Name______________________
Date__________Period______

AP Calculus AB MID-YEAR REVIEW
Review Packet for Mid-Year Exam

PART I: Multiple Choice- No Calculators. Show all work on a separate sheet of paper.

1. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to *f(x)* at x =2 is
(a) $y - 2 = 45(x - 25)$
(b) $y - 25 = -\frac{1}{45}(x - 2)$
(c) $y - 25 = 45(x - 2)$
(d) $y - 25 = (12x^2 - 3)(x - 2)$

2. An equation of the normal line to the graph of $f(x) = \frac{x}{2x-3}$ $at\ (1, f(1))\ is$

(a) $y - 1 = \frac{1}{3}(x + 1)$
(b) $y + 1 = -3(x - 1)$
(c) $y + 1 = \frac{(2x-3)^2}{-3}(x - 1)$
(d) $y + 1 = \frac{1}{3}(x - 1)$
(e) $y + 1 = 3(x - 1)$

3. Find the derivative of $y - 2 = ln\frac{\frac{1}{2}y}{x^2}$ at (1,2) .
(a) -4
(b) -2
(c) $\frac{1}{2}$
(d) 2

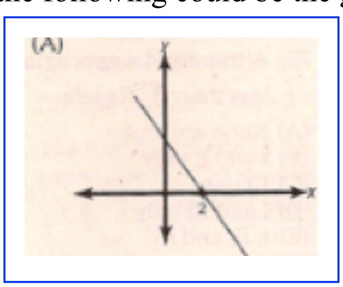
4. Find $\lim\limits_{x\to 1}\frac{x-1}{x^2-1}$
(a) 0
(b) ½
(c) 1
(d) ∞
(e) The limit does not exist

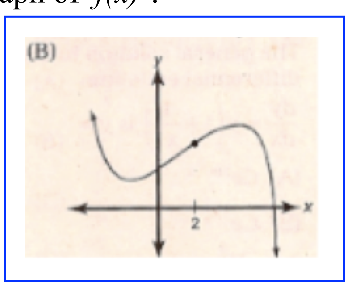
5. The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is
(a) 0
(b) 2
(c) 6
(d) -2

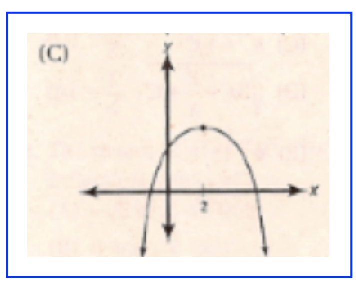
6. If $h(x) = \left(x^2-4\right)^{\frac{3}{4}} + 1$, then the value of $h'(2)$ is
(a) 3
(b) 2
(c) 1
(d) 0
(e) does not exist

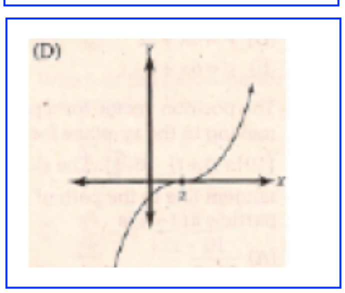
7. An equation for a tangent to the curve $f(x) = e^{\sin x} + x$ at $x = 0$ is
(a) $y = 2x - 1$
(b) $y = 2x + 1$
(c) $y = 2x$
(d) $y = 1$
(e) $y = 0$

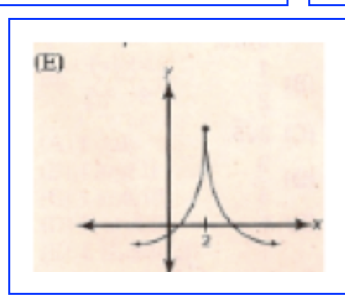
8. If *f(x)* is a function such that $f'(x)$ is increasing for x < 2 and $f'(x)$ is decreasing for x > 2, then which of the following could be the graph of *f(x)* ?











9.What is $\lim_{x\to\infty} \frac{x^2-6}{2+x-3x^2}$?

(a) -3 (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) 2 (e) Does not exist

10. Find $\frac{dy}{dx}$ at (1, π/4) if $x\sec y = \ln x$

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1-\sqrt{2}}{\sqrt{2}}$ (e) does not exist

11. Find $f'(1)$ if $f(x) = \ln\left(\frac{1}{1-3x^2}\right)$.

(a) 1 (b) -1 (c) -3 (d) 3 (e) DNE

12. An equation for a tangent to the curve $y = \tan\frac{x}{3}$ at the origin is

(a) $y = \frac{1}{3}x$ (b) x = 0 (c) y = 0 (d) y = -3x (e) $y = \sec^2 x\,(\frac{1}{3}\,x)$

13. Estimate *f(1.02)* by using the linearization of $f(x) = -\frac{2}{x^4}$ at x = 1.

(a) -1.84 (b) -2.16 (c) 0.02 (d) 1.84 (e) 2.16

14. The function $f(x) = x^4 - 4x^2$ has

(a) one relative minimum and two relative maxima
(b) one relative minimum and one relative maximum
(c) two relative minima and no relative maximum
(d) two relative maxima and no relative minimum
(e) two relative minima and one relative maximum

15. Use the table below to answer the question that follows.

x	-2	-1	0	1	2
$f'(x)$	-10	0	0	-8	-15
$f''(x)$	-1	5	-2	-3	1

Given the differentiable curve $y = f(x)$ and the table of values above, the relative minimum occurs at x =

(a) -1 (b) 0 (c) 1 (d) 2 (e) no relative minimum

16. $\lim_{h\to 0} \frac{\tan(\frac{\pi}{4}+h)-1}{h} =$

(a) 0 (b) 2 (c) $\frac{\sqrt{2}}{2}$ (d) 4 (e) DNE

17. Find $\frac{dy}{dx}$ if $y = \sin^2 3x + \cos^2 2x$.

(a) 0 (b) 6sin3xcos3x – 4sin2xcos2x (c) 1 (d) 6cos3x – 4 sin2x (e) $3\cos^2 3x - 2\sin^2 2x$

18. What is the total number of relative maximum and minimum points of $f(x)$ if the derivative is given as $f'(x) = x(x-3)^2(x+1)^4$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) none of these

19. Let $f(x) = \begin{cases} x^2 - 1 & if \quad x \neq 1 \\ 4 & if \quad x = 1 \end{cases}$. Which of the following are true?

I. $\lim_{x \to 1} f(x)$ exists

II. $f(1)$ exists

III. f is continuous at x = 1

(a) I only (b) II only (c) I and II (d) none of them (e) I, II, and III

20. Find the slope on the curve at $t = 1$ if $x = t^2 - 1$ and $y = t^4 - 2t^3$.

(a) 1 (b) -1 (c) 0 (d) 3 (e) ½

21. Find the derivative of $f(x) = e^{-\tan x + \cos x}$ at x = 0.

(a) e (b) $-e$ (c) 1 (d) -1 (e) does not exist

22. Find the derivative of $f(x) = \ln(1 - e^x)$ at x = *ln2*.

(a) 0 (b) 2 (c) -2 (d) 1 (e) does not exist

In Questions 23-24, the position of a particle on a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

23. The acceleration is positive for

(a) t > 2 (b) for all t, t ≠ 2 (c) t < 2 (d) 1 < t < 3 (e) 1 < t < 2

24. The particle is moving to the left

(a) for t < 2 (b) for t > 2 (c) for all t, t ≠ 2 (d) 0 < t < 3 (e) no where

25. Which statement below is true about the curve $y = \dfrac{x^2 + 4}{2 + 7x - 4x^2}$?

(a) The line x = -1/4 is a vertical asymptote (b) The line x = 1 is a vertical asymptote

(c) The line y = ¼ is a horizontal asymptote (d) The line y = 2 is a horizontal asymptote

26. Find all values of *c*, if any, such that the rate of change is the same as the average rate of change on the curve $f(x) = \dfrac{x+1}{x}$ for t $t \in [\ \dfrac{1}{2}\ , 2\ ]$.

(a) 1 (b) $\dfrac{3}{2}$ (c) 2 (d) 0 (e) none of these

27. If $h''(x) = e^x(2x-1)^2(x-3)^3(4x+5)$ then $h(x)$ has how many points of inflection?

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

28. The slope of the curve $y^2 - xy - 3x = 1$ at the point (0, -1) is

(a) -1 (b) -2 (c) 1 (d) 2 (e) -3

29. Let $f(x)=\begin{cases} \dfrac{x^2-x}{2x} & for\ \ x\neq 0 \\ k & for\ \ x=0 \end{cases}$ If f is continuous at x = 0, then k =

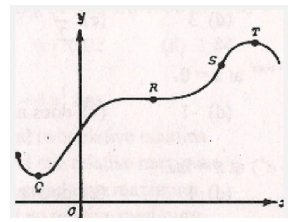
(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) DNE

30. $\lim_{x\to 0}\frac{\sin(2x)}{x}$ =
(a) 0 (b) 1 (c) 2 (d) -2 (e) DNE

31. $\lim_{x\to\infty}\frac{30x^3-2x+1}{8-15x^3+x^4}=$

(a) 0 (b) -2 (c) 30/8 (d) ∞ (e) DNE

32. At which point on the graph shown do both $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ equal zero?
(a) P (b) Q (c) R (d) S (e) T

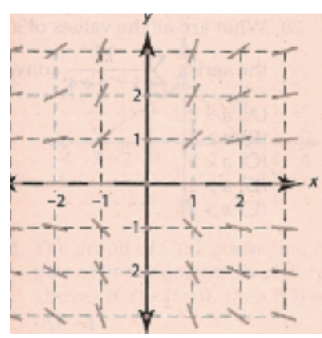


33. $\lim_{x\to 0}\frac{|x|}{x}=$
(a) 1 (b) -1 (c) 0 (d) DNE

34. Find $h'(2)$ if $h(x) = g(f(3x) - 6x$, and the values of f(x) and g(x) are provided in the table:
(a) -9 (b) -7 (c) -3
(d) 0 (e) 3

x	*g(x)*	*g'(x)*	*f(x)*	*f'(x)*
2	3	-1	-4	-5
3	5	4	1	-2
6	1/2	-1/2	2	-3

35. The slope field represents an approximation to the general solution to which differential equation?
(a) $\frac{dy}{dx}=\frac{y}{x}$ (b) $\frac{dy}{dx}=\frac{x}{y^2}$
(c) $\frac{dy}{dx}=\frac{y}{x^2}$ (d) $\frac{dy}{dx}=\frac{y^3}{x}$
(e) $\frac{dy}{dx}=\frac{y^2}{x^2}$



36. Let $f(x)=x^2e^x$ on the interval $-10\le x\le 0$. The absolute maximum of *f(x)* on the interval is
(a) $\frac{100}{e^{10}}$ (b) $\frac{4}{e^2}$ (c) 1 (d) e (e) $2e$

37. If $f(x)=\ln x$ on the interval $1\le x\le e$, then what is the value of *c* on the interval $1 < x < e$ that satisfies the Mean Value Theorem?
(a) $\frac{1}{e-1}$ (b) $\frac{e}{e-1}$ (c) $\frac{1+e}{2}$ (d) $e-1$ (e) $e^{\frac{1}{e-1}}$

38. Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ when $y = 2\cos\theta$ and $x = 4\sin\theta$
(a) 1/2 (b) -1/2 (c) 1 (d) -1 (e) DNE

39. Find any t-values where horizontal tangents exist on t ∈ [0, π/2] for the set of parametric equations: $x(t) = \cos t^2$ *and* $y(t) = \sin 2t$

(a) t = 0, $\sqrt{\pi}$ (b) π/2 (c) π (d) 0, $\sqrt{\pi}$, π/4 (e) π/4

40. Find $\frac{dy}{dx}$ when x = $\frac{e}{2}$ and y = 0: $e^{2xy} = \frac{1}{\ln 2x}$

(a) -2 (b) 2 (c) $-\frac{2}{e}$ (d) $-2e$ (e) $-\frac{2}{e^2}$

Part 2: No Calculators. Show all work to justify each answer.

41. Graph the function $f(x)= \begin{cases} x & if\ 0 \le x \le 1 \\ 2-x & if\ 1 < x \le 2 \end{cases}$

Is *f* continuous at x = 1? Does *f* have a derivative at x = 1? Justify your answers. (*graph on last sheet of packet)*

42. Let $f(x) = |\ 2x-1\ | + 3x$
   (a) Define the function as a piece-wise function. Then graph it. (*graph on last sheet of packet)*
   (b) Is the function continuous and differentiable for x ∈ $\Re$? Justify both answers.
   (c) Is the function even, odd or neither? Explain.
   (d) Find $f'(0)$
   (e) Find the range of $f(x)$. Write your answer in interval notation.

43. Sketch each without the use of a graphing calculator. (*graph on last sheet of packet)*
   (a) $x = y^2$
   (b) $y = \ln x$
   (c) $y = e^x$

Part 3: Multiple Choice-Calculator Section. Show work on a separate sheet of paper.

44. Let f and g be the functions given by $f(x) = e^x$ *and* $g(x) = \ln x$. Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value, *m*, and absolute maximum value, *M*, of *h(x)* on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

(a) $m\ value = .567,\ M\ value = 1$
(b) $m\ value = 2.33,\ M\ value = 2.34$
(c) $m\ value = 2.33,\ M\ value = 2.72$
(d) $m\ value = .567,\ M\ value = .5$
(e) $m\ value = 2.33,\ no\ M\ value$

45. Find the limit: $\lim_{x \to 3^-} \frac{x^3 - 2x + 6}{5x - 15}$

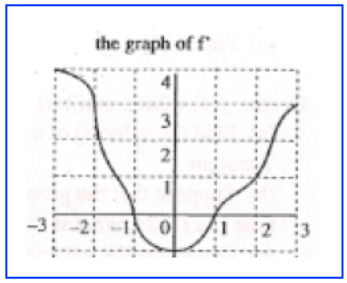
(a) 0 (b) ∞ (c) -∞ (d) DNE (e) $\frac{1}{5}$

46. The graph of $f'(x)$, the derivative of a function f, is shown. Which of the following statements are true about $f(x)$?

I. f is increasing on the interval (-2, -1)

II. f has an inflection point at x = 0

III. f is concave up on the interval (-1, 0)



(a) I only (b) II only (c) III only (d) I and II (e) II and III

47. $f(x)=x^5-2x^4+x^3-1$ and $g(x)=2x^3-8x+1$ have the same slope at one point. Find the x- value of the point.

(a) 0.253 (b) 1.690 (c) 1.840 (d) 1.570 (e) 1.910

48. Find the time the particle is at rest if the position of the particle is given by the set of parametric equations:

$x(t)=\frac{1}{2}t^2+\frac{1}{2}\ln 2t-2t$ and $y(t)=e^{2t-4}+t$

(a) 0 (b) 0.174 (c) 0.293 (d) 3.439 (e) no where

49. Let f(x) be a function defined for $1.6 \le x \le 11.6$ such that $f'(x)=\ln x \sin x$. How many inflection points does the graph of f(x) have on this interval?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

50. A particle moves along a line for time $t \ge 0$ such that its velocity is $v(t)=10e^{-t}\cos t$. What is the velocity of the particle when its acceleration is zero for the first time?

(a) -2.709 (b) -0.670 (c) 2.356 (d) 3.185 (e) 10.000

51. Let f(x) be a function which is continuous on the interval $0 \le x \le 4$ and differentiable on $0 < x < 4$, with selected values of f(x) given in the table. Which of the following statements is true?

x	0	1	2	3	4
f(x)	10	30	46	58	66

(a) There is some value c between 0 and 4 such that $f'(c)=0$

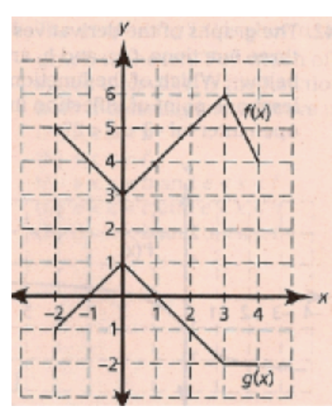
(b) $f'(x)>0$ for $0 < x < 4$

(c) There is some value of c between 0 and 4 such that $f'(c)=14$

(d) $f''(x)<0$ for $0<x<4$

(e) The maximum value of f(x) on $0 \le x \le 4$ is 66.

52. The graphs of *f(x)* and *g(x)* are shown on the right.



If $h(x)=\frac{g(2x)}{f(x)}$, use the graphs to find $h'(1)$.

(a) $-\frac{7}{4}$ (b) $-\frac{9}{16}$ (c) $-\frac{7}{16}$

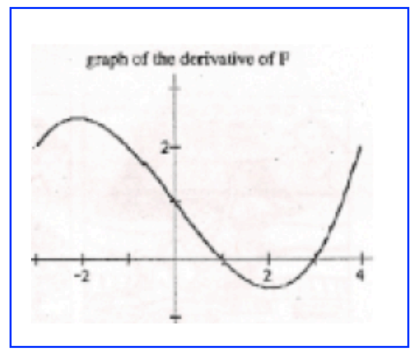
(d) $-\frac{5}{16}$ (e) $-\frac{3}{16}$

Part 4: Calculator Section. Show all work to justify your answer.

53. Given: $f(x) = e^{-\sin x}$ *and* $g(x) = \ln(2x+1)$
(a) Find the solutions to the equation $f(x) = g(x)$
(b) If $h(x) = f(x) - g(x)$, find the minimum value of $h(x)$ on the interval [0, 3]
(c) Find $h'(2)$

54. Given: $f(t) = 2\pi t + \sin(2\pi t)$
(a) Find the value(s) of c when c is contained in [0, 1] that satisfies the Mean Value Theorem.
(b) Suppose that the given function describes the position of a particle on the x-axis for time $0 \le t \le 2$. What is the average velocity ?
(c) Determine the velocity and the acceleration of the particle at $t = 1$ if *f(t)* represents the position.

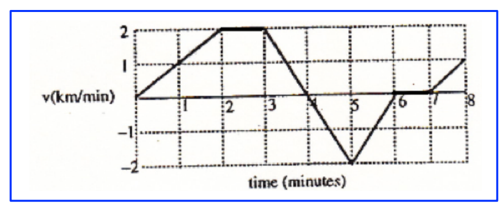
55. (a) Find the derivative of $f(x) = xe^{-\cot 2x}$
(b) Find the derivative of $g(x) = \ln(5e^{2x})$
(c) Find f^{-1}, the inverse of *f,* in terms of x: $f(x) = \ln\dfrac{2x}{x+3}$



56. A function F is defined for x on the closed interval[-3, 4].
The graph of the derivative of F is shown to the right.

(a) Find the interval(s) for which the graph of F is increasing. Explain.
(b) Find the possible x-coordinates for the absolute minimum value and the absolute maximum value of F on [-3, 4]
(c) Find the interval(s) for which the graph of F is concave down. Explain.

57. A car is moving along a straight road from A to B, starting from A at time t = 0.
Shown is a graph of the car's velocity plotted against time:

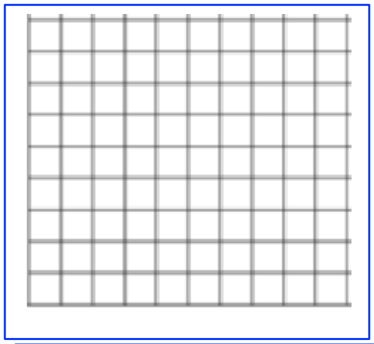


(a) At what time does the car change direction? Explain.
(b) On the axes provided, sketch a graph of the acceleration of the car.
(c) On the velocity graph provided below, sketch a graph of the speed of the car.
Note: (b) and (c) graphs provided on additional sheet

58. Given the differential equation $\dfrac{dy}{dx} = \dfrac{x}{2y}$
(a) Create a visualization of solutions by sketching a slope field at the points indicated.
(b) Sketch a solution through the point (1, 1)
(c) Explain what a general solution is to a differential equation.
(slope field on additional sheet)

Graphs for Answers to Certain Review Problems:

41.



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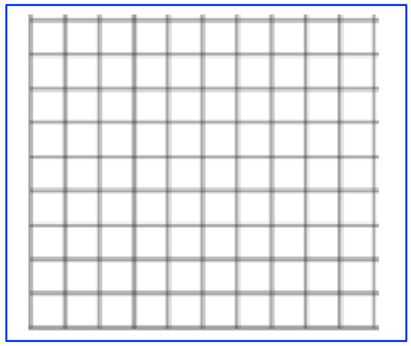
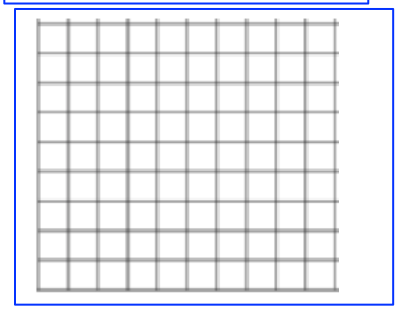


43 (a)



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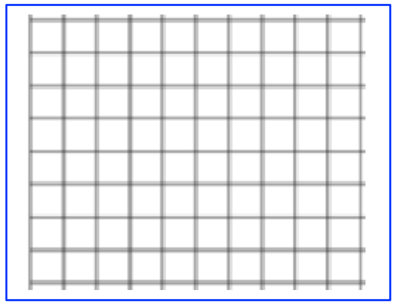
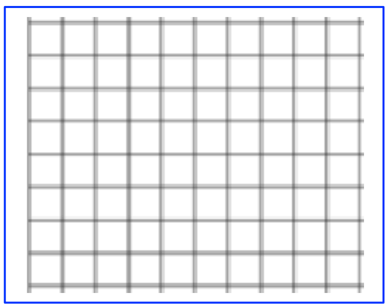
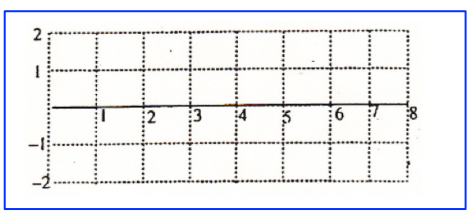


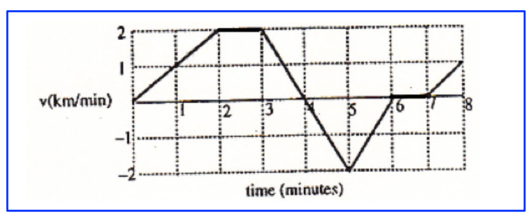
(c)



57. (b) Place answer into graph:



(c) Place answer into graph:



58.