frac{\partial^2 \delta w}{\partial x^2}-M_y\frac{\partial^2 \delta w}{\partial y^2}-2M_{xy}\frac{\partial^2 \delta w}{\partial x\partial y}\Bigg\}\,dxdy$$

$$-\int_s\Bigg\{[\overline{N}_x\cos(\ell,x)+\overline{N}_{xy}\cos(\ell,y)]\,\delta u+[\overline{N}_{xy}\cos(\ell,x)+\overline{N}_y\cos(\ell,y)]\,\delta v$$

$$+\left[\left(\overline{Q}_x+\frac{\partial \overline{M}_{xy}}{\partial y}\right)\cos(\ell,x)+\left(\overline{Q}_y+\frac{\partial \overline{M}_{xy}}{\partial x}\right)\cos(\ell,y)\right]\delta w-\overline{M}_x\cos(\ell,x)\frac{\partial \delta w}{\partial x}$$

$$-\overline{M}_y\cos(\ell,y)\frac{\partial \delta w}{\partial y}\Bigg\}\,ds=0 \qquad \text{(A-30)}$$

Employing the identity

$$C \frac{\partial B}{\partial \xi} = \frac{\partial CB}{\partial \xi} - B \frac{\partial C}{\partial \xi} \tag{A-31a}$$

and then making use of Green's theorem

$$\iint_R \left[\frac{\partial C}{\partial x} + \frac{\partial B}{\partial y} \right] dR = \int_s (Cdy - Bdx) \tag{A-31b}$$

with the relations A-27b, Equation A-31b becomes

$$\int_s (Cdy-Bdx) = \int \Big(C \cos(\ell, x) + B \cos(\ell, y) \Big) ds \tag{A-31c}$$

and the final form of Equation A-29 can be written as

$$\iint \Big\{ -\delta u \left[\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right] - \delta v \left[\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right]$$

$$- \delta w \left[\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{N_x}{R_x} - \frac{N_y}{R_y} + p \right] \Big\} dxdy$$

$$+ \int_s \Big\{ \Big[N_x \cos(\ell, x) + N_{xy} \cos(\ell, y) - \Big(\overline{N}_x \cos(\ell, x) + \overline{N}_{xy} \cos(\ell, y) \Big) \Big] \delta u$$

$$+ \Big[N_{xy} \cos(\ell, x) + N_y \cos(\ell, y) - \overline{N}_{xy} \cos(\ell, x) + \overline{N}_y \cos(\ell, y) \Big] \delta v$$

$$- \Big[(M_x - \overline{M}_x) \cos(\ell, x) \frac{\partial \delta w}{\partial x} + (M_y - \overline{M}_y) \cos(\ell, y) \frac{\partial \delta w}{\partial y} \Big]$$

$$+ \Big[\Big(\frac{\partial M_x}{\partial x} + \frac{2 \partial M_{xy}}{\partial y} - \overline{Q}_x - \frac{\partial \overline{M}_{xy}}{\partial y} \Big) \cos(\ell, x)$$

$$+ \Big(\frac{2 \partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - \overline{Q}_y - \frac{\partial \overline{M}_{xy}}{\partial x} \Big) \cos(\ell, y) \Big] \delta w \Big\} ds = 0 \tag{A-32}$$

Since the domain of integration is arbitrary, then satisfaction of Equation A-32 requires the vanishing of the integrands. The area integrals, lead to the field equations

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{A-33a}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{A-33b}$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2\,\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{N_x}{R_x} - \frac{N_y}{R_y} + p = 0 \tag{A-33c}$$

and the contour integral stipulates the associated boundary conditions. A consistent set must include

$$u \quad \text{or} \quad N_x \cos(\ell, x) + N_{xy} \cos(\ell\, y) \tag{A-34a}$$

$$v \quad \text{or} \quad N_{xy} \cos(\ell, x) + N_y \cos(\ell\, y) \tag{A-34b}$$

$$w \quad \text{or} \quad \left(Q_x + \frac{\partial M_{xy}}{\partial y}\right) \cos(\ell\, x) + \left(Q_y + \frac{\partial M_{xy}}{\partial x}\right) \cos(\ell\, y) \tag{A-34c}$$

$$\frac{\partial w}{\partial x} \quad \text{or} \quad M_x \tag{A-34d}$$

$$\frac{\partial w}{\partial y} \quad \text{or} \quad M_y \tag{A-34e}$$

In addition, the arbitrariness of the boundaries leads to the field equations for the transverse shear resultants

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \tag{A-35a}$$

$$Q_y = \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} \tag{A-35b}$$

The formulation is now completed and the coupled linear system of fourteen partial differential equations for the fourteen unknowns, u, v, w, ϵ_x, ϵ_y, γ_{xy}, N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, and Q_y, consists of Equations A-16, A-21, A-33, and A-35, subject to the boundary conditions listed in Equations A-34.

E. COMPLEX DEFLECTION-STRESS FUNCTION

The governing equations for the stresses and deflections (which are fourteen in number) in a thin, shallow elliptic paraboloidal shell can be reduced to one fourth-order partial differential equation in terms of a complex deflection-stress function. To this end, one may define the auxiliary stress function, F, which is related to the force resultants by

$$\begin{aligned} N_x &= \frac{\partial^2 F}{\partial y^2} \\ N_{xy} &= -\frac{\partial^2 F}{\partial x \partial y} \\ N_y &= \frac{\partial^2 F}{\partial x^2} \end{aligned} \tag{A-36}$$

and satisfies Equations A-33a and A-33b identically. Then, making use of the last three relations of Equations A-21 in A-33c in conjunction with Equation A-36 results in

$$D \nabla^4 W + \nabla_k^2 F = p(x,y) \tag{A-37}$$

where

$$\begin{aligned} \nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \\ \nabla_k^2 &= \frac{1}{R_x} \frac{\partial^2}{\partial y^2} + \frac{1}{R_y} \frac{\partial^2}{\partial x^2} \end{aligned} \tag{A-38}$$

$$\nabla^4 F - (1 - \nu^2) A \nabla_k^2 W = 0 \tag{A-39}$$

The system of Equations A-37 and A-39 for the deflection and stress function will be reduced to a single fourth-order partial differential equation by introducing the complex function φ, defined by

$$\varphi = w + \frac{i F}{\left[(1 - \nu^2) AD \right]^{1/2}} \tag{A-40}$$

where now $i = \sqrt{-1}$. Substituting from Equations A-40 into Equations A-37 and A-39 leads to the controlling equation for φ, which is

$$\nabla^4 \varphi - i \delta^2 \nabla_k^2 \varphi = \frac{p(x, y)}{D} \tag{A-41a}$$

with

$$\delta^2 = \frac{(1 - \nu^2) A}{D} \tag{A-41b}$$

Thus, the solution to a problem involving a thin, shallow elliptic-paraboloidal shell reduces to the solution of Equation A-41a subject to the admissible boundary conditions (Equations A-34). With φ determined, all the field quantities can be evaluated by direct application of the interconnecting relations, Equations A-40, A-36, A-21 and A-35.

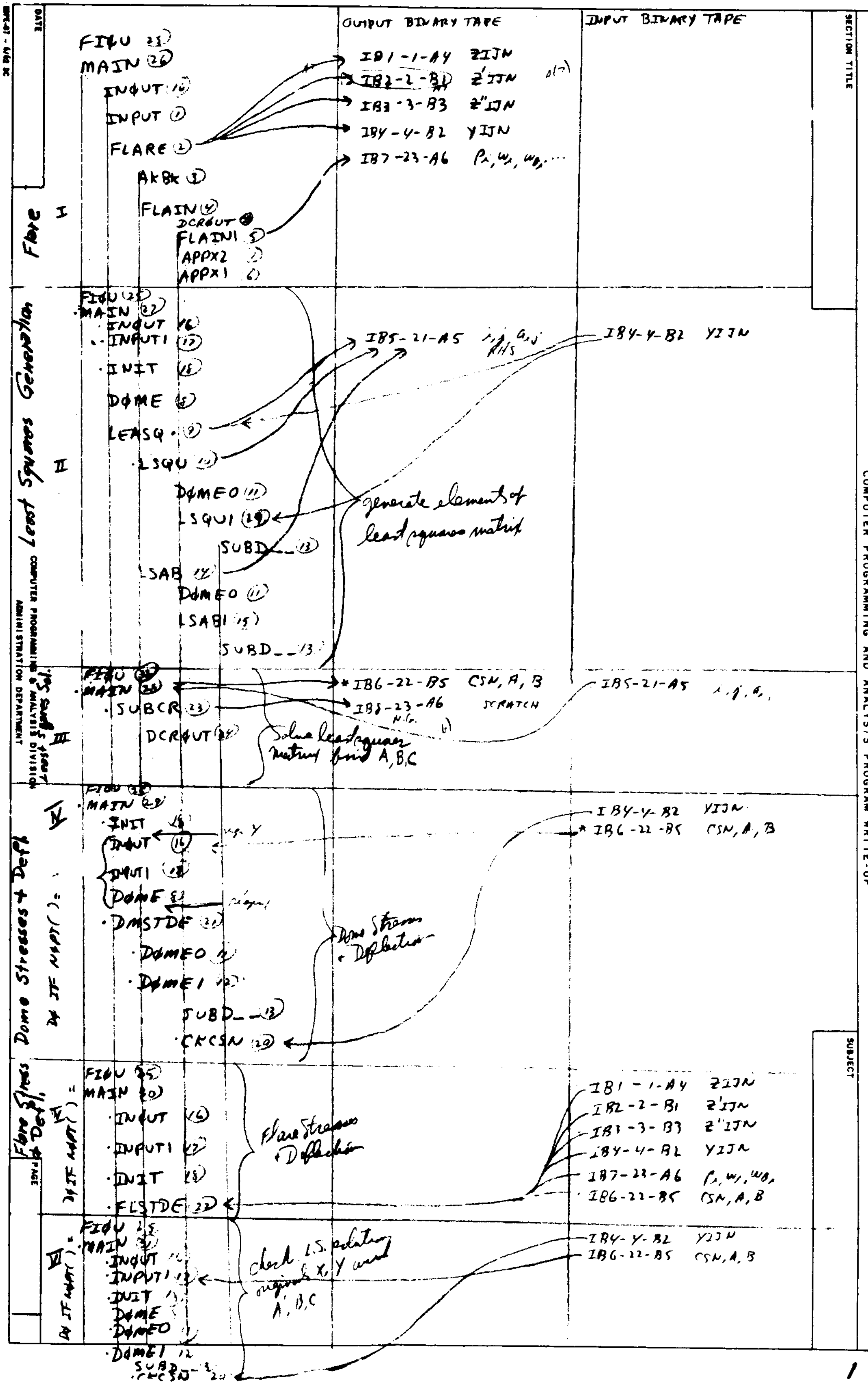
APPENDIX IV

PROGRAM FLOW CHARTS

SECTION TITLE: TABLE OF CONTENTS 61H623 - FLARED NOZZLE - C3442

SUBJECT

DATE



COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE

SUBJECT

DATE

PAGE

COMPUTER PROGRAMMING & ANALYSIS DIVISION
ADMINISTRATION DEPARTMENT

COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE: FLAGS FOR PRINT NC(I), I=1,144

NC(I)		NC(I)		NC(I)		NC(I)	
1		37	DØME1	73		109	
2		38	DØME1	74	APPX2	110	
3		39	DØME1	75	APPX1	111	FLSTDE
4		40	DØME1	76	DASTDE	112	FLSTDE
5		41	DØME1	77	DMSTDE	113	FLSTDE
6		42	DØME1	78	DMSTDE	114	FLSTDE
7		43	DØME1	79		115	FLSTDE
8		44	DØME1	80		116	FLSTDE
9		45	DØME1	81		117	FLSTDE
10		46	DØME1	82		118	FLSTDE
11	DØME	47	DØME1	83		119	FLSTDE
12	DØME	48	DØME1	84		120	FLSTDE
13	DØME	49	DØME1	85	LEASQ	121	SUBDVW
14	DØME0	50		86	LSQU1	122	SUBDXY
15	DØME0	51		87	LSQU1	123	SUBDZ
16	DØME	52		88	LSQU1	124	SUBD1A
17	DØME	53		89		125	SUBD2A
18		54		90	LSQU1	126	SUBD3A
19		55		91	LSQU1	127	
20	SUBD12	56		92		128	
21	SUBD34	57		93	LSAB	129	
22	SUBD56	58		94	LSAB1	130	
23	SUBD78	59		95	LSAB1	131	
24	SUBD9A	60	FLARE	96	LEASQ	132	
25	SUBDBC	61	AKBK	97		133	
26	SUBDDE	62	FLARE	98	LSQU LMN,NC,RHSLS(NI)	134	
27	SUBDFG	63	FLARE	99	LSQU	135	
28	SUBDHI	64	FLARE	100		136	
29	SUBDJK	65	FLARE	101		137	
30	SUBDLM	66	FLARE	102		138	
31	SUBDNØ	67	FLARE	103		139	
32	SUBDPQ	68		104		140	CHKSN
33		69	FLAIN1	105		141	CHKSN
34	DØME1, SUBDRS, SUBDTU	70	FLAIN	106		142	CHKSN
35	DØME1	71		107		143	CHKSN
36	DØME1	72	FLAIN	108		144	LSQU

COMPUTER PROGRAMMING & ANALYSIS DIVISION
ADMINISTRATION DEPARTMENT

PAGE: NC(I) PRINTOUT FLAGS

SECTION TITLE: ØPTIØNS NØPT(I) I=1, 72

SUBJECT

COL. NØ.

NØPT(I)	Value	RØUTINE	PURPOSE
2	0	FLARE, FLAIN	ELLIPSE OPTIØN
	1	" "	CUBIC CURVE FIT
63	0	LEASQ-LSQU	NORMAL
	1	" "	ØMIT LMN = 1,3,4 (TEST RUN)
67	0	LEASQ-LSQU	NØRMAL
	1	" "	ØMIT LMN = 2,3,5,6,8 (TEST RUN)
70	0	INPUT1	READ IN COEFFICIENTS A', B, CSN FRØM LEAST SQ. SOL. FRØM BIN TAPE IB6
	1	"	" " " " " " " " " " " CARDS
	2	"	SKIP READING IN COEFFICIENTS A', B, CSN, FRØM LEAST SQUARES SOLUTION
69	0	DMSTDE	DØ NØT COMPUTE $\sigma_{x\pm}$ $\sigma_{y\pm}$, $\tau_{xy\pm}$, etc
	1	DMSTDE	CØMPUTE $\sigma_{x\pm}$, $\sigma_{y\pm}$, $\tau_{\sigma xy\pm}$, etc USING INPUT X_0, ΔX, ΔY CARD 11 IN INPUT1 (NORMAL)

COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE

INPUT SYMBOLS AND EXPLANATIONS FOR INPUT #17, INPUT #1

PHYSICAL SYMBOLS	PROGRAM SYMBOLS	PHYSICAL DESCRIPTION INPUT #17 PARTS II, III, IV	PHYSICAL SYMBOLS	PROGRAM SYMBOLS	PHYSICAL DESCRIPTION INPUT #1 PART I FLARE
L_x	LX	Half Length of Dome - x Direction	E	EF	Flare Modulus of Elasticity
L_y	LY	" " " " - Y "	ν	NU	Poisson's Ratio
t_D	TD	Dome Thickness	p_{ai}	PSI	Internal Pressure
$1/R_x$	RATIØX	Curvature - x Direction	r_c	RC	Characteristic Radius
$1/R_y$	RATIØY	" - Y "	t_c	TC	" Thickness
E	ED-ELAS	Dome Modulus of Elasticity	σ_c	SIGC	" stress
A	AD	Dome Stretching Stiffness		NINIT	
D	DD	Dome Bending "		NBAR	
ν	NUD-NU	Poissons Ratio	$\bar{N}$	NB	
N_F	NF	Number of Flare Difference Pts	$\bar{\bar{N}}$	NBB	
	NINIT		a	A	Flare Radius at "0" station
ψ°	PSID	Angle Flare tangent makes with Dome	r'_k	RPK( )	
p_{ai}	PSI	Internal Pressure	z'_k	ZPK( )	
r_c	RC	Characteristic Radius	t'_0	TZP	
t_c	TC	" Thickness	t'_k	TPK( )	
σ_c	SIGC	" stress	ψ°	PSID	Angle flare tangent makes with dome
t'_0	TZP	Flare Thickness at "0" station	α	ALPHA	Ellipse semi-major axis
a	ACR	" Radius at " "	β	BETA	" " -minor "
R_i	RINIT	Radius of Flare-Dome Junction	T_F	TF	Flare Thickness
ΔR	DELTAR	Dome Stations at which Derived Quant. are computed		SC	Length of Cylinder Above Flare
X_0	XO	PART III ONLY IF NØPT(LA)≠0 DMSTDE	t_0	TO	
Δx	DELTAX	" " " " " "		TTDD	
Δy	DELTAY	" " " " " "	z_F	ZF	
θ_0°	THETAO	1st Dome station at which Derived Quant. Comp.		AF	
$\Delta\theta^\circ$	DTHETO	" stations " "		DF	
θ°_{last}	THLAST	Last " " " " " "	m	M	No. Diff. stations in Cylinder
	XD ( )		N_F	NF	No. Flare Diff. stations
	YD ( )		ϵ	CREPS	EPSILON FOR CROUT ROUTINE
	XPD ( )		ω'_0	ØMZP	
	YPD ( )		ω'_k	ØMPK	
	THD ( )				
	THDP( )				
A_0	AOLS	II, III, IV ONLY			
A_i	AILS( )				
B_0	BOLS				
B_j	BJLS( )				
C_s^m	CSN(,)				

DATE

PAGE

SUBJECT

COMPUTER PROGRAMMING & ANALYSIS DIVISION
ADMINISTRATION DEPARTMENT

DPF-47 - 6/62 DC

INPUT #1 FORMAT (FLARE)

SUBMITTED BY
EXT.
CHARGE NO.
PROB.

DATE

	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	COL 1-72	←	TITLE →						
2	FORMAT (72I1) PRINT FLAGS		NC(I) I = 1, 72						
3	FØRMAT (72I1) " "		NC(I) I = 73, 144						
4	FØRMAT (72I1) OPTION FLAGS		NØPT(I) I = 1, 72						
5	FØRMAT (15I5) INPUT INDICATØR	1							
6	FØRMAT (7E10.0)	EF	ν						
7	FØRMAT (15I5) INPUT INDICATOR	2							
8	FØRMAT (7E10.0)	ν							
9	FORMAT (15I5)	3							
10	FØRMAT (7E10.0)	λc	Tc	σc					
11	FØRMAT (15I5)	4							
12	" "	HINIT HBAR							
13	" "	5							
14	" "	$\bar{N}$ $\bar{M}$							
15	" "	6							
16	FØRMAT (7E10.0)	a	RPK(1)	RPK(2) →			RPK($\bar{N}$)		
17	FØRMAT (15I5)	7							
18	FØRMAT (7E10.0)	ZPK(1)	ZPK(2) →				ZPK($\bar{N}$)		
19	FØRMAT (15I5)	8							
20	FØRMAT (7E10.0)	TZP	TPK(1)	TPK(2) →			TPK($\bar{N}$)		
21	FØRMAT (15I5)	9							
22	FØRMAT (7E10.0)	PSID							
23	FØRMAT (15I5)	10							
24	FØRMAT (7E10.0)	a	PSID	α	β	TF	SC	TZERØ	
25	" "	TTDD	ZF						
26	FØRMAT (15I5)	11							
27	FØRMAT (7E10.0)	AF	DF						
28	FØRMAT (15I5)	12							
29	" "	M NF							
30	" "	13							

RAC E-1-208 REV. 3/56

INPUT #1 FORMAT (FLARE)

SUBMITTED BY　　　　PROB.　　　　PAGE 2 OF 2

EXT.　　　　CHARGE NO.　　　　DATE

	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	FØRMAT (7E10.0)	CREPS							
2	FØRMAT (15I5)	14							
3	FØRMAT (7E10.0)	ØMZP	ØMPK(1)	ØMAK(2) →	→	→	ØMPK($\overline{N}$)		
4	FØRMAT (15I5) RETURN	15							
5									
6									
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RAC E-1-208 REV. 3/56

CARD INPUT TØ PART III CALLED FROM MAIN #28

SUBMITTED BY | **EXT.** | **CHARGE NO.** | **PROB.** | **PAGE** | **OF** | **DATE**

	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	MNCC(I) I=1,10 FORMAT (10I1) PRINT FLAGS + OPTIONS	MNCK(I)							
2	Ea, Eb, IIMAX FORMAT (2E10.0, I10)	Ea	Eb	IIMAX					
3	USED IN SUBCR #23								
4									
5	SEE FLOW CHART MAIN #28								
6									
7									
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INPUT1 #17 FORMAT

SUBMITTED BY
EXT.
CHARGE NO.
PROB.
PAGE OF
DATE

	OPERATION NOTES	WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
1	Col 1-72	←	TITLE					→	
2	FORMAT (72I1) PRINT FLAGS		NC(I) I=1, 72						
3	FORMAT (72I1) " "		NC(I) I=73, 144						
4	FORMAT (72I1) OPTION FLAGS		NAPT(I) I=1, 72						
5	FORMAT (7E10.0)	Lx	Ly	TD	RATIOX	RATIOY	ID	JD	
6	" "	ED	AD	DD	ν				
7	" "	NDY	NDX	NDYP	NDXP	NDθ	NDθ'	M̄	
8	" "	NF	NINIT	ψ0					
9	" "	PSI	RC	TC	σc	TD'	ACR		
10	" "	RINIT	ΔR						
11	" " (DMSTDE) NOPT(69)≠0	X0	ΔX	ΔY					
12	" " (DMSTDE)	θ0	Δθ	θLAST					
13	" "	XD(1)	XD(2)	←		→	XD(NDX)		
14	" "	YD(1)	YD(2)	←		→	YD(NDY)		
15	" "	X'D(1)	X'D(2)	←		→	X'D(NDXP)		
16	" "	Y'D(1)	Y'D(2)	←		→	Y'D(NDYP)		
17	" "	θD(1)	θD(2)	←		→	θD(NDθ)		
18	" "	θ'D(1)	θ'D(2)	←		→	θ'D(NDθ')		
19	FORMAT (4E15.0)	AOLS	AILS(1)		AILS(2)		AILS(3)		
20	"	AILS(4)	←		→		AILS(ID)		
21	"	BOLS	BJLS(1)		BJLS(2)		BJLS(3)		
22	"	BJLS(4)	←		→		BJLS(JD)		
23	"	CSN(1,1)	CSN(1,2)		CSN(1,3)		CSN(1,4)		
24	"	CSN(2,1)	CSN(2,2)		CSN(2,3)		CSN(2,4)		
25	"	↓	↓		↓		↓		
26	"								
27	"	CSN(N/2+1, 1)	CSN(N/2+1, 2)		CSN(N/2+1, 3)		CSN(N/2+1, 4)		
28									
29									
30									

READ IN ONLY IF NAPT(20) = 1

RAC E-1-208 REV. 3/56

SECTION TITLE: LMN

PART II EQUATIONS FOR LEAST SQUARES

SUBJECT:

THESE 8 SETS OF EQUATIONS FOR XX = RINIT · COS θ'; YY : RINIT · sin θ' FOR θ'_1, θ'_2, θ'_3 . . . θ'_{NDTHP} = 8 · NDTHP EQUATIONS [5]

1 U_D
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \left[\cos\psi\,({}_s\hat{U}_{FN}^n) + \sin\psi\,({}_s\hat{W}_{FN}^n)\right] - \frac{E}{r_c \cdot \sigma_c}\left\{\frac{D_D}{A_D(1-\nu^2)}\right\}^{1/2}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x}\left(\bar{U}_{1i}\cdot\cos\theta + \bar{V}_{1i}\cdot\sin\theta\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y}\left(\bar{U}_{2j}\cdot\cos\theta + \bar{V}_{2j}\cdot\sin\theta\right)\right\rangle$$

2 V_D
$$\sum_{n=0,2,4}^{\bar{n}} \sin n\theta \cdot \sum_{s=1}^{4} C_s^n \cdot ({}_s\hat{V}_{FN}^n) - \frac{E}{r_c \cdot \sigma_c}\left\{\frac{D_D}{A_D(1-\nu^2)}\right\}^{1/2}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x}\left(-\bar{U}_{1i}\cdot\sin\theta + \bar{V}_{1i}\cdot\cos\theta\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y}\cdot\left(-\bar{U}_{2j}\cdot\sin\theta + \bar{V}_{2j}\cdot\cos\theta\right)\right\rangle$$

3 W_D
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \cdot \left(\cos\psi\cdot{}_s\hat{W}_{FN}^n - \sin\psi\cdot{}_s\hat{U}_{FN}^n\right) - \frac{E}{r_c \cdot \sigma_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x}\cdot L_x\cdot W_{1i} + \sum_{j=0}^{JD}\frac{B_j}{L_y}\cdot L_y\cdot W_{2j}\right\rangle$$

4 Φ_R
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \cdot \left(-{}_s\hat{\Phi}_N^n\right) - \frac{E}{\sigma_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x}\left(\Phi_{1ix}\cdot\cos\theta + \Phi_{1iy}\cdot\sin\theta\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y}\left(\Phi_{2jx}\cdot\cos\theta + \Phi_{2jy}\cdot\sin\theta\right)\right\rangle$$

5 N_R
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \cdot \left(\cos\psi\cdot{}_s\hat{t}_N^n - \sin\psi\cdot{}_s\hat{f}_N^n\right) - \frac{\sqrt{(1-\nu^2)A_D D_D}}{\sigma_c \cdot t_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\left(N_{1ix}\cdot\frac{1+\cos 2\theta}{2} + N_{1iy}\cdot\frac{1-\cos 2\theta}{2} + N_{1ixy}\sin 2\theta\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\left(N_{2jx}\frac{1+\cos 2\theta}{2} + N_{2jy}\frac{1-\cos 2\theta}{2} + N_{2jxy}\sin 2\theta\right)\right\rangle$$

6 $N_{R\theta}$
$$\sum_{n=0,2,4}^{\bar{n}} \sin n\theta \cdot \sum_{s=1}^{4} C_s^n \cdot {}_s\hat{t}_{N\theta}^n - \frac{\sqrt{(1-\nu^2)A_D D_D}}{\sigma_c \cdot t_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\left(\frac{\sin 2\theta}{2}(N_{1iy}-N_{1ix}) + \cos 2\theta\cdot N_{1ixy}\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\left((N_{2jy}-N_{2jx})\frac{\sin 2\theta}{2} + \cos 2\theta\cdot N_{2jxy}\right)\right\rangle$$

7 $\bar{Q}_R$
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \left(-\cos\psi\cdot{}_s\hat{f}_N^n - \sin\psi\cdot{}_s\hat{t}_N^n\right) + -\frac{DD}{\sigma_c \cdot t_c}\sum_{i=0}^{ID}\frac{A_i}{L_x^3}\left\{\begin{matrix} Q_{1ix}\cos\theta + Q_{1iy}\cdot\sin\theta + L_x/(xx^2+yy^2)^{1/2}\cdot\left[-2\cdot M_{1ixy}\cdot\sin 2\theta + (M_{1ix}+M_{1iy})\cdot\cos 2\theta\right] + \\ (QB3\cdot\cos 2\theta\cdot\cos\theta - QB4\cdot\cos 2\theta\cdot\sin\theta + QB6\cdot\sin 2\theta\,(\cos\theta/2)) + (-QB1\cdot\sin 2\theta\cdot\sin\theta - QB5\cdot\sin 2\theta\cdot\cos\theta + QB2\cdot\sin 2\theta\cdot\sin\theta)/2 \end{matrix}\right\}$$
$$+ -\frac{DD\cdot r_c}{\sigma_c \cdot t_c^3}\left\langle \sum_{j=0}^{JD}\frac{B_j}{L_y^3}\left\{Q_{2jx}\cdot\cos\theta + Q_{2jy}\cdot\sin\theta + \frac{L_y}{(xx^2+yy^2)^{1/2}}\cdot\left[2\cdot M_{2jxy}\cdot\sin 2\theta + (M_{2jx} - M_{2jy})\cdot\cos 2\theta\right] + \left[QB9\cdot\cos 2\theta\cdot\cos\theta - QB10\cdot\cos 2\theta\cdot\sin\theta + QB8\cdot\sin 2\theta\cdot\cos\theta\right]/2 + \left[-QB11\cdot\sin 2\theta\cdot\sin\theta - QB7\cdot\sin 2\theta\cdot\cos\theta + QB12\cdot\sin 2\theta\cdot\sin\theta\right]/2\right\}$$

8 M_R
$$\sum_{n=0,2,4}^{\bar{n}} \cos n\theta \cdot \sum_{s=1}^{4} C_s^n \left(-{}_s\hat{m}_{sN}^n\right) \quad -\frac{DD\cdot r_c}{\sigma_c \cdot t_c^3}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot\left(M_{1ix}\frac{1+\cos 2\theta}{2} + M_{1iy}\frac{1-\cos 2\theta}{2} + M_{1ixy}\sin 2\theta\right) + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\left(M_{2jx}\cdot\left(\frac{1+\cos 2\theta}{2}\right) + M_{2jy}\cdot\left(\frac{1-\cos 2\theta}{2}\right) + M_{2jxy}\sin 2\theta\right)\right\rangle$$

9 $\bar{N}_{xy_x}$ — $X = L_x$; $Y_0', Y_1', Y_2' \cdot Y_{NDYP}'$ [3]
$$-\frac{1}{\sigma_c \cdot t_c}\cdot\sqrt{(1-\nu^2)A_D D_D}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot N_{1ixy} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2} N_{2jxy}\right.$$

10 $\bar{N}_{xy_y}$ — $Y = L_y$; $X = X_0', X_1' \; X_{NDXP}'$ [4]
$$-\frac{1}{\sigma_c \cdot t_c}\cdot\sqrt{(1-\nu^2)A_D D_D}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot N_{1ixy} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\cdot N_{2jxy}\right.$$

11 $\bar{Q}_x$ — $X = L_x$; $Y = Y_0', Y_1' \cdots Y_{NDYP}'$ [3]
$$-\frac{D_D}{\sigma_c \cdot t_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^3}\cdot QB1ix + \sum_{j=0}^{JD}\frac{B_j}{L_y^3}\cdot QB2jx\right\rangle$$

12 $\bar{Q}_y$ — $Y = L_y$; $X = X_0', X_1' \cdots X_{NDXP}'$ [4]
$$-\frac{D_D}{\sigma_c \cdot t_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^3}\cdot QB1iy + \sum_{j=0}^{JD}\frac{B_j}{L_y^3}\cdot QB2jy\right\rangle$$

13 M_x — $X = L_x$; $Y = Y_0', Y_1' \; Y_{NDYP}'$ [3]
$$-\frac{D_D\cdot r_c}{\sigma_c \cdot t_c^3}\cdot\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot M_{1ix} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\cdot M2JX\right\rangle$$

14 M_y — $Y = L_y$; $X = X_0', X_1' \; X_{NDXP}'$ [4]
$$-\frac{D_D\cdot r_c}{\sigma_c \cdot t_c^3}\cdot\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_y^2}\cdot M_{1iy} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\cdot M2JY\right\rangle$$

15 $\bar{Q}_x$ — $X = 0$; $Y = Y_0, Y_1 \; Y_{NDY}$ [1]
$$-\frac{D_D}{\sigma_c \cdot t_c}\cdot\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^3}\cdot QB1iX + \sum_{j=0}^{JD}\frac{B_j}{L_y^3}\cdot QB2JX\right\rangle$$

16 $\bar{Q}_y$ — $Y = 0$; $X = Y_0, Y_1 \cdot X_{NDX}$ [2]
$$-\frac{D_D}{\sigma_c \cdot t_c}\cdot\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^3}\cdot QB1iY + \sum_{j=0}^{JD}\frac{B_j}{L_y^3}\cdot QB2JY\right\rangle$$

17 N_x — $X = L_x$; $Y = Y_0', Y_1' \; Y_{NDYP}'$ [3]
$$-\frac{\sqrt{(1-\nu^2)\cdot A_D\cdot D_D}}{\sigma_c \cdot t_c}\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot N_{1ix} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\cdot N2JX\right\rangle$$

18 N_y — $Y = L_y$; $X = X_0', X_1' \cdots X_{NDXP}'$ [4]
$$-\frac{\sqrt{(1-\nu^2)\cdot A_D\cdot D_D}}{\sigma_c \cdot t_c}\cdot\left\langle \sum_{i=0}^{ID}\frac{A_i}{L_x^2}\cdot N_{1iy} + \sum_{j=0}^{JD}\frac{B_j}{L_y^2}\cdot N2JY\right\rangle$$

DATE

A

SECTION TITLE

SUBJECT: RHSLS (1e)

RIGHT HAND SIDE

1) $\frac{PSI\cdot R_y}{2\cdot E\cdot T_D}\cdot\left\langle\left(1-2\nu+\nu\frac{R_y}{R_x}\right)\cdot XX\cdot\cos\theta + \left(2-\nu-\frac{R_y}{R_x}\right)\cdot YY\cdot\sin\theta\right\rangle\frac{E}{r_c\cdot\sigma_c}$

2) $\frac{PSI\cdot R_y}{2\cdot E\cdot T_D}\left\langle -\left(1-2\nu+\nu\cdot\frac{R_y}{R_x}\right)\cdot XX\cdot\sin\theta + \left(2-\nu-\frac{R_y}{R_x}\right)\cdot YY\cdot\cos\theta\right\rangle\cdot\frac{E}{r_c\cdot\sigma_c}$

3) $= 0$

4) $= \hat{\Phi}_{0N}^{0} \cdot YIJN(4)$

5) $= \frac{PSI\cdot R_y}{2}\cdot\left\langle\left(\frac{1+\cos 2\theta}{2}\right) + \left(2-\frac{R_y}{R_x}\right)\cdot\left(\frac{1-\cos 2\theta}{2}\right)\right\rangle\cdot\frac{1}{\sigma_c\cdot t_c} - \cos\psi\cdot\hat{t}_{0N}^{0} + \sin\psi\cdot\hat{f}_{0N}^{0}$

6) $= \frac{PSI\cdot R_y}{4}\left(1-\frac{R_y}{R_x}\right)\sin 2\theta\cdot\frac{1}{\sigma_c\cdot t_c}$

7) $= \cos\psi\cdot\hat{f}_{0N}^{0} + \sin\psi\cdot\hat{t}_{0N}^{0}$

8) $= 0$

9) $= 0$

10) $= 0$

11) $= 0$

12) $= 0$

13) $= 0$

14) $= 0$

15) $= 0$

16) $= 0$

17) $= 0$

18) $= 0$

DATE

B

SECTION TITLE: PART II - COMPUTE COEFFICIENTS OF LEAST SQUARES MATRIX

SUBJECT:

FIØU 25
MAIN 27
INØUT 16
INPUT1 17
INIT 18
DØME 19
LEASQ 9
LSQU 10
DØMEO 11
LSQU1 19
SUBD-- 13
LSAB 14
DØMEO 11
LSAB1 15
SUBD-- 13

INPUT BINARY TAPE IB4 CONTAINS YIJN(1,2,3,4) FROM FLARE PT.I

IDD m j i $\{\hat{t}^m_{ji}\ \hat{t}^m_{\theta ji}\ \hat{f}^m_{ji}\ \hat{g}^m_{ji}\}$ = YIJN

IDD	m	j	i
4	0	0	0, 1, 2 … N_F
		1	0, 1, 2 … N_F
		2	0, 1, 2 … N_F
		3	0, 1, 2 … N_F
		4	0, 1, 2 … N_F
	2	0	0, 1, 2 … N_F
		1	0, 1, 2 … N_F
	$\bar{m}$		N_F

$m = m_0^{init}, \bar{m}$

$j = 0,1,2,3,4$

$i = 0,1,2, \ldots N_F$

OUTPUT BINARY TAPE IB5 CONTAINS LEAST SQUARES MATRIX INPUT TO PART III

NN 28

TITLE

NCEQ, NABEQ, NEQ, NCC, NCCI, MUN, ID, JD, NBAR, NC(I), I=1,NY

i_e j_e ABC(i_e, j_e)

0 0 0

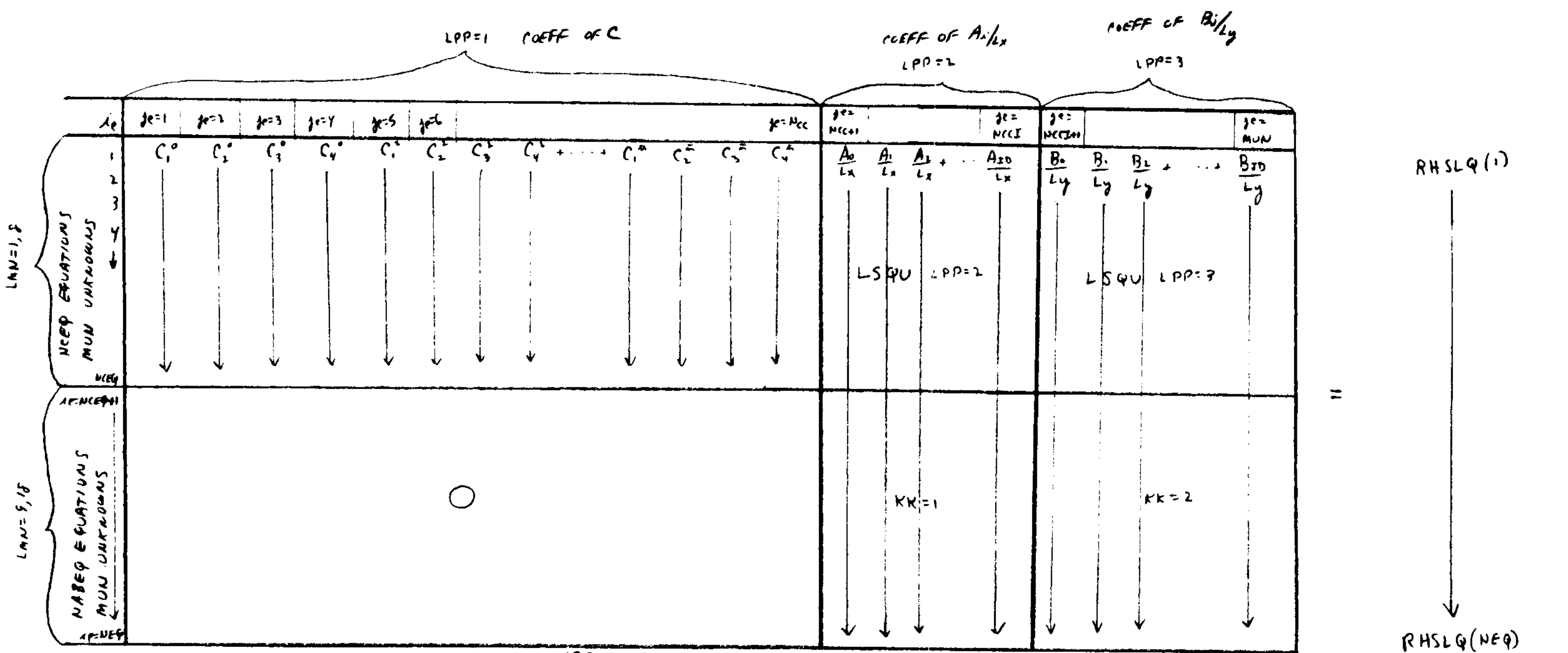
RHSLS(i_e) (i_e=1, NEQ)

DATE

COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE: PART II — LEAST SQUARES MATRIX ABC(IE, JE)   IE = 1, NEQ; JE = 1, MUN

SUBJECT



TOTAL EQUATIONS: $\underbrace{8 \cdot NDTHP}_{NCEQ} + \underbrace{4 \cdot NDYP + 4 \cdot NDXP + NDY + NDX}_{NABEQ} = NEQ$

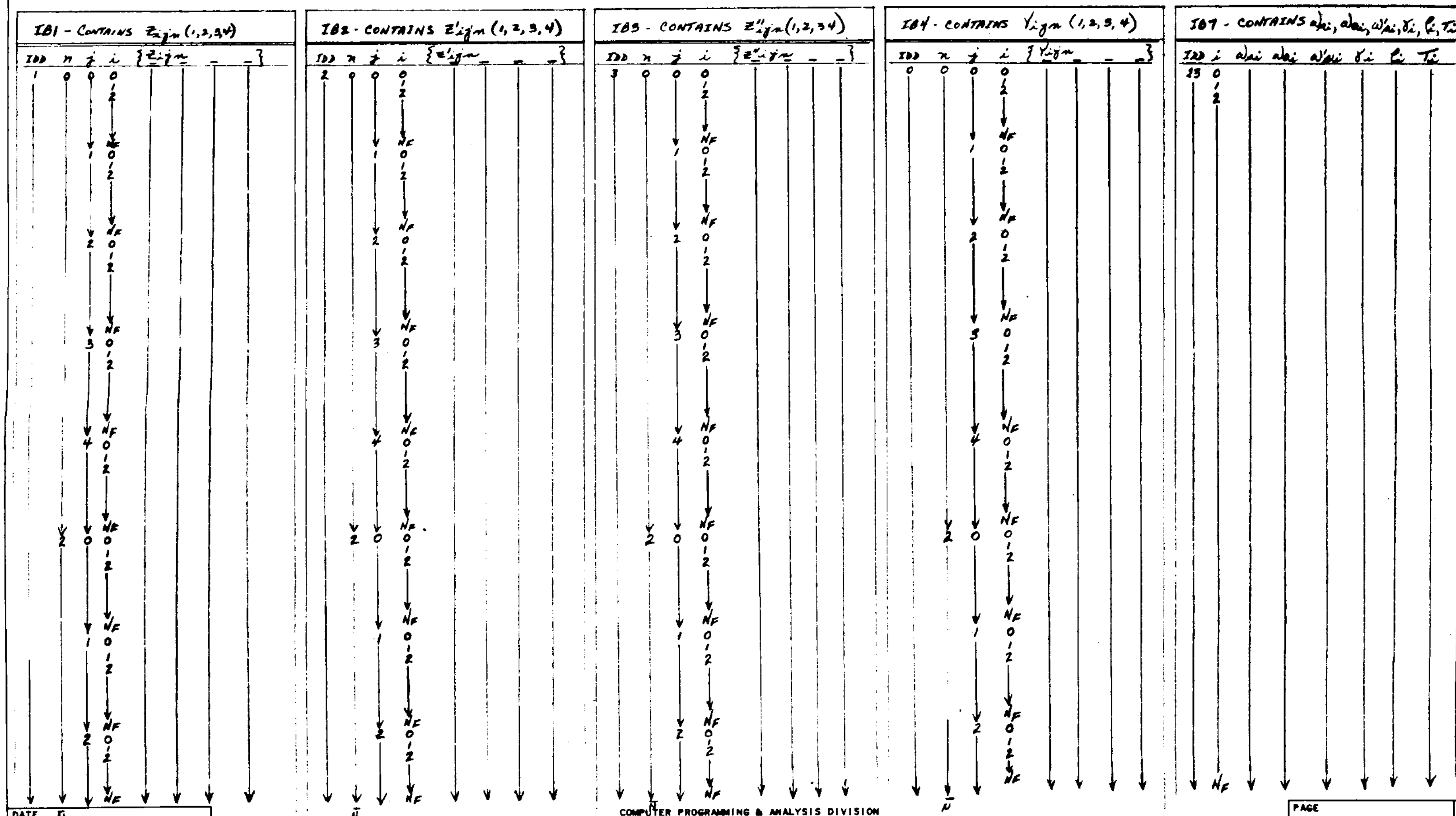
TOTAL UNKNOWNS: $\underbrace{\underbrace{\left(\frac{\bar{IT}}{2}+1\right) \cdot 4}_{NCC} + (I_D+1)}_{NCCI} + (J_D+1) = MUN$

SECTION TITLE

PART I - FLARE #2

SUBJECT

BINARY TAPE · OUTPUT



DATE $\bar{N}$

SECTION TITLE

SUBJECT

SUBROUTINE INIT #18 SUBROUTINE INPUT ( ) #15 PTS I, II, V, VI

SUBROUTINE INIT #18

$\beta^* = \tan^{-1} L_y/L_x$

$\left.\begin{array}{l} \alpha_{1x}(i)=0 \\ \alpha_{2y}(i)=0 \\ A_i(i)=0 \\ B_i(i)=0 \\ C_j(i)=0 \\ D_j(i)=0 \end{array}\right\} i=1\text{-}50$

$w_{10}=0$ $M_{10x}=0$
$w_{20}=0$ $M_{20y}=0$
$\bar{V}_{10}=0$ $q_{10y}=0$
$\bar{U}_{20}=0$ $qB_{10y}=0$
$N_{10x}=0$ $Q_{20x}=0$
$N_{20y}=0$ $qB_{20x}=0$
$Q_{10y}=0$ $M_{10y}=0$
$Q_{20x}=0$ $M_{20x}=0$
$M_{10xy}=0$ $M_{20xy}=0$

$\left.\begin{array}{l} P(j,i)=0 \\ Q(j,i)=0 \\ AL(j,i)=0 \\ BL(j,i)=0 \end{array}\right\} \begin{array}{l} j=1,4 \\ i=1,50 \end{array}$

$\left.\begin{array}{l} DD=E_D \cdot T_D^3 / 12(1-\nu^2) \\ A_D=E_D \cdot T_D / (1-\nu^2) \end{array}\right\}$ if A_D is read in as 0

$FACT=\{(1-\nu^2) \cdot A_D \cdot D_D\}^{1/2}$

$R_x = 1/RATIOX$

$R_y = 1/RATIOY$

RETURN

SUBROUTINE INPUT (IN, IO, IB1, IB2, IB3, IB4, IB5, IB6, IB7) # 16

IN=5	NORMAL CARD INPUT
IO=6	NORMAL PRINT OUTPUT
IB1=1 (A4)	{ZIJN} GENERATED IN FLARE
IB2=2 (B1)	{ZPIJN} " " "
IB3=3 (B3)	{ZPPIJN} " " "
IB4=4 (B2)	{YIJN} " " "
IB5=21 (A5)	λ, η, a_{ij} LEAST SQUARES elements GENERATED IN PART II
IB6=22 (B5)	CSN, AI, BJ COEFFICIENTS FOUND IN PART III SOLUTION OF LEAST SQ. MATRIX
IB7=23 (A6)	23, $\lambda, w_{S\lambda}, w_{B\lambda}, w'_{S\lambda}, \gamma_\lambda, P_\lambda, T_\lambda$ GENERATED IN FLARE - USED IN FLARE STRESSES + DEFLECTIONS FLSTDE # 22

DATE

SECTION TITLE

SUBROUTINE INPUT1 #17 PART II, IV, V, VI

SUBJECT

CARD NO

READ IN :

1)	TITLE(I) I=1,12		
2)	N(I) I=1,72		
3)	N(I) I=73,144		
4)	NØPT(I) I=1,72		
5)	L_x, L_y, T_D, RATIØX, RATIØY, ID, JD		
6)	E_D, A_D, D_D, ν		
7)	N_{Dy}, N_{Dx}, N_{Dyp}, N_{Dxp}, $N_{D\theta}$, $N_{D\theta'}$, $\bar{N}$		
8)	N_F, MNIT, ψ_D°		
9)	PSI, R_C, T_C, σ_C, T_0', A_{CR}		
10)	R_{INIT}, ΔR		USED IN DOME STRESSES + DEFLECTIONS (DMSTDE #21)
11)	X_0, Δx, Δy		USED IN DOME STRESSES + DEFLECTIONS (DMSTDE #21) OPTION IF NØPT(64) ≠ 0
12)	θ_0, $\Delta\theta$, θ_{last}		USED IN DOME STRESSES + DEFLECTIONS (DMSTDE #21)
13)	$X_D(i)$ i=1, N_{Dx}	2	X'S USED TO GENERATE EQUATIONS IN LMN=16 $\bar{Q}_y$
14)	$Y_D(i)$ i=1, N_{Dy}	1	Y'S USED TO GENERATE EQUATIONS IN LMN=15 $\bar{Q}_x$
15)	$X_D'(i)$ i=1, N_{Dxp}	4	X'S USED TO GENERATE EQUATIONS IN LMN=10, 12, 14, 18; ($\bar{N}_{xy}$, $\bar{Q}_y$, M_y, N_y)
16)	$Y_D'(i)$ i=1, N_{Dyp}	3	Y'S USED TO GENERATE EQUATIONS IN LMN=9, 11, 13, 17; ($\bar{N}_{yx}$, $\bar{Q}_x$, M_x, N_x)
17)	$\theta_D(i)$ i=1, $N_{D\theta}$		
18)	$\theta_D'(i)$ i=1, $N_{D\theta'}$		θ'S USED TO GENERATE EQUATIONS IN LMN=1–8; (U_D, V_D, W_D, ϕ_R, N_R, $\bar{N}_{R\theta}$, $\bar{Q}_R$, M_R)

READ FROM CARDS IF NØPT(70)=1
" " TAPE IB6 " " =0
OMIT READ IN IF NØPT(70)=2

- AOLS, AILS(i) i=1, ID
- BOLS, BILS(i) i=1, JD
- CSN(i, is) is=1,4; i=1, $\frac{\bar{N}}{2}+1$

GENERATED IN PART III SOLUTIONS TO LEAST SQUARES
INPUT TO CHECK DOME EQUATIONS

DATE

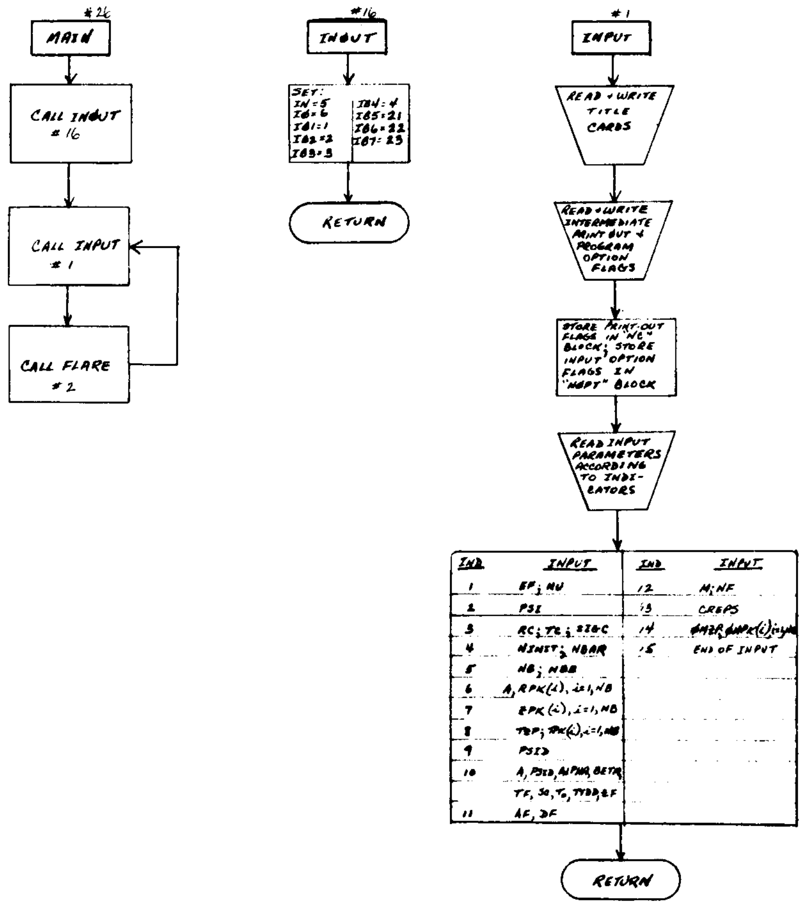
SECTION TITLE

PART I - FLARE #2

#26, #16, #1

SUBJECT

MAIN, INØUT, INPUT FLØW CHARTS



SECTION TITLE: PART I – ONLY   INPUT #1

SUBJECT

FIØU 27
MAIN 26
 INØUT 26
 INPUT 1
 FLARE 2
  AKBK 3
  FLAIN 4
   FLAIN1 5
   APPX2 7
   APPX1 6

CARD INPUT - READ BY SUBROUTINE INPUT 1 *

CARD NO.	BLOCK	DESCRIPTION
1	TITLE(I), I=1,···,12	TITLE CARD - BCD
2	NC(I), I=1,···,72	PRINT OUT OPTIONS · INTEGER BLANK OR 1
3	NC(J), J=73,···,144	"
4	NOPT(I), I=1,···,72	PROGRAM FLOW OPTIONS - INTEGER BLANK OR 1
	THE REMAINING CARDS ARE READ IN UNDER OPTION - A CARD CONTAINING THE OPTION NUMBER IN COLUMNS 1-5 MUST PRECEDE EACH INPUT CARD OR	
OPTION NO.	GROUP OF CARDS	
1	EF, NU	F, ν - FLOATING PT.
2	PSI	ψ_i - FLOATING PT.
3	RC, TC, SIGC	r_c, t_c, σ_c - FLOATING PT.
4	NINIT, NBAR	NINIT, NBAR - INTEGER
5	NB, NBB	$\bar{N}, \bar{\bar{N}}$ - INTEGER
6	A, RPK(I), I=1,···,NB	$a, r_{PR}(i), i=1,\cdots,\bar{N}$ - FLOATING PT.
7	ZPK(I), I=1,···,NB	$z_{PR}(i), i=1,\cdots,\bar{N}$ - FLOATING PT.
8	TZP, TPK(I), I=1,···,NB	$T_{PR}(0), T_{PR}(i), i=1,\cdots,\bar{N}$ - FLOATING PT.
9	PSID	ψ° - FLOATING PT.
10	A, PSID, ALPHA, BETA, TF, SC, TZERO, TDD, ZF	$a, \psi_d, \alpha, \beta, t_f, s_c, t_0, t^2 d^2, z_f$ - FLOATING PT.
11	AF, DF	a_f, d_f - FLOATING PT.
12	M, NF	m, n_f - INTEGER
13	CREPS	ϵ for subroutine DCROUT - FLOATING PT.
14	ØMZP, ØMPK(I), I=1,···,NB	$\omega_{p0}, \omega_{PR}(i), i=1,\cdots,\bar{N}$
15	RETURN FROM SUBROUTINE TO MAIN PROGRAM	

* SEE ALSO INPUT #1 FORMAT

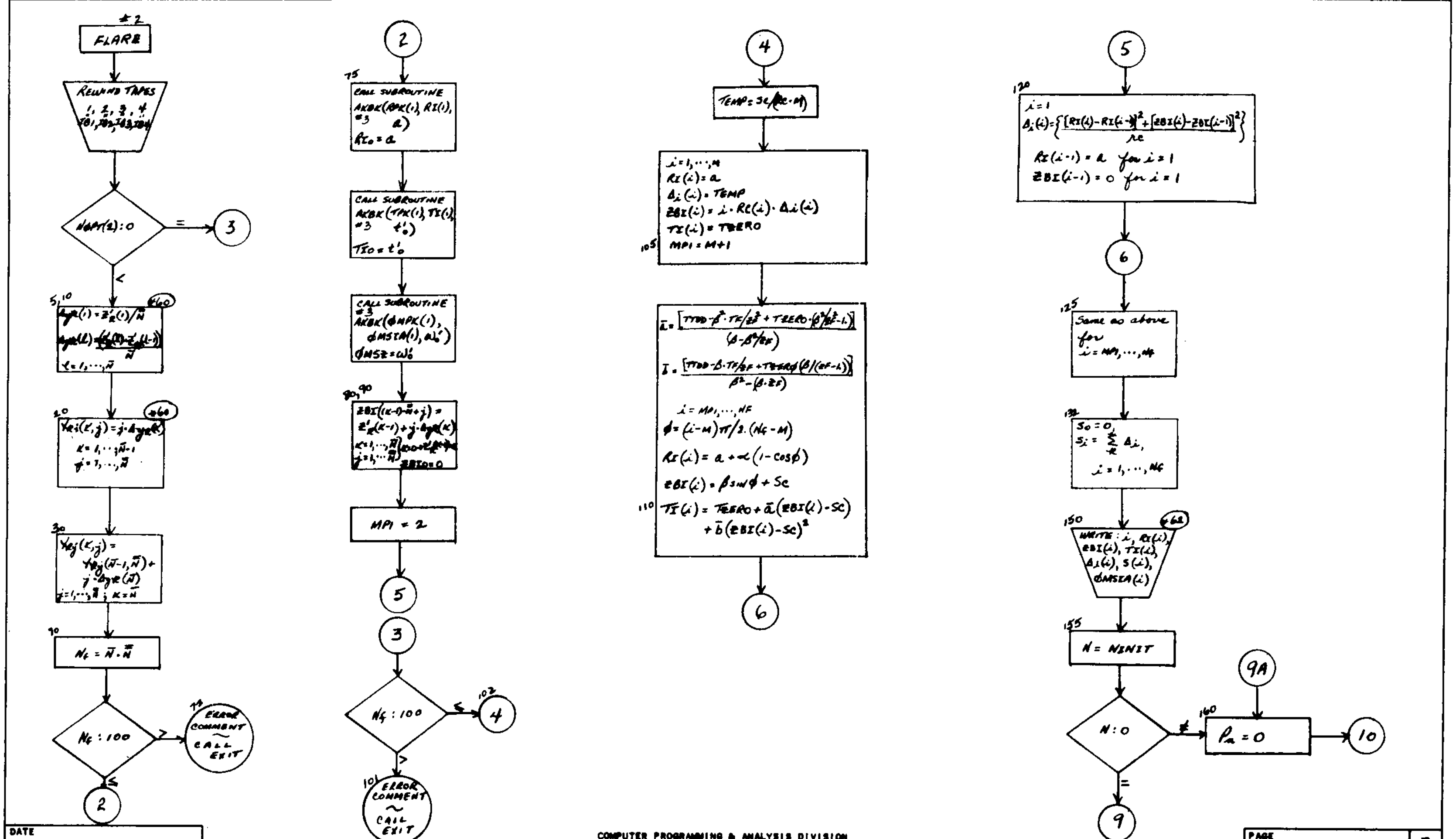
SECTION TITLE

PART I - FLARE #2

#2

SUBJECT

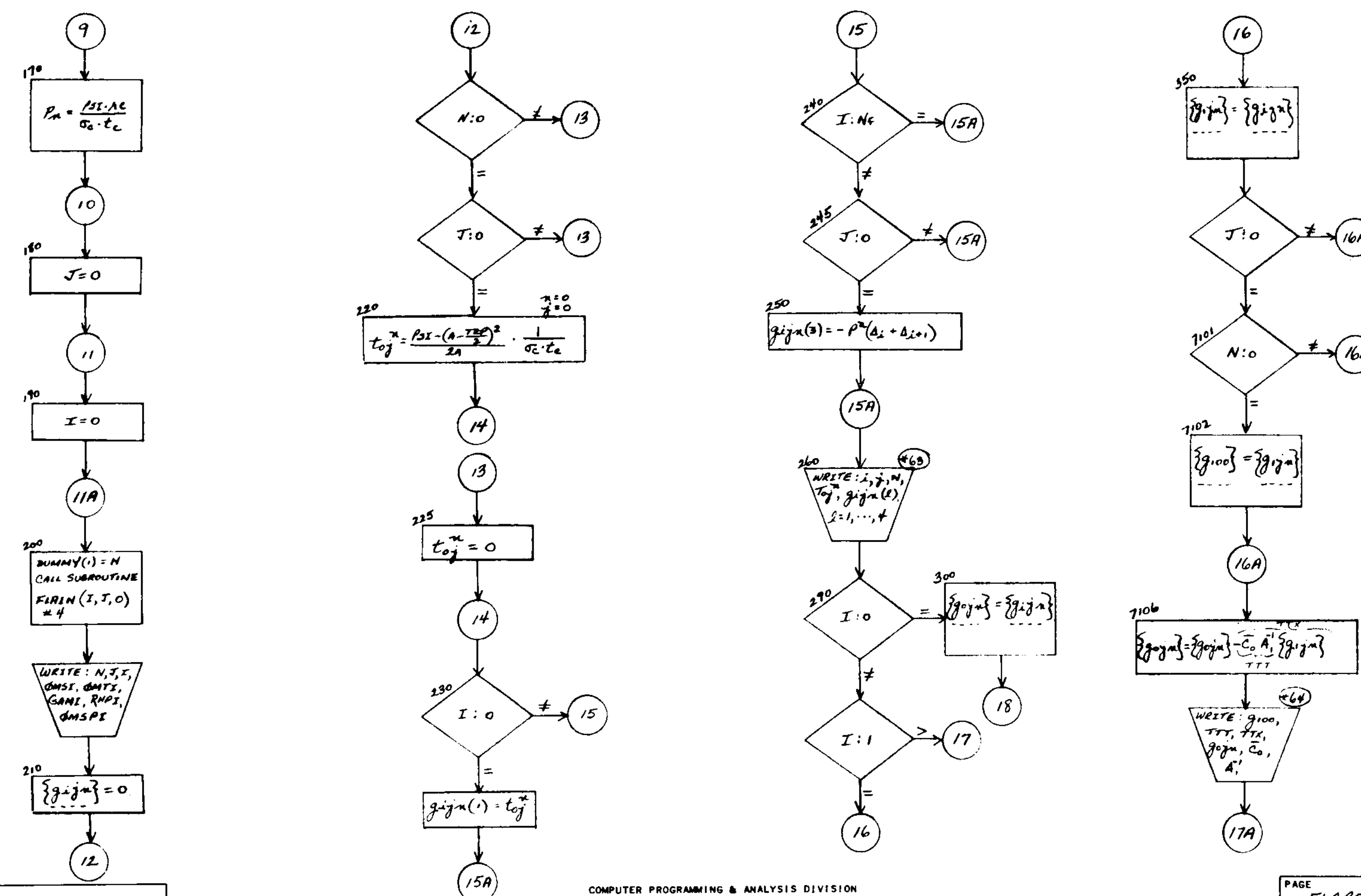
FLARE FLOW CHART

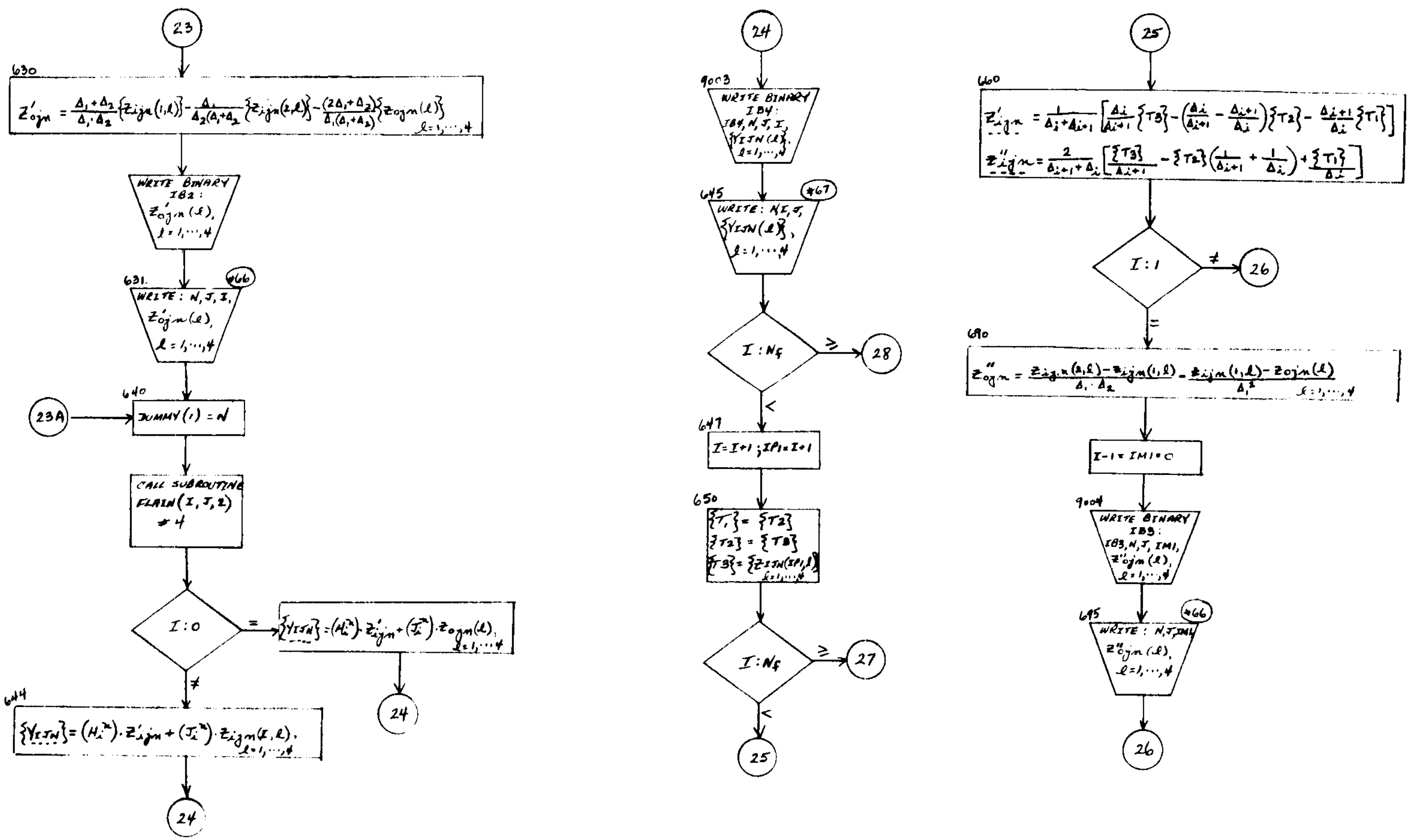


DATE

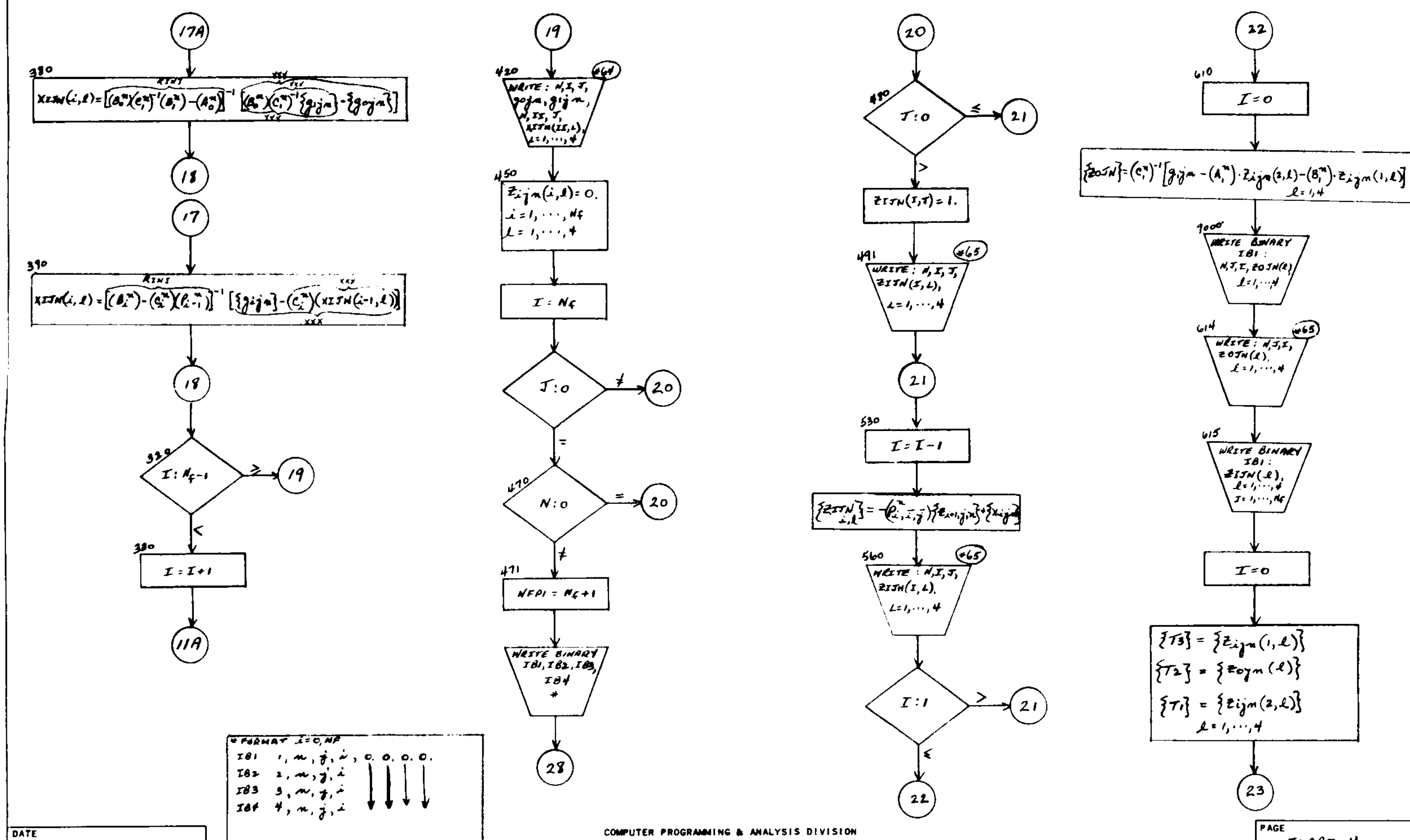
COMPUTER PROGRAMMING & ANALYSIS DIVISION
ADMINISTRATION DEPARTMENT

EDPE-87 - 6/62 DC


9
170
$P_n = \frac{PSI \cdot AC}{\sigma_c \cdot t_c}$
10
180
J=0
11
190
I=0
11A
200
DUMMY(1) = N
CALL SUBROUTINE
FLAIN (I,J,O)
#4
WRITE: N,J,I, OMSI, OMTI, GAMI, RHPI, OMSPI
210
$\{g_{ijn}\} = 0$
12
N:0
J:0
13
220
$t_{oj}^{n} = \frac{PSI-(A-\frac{TBC}{3})^2}{2A} \cdot \frac{1}{\sigma_c \cdot t_c}$
14
225
$t_{oj}^{n} = 0$
230
I : 0
15
$g_{ijn}(1) = t_{oj}^{n}$
15A
240
I:NG
245
J:0
250
$g_{ijn}(3) = -\rho^2(\Delta_i + \Delta_{i+1})$
260
#63
WRITE: i, j, N, T_{oj}^{n}, $g_{ijn}(l)$, $l=1,\cdots,4$
290
I:0
300
$\{g_{ojn}\} = \{g_{ijn}\}$
18
I:1
17
16
350
$\{g_{ijn}\} = \{g_{ijn}\}$
16A
7101
7102
$\{g_{i00}\} = \{g_{ijn}\}$
7106
$\{g_{ojn}\} = \{g_{ojn}\} - \bar{C}_0 A_1' \{g_{ijn}\}$
TTT
#64
WRITE: g_{100}, TTT, TTX, g_{ojn}, $\bar{C}_0$, A_1'
17A

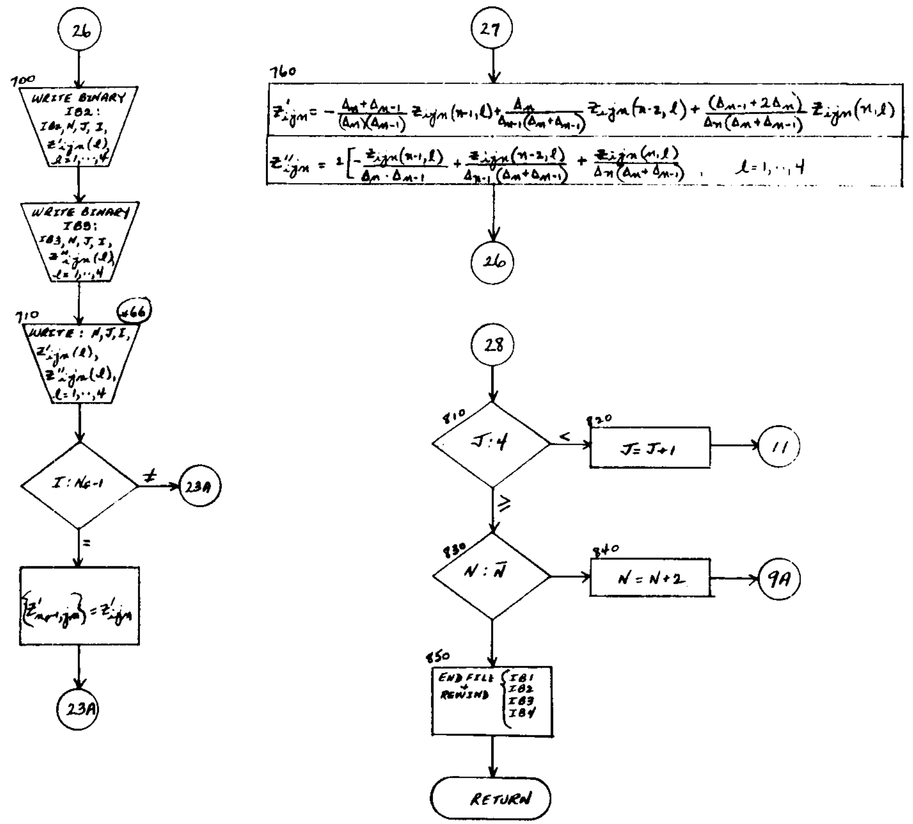

23
630
WRITE BINARY IB2: $Z'_{ojn}(\ell)$, $\ell=1,\dots,4$
631 WRITE: N, J, I, $Z'_{ojn}(\ell)$, $\ell=1,\dots,4$
#66
640
23A
DUMMY(1) = N
CALL SUBROUTINE FLAIN(I, J, 2) #4
I : 0
=
≠
644
24
24
9003 WRITE BINARY IB4: IB4, N, J, I, $\{Y_{IJN}(\ell)\}$, $\ell=1,\dots,4$
645 WRITE: N, I, J, $\{Y_{IJN}(\ell)\}$, $\ell=1,\dots,4$
#67
I : N_f
≥
28
<
647 I=I+1; IP1=I+1
650
I : N_f
≥
27
<
25
660
I : 1
≠
26
=
690
I-1 = IM1 = 0
9004 WRITE BINARY IB3: IB3, N, J, IM1, $Z''_{ojn}(\ell)$, $\ell=1,\dots,4$
695 WRITE: N, J, IM1, $Z''_{ojn}(\ell)$, $\ell=1,\dots,4$
#66
26

SECTION TITLE: PART I - FLARE #2 | #2 | SUBJECT: FLARE FLOW CHART



SECTION TITLE: PART I - FLARE #2

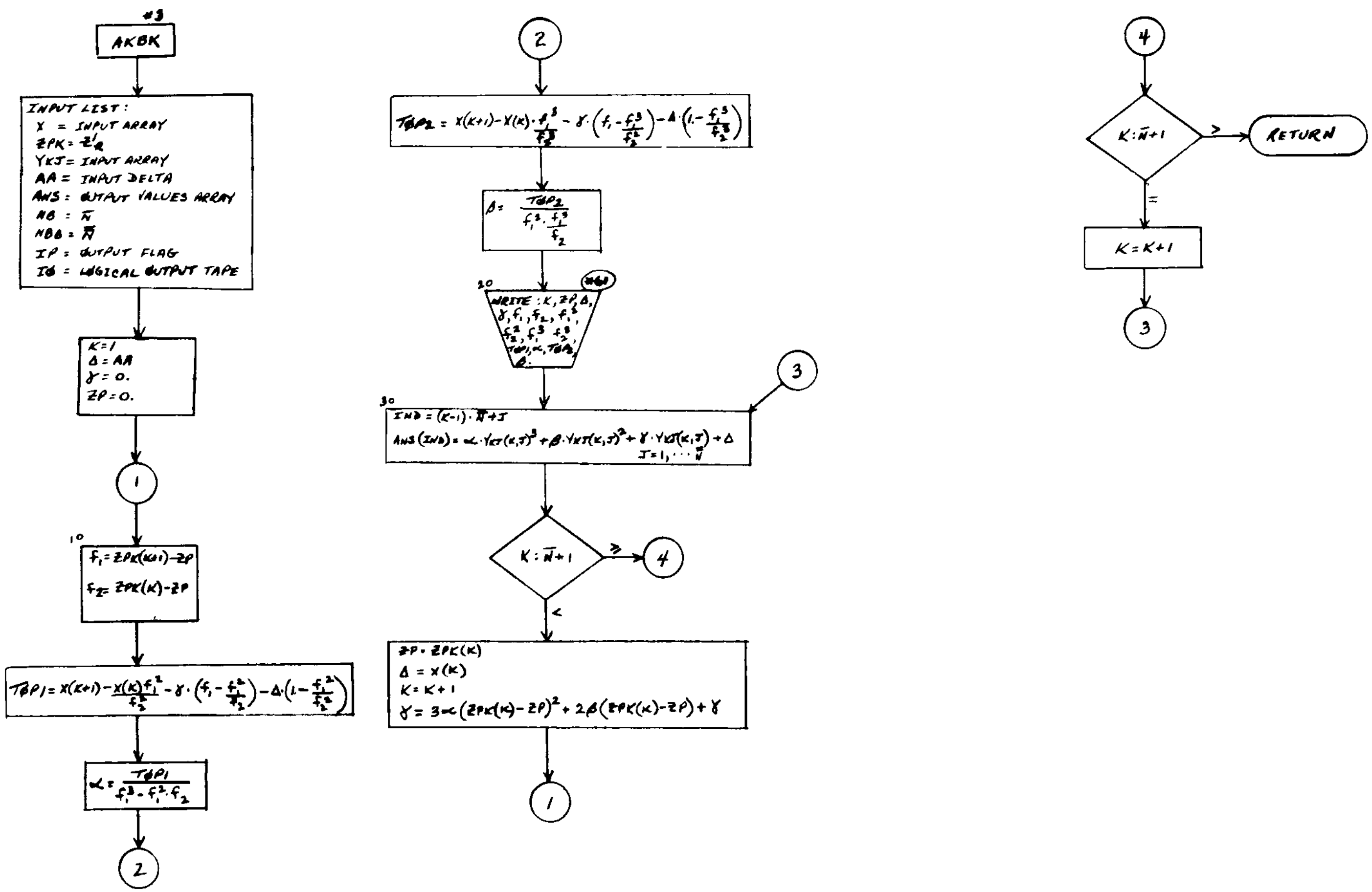
SUBJECT: FLARE FLOW CHART



SECTION TITLE

PART I - FLARE SUBROUTINE AKBK #3 #3

SUBJECT AKBK FLOW CHART



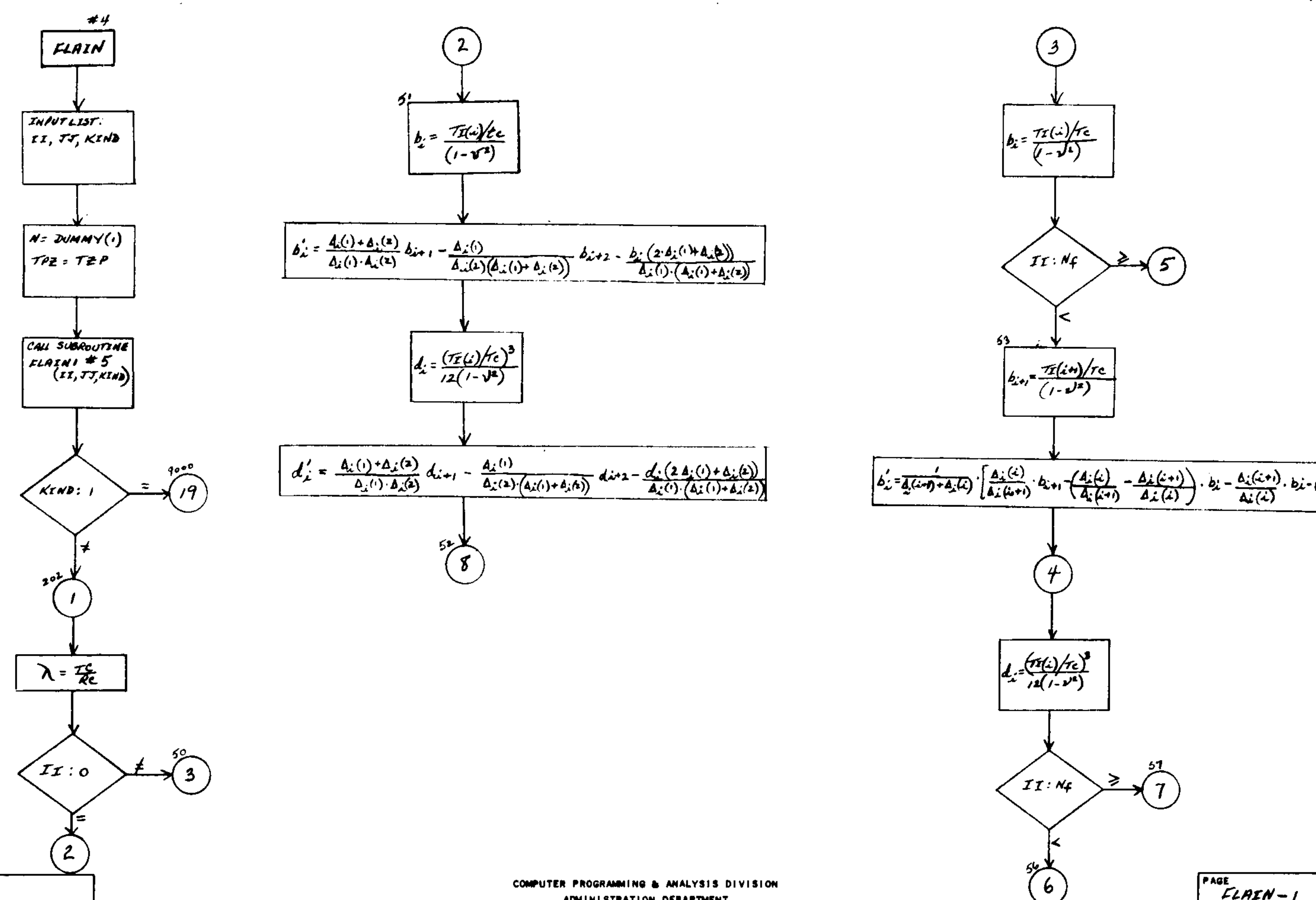
DATE

SECTION TITLE

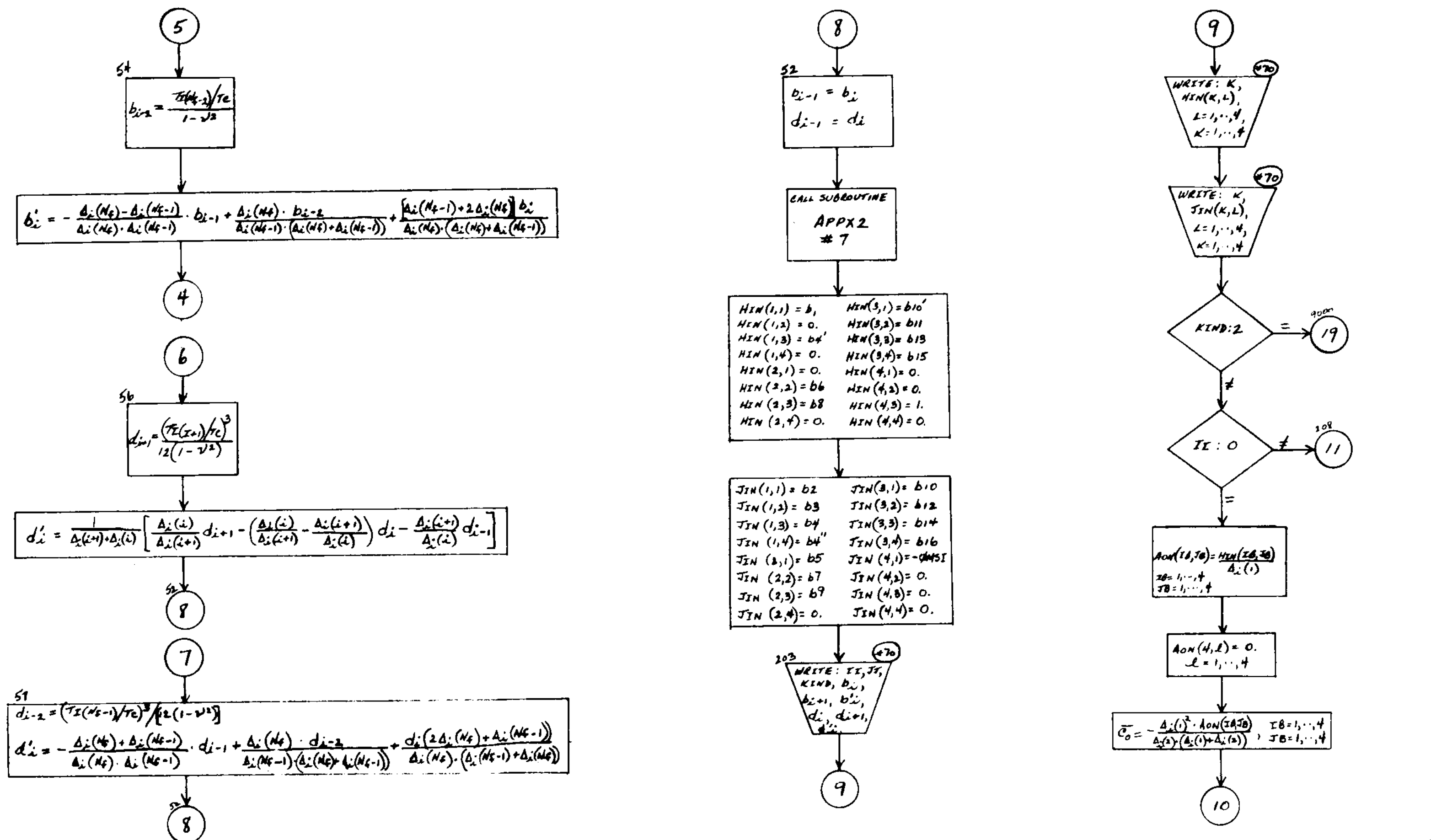
PART I - FLARE SUBROUTINE FLAIN #4 #4

SUBJECT

FLAIN FLOW CHART

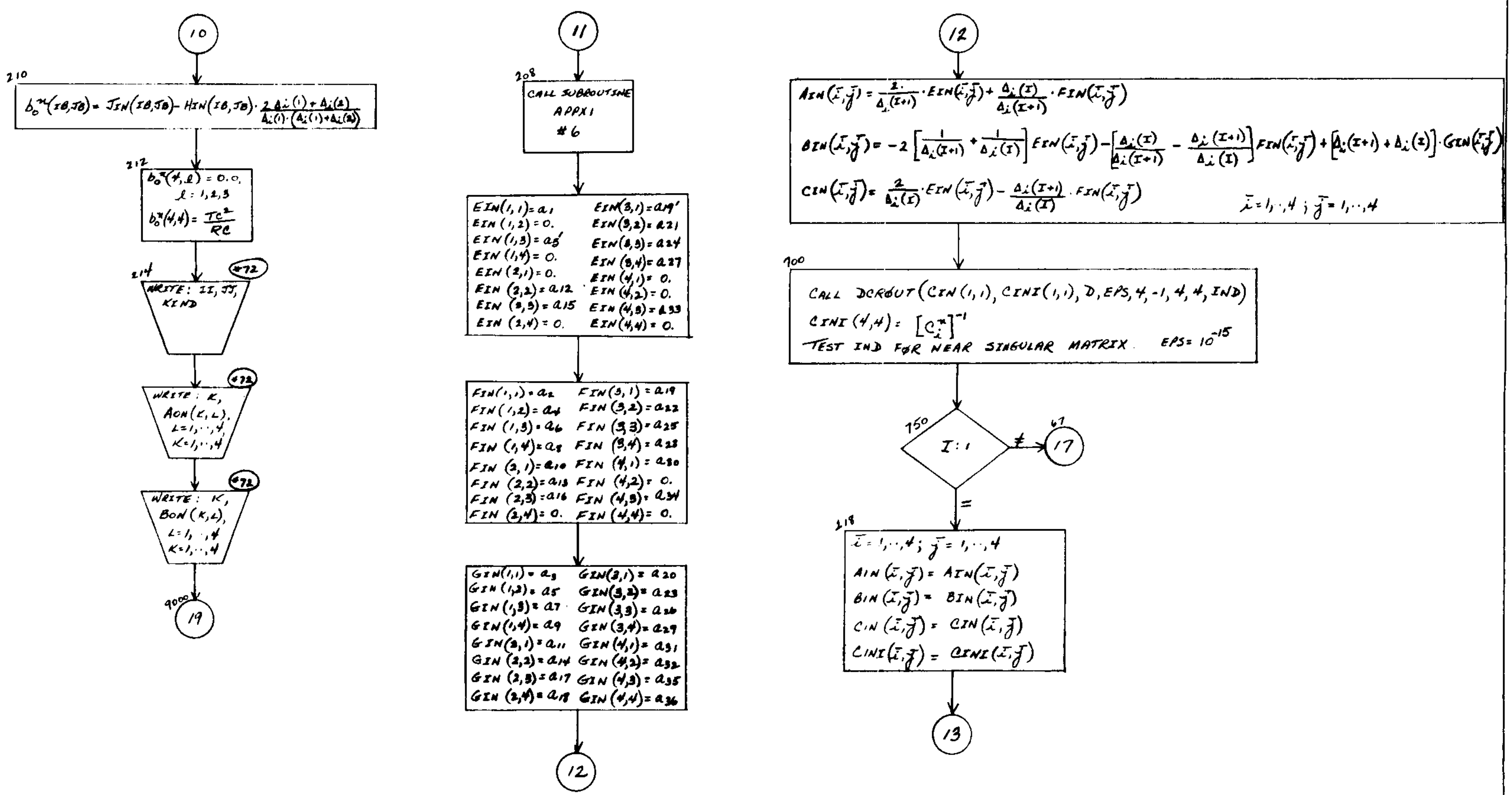


DATE


5
54
$b_{i-2} = \frac{T_I(N_f-2)/T_C}{1-\nu^2}$
$b_i' = -\frac{\Delta_i(N_f)-\Delta_i(N_f-1)}{\Delta_i(N_f)\cdot\Delta_i(N_f-1)}\cdot b_{i-1} + \frac{\Delta_i(N_f)\cdot b_{i-2}}{\Delta_i(N_f-1)\cdot(\Delta_i(N_f)+\Delta_i(N_f-1))} + \frac{[\Delta_i(N_f-1)+2\Delta_i(N_f)]\, b_i}{\Delta_i(N_f)\cdot(\Delta_i(N_f)+\Delta_i(N_f-1))}$
4
6
56
$d_{i-1} = \frac{(T_E(I+1)/T_C)^3}{12(1-\nu^2)}$
$d_i' = \frac{1}{\Delta_i(i+1)+\Delta_i(i)}\left[\frac{\Delta_i(i)}{\Delta_i(i+1)}d_{i+1} - \left(\frac{\Delta_i(i)}{\Delta_i(i+1)} - \frac{\Delta_i(i+1)}{\Delta_i(i)}\right)d_i - \frac{\Delta_i(i+1)}{\Delta_i(i)}d_{i-1}\right]$
52
8
7
51
$d_{i-2} = (T_I(N_f-1)/T_C)^3 / [12(1-\nu^2)]$
$d_i' = -\frac{\Delta_i(N_f)+\Delta_i(N_f-1)}{\Delta_i(N_f)\cdot\Delta_i(N_f-1)}\cdot d_{i-1} + \frac{\Delta_i(N_f)\cdot d_{i-2}}{\Delta_i(N_f-1)\cdot(\Delta_i(N_f)+\Delta_i(N_f-1))} + \frac{d_i(2\Delta_i(N_f)+\Delta_i(N_f-1))}{\Delta_i(N_f)\cdot(\Delta_i(N_f-1)+\Delta_i(N_f))}$
52
8
8
52
$b_{i-1} = b_i$
$d_{i-1} = d_i$
CALL SUBROUTINE
APPX2
#7
HIN(1,1) = b1
HIN(1,2) = 0.
HIN(1,3) = b4'
HIN(1,4) = 0.
HIN(2,1) = 0.
HIN(2,2) = b6
HIN(2,3) = b8
HIN(2,4) = 0.
HIN(3,1) = b10'
HIN(3,2) = b11
HIN(3,3) = b13
HIN(3,4) = b15
HIN(4,1) = 0.
HIN(4,2) = 0.
HIN(4,3) = 1.
HIN(4,4) = 0.
JIN(1,1) = b2
JIN(1,2) = b3
JIN(1,3) = b4
JIN(1,4) = b4''
JIN(2,1) = b5
JIN(2,2) = b7
JIN(2,3) = b9
JIN(2,4) = 0.
JIN(3,1) = b10
JIN(3,2) = b12
JIN(3,3) = b14
JIN(3,4) = b16
JIN(4,1) = -GMSI
JIN(4,2) = 0.
JIN(4,3) = 0.
JIN(4,4) = 0.
203
#70
WRITE: II, JI, KIND, b_i, b_{i+1}, b_i', d_i, d_{i+1}, d_i'
9
9
#70
WRITE: K, HIN(K,L), L=1,...,4, K=1,...,4
#70
WRITE: K, JIN(K,L), L=1,...,4, K=1,...,4
KIND:2
=
≠
900
19
II : 0
≠
=
108
11
AON(IB,JB) = HIN(IB,JB) / $\Delta_i(1)$
IB = 1,...,4
JB = 1,...,4
AON(4,L) = 0.
L = 1,...,4
$\bar{C}_0 = -\frac{\Delta_i(1)^2\cdot AON(IB,JB)}{\Delta_i(2)\cdot(\Delta_i(1)+\Delta_i(2))}$, IB = 1,...,4, JB = 1,...,4
10

SECTION TITLE: PART I - FLARE #4

SUBJECT: FLAIN FLOW CHART

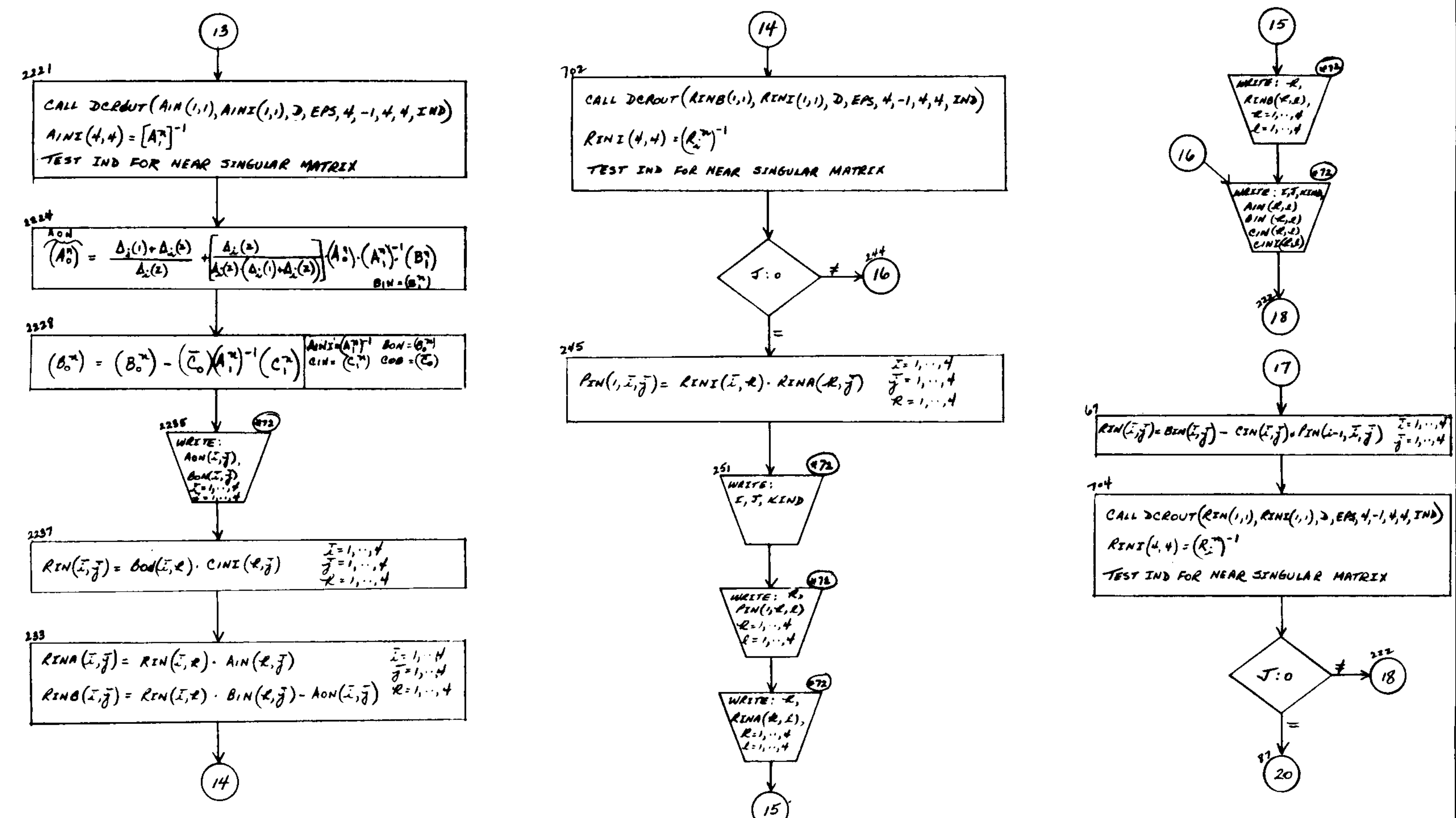


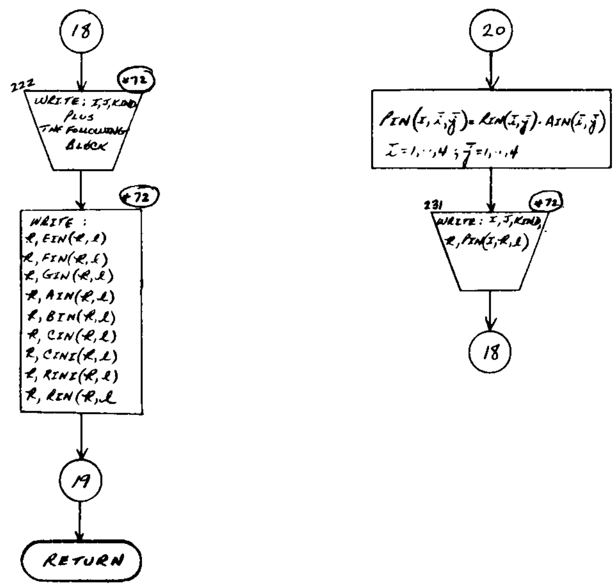
SECTION TITLE

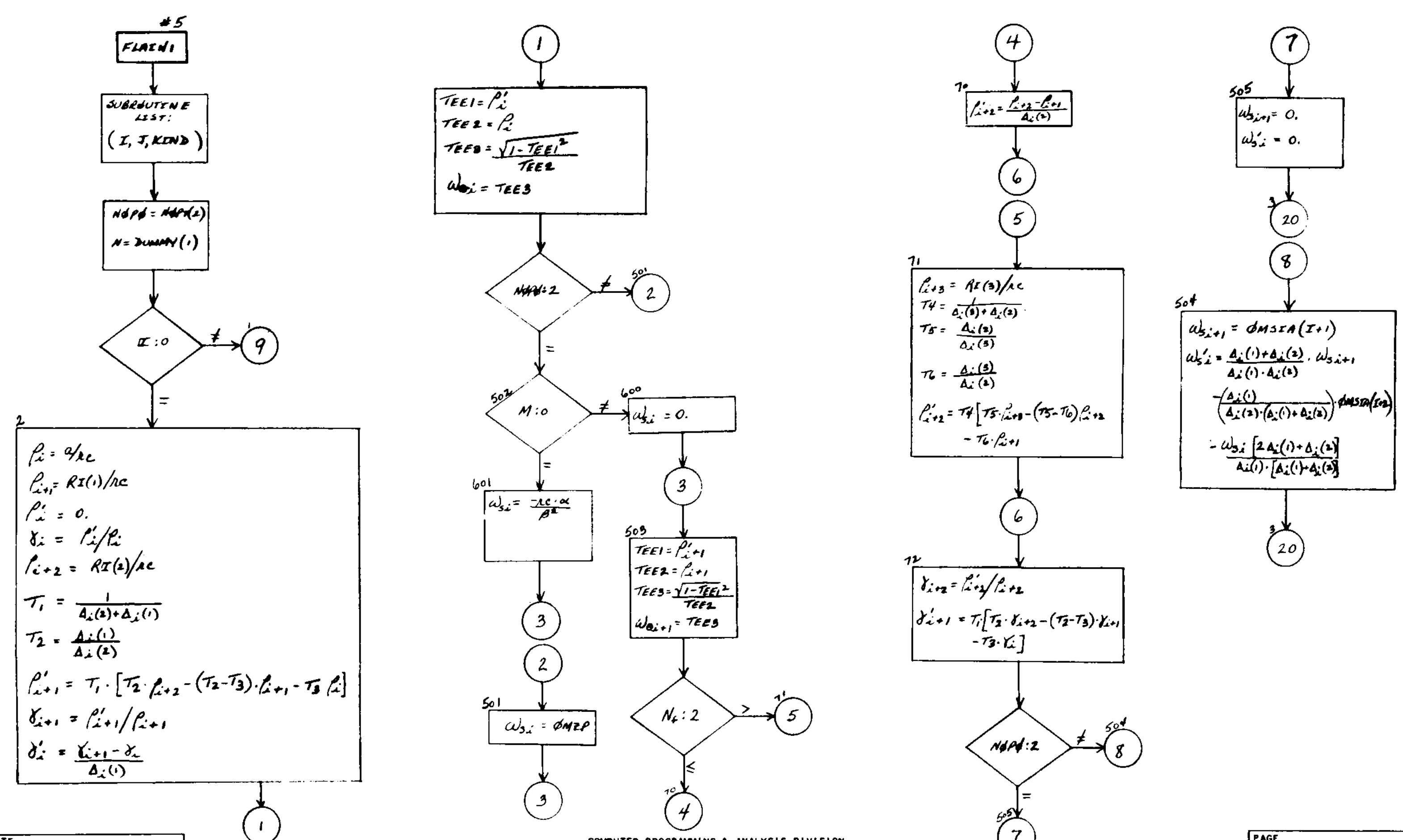
PART I - FLARE #4

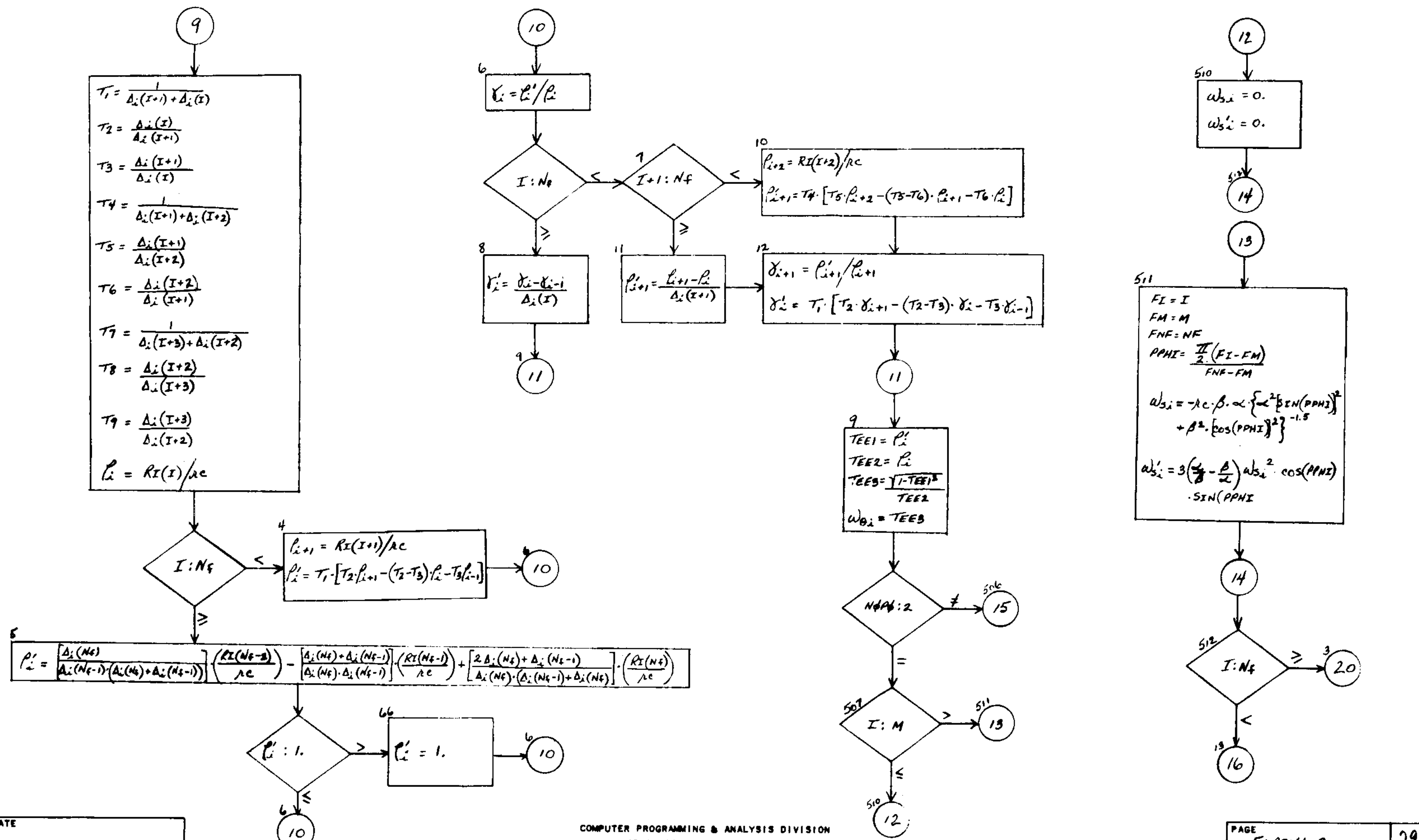
SUBJECT

FLAIN FLOW CHART



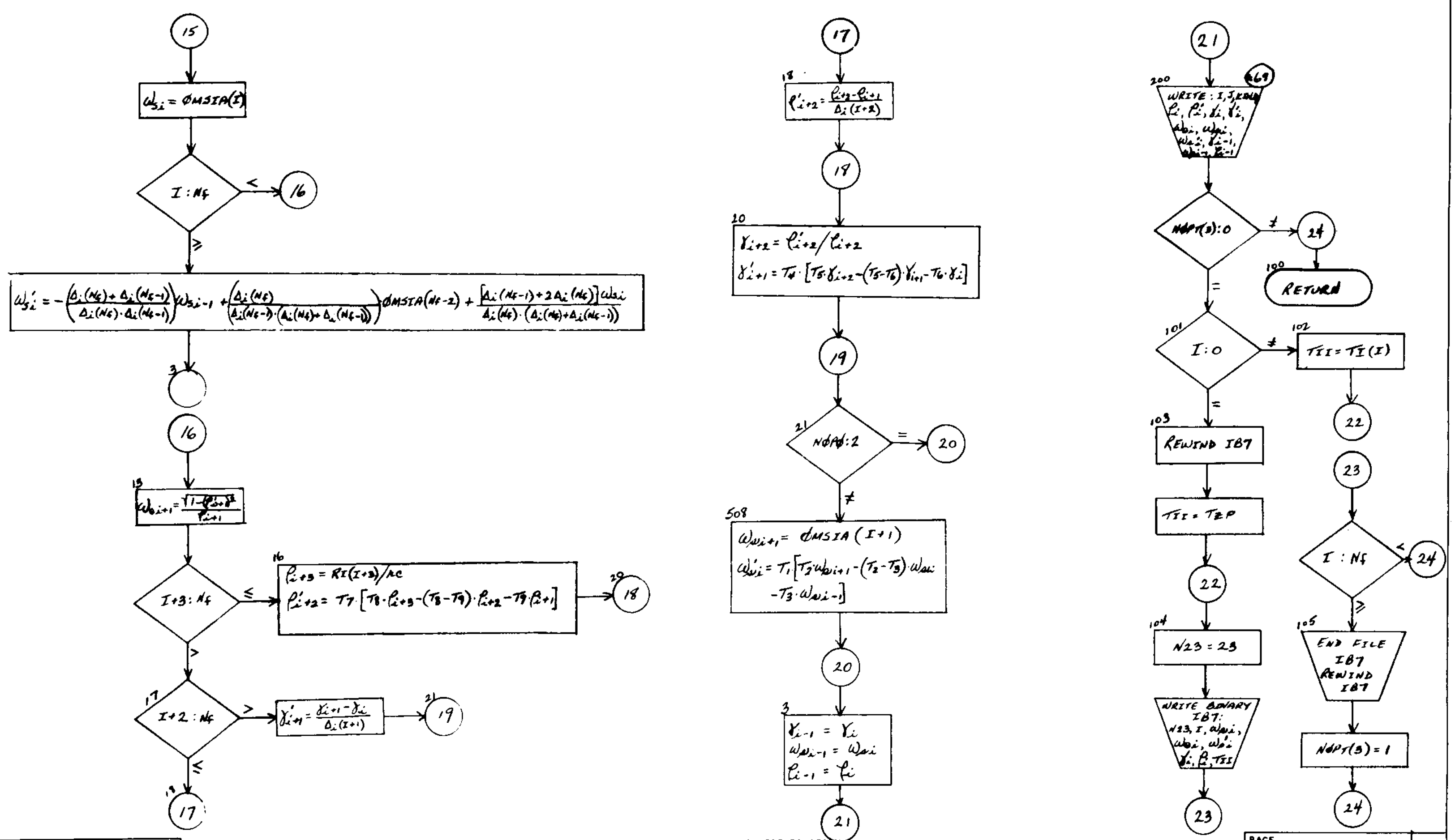

18
222
#72
WRITE: I,J,KIND PLUS THE FOLLOWING BLOCK
#72
WRITE :
k, EIN(k,l)
k, FIN(k,l)
k, GIN(k,l)
k, AIN(k,l)
k, BIN(k,l)
k, CIN(k,l)
k, CINI(k,l)
k, RINI(k,l)
k, RIN(k,l
19
RETURN
20
PIN(I,ī,j̄) = RIN(I,j̄)·AIN(ī,j̄)
ī=1,…,4 ; j̄=1,…,4
231
#72
WRITE: I,J,KIND, k,PIN(I,k,l)
18


#5
FLAINI
SUBROUTINE LIST: (I, J, KIND)
NOPØ = NOPO(2)
N = DUMMY(1)
I:0
9
TEE1 = ρ'i
TEE2 = ρi
TEE3 = √(1-TEE1²) / TEE2
ω3i = TEE3
M:0
ω3i = 0.
ω3i = -RC·α / β²
ω3i = ØMZP
TEE1 = ρ'i+1
TEE2 = ρi+1
TEE3 = √(1-TEE1²) / TEE2
ω3i+1 = TEE3
Ni:2
ω3i+1 = ØMSIA(I+1)
ω3i' = 0.
20


9
I : NF
10
I : NF
I+1 : NF
11
TEE1 = ρ'i
TEE2 = ρi
ωθi = TEE3
NØP0 : 2
15
I : M
13
12
ωsi = 0.
ωs'i = 0.
14
FI = I
FM = M
FNF = NF
I : NF
20
16

SECTION TITLE: PART I - FLARE #5

SUBJECT: FLAINI FLOW CHART



#6

APPXI

$N = DUMMY(1)$

$a_1 = b_i + (d_i \lambda^2)(\omega_{si} - \omega_{\theta i})(\omega_{\theta i})$

$a_2 = \gamma_i b_i + b_i' + (\gamma_i d_i \lambda^2)(\omega_{si} - \omega_{\theta i})^2 + d_i' \cdot \lambda^2 \cdot \omega_{\theta i} \cdot (\omega_{si} - \omega_{\theta i}) + d_i \lambda^2 \cdot \omega_{si}' \cdot \omega_{\theta i}$

$a_3 = \gamma_i \cdot (\nu \cdot b_i' - \gamma_i b_i) - \nu \cdot b_i \cdot \omega_{si} \cdot \omega_{\theta i} - (b_i N^2)\left(\frac{1-\nu}{2\rho^2}\right) + \nu \lambda^2 \left\{ \left[\gamma_i \cdot d_i' \cdot \omega_{si} - \gamma_i^2 \cdot d_i \cdot \omega_{si} - d_i(\omega_{si})^2 \omega_{\theta i}\right](\omega_{si} - \omega_{\theta i}) + \gamma_i d_i \omega_{si}' (2\omega_{si} - \omega_{\theta i}) + \nu d_i \gamma_i^2 \cdot \omega_{si}^2 \right\} - \lambda^2 d_i \left[\gamma_i^2 + \frac{N^2(1-\nu)}{2\rho^2}\right] \cdot (3\omega_{si}^2 - 3\omega_{\theta i} \cdot \omega_{si} + \omega_{\theta i}^2)$

$a_4 = \left(\frac{1+\nu}{2}\right)\left(\frac{N}{\rho_i}\right) \cdot b_i + d_i \lambda^2 \left(\frac{N}{\rho_i}\right) \cdot \left[\left(\frac{1+\nu}{2}\right) \cdot \omega_{si} \cdot \omega_{\theta i} - \nu \cdot \omega_{\theta i}^2\right]$

$T_1 = \left[\nu b_i' - \left(\frac{3-\nu}{2}\right) \cdot \gamma_i \cdot b_i\right] \cdot \left(\frac{N}{\rho_i}\right)$

$T_2 = \nu \lambda^2 \left(\frac{N}{\rho_i}\right) \cdot \left[d_i' \cdot \omega_{\theta i} \cdot (\omega_{si} - \omega_{\theta i}) + d_i \cdot \omega_{\theta i} \cdot \omega_{si}'\right]$

$T_3 = \lambda^2 \left(\frac{N}{\rho_i}\right) \cdot \gamma_i \cdot d_i$

$T_4 = -(1-\nu)\omega_{si}^2 - \left[\left(\frac{1+5\nu}{2}\right) - \nu^2\right] \cdot \omega_{si} \cdot \omega_{\theta i} + 2\nu \cdot \omega_{\theta i}^2$

$a_5 = T_1 + T_2 + T_3 \cdot T_4$

$a_5' = -\nu d_i \cdot \lambda^2 \cdot \gamma_i (\omega_{si} - \omega_{\theta i})$

$T_1 = b_i(\omega_{si} + \nu + \omega_{\theta i})$

$T_2 = \nu \cdot \gamma_i \cdot \lambda^2 \left[d_i'(\omega_{si} - \omega_{\theta i}) + d_i \cdot \omega_{si}'\right]$

$T_3 = d_i \lambda^2$

$T_4 = (1+\nu)(\omega_{si} - \omega_{\theta i}) \cdot (\omega_{si} \cdot \omega_{\theta i} + \gamma_i^2) + (1-\nu^2)\gamma_i^2 \cdot \omega_{si}$

$T_5 = \left(\frac{3-\nu}{2}\right)\left(\frac{N^2}{\rho^2}\right)\omega_{si} - 5(1+\nu)\omega_{\theta i}\left(\frac{N^2}{\rho^2}\right)$

$a_6 = T_1 - T_2 + T_3(T_4 + T_5)$

$T_1 = b_i'(\omega_{si} + \nu \cdot \omega_{\theta i})$

$T_2 = \lambda^2 (\omega_{si} \cdot \omega_{\theta i} + \frac{\nu N^2}{\rho^2})$

$T_3 = d_i'(\omega_{si} - \omega_{\theta i}) + d_i \cdot \omega_{si}'$

1

1

$T_4 = b_i \left[\gamma_i(\omega_{si} - \omega_{\theta i}) + \omega_{si}'\right]$

$T_5 = d_i \cdot \lambda^2 \cdot \omega_{\theta i}(\omega_{si} - \omega_{\theta i}) \cdot \omega_{si}'$

$T_6 = d_i \lambda^2 \gamma_i$

$T_7 = (\omega_{si} - \omega_{\theta i})^3 - \nu \omega_{si}^2(\omega_{si} - \omega_{\theta i})$

$T_8 = \left(\frac{N^2}{\rho^2}\right) \cdot \left[(\omega_{si} - \omega_{\theta i}) - .5(1-5\nu)(\omega_{si} - \omega_{\theta i}) + (1-\nu^2) \cdot \omega_{si} + (1-\nu) \cdot (2 \cdot \omega_{si} - \omega_{\theta i})\right]$

$a_7 = T_1 + T_2 \cdot T_3 + T_4 + T_5 + T_6(T_7 - T_8)$

$a_8 = -\lambda^2 \cdot \omega_{\theta i}$

$a_9 = \lambda^2 \left[\omega_{si}' - \gamma_i(1-\nu) \cdot \omega_{si}\right]$

$a_{10} = \left(-\frac{b_i \cdot N}{2\rho_i}\right)(1+\nu) - \lambda^2 \cdot N \cdot d_i \cdot \frac{\omega_{\theta i}}{\rho_i} \cdot \left[.5(1+\nu) \cdot \omega_{si} - \nu \cdot \omega_{\theta i}\right]$

$T_1 = \left(\frac{N}{\rho_i}\right) \cdot \left[.5 \cdot b_i' \cdot (1-\nu) + b_i \cdot \gamma_i\right]$

$T_2 = \left(\frac{N}{\rho_i}\right)\left[\lambda^2 \cdot \omega_{\theta i} \cdot .5(1-\nu)(d_i' \cdot \omega_{si} + d_i \cdot \omega_{si}')\right]$

$T_3 = \left(\frac{N}{\rho_i}\right) \cdot \lambda^2 \cdot \gamma_i \cdot d_i \cdot \omega_{si} \cdot \left[.5(3-\nu)\omega_{si} - \nu^2 \cdot \omega_{\theta i}\right]$

2

(2)

$a_{11} = -T_1 - T_2 - T_3$

$N\phi\phi = .5(1-\nu)$

$a_{12} = N\phi\phi\left[b_i + d_i \cdot \lambda^2(\omega_{si}^2 - 3\omega_{\theta i} \cdot \omega_{si} + 3\omega_{\theta i}^2)\right]$

$T_1 = b_i' + d_i' \cdot \lambda^2(\omega_{si}^2 - 3\omega_{si} \cdot \omega_{\theta i} + 3\omega_{\theta i}^2)$

$T_2 = d_i \lambda^2 \cdot \omega_{si}'(2\omega_{si} - 3\omega_{\theta i})$

$T_3 = d_i \cdot \lambda^2 \cdot \gamma_i(3\omega_{si} - 2\omega_{\theta i})(\omega_{si} - 2\omega_{\theta i})$

$a_{13} = N\phi\phi(T_1 + T_2 - T_3)$

$T_1 = b_i \cdot \omega_{si} \cdot \omega_{\theta i} - b_i'\gamma_i - d_i' \cdot \lambda^2 \cdot \gamma_i \cdot (\omega_{si}^2 - 3\omega_{si} \cdot \omega_{\theta i} + 3\omega_{\theta i}^2)$

$T_2 = d_i \cdot \lambda^2 \cdot \gamma_i \cdot \omega_{si}'(3 \cdot \omega_{\theta i} - 2\omega_{si}) \cdot d_i \cdot \lambda^2 \cdot \gamma_i^2 \cdot (3 \cdot \omega_{si} - 2 \cdot \omega_{\theta i})(\omega_{si} - 2\omega_{\theta i})$

$T_3 = d_i \cdot \lambda^2 \cdot \omega_{si} \cdot \omega_{\theta i}(\omega_{si}^2 - 3\omega_{si} \cdot \omega_{\theta i} + 3 \cdot \omega_{\theta i}^2)$

$T_4 = \left(\frac{N^2}{\rho_i^2}\right)\left[b_i + \lambda^2 \cdot d_i \cdot \omega_{\theta i} \cdot (\omega_{si} - \nu^2 \cdot \omega_{\theta i})\right]$

$a_{14} = N\phi\phi(T_1 + T_2 + T_3) - T_4$

$a_{15} = N\phi\phi\left(\frac{N}{\rho_i}\right) \cdot \lambda^2 \cdot d_i \cdot (3\omega_{\theta i} - \omega_{si})$

$T_1 = \left(\frac{N}{\rho_i}\right) \cdot \lambda^2 \cdot \left[d_i' \cdot (3\omega_{\theta i} - \omega_{si}) - d_i \cdot \omega_{si}'\right]$

$T_2 = \left(\frac{N}{\rho_i}\right) \cdot \gamma_i \cdot d_i \cdot \lambda^2 \cdot \left[(3\omega_{si} - 2\omega_{\theta i}) - 2\nu \cdot (\omega_{si} - \omega_{\theta i}) - \nu^2 \cdot \omega_{\theta i}\right]$

$a_{16} = N\phi\phi \cdot T_1 + T_2$

$T_1 = \left(\frac{N}{\rho_i}\right) \cdot \gamma_i \cdot \lambda^2 \cdot \left[d_i' \cdot (\omega_{si} - 3\omega_{\theta i}) + d_i \cdot \omega_{si}'\right]$

$T_2 = \left(\frac{b_i \cdot N}{\rho_i}\right)(\nu \cdot \omega_{si} + \omega_{\theta i})$

$T_3 = 2 \cdot (1-\nu)\left(\frac{N}{\rho_i}\right) \cdot d_i \cdot \gamma_i^2 \cdot \lambda^2 \cdot (\omega_{si} - \omega_{\theta i})$

$T_4 = \left(\frac{N}{\rho_i}\right)^3 \cdot d_i \cdot \lambda^2 \cdot (\omega_{si} - \nu^2 \cdot \omega_{\theta i})$

$T_5 = \left(\frac{N}{2\rho_i}\right) \cdot d_i \cdot \lambda^2 \cdot \omega_{si} \cdot \omega_{\theta i}\left[(3\omega_{\theta i} - \omega_{si}) - \nu(\omega_{si} + \omega_{\theta i})\right]$

(3)

(3)

$a_{17} = N\phi\phi \cdot T_1 - T_2 - T_3 - T_4 + T_5$

$a_{18} = \nu\left(\frac{N}{\rho_i}\right) \cdot \lambda^2 \cdot \omega_{\theta i}$

$a_{19}' = -\left[\nu \cdot \lambda^2 \cdot \gamma_j \cdot d_i \cdot (\omega_{si} - \omega_{\theta i})\right]$

$T_1 = b_i(\omega_{si} + \nu \cdot \omega_{\theta i}) + \nu \cdot \lambda^2 \cdot \gamma_i \cdot d_i'(\omega_{si} - \omega_{\theta i})$

$T_2 = (1-\nu) \cdot d_i \cdot \lambda^2 \cdot (\omega_{si} \cdot \omega_{\theta i}) \cdot (\omega_{\theta i} - \omega_{si})$

$T_3 = \left[\frac{d_i \cdot \lambda^2 \cdot N^2}{2\rho^2}\right] \cdot \left[3\omega_{si} - \omega_{\theta i} - \nu(\omega_{si} + \omega_{\theta i})\right]$

$T_4 = d_i \cdot \lambda^2 \cdot \gamma_i^2 \cdot \left[(2-\nu-\nu^2) \cdot \omega_{si} - (1-\nu) \cdot \omega_{\theta i}\right]$

$T_5 = \nu \cdot d_i \cdot \lambda^2 \cdot \gamma_i \cdot \omega_{si}'$

$a_{19} = -(T_1 + T_2 + T_3 + T_4 + T_5)$

$T_1 = b_i \cdot \gamma_i \cdot (\nu \cdot \omega_{si} + \omega_{\theta i})$

$T_2 = \lambda^2 \cdot d_i \cdot \gamma_i \cdot \left[\nu \cdot \omega_{si}^3 - (3+\nu-2\nu^2) \cdot \omega_{si}^2 \cdot \omega_{\theta i} + \omega_{\theta i}^3\right]$

$T_3 = \lambda^2 \cdot \gamma_i^2 \cdot d_i' \cdot \left[(2-\nu^2) \cdot \omega_{si} - \omega_{\theta i}\right]$

$T_4 = N\phi\phi \cdot \lambda^2 \cdot \left(\frac{N^2}{\rho^2}\right) \cdot d_i' \cdot (3\omega_{si} - \omega_{\theta i})$

$T_5 = \lambda^2 \cdot \gamma_i^3 \cdot d_i \cdot \left[2\omega_{\theta i} - (3-\nu^2) \cdot \omega_{si}\right]$

$T_6 = \lambda^2 \cdot d_i \cdot \omega_{si}' \cdot \left[(2-\nu^2) \cdot \gamma_i^2 + 3 \cdot N\phi\phi \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$T_7 = \lambda^2 \cdot \gamma_i \cdot d_i\left(\frac{N^2}{\rho_i^2}\right) \cdot \left[(1.5 + .5\nu - \nu^2) \cdot \omega_{si} - .5(1+\nu) \cdot \omega_{\theta i}\right]$

$a_{20} = -(T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7)$

$a_{21} = -N\phi\phi \cdot \lambda^2 \cdot d_i \cdot \left(\frac{N}{\rho_i}\right) \cdot (3\omega_{\theta i} -$

$T_1 = \lambda^2 \cdot \gamma_i \cdot d_i\left(\frac{N}{\rho_i}\right)$

$T_2 = -.5(1-3\nu) \cdot \omega_{si} + (1.5 - 1.5\nu - \nu^2) \cdot \omega_{\theta i}$

$T_3 = N\phi\phi \cdot \lambda^2 \cdot \left(\frac{N}{\rho_i}\right)$

$T_4 = -d_i' \cdot (3\omega_{\theta i} - \omega_{si}) + d_i \cdot \omega_{si}'$

(4)

4

$a_{22} = -(T1 \cdot T2 + T3 \cdot T4)$

$T1 = b_i \left(\frac{N}{\rho_i}\right) \cdot (\nu \cdot \omega_{si} + \omega_{\theta i})$

$T2 = \left[-\lambda^2 \cdot d_i \cdot \left(\frac{N}{\rho_i}\right) \cdot \omega_{si} \cdot \omega_{\theta i}\right] \cdot \left\{N\phi\phi \cdot \omega_{si} + [.5(3-\nu)-\nu^2] \cdot \omega_{\theta i}\right\}$

$T3 = \lambda^2 \cdot \gamma_i \cdot d_i' \cdot \left(\frac{N}{\rho_i}\right)$

$T4 = .5(1+\nu) \cdot \omega_{si} + (1.5 - 1.5\nu - \nu^2) \cdot \omega_{\theta i}$

$T5 = \lambda^2 \cdot \gamma_i^2 \cdot d_i \cdot \left(\frac{N}{\rho_i}\right)$

$T6 = [.5(1-3\nu)-\nu^2] \cdot \omega_{si} + (1.5 + 1.5\nu + 2\nu^2) \cdot \omega_{\theta i}$

$T7 = \lambda^2 \cdot \gamma_i \cdot d_i \cdot \left(\frac{N}{\rho_i}\right) \cdot \omega_{si}' \cdot 5(1+\nu)$

$T8 = \lambda^2 \cdot d_i \left(\frac{N^3}{\rho_i^3}\right)(\omega_{si} - \nu^2 \cdot \omega_{\theta i})$

$a_{23} = -(T1 + T2 + T3 \cdot T4 + T5 \cdot T6 + T7 + T8)$

$a_{24} = \lambda^2 d_i \left[(1-\nu^2) \cdot \gamma_i^2 + 2(1-\nu) \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$T1 = \lambda^2 \cdot \gamma_i \cdot d_i \left[(1-\nu^2)(\gamma_i^2 + 2\omega_{si} \cdot \omega_{\theta i}) + 2(1-\nu) \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$T2 = \lambda^2 \cdot d_i' \left[(1-\nu^2) \cdot \gamma_i^2 + 2(1-\nu) \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$a_{25} = T2 - T1$

$T1 = b_i (\omega_{si}^2 + 2\nu \cdot \omega_{si} \cdot \omega_{\theta i} + \omega_{\theta i}^2)$

$T2 = d_i \lambda^2 (\omega_{si} - \omega_{\theta i}) \omega_{\theta i} \cdot [(1-\nu) \cdot \omega_{si}^2 - \omega_{si} \cdot \omega_{\theta i} - \omega_{\theta i}^2]$

$T3 = d_i \lambda^2 \left(\frac{N^2}{\rho_i^2}\right)$

$T4 = 2(\omega_{si} - \omega_{\theta i}) \cdot (\nu \cdot \omega_{si} + \omega_{\theta i}) - (3 - 2\nu - \nu^2) \cdot \omega_{si} \cdot \omega_{\theta i} + (1-\nu^2)\left(\frac{N^2}{\rho_i^2}\right)$

$T5 = d_i' \lambda^2 \gamma_i$

$T6 = (\omega_{si} - \omega_{\theta i}) \cdot (\omega_{\theta i} + \nu \cdot \omega_{si}) + (3 - 2\nu - \nu^2)\left(\frac{N^2}{\rho_i^2}\right)$

$T7 = \lambda^2 \cdot \gamma_i \cdot d_i \left\{\gamma_i (\omega_{si} - \omega_{\theta i}) \cdot [(1-\nu)\omega_{si} - 2\omega_{\theta i}] + \omega_{si}' \cdot [(1-\nu)\omega_{\theta i} + 2\nu \cdot \omega_{si}] - 2(2 - \nu - \nu^2) \cdot \left(\frac{N^2}{\rho_i^2}\right) \cdot \gamma_i\right\}$

$a_{26} = -(T1 + T2 + T3 \cdot T4 + T5 \cdot T6 + T7)$

$a_{27} = -\lambda^2$

5

5

$a_{28} = -\gamma_i \cdot \lambda^2 \cdot (2-\nu)$

$a_{29} = -\lambda^2 \left[\omega_{si}^2 - (2-\nu) \cdot \omega_{si} \cdot \omega_{\theta i} - \nu \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$a_{30} = d_i \cdot \omega_{\theta i}$

$a_{31} = d_i (\omega_{si}' + \nu \cdot \gamma_i \cdot \omega_{si})$

$a_{32} = \nu \cdot d_i \cdot \left(\frac{N}{\rho_i}\right) \cdot \omega_{\theta i}$

$a_{33} = -d_i$

$a_{34} = -\nu \cdot d_i \cdot \gamma_i$

$a_{35} = -d_i \left[(\omega_{si} \cdot (\omega_{si} - \omega_{\theta i}) - \nu \cdot \left(\frac{N^2}{\rho_i^2}\right)\right]$

$a_{36} = 1.$

#75

WRITE: $a_1, \cdots, a_{36}$

RETURN

#7

APPX2

$N = DUMMY(1)$

$$b_1 = b_i + (d_i \cdot \lambda^2)(\omega_{si} - \omega_{\theta i}) \cdot \omega_{\theta i}$$

$$b_2 = \nu \cdot b_i \cdot \gamma_i + \nu \cdot \gamma_i \cdot d_i \cdot \lambda^2 \cdot \omega_{si} \cdot (\omega_{si} - \omega_{\theta i})$$

$$b_3 = \frac{\nu \cdot b_i \cdot N}{\rho_i} + \left[\frac{(\nu \cdot d_i \cdot \lambda^2 \cdot N \cdot \omega_{\theta i})}{\rho_i}\right](\omega_{si} - \omega_{\theta i})$$

$$b_4' = -\nu \cdot \gamma_i \cdot d_i \cdot \lambda^2 \cdot (\omega_{si} - \omega_{\theta i})$$

$$b_4 = b_i \cdot (\omega_{si} + \nu \cdot \omega_{\theta i}) + d_i \cdot \lambda^2 \cdot (\omega_{si} - \omega_{\theta i})\left[\omega_{si} \cdot \omega_{\theta i} + \left(\frac{\nu \cdot N^2}{\rho_i^2}\right)\right]$$

$$b_4'' = \lambda^2 \cdot (\omega_{si} - \omega_{\theta i})$$

$$b_5 = \frac{\left[-(1-\nu) \cdot N(b_i + \lambda^2 \cdot d_i \cdot \omega_{si} \cdot \omega_{\theta i})\right]}{2\rho_i}$$

$$c_1 = (\omega_{si} - \omega_{\theta i})^2$$

$$c_2 = 2\omega_{\theta i} - \omega_{si}$$

$$c_3 = \frac{1-\nu}{2}$$

$$b_6 = c_3 \cdot \left[b_i + d_i \lambda^2 \cdot (c_1 + \omega_{\theta i} \cdot c_2)\right]$$

$$b_7 = -\gamma_i \cdot b_6$$

$$b_8 = c_3 \cdot d_i \cdot \lambda^2 \cdot \left(\frac{N}{\rho_i}\right)(3\omega_{\theta i} - \omega_{si})$$

$$b_9 = -\gamma_i \cdot b_8$$

$$b_{10}' = \nu \cdot d_i \cdot \lambda^2 \cdot \gamma_i \cdot (\omega_{si} - \omega_{\theta i})$$

$$b_{10} = d_i \lambda^2 \cdot \gamma_i^2 \cdot (2\omega_{si} - \omega_{\theta i} - \nu^2 \cdot \omega_{si}) + c_3 \cdot d_i \lambda^2 \left(\frac{N^2}{\rho_i^2}\right) \cdot (3\omega_{si} - \omega_{\theta i})$$

(1)

(1)

$$b_{11} = -c_3 \cdot d_i \cdot \lambda^2 \left(\frac{N}{\rho_i}\right)(3\omega_{\theta i} - \omega_{si})$$

$$b_{12} = d_i \lambda^2 \cdot \gamma_i \left(\frac{N}{\rho_i}\right)\left[\omega_{si} - \nu^2 \cdot \omega_{\theta i} + c_3(3\omega_{\theta i} - \omega_{si})\right]$$

$$b_{13} = -2(1-\nu) d_i \cdot \lambda^2 \left(\frac{N^2}{\rho_i^2}\right) - d_i \cdot \lambda^2 \gamma_i^2 \cdot (1-\nu^2)$$

$$b_{14} = d_i \cdot \lambda^2 \gamma_i \left[(\omega_{\theta i} + \nu \cdot \omega_{si})(\omega_{si} - \omega_{\theta i}) + (3 - 2\nu - \nu^2)\left(\frac{N^2}{\rho_i^2}\right)\right]$$

$$b_{15} = \lambda^2$$

$$b_{16} = \lambda^2 \gamma_i (1-\nu)$$

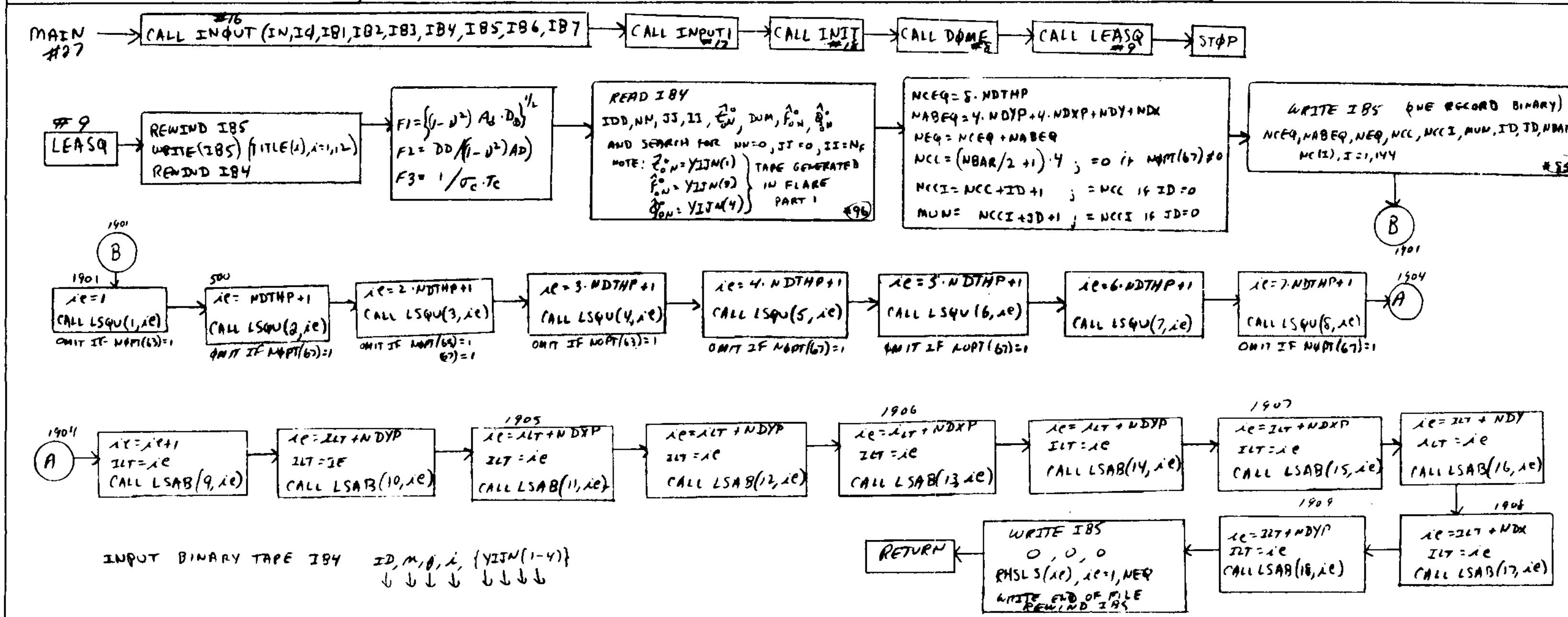
1 WRITE: $b_1, \cdots, b_{16}$ #74

RETURN

SECTION TITLE

PART II GENERATE COEFFICIENTS OF LEAST SQUARES MATRIX - LEASQ #9

SUBJECT



INPUT BINARY TAPE IB4 ID, m, j, i, {YIJN(1-4)}

OUTPUT BINARY TAPE IB5 TITLE

NCEQ, NABEQ, ... , NBAR, NC(i) i=1,144

i, j, a_{ij}

0 0 0

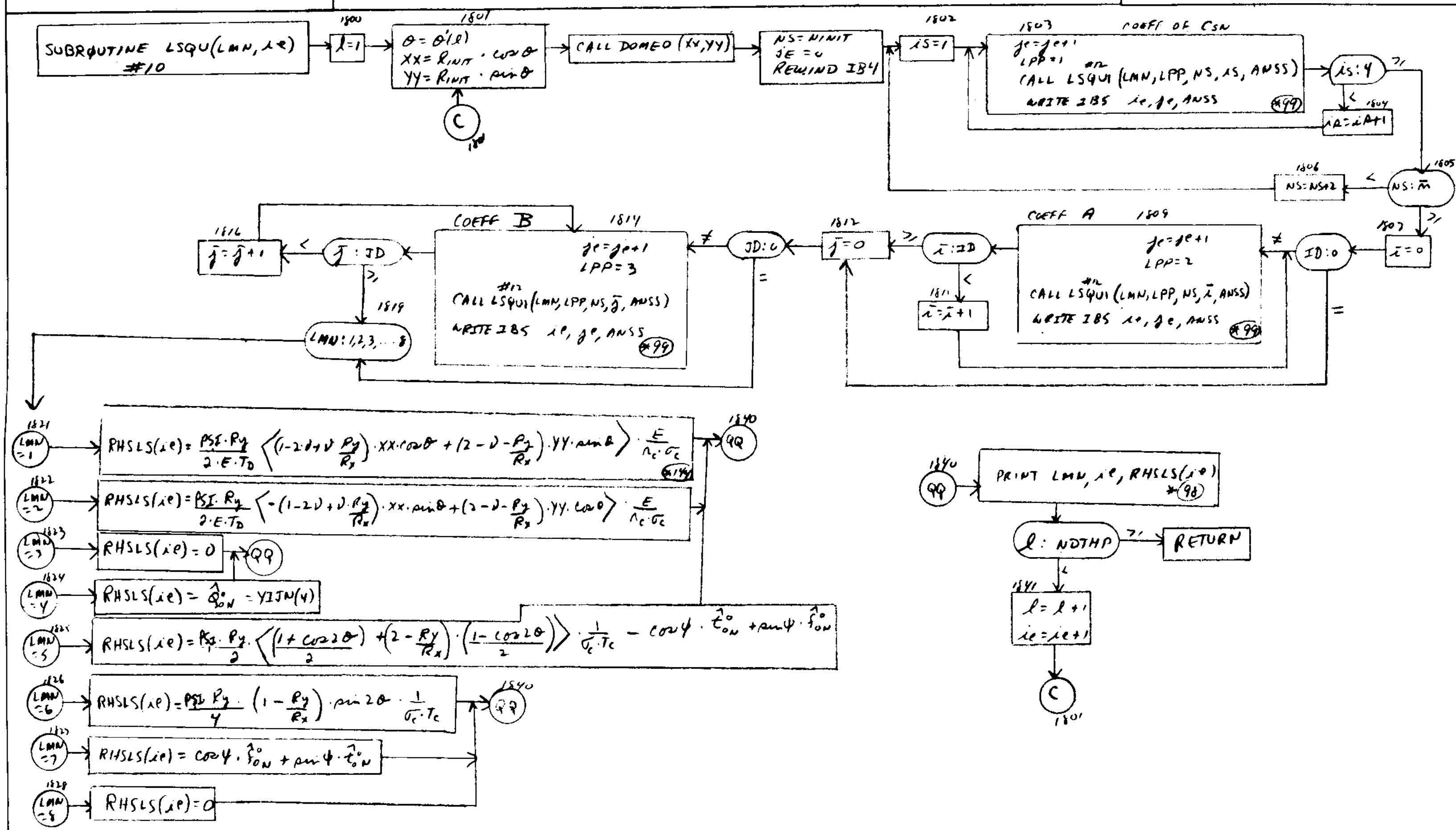
PHSLS(i) (i=1, NEQ)

DATE

COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE: PART II CONTINUED SUBROUTINE LSQU #10

SUBJECT



COMPUTER PROGRAMMING & ANALYSIS DIVISION
ADMINISTRATION DEPARTMENT

DATE

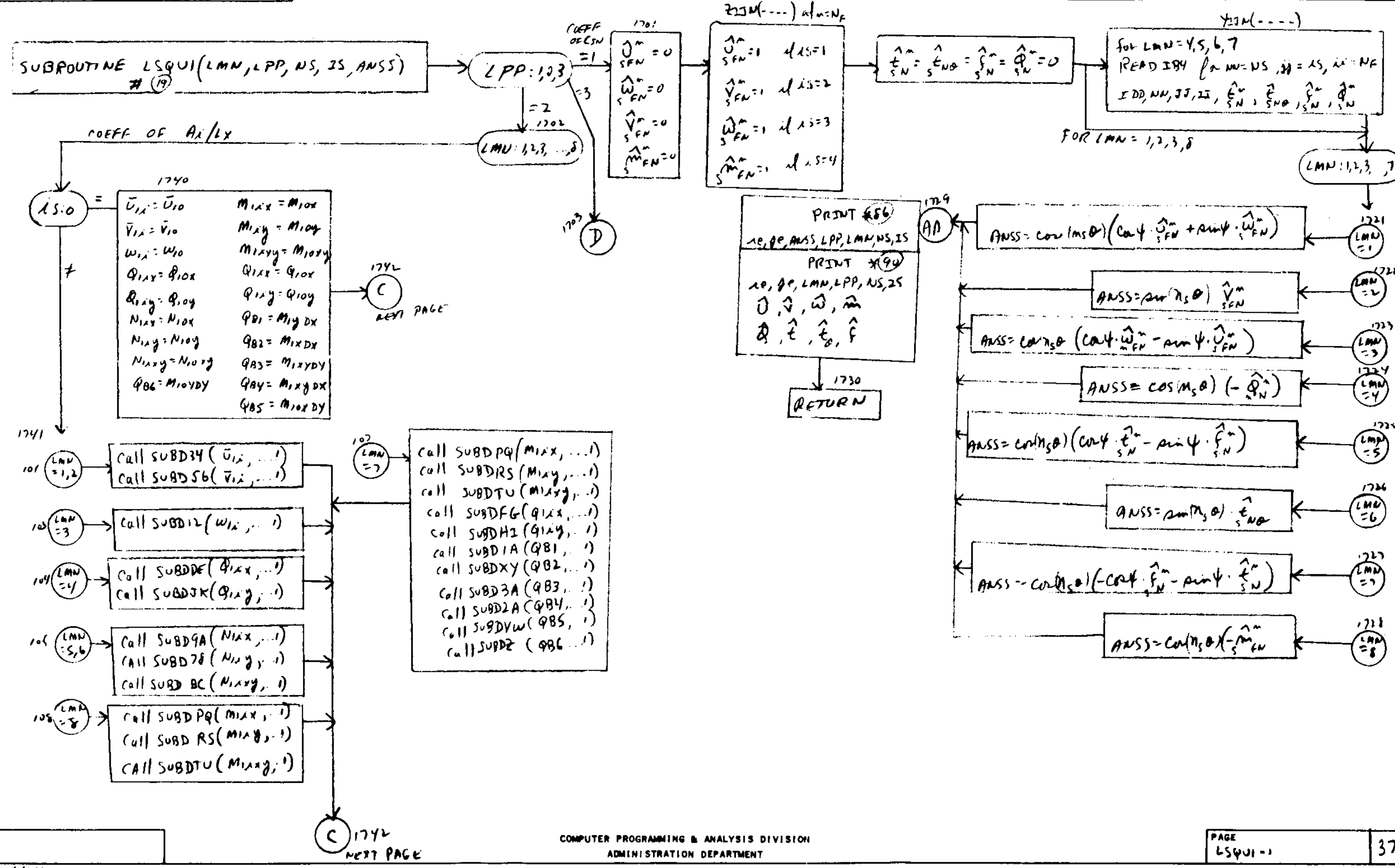
PAGE LSQU - 1

EDPE-87 - 6/62 DC

SECTION TITLE

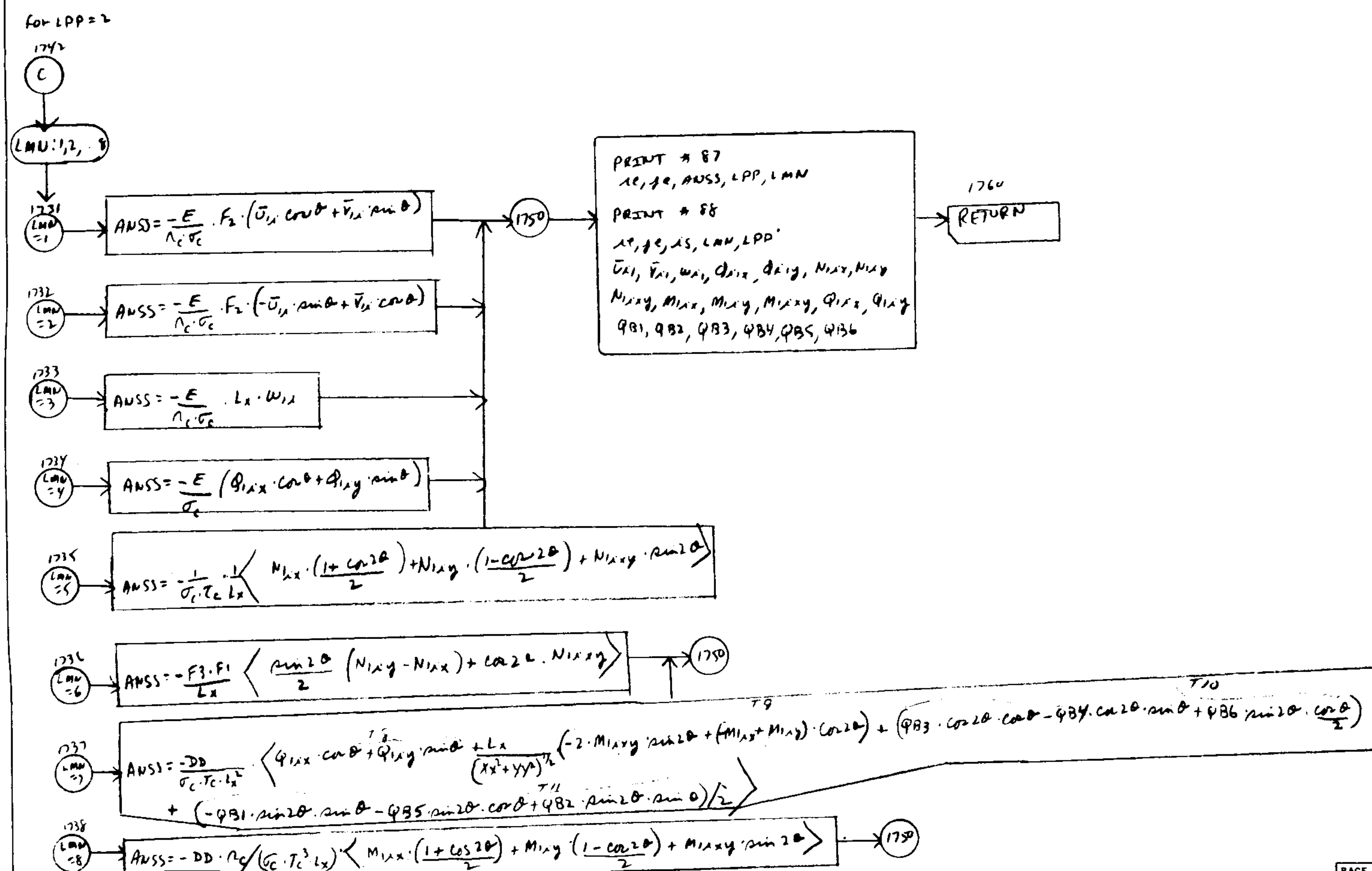
PART II CONTINUED SUBROUTINE LSQUI # 19.

SUBJECT



SECTION TITLE: PART II CONTINUED - SUBROUTINE LSQUI #19 CONTINUED

SUBJECT

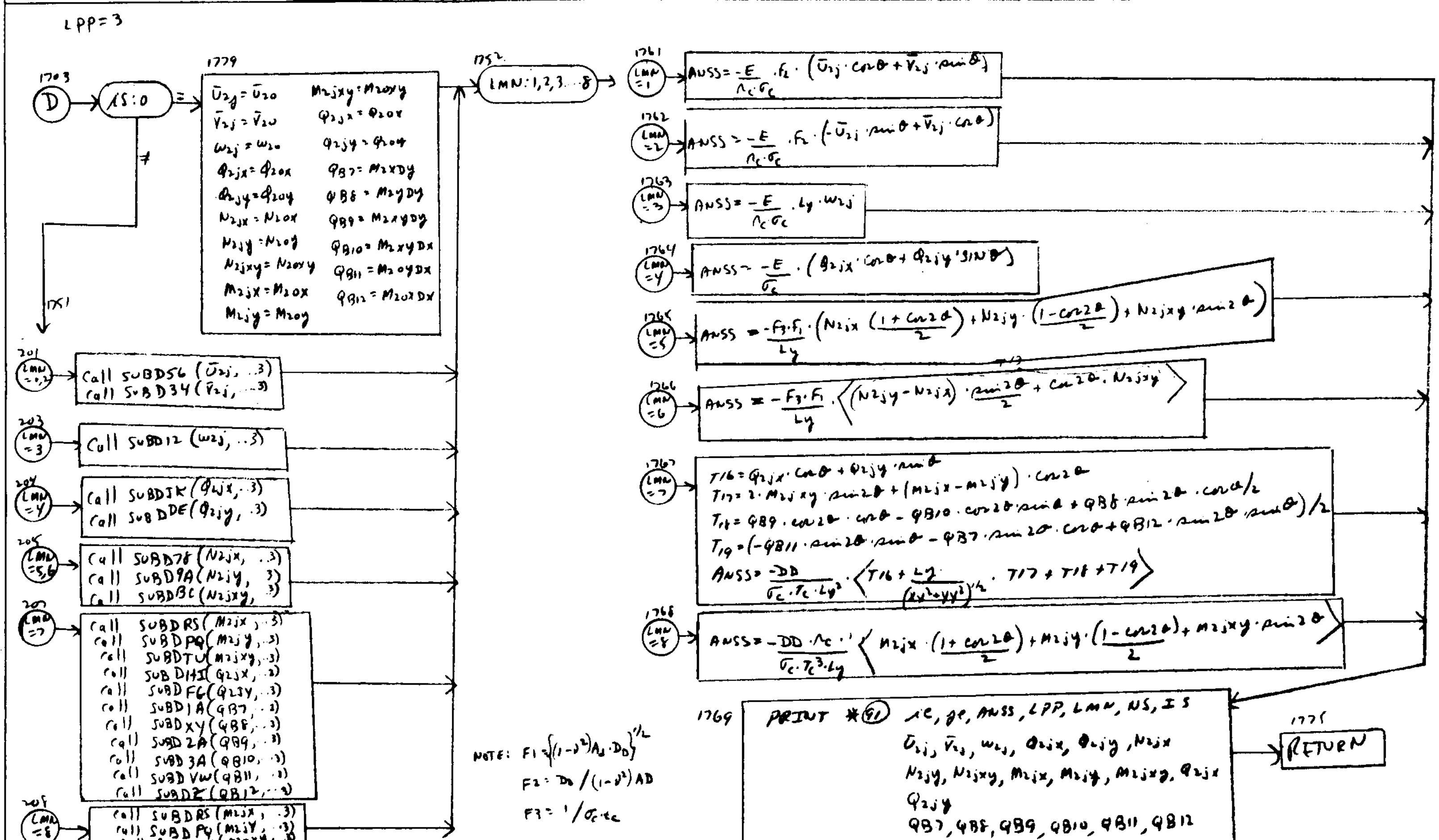


DATE

SECTION TITLE

PART II CONTINUED — SUBROUTINE LSQUI CONTINUED #19 FOR LPP = 3 COEFF FOR BJ/Ly

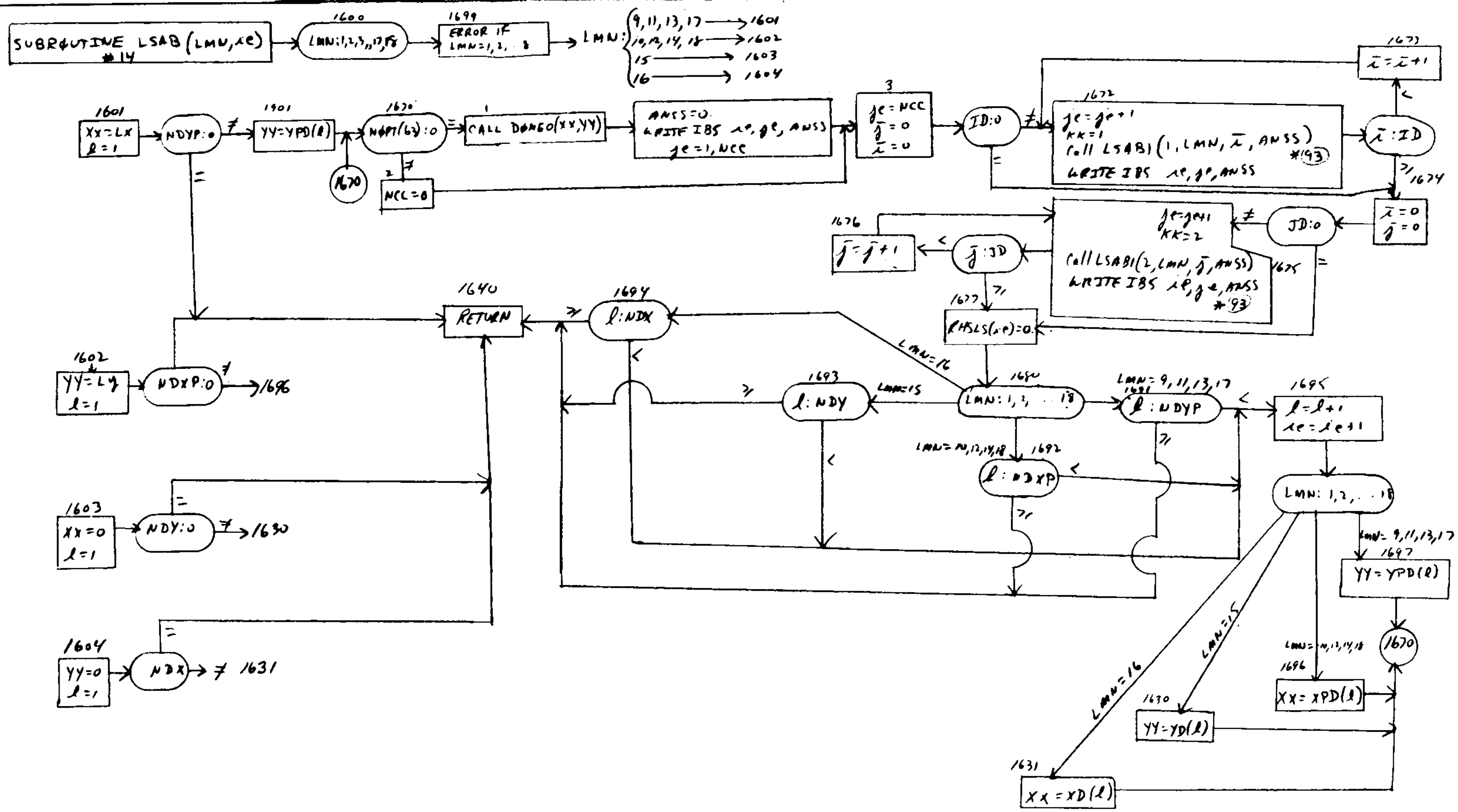
SUBJECT



DATE

SECTION TITLE: PART II CONTINUED - SUBROUTINE LSAB #14

SUBJECT-



DATE

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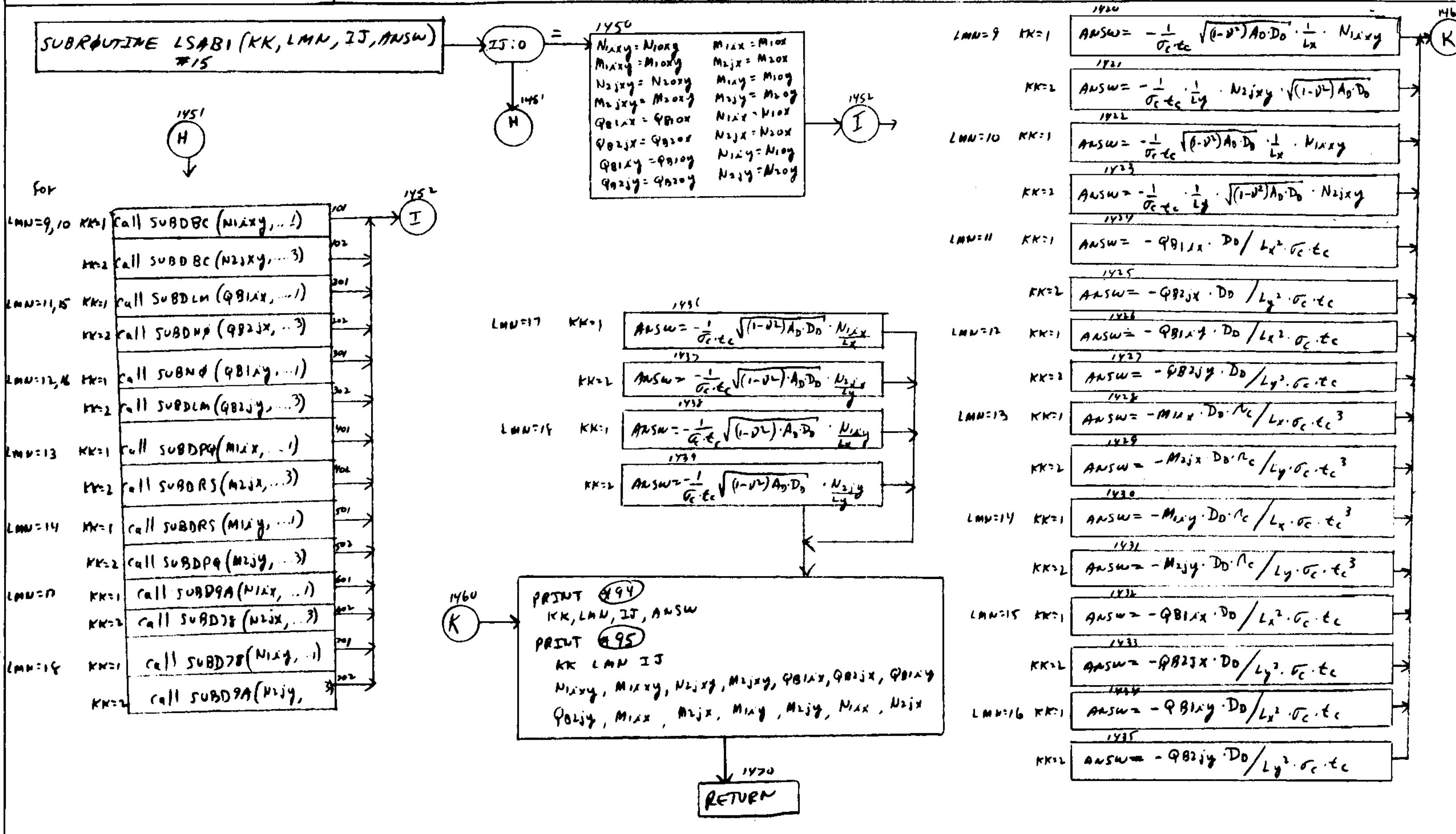
SECTION TITLE: PART II CONTINUED - SUBROUTINE LSABI #15

SUBJECT



DATE

SECTION TITLE: SUBRØUTINE DØME #8 PARTS II, IV, VI

SUBJECT

SUBRØUTINE DØME #8 →

$DX2 = (1-\nu^2)\cdot A_D \cdot L_x^2 / D_D$
$DY2 = (1-\nu^2)\cdot A_D \cdot L_y^2 / D_D$
*(11)

→ 6

DØ 3 FØR $i = 1, ID$

$ALIX(i) = i\cdot\pi/2$

$MIX = 4\cdot\left(\frac{1}{L_x \cdot RATIØX}\right)^2 \cdot ALIX(i)^2 \cdot (L_x \cdot RATIØX - L_x \cdot RATIØY)/DX2$

$AI(i) = \left[\frac{1}{2} + \frac{1}{2}\left(1 + MIX^2\right)^{1/2}\right]^{1/2}$

$BI(i) = -MIX/(2\cdot AI(i))$

*(13)

↓

DØ 4 FØR $j = 1, JD$

$ALJY(j) = j\cdot\pi/2$

$MYJ = 4\cdot\frac{1}{(LY\cdot RATIØY)^2}\cdot ALJY(j)^2\cdot(Ly\cdot RATIØY - LY\cdot RATIØX)/DY2$

$CJ(j) = \left[\frac{1}{2} + \frac{1}{2}\left(1 + MYJ^2\right)^{1/2}\right]^{1/2}$

$DJ(j) = -MYJ/(2\cdot CJ(j))$

*(13)

←

DØ 9 FØR $i = 1, ID$

$T1 = (ALIX(i))^2 - DX2\cdot BI(i)\cdot L_x\cdot RATIØX/2)$

$CØEF = -1$ IF $T1 < 0$
$CØEF = +1$ " " $\geq$

$TEM4 = [1 + AI(i)]^2$

$TEM5 = 2\cdot\left(\frac{1}{L_x \cdot RATIØX}\right)\cdot ALIX(i)^2 - DY2\cdot BI(i)^2$

$T2 = \left[1 + (DY2)^2\cdot TEM4/TEM5\right]^{1/2}$

$T3 = (ALIX(i))^2 + DX2\cdot BI(i)\cdot L_x\cdot RATIØX/2)$

$CØEF2 = -1$ IF $T3 < 0$
$CØEF2 \ +1$ " " ≥ 0

$TEM4 = (1 - AI(i))^2$

$TEM55 = 2\cdot\left(\frac{1}{L_x\cdot RATIØX}\right)\cdot ALIX(i)^2 + DY2\cdot BI(i)^2$

$TT2 = \left(1 + (DX2)^2\cdot TEM4/TEM55\right)^{1/2}$

$P(1,i) = \left[\frac{1}{2}\cdot T1\cdot(1 + CØEF\cdot T2)\right]^{1/2}$

$P(2,i) = \left[\frac{1}{2}\cdot T3\cdot(1 + CØEF2\cdot TT2)\right]^{1/2}$

$Q(1,i) = \left[\frac{1}{2}\cdot T1\cdot(-1 + CØEF\cdot T2)\right]^{1/2}$

$Q(2,i) = \left[\frac{1}{2}\cdot T3\cdot(-1 + CØEF\cdot TT2)\right]^{1/2}$

*(12)

→ A NEXT PAGE

DATE

SECTION TITLE | SUBRØUTINE DIME #8 CON'T PARTS II, IV, VI | SUBJECT-

(A)

DØ 16 FØR $j = 1, JD$

$T4 = (ALJY(j)^2 - DY2 \cdot DJ(j) \cdot LY \cdot RATIØ/2.)$

$CØEF1 = -1$ IF $T4 < 0$

$CØEF1 = 1$ " " ≥ 0

$TEM4 = (1 + CJ(j))^2$

$TEM5 = \left[2 \cdot \left(\frac{1}{LY \cdot RATIØY}\right) \cdot ALJY(j)^2 - DY2 \cdot DJ(j)\right]^2$

$T5 = \left[1 + (DY2)^2 \cdot TEM4 / TEM5\right]^{1/2}$

$T6 = ALJY(j)^2 + DY2 \cdot DJ(j) \cdot LY \cdot RATIØY/2.$

$CØEF3 = -1$ IF $T6 < 0$

$CØEF3 + 1$ IF $T6 \geq 0$

$TEM4 = (1 - CJ(j))^2$

$TEM55 = \left[2 \cdot \left(\frac{1}{LY \cdot RATIØY}\right) \cdot ALJY(j)^2 + DY2 \cdot DJ(j))\right]^2$

$TT5 = \left[1 + (DY2)^2 \cdot TEM4 / TEM55\right]^{1/2}$

$P(3,j) = \left[\frac{1}{2} \cdot T4 \cdot (1 + CØEF1 \cdot T5))\right]^{1/2}$

$P(4,j) = \left[\frac{1}{2} \cdot T6 \cdot (1 + CØEF3 \cdot TT5)\right]^{1/2}$

$Q(3,j) = \left[\frac{1}{2} \cdot T4 \cdot (-1 + CØEF1 \cdot T5)\right]^{1/2}$

$Q(4,j) = \left[\frac{1}{2} \cdot T6 \cdot (-1 + CØEF3 \cdot TT5)\right]^{1/2}$

* (12)

DØ 23 $i = 1, ID$ for $l = 1$

$T7 = [P(l+1,i)^2 + Q(l+1,i)^2] \cdot [Q(l,i)^3 - 3 \cdot Q(l,i) \cdot P(l,i)^2]$

$T8 = [Q(l+1,i)^3 - 3 \cdot Q(l+1,i) \cdot (P(l+1,i)^2) \cdot [P(l,i) \cdot P(l+1,i) + Q(l,i) \cdot Q(l+1,i)]]$

$T9 = [P(l+1,i)^3 - 3 \cdot Q(l+1,i)^2 \cdot P(l+1,i)) \cdot [P(l,i) \cdot Q(l+1,i) - P(l+1,i) \cdot Q(l,i)]]$

$T10 = [P(l+1,i)^2 + (Q(l+1,i)^2) \cdot [P(l,i)^3 - 3 \cdot (Q(l,i)^2) \cdot P(l,i)]]$

$T11 = [Q(l+1,i)^3 - 3 \cdot Q(l+1,i) \cdot (P(l+1,i)^2) \cdot [P(l+1,i) \cdot Q(l,i) - P(l,i) \cdot Q(l+1,i)]]$

$T12 = [P(l+1,i)^3 - 3 \cdot (Q(l+1,i)^2) \cdot P(l+1,i)] \cdot [P(l,i) \cdot P(l+1,i) + Q(l,i) \cdot Q(l+1,i)]]$

$BL(l,i) = (T7 - T8 - T9) / (T10 - T11 - T12)$

$T13 = [P(l,i) \cdot P(l+1,i) + Q(l,i) \cdot Q'(l+1,i)] / [P(l+1,i)^2 + Q(l+1,i)^2]$

$T14 = [P(l+1,i) \cdot Q(l,i) - P(l,i) \cdot Q(l+1,i)] / [P(l+1,i)^2 + Q(l+1,i)^2]$

$AL(l+1,i) = -T13 + T14 \cdot BL(l,i)$

$T15 = [P(l,i) \cdot Q(l+1,i) - P(l+1,i) \cdot Q(l,i)] / [P(l+1,i)^2 + Q(l+1,i)^2]$

$T16 = [P(l,i) \cdot P(l+1,i) + Q(l,i) \cdot Q(l+1,i)] / [P(l+1,i)^2 + Q(l+1,i)^2]$

$BL(l+1,i) = T15 - T16 \cdot BL(l,i)$

* (16) (17)

(B)

NEXT PAGE

DATE

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SECTION TITLE: SUBROUTINE DØME #8 CON'T PARTS II, IV, VI

SUBJECT

(B)

For $\ell = 3$

DO 28 $\lambda = 1, 27$

$T7 = [P(\ell+1,\lambda)^2 + Q(\ell+1,\lambda)^2] \cdot [Q(\ell,\lambda)^3 - 3 \cdot Q(\ell,\lambda) \cdot (P(\ell,\lambda)^2)]$

$T8 = [Q(\ell+1,\lambda)^3 - 3 \cdot Q(\ell+1,\lambda) \cdot [P(\ell+1,\lambda)^2] \cdot [P(\ell,\lambda) \cdot P(\ell+1,\lambda) + Q(\ell,\lambda) \cdot Q(\ell+1,\lambda))]$

$T9 = [P(\ell+1,\lambda)^3 - 3 \cdot [Q(\ell+1,\lambda)^2] \cdot P(\ell+1,\lambda)] \cdot [P(\ell,\lambda) \cdot Q(\ell+1,\lambda) - P(\ell+1,\lambda) \cdot Q(\ell,\lambda))]$

$T10 = [P(\ell+1,\lambda)^2 + (Q(\ell+1,\lambda)^2) \cdot P(\ell,\lambda)^3 - 3 \cdot (Q(\ell,\lambda)^2) \cdot P(\ell,\lambda)]$

$T11 = [Q(\ell+1,\lambda)^3 - 3 \cdot Q(\ell+1,\lambda) \cdot (P(\ell+1,\lambda)^2) \cdot [P(\ell+1,\lambda) \cdot Q(\ell,\lambda) - P(\ell,\lambda) \cdot Q(\ell+1,\lambda))]$

$T12 = [P(\ell+1,\lambda)^3 - 3 \cdot (Q(\ell+1,\lambda)^2) \cdot P(\ell+1,\lambda)] \cdot [P(\ell,\lambda) \cdot P(\ell+1,\lambda) + Q(\ell,\lambda) \cdot Q(\ell+1,\lambda))]$

$BL(\ell,\lambda) = (T7 - T8 - T9) / (T10 - T11 - T12)$

$T13 = [P(\ell,\lambda) \cdot P(\ell+1,\lambda) + Q(\ell,\lambda) \cdot Q(\ell+1,\lambda))] / [P(\ell+1,\lambda)^2 + Q(\ell+1,\lambda)^2]$

$T14 = [P(\ell+1,\lambda) \cdot Q(\ell,\lambda) - P(\ell,\lambda) \cdot Q(\ell+1,\lambda))] / [P(\ell+1,\lambda)^2 + Q(\ell+1,\lambda)^2]$

$AL(\ell+1,\lambda) = -T13 + T14 \cdot BL(\ell,\lambda)$

$T15 = [P(\ell,\lambda) \cdot Q(\ell+1,\lambda) - P(\ell+1,\lambda) \cdot Q(\ell,\lambda))] / [P(\ell+1,\lambda)^2 + Q(\ell+1,\lambda)^2]$

$T16 = [P(\ell,\lambda) \cdot P(\ell+1,\lambda) + Q(\ell,\lambda) \cdot Q(\ell+1,\lambda))] / [P(\ell+1,\lambda)^2 + Q(\ell+1,\lambda)^2]$

$BL(\ell+1,\lambda) = T15 - T16 \cdot BL(\ell,\lambda)$

* 16 17

31 RETURN

SECTION TITLE: SUBRØUTINE DØMEO(XX,YY) PARTS II, IV, VI #11

SUBJECT

SUBRØUTINE DØMEO(XX,YY) #11

31

$P10 = (DX2 \cdot Lx \cdot RATIØX/2)^{1/2}$
$P30 = (DY2 \cdot Ly \cdot RATIØY/2)^{1/2}$
*(14)

34

$\left.\begin{array}{l} \bar{U}_{10} \\ \bar{V}_{20} \\ N_{10y} \\ N_{20x} \\ N_{10xy} \\ N_{20xy} \\ Q_{10x} \\ Q_{20y} \\ \bar{Q}_{0x} \\ \bar{Q}_{10x} \\ M1XYDX \end{array}\right\} = 0$

$\left.\begin{array}{l} Q_{20y} \\ \bar{Q}_{20y} \\ M_{10xy} \\ M_{20xy} \\ M_{1}YDX \\ M_{2}XDY \\ M_{1}XDX \\ M_{2}YDY \\ M_{1}XYDY \\ M_{2}XYDY \\ M_{2}XYDX \end{array}\right\} = 0$

$CP10YL = \cos(P10 \cdot YY/Lx)$
$SP10YL = \sin(P10 \cdot YY/Lx)$
$EXPP10 = e^{(-P10 \cdot YY/Lx)}$
$M10XDY = -4 \cdot \nu \cdot P10^3 \cdot EXPP10 \cdot CP10YL$
$M10YDY = -4 \cdot P10^3 \cdot EXPP10 \cdot CP10YL$
$\omega_{10} = CP10YL + SP10YL \cdot EXPP10$
$\bar{V}_{10} = L_x^2/P_{10} \cdot (RATIØY + \nu \cdot RATIØX)\ (CP10YL \cdot EXPP10 - 1) \cdot \{A_D \cdot (1-\nu^2)/D_D\}^{1/2}$
$N_{10x} = L_x^2\ \{A_D \cdot (1-\nu^2) \cdot RATIØX \cdot RATIØY/D_D\}^{1/2}$
$N_{10x} = N_{10x} \cdot (CP10YL + SP10YL) \cdot EXPP10$
$Q_{10y} = -2 \cdot P_{10} \cdot EXPP10 \cdot SP10YL$
$M_{10x} = 2 \cdot \nu \cdot P_{10}^2 \cdot EXPP10 \cdot (CP10YL - SP10YL)$
$M_{10y} = M_{10x}/\nu$
$Q_{10y} = -4 \cdot P_{10}^3 \cdot EXPP10 \cdot CP10YL$
$\bar{Q}_{10y} = Q_{10y}$
*(15)

35

$CP30XL = \cos(P30 \cdot XX/Ly)$
$SP30XL = \sin(P30 \cdot XX/Ly)$
$EXPP30 = e^{-P30 \cdot XX/Ly}$
$M20YDX = -4 \cdot \nu \cdot P_{30}^3 \cdot EXPP30 \cdot CP30XL$
$M20XDX = -4 \cdot P_{30}^3 \cdot EXPP30 \cdot CP30XL$
$\omega_{20} = (CP30XL + SP30XL) \cdot EXPP30$
$\bar{U}_{20} = Ly^2/P_{30} \cdot (RATIØX + \nu \cdot RATIØY) \cdot (CP30XL \cdot EXPP30 - 1) \cdot (A_D \cdot (1-\nu^2)/D_D)^{1/2}$
$N_{20y} = Ly^2 \cdot (A_D \cdot (1-\nu^2) \cdot RATIØX \cdot RATIØY/DD)^{1/2}$
$N_{20y} = N_{20y} \cdot (CP30XL + SP30XL) \cdot EXPP30$
$Q_{20x} = -2 \cdot P_{30} \cdot EXPP30 \cdot SP30XL$
$M_{20y} = 2 \cdot \nu \cdot P_{30}^2 \cdot EXPP30 \cdot (CP30XL - SP30XL)$
$M_{20x} = M_{20y}/\nu$
$Q_{20x} = -4 \cdot P_{30}^3 \cdot EXPP30 \cdot CP30XL$
$\bar{Q}_{20x} = Q_{20x}$
*(15)

35

RETURN

$*$ NOTE: $\left.\begin{array}{l} DX2 = \left[(1-\nu^2) \cdot A_D \cdot L_x^2/D_D\right]^{1/2} \\ DY2 = \left[(1-\nu^2) \cdot A_D \cdot L_y^2/D_D\right]^{1/2} \end{array}\right\}$ COMPUTED IN DØME #8

DATE

#	PARTS II, V, VI	INDEX = 1	INDEX = 3	
13.1	SUBD12(W1W2, ALXY, FX, FY, LXLY, IJ, INDEX)	w_{1i}	w_{2j}	
13.2	SUBD34(UBIVB2, ALXY, FX, FY, LXLY, RXRY, IJ, INDEX)	$\bar{U}_{1i}$	$\bar{V}_{2j}$	
13.3	SUBD56(VBIUB2, ALXY, FX, FY, LXLY, RXRY, IJ, INDEX)	$\bar{V}_{1i}$	$\bar{U}_{2j}$	
13.4	SUBD78 (N1YN2X, ALXY, FX, FY, LXLY, IJ, INDEX)	N_{1iy}	N_{2jx}	
13.5	SUBD9A (N1XN2Y, ALXY, FX, FY, LXLY, IJ, INDEX)	N_{1ix}	N_{2jy}	
13.6	SUBDBC (N12XY, ALXY, FX, FY, LXLY, IJ, INDEX)	N_{1ixy}	N_{2jxy}	
13.7	SUBDDE (PHIXY, ALXY, FX, FY, LXLY, IJ, INDEX)	φ_{1ix}	Q_{2jy}	
13.8	SUBDFG (Q1XQ2Y, ALXY, FX, FY, LXLY, IJ, INDEX)	q_{1ix}	q_{2jy}	
13.9	SUBDHI (Q1YQ2X, ALXY, FX, FY, LXLY, IJ, INDEX)	q_{1iy}	q_{2jx}	
13.10	SUBDJK (PHIYX, ALXY, FX, FY, LXLY, IJ, INDEX)	φ_{1iy}	φ_{2jx}	
13.11	SUBDLM (QBARXY, ALXY, FX, FY, LXLY, IJ, INDEX)	$\bar{q}_{1ix}$	$\bar{q}_{2jy}$	
13.12	SUBDNØ (QBARYX, ALXY, FX, FY, LXLY, IJ, INDEX)	$\bar{q}_{1iy}$	$\bar{q}_{2jx}$	
13.13	SUBDPQ (M1XM2Y, ALXY, FX, FY, LXLY, IJ, INDEX)	M_{1ix}	M_{2jy}	
13.14	SUBDRS (M1YM2X, ALXY, FX, FY, LXLY, IJ, INDEX)	M_{1iy}	M_{2jx}	
13.15	SUBDTU (M1IJXY, ALXY, FX, FY, LXLY, IJ, INDEX)	M_{1ixy}	M_{2jxy}	
13.16	SUBDVW (MJXMJY, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1ix}D_y$ 985	$M_{2jy}D_x$ 9811	
13.17	SUBDXY (MXXMYY, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1ix}D_x$ 982	$M_{2jy}D_y$ 988	
13.18	SUBDZ (MYYMXX, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1ixy}D_x$ 996	$M_{2jxy}D_y$ 9812	
13.19	SUBD1A (MYXMXY, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1ixy}D_y$ 981	$M_{2jxy}D_x$ 987	
13.20	SUBD2A (MXYMXY, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1iy}D_y$ 984	$M_{2jx}D_x$ 989	
13.21	SUBD3A (MXYMYX, ALXY, FX, FY, LXLY, IJ, INDEX)	$M_{1iy}D_x$ 983	$M_{2jx}D_y$ 9810	

	INDEX=1	INDEX=3
ALXY	ALIX	ALIY
FX	XX	YY
FY	YY	XX
LXLY	LX	LY
RXRY	RX	RY

SECTION TITLE: SUBD12 #13.1 PARTS II, IV, VI
SUBD34 #13.2

SUBJECT

```
SUBROUTINE SUBD12 (W1W2, ALXY, FX, FY, LXLY, IJ, INDEX)
  #13.1
```

```
T1 = COS(ALXY(λj) · FY/LXLY)
T2 = COS(φ(INDEX, λj) · FY/LXLY)
T3 = BL(INDEX, λj) · SIN(φ(INDEX+1, λj) · FY/LXLY)
T4 = EXP{-P(INDEX, λj) · FY/LXLY}
T5 = AL(INDEX+1, λj) · COS(φ(INDEX+1, λj) · FY/LXLY)
T6 = BL(INDEX+1, λj) · SIN(φ(INDEX+1, λj) · FY/LXLY)
T7 = EXP(-P(INDEX+1, λj) · FY/LXLY)
T8 = (T2+T3) · T4 + (T5+T6) · T7
W1W2 = T1·T8
* (22)
```

RETURN

```
SUBROUTINE SUBD34 (UB1VB2, ALXY, FX, FY, LXLY, RXRY, IJ, INDEX)
  #13.2
```

```
T1 = SIN(ALXY(λj) · FX/LXLY) / ALXY(λj))
T2 = EXP(-P(INDEX, λj) · FY/LXLY)
T3 = φ(INDEX, λj)² - P(INDEX, λj)² - ALXY(λj)² · ν
T4 = 2 · φ(INDEX, λj) · P(INDEX, λj) - ((1 - ν²) · A0 · LXLY²) / (FACT · RXRY)
T5 = SIN(φ(INDEX, λj) · FY/LXLY)
T6 = T4
T7 = T3
T8 = COS(φ(INDEX, λj) · FY/LXLY)
T9 = EXP(-P(INDEX+1, λj) · FY/LXLY)
T10 = φ(INDEX+1, λj)² - P(INDEX+1, λj)² - ALXY(λj)² · ν
T11 = 2 · φ(INDEX+1, λj) · P(INDEX+1, λj) - ((1 - ν²) · A0 · LXLY²) / (FACT · RXRY)
```

```
T12 = SIN(φ(INDEX+1, λj) · FY/LXLY)
T13 = T11
T14 = T10
T15 = COS(φ(INDEX+1, λj) · FY/LXLY)
T16 = T2 · ((T3+T4 · BL(INDEX, λj)) · T5 + (T6-T7 · BL(INDEX, λj)) · T8) +
      T9 · ((T10 · AL(INDEX+1, λj) + T11 · BL(INDEX+1, λj)) · T12 + (T13 · AL(INDEX+1, λj)
      - T14 · BL(INDEX+1, λj)) · T15)
UB1VB2 = T1 · T16
* (21)
```

RETURN

DATE

```
SUBROUTINE SUBDS6 (VB1UB2, ALXY, FX, FY, LXLY, RXRY, IJ, INDEX)
#13.3
```

```
FACT1 = ((1-ν²)·A0)^(1/2) · LXLY² / (RXRY · D0^(1/2))
T1 = cos (ALXY(IJ) · FX / LXLY)
T1 = T1 / ALXY(IJ)²
T2 = EXP(-P(INDEX,IJ) · FY/LXLY)
T3 = P(INDEX,IJ)³ - 3 · P(INDEX,IJ) · Q(INDEX,IJ)²
T4 = ALXY(IJ)² · P(INDEX,IJ) · ν
T5 = Q(INDEX,IJ) · FACT1
T6 = 2 · (1+ν) ALXY(IJ)² · P(INDEX,IJ)
T7 = Q(INDEX,IJ)³ - 3 · P(INDEX,IJ)² · Q(INDEX,IJ)
T8 = Q(INDEX,IJ) · ALXY(IJ)² · ν
T9 = P(INDEX,IJ) · FACT1
T10 = 2. · (1+ν) · ALXY(IJ)² · Q(INDEX,IJ)
```

```
T11 = sin(Q(INDEX,IJ) · FY/LXLY)
T12 = Q(INDEX,IJ)³ - 3 · Q(INDEX,IJ) P(INDEX,IJ)²
T13 = T8
T14 = T9
T15 = T10
T16 = T3
T17 = T4
T18 = T5
T19 = T6
T20 = cos(Q(INDEX,IJ) · FY/LXLY
T21 = T2 · (((T3+T4+T5-T6)+(T7-T8+T9+T10) · BL(INDEX,IJ)) · T11
      + ((T12-T13+T14+T15) - (T16+T17+T18-T19) · BL(INDEX,IJ)) · T20)
T22 = EXP{-P(INDEX+1,IJ) · FY/LXLY}
T3 = P(INDEX+1,IJ)³ - 3 · P(INDEX+1,IJ) · Q(INDEX+1,IJ)²
T24 = ALXY(IJ)² · P(INDEX+1,IJ) · ν
```

```
T25 = Q(INDEX+1,IJ) · FACT1
T26 = 2 · (1+ν) · ALXY(IJ)² · P(INDEX+1,IJ)
T27 = Q(INDEX+1,IJ)³ - 3 · P(INDEX+1,IJ)² · Q(INDEX+1,IJ)
T28 = Q(INDEX+1,IJ) · ALXY(IJ)² · ν
T29 = P(INDEX+1,IJ) · FACT1
T30 = 2 · (1+ν) · ALXY(IJ)² · Q(INDEX+1,IJ)
T31 = sin(Q(INDEX+1,IJ) · FY/LXLY)
T32 = T27
T33 = T28
T34 = T29
T35 = T30
T36 = T23
T37 = T24
T38 = T25
T39 = T26
T40 = cos(Q(INDEX+1,IJ) · FY/LXLY)
T41 = T22 · (((T23+T24+T25-T26) · AL(INDEX+1,IJ)
      + (T27-T28+T29+T30) · BL(INDEX+1,IJ)) · T31
      + ((T32-T33+T34+T35) · AL(INDEX+1,IJ)
      - (T36+T37+T38-T39) · BL(INDEX+1,IJ)) · T40)
VB1UB2 = T1 · (T21+T41)
*(23)
```

RETURN

```
SUBROUTINE SUBD78 (NYNDX, ALXY, FX, FY, LXLY, IJ, INDEX)
#13.4
```

```
T1 = - ALXY(IJ)² · cos(ALXY(IJ) · FX/LXLY)
T2 = -sin(Q(INDEX,IJ) · FY/LXLY) + BL(INDEX,IJ) · COS(Q(INDEX,IJ) · FY/LXLY)
T3 = EXP(-P(INDEX,IJ) · FY/LXLY)
T5 = EXP(-P(INDEX+1,IJ)) · FY/LXLY
T4 = -AL(INDEX+1,IJ) · sin(Q(INDEX+1,IJ) · FY/LXLY) + BL(INDEX+1,IJ) · COS(Q(INDEX+1,IJ) · FY/LXLY)
T6 = T2 · T3 + T4 · T5
NYNDX = T1 · T6
*(23)
```

RETURN

SECTION TITLE: SUBD9A #13.5 / SUBDBC #13.6

SUBJECT:

SUBROUTINE SUBD9A(N1XN2Y, ALXY, FX, FY, LXLY, IJ, INDEX) #13.5

```
T1 = COS(ALXY(IJ)·FX/LXLY)
T2 = (Q(INDEX,IJ)^2 - P(INDEX,IJ)^2) + 2·Q(INDEX,IJ)·P(INDEX,IJ)·BL(INDEX,IJ)
T3 = SIN(Q(INDEX,IJ)·FY/LXLY)
T4 = 2·Q(INDEX,IJ)·P(INDEX,IJ) - (Q(INDEX,IJ)^2 - P(INDEX,IJ)^2)·BL(INDEX,IJ)
T5 = COS(Q(INDEX,IJ)·FY/LXLY)
T6 = EXP(-P(INDEX,IJ)·FY/LXLY)
T7 = ((Q(INDEX+1,IJ)^2 - P(INDEX+1,IJ)^2)·AL(INDEX+1,IJ) + 2·Q(INDEX+1,IJ)·P(INDEX+1,IJ)·BL(INDEX+1,IJ))
T8 = SIN(Q(INDEX+1,IJ)·FY/LXLY)
T9 = (2·Q(INDEX+1,IJ)·P(INDEX+1,IJ)·AL(INDEX+1,IJ) - (Q(INDEX+1,IJ)^2 - P(INDEX+1,IJ)^2)·BL(INDEX+1,IJ))
T10 = COS(Q(INDEX+1,IJ)·FY/LXLY)
T11 = EXP{-P(INDEX+1,IJ)·FY/LXLY)
T12 = (T2·T3+T4·T5)·T6+(T7·T8+T9·T10)·T11
N1XN2Y = T1·T12
*(24)
```

RETURN

SUBROUTINE SUB9BC(N12XY, ALXY, FX, FY, LXLY, IJ, INDEX) #13.6

```
T1 = ALXY(IJ)·SIN(ALXY(IJ)·FX/LXLY)
T2 = EXP{-P(INDEX,IJ)·FY/LXLY}
T3 = P(INDEX,IJ) - Q(INDEX,IJ)·BL(INDEX,IJ)
T4 = SIN(Q(INDEX,IJ)·FY/LXLY)
T5 = Q(INDEX,IJ) + P(INDEX,IJ)·BL(INDEX,IJ)
T6 = COS(Q(INDEX,IJ)·FY/LXLY)
T7 = EXP(-P(INDEX+1,IJ)·FY/LXLY)
T8 = P(INDEX+1,IJ)·AL(INDEX+1,IJ) - Q(INDEX+1,IJ)·BL(INDEX+1,IJ)
T9 = SIN(Q(INDEX+1,IJ)·FY/LXLY)
T10 = Q(INDEX+1,IJ)·AL(INDEX+1,IJ) + P(INDEX+1,IJ)·BL(INDEX+1,IJ)
T11 = COS(Q(INDEX+1,IJ)·FY/LXLY)
N12XY = T1·(T2·(T3·T4-T5·T6)+T7·(T8·T9-T10·T11))
*(25)
```

RETURN

DATE

SUBROUTINE SUBDDE (PHIXY, ALXY, FX, FY, LXLY, IJ, INDEX)
13.7

T1 = ALXY(IJ) · sin (ALXY(IJ) · FX/LXLY)
T2 = cos (Q(INDEX, IJ) · FY/LXLY)
T3 = BL(INDEX, IJ) · sin (Q(INDEX, IJ) · FY/LXLY)
T4 = EXP(−P(INDEX, IJ) · FY/LXLY)
T5 = AL(INDEX+1, IJ) · cos (Q(INDEX+1, IJ) · FY/LXLY)
T6 = BL(INDEX+1, IJ) · sin (Q(INDEX+1, IJ) · FY/LXLY)
T7 = EXP(−P(INDEX+1, IJ) · FY/LXLY)
PHIXY = T1 · ((T2+T3) · T4 + (T5+T6) · T7)   *(26)

RETURN

SUBROUTINE SUBDFG (Q1XQ2Y, ALXY, FX, FY, LXLY, IJ, INDEX)
13.8

T1 = ALXY(IJ) · sin (ALXY(IJ) · FX/LXLY)
T2 = EXP{−P(INDEX, IJ) · FY/LXLY}
T3 = P(INDEX, IJ)² − Q(INDEX, IJ)² − ALXY(IJ)²
T4 = 2 · P(INDEX, IJ) · Q(INDEX, IJ) · BL(INDEX, IJ)
T5 = cos(Q(INDEX, IJ) · FY/LXLY)
T6 = 2 · P(INDEX, IJ) · Q(INDEX, IJ) + (P(INDEX, IJ)² − Q(INDEX, IJ)² − ALXY²) · BL(INDEX, IJ)
T7 = sin (Q(INDEX, IJ) · FY/LXLY)
T8 = EXP{−P(INDEX+1, IJ) · FY/LXLY}
T9 = (P(INDEX+1, IJ)² − Q(INDEX+1, IJ)² − ALXY(IJ)²) · AL(INDEX+1, IJ)
T10 = P(INDEX+1, IJ) · Q(INDEX+1, IJ) · BL(INDEX+1, IJ) · 2
T11 = cos (Q(INDEX+1, IJ) · FY/LXLY)
T12 = 2 · P(INDEX+1, IJ) · Q(INDEX+1, IJ) · AL(INDEX+1, IJ) + (P(INDEX+1, IJ)² − Q(INDEX+1, IJ)²
− ALXY(IJ)²) · BL(INDEX+1, IJ)
T13 = sin (Q(INDEX+1, IJ) · FY/LXLY)
T14 = (T3 − T4) · T5 + T6 · T7
T15 = (T9 − T10) · T11 + T12 · T13
Q1XQ2Y = T1 · (T2 · T14 + T8 · T15)   *(27)

RETURN

SUBROUTINE SUBDHI (QIYQ2X, ALXY, FX, FY, LXLY, IJ, INDEX)
13.9

T1 = COS(ALXY(IJ)·FX/LXLY)
T2 = EXP(-P(INDEX,IJ)·FY/LXLY)
T3 = P(INDEX,IJ)·{P(INDEX,IJ)² - 3·Q(INDEX,IJ)² - ALXY(IJ)²}
T4 = (Q(INDEX,IJ)·(Q(INDEX,IJ)² - 3·P(INDEX,IJ)² + ALXY(IJ)²))
T5 = T4·BL(INDEX,IJ)   T6 = COS(Q(INDEX,IJ)·FY/LXLY)   T7 = -T4
T8 = T3·BL(INDEX,IJ)
T9 = SIN(Q(INDEX,IJ)·FY/LXLY
T10 = EXP(-P(INDEX+1,IJ)·FY/LXLY)
T11 = P(INDEX+1,IJ)·{P(INDEX+1,IJ)² - 3·Q(INDEX+1,IJ)² - ALXY(IJ)²}
T12 = T11·AL(INDEX+1,IJ)
T13 = Q(INDEX+1,IJ)·{Q(INDEX+1,IJ)² - 3·P(INDEX+1,IJ)² + ALXY(IJ)²}
T14 = T13·BL(INDEX+1,IJ) ; T15 = COS(Q(INDEX+1,IJ)·FY/LXLY)
T16 = -T13·AL(INDEX+1,IJ) ; T17 = T11·BL(INDEX+1,IJ) ; T18 = SIN(Q(INDEX+1,IJ)·FY/LXLY)
T19 = (T3+T5)·T6 ; T20 = (T7+T8)·T9 ; T21 = (T12+T14)·T15 ; T22 = (T16+T17)·T18
QIYQ2X = T1·(T2·(T19+T20) + T10·(T21+T22))   * 28

RETURN

SUBROUTINE SUBDJK (PHIYX, ALXY, FX, FY, LXLY, IJ, INDEX)
13.10

T1 = COS(ALXY(IJ)·FX/LXLY)
T2 = (-P(INDEX,IJ)·FY/LXLY)
T3 = COS(Q(INDEX,IJ)·FY/LXLY)
T4 = -P(INDEX,IJ) + BL(INDEX,IJ)·Q(INDEX,IJ)
T5 = SIN(Q(INDEX,IJ)·FY/LXLY)
T6 = Q(INDEX,IJ) + BL(INDEX,IJ)·P(INDEX,IJ)
T7 = EXP(-P(INDEX+1,IJ)·FY/LXLY)
T8 = COS(Q(INDEX+1,IJ)·FY/LXLY)
T9 = -P(INDEX+1,IJ)·AL(INDEX+1,IJ) + BL(INDEX+1,IJ)·Q(INDEX+1,IJ)
T10 = SIN(Q(INDEX+1,IJ)·FY/LXLY)
T11 = AL(INDEX+1,IJ)·Q(INDEX+1,IJ) + BL(INDEX+1,IJ)·P(INDEX+1,IJ)

PHIYX = T1·(T2·(T3·T4 - T5·T6) + T7·(T8·T9 - T10·T11))
* 29

RETURN

SUBROUTINE SUBDLM(QBARXY,ALXY,FX,FY,LXLY,IJ,INDEX) #13.11

```
T1 = ALXY(IJ)·sin(ALXY(IJ)·FX/LXLY)
T2 = EXP{-P(INDEX,IJ)·FY/LXLY}
T3 = (2-D)(P(INDEX,IJ)^2 - Q(INDEX,IJ)^2) - ALXY(IJ)^2)
T4 = 2·(2-D)·P(INDEX,IJ)·Q(INDEX,IJ))
T5 = BL(INDEX,IJ))
T6 = cos{Q(INDEX,IJ)·FY/LXLY}
T7 = T3·T5 ; T8 = sin(Q(INDEX,IJ)·FY/LXLY) ; T9 = EXP{-P(INDEX+1,IJ)·FY/LXLY}
T10 = ((2-D)·(P(INDEX+1,IJ)^2 - Q(INDEX+1,IJ)^2)) - ALXY(IJ)^2
T11 = AL(INDEX+1,IJ) ; T12 = T10·T11 ; T13 = 2·(2-D)·P(INDEX+1,IJ)·Q(INDEX+1,IJ))
T14 = BL(INDEX+1,IJ) ; T15 = T13·T14 ; T16 = cos(Q(INDEX+1,IJ)·FY/LXLY)
T17 = T13·T11 ; T18 = T10·T14 ; T19 = sin(Q(INDEX+1,IJ)·FY/LXLY)
QBARXY = T1·(T2·((T3-T4·T5)·T6 + (T4+T7)·T8) + T9·((T12-T15)·T16 + (T17+T18)·T19))
* 30
```

RETURN

SUBROUTINE SUBDNØ(QBARYX,ALXY,FX,FY,LXLY,IJ,INDEX) #13.12

```
T1 = cos(ALXY(IJ)·FX/LXLY)
T2 = EXP(-P(INDEX,IJ)·FY/LXLY)
T3 = P(INDEX,IJ)·(P(INDEX,IJ)^2 - 3·Q(INDEX,IJ)^2 - (2-D)·ALXY(IJ)^2)
T4 = Q(INDEX,IJ)·(Q(INDEX,IJ)^2 - 3·P(INDEX,IJ)^2 + (2-D)·ALXY(IJ)^2)
T5 = BL(INDEX,IJ) ; T6 = cos(Q(INDEX,IJ)·FY/LXLY) ; T7 = T3·T5 ; T8 = sin(Q(INDEX,IJ)·FY/LXLY)
T9 = EXP(-P(INDEX+1,IJ)·FY/LXLY)
T10 = P(INDEX+1,IJ)·{P(INDEX+1,IJ)^2 - 3·Q(INDEX+1,IJ)^2 - (2-D)·ALXY(IJ)^2} ; T11 = AL(INDEX+1,IJ)
T12 = Q(INDEX+1,IJ)·{Q(INDEX+1,IJ)^2 - 3·P(INDEX+1,IJ)^2 + (2-D)·ALXY(IJ)^2} ; T13 = BL(INDEX+1,IJ)
T14 = cos(Q(INDEX+1,IJ)·FY/LXLY
T15 = -{Q(INDEX+1,IJ)·(Q(INDEX+1,IJ)^2 - 3·P(INDEX+1,IJ)^2 + (2-D)·ALXY(IJ)^2} ·T11
T16 = {P(INDEX+1,IJ)·(P(INDEX+1,IJ)^2 - 3·Q(INDEX+1,IJ)^2 - (2-D)·ALXY(IJ)^2)·T13
T17 = sin(Q(INDEX+1,IJ)·FY/LXLY)

QBARYX = T1·(T2·((T3+T4·T5)·T6 + (T7-T4)·T8)
     + T9·(((T10·T11) + (T12·T13))·T14 + (T15+T16)·T17))
* 31
```

RETURN

SECTION TITLE: SUBDPQ # 13.13
SUBDRS # 13.14

SUBJECT

```
SUBROUTINE SUBDPQ (MIXM2Y, ALXY, FX, FY, LXLY, IJ, INDEX)
#13.13

T1 = COS(ALXY(IJ) · FX/LXLY)
T2 = EXP(-P(INDEX,IJ)) · FY/LXLY
T3 = ALXY(IJ)^2 - ν · (P(INDEX,IJ)^2 - Q(INDEX,IJ)^2)
T4 = 2 · ν · P(INDEX,IJ) · Q(INDEX,IJ) · BL(INDEX,IJ)
T5 = COS(Q(INDEX,IJ) · FY/LXLY)
T6 = -2 · ν · P(INDEX,IJ) · Q(INDEX,IJ)   ; T7 = T3
T8 = SIN(Q(INDEX,IJ) · FY/LXLY)
TT8 = T2 · ((T3+T4) · T5 + (T6+T7 · BL(INDEX,IJ)) · T8)
T9 = EXP(-P(INDEX+1,IJ) · FY/LXLY)
T10 = ALXY(IJ)^2 - ν · (P(INDEX+1,IJ)^2 - Q(INDEX+1,IJ)^2)
T11 = 2 · ν · P(INDEX+1,IJ) · Q(INDEX+1,IJ) · BL(INDEX+1,IJ)
T12 = COS(Q(INDEX+1,IJ) · FY/LXLY
T13 = -2 · ν · P(INDEX+1,IJ) · Q(INDEX+1,IJ) · AL(INDEX+1,IJ)   ; T14 = T10
T15 = SIN(Q(INDEX+1,IJ) · FY/LXLY
TT15 = T9 · ((T10 · AL(INDEX+1,IJ) + T11) · T12 + (T13+T14 · BL(INDEX+1,IJ)) · T15)
T16 = TT8 + TT15
MIXM2Y = T1 · T16        * (33)

RETURN
```

```
SUBROUTINE SUBDRS (MIYMLX, ALXY, FX, FY, LXLY, IJ, INDEX
#13.14

T1 = -COS(ALXY(IJ) · FX/LXLY)
T2 = EXP(-P(INDEX,IJ) · FY/LXLY)
T3 = P(INDEX,IJ)^2 - Q(INDEX,IJ)^2 - ν · ALXY(IJ)^2
T4 = -2 · P(INDEX,IJ) · Q(INDEX,IJ) · BL(INDEX,IJ)
T5 = COS(Q(INDEX,IJ) · FY/LXLY
T6 = 2 · P(INDEX,IJ) · Q(INDEX,IJ)
T7 = T3
T8 = SIN(Q(INDEX,IJ) · FY/LXLY

TT8 = T2((T3+T4) · T5 + (T6+T7 · BL(INDEX,IJ)) · T8)
T9 = EXP(-P(INDEX+1,IJ) · FY/LXLY
T10 = P(INDEX+1,IJ)^2 - Q(INDEX+1,IJ)^2 - ν · ALXY(IJ)^2
T11 = -2 · P(INDEX+1,IJ) · Q(INDEX+1,IJ) · BL(INDEX+1,IJ)
T12 = COS(Q(INDEX+1,IJ) · FY/LXLY
T13 = 2 · P(INDEX+1,IJ) · Q(INDEX+1,IJ) · AL(INDEX+1,IJ)
T14 = T13   ; T15 = SIN(Q(INDEX+1,IJ) · FY/LXLY
TT15 = T9 · ((T10 · AL(INDEX+1,IJ) + T11) · T12 + (T13 + T14 · BL(INDEX+1,IJ)) · T15)
T16 = TT8 + TT15
MIYMLX = T1 · T16        * (34)

RETURN
```

DATE

SECTION TITLE: SUBDTU #13.15
SUBDVW #13.16

SUBJECT

SUBROUTINE SUBDTU(MIIJXY, ALXY, FX, FY, LXLY, IJ, INDEX)
#13.15

T1 = (ALXY(IJ) · sin(ALXY(IJ) · FX/LXLY)) · (1-V)
T2 = EXP(-P(INDEX,IJ) · FY/LXLY
T3 = -P(INDEX,IJ) + Q(INDEX,IJ) · BL(INDEX,IJ)
T4 = cos(Q(INDEX,IJ) · FY/LXLY)
T5 = Q(INDEX,IJ) + P(INDEX,IJ) · BL(INDEX,IJ)
T6 = sin(Q(INDEX,IJ)) · FY/LXLY
T7 = EXP(-P(INDEX+1,IJ)) · FY/LXLY
T8 = -P(INDEX+1,IJ) · AL(INDEX+1,IJ) + Q(INDEX+1,IJ) · BL(INDEX+1,IJ)
T9 = cos(Q(INDEX+1,IJ) · FY/LXLY
T10 = Q(INDEX+1,IJ) · AL(INDEX+1,IJ) + P(INDEX+1,IJ) · BL(INDEX+1,IJ)
T11 = sin(Q(INDEX+1,IJ) · FY/LXLY)
MIIJXY = T1 · (T2 · (T3 · T4 - T5 · T6) + T7 · (T8 · T9 - T10 · T11)) *(34)

RETURN

SUBROUTINE SUBDVW(MIXMIY, ALXY, FX, FY, LXLY, IJ, INDEX)
#13.16

T1 = cos(ALXY(IJ) · FX/LXLY
T2 = EXP(-P(INDEX,IJ) · FY/LXLY) · sin(Q(INDEX,IJ) · FY/LXLY)
T3 = -Q(INDEX,IJ) · (ALXY(IJ)^2 + V · Q(INDEX,IJ)^2 - 3 · V · P(INDEX,IJ)^2)
T4 = -P(INDEX,IJ) · (ALXY(IJ)^2 - V · P(INDEX,IJ)^2 + 3 · V · Q(INDEX,IJ)^2)
T5 = EXP(-P(INDEX,IJ) · FY/LXLY) · cos(Q(INDEX,IJ) · FY/LXLY)
T6 = T4
T7 = -T3 · BL(INDEX,IJ)
T8 = EXP(-P(INDEX+1,IJ) · FY/LXLY) · sin(Q(INDEX+1,IJ) · FY/LXLY)
T9 = -Q(INDEX+1,IJ) · (ALXY(IJ)^2 + V · Q(INDEX+1,IJ)^2 - 3 · V · P(INDEX+1,IJ)^2)
T10 = T9 · AL(INDEX+1,IJ)
T11 = -P(INDEX+1,IJ) · (ALXY(IJ)^2 - V · P(INDEX+1,IJ)^2 + 3 · V · Q(INDEX,IJ)^2)
T12 = T11 · BL(INDEX+1,IJ)
T13 = EXP(-P(INDEX+1,IJ) · FY/LXLY) · cos(Q(INDEX+1,IJ) · FY/LXLY)
T14 = T11 AL(INDEX+1,IJ)
T15 = -T9 · BL(INDEX+1,IJ)
MIXMIY = T1 · (T2 · (T3 + T4 · BL(INDEX,IJ)) + T5 · (T6 + T7) + T8 · (T10 + T12) + T13 · (T14 + T15)) *(21)

RETURN

SUBROUTINE SUBDXY(MXXMYY,ALXY,FX,FY,LXLY,IJ,INDEX)
13.17

```
T1 = -ALXY(i,j) · SIN(ALXY(i,j) · FX/LXLY)
T2 = EXP(-P(INDEX,i,j) · FY/LXLY) · SIN(Q(INDEX,i,j) · FY/LXLY)
T3 = -2 · ν · P(INDEX,i,j) · Q(INDEX,i,j)
T4 = ALXY(i,j)^2 - ν · (P(INDEX,i,j)^2 - Q(INDEX,i,j)^2)
T5 = T4 · BL(INDEX,i,j)
T6 = EXP(-P(INDEX,i,j) · FY/LXLY) · COS(Q(INDEX,i,j) · FY/LXLY
T7 = -T3 · BL(INDEX,i,j)
```

```
T8 = EXP(-P(INDEX+1,i,j) · FY/LXLY) · SIN(Q(INDEX+1,i,j) · FY/LXLY)
T9 = -2 · ν · P(INDEX+1,i,j) · Q(INDEX+1,i,j)
T10 = AL(INDEX+1,i,j)
T11 = ALXY(i,j)^2 - ν · (P(INDEX+1,i,j)^2 - Q(INDEX+1,i,j)^2)
T12 = BL(INDEX+1,i,j)
T13 = EXP(-P(INDEX+1,i,j) · FY/LXLY) · COS(Q(INDEX+1,i,j) · FY/LXLY)
T14 = T11 · T10
T15 = T9 · T12
MXXMYY = T1 · (T2 · (T3+T5) + T6 · (T4+T7) + T8 · (T9 · T10 + T11 · T12) + T13 · (T14+T15))
* (122)
```

RETURN

SUBROUTINE SUBDZ(MYYMXX,ALXY,FX,FY,LXLY,IJ,INDEX)
13.18

```
T1 = COS(ALXY(i,j) · FX/LXLY
T2 = EXP(-P(INDEX,i,j) · FY/LXLY) · SIN(Q(INDEX,i,j) · FY/LXLY)
T3 = -Q(INDEX,i,j) · (Q(INDEX,i,j)^2 - 3 · P(INDEX,i,j)^2 + ν · ALXY(i,j)^2)
T4 = -P(INDEX,i,j) · (3 · Q(INDEX,i,j)^2 - P(INDEX,i,j)^2 + ν · ALXY(i,j)^2)
T5 = BL(INDEX,i,j)
T6 = EXP(-P(INDEX,i,j) · FY/LXLY) · COS(Q(INDEX,i,j) · FY/LXLY)
T7 = EXP(-P(INDEX+1,i,j) · FY/LXLY) · SIN(Q(INDEX+1,i,j) · FY/LXLY)
T8 = -Q(INDEX+1,i,j) · (Q(INDEX+1,i,j)^2 - 3 · P(INDEX+1,i,j)^2 + ν · ALXY(i,j)^2)
```

```
T9 = AL(INDEX+1,i,j)
T10 = -P(INDEX+1,i,j) · (3 · Q(INDEX+1,i,j)^2 - P(INDEX+1,i,j)^2 + ν · ALXY(i,j)^2)
T11 = BL(INDEX+1,i,j)
T12 = EXP(-P(INDEX+1,i,j) · FY/LXLY) · COS(Q(INDEX+1,i,j) · FY/LXLY)
MYYMXX = T1 · (T2 · (T3+T4 · T5) + T6 · (T4-T3 · T5) + T7 · (T8 · T9 + T10 · T11)
         + T12 · (T10 · T9 - T8 · T11)) * (123)
```

RETURN

SECTION TITLE: SUBDIA #13.19
SUBD2A # 13.20

SUBJECT

SUBROUTINE SUBDIA (MYXMXY, ALXY, FX, FY, LXLY, 2J, INDEX)
#13.19

T1 = (FY/LXLY) · Q(INDEX, λj)
T2 = (FY/LXLY) · P(INDEX, λj)
T3 = -ALXY(λj) · sin(ALXY(λj) FX/LXLY)
T4 = EXP(-T2) · sin(T1)
T5 = Q(INDEX, λj); T6 = P(INDEX, λj); T7 = Q(INDEX+1, λj); T8 = P(INDEX+1, λj); T9 = -2 · T5 · T6
T10 = T5² - T6² + ν · ALXY(λj)² ; T11 = BL(INDEX, λj); T12 = EXP(-T2) · cos(T1); T13 = T9 · T11
T14 = EXP(-T8 · FY/LXLY) · sin(T7 · FY/LXLY) ; T15 = -2 · T8 · T7
T16 = AL(INDEX+1, λj) ; T17 = T7² - T8² + ν · ALXY(λj)²
T18 = BL(INDEX+1, λj)
T19 = EXP(-T8 · FY/LXLY) · cos(T7 · FY/LXLY)
MYXMXY = T3 · (T4 · (T9 + T10 · T11) + T12 · (T10 - T13) + T14 · (T15 · T16 + T17 · T18) + T19 · (T17 · T16 - T15 · T18))
* 124

RETURN

SUBROUTINE SUBD2A (MXYMXY, ALXY, FX, FY, LXLY, 2J, INDEX)
#13.20

T1 = (1-ν) · ALXY(λj)² · cos(ALXY(λj) · FX/LXLY)
T2 = EXP(-P(INDEX, λj) · FY/LXLY)
T3 = T2 · sin(Q(INDEX, λj) · FY/LXLY)
T4 = -Q(INDEX, λj) - P(INDEX, λj) · BL(INDEX, λj)
T5 = T2 · cos(Q(INDEX, λj) · FY/LXLY)
T6 = -P(INDEX, λj) + Q(INDEX, λj) · BL(INDEX, λj)
T7 = EXP(-P(INDEX+1, λj) · FY/LXLY)
T8 = T7 · sin(Q(INDEX+1, λj) · FY/LXLY)
T9 = -Q(INDEX+1, λj) · AL(INDEX+1, λj) - P(INDEX+1, λj) · BL(INDEX+1, λj)
T10 = T7 · cos(Q(INDEX+1, λj) · FY/LXLY)
T11 = -P(INDEX+1, λj) · AL(INDEX+1, λj) + Q(INDEX+1, λj) · BL(INDEX+1, λj)
MXYMXY = T1 · (T3 · T4 + T5 · T6 + T8 · T9 + T10 · T11)
* 125

RETURN

DATE

SECTION TITLE: SUBD3A #13.21

SUBJECT

SUBROUTINE SUBD3A (MXYMYX, ALXY, FX, FY, LXLY, IJ, INDEX)
#13.21

```
T1 = (1-ν)·ALXY(IJ)·sin(ALXY(IJ)·FX/LXLY)
T2 = EXP(-P(INDEX,IJ)·FY/LXLY)
T3 = T2·(sin(Q(INDEX,IJ)·FY/LXLY
T4 = 2·P(INDEX,IJ)·Q(INDEX,IJ)+(P(INDEX,IJ)^2 - Q(INDEX,IJ)^2)·BL(INDEX,IJ)
T5 = T2·(cos(Q(INDEX,IJ)·FY/LXLY
T6 = P(INDEX,IJ)^2 - Q(INDEX,IJ)^2 - 2·P(INDEX,IJ)·Q(INDEX,IJ)·BL(INDEX,IJ)
T7 = EXP(-P(INDEX+1,IJ)·FY/LXLY
T8 = T7·sin(Q(INDEX+1,IJ)·FY/LXLY)
T9 = 2·P(INDEX+1,IJ)·Q(INDEX+1,IJ)·AL(INDEX+1,IJ)+(P(INDEX+1,IJ)^2
     - Q(INDEX+1,IJ)^2)·BL(INDEX+1,IJ)
T10 = T7·(cos(Q(INDEX+1,IJ)·FY/LXLY))
T11 = (P(INDEX+1,IJ)^2 - Q(INDEX+1,IJ)^2)·AL(INDEX+1,IJ)
      -2·P(INDEX+1,IJ)·Q(INDEX+1,IJ)·BL(INDEX+1,IJ)

MXYMYX = T1·(T3·T4+T5·T6+T8·T9+T10·T11)
      *(126)
```

RETURN

DATE

SECTION TITLE

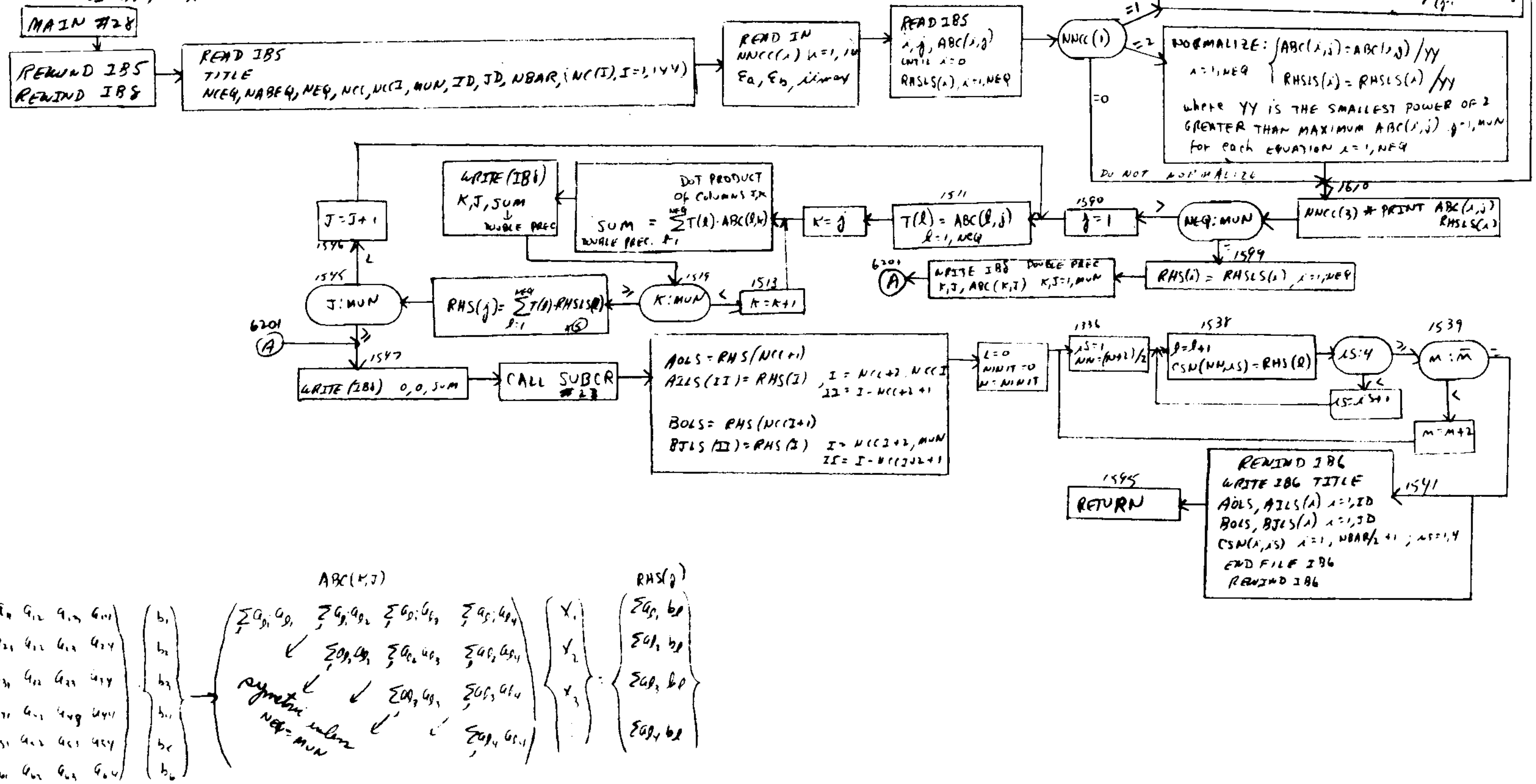
PART III SOLVE LEAST SQUARES MATRIX - OBTAIN CSN, A'S, B'S MAIN #28

SUBJECT

INPUT BINARY TAPE IB5 — CONTAINS ELEMENTS OF MATRIX

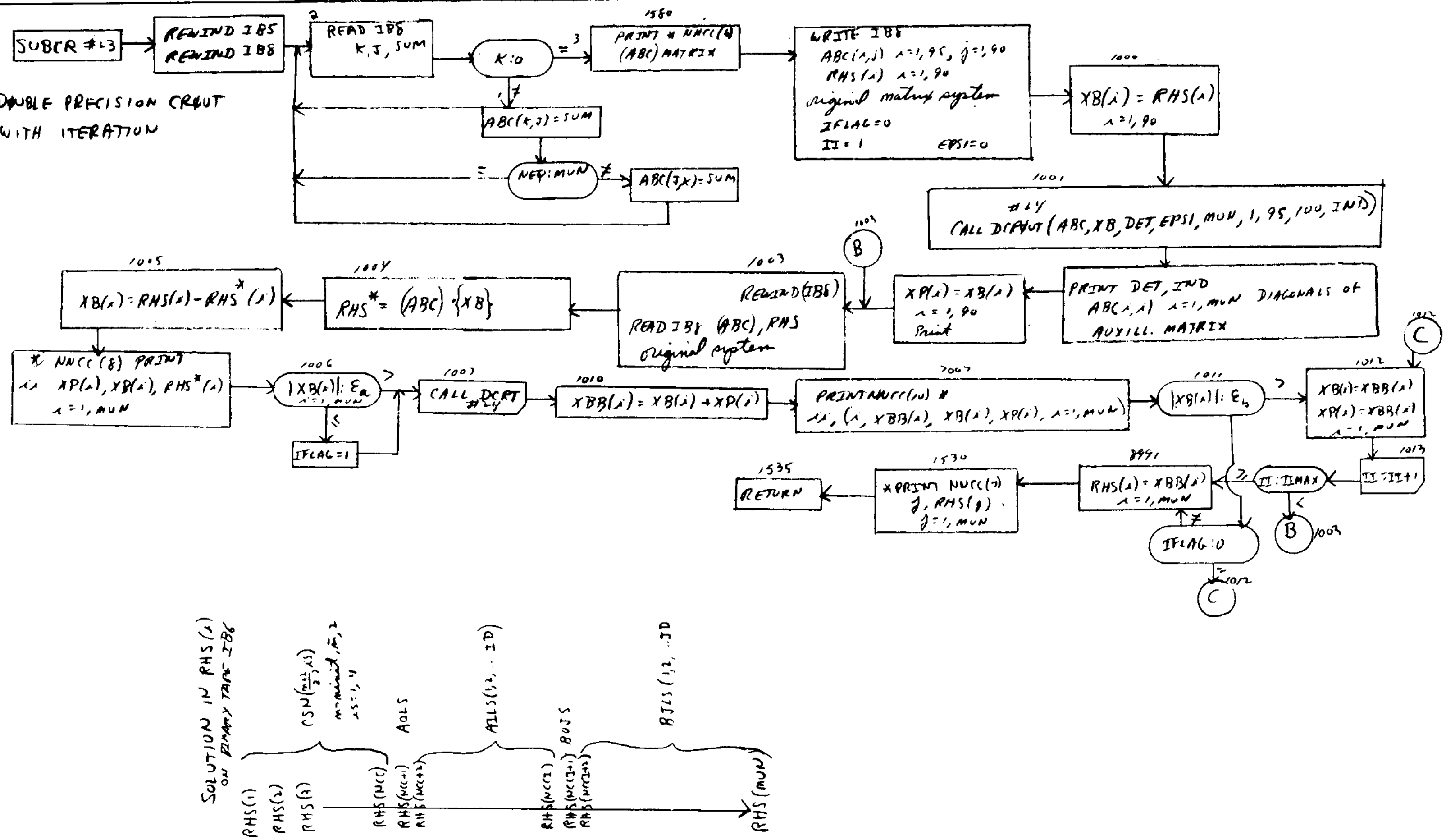
OUTPUT BINARY TAPE IB6 — CONTAINS A'S, B'S, CSN

BINARY TAPE IB8 — TEMPORARY SCRATCH TAPE



SECTION TITLE: PART III LEAST SQUARES SOLUTION - SUBROUTINE SUBCR #23

SUBJECT



SECTION TITLE

PART IV DOME STRESSES + DEFLECTIONS - MAIN # 29

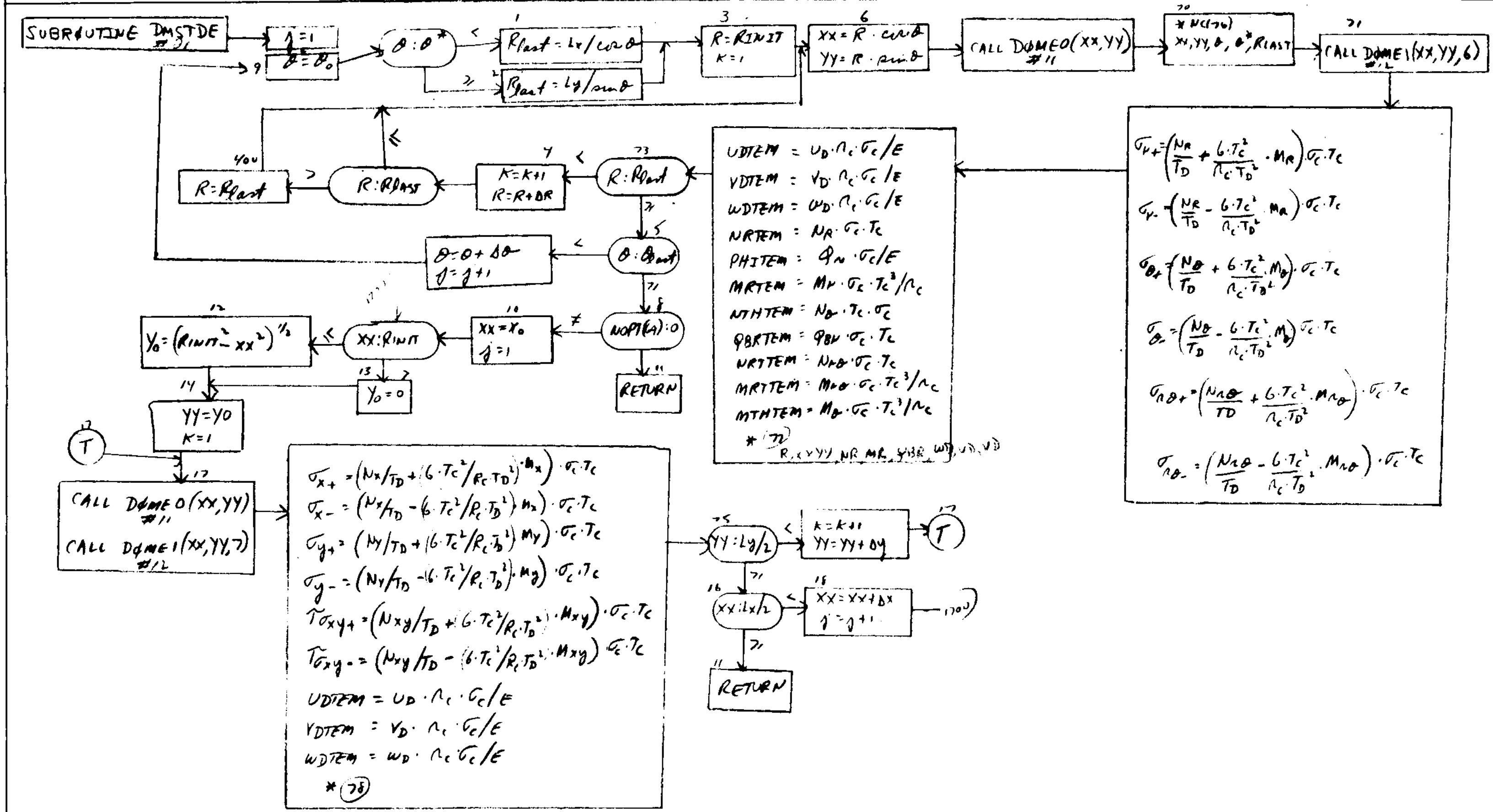
SUBJECT

```
MAIN 29
   INØUT(IN,IØ,IB1,IB2,IB3,IB4,IB5,IB6,IB7) 16
   INPUT1 12
   INIT 11
   DØME 30
   DMSTDE 21
      DØME0 11
      DØME1 12
         SUBD-- 13
         ØKCSN 20  -> USES-IB4-4-B2 - {YIJN}
```

DATE

SECTION TITLE: PART II DOME STRESSES + DEFLECTIONS – DMSTDE #21

SUBJECT

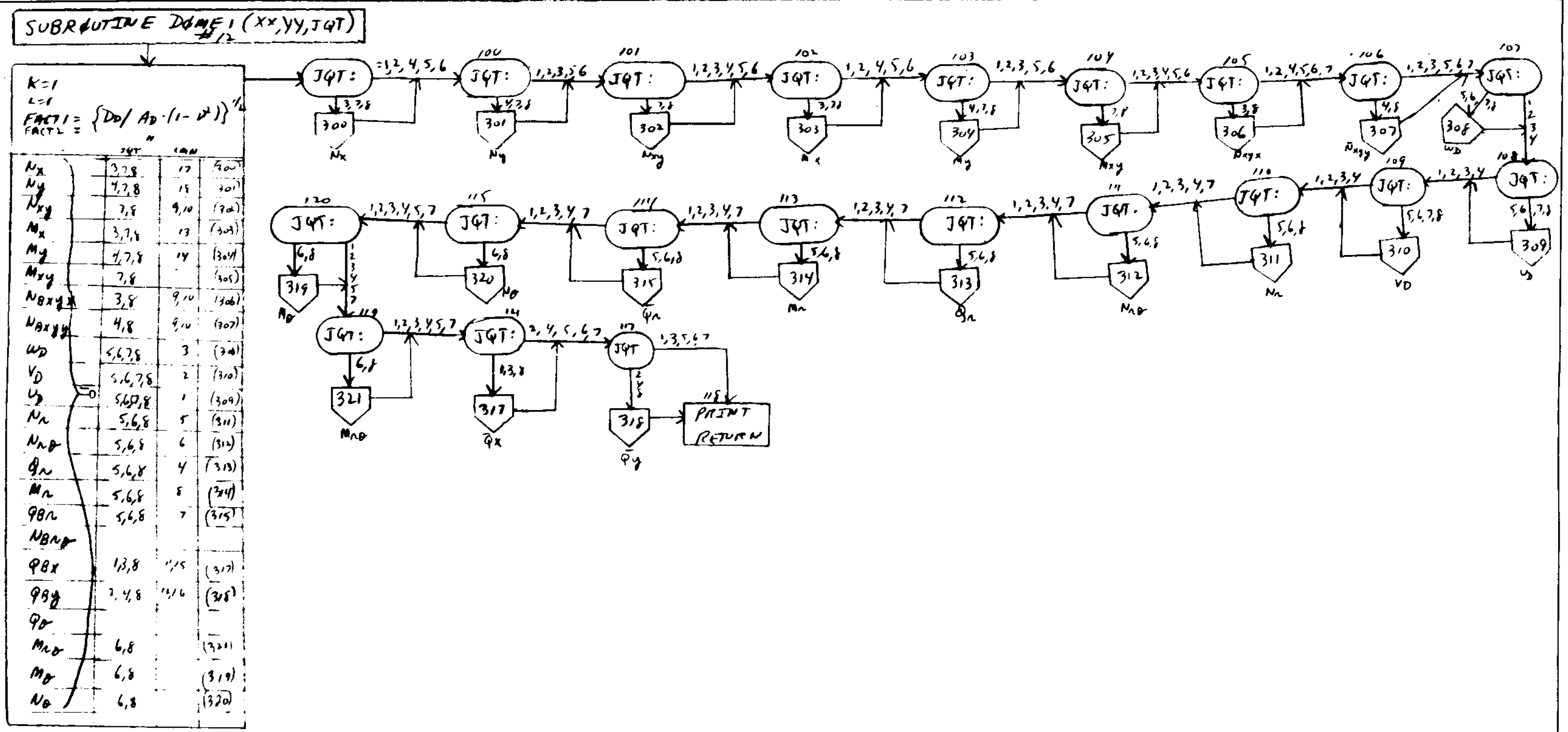


DATE

SECTION TITLE

SUBROUTINE DOME1 (XX, YY, JQT) #12 PART IV, VI

SUBJECT



DATE

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SECTION TITLE: SUBROUTINE DOME1(XX,YY,JQT) #12 CONTINUED PART V, VI

SUBJECT

N_x 300 (3,7,8) → SUMI = AOLS · N10X/LX → 3000 CALL SUBD9A(ANS, ALIX, XX, YY, LX, i, 1); SUMI = SUMI + ANS · AILS(I)/LX *(34) {i = 1, ID; by PASS IF ID=0} → 3001 SUM2 = BOLS · N20X/LY → 5001 CALL SUBD7F(ANS, ALJY, YY, XX, LY, J, 3); SUM2 = SUM2 + ANS · BJLS(J)/LY *(34) {J = 1, JD; by PASS IF JD=0} → $N_x = \frac{1}{\sigma_c T_c} \cdot FACT \cdot (\Sigma_1 + \Sigma_2)$ → 100

N_y 301 (4,7,8) → Σ_1 = AOLS · N10Y/LX → 3002 Call SUBD7F(ANS, ALIX, XX, YY, LX, i, 1); $\Sigma_1 = \Sigma_1$ + ANS · AILS(I)/LX *(35) {i = 1, ID; by PASS IF ID=0} → 3003 Σ_2 = BOLS · N20Y/LY → 5002 CALL SUBD9A(ANS, ALJY, YY, XX, LY, J, 3); $\Sigma_2 = \Sigma_2$ + ANS · BJLS(J)/LY *(35) {J = 1, JD} → $N_y = \frac{1}{\sigma_c T_c} \cdot FACT\ (\Sigma_1 + \Sigma_2)$ → 101

N_{xy} 302 (7,8) → Σ_1 = AOLS · N10XY/LX → 3004 CALL SUBDBC(ANS, ALIX, XX, YY, LX, i, 1); $\Sigma_1 = \Sigma_1$ + AILS(I) · ANS/LX *(36) {i = 1, ID} → 3005 Σ_2 = BOLS · N20XY/LY → 5004 CALL SUBDBC(ANS, ALJY, YY, XX, LY, J, 3); $\Sigma_2 = \Sigma_2$ + ANS · BJLS(J)/LY *(36) {J = 1, JD} → $N_{xy} = \frac{1}{\sigma_c T_c} \cdot FACT \cdot (\Sigma_1 + \Sigma_2)$ → 102

M_x 303 (3,7,8) → Σ_1 = AOLS · M10X/LX → 3006 Call SUBDPQ(ANS, ALIX, XY, YY, LX, i, 1) i = 1, ID; $\Sigma_1 = \Sigma_1$ + ANS · AILS(i)/LX *(37) → 3007 Σ_2 = BOLS · M20X/LY → 5006 CALL SUBDRS(ANS, ALJY, YY, XX, LY, J, 3) J = 1, JD; $\Sigma_2 = \Sigma_2$ + ANS · BJLS(J)/LY *(37) → $M_x = D_0 \cdot \Lambda_c / \sigma_c \cdot t_c \cdot \tau_c^2 (\Sigma_1 + \Sigma_2)$ → 103

M_y 304 (4,7,8) → Σ_1 = AOLS · M10Y/LX → 3008 Call SUBDRS(ANS, ALIX, XX, YY, LX, i, 1) i = 1, ID; $\Sigma_1 = \Sigma_1$ + AILS(i) · ANS/LX *(38) → 3009 Σ_2 = BOLS · M20Y/LY → 5008 CALL SUBDPQ(ANS, ALJY, YY, XX, LY, J, 3), j = 1, JD; $\Sigma_2 = \Sigma_2$ + ANS · BJLS(j)/LY *(38) → $M_y = D_0 \cdot \Lambda_c / \sigma_c \cdot t_c \cdot \tau_c^2 (\Sigma_1 + \Sigma_2)$ → 104

M_{xy} 305 (7,8) → Σ_1 = AOLS · M10XY/LX → 3010 Call SUBDTU(ANS, , , , , i, 1) i = 1, ID; $\Sigma_1 = \Sigma_1$ + AILS(i) · ANS/LX *(39) → 3011 Σ_2 = BOLS · M20XY/LY → 5010 Call SUBDTU(ANS, , , , , j, 3) j = 1, JD; $\Sigma_2 = \Sigma_2$ + BJLS(j) · ANS/LY *(39) → $M_{xy} = D_0 \cdot \Lambda_c \cdot (\Sigma_1 + \Sigma_2) / \sigma_c \cdot t_c^3$ → 105

N_{BXYX} 306 (3,8) → Σ_1 = AOLS · N10XY/LX → 8001 Call SUBDBC(ANS, , , , , i, 1) i = 1, ID; $\Sigma_1 = \Sigma_1$ + AILS(i) · ANS/LX *(40) → 8002 Σ_2 = BOLS · N20XY/LY → 5005 Call SUBDBC(ANS, , , , , j, 3); $\Sigma_2 = \Sigma_2$ + BJLS(j) · ANS/LY *(40) → 5006 $N_{BXYX} = \frac{1}{\sigma_c t_c} (FACT \cdot (\Sigma_1 + \Sigma_2))$ → 106

N_{BXYY} 307 (4,8) → Σ_1 = AOLS · N1XY/LX → 8011 Call SUBDBC(ANS, , , , , i, 1); $\Sigma_1 = \Sigma_1$ + AILS(i) · ANS/LX i = 1, ID *(41) → 5012 Σ_2 = BOLS · N20XY/LY → 5015 Call SUBDBC(ANS, , , , , j, 3); $\Sigma_2 = \Sigma_2$ + BJLS(j) · ANS/LY (41) j = 1, JD → $N_{BXYY} = \frac{-1}{\sigma_c T_c} (FACT \cdot (\Sigma_1 + \Sigma_2))$ → 107

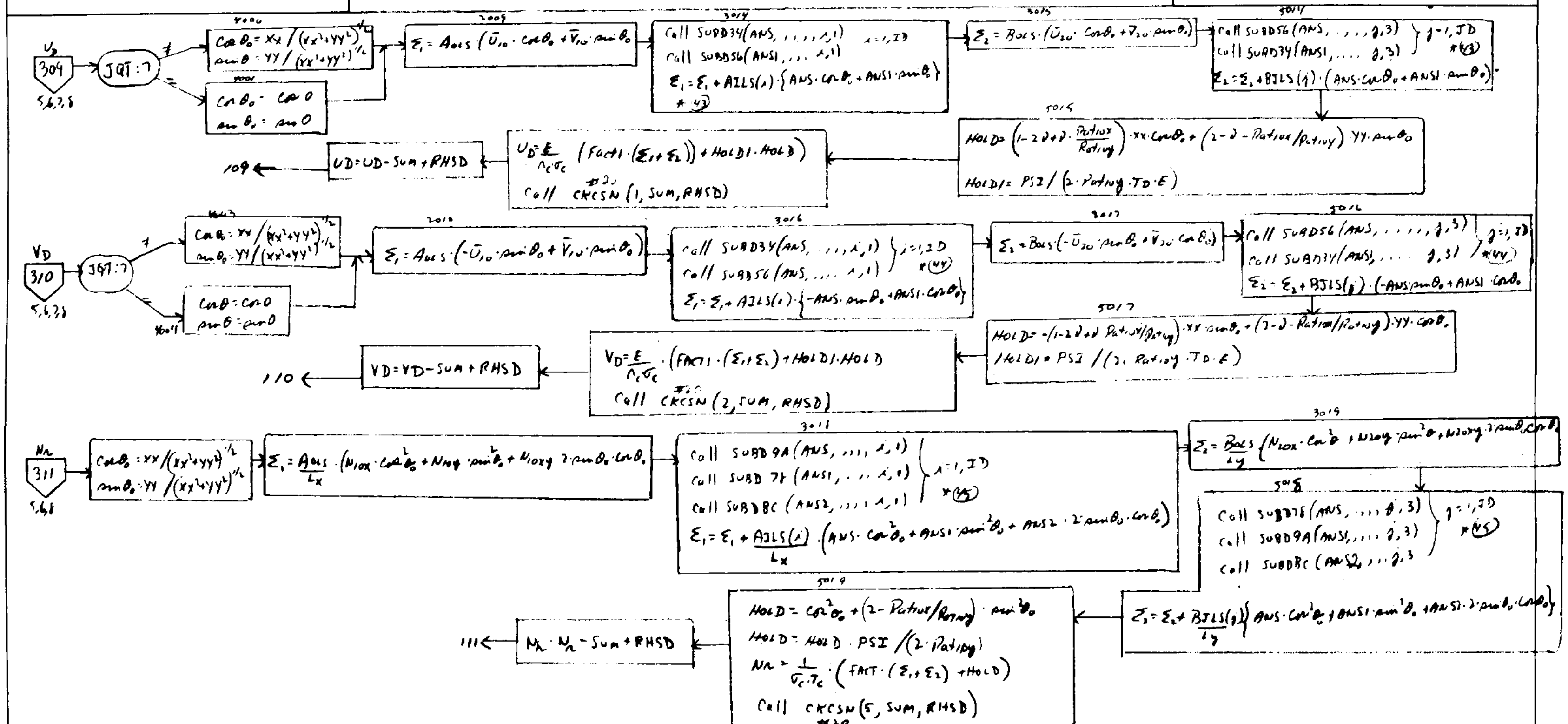
W_D 308 (5,6,7,8) → Σ_1 = AOLS · W10 · LX → 3012 Call SUBD12(ANS, , , , , i, 1); $\Sigma_1 = \Sigma_1$ + AILS(i) · ANS · LX i = 1, ID *(42) → 3013 Σ_2 = BOLS · W20 · LY → 5012 Call SUBD12(ANS, , , , , j, 3); $\Sigma_2 = \Sigma_2$ + ANS · BJLS(j) · LY j = 1, JD *(42) → 5013 $W_D = E \cdot (\Sigma_1 + \Sigma_2) / \Lambda_c \sigma_c$; Call CKCSW(3, SUM, RHSD) #20; $W_D = W_D$ − SUM + RHSD → 108

DATE

SECTION TITLE

SUBROUTINE DAME1 (XX, YY, JGT) #12 CONTINUED PART V, VI

SUBJECT



SECTION TITLE

SUBROUTINE DOME1 (XX, YY, JGT) #12 CONTINUED PART V, VI

SUBJECT

$N_{\theta\theta}$ 312 5,6,8

$\cos\theta_0 = XX/(XX^2+YY^2)^{1/2}$
$\sin\theta_0 = YY/(XX^2+YY^2)^{1/2}$

$\Sigma_1 = \frac{A_{0LS}}{L_X}\cdot((N10Y - N10X)\cdot 2\sin\theta_0\cos\theta_0/2 + N10XY\cdot(\cos^2\theta - \sin^2\theta))$

3020
Call SUBDJS (ANS, ..., i, 1)
Call SUBD9A (ANS1, ..., i, 1)
Call SUBDBC (ANS2, ..., i, 1)
$i = 1, ID$ $\times(46)$
$\Sigma_1 = \Sigma_1 + \frac{AILS(i)}{L_X}\cdot((ANS - ANS1)\cdot\frac{\sin 2\theta_0}{2} + ANS2\cdot\cos 2\theta_0$

3021
$\Sigma_2 = \frac{B_{0LS}}{L_Y}\cdot((N20Y - N20X)\cdot\frac{\sin 2\theta_0}{2} + N20XY\cdot\cos 2\theta_0$

Call SUBD9A (ANS, ..., j, 3)
Call SUBD78 (ANS1, ..., j, 3)
Call SUBDBC (ANS2, ..., j, 3)
$j = 1, JD$ $\times(48)$
$\Sigma_2 = \Sigma_2 + \frac{BJLS(j)}{L_Y}\cdot((ANS - ANS1)\cdot\frac{\sin 2\theta_0}{2} + ANS2\cdot\cos 2\theta)$

5021
$HOLD = \frac{PSI}{\theta_{ratio}}\cdot\sin 2\theta_0\cdot(1 - \frac{\theta_{ratio}}{\theta_{ratio}})$
$N_{r\theta} = \frac{1}{\sigma_c\cdot r_c}\cdot(Fact\cdot(\Sigma_1 + \Sigma_2) + HOLD$
Call CKCSN (6, SUM, RHSD) #20
$N_{r\theta} = N_{r\theta} - SUM + RHSD$

112

Q_r 313 5,6,8

$\sin\theta_0 = YY/(XX^2+YY^2)^{1/2}$
$\cos\theta_0 = XX/(XX^2+YY^2)^{1/2}$

$\Sigma_1 = A_{0LS}\cdot(Q_{0X}\cdot\cos\theta_0 + Q_{10Y}\cdot\sin\theta_0)$

3024
Call SUBDDE (ANS, ..., i, 1)
Call SUBDJK (ANS1, ..., i, 1)
$\times(47)$ $i = 1, ID$
$\Sigma_1 = \Sigma_1 + AILS(i)\cdot(ANS\cdot\cos\theta_0 + ANS1\cdot\sin\theta_0)$

$\Sigma_2 = B_{0LS}\cdot(Q_{0X}\cdot\cos\theta_0 + Q_{20Y}\cdot\sin\theta_0)$

3022
Call SUBDJK (ANS, ..., j, 3)
Call SUBDDE (ANS1, ..., j, 3)
$j = 1, JD$ $\times(47)$
$\Sigma_2 = \Sigma_2 + BJLS(j)\cdot(ANS\cdot\cos\theta_0 + ANS1)\cdot\sin\theta_0)$
$Q_r = \frac{E}{\sigma_c}(\Sigma_1 + \Sigma_2$
Call CKCSN (4, SUM, RHSD)
$Q_r = Q_r - SUM + RHSD$

314

M_r 314 5,6,8

$\sin\theta = YY/(XX^2+YY^2)^{1/2}$
$\cos\theta = XX/(XX^2+YY^2)^{1/2}$

$\Sigma_1 = \frac{A_{0LS}}{L_X}\cdot(M_{10X}\cdot\cos^2\theta_0 + M_{10Y}\cdot\sin^2\theta_0 + M_{10XY}\cdot\sin 2\theta_0)$

3024
Call SUBDPQ (ANS, ..., i, 1)
Call SUBDRS (ANS1, ..., i, 1)
Call SUBDTU (ANS2, ..., i, 1)
$\times(48)$ $i = 1, ID$
$\Sigma_1 = \Sigma_1 + \frac{AILS(i)}{L_X}\cdot(ANS\cdot\cos^2\theta_0 + ANS1\cdot\sin^2\theta_0 + ANS2\cdot\sin 2\theta_0)$

3025
$\Sigma_2 = \frac{B_{0L}S}{L_Y}\cdot(M_{20X}\cdot\cos^2\theta_0 + M_{20Y}\cdot\sin^2\theta_0 + M_{20XY}\cdot\sin 2\theta_0)$

3024
Call SUBDRS (ANS, ..., j, 3)
Call SUBDPQ (ANS1, ..., j, 3)
Call SUBDTU (ANS2, ..., j, 3)
$j = 1, JD$ $\times(49)$
$\Sigma_2 = \Sigma_2 + \frac{BJLS(j)}{L_Y}(ANS\cdot\cos^2\theta_0 + ANS1\cdot\sin^2\theta_0 + ANS2\cdot\sin 2\theta)$

$M_r = \frac{D_0\cdot r_c}{\sigma_c\cdot r_c^3}\cdot(\Sigma_1 + \Sigma_2)$
Call CKCSN (8, SUM, RHSD) #20
$M_r = M_r - SUM + RHSD$

114

COMPUTER PROGRAMMING AND ANALYSIS PROGRAM WRITE-UP

SECTION TITLE

SUBROUTINE DAME1 (XX,YY,JQT) #12 PART II, VI

SUBJECT

$\bar{Q}_n$ 315 5,45

$\cos\theta_0 = XX/(XX^2+YY^2)^{1/2}$

$\sin\theta_0 = YY/(XX^2+YY^2)^{1/2}$

3015

$\Sigma_1 = \frac{A_{OLS}}{L_x^2}\Big\{ Q_{10x}\cdot\cos\theta_0 + Q_{10y}\cdot\sin\theta_0 + \frac{L_x}{(XX^2+YY^2)^{1/2}}\cdot M_{10y}\cdot\cos 2\theta_0 - M_{10x}\cdot\cos 2\theta_0 - 2\cdot M_{10xy}\cdot\sin 2\theta_0 + M_{1xyDY}\cdot\cos 2\theta_0\cdot\cos\theta_0 - M_{1xyDX}\cdot\cos 2\theta_0\cdot\sin\theta_0 + .5\cdot M_{10yDY}\cdot\sin 2\theta_0\cdot\cos\theta_0$
$-.5\cdot M_{1YDX}\cdot\sin 2\theta_0\cdot\sin\theta_0 - .5\cdot M_{10xDY}\cdot\sin 2\theta_0\cdot\cos\theta_0 + .5\cdot M_{1XDX}\cdot\sin 2\theta_0\cdot\sin\theta_0 \Big\}$

3026

Call SUBDFG (ANS, ... i,1)
Call SUBDHI (ANS1, ... i,1)
Call SUBDRS (ANS2, ... i,1)
Call SUBDPQ (ANS3, ... i,1)
Call SUBDTU (ANS4, ... i,1)
Call SUBD3A (ANS5, ... i,1)
Call SUBD2A (ANS6, ... i,1)
Call SUBDZ (ANS7, ... i,1)
Call SUBD1A (ANS8, ... i,1)
Call SUBDVW (ANS9, ... i,1)
Call SUBDXY (ANS10, ... i,1)

$i = 1, I_D$ * 49

$\Sigma_1 = \Sigma_1 + AILS(i)/L_x^2 \Big\{ ANS\cdot\cos\theta_0 + ANS1\cdot\sin\theta_0 + L_x/(XX^2+YY^2)^{1/2}(ANS2\cdot\cos 2\theta_0 - ANS3\cdot\cos 2\theta_0 - 2\cdot ANS4\cdot\sin 2\theta_0) + ANS5\cdot\cos 2\theta_0\cdot\cos\theta_0$
$- ANS6\cdot\cos 2\theta_0\cdot\sin\theta_0 + .5\cdot ANS7\cdot\sin 2\theta_0\cdot\cos\theta_0 - .5\cdot ANS8\cdot\sin 2\theta_0\cdot\sin\theta_0 - .5\cdot ANS9\cdot\sin 2\theta_0\cdot\cos\theta_0 + .5\cdot ANS10\cdot\sin 2\theta_0\cdot\sin\theta_0 \Big\}$

3027

$\Sigma_2 = \frac{B_{OLS}}{L_y^2}\Big\{ Q_{20x}\cdot\cos\theta_0 + Q_{20y}\cdot\sin\theta_0 + (L_y/(XX^2+YY^2)^{1/2})(M_{20y}\cdot\cos 2\theta_0 - M_{20x}\cdot\cos 2\theta_0 - 2\cdot M_{20xy}\cdot\sin 2\theta_0) + M_{2xyDY}\cdot\cos 2\theta_0\cdot\cos\theta_0 - M_{2xyDX}\cdot\cos 2\theta_0\cdot\sin\theta_0 + .5\cdot M_{2yDY}\cdot\sin 2\theta_0\cdot\cos\theta_0$
$-.5\cdot M_{20yDX}\cdot\sin 2\theta_0\cdot\sin\theta_0 - .5\cdot M_{2XDY}\cdot\sin 2\theta\cdot\cos\theta_0 + .5\cdot M_{20XDX}\cdot\sin 2\theta_0\cdot\sin\theta_0 \Big\}$

5026

Call SUBDHI (ANS, ... j,3)
FG (ANS1, ... j,3)
PQ (ANS2, ... j,3)
RS (ANS3, ... j,3)
TU (ANS4, ... j,3)
2A (ANS5, ... j,3)
3A (ANS6, ... j,3)
XY (ANS7, ... j,3)
VW (ANS8, ... j,3)
1A (ANS9, ... j,3)
Z (ANS10, j,3)

$j = 1, J_D$ * 49

$\Sigma_2 = \Sigma_2 + \frac{BJLS(j)}{L_y^2}\Big\{ ANS\cdot\cos\theta_0 + ANS1\cdot\sin\theta_0 + L_y/(XX^2+YY^2)^{1/2}\cdot(ANS2\cdot\cos 2\theta_0 - ANS3\cdot\cos 2\theta_0 - 2\cdot ANS4\cdot\sin 2\theta) + ANS5\cdot\cos 2\theta\cdot\cos\theta_0$
$- ANS6\cdot\cos 2\theta_0\cdot\sin\theta_0 + .5\cdot ANS7\cdot\sin 2\theta_0\cdot\cos\theta_0 - .5\cdot ANS8\cdot\sin 2\theta_0\cdot\sin\theta_0 - .5\cdot ANS9\cdot\sin 2\theta_0\cdot\cos\theta_0$
$+ .5\cdot ANS10\cdot\sin 2\theta_0\cdot\sin\theta_0 \Big\}$

$\bar{Q}_n = \frac{D_D\cdot(\Sigma_1 + \Sigma_2)}{\sigma_c\cdot T_c}$

CALL CKCSN(7,SUM,RHSD) #20

$\bar{Q}_n = \bar{Q}_n - SUM + RHSD$

115

DATE

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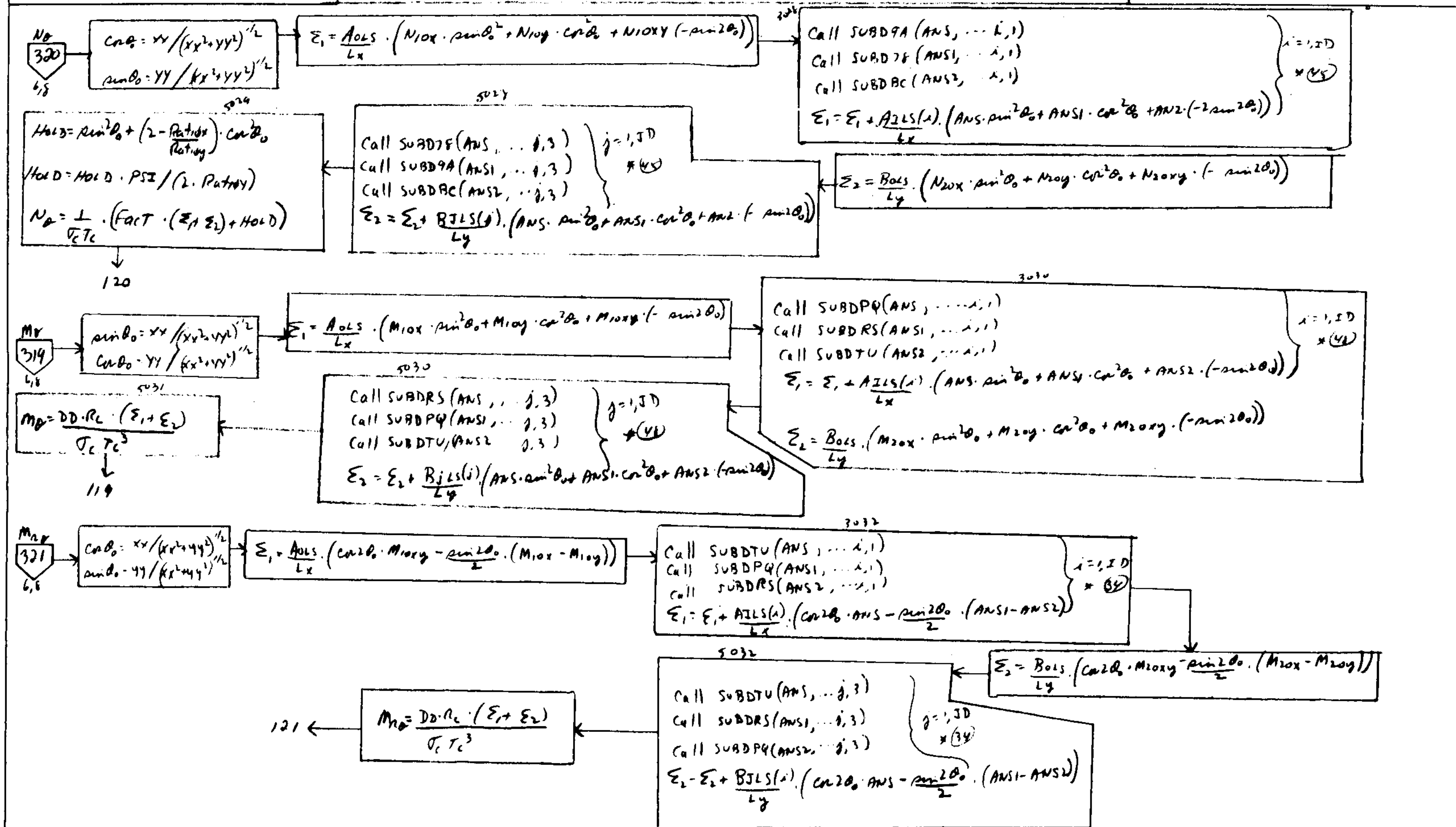
SECTION TITLE: SUBROUTINE DOME1(XX, YY, JGT) #12 — PART IV, V

SUBJECT

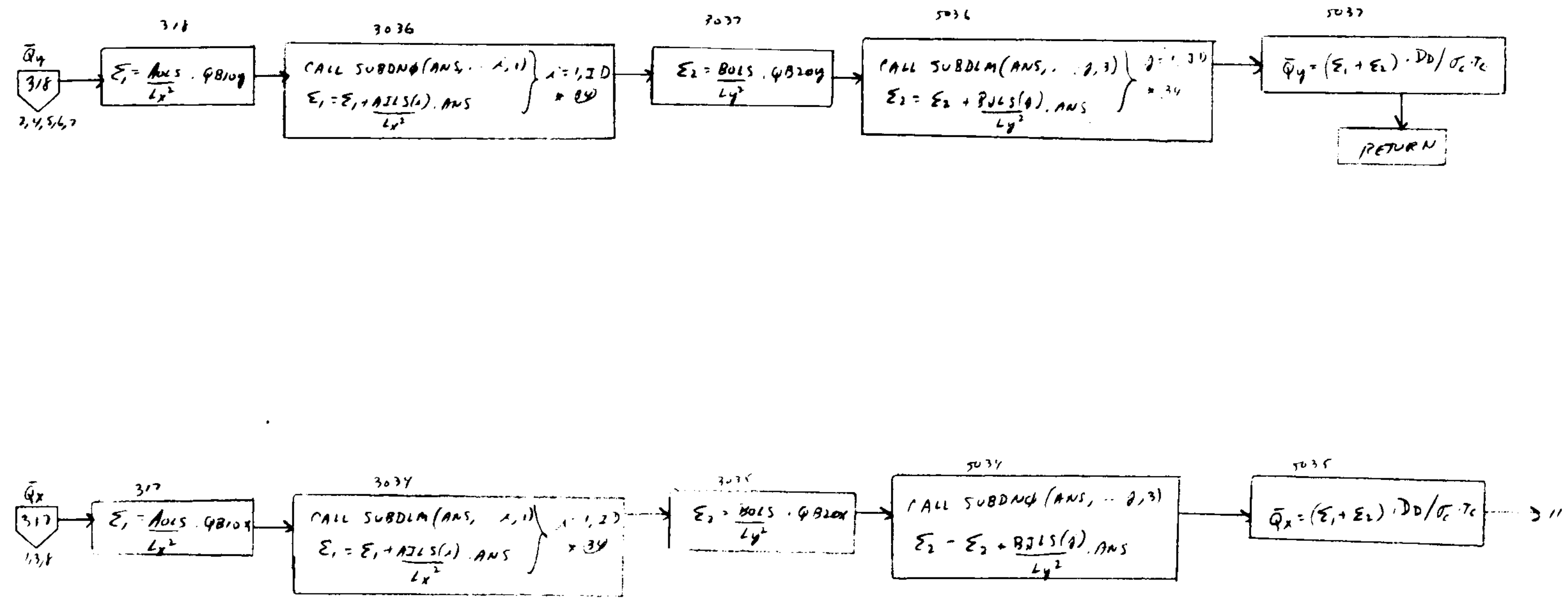


DATE

SECTION TITLE

SUBRØUTINE DØME1(XY,YY,JQT) #12 PART Ⅱ Ⅵ

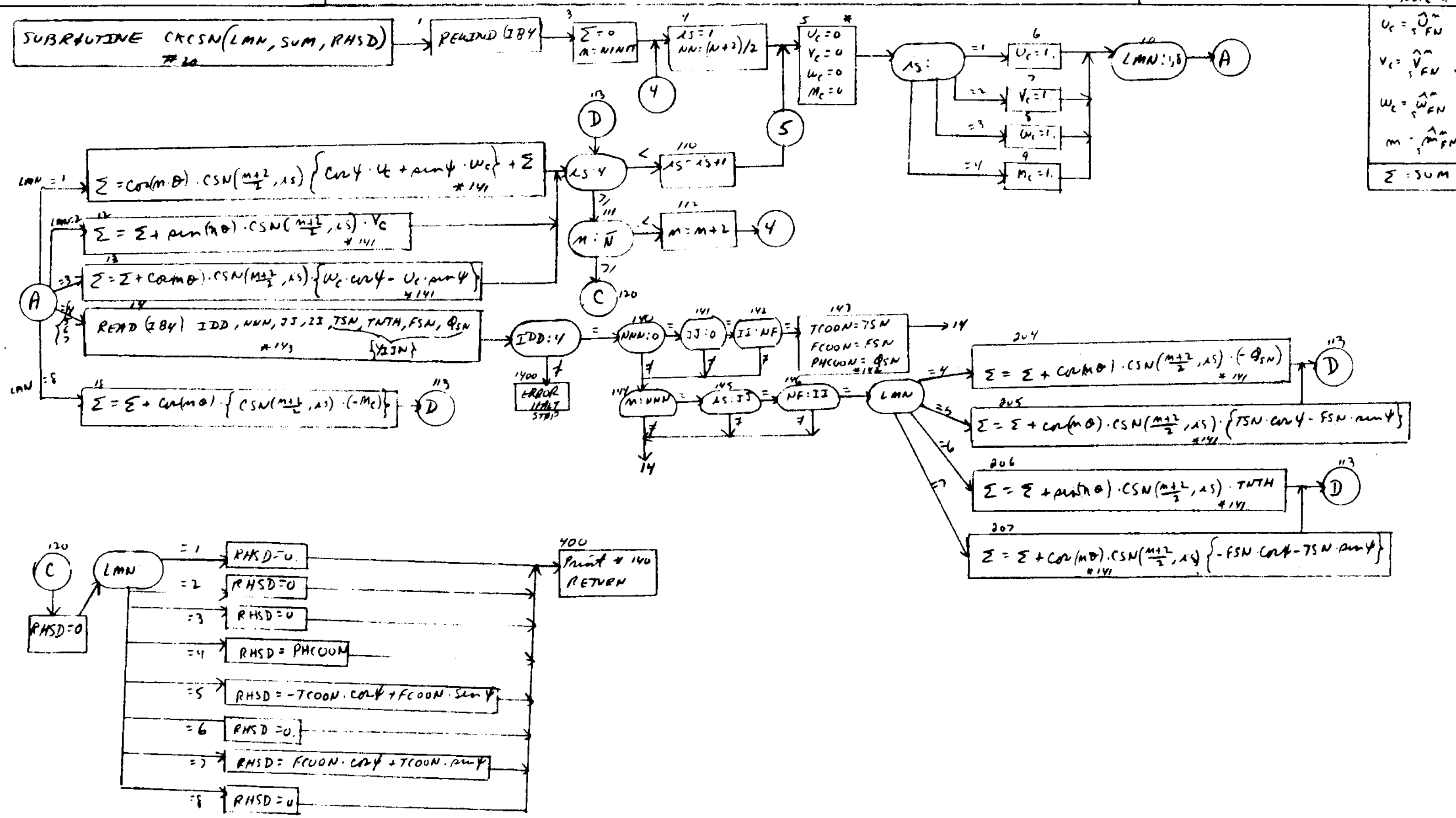
SUBJECT



DATE

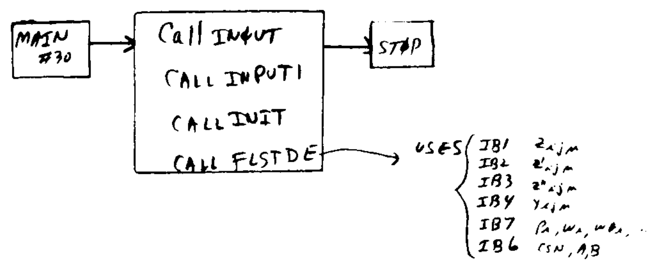
SECTION TITLE: SUBROUTINE CKCSN(LMN, SUM, RHSD) #20 PART V, VI $\Sigma CSN(\ ,\ )$

SUBJECT



SECTION TITLE: MAIN #30 FLARE STRESSES + DEFLECTIONS PART V

SUBJECT

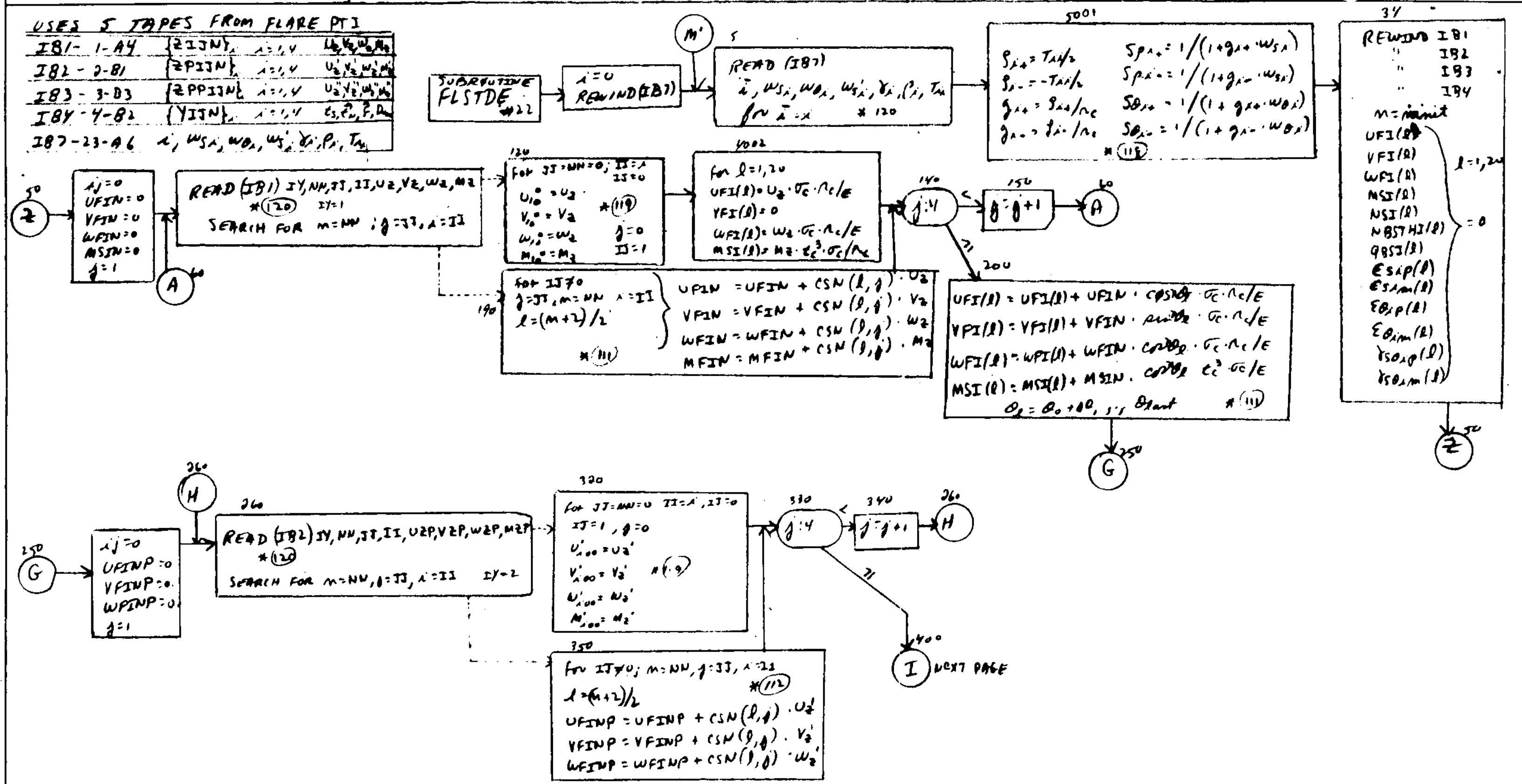


DATE

SECTION TITLE

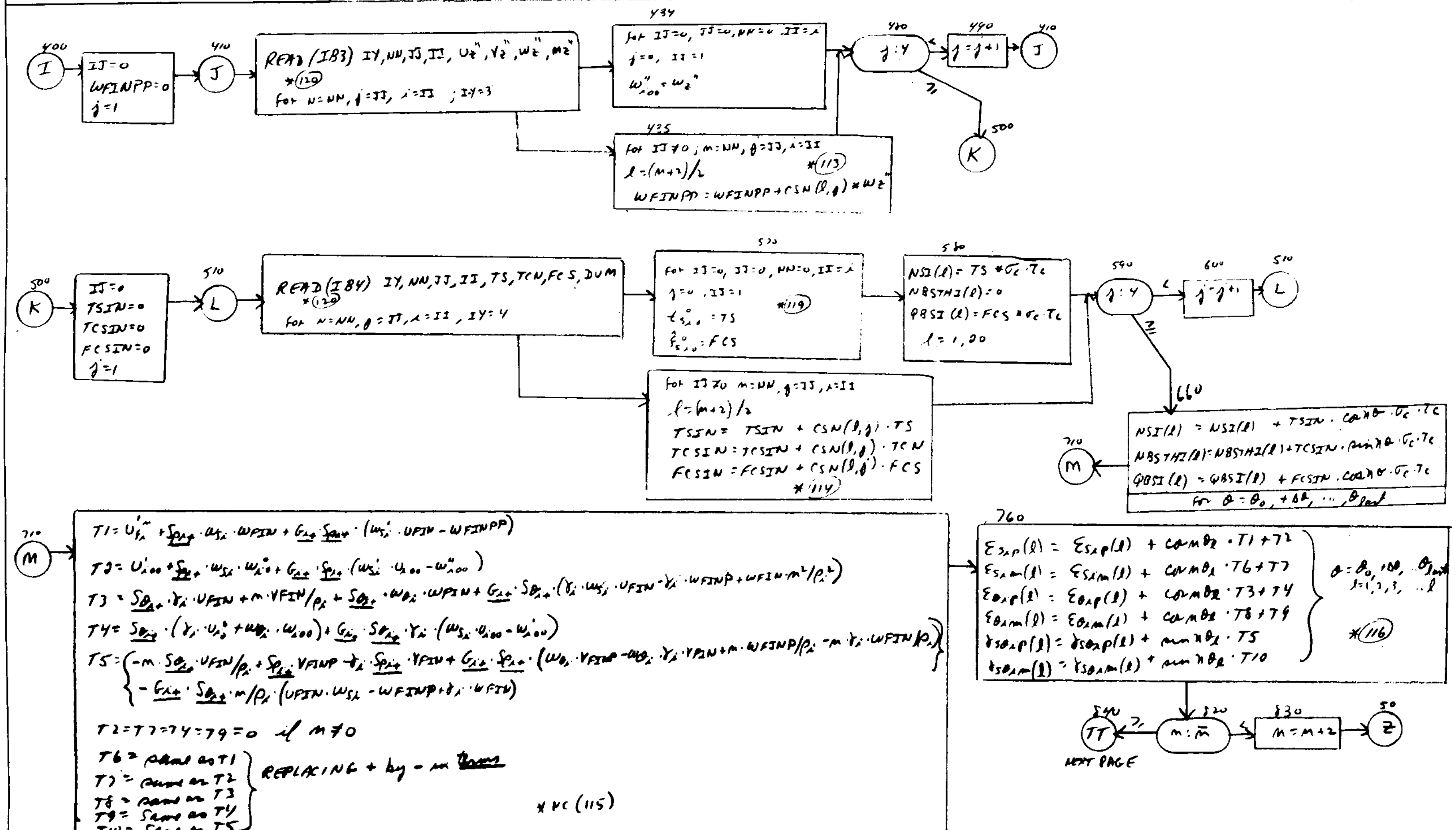
PART II FLARE STRESSES + DEFLECTIONS FLSTDE #22

SUBJECT



SECTION TITLE: PART V CONTINUED FLSTDE #22

SUBJECT



SECTION TITLE | PART V FLSTDE #22 CONTINUED | SUBJECT

840 (TT) →

$$\varepsilon_{sxp}(ll) = \varepsilon_{sxp}(ll) \cdot \sigma_c/E \qquad \text{for } ll = 1,2$$
$$\varepsilon_{sxm}(ll) = \varepsilon_{sxm}(ll) \cdot \sigma_c/E$$
$$\varepsilon_{\theta xp}(ll) = \varepsilon_{\theta xp}(ll) \cdot \sigma_c/E$$
$$\varepsilon_{\theta xm}(ll) = \varepsilon_{\theta xm}(ll) \cdot \sigma_c/E$$
$$\gamma_{s\theta xp}(ll) = \gamma_{s\theta xp}(ll) \cdot \sigma_c/E$$
$$\gamma_{s\theta xm}(ll) = \gamma_{s\theta xm}(ll) \cdot \sigma_c/E$$
$$\sigma_{sxp}(ll) = E/(1-\nu^2) \cdot (\varepsilon_{sxp}(ll) + \nu \cdot \varepsilon_{\theta xp}(ll))$$
$$\sigma_{sxm}(ll) = E/(1-\nu^2) \cdot (\varepsilon_{sxm}(ll) + \nu \cdot \varepsilon_{\theta xm}(ll))$$
$$\sigma_{\theta xp}(ll) = E/(1-\nu^2) \cdot (\varepsilon_{\theta xp}(ll) + \nu \cdot \varepsilon_{sxp}(ll))$$
$$\sigma_{\theta xm}(ll) = E/(1-\nu^2) \cdot (\varepsilon_{\theta xm}(ll) + \nu \cdot \varepsilon_{sxm}(ll))$$
$$\sigma_{xp}(ll) = E/(1-\nu^2) \cdot \gamma_{s\theta xp}(ll)$$
$$\sigma_{xm}(ll) = E/(1-\nu^2) \cdot \gamma_{s\theta xm}(ll)$$

*(117)

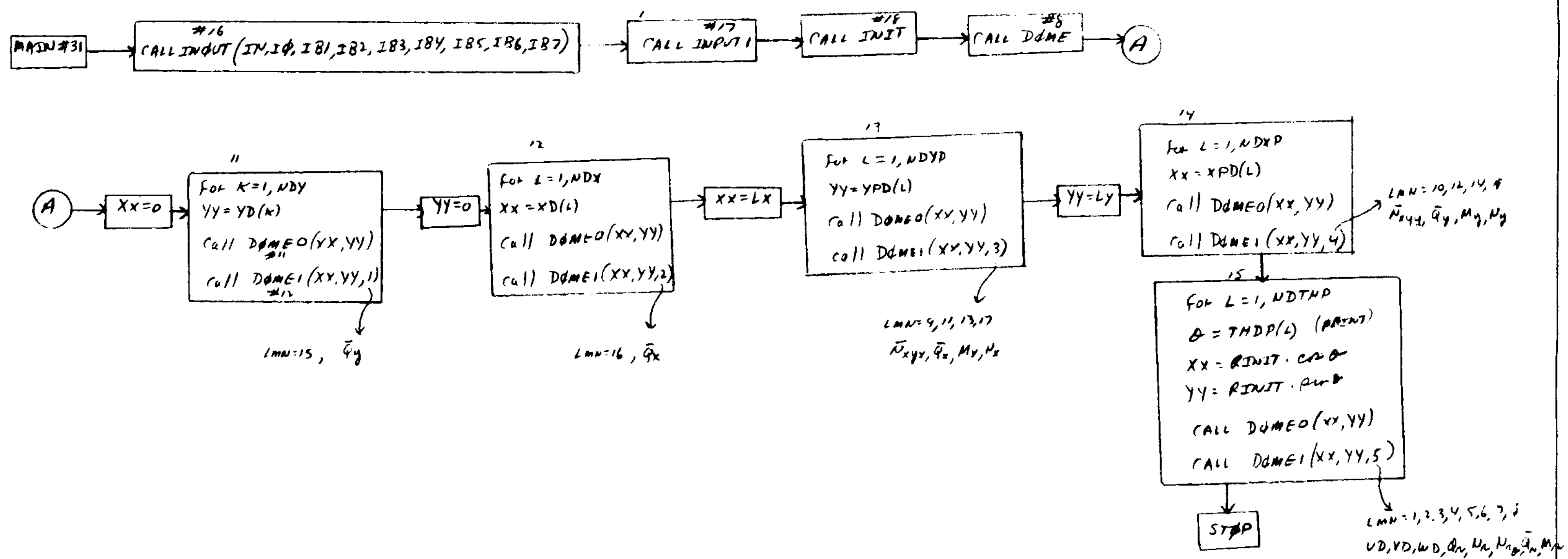
890 $x : N_F$

< → 900 $x = x+1$ → 5 (M)

≥ → 910 RETURN

SECTION TITLE: MAIN #31 CHECK L.S. SOLUTIONS - PART VI

SUBJECT



DATE