TRIGONOMETRI

NAMA :
KELAS :

BAB 2 TRIGONOMETRI

Pengukuran Sudut

Ada dua satuan pengukuran sudut yaitu : derajat dan radian

Satuan derajat

Definisi :

$1^0 = \frac{1}{360}\, putaran$

Ingat : 1 putaran = 360°

Jadi : $\frac{1}{2}$ putaran = 180^0

$\frac{1}{4}$ putaran = 90^0

$\frac{3}{4}$ putaran = 270^0

1 putaran = 360^0

$\frac{1}{6}$ putaran = $\frac{1}{6} \times 360^0 = 60^0$

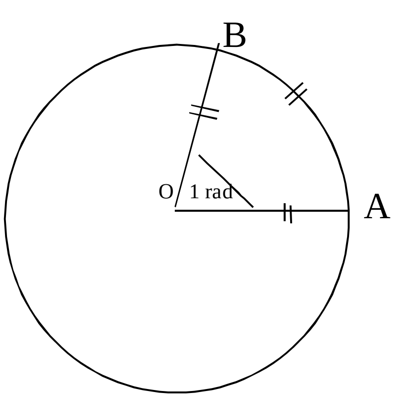
Satuan radian

Definisi:

1 radian : besar sudut pusat lingkaran yang menghadapi busur sepanjang jari-jari

ukuran sudut (dalam radian) dihitung = $\frac{panjang\ busur}{panjang\ \ jari-jari}$ atau

Suatu sudut dikatakan besarnya 1 radian jika: **panjang busurnya = panjang jari-jarinya**



Hubungan derajat dan radian

1 putaran penuh = 360^0

1 putaran penuh menghadapi busur sepanjang $2\pi R$

Jadi 360^0 = 2π radian

180^0 = π radian

$$60^0 = \frac{60^0}{180^0} \times \pi \text{ rad} = \frac{1}{3}\pi \text{ rad}$$

$$1^0 = \frac{1}{180}\pi \text{ rad}$$

$$1 \text{ rad} = \frac{1}{\pi}180^0$$

Contoh :

1. Ubah sudut –sudut ini kedalam satuan radian :

90^0 ; 45^0 ; 30^0 ; 120^0 ; 240^0

Jawab :

$$90^0 = \frac{90^0}{180^0} \times \pi \text{ rad} = \frac{1}{2}\pi \text{ rad}$$

$$45^0 = \frac{45^0}{180^0} \times \pi\ rad = \frac{1}{4}\pi\ rad$$

$$30^0 = \frac{30^0}{180^0} \times \pi\ rad = \frac{1}{6}\pi\ rad$$

$$120^0 = \frac{120^0}{180^0} \times \pi\ rad = \frac{2}{3}\pi\ rad$$

$$240^0 = \frac{240^0}{180^0} \times \pi\ rad = \frac{4}{3}\pi\ rad$$

2. Ubah sudut-sudut ini kedalam satuan derajat:

$\frac{1}{3}\pi$; $\frac{3}{4}\pi$; $\frac{7}{6}\pi$; 2π ;

Jawab :

$$\frac{1}{3}\pi = \frac{1}{3}180^0 = 60^0$$

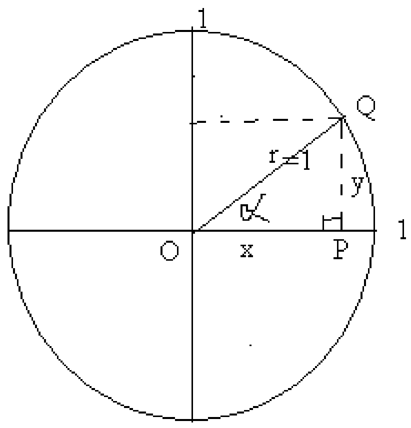
$$\frac{3}{4}\pi = \frac{3}{4}180^0 = 135^0$$

$$\frac{7}{6}\pi = \frac{7}{6}180^0 = 210^0$$

$$2\pi = 2 \times 180^0 = 360^0$$

PERBANDINGAN TRIGONOMETRI SUDUT SUDUT KHUSUS : 0^0 dan 90^0

Untuk menentukan perbandingan trigonometri sudut 0^0 dan 90^0 ,kita menggunakan lingkaran satuan di koordinat Cartesius



Dari gambar kita dapat :

Sin α = ……..

Cos α = ……..

Tan α = ……..

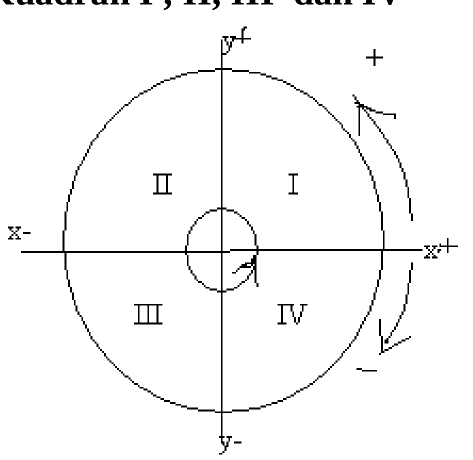
1. Jika sudut 0^0 titik Q berimpit dengan titik P pada sumbu x :
x = 1, y = 0 dan r = 1 sehingga :
sin 0^0 = ………. ; cos 0^0 = …….. dan tan 0^0 = . ……

2. Jika kita gambarkan sudut 90^0 , titik Q di sumbu y , titik P di O
x = …………
., y = …….. dan r = …….. , sehingga
sin 90^0= ………..; cos 90^0 = ………., dan tan 90° = ………

Perbandingan Trigonometri Sudut Berelasi

Sekarang kita akan mempelajari perbandingan trigonometri sudut dikuadran I, I, III, dan IV dan hubungannya satu sama lain.

Kuadran I , II, III dan IV



Besar sudut positip di ukur dari sumbu x^+ **berlawanan arah dengan putaran jarum jam**
Besar sudut negatip diukur dari sumbu x^- **searah putaran jarum jam** .

Sudut dari 0^0 sampai 360^0 dibagi menjadi empat kuadran :

Sudut antara 0^0 dan 90^0 terletak di kuadran I

Sudut antara 90^0dan 180^0 terletak di kuadran II

Sudut antara 180^0dan 270^0 terletak dikuadran III

Sudut antara 270^0dan 360^0 terletak di kuadran IV

Contoh :

Dikuadran manakah sudut-sudut ini ?

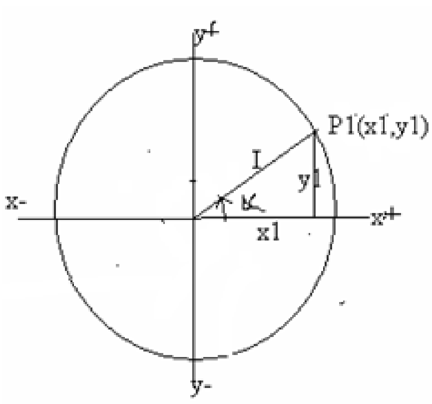
65^0 ; 117^0 ; 326^0 ; 234^0; 95^0; 355^0

Jawab:

65^0 terletak di kuadran	326^0 terletak di kuadran	95^0 terletak di kuadran ......
117^0 terletak di kuadran	234^0 terletak di kuadran	355^0 terletak di kuadran ......

Tanda Perbandingan Trigonometri Sudut di kuadran I , II , III , IV

1. **Sudut di-kuadran I**

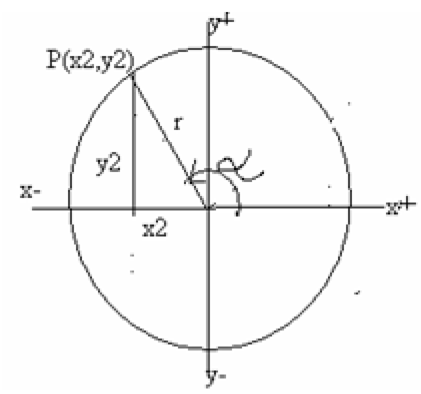


sin α = ...

cos α = ...

tan α = ...

2. **Sudut di kuadran II**

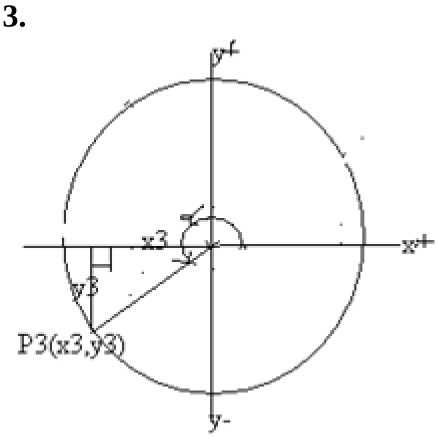


sin α = ...

cos α = ...

tan α = ...

3. **Sudut di kuadran III**

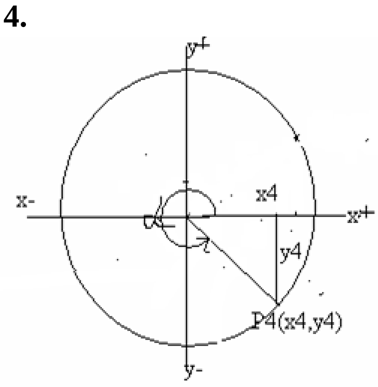


sin α = ...

cos α = ...

tan α = ...

4. **Sudut di kuadran IV**



sin α = ...

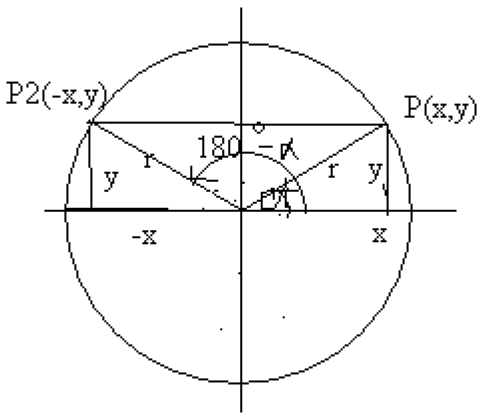
cos α = ...

tan α = ...

Jadi : tanda perbandingan trigonometri di kuadran adalah sbb (hafalkan !) :
Di kuadran I : semua (perbandingan trigonometri) positif
Di kuadran II: hanya sin (dan cosec) positif
Di kuadran III : hanya tan (dan cotan) positif
Di kuadran IV : hanya cos (dan sec) positif

Sudut Berelasi (1)

1. α dan (180° - α)

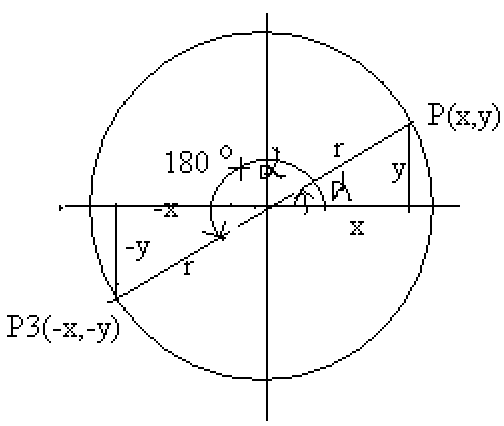


sin (180° - α) = ...

cos (180° - α) = ...

tan (180° - α) = ...

2. α dan (180° + α)

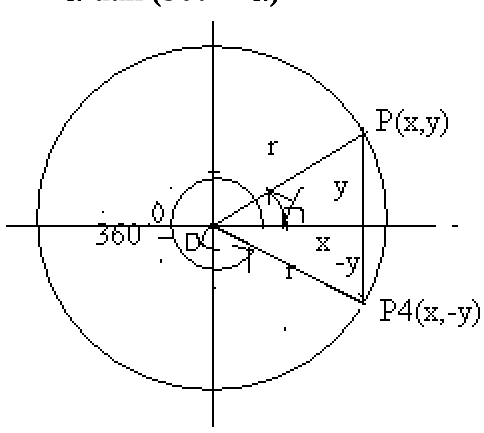


sin (180° + α) = ...

cos (180° + α) = ...

tan (180° + α) = ...

3. α dan (360° - α)



sin (360° - α) = ...

cos (360° - α) = ...

tan (360° - α) = ...

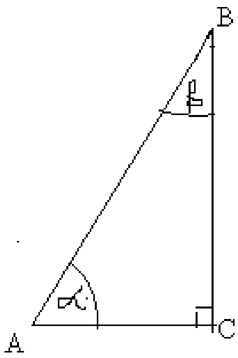
Contoh :

Tentukan nilai : sin 225^0, sin 120^0, sin 330^0, sin 203^0, cos 95^0, cos 240^0, cos 350^0, tan 135^0, tan 187^0, tan 300^0!

Jawab :

Sudut Berelasi (2)

1. **α dan ($90° - \alpha$)**

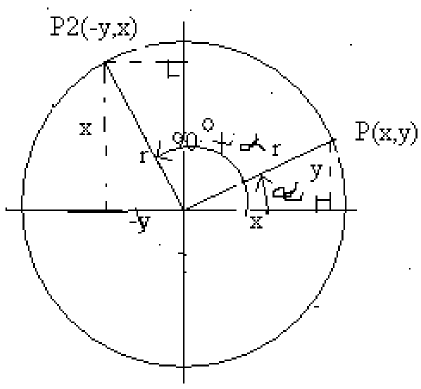


sin α = ...

cos α = ...

tan α = ...

2. **α dan ($90° + \alpha$)**

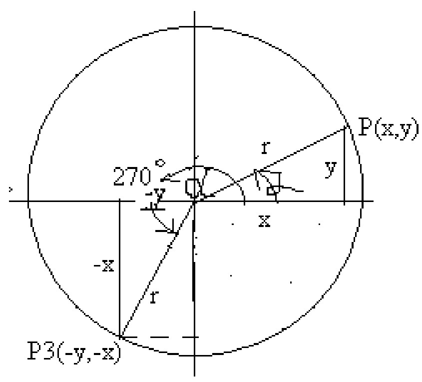


sin ($90° + \alpha$) = ...

cos ($90° + \alpha$) = ...

tan ($90° + \alpha$) = ...

3. **α dan ($270° - \alpha$)**

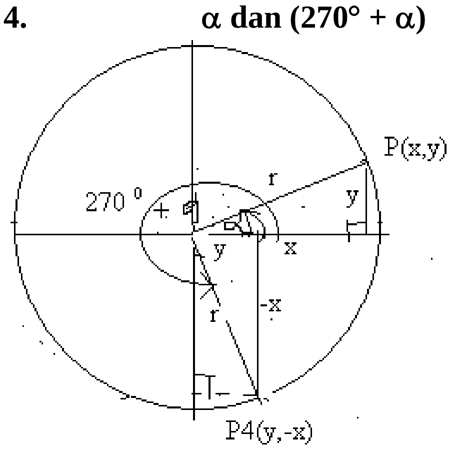


sin ($270° - \alpha$) = ...

cos ($270° - \alpha$) = ...

tan ($270° - \alpha$) = ...

4. **α dan ($270° + \alpha$)**

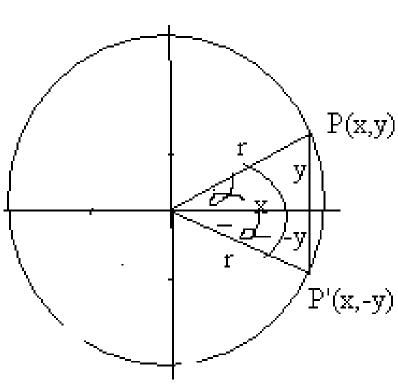


sin ($270° + \alpha$) = ...

cos ($270° + \alpha$) = ...

tan ($270° + \alpha$) = ...

5. **α dan $-\alpha$**



sin ($-\alpha$) = ...

cos ($-\alpha$) = ...

tan ($-\alpha$) = ...

6. α **dan** $\alpha \pm k.360^0$

Sudut $\alpha + k.360^0$ gambarnya sama dengan gambar sudut α

Maka:

$\sin(\alpha + k.360^0) = \sin\alpha$

$\cos(\alpha + k.360^0) = \cos\alpha$

$\tan(\alpha + k.360^0) = \tan\alpha$

Contoh:

1. Ubah perbandingan trigonometri ini menjadi perbandingan trigonometri sudut lancip, hitunglah tan 120^0 , cos 280^0 , cos 240^0 , cos $(-135)^0$, sin $(-45)^0$, tan 390^0!

Jawab :

Rangkuman

	Dalam Derajat	Dalam Radian
1.	$\sin(180^0-\alpha)=\sin\alpha$ $\cos(180^0-\alpha)=-\cos\alpha$ $\tan(180^0-\alpha)=-\tan\alpha$	$\sin(\pi-\theta)=\sin\theta$ $\cos(\pi-\theta)=-\cos\theta$ $\tan(\pi-\theta)=-\tan\theta$
2.	$\sin(180^0+\alpha)=-\sin x$ $\cos(180^0+\alpha)=-\cos\alpha$ $\tan(180^0+\alpha)=\tan\alpha$	$\sin(\pi+\theta)=-\sin\theta$ $\cos(\pi+\theta)=-\cos\theta$ $\tan(\pi+\theta)=\tan\theta$
3.	$\sin(360^0-\alpha)=-\sin\alpha$ $\cos(360^0-\alpha)=\cos\alpha$ $\tan(360^0-\alpha)=-\tan x$	$\sin(2\pi-\theta)=-\sin\theta$ $\cos(2\pi-\theta)=\cos\theta$ $\tan(2\pi-\theta)=-\tan\theta$
4.	$\sin\alpha=\cos(90^0-\alpha)$ $\cos\alpha=\sin(90^0-\alpha)$ $\tan\alpha=\cot(90^0-\alpha)$	$\sin(\frac{1}{2}\pi-\theta)=\cos\theta$ $\cos(\frac{1}{2}\pi-\theta)=\sin\theta$ $\tan(\frac{1}{2}\pi-\theta)=\cot\theta$
5.	$\sin(90^0+\alpha)=\cos\alpha$ $\cos(90^0+\alpha)=-\sin\alpha$ $\tan(90^0+\alpha)=-\cot\alpha$	$\sin(\frac{1}{2}\pi+\theta)=\cos\theta$ $\cos(\frac{1}{2}\pi+\theta)=\sin\theta$ $\tan(\frac{1}{2}\pi+\theta)=\cot\theta$
6.	$\sin(270^0+\alpha)=-\cos\alpha$ $\cos(270^0+\alpha)=+\sin\alpha$ $\tan(270^0+\alpha)=-\cot\alpha$	$\sin(\frac{3}{2}\pi+\theta)=-\cos\theta$ $\cos(\frac{3}{2}\pi+\theta)=\sin\theta$ $\tan(\frac{3}{2}\pi+\theta)=-\cot\theta$
7.	$\sin(-\alpha)=-\sin\alpha$ $\cos(-\alpha)=\cos\alpha$ $\tan(-\alpha)=-\tan\alpha$	-
8.	$\sin(\alpha+k.360^0)=\sin\alpha$ $\cos(\alpha+k.360^0)=\cos\alpha$ $\tan(\alpha+k.360^0)=\tan\alpha$	$\sin(\theta+k.2\pi)=\sin\theta$ $\cos(\theta+k.2\pi)=\cos\theta$ $\tan(\theta+k.2\pi)=\tan\theta$

Kesimpulan :

1. fungsi trigonometri $180^0 \pm\alpha$ } nama tetap
   $360^0\pm\alpha$ } tanda sesuai kuadran

2. fungsi trigonometri $90^0\pm\alpha$ } nama berubah
   $270^0\pm\alpha$ } tanda sesuai kuadran

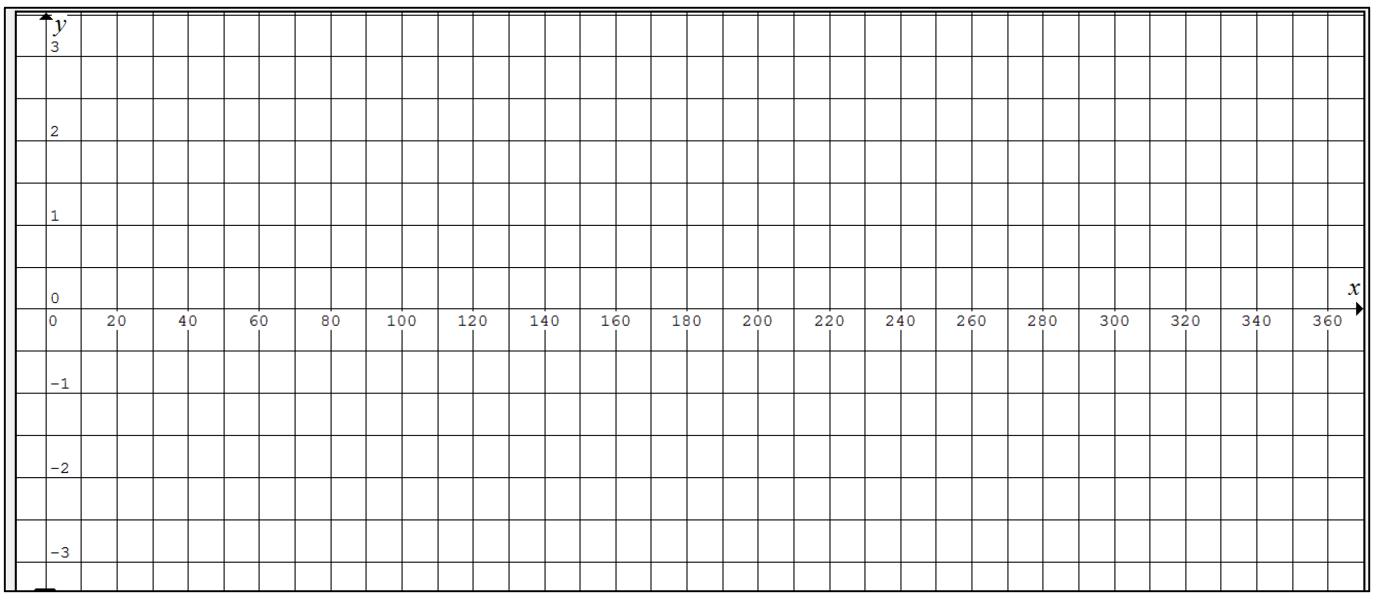
GRAFIK FUNGSI TRIGONOMETRI

Grafik fungsi trigonometri $f(x) = \sin x^0$, $f(x) = \cos x^0$ dan $f(x) = \tan x^0$ didalam domain $0 \le x \le 360$ dapat digambarkan ,dengan membuat daftar terlebih dahulu .

1. Grafik $y = f(x) = \sin x^0$, $0 \le x \le 360$

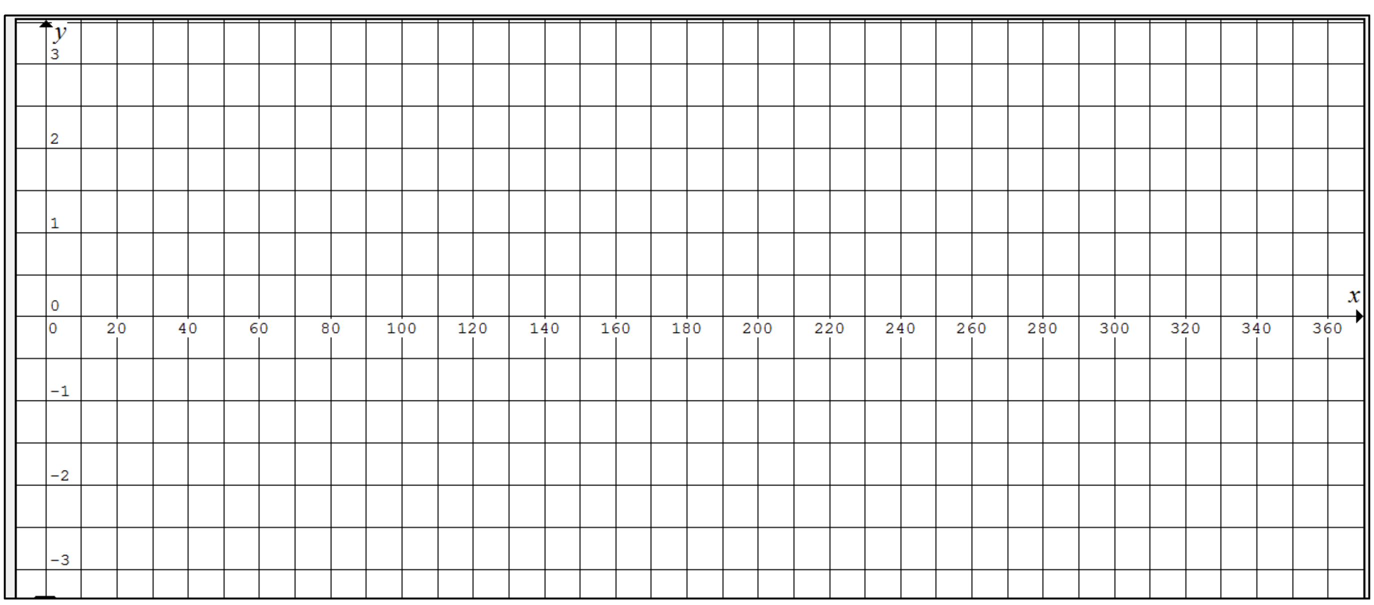
x	0	30	60	90	120	150	180	210	240	270	300	330	360
y = f(x) $= \sin x^0$													

Catatan: $\frac{\sqrt{3}}{2} = 0,8660$



2. Grafik $y = f(x) = \cos x^0$, $0 \le x \le 360$

x	0	30	60	90	120	150	180	210	240	270	300	330	360
y = f(x) $= \cos x^0$													

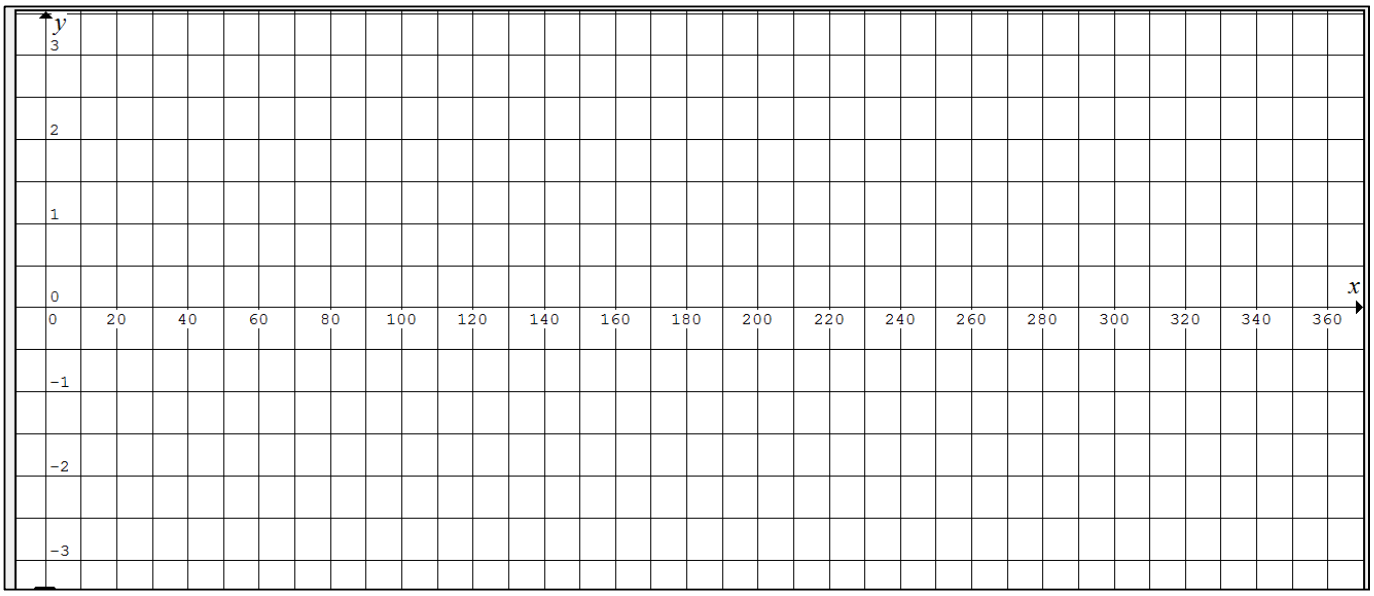


3. Grafik $y = f(x) = \tan x^0$, $0 \le x \le 360$

x	**0**	**45**	**30**	**60**	**90**	**120**	**135**	**150**	**180**	**210**	**215**	**240**	**270**	**300**	**315**	**330**	**360**
y = f(x) $= \tan x^0$																	

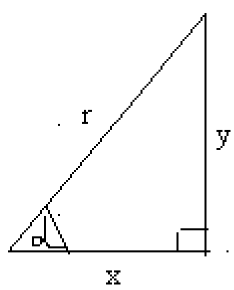
Catatan: $\sqrt{3} = 1{,}732$

$\frac{\sqrt{3}}{3} = 0{,}577$



IDENTITAS TRIGONOMETRI

HUBUNGAN ANTAR PERBANDINGAN TRIGONOMETRI SUATU SUDUT



$$r^2 = \sqrt{x^2 + y^2}$$

$$\sin \alpha = \frac{y}{r} \rightarrow \sin^2 \alpha = \frac{y^2}{r^2}$$

$$\cos \alpha = \frac{x}{r} \rightarrow \cos^2 \alpha = \frac{x^2}{r^2}$$

Identitas trigonometri:

1. $\sin^2 \alpha + \cos^2 \alpha = 1$

Bukti:

$$\sin^2 \alpha + \cos^2 \alpha = \frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{y^2 + x^2}{r^2} = \frac{r^2}{r^2} = 1$$

2. $1 + \tan^2 \alpha = \sec^2 \alpha$

Bukti:

$$1 + \tan^2 \alpha = 1 + \frac{y^2}{x^2} = \frac{x^2 + y^2}{x^2} = \frac{r^2}{x^2} = \sec^2 \alpha$$

3. $1 + \cot^2 \alpha = \cos ec^2 \alpha$

Bukti:

$$1 + \cot^2 \alpha = 1 + \frac{x^2}{y^2} = \frac{y^2 + x^2}{y^2} = \frac{r^2}{y^2} = \cos ec^2 \alpha$$

4. $\cot \alpha = \frac{1}{\tan \alpha}$
5. $\sec \alpha = \frac{1}{\cos \alpha}$
6. $\cos ec \alpha = \frac{1}{\sin \alpha}$
7. $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$
8. $\cot \alpha = \frac{\cos \alpha}{\sin \alpha}$

Membuktikan Identitas Trigonometri

Dalam membuktikan identitas Trigonometri terdapat beberapa cara yang dilakukan ,yaitu

1.Menyederhanakan ruas kiri menjadi seperti ruas kanan
2.Menyederhanakan ruas kanan supaya seperti ruas kiri
3.Menyederhanakan kedua ruas unutk memperoleh bentuk yang sama

Pemilihan cara mana yang akan dilakukan tergantung dari kerumitan soal.

Latihan
Buktikan identitas trigonometri berikut:

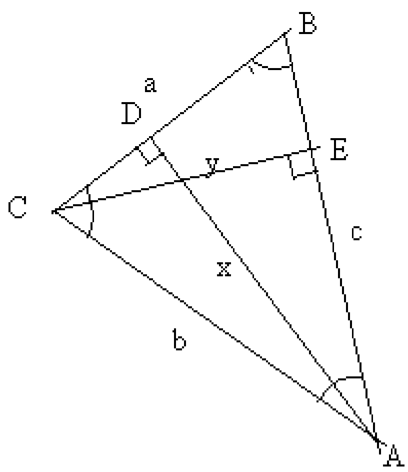
1. $\sin\theta\cos\theta\tan\theta = (1-\cos\theta)(1+\cos\theta)$
2. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$
3. $\dfrac{1}{\cos x\sin x} - \dfrac{\cos x}{\sin x} = \tan x$
4. $\dfrac{\sin x}{1+\cos x} = \dfrac{\sec x - 1}{\tan x}$
5. $\sin\theta\tan\theta + \cos\theta = \sec\theta$
6. $(1 + \sin x)^2 – (1 - \sin x)^2 = 4 \sin x$
7. $\dfrac{\tan A + \cot B}{\cot A + \tan B} = \tan A\cot B$

Jawab:

ATURAN SINUS,ATURAN COSINUS DAN RUMUS LUAS SEGITIGA

Pada subbab ini kita akan membahas segitiga sembarang (bukan segitiga siku-siku). Sebuah segitiga tertentu apabila 3 unsurnya diketahui (asalkan bukan sudut ketiga-tiganya) dapat dicari unsur lain yang belum diketahui. Untuk mencari unsur yang lain, dapat menggunakan salah satu dari dua rumus ini : aturan Sinus, aturan Cosinus

1. Aturan Sinus



Perhatikan segitiga ABC berikut :

$$\sin B = \frac{y}{a} \Rightarrow y = a \sin B \text{........1)}$$

$$\sin A = \frac{y}{b} \Rightarrow y = b \sin A \text{.........2)}$$

$$a \sin B = b \sin A \Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin C = \frac{x}{b} \Rightarrow x = b \sin C \text{.......3)}$$

$$\sin B = \frac{x}{c} \Rightarrow x = c \sin B \text{.........4)}$$

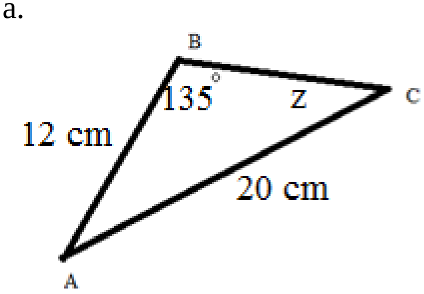
$$b \sin C = c \sin B \qquad \frac{b}{\sin B} = \frac{c}{\sin C}$$

Jadi : $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

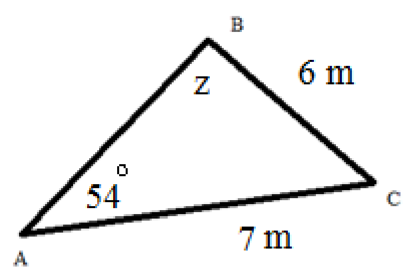
Contoh:

Tentukan besar **z** pada gambar berikut!

a.



b.



Jawab:

2. LUAS SEGITIGA

Luas segitiga ABC $= \dfrac{a \times x}{2} = \dfrac{1}{2}(a \times b \sin C) = \dfrac{1}{2} ab \sin C$

Dengan cara sama dapat dibuktikan bahwa :

Luas $\Delta ABC = \dfrac{1}{2} ab \sin C$ **(jika yang diketahui ss,sd,ss)** $= \dfrac{1}{2} ac \sin B$ $= \dfrac{1}{2} bc \sin A$

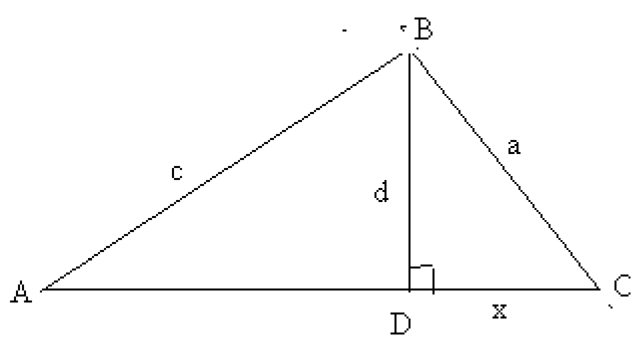
Jika yang diketahui sd.ss.sd

$$L\Delta ABC = \frac{1}{2} ab \sin C = \frac{1}{2} a . \frac{a \sin B}{\sin A} . \sin C = \frac{a^2 \sin B \sin C}{2 \sin A}$$

Dengan cara sama diperoleh :

$L\Delta ABC = \dfrac{a^2 \sin B \sin C}{2 \sin A}$ $L\Delta ABC = \dfrac{b^2 \sin A \sin C}{2 \sin B}$ $L\Delta ABC = \dfrac{c^2 \sin A \sin B}{2 \sin C}$

Jika yang diketahui ss, ss,ss



$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$a \cos C = x$$

$$c^2 = a^2 + b^2 - 2bx$$
$$2bx = a^2 + b^2 - c^2$$
$$2b\sqrt{a^2 - d^2} = a^2 + b^2 - c^2$$
$$4b^2(a^2 - d^2) = (a^2 + b^2 - c^2)^2$$

$$
\begin{aligned}
4b^2a^2 - 4b^2d^2 &= (a^2 + b^2 - c^2)^2 \\
4b^2d^2 &= 4b^2a^2 - (a^2 + b^2 - c^2)^2 \\
&= (2ab + a^2 + b^2 - c^2)(2ab - a^2 - b^2 + c^2) \\
&= \{(a+b)^2 - c^2\}\{c^2 - (a^2 + b^2 - 2ab)\} \\
&= \{(a+b)^2 - c^2\}\{c^2 - (a-b)^2\} \\
&= (a+b-c)(a+b+c)(c+a-b)(c-a+b)
\end{aligned}
$$

$a + b + c = keliling\Delta = 2s$

$a + b - c = a + b + c - 2c = 2s - 2c$

$a + c - b = 2s - 2b$

$b + c - a = 2s - 2a$

$4b^2d^2 = 2s.2(s-a).2(s-b).2(s-c)$

$b^2d^2 = 4s(s-a)(s-b)(s-c)$

$b^2 = \frac{4s(s-a)(s-b)(s-c)}{d^2}$

$b = \frac{2}{d}\sqrt{s(s-a)(s-b)(s-c)}$

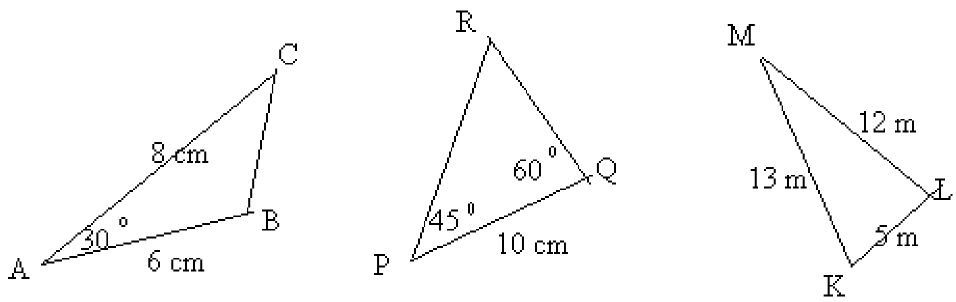
$\frac{1}{2}b.d = \sqrt{s(s-a)(s-b)(s-c)}$

$$\boxed{Luas\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}}$$

Catatan: s = $\frac{1}{2}$ keliling segitiga = $\frac{1}{2}$ (a + b + c)

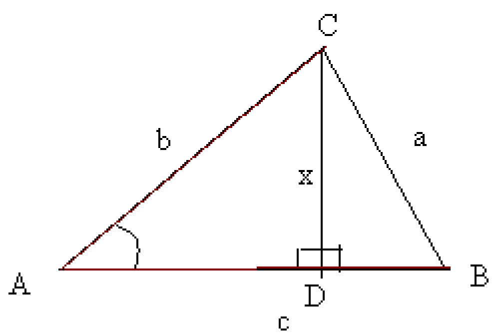
Contoh :

1. Hitung luas segitiga yang tergambar sbb:



Jawab :

3. Aturan Cosinus



Perhatikan gambar!
Pada segitiga ABC ,
Diketahui AB , AC dan sudut A

Berapa panjang sisi BC ?

$x^2 = b^2 - AD^2$

$x^2 = b^2 - (b\cos A)^2 = b^2 - b^2\cos^2 A$

$BD^2 = (c - b\cos A)^2 = c^2 - 2bc\cos A + b^2\cos^2 A$

$a^2 = \quad CD^2 + BD^2 = x^2 + BD^2 = b^2 - b^2\cos^2 A + c^2 - 2bc\cos A + b^2\cos^2 A$

$a^2 = b^2 + c^2 - 2bc\cos A$ (Aturan Kosinus)

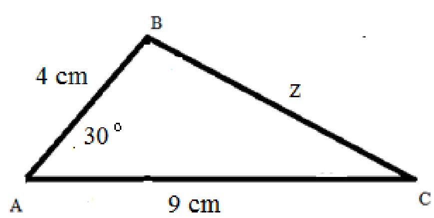
Dengan cara yang sama didapat ;

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

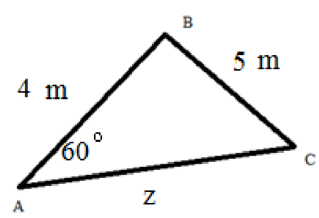
Contoh :

1. Tentukan besar z, kemudian tentukan luasnya!

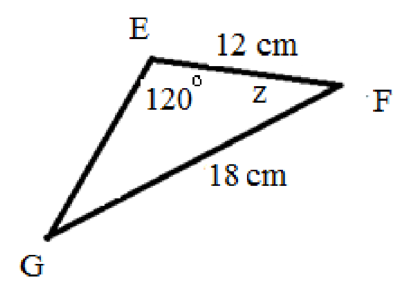
a.



b.



c.



2. Tentukan besar sudut Q dan tentukan luasnya!