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Relationships

S4 Block 1

Homework

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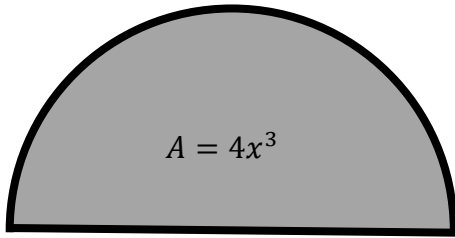
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N5**Homework 1**
(Applying the content of Surds and Indices)

1.

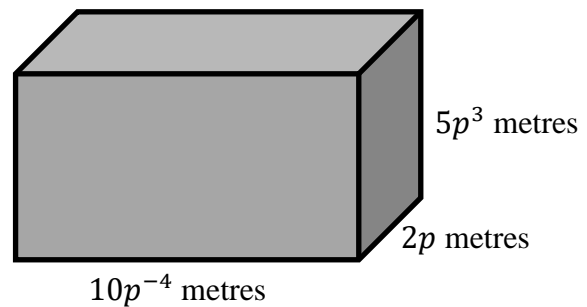
The area of the semicircle shown is $4x^3$.

Show that the radius of the semicircle is given by $r = \sqrt{\frac{8x^3}{\pi}}$

2.

A cuboid has dimensions as shown .

Show that the volume of the cuboid is 100 cubic metres.



3.

A particle travels $3ab^2$ metres in $12a^2c$ seconds.

Calculate the particles average speed in metres per second.

4.

(a) Evaluate $(2^4)^2$.(b) Hence find n , when $(2^4)^n = \frac{1}{256}$

5.

Lauren writes down the following statement.

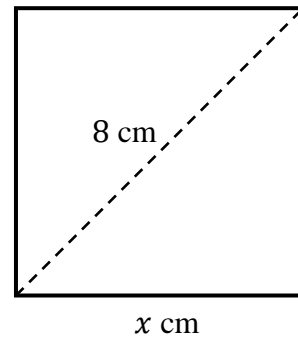
$$p^{\frac{1}{3}}\left(p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right) = p - 1$$

Is the statement true?

Justify your answer with working.

6. A square of side x centimetres has a diagonal which is 8 centimetres long.

Show that $x = 4\sqrt{2}$ cm.



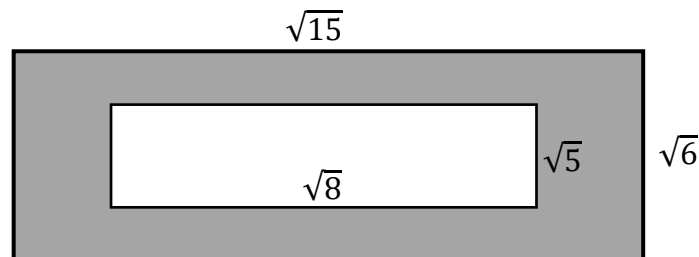
7. Sam's homework jotter has the following statement.

$$\frac{6}{\sqrt{3}} = 2\sqrt{3}$$

Is the statement true?

Justify your answer with working.

8. A small rectangle is drawn completely enclosed in a larger rectangle as show.



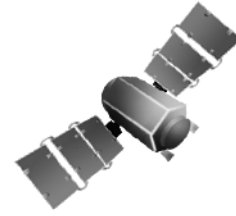
With the dimensions given, **show that** the shaded area is $\sqrt{10}$ square units.

9. **Show that** $\frac{\sqrt{3} \times \sqrt{12}}{\sqrt{3} + \sqrt{12}} = \frac{2\sqrt{3}}{3}$.

N5**Homework 2**
(Applying the content of Brackets and Factors)

1. A satellite travels at a speed of $3x + 2$ kilometres per hour for $4x - 1$ hours.

Show that the distance travelled by the satellite (D kilometres) is given by $= 12x^2 + 5x - 2$.

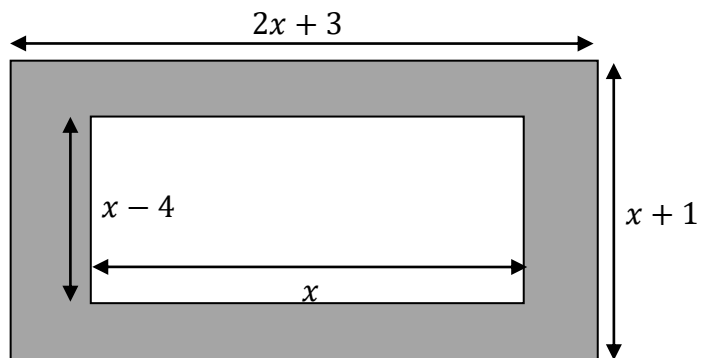


2. Expand $(x^{\frac{1}{2}} + 1)(x^{\frac{1}{2}} + x^{-\frac{1}{2}})$

3. Look at the diagram.

Show that the shade area can be represented by

$$A = x^2 + 9x + 3.$$

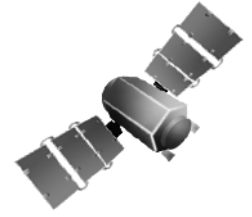


4. (a) Factorise $a^2 - b^2$
(b) Hence find the value of $2 \cdot 4^2 - 1 \cdot 6^2$.
5. Gillian says that $(x + 3)^2 - (x + 1)^2 = 4(x + 2)$.
Kenny says that $(x + 3)^2 - (x + 1)^2 = 2(4x + 5)$.
Which of the two students is correct?

Justify your answer with working.

N5**Homework 3**
(Applying the content of Algebraic fractions)

1. A satellite travels a distance of $3xy^3$ kilometres in $12x^2y$ hours. Calculate the speed of the satellite in kilometres per hour. *Give your answer as a fraction in its simplest form.*

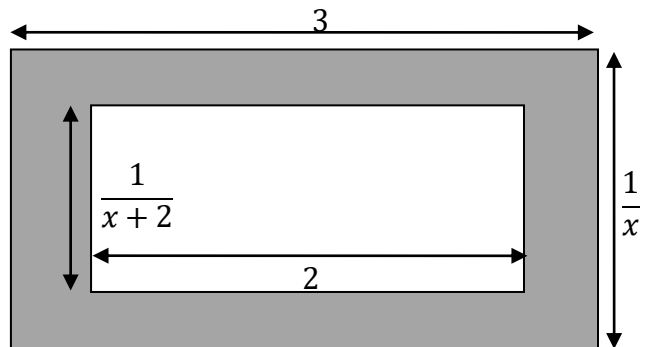


2. A particle travels at a speed of $\frac{5x^2}{2}$ metres per second for $\frac{4t^3}{3x}$ seconds.

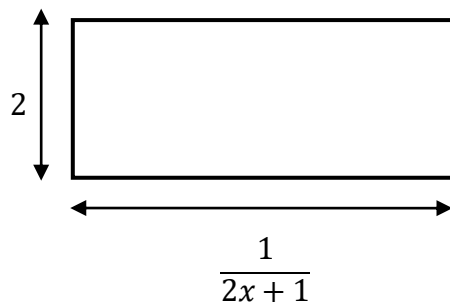
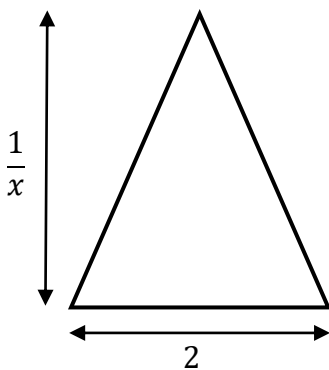
Calculate the distance travelled by the particle.

3. Look at the diagram.
Show that the shade area can be represented by

$$A = \frac{x+6}{x(x+2)}$$



4. In the diagram below, the triangle has an area 3 square units greater than the area of the rectangle.



Show that $\frac{1}{x(2x+1)} = 3$.

N5**Homework 4**

(Applying the content of Expressions and Formulae 1.4)

1.

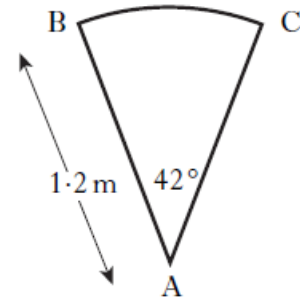


A spiral staircase is being designed.

Each step is made from a sector of a circle as shown.

The radius is 1.2 metres.

Angle BAC is 42° .

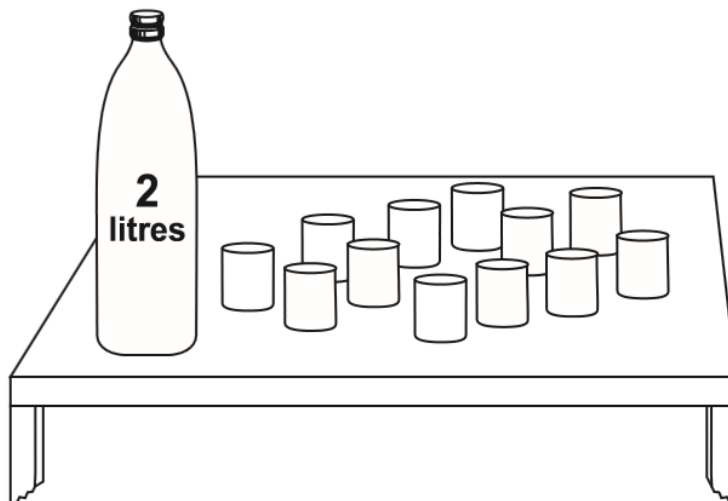


For the staircase to pass safety regulations, the arc BC must be at least 0.9 metres.

Will the staircase pass safety regulations?

2. Lemonade is to be poured from a 2 litre bottle into glasses.

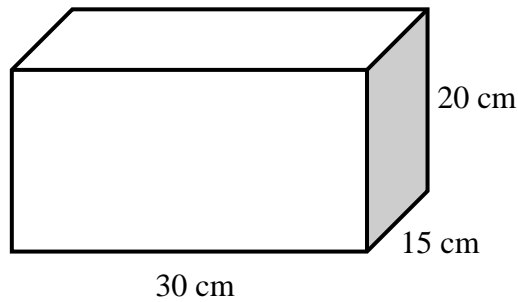
Each glass is in the shape of a cylinder of radius 3 centimetres and height 8 centimetres.



How many full glasses can be poured from the bottle?

3. Marie makes candles from wax.

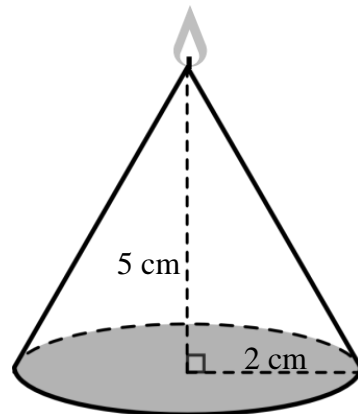
She buys the wax for the candle in large cuboid blocks with dimensions shown below.



One of the candles that Marie makes is a cone shaped candle with a radius of 2 centimetres and a height of 5 centimetres.

Marie says she will be able to make 430 complete candles from the block of wax.

Is she correct?



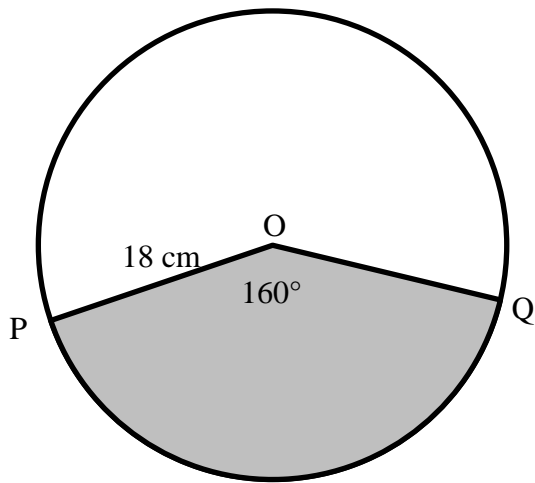
4. A container, in the shape of a cylinder, is full of fluid.

The container has a radius of 4 centimetres and a height of 6.5 centimetres.

The fluid in the container is poured into a hemispherical mould with a radius of 5.3 centimetres.

Will the mould hold the fluid or will it overflow?

5. The guard on a garden strimmer is in the shape of a circle with a piece, represented by the shaded sector in the diagram, cut out.



PQ is a minor arc of the circle with centre O.
The radius OP is 18 cm.
Angle POQ is 160° .

- (a) Calculate the length of minor arc PQ.
- (b) Calculate the area of the waste plastic, the shaded sector, in square metres.
- (c) The manufacturers of the guards also make magazine racks.

A magazine rack is designed which uses two pieces of waste plastic in each magazine rack. The cost of the plastic is £15.73 per square metre and the company claim to save over £140 every 100 magazine racks by recycling the plastic.

Is the company's claim true?

1. Suzie has a new mobile phone. She is charged x pence per minute for calls and y pence for each text she sends. During the first month her calls last a total of 280 minutes and she sends 70 texts. Her bill is £52.50.

(a) Write down an equation in x and y which satisfies the above condition.

The next month she reduces her bill. She restricts her calls to 210 minutes and sends 40 texts. Her bill is £38.00.

(b) Write down a second equation in x and y which satisfies this condition.

(c) Calculate the price per minute for a call and the price for each text.

2. Alan is taking part in a quiz. He is awarded x points for each correct answer and y points for each wrong answer. During the quiz, Alan gets 24 questions correct and 6 wrong. He scores 60 points.

(a) Write down an equation in x and y which satisfies the above condition.

Helen also takes part in the quiz. She gets 20 questions correct and 10 wrong. She scores 40 points.

(b) Write down a second equation in x and y which satisfies this condition.

(c) Calculate the score for David who gets 17 correct and 13 wrong.

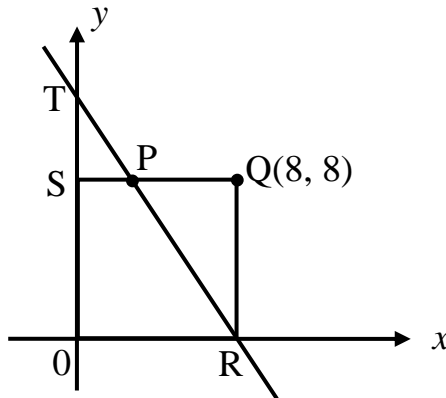
3. Two straight lines, $y = 2x - 3$ and $x + 2y = 14$ intersect at the point P.

Find, algebraically, the coordinates of the point P.

4. A straight line has equation $y = mx + c$, where m and c are constants.
- (a) The point $(2, 7)$ lies on this line.
Write down an equation in m and c to illustrate this information.
- (b) A second point $(4, 17)$ also lies on this line.
Write down another equation in m and c to illustrate this information.
- (c) Hence calculate the values of m and c .
- (d) Write down the gradient and the coordinates of the y intercept of this line.

5. A straight line with equation $y = \frac{1}{3}x + 2$ cuts the x -axis at B.
- (a) Find the coordinates of B.
- (b) For what values of x is $y < 0$.

6. A square, OSQR, is shown below.
Q is the point $(8, 8)$



The straight line TR cuts the y -axis at $T(0, 12)$ and the x -axis at R.

- (a) Find the equation of the line TR.

The line TR also cuts SQ at P.

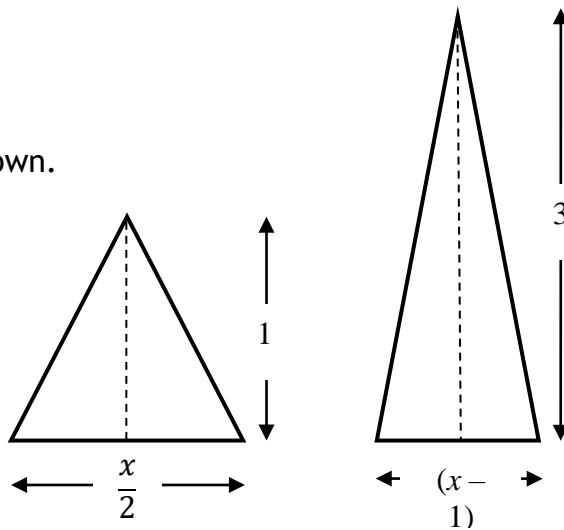
- (b) Find the coordinates of P.

7. Jane enters a two-part race.
- (a) She cycles for 2 hours at a speed of $(x + 8)$ kilometres per hour.
Write down an expression in x for the distance cycled.
- (b) She then runs for 30 minutes at a speed of x kilometres per hour.
Write down an expression in x for the distance run.
- (c) The total distance of the race is 46 kilometres.
Calculate Jane's running speed.

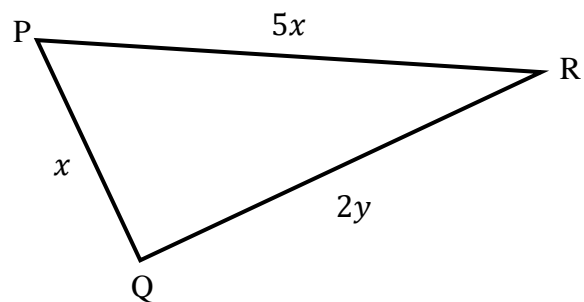
8. Two triangles have dimensions as shown.

The triangles are equal in area.

Calculate the value of x .

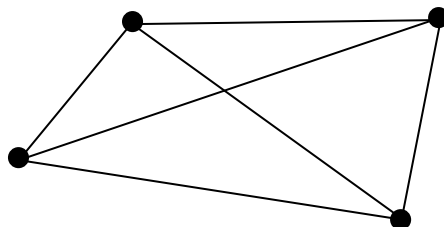


9. The diagram show triangle PQR.



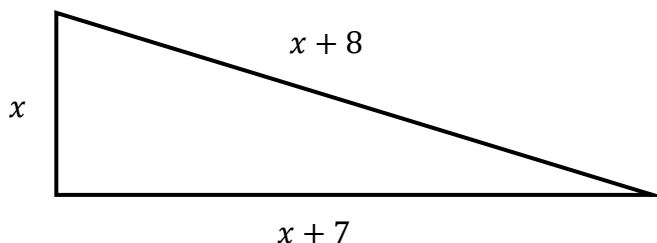
- (a) The perimeter of PQR is 42 centimetres.
Write down an equation in x and y to illustrate this information.
- (b) PR is 2 centimetres longer than QR.
Write down a second equation in x and y to illustrate this information.
- (c) Hence calculate the values of x and y .

1. The minimum number of roads joining 4 towns to each other is 6 as shown.



The minimum number of roads, r , joining n towns to each other is given by the formula $r = \frac{1}{2}n(n - 1)$.

- (a) State the minimum number of roads needed to join 7 towns to each other.
- (b) When $r = 55$, show that $n^2 - n - 110 = 0$.
- (c) Hence find algebraically the value of n .
2. A rectangular garden has a length of $(x + 7)$ metres and a breadth of $(x + 3)$ metres.
- (a) Show that the area, A square metres, of the garden is given by $A = x^2 + 10x + 21$.
- (b) If the area of the garden is 45 square metres, find x .
3. A right angled triangle has dimensions, in centimetres, as shown.



Calculate the value of x .

4. The weight, W kilograms, of a giraffe is related to its age, M months, by the formula

$$W = \frac{1}{4}(M^2 - 4M + 272).$$

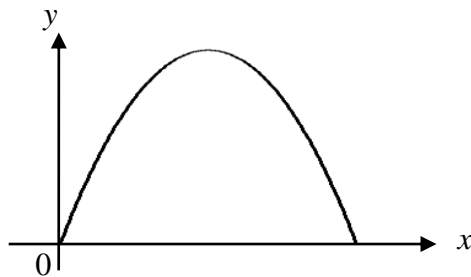
At what age will the giraffe weigh 83 kilograms?

5. The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x)$$

where y is the profit (in pounds) and x is selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x .



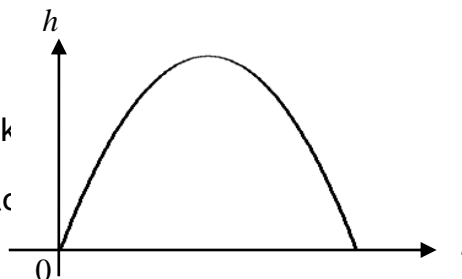
Find the maximum profit the company can make from the sale of the magazine.

6. The diagram below shows the path of a rocket which is fired into the air.

The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$

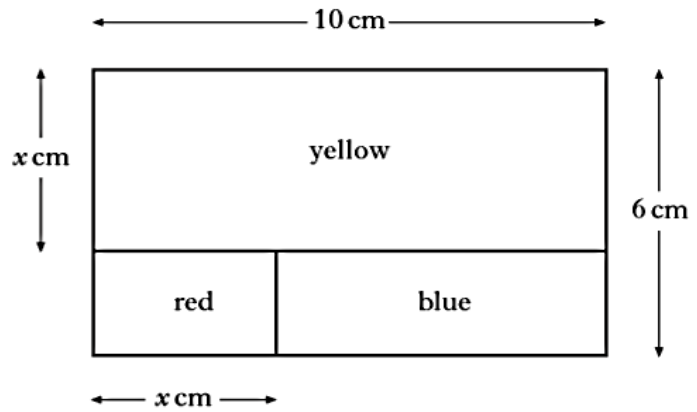
- (a) For how many seconds is the rock
(b) What is the maximum height reac



7. A decorator's logo is rectangular and measures 10 centimetres by 6 centimetres.

It consists of three rectangles: one red, one yellow and one blue.

The yellow rectangle measures 10 centimetres by x centimetres. The width of the red rectangle is x centimetres.



- (a) Show that the area, A , of the blue rectangle is given by the expression

$$A = x^2 - 16x + 60.$$

- (b) The area of the blue rectangle is equal to $\frac{1}{5}$ of the total area of the logo.

Calculate the value of x .

8. Given that $ax^2 + 4x - 2 = 0$ has equal roots. Find a .

9. It is known that the equation $x^2 + bx + 25 = 0$ has 1 root.

Find 2 values for b .

10. The equation $px^2 + 8x - 2 = 0$ has 2 real roots.

Set up an inequality in p , and solve it.

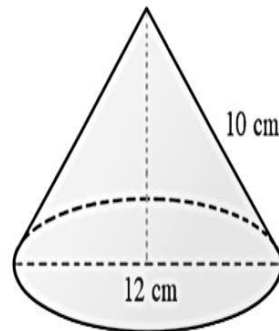
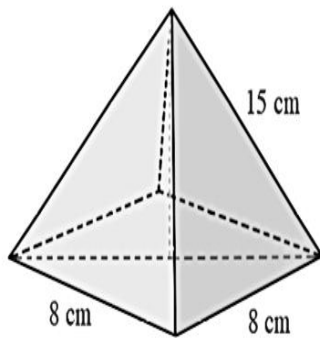
11. Given that $x^2 + x - t = 0$ has no real roots.

Find the value of t .

N5**Homework 7**
(Applying the content of Relationships 1.4)

1. Shown are a square based pyramid and a cone.

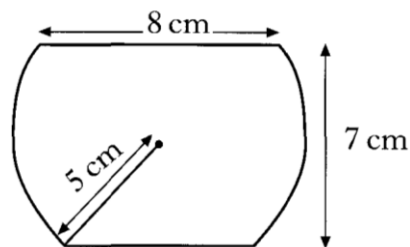
By calculating the height of both, decide which has the greater **volume** and by how much.



2. A badge is made from a circle of radius 5 centimetres.

Segments are taken off the top and the bottom of the circle as shown.

The straight edges are parallel.



The badge measures 7 centimetres from the top to the bottom.

The top is 8 centimetres wide.

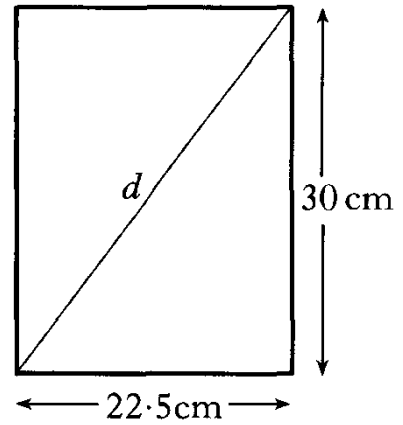
Calculate the width of the base.

3. A rectangular picture frame is to be made.
It is 30 centimetres high and 22.5 centimetres wide, as shown.

To check that the frame is rectangular, the diagonal, d , is measured.

It is 37.3 centimetres long.

Is the frame rectangular?



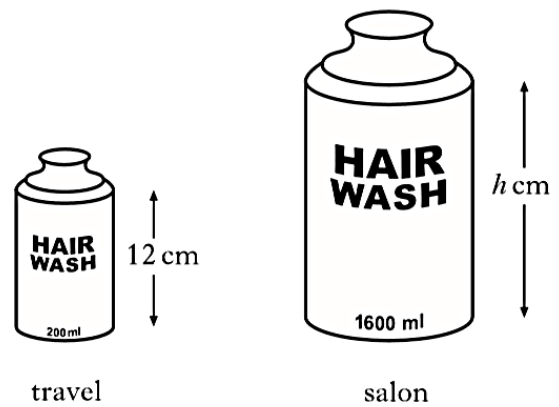
4. Shampoo is available in travel size and salon size bottles.

The bottles are mathematically similar.

The travel size contains 200 millilitres and is 12 centimetres in height.

The salon size contains 1600 millilitres.

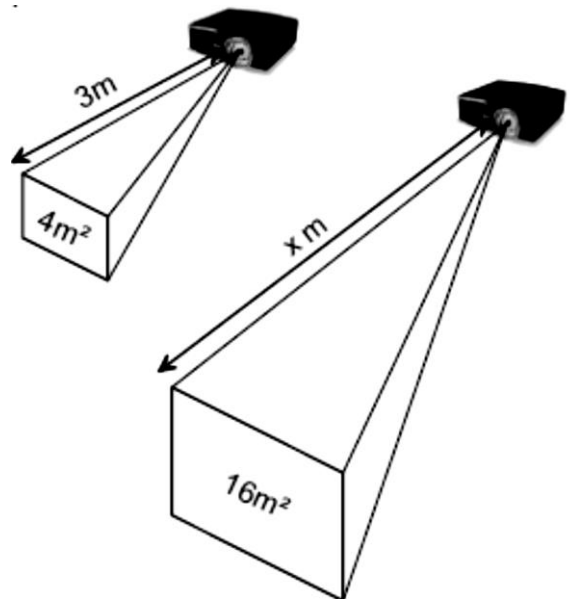
Calculate the height of the salon size bottle.



5. A projector positioned 3 metres from a screen produces a rectangular image of 4 square metres.

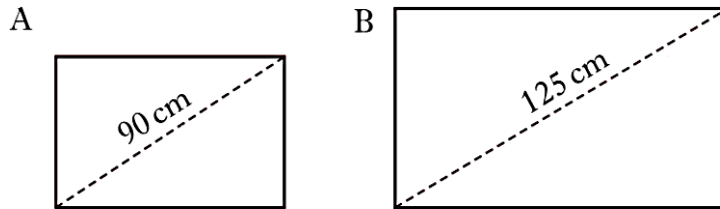
The projector is moved further back, as shown opposite, and the rectangular image now produced is 16 square metres.

Calculate how far the projector is from the image now.



6. Two rectangular solar panels, A and B, are mathematically similar.

Panel A has a diagonal of 90 centimetres and an area of 4020 square centimetres.



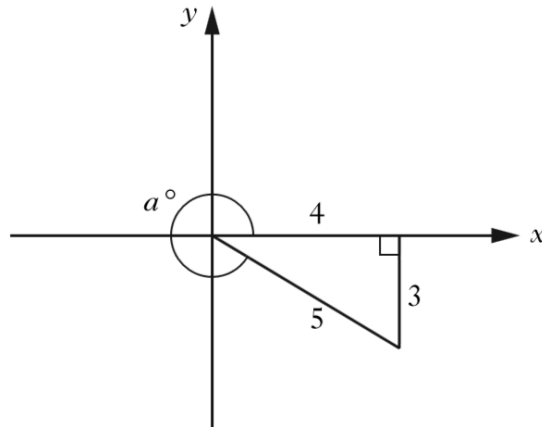
A salesman claims that panel B, with a diagonal of 125 centimetres, will be double the area of panel A.

Is this claim justified?

Show all your working.

N5**Homework 8**
(Applying the content of Trig equations and Graphs)

1.



Write down the value of $\cos a^\circ$.

2. If $f(x) = 3 \sin x^\circ$, $0 \leq x \leq 360$ (a) Find $f(270)$.(b) $f(t) = 0 \cdot 6$.

Find two possible values of t .

3. An angle, a° , can be described by the following statements.

- a° is greater than 0 and less than 360
- $\sin a^\circ$ is negative
- $\cos a^\circ$ is positive
- $\tan a^\circ$ is negative

Write down a possible value for a .

4. The depth of water, D metres, in a harbour is given by the formula

$$D = 3 + 1 \cdot 75 \sin 30h^\circ$$

Where h is the number of hours after midnight.

(a) Calculate the depth of water at 5am.

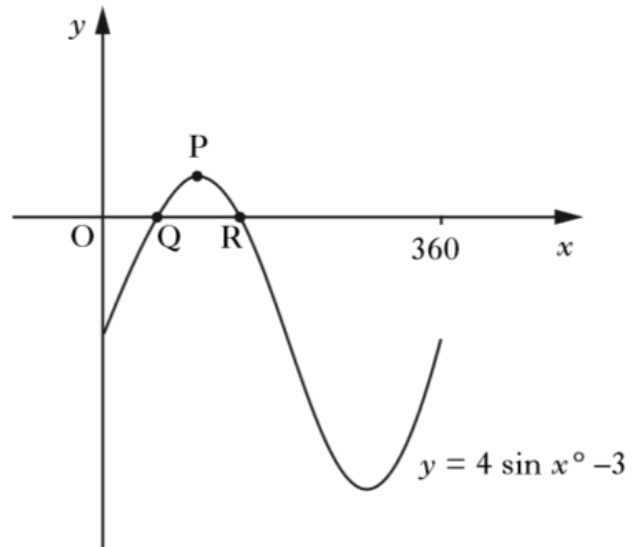
(b) Calculate the maximum difference in depth of water in the harbour.

5. Part of a graph of $y = 4\sin x^\circ - 3$ is shown.

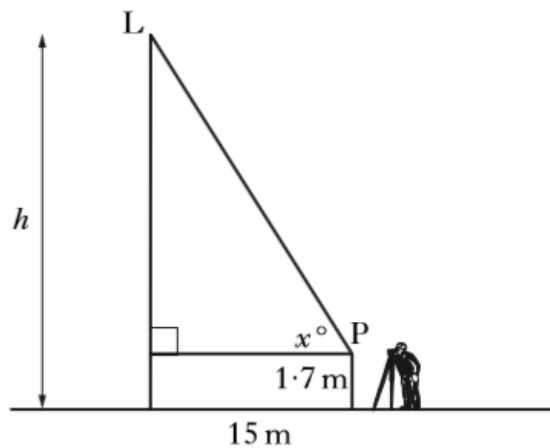
The graph cuts the x -axis at Q and R.

P is the maximum turning point.

- (a) Write down the coordinates of P.
- (b) Calculate the x -coordinates of Q and R.



6. In the diagram below, the point L represents the lift.



The height, h metres, of the lift above the ground is given by the formula

$$h = 15\tan x^\circ + 1.7$$

Where x° is the angle of elevation from the surveyor at point P.

- (a) What is the height of the lift above the ground when the angle of elevation from P is 25° ?
- (b) What is the angle of elevation at the point P when the height of the lift above the ground is 18.4 metres?

Answers

Homework 1

- Q1 Proof Q2 Proof Q3 $\frac{b^2}{4ac}$ Q4 $n = -2$
Q5 - 9 Proof.

Homework 2

- Q1 $12x^2 + 5x - 2$ as required.
Q2 $x + x^{1/2} + x^{-1/2} + 1$
Q3 $x^2 + 9x + 3$ as required.
Q4 (a) $(a - b)(a + b)$ (b) $3 \cdot 2$
Q5 Gillian is correct as Kenny missed the negative outside the second bracket.

Homework 3

- Q1 $S = \frac{y^2}{4x} \text{ km/h}$
Q2 $D = \frac{10mt^3}{3} \text{ metres}$
Q3 $A = \frac{x+6}{x(x+2)}$ as required.
Q4 $\frac{1}{x(2x+1)}$ as required.

Homework 4

1. No it will not pass safety regulations as $0.88\text{m} < 0.9\text{m}$.
2. 8 full glasses can be filled.
3. Yes she can make 430 complete candles as $9000 \div 20.94 = 439$ complete candles can be made.

OR

Yes she can make 430 complete candles as $8987\text{cm}^3 < 9000\text{cm}^3$.

4. The mould will overflow as 311.8cm^3 (mould) $<$ 326.7cm^3 (container)
5. (a) 50.3cm (b) 0.045m^2
(c) Yes the company's claim is true as they will save $\pounds 141.57$ which is greater than $\pounds 140$ for every 100 magazine racks.

Homework 5

- Q1 (a) $280x + 70y = 52 \cdot 50$ (b) $210x + 40y = 38 \cdot 00$
(c) The cost of a call = £0.16 and the cost of a text = £0.11.
- Q2 (a) $24x + 6y = 60$ (b) $20x + 10y = 40$
(c) David's score would be 25 points ($17 \times 3 + 13 \times (-2)$).
- Q3 (4, 5)
- Q4 (a) $2m + c = 7$ (b) $4m + c = 17$
(c) $m = 5, c = -3$
(d) *gradient* = 5, *y* - *intercept* at (0, -3).
- Q5 (a) $B(-6, 0)$ (b) $x < -6$
- Q6 (a) $y = -\frac{3}{2}x + 12$ (b) $P\left(\frac{8}{3}, 8\right)$
- Q7 (a) $D_1 = 2x + 16$ (b) $D_2 = \frac{1}{2}x$
(c) *Jane's running speed* = 12 km/h.
- Q8 $x = \frac{6}{5}$
- Q9 (a) $6x + 2y = 42$ (b) $5x = 2y + 2$
(c) $x = 4, y = 9$

Homework 6

- Q1 (a) $r = 21$ (b) $n^2 - n - 110 = 0$ as required (c) $n = 11$
- Q2 (a) $A = x^2 + 10x + 21$ as required (b) $x = 2$
- Q3 $x = 5$
- Q4 10 months
- Q5 £19 600
- Q6 (a) 14 seconds (b) 98 metres
- Q7 (a) $A = x^2 - 16x + 60$ as required (b) $x = 4$
- Q8 $a = -2$
- Q9 $b = -10$ and $b = 10$
- Q10 $p > -8$
- Q11 $t < -\frac{1}{4}$

Homework 7

- Q1 *Volume of cone is greater by* $(301 \cdot 59 - 296 \cdot 53) = 5 \cdot 06 \text{cm}^3$
- Q2 *width* = 6cm
- Q3 *The frame is not rectangular as* $22 \cdot 5^2 + 30^2 \neq 37 \cdot 3^2$
- Q4 *salon height* = 24cm
- Q5 $x = 6m$ Q6 *No as* $7754 \cdot 6 \neq 8040$

Homework 8

1. $\frac{4}{5}$ 2. (a) -3 (b) $t = 11 \cdot 5$ and $168 \cdot 5$
3. Any answer $270 < a < 360$ 4. (a) 3.875 m (b) 3.50 m
5. (a) P(90, 1) (b) Q(48.6, 0) R(131.4, 0)
6. (a) 8.69 m (b) 48.1°