Name _____ Class _____ Date _____

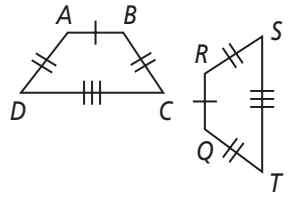
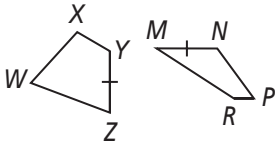
# 4-1 ELL Support

## Congruent Figures

### Concept List

| | | |
|---|---|---|
| algebraic equation | angle measure | congruency statement |
| congruent angles | congruent polygons | congruent segments |
| congruent triangles | proof | segment measure |

**Choose the concept from the list above that best represents the item in each box.**

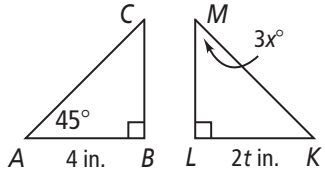
| | | |
|---|---|---|
| **1.** $\overline{GH} \cong \overline{ST}$<br>congruency statement | **2.** $m\angle A = 45$<br>angle measure | **3.**  <br>congruent polygons |
| **4.** $YZ = MN$<br>  <br>congruent segments | **5.** $\triangle ABC \cong \triangle XYZ$<br>congruent triangles | **6.** **Given:** $\overline{BD}$ is the angle bisector of $\angle ABC$, and $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.<br>**Prove:** $\triangle ADB \cong \triangle CDB$<br>proof |
| **7.** $m\angle H = 5x$<br>$m\angle W = x + 28$<br>Solve $5x = x + 28$ to find the measures of $\angle H$ and $\angle W$.<br>algebraic equation | **8.** $BC = 3$ cm<br>segment measure | **9.** $\angle ADB$ and $\angle SDT$ are vertical angles. So, $\angle ADB \cong \angle SDT$.<br>congruent angles |

Name _____ Class _____ Date _____

# 4-1 Think About a Plan

## Congruent Figures

**Algebra** Find the values of the variables.




$\triangle ABC \cong \triangle KLM$

### Know

1. What do you know about the measure of each of the non-right angles?

   **The measure of each of the non-right angles are complementary.**

2. What do you know about the length of each of the legs?

   **All of the legs are equal in length.**

3. What types of triangles are shown in the figure?

   **isosceles right triangles**

### Need

4. What information do you need to know to find the value of $x$?

   **You need to know that the measure of each of the non-right angles is 45.**

5. What information do you need to know to find the value of $t$?

   **You need to know that the length of each of the legs is 4 in.**

### Plan

6. How can you find the value of $x$? What is its value?

   **Answers may vary. Sample: Set $3x$ equal to 45. So, $3x = 45$; $x = 15$.**

7. How do you find the value of $t$? What is its value?

   **Answers may vary. Sample: Set $2t$ equal to 4. So, $2t = 4$; $t = 2$.**

Name _____ Class _____ Date _____

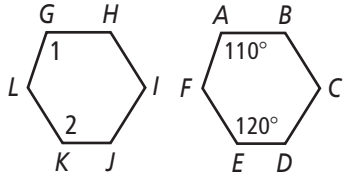
# 4-1 Practice

*Form G*

## Congruent Figures

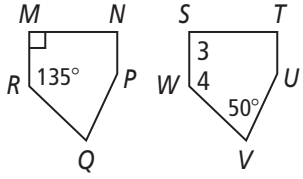
**Each pair of polygons is congruent. Find the measures of the numbered angles.**

**1.**



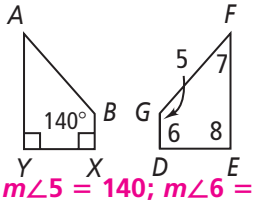

$m\angle 1 = 110$; $m\angle 2 = 120$

**2.**



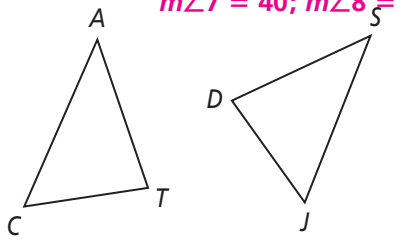

$m\angle 3 = 90$; $m\angle 4 = 135$

**3.**




$m\angle 5 = 140$; $m\angle 6 = 90$; $m\angle 7 = 40$; $m\angle 8 = 90$

**$\triangle CAT \cong \triangle JSD$. List each of the following.**



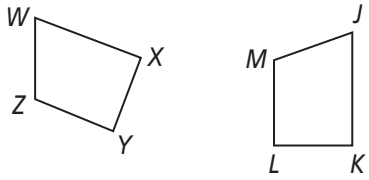

**4.** three pairs of congruent sides

$\overline{CA} \cong \overline{JS}$, $\overline{AT} \cong \overline{SD}$, $\overline{CT} \cong \overline{JD}$

**5.** three pairs of congruent angles

$\angle C \cong \angle J$, $\angle A \cong \angle S$, $\angle T \cong \angle D$

**$WXYZ \cong JKLM$. List each of the following.**




**6.** four pairs of congruent sides

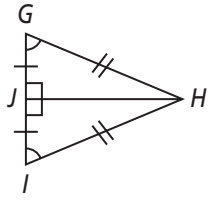
$\overline{WZ} \cong \overline{JM}$, $\overline{WX} \cong \overline{JK}$, $\overline{XY} \cong \overline{KL}$, $\overline{ZY} \cong \overline{ML}$

**7.** four pairs of congruent angles

$\angle W \cong \angle J$, $\angle X \cong \angle K$, $\angle Y \cong \angle L$, $\angle Z \cong \angle M$

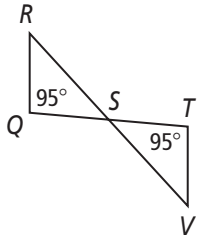
**For Exercises 8 and 9, can you conclude that the triangles are congruent? Justify your answers.**

**8.** $\triangle GHJ$ and $\triangle IHJ$



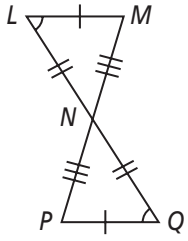

Yes; $\angle GHJ \cong \angle IHJ$ by Third Angles Thm. and by the Refl. Prop. $\overline{JH} \cong \overline{JH}$. Therefore, $\triangle GHJ \cong \triangle IHJ$ by the Def. of $\cong$ triangles.

**9.** $\triangle QRS$ and $\triangle TVS$




No; $\angle QSR \cong \angle TSV$ because vert. angles are congruent, and $\angle QRS \cong \angle TVS$ by Third Angles Thm., but none of the sides are necessarily congruent.

**10. Developing Proof** Use the information given in the diagram. Give a reason that each statement is true.




**a.** $\angle L \cong \angle Q$ Given

**b.** $\angle LNM \cong \angle QNP$ Vert. angles are $\cong$.

**c.** $\angle M \cong \angle P$ Third Angles Thm.

**d.** $\overline{LM} \cong \overline{QP}$, $\overline{LN} \cong \overline{QN}$, $\overline{MN} \cong \overline{PN}$ Given

**e.** $\triangle LNM \cong \triangle QNP$ Def. of $\cong$ triangles

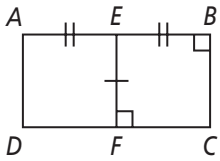
Name _____ Class _____ Date _____

# 4-1 Practice (continued) Form G

## Congruent Figures

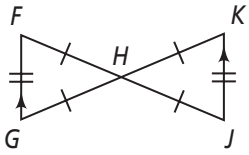
**For Exercises 11 and 12, can you conclude that the figures are congruent? Justify your answers.**

**11.** $AEFD$ and $EBCF$




No; answers may vary. Sample: $\angle D$ does not have to be a right angle.

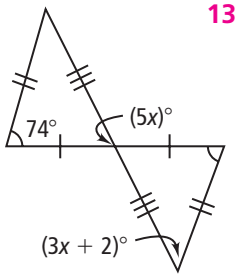
**12.** $\triangle FGH$ and $\triangle JKH$



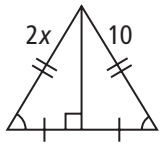

Yes; answers may vary. Sample: $\angle F \cong \angle J$ and $\angle G \cong K$ by the Alt. Int. Angles Thm. and $\angle FHG \cong \angle JHK$ by the Vert. Angles Thm., so all corresp. parts are congruent.

**Algebra** **Find the values of the variables.**

**13.** 13




**14.** 5




**Algebra** $ABCD \cong FGHJ$. **Find the measures of the given angles or lengths of the given sides.**

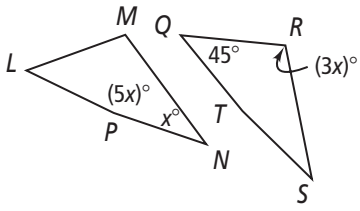
**15.** $m\angle B = 3y$, $m\angle G = y + 50$ 75

**16.** $CD = 2x + 3$; $HJ = 3x + 2$ 5

**17.** $m\angle C = 5z + 20$, $m\angle H = 6z + 10$ 70

**18.** $AD = 5b + 4$; $FJ = 3b + 8$ 14

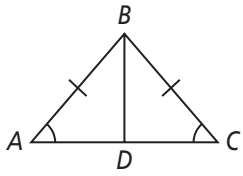
**19.** $LMNP \cong QRST$.
Find the value of $x$. 35




**20.** **Given:** $\overline{BD}$ is the angle bisector of $\angle ABC$.
$\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

**Prove:** $\triangle ADB \cong \triangle CDB$

Because $\overline{BD}$ is the angle bisector of $\angle ABC$, $\angle ABD \cong \angle CBD$. Because $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$, $\overline{AD} \cong \overline{CD}$ and $\angle ADB \cong \angle CDB$. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence. So, because the corresponding parts are all congruent, $\triangle ABD \cong \triangle CBD$.



Name _____ Class _____ Date _____

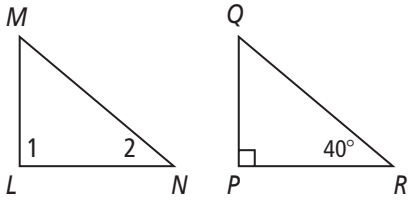
# 4-1 Practice

*Form K*

## Congruent Figures

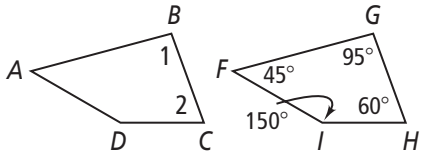
**Each pair of polygons is congruent. Find the measures of the numbered angles.**

**1.**



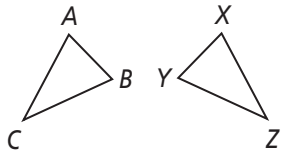

90; 40

**2.**




95; 60

**Use the diagram at the right for Exercises 3–7. $\triangle ABC \cong \triangle XYZ$. Complete the congruence statements.**




**3.** $\overline{AB} \cong$ [$\overline{XY}$]

To start, use the congruence statement to identify the points that correspond to $A$ and $B$.

$A$ corresponds to [$X$]. $B$ corresponds to [$Y$].

**4.** $\overline{ZY} \cong$ [$\overline{CB}$]

**5.** $\angle Z \cong$ [$\angle C$]

**6.** $\angle BAC \cong$ [$\angle YXZ$]

**7.** $\angle B \cong$ [$\angle Y$]

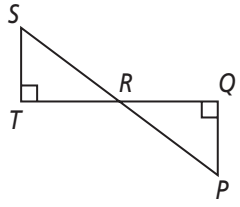
***FOUR* ≅ *MANY*. List each of the following.**

**8.** four pairs of congruent angles $\angle F \cong \angle M$; $\angle O \cong \angle A$; $\angle U \cong \angle N$; $\angle R \cong \angle Y$

**9.** four pairs of congruent sides $\overline{FO} \cong \overline{MA}$; $\overline{OU} \cong \overline{AN}$; $\overline{UR} \cong \overline{NY}$; $\overline{RF} \cong \overline{YM}$

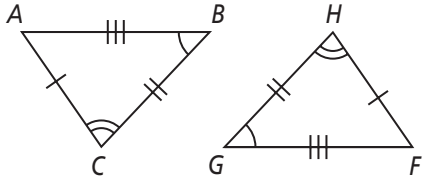
**For Exercises 10 and 11, can you conclude that the figures are congruent? Justify your answers.**

**10.** $\triangle SRT$ and $\triangle PRQ$




No; $\angle SRT \cong \angle PRQ$ because vert. ∡s are ≅, and $\angle RST \cong \angle RPQ$ by Third Angles Thm., but none of the sides are necessarily congruent.

**11.** $\triangle ABC$ and $\triangle FGH$

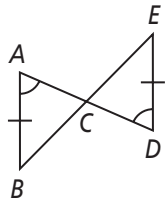



Yes; $\angle BAC \cong \angle GFH$ by Third Angles Thm. Therefore, $\triangle ABC \cong \triangle FGH$ by the def. of ≅ triangles.

# 4-1 Practice (continued) Form K

## Congruent Figures

**12.** **Given:** $\overline{AD}$ and $\overline{BE}$ bisect each other.
$\overline{AB} \cong \overline{DE}$; $\angle A \cong \angle D$
**Prove:** $\triangle ACB \cong \triangle DCE$

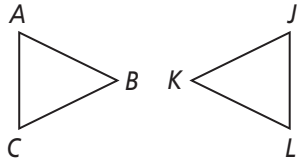



| Statements | Reasons |
|---|---|
| 1) $\overline{AD}$ and $\overline{BE}$ bisect each other. $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$ | 1) Given |
| 2) $\overline{AC} \cong \overline{CD}$, $\overline{BC} \cong \overline{CE}$ | 2) ? **Definition of bisect** |
| 3) $\angle ACB \cong \angle DCE$ | 3) ? **Vertical angles are ≅.** |
| 4) $\angle B \cong \angle E$ | 4) ? **Third Angles Theorem** |
| 5) $\triangle ACB \cong \triangle DCE$ | 5) ? **Definition of ≅ Triangles** |

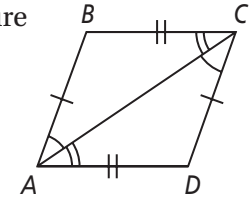
**13.** If $\triangle ABC \cong \triangle JKL$, which of the following must be a correct congruence statement? **C**

(A) $\angle A \cong \angle L$ (C) $\angle B \cong \angle K$

(B) $\overline{AB} \cong \overline{JL}$ (D) $\triangle BAC \cong \triangle LKJ$

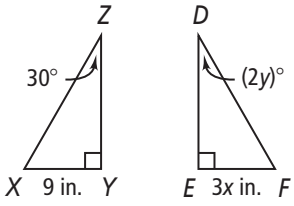



**14.** **Reasoning** A student says she can use the information in the figure to prove $\triangle ACB \cong \triangle ACD$. Is she correct? Explain.
**No; explanations may vary. Sample: Corresponding parts are not congruent. The figure can be used to prove $\triangle ACB \cong \triangle CAD$.**

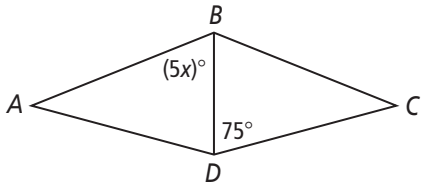



**Algebra** **Find the values of the variables.**

**15.** $\triangle XYZ \cong \triangle FED$ **3; 15**




**16.** $\triangle ABD \cong \triangle CDB$ **15**




**Algebra** **$\triangle FGH \cong \triangle QRS$. Find the measures of the given angles or the lengths of the given sides.**

**17.** $m\angle F = x + 24$; $m\angle Q = 3x$ **36; 36**

**18.** $\overline{GH} = 3x - 2$; $\overline{RS} = x + 6$ **10; 10**

Name _____ Class _____ Date _____

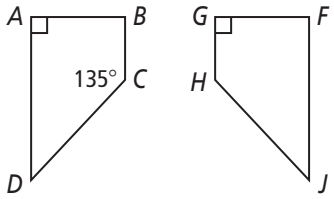
# 4-1 Standardized Test Prep

Congruent Figures

## Multiple Choice

**For Exercises 1–6, choose the correct letter.**

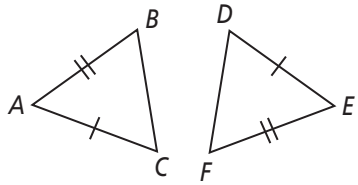
**1.** The pair of polygons at the right is congruent. What is $m\angle J$? **A**




(A) 45 (C) 135

(B) 90 (D) 145

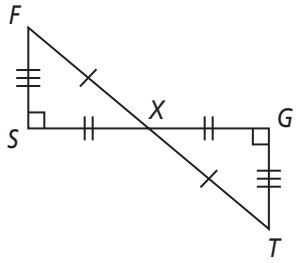
**2.** The triangles at the right are congruent. Which of the following statements must be true? **I**




(F) $\angle A \cong \angle D$ (H) $\overline{AB} \cong \overline{DE}$

(G) $\angle B \cong \angle E$ (I) $\overline{BC} \cong \overline{FD}$

**3.** Given the diagram at the right, which of the following must be true? **B**




(A) $\triangle XSF \cong \triangle XTG$ (C) $\triangle FXS \cong \triangle XGT$

(B) $\triangle SXF \cong \triangle GXT$ (D) $\triangle FXS \cong \triangle GXT$

**4.** If $\triangle RST \cong \triangle XYZ$, which of the following need not be true? **I**

(F) $\angle R \cong \angle X$ (G) $\angle T \cong \angle Z$ (H) $\overline{RT} \cong \overline{XZ}$ (I) $\overline{SR} \cong \overline{YZ}$

**5.** If $\triangle ABC \cong \triangle DEF$, $m\angle A = 50$, and $m\angle E = 30$, what is $m\angle C$? **C**

(A) 30 (B) 50 (C) 100 (D) 120

**6.** If $ABCD \cong QRST$, $m\angle A = x - 10$, and $m\angle Q = 2x - 30$, what is $m\angle A$? **F**

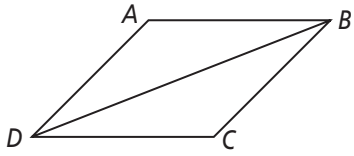
(F) 10 (G) 20 (H) 30 (I) 40

## Short Response

**[2]** ***$\angle ABD \cong \angle CDB$*** **and** ***$\angle ADB \cong \angle CBD$*****, both by the Alt. Int Angles Thm. So, by Third Angles Thm.,** ***$\angle A \cong \angle C$*****. Because** ***$\overline{DB} \cong \overline{BD}$*** **by the Refl. Prop. of Congruence, and we know** ***$\overline{AB} \cong \overline{CD}$*** **and** ***$\overline{AD} \cong \overline{CB}$*****, then all the corresponding parts are congruent and** ***$\triangle ABD \cong \triangle CDB$*****.**
**[1] incomplete proof [0] no proof or incorrect proof**

**7.** **Given:** $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

**Prove:** $\triangle ABD \cong \triangle CDB$



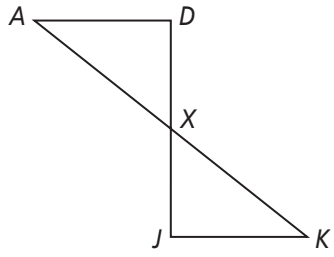
Name _____ Class _____ Date _____

# 4-1 Enrichment

## Congruent Figures

### Shared Implications

Sometimes different statements share one or more implications. For example, "$\overline{QR} \perp \overline{ST}$" and "$\overline{QR}$ is the perpendicular bisector of $\overline{ST}$" share the implication that $\overline{QR}$ meets $\overline{ST}$ at a right angle. The statements below refer to the diagram at the right.




1. $\overline{DJ} \perp \overline{JK}$;
2. $\overline{DJ} \perp \overline{AD}$;
3. $\overline{AD} \parallel \overline{JK}$;
4. $\angle A \cong \angle K$;
5. $\overline{DX} \cong \overline{JX}$;
6. $\overline{AD} \cong \overline{KJ}$;
7. $\overline{AK}$ bisects $\overline{DJ}$;
8. $\overline{DJ}$ bisects $\overline{AK}$;
9. $m\angle D = m\angle J = 90$

**Identify shared implications and reduce the number of given statements.**

1. What implication is shared by Statement 5 and Statement 7?
   **Answers may vary. Sample: Both statements imply that $\overline{DX} \cong \overline{JX}$.**

2. What implication is shared by Statement 3 and Statement 4?
   **Answers may vary. Sample: Both statements imply that $\angle A \cong \angle K$.**

3. Which two statements share at least one implication with Statement 9?
   **Answers may vary. Sample: Statements 1 and 2**

4. Can you prove $\triangle ADX \cong \triangle KJX$ using only five of the statements above? If so, identify them, then complete the proof. **Yes; Sample: statements 4, 6, 7, 8, and 9; $\overline{AK}$ bisects $\overline{DJ}$, so $\overline{DX} \cong \overline{JX}$. $\overline{DJ}$ bisects $\overline{AK}$, so $\overline{AX} \cong \overline{KX}$. $\overline{AD} \cong \overline{KJ}$ is given, so all corresponding sides are congruent. $\angle AXD$ is congruent to $\angle KXJ$ by the Vert. Angles Thm. $m\angle D = m\angle J = 90$ and $\angle A \cong \angle K$ are given, so all corresponding angles are congruent. So, $\triangle ADX \cong \triangle KJX$.**
5. Can you prove $\triangle ADX \cong \triangle KJX$ using only four of the statements above? If so, identify them, then complete the proof.
   **Yes; Sample: statements 6, 7, 8, and 3. $\overline{AK}$ bisects $\overline{DJ}$, so $\overline{DX} \cong \overline{JX}$. $\overline{DJ}$ bisects $\overline{AK}$, so $\overline{AX} \cong \overline{KX}$. $\overline{AD} \cong \overline{JK}$ is given, so all corresp. sides are congruent. $\angle AXD \cong \angle KXJ$ by the Vert. Angles Thm. Because $\overline{AD} \parallel \overline{JK}$, $\angle D \cong \angle J$, and $\angle A \cong \angle K$, by the Alt. Int. Angles Thm., all corresp. angles are congruent. So, $\triangle ADX \cong \triangle KJX$.**
6. Can you prove $\triangle ADX \cong \triangle KJX$ using only three of the statements above if the only way to prove triangles congruent is through the definition of congruent triangles? If so, identify them, then complete the proof. **no**

Name _____ Class _____ Date _____

# 4-1 Reteaching

## Congruent Figures

Given $ABCD \cong QRST$, find corresponding parts using the names. Order matters.

For example, $\begin{matrix} ABCD \\ QRST \end{matrix}$ This shows that $\angle A$ corresponds to $\angle Q$.

Therefore, $\angle A \cong \angle Q$.

For example, $\begin{matrix} ABCD \\ QRST \end{matrix}$ This shows that $\overline{BC}$ corresponds to $\overline{RS}$.

Therefore, $\overline{BC} \cong \overline{RS}$.

## Exercises

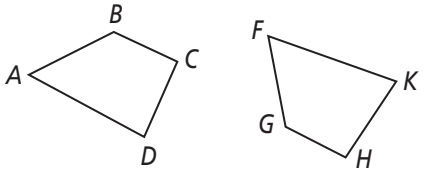
**Find corresponding parts using the order of the letters in the names.**

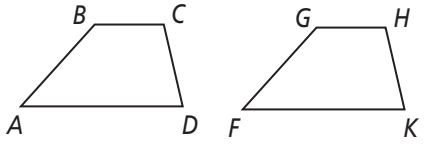
1. Identify the remaining three pairs of corresponding angles and sides between $ABCD$ and $QRST$ using the circle technique shown above.
   $\angle B \cong \angle R$, $\angle C \cong \angle S$, $\angle D \cong \angle T$, $\overline{AB} \cong \overline{QR}$, $\overline{CD} \cong \overline{ST}$, and $\overline{DA} \cong \overline{TQ}$
   Angles: $\begin{matrix} ABCD \\ QRST \end{matrix}$ $\begin{matrix} ABCD \\ QRST \end{matrix}$ $\begin{matrix} ABCD \\ QRST \end{matrix}$ Sides: $\begin{matrix} ABCD \\ QRST \end{matrix}$ $\begin{matrix} ABCD \\ QRST \end{matrix}$ $\begin{matrix} ABCD \\ QRST \end{matrix}$

2. Which pair of corresponding sides is hardest to identify using this technique?
   Answers may vary. Sample: $\overline{AD}$ and $\overline{QT}$

**Find corresponding parts by redrawing figures.**

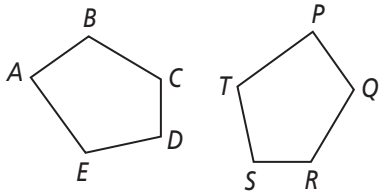
3. The two congruent figures below at the left have been redrawn at the right. Why are the corresponding parts easier to identify in the drawing at the right?



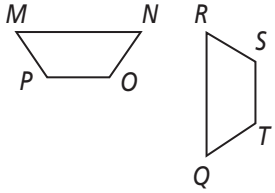




Answers may vary. Sample: The drawing at the right shows figures in same orientation.

4. Redraw the congruent polygons at the right in the same orientation. Identify all pairs of corresponding sides and angles. Check students' work. $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$, $\angle D$ and $\angle S$, $\angle E$ and $\angle T$, $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$, $\overline{CD}$ and $\overline{RS}$, $\overline{DE}$ and $\overline{ST}$, and $\overline{EA}$ and $\overline{TP}$ all correspond.




5. $MNOP \cong QRST$. Identify all pairs of congruent sides and angles.
   $\angle M \cong \angle Q$, $\angle N \cong \angle R$, $\angle O \cong \angle S$, $\angle P \cong \angle T$, $\overline{MN} \cong \overline{QR}$, $\overline{NO} \cong \overline{RS}$, $\overline{OP} \cong \overline{ST}$, and $\overline{PM} \cong \overline{TQ}$



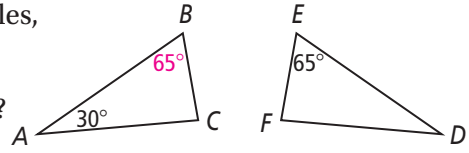
# 4-1 Reteaching (continued)

## Congruent Figures

**Problem**

Given $\triangle ABC \cong \triangle DEF$, $m\angle A = 30$, and $m\angle E = 65$, what is $m\angle C$?

How might you solve this problem? Sketch both triangles, and put all the information on both diagrams.




$m\angle A = 30$; therefore, $m\angle D = 30$. How do you know?
Because $\angle A$ and $\angle D$ are corresponding parts of congruent triangles.

## Exercises

**Work through the exercises below to solve the problem above.**

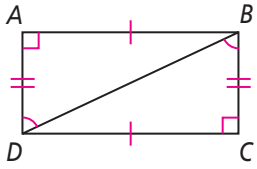
6. What angle in $\triangle ABC$ has the same measure as $\angle E$? What is the measure of that angle? Add the information to your sketch of $\triangle ABC$.
   **$\angle B$; 65**

7. You know the measures of two angles in $\triangle ABC$. How can you find the measure of the third angle?
   **Answers may vary. Sample: Use Triangle Angle-Sum Thm. Set sum of all three angles equal to 180.**
8. What is $m\angle C$? How did you find your answer?
   **85; answers may vary. Sample: $m\angle C = 180 - (60 + 35)$**

Before writing a proof, add the information implied by each given statement to your sketch. Then use your sketch to help you with Exercises 9–12.

**Add the information implied by each given statement.**

9. Given: $\angle A$ and $\angle C$ are right angles.
   **$m\angle A = m\angle C = 90$, $\overline{DA} \perp \overline{AB}$ and $\overline{DC} \perp \overline{BC}$**




10. Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$.
    **$ABCD$ is a parallelogram because it has opposite sides that are congruent.**

11. Given: $\angle ADB \cong \angle CBD$.
    **$\overline{AD} \parallel \overline{BC}$**

12. Can you conclude that $\angle ABD \cong \angle CDB$ using the given information above? If so, how?
    **Yes; use the Third Angles Thm.**

13. How can you conclude that the third side of both triangles is congruent?
    **The third side is shared by both triangles and is congruent by the Refl. Prop. of Congruence.**

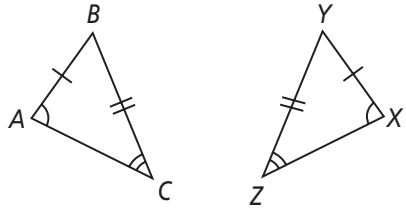
Name _____ Class _____ Date _____

# 4-2 ELL Support

## Triangle Congruence by SSS and SAS

**Problem**

Use the figure at the right. How can you prove that $\triangle ABC \cong \triangle XYZ$? Justify each step.



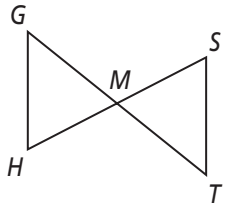

**Given:** The figure at the right

**Prove:** $\triangle ABC \cong \triangle XYZ$

| | |
|---|---|
| 1) $\overline{AB} \cong \overline{XY}$ | 1) Given |
| 2) $\overline{BC} \cong \overline{YZ}$ | 2) Given |
| 3) $\angle A \cong \angle X$ | 3) Given |
| 4) $\angle C \cong \angle Z$ | 4) Given |
| 5) $\angle B \cong \angle Y$ | 5) Third Angles Theorem |
| 6) $\triangle ABC \cong \triangle XYZ$ | 6) Side-Angle-Side (SAS) Postulate |

## Exercises

**1. Use the figure at the right. How can you prove that $\triangle GMH \cong \triangle TMS$? Justify each step.**

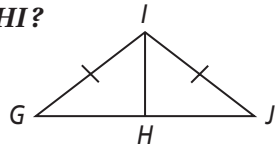



**Given:** $M$ is the midpoint of $\overline{HS}$ and $\overline{GT}$.

**Prove:** $\triangle GMH \cong \triangle TMS$

| | |
|---|---|
| 1) $M$ is the midpoint of $\overline{HS}$ and $\overline{GT}$. | 1) Given |
| 2) $\overline{GM} \cong \overline{TM}$ | 2) Definition of the midpoint |
| 3) $\overline{HM} \cong \overline{SM}$ | 3) Definition of the midpoint |
| 4) $\angle GMH \cong \angle TMS$ | 4) Vertical angles are congruent. |
| 5) $\triangle GMH \cong \triangle TMS$ | 5) Side-Angle-Side (SAS) Postulate |

**2. Use the figure at the right. How can you prove that $\triangle GHI \cong \triangle JHI$? Justify each step.**




**Given:** $H$ is the midpoint of $\overline{GJ}$.

**Prove:** $\triangle GHI \cong \triangle JHI$

| | |
|---|---|
| 1) $H$ is the midpoint of $\overline{GJ}$. | 1) Given |
| 2) $\overline{GH} \cong \overline{JH}$ | 2) Definition of the midpoint |
| 3) $\overline{HI} \cong \overline{HI}$ | 3) Reflexive Property of $\cong$ |
| 4) $\overline{GI} \cong \overline{JI}$ | 4) Third Side Postulate |
| 5) $\triangle GHI \cong \triangle JHI$ | 5) Side-Side Side (SSS) Postulate |

Name _____ Class _____ Date _____

# 4-2 Think About a Plan

## Triangle Congruence by SSS and SAS

Use the Distance Formula to determine whether $\triangle ABC$ and $\triangle DEF$ are congruent. Justify your answer.

$A(1, 4), B(5, 5), C(2, 2)$

$D(-5, 1), E(-1, 0), F(-4, 3)$

## Understanding the Problem

1. You need to determine if $\triangle ABC \cong \triangle DEF$. What are the three ways you know to prove triangles congruent?
   **If all corresponding parts are congruent, if all three sides are congruent, or, if two sides and the included angle are congruent, then the triangles are congruent.**

2. What information is given in the problem?
   **the coordinates for each vertex of each triangle**

## Planning the Solution

3. If you use the SSS Postulate to determine whether the triangles are congruent, what information do you need to find?
   **the lengths of the three sides of each triangle**

4. How can you find distances on a coordinate plane without measuring?
   **Use the Distance Formula.**

5. In an ordered pair, which number is the $x$-coordinate? Which is the $y$-coordinate?
   **The *x*-coordinate is the first number and the *y*-coordinate is the second number.**

## Getting an Answer

6. Find the length of each segment using the Distance Formula, $D = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$. Your answers may be in simplest radical form.

| | | |
|---|---|---|
| $\overline{AB}$ $\sqrt{17}$ | $\overline{BC}$ $3\sqrt{2}$ | $\overline{CA}$ $\sqrt{5}$ |
| $\overline{DE}$ $\sqrt{17}$ | $\overline{EF}$ $3\sqrt{2}$ | $\overline{FD}$ $\sqrt{5}$ |

7. Using the SSS Postulate, are the triangles congruent? Explain.
   **Yes; the triangles are congruent, because three pairs of sides are congruent.**

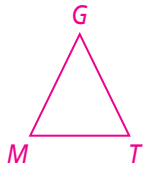
Name _____ Class _____ Date _____

# 4-2 Practice

*Form G*

## Triangle Congruence by SSS and SAS

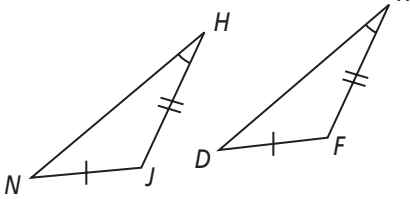
**Draw $\triangle MGT$. Use the triangle to answer the questions below.**




1. What angle is included between $\overline{GM}$ and $\overline{MT}$? ***∠M***
2. Which sides include $\angle T$? ***$\overline{GT}$ and $\overline{TM}$***
3. What angle is included between $\overline{GT}$ and $\overline{MG}$? ***∠G***

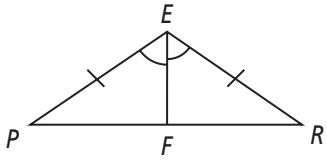
**Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information.* Explain your answer.**

**4.**



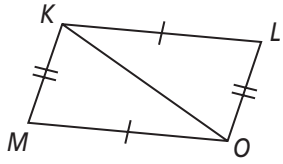

**Not enough information; two pairs of corresponding sides are congruent, but the congruent angle is not included.**

**5.**



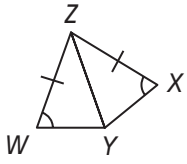

**SAS; two pairs of corresponding sides and their included angle are congruent.**

**6.**



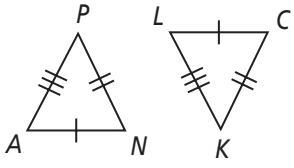

**SSS; three pairs of corresponding sides are congruent.**

**7.**



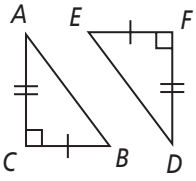

**Not enough information; two pairs of corresponding sides are congruent, but the congruent angle is not the included angle.**

**8.**



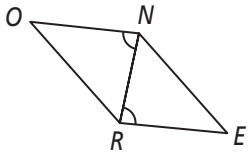

**SSS; three corresponding sides are congruent.**

**9.**



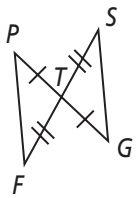

**SAS; two pairs of corresponding sides and their included right angle are congruent.**

**10.**



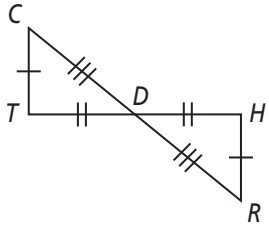

**Not enough information; one pair of corresponding sides and corresponding angles are congruent, but the other pair of corresponding sides that form the included angle must also be congruent.**

**11.**




**SAS; two pairs of corresponding sides and their included vertical angles are congruent.**

**12.**




**SSS or SAS; three pairs of corresponding sides are congruent, or, two pairs of corresponding sides and their included vertical angles are congruent.**

# 4-2 Practice (continued) Form G

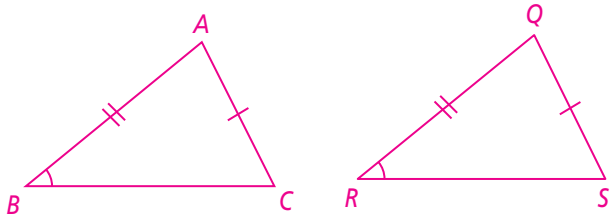
## Triangle Congruence by SSS and SAS

**13. Draw a Diagram** A student draws $\triangle ABC$ and $\triangle QRS$. The following sides and angles are congruent:

$\overline{AC} \cong \overline{QS}$ $\overline{AB} \cong \overline{QR}$ $\angle B \cong \angle R$

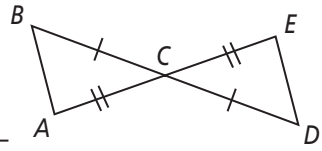
Based on this, can the student use either SSS or SAS to prove that $\triangle ABC \cong \triangle QRS$? If the answer is no, explain what additional information the student needs. Use a sketch to help explain your answer.

**No; $\angle B$ and $\angle R$ are not the included angles for the sides given. To prove congruence, you would need to know either that $\overline{BC} \cong \overline{RS}$ or $\angle Q \cong \angle A$.**




**14. Given:** $\overline{BC} \cong \overline{DC}$, $\overline{AC} \cong \overline{EC}$

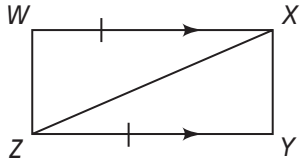
**Prove:** $\triangle ABC \cong \triangle EDC$




| Statements | Reasons |
|---|---|
| 1) $\overline{BC} \cong \overline{DC}$ | 1) Given |
| 2) $\overline{AC} \cong \overline{EC}$ | 2) Given |
| 3) $\angle BCA \cong \angle DCE$ | 3) Vertical $\measuredangle$s are $\cong$. |
| 4) $\triangle ABC \cong \triangle EDC$ | 4) SAS |

**15. Given:** $\overline{WX} \parallel \overline{YZ}$, $\overline{WX} \cong \overline{YZ}$

**Prove:** $\triangle WXZ \cong \triangle YZX$




| Statements | Reasons |
|---|---|
| 1) $\overline{WX} \parallel \overline{YZ}$ | 1) Given |
| 2) $\angle WXZ \cong \angle YZX$ | 2) Alternate Interior $\measuredangle$s are $\cong$. |
| 3) $\overline{WX} \cong \overline{YZ}$ | 3) Given |
| 4) $\overline{ZX} \cong \overline{XZ}$ | 4) Reflexive Property |
| 5) $\triangle WXZ \cong \triangle YZX$ | 5) SAS |

**16. Error Analysis** $\triangle FGH$ and $\triangle PQR$ are both equilateral triangles. Your friend says this means they are congruent by the SSS Postulate. Is your friend correct? Explain. **Incorrect; both triangles being equilateral means that the three angles and sides of each triangle are congruent, but there is no information comparing the side lengths of the two triangles.**

**17.** A student is gluing same-sized toothpicks together to make triangles. She plans to use these triangles to make a model of a bridge. Will all the triangles be congruent? Explain your answer. **Yes; because all the triangles are made from the same-sized toothpick, all three corresponding sides will be congruent.**

Name _____ Class _____ Date _____

# 4-2 Practice

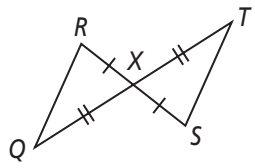
*Form K*

## Triangle Congruence by SSS and SAS

**1. Developing Proof** Copy and complete the flow proof.

**Given:** $\overline{RX} \cong \overline{SX}$, $\overline{QX} \cong \overline{TX}$

**Prove:** $\triangle QXR \cong \triangle TXS$




$\overline{RX} \cong \overline{SX}$ and $\overline{QX} \cong \overline{TX}$

? Given

$\angle QXR \cong \angle TXS$

?

Vert. ∡s are ≅.

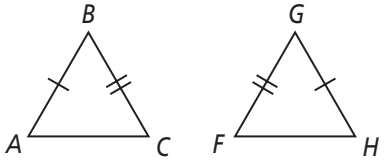
$\triangle QXR$ $\triangle TXS$

? ≅ ?

SAS

**What other information, if any, do you need to prove the two triangles congruent by SAS? Explain. To start, list the pairs of congruent, corresponding parts you already know.**

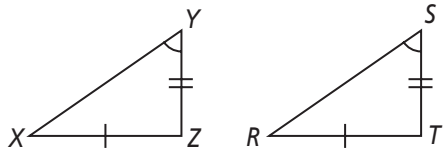
**2.**




$\overline{AB} \cong$ $\overline{HG}$ and $\overline{BC} \cong$ $\overline{GF}$

Need $\angle B \cong \angle G$; these are the included ∡s.

**3.**

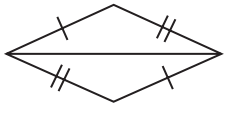



$\overline{XZ} \cong \overline{RT}$ and $\overline{ZY} \cong \overline{TS}$ and $\angle Y \cong \angle S$

need $\angle Z \cong \angle T$ or $\overline{XY} \cong \overline{RS}$

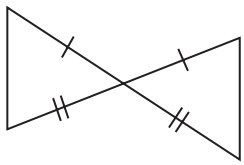
**Would you use SSS or SAS to prove these triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information.* Explain your answer.**

**4.**



SSS; third side is shared by both ∡s and is ≅ to itself by Refl. Prop. of ≅.

**5.**



SAS; vertical ∡s are ≅.

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# 4-2 Practice (continued) Form K

## Triangle Congruence by SSS and SAS

**Use the Distance Formula to determine whether $\triangle FGH$ and $\triangle JKL$ are congruent. Justify your answer.**

**6.** $F(0, 0)$, $G(0, 4)$, $H(3, 0)$ To start, find the lengths of the corresponding sides.
$J(1, 4)$, $K(-3, 4)$, $L(1, 1)$

$FG = \sqrt{(0 - 4)^2 + (0 - 0)^2} =$ **4**

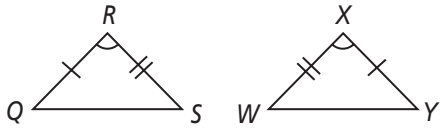
$JK = \sqrt{(4 - 4)^2 + (1-(-3))^2} =$ **4** Yes; they are ≅ by SSS.

$GH =$ **5** $KL =$ **5** $HF =$ **3** $LJ =$ **3**

**7.** $F(-2, 5)$, $G(4, -3)$, $H(4, 3)$ No; they are not ≅ because $\overline{FH}$ and $\overline{JL}$ have different lengths.
$J(2, 1)$, $K(-6, 7)$, $L(-6, 1)$

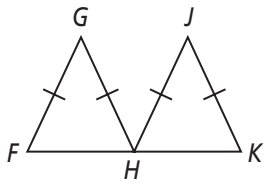
**Can you prove the triangles congruent? If so, write the congruence statement and name the postulate you would use. If not, write *not enough information* and tell what other information you would need.**

**8.**




Yes; $\triangle QRS \cong \triangle YXW$ by SAS.

**9.**

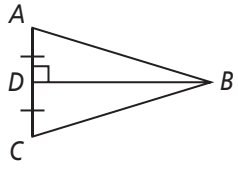



not enough information; need $\overline{FH} \cong \overline{HK}$ to apply SSS or $\angle G \cong \angle J$ to apply SAS

**10. Reasoning** Suppose $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, and $\overline{AB} \cong \overline{BC}$. Is $\triangle ABC$ congruent to $\triangle DEF$? Explain. Not necessarily; $\overline{EF}$ may not be ≅ to $\overline{BC}$.

**11. Given:** $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

**Prove:** $\triangle BAD \cong \triangle BCD$




| Statements | Reasons |
|---|---|
| 1) $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. | 1) Given |
| 2) $\overline{AD} \cong \overline{CD}$ | 2) Definition of segment bisector |
| 3) $\angle ADB$ and $\angle CDB$ are right $\measuredangle$. | 3) Definition of perpendicular |
| 4) ? $\angle ADB \cong \angle CDB$ | 4) ? All right angles are ≅. |
| 5) ? $\overline{BD} \cong \overline{BD}$ | 5) ? Reflexive Property of ≅ |
| 6) ? $\triangle BAD \cong \triangle BCD$ | 6) ? SAS |

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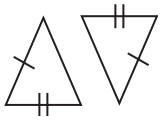
# 4-2 Standardized Test Prep

Triangle Congruence by SSS and SAS

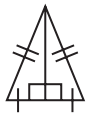
## Multiple Choice

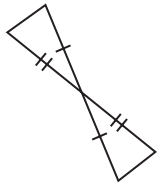
**For Exercises 1–4, choose the correct letter.**

**1.** Which pair of triangles can be proved congruent by SSS? **C**

Ⓐ 

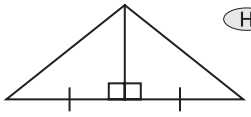
Ⓑ 

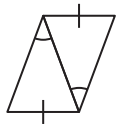
Ⓒ 

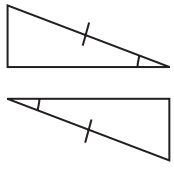
Ⓓ 

**2.** Which pair of triangles can be proved congruent by SAS? **G**

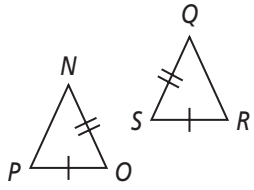
Ⓕ 

Ⓖ 

Ⓗ 

Ⓘ 

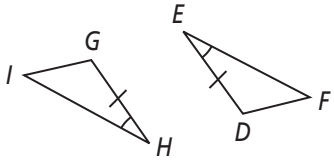
**3.** What additional information do you need to prove $\triangle NOP \cong \triangle QSR$? **D**




Ⓐ $\overline{PN} \cong \overline{SQ}$ Ⓒ $\angle P \cong \angle S$

Ⓑ $\overline{NO} \cong \overline{QR}$ Ⓓ $\angle O \cong \angle S$

**4.** What additional information do you need to prove $\triangle GHI \cong \triangle DEF$? **F**

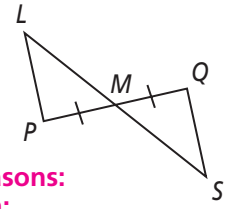



Ⓕ $\overline{HI} \cong \overline{EF}$ Ⓗ $\angle F \cong \angle G$

Ⓖ $\overline{HI} \cong \overline{ED}$ Ⓘ $\overline{GI} \cong \overline{DF}$

## Short Response

**5.** Write a two-column proof.




**Given:** $M$ is the midpoint of $\overline{LS}$, $\overline{PM} \cong \overline{QM}$.

**Prove:** $\triangle LMP \cong \triangle SMQ$

**[2] Statements: 1) *M* is the midpoint of $\overline{LS}$; 2) $\overline{LM} \cong \overline{SM}$; 3) $\angle LMP \cong \angle SMQ$; 4) $\overline{PM} \cong \overline{QM}$; 5) $\triangle LMP \cong \triangle SMQ$; Reasons: 1) Given; 2) Def. of a midpoint; 3) Vert. ∡s are ≅; 4) Given; 5) SAS [1] incomplete proof [0] incorrect or no proof**

Name _____ Class _____ Date _____

# 4-2 Enrichment

## Triangle Congruence by SSS and SAS

**Complete each proof using the given information and figure *ADGJ*.**

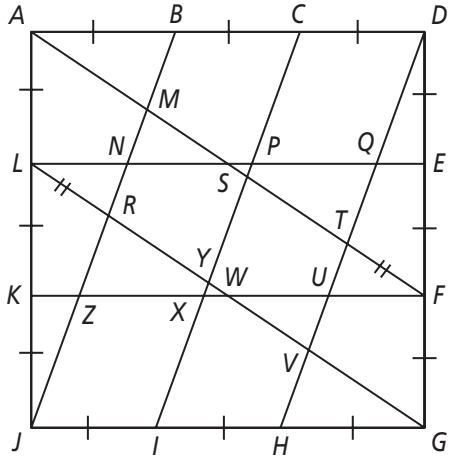
**Given:** Rectangle *ADGJ* is a square.

$\overline{AD} \parallel \overline{LE} \parallel \overline{KF} \parallel \overline{JG}$ $\quad \overline{LE} \perp \overline{DG}$

$\overline{AF} \parallel \overline{LG}$ $\quad \overline{FA} \cong \overline{LG}$

$\overline{BJ} \parallel \overline{CI} \parallel \overline{DH}$ $\quad \overline{BJ} \cong \overline{CI} \cong \overline{DH}$

$\overline{AJ} \perp \overline{KF}$ $\quad \overline{LR} \cong \overline{FT}$




**1. Prove:** $\triangle AKF \cong \triangle GEL$

| Statements | Reasons |
|---|---|
| 1) $\overline{LA} \cong \overline{KL} \cong \overline{EF} \cong \overline{FG}$ | 1) ? Given |
| 2) $LA + KL = AK$, $EF + FG = GE$ | 2) ? Segment Addition Postulate |
| 3) $\overline{AK} \cong \overline{GE}$ | 3) ? Substitution Property |
| 4) $\overline{LE} \perp \overline{DG}$, $\overline{AJ} \perp \overline{KF}$, $\overline{LE} \parallel \overline{KF}$ | 4) ? Given |
| 5) $\overline{LE} \perp \overline{AJ}$, $\overline{DG} \perp \overline{KF}$ | 5) ? Perpendicular Transversal Theorem |
| 6) *LEFK* is a rectangle | 6) Definition of a rectangle |
| 7) $\overline{LE} \cong \overline{FK}$ | 7) Opposite sides of a rectangle are $\cong$. |
| 8) $m\angle AKF = 90$, $m\angle GEL = 90$ | 8) ? Definition of perpendicular lines |
| 9) ? $\angle AKF \cong \angle GEL$ | 9) Definition of congruence |
| 10) $\triangle AKF \cong \triangle GEL$ | 10) ? SAS |

**2. Prove:** $\triangle JRG \cong \triangle DTA$

| Statements | Reasons |
|---|---|
| 1) $\overline{LR} \cong \overline{FT}$, $\overline{FA} \cong \overline{LG}$ | 1) Given |
| 2) $LR + RG = LG$, $TF + TA = FA$ | 2) ? Segment Addition Postulate |
| 3) ? $LR + RG = TF + TA$ | 3) Substitution Property |
| 4) ? $\overline{RG} \cong \overline{TA}$ | 4) Segment Subtraction Postulate |
| 5) $\overline{AD} \parallel \overline{KF}$, $\overline{AF} \parallel \overline{LG}$, $\overline{JG} \parallel \overline{KF}$ | 5) ? Given |
| 6) $\angle DAT \cong \angle AFK$, $\angle AFK \cong \angle FWG$ | 6) ? Alternate interior angles are $\cong$. |
| 7) $\angle FWG \cong \angle RGJ$ | 7) ? Alternate interior angles are $\cong$. |
| 8) ? $\angle DAT \cong \angle RGJ$ | 8) Transitive Property of Congruence |
| 9) *ADGJ* is a square. | 9) ? Given |
| 10) ? $\overline{AD} \cong \overline{GJ}$ | 10) Definition of a square |
| 11) $\triangle JRG \cong \triangle DTA$ | 11) ? SAS |

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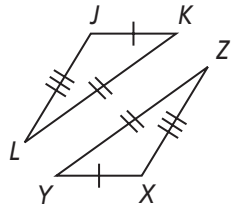
# 4-2 Reteaching

## Triangle Congruence by SSS and SAS

You can prove that triangles are congruent using the two postulates below.

### Postulate 4-1: Side-Side-Side (SSS) Postulate

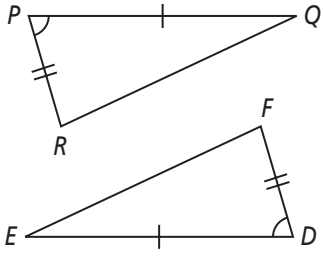



If all three sides of a triangle are congruent to all three sides of another triangle, then those two triangles are congruent.

If $\overline{JK} \cong \overline{XY}$, $\overline{KL} \cong \overline{YZ}$, and $\overline{JL} \cong \overline{XZ}$, then $\triangle JKL \cong \triangle XYZ$.

In a triangle, the angle formed by any two sides is called the *included angle* for those sides.

### Postulate 4-2: Side-Angle-Side (SAS) Postulate

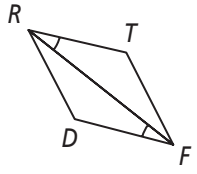



If two sides and the included angle of a triangle are congruent to two sides and the included angle of another triangle, then those two triangles are congruent.

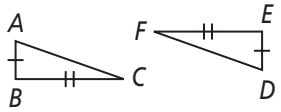
If $\overline{PQ} \cong \overline{DE}$, $\overline{PR} \cong \overline{DF}$, and $\angle P \cong \angle D$, then $\triangle PQR \cong \triangle DEF$.

$\angle P$ is included by $\overline{QP}$ and $\overline{PR}$. $\angle D$ is included by $\overline{ED}$ and $\overline{DF}$.

## Exercises




1. What other information do you need to prove $\triangle TRF \cong \triangle DFR$ by SAS? Explain. **$\overline{DF} \cong \overline{TR}$; by the Reflexive Property of Congruence, $\overline{RF} \cong \overline{FR}$. It is given that $\angle TRF \cong \angle DFR$. These are the included angles for the corresponding congruent sides.**

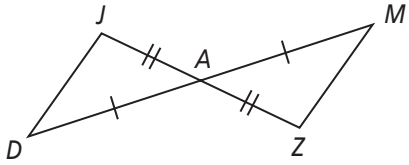


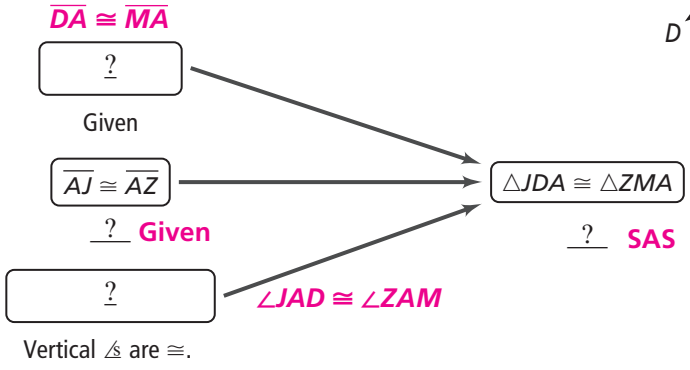

2. What other information do you need to prove $\triangle ABC \cong \triangle DEF$ by SAS? Explain.
**$\angle B \cong \angle E$; These are the included angles between the corresponding congruent sides.**

3. **Developing Proof** Copy and complete the flow proof.

**Given:** $\overline{DA} \cong \overline{MA}$, $\overline{AJ} \cong \overline{AZ}$

**Prove:** $\triangle JDA \cong \triangle ZMA$



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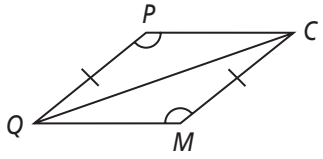
# 4-2 Reteaching (continued)

## Triangle Congruence by SSS and SAS

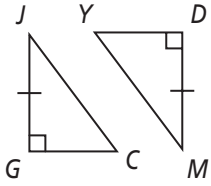
**Would you use SSS or SAS to prove the triangles congruent? If there is not enough information to prove the triangles congruent by SSS or SAS, write *not enough information.* Explain your answer.**

**4.**



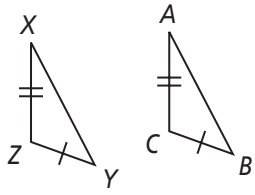

Not enough information; two pairs of corresponding sides are congruent, but the congruent angles are not the included angles.

**5.**




Not enough information; you need to know if $\overline{GC} \cong \overline{DY}$.

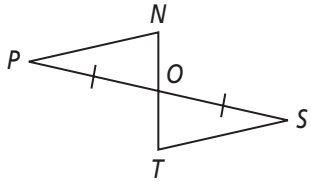
**6.**




Not enough information; only two pairs of corresponding sides are congruent. You need to know if $\overline{AB} \cong \overline{XY}$ or $\angle Z \cong \angle C$.

**7. Given:** $\overline{PO} \cong \overline{SO}$, $O$ is the midpoint of $\overline{NT}$.

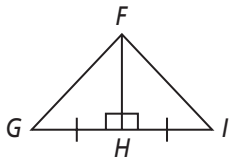
**Prove:** $\triangle NOP \cong \triangle TOS$




Statements: 1) $\overline{PO} \cong \overline{SO}$; 2) $O$ is the midpoint of $\overline{NT}$; 3) $\overline{NO} \cong \overline{TO}$; 4) $\angle NOP \cong \angle TOS$; 5) $\triangle NOP \cong \triangle TOS$; Reasons: 1) Given; 2) Given; 3) Def. of midpoint; 4) Vert. $\measuredangle$s are $\cong$; 5) SAS

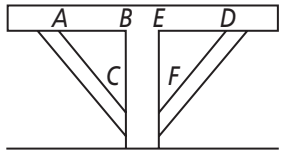
**8. Given:** $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$

**Prove:** $\triangle FHI \cong \triangle FHG$




Statements: 1) $\overline{FH} \cong \overline{FH}$; 2) $\overline{HI} \cong \overline{HG}$, $\overline{FH} \perp \overline{GI}$; 3) $\angle FHG$ and $\angle FHI$ are rt. $\measuredangle$s; 4) $\angle FHG \cong \angle FHI$; 5) $\triangle FHI \cong \triangle FHG$; Reasons: 1) Refl. Prop.; 2) Given; 3) Def. of perpendicular; 4) All rt. $\measuredangle$ are $\cong$; 5) SAS

**9.** A carpenter is building a support for a bird feeder. He wants the triangles on either side of the vertical post to be congruent. He measures and finds that $\overline{AB} \cong \overline{DE}$ and that $\overline{AC} \cong \overline{DF}$. What would he need to measure to prove that the triangles are congruent using SAS? What would he need to measure to prove that they are congruent using SSS?




For SAS, he would need to determine if $\angle BAC \cong \angle EDF$; for SSS, he would need to determine if $\overline{BC} \cong \overline{EF}$.

**10.** An artist is drawing two triangles. She draws each so that two sides are 4 in. and 5 in. long and an angle is 55°. Are her triangles congruent? Explain.

Answers may vary. Sample: Maybe; if both the 55° angles are between the 4-in. and 5-in. sides, then the triangles are congruent by SAS.

Name _____ Class _____ Date _____

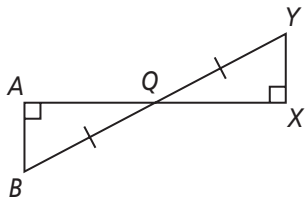
# 4-3 ELL Support

## Triangle Congruence by ASA and AAS

**Problem**

**Given:** The figure at the right

**Prove:** $\triangle ABQ \cong \triangle XYQ$

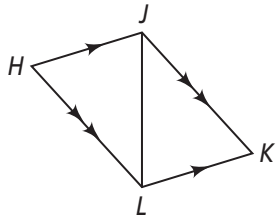



| Explain | Work | Justify |
|---|---|---|
| **First,** list the information that is given directly in the diagram. | $\overline{BQ} \cong \overline{YQ}$<br>$\angle A$ and $\angle X$ are right $\measuredangle$s. | Given |
| **Second,** use the fact that all right angles are congruent to each other. | $\angle A \cong \angle X$ | All right angles are congruent. |
| **Next,** use the fact that vertical angles are congruent. | $\angle AQB \cong \angle XQY$ | Vertical Angles Theorem |
| **Finally,** determine which theorem can be used to prove the triangles congruent using the information listed above. | $\triangle ABQ \cong \triangle XYQ$ | Angle-Angle-Side (AAS) Theorem |

## Exercise

**Given:** The figure at the right

**Prove:** $\triangle HJL \cong \triangle KLJ$




| Explain | Work | Justify |
|---|---|---|
| **First,** list the information that is given directly in the diagram. | $\overline{HJ} \parallel \overline{KL}$<br>$\overline{HL} \parallel \overline{KJ}$ | Given |
| **Second,** use the fact that alternate interior angles are congruent when parallel lines are cut by a transversal. | $\angle HJL \cong \angle KLJ$<br>$\angle HLJ \cong \angle KJL$ | Alternate Interior Angles Theorem |
| **Next,** recall that any line segment is congruent to itself. | $\overline{JL} \cong \overline{LJ}$ | Reflexive Property of Congruence |
| **Finally,** determine which theorem can be used to prove the triangles congruent using the information listed above. | $\triangle HJL \cong \triangle KLJ$ | Angle-Side-Angle (ASA) Postulate |

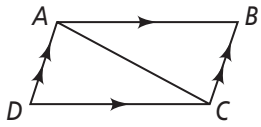
Name _____ Class _____ Date _____

# 4-3 Think About a Plan

## Triangle Congruence by ASA and AAS

**Given:** $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$

**Prove:** $\triangle ABC \cong \triangle CDA$




**1.** What do you need to find to solve the problem?

**Sample: at least three corresponding pairs of sides or angles that I can prove to be congruent**

**2.** What are the corresponding parts of the two triangles?

**$\angle CAB$ and $\angle ACD$; $\angle D$ and $\angle B$; $\angle DAC$ and $\angle BCA$; $\overline{AC}$ and $\overline{CA}$; $\overline{AB}$ and $\overline{CD}$; and $\overline{BC}$ and $\overline{DA}$**

**3.** What word would you use to describe $\overline{AC}$? **transversal**

**4.** What can you show about angles in the triangles that can indicate congruency?

**I can find congruent angles using alternate interior angles of the transversal $\overline{AC}$.**

**5.** What do you know about a side or sides of the triangles that can be used to show congruency?

**The transversal is part of both triangles, so it is congruent to itself by the Reflexive Property of Congruence.**

**6.** Write a proof in paragraph form.

**Answers may vary. Sample: $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ are given. Therefore $\overline{AC}$ is a transversal. $\angle CAB \cong \angle ACD$ and $\angle DAC \cong \angle BCA$ by the Alternate Interior Angles Theorem. The transversal is part of both triangles, so $\overline{AC} \cong \overline{CA}$ by the Reflexive Property of Congruence. $\triangle ABC \cong \triangle CDA$ by the ASA Postulate.**

Name _____ Class _____ Date _____

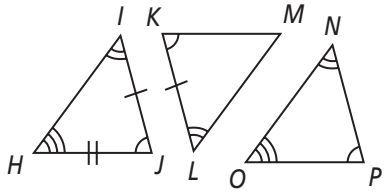
# 4-3 Practice

*Form G*

## Triangle Congruence by ASA and AAS

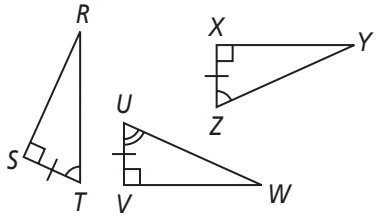
**Name two triangles that are congruent by ASA.**

**1.**



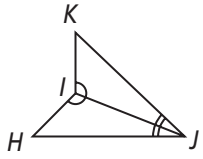

△*HIJ* ≅ △*MLK*

**2.**




△*RST* ≅ △*YXZ*

**3. Developing Proof** Complete the proof by filling in the blanks.




**Given:** $\angle HIJ \cong \angle KIJ$

$\angle IJH \cong \angle IJK$

**Prove:** $\triangle HIJ \cong \triangle KIJ$

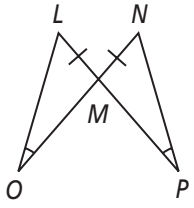
**Proof:** $\angle HIJ \cong \angle KIJ$ and $\angle IJH \cong \angle IJK$ are given.

$\overline{IJ} \cong \overline{IJ}$ by ? . **Refl. Prop. of Congruence**

So, $\triangle HIJ \cong \triangle KIJ$ by ? . **ASA**

**4. Given:** $\angle LOM \cong \angle NPM$, $\overline{LM} \cong \overline{NM}$

**Prove:** $\triangle LOM \cong \triangle NPM$

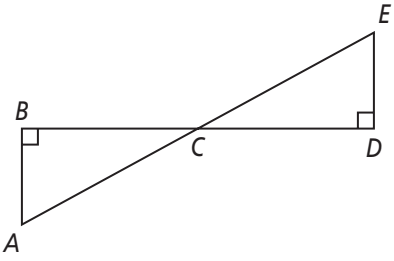



**Proof: ∠*LOM* ≅ ∠*NPM* and $\overline{LM} \cong \overline{NM}$ are given. ∠*LMO* ≅ ∠*NMP* because vert. ∡s are ≅. So, △*LOM* ≅ △*NPM* by AAS.**

**5. Given:** $\angle B$ and $\angle D$ are right angles.

$\overline{AE}$ bisects $\overline{BD}$

**Prove:** $\triangle ABC \cong \triangle EDC$




| Statements | Reasons |
|---|---|
| 1) ∠*B* and ∠*D* are right angles. | 1) Given |
| 2) ∠*B* ≅ ∠*D* | 2) All right angles are congruent. |
| 3) ∠*BCA* ≅ ∠*DCE* | 3) Vertical angles are congruent. |
| 4) $\overline{AE}$ bisects $\overline{BD}$ | 4) Given |
| 5) $\overline{BC} \cong \overline{CD}$ | 5) Def. of bisector |
| 6) △*ABC* ≅ △*EDC* | 6) ASA |

Name _____ Class _____ Date _____

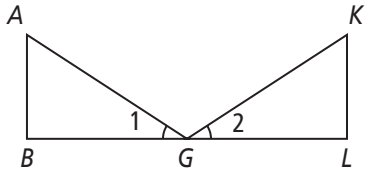
# 4-3 Practice (continued) Form G

## Triangle Congruence by ASA and AAS

**6. Developing Proof** Complete the proof.

**Given:** $\angle 1 \cong \angle 2$, $\overline{AB} \perp \overline{BL}$, $\overline{KL} \perp \overline{BL}$, $\overline{AB} \cong \overline{KL}$

**Prove:** $\triangle ABG \cong \triangle KLG$




**Proof:**

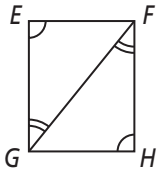
- $\overline{AB} \perp \overline{BL}$ — a. Given
- $\overline{KL} \perp \overline{BL}$ — b. Given
- $\angle B$ is a right $\angle$. — c. $\perp$ lines form right $\measuredangle$
- $\angle L$ is a right $\angle$. — d. $\perp$ lines form right $\measuredangle$
- f. all right $\measuredangle$ are $\cong$
- $\angle 1 \cong \angle 2$ — e. Given
- $\angle B \cong \angle L$ — f.
- $\overline{AB} \cong \overline{KL}$ — g. Given
- $\triangle ABG \cong \triangle KLG$ — h. AAS

**7.** Write a flow proof.

**Given:** $\angle E \cong \angle H$

$\angle HFG \cong \angle EGF$

**Prove:** $\triangle EGF \cong \triangle HFG$



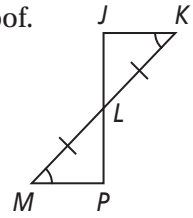

- $\angle E \cong \angle H$ — Given
- $\angle HFG \cong \angle EGF$ — Given
- $\overline{FG} \cong \overline{GF}$ — Reflexive Prop. of $\cong$
- $\triangle EGF \cong \triangle HFG$ — AAS Theorem

**8.** Write a two-column proof.

**Given:** $\angle K \cong \angle M$

$\overline{KL} \cong \overline{ML}$

**Prove:** $\triangle JKL \cong \triangle PML$



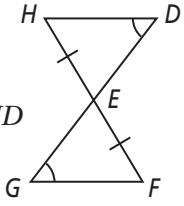

| Statements | Reasons |
|---|---|
| $\angle K \cong \angle M$ | Given |
| $\overline{KL} \cong \overline{ML}$ | Given |
| $\angle JLK \cong \angle PLM$ | Vert. $\measuredangle$ are $\cong$. |
| $\triangle JKL \cong \triangle PML$ | ASA Postulate |

**For Exercises 9 and 10, write a paragraph proof.**

**9. Given:** $\angle D \cong \angle G$

$\overline{HE} \cong \overline{FE}$

**Prove:** $\triangle EFG \cong \triangle EHD$

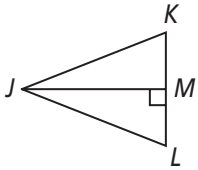



$\angle D \cong \angle G$ is given. $\angle DEH \cong \angle GEF$ because vert. $\measuredangle$ are $\cong$. $\overline{HE} \cong \overline{FE}$ is given. So, $\triangle EFG \cong \triangle EHD$ by AAS.

**10. Given:** $\overline{JM}$ bisects $\angle J$.

$\overline{JM} \perp \overline{KL}$

**Prove:** $\triangle JMK \cong \triangle JML$




$\overline{JM}$ bisects $\angle J$ is given. $\angle KJM \cong \angle LJM$ by def. of an $\angle$ bisector. $\overline{JM} \cong \overline{JM}$ by the Refl. Prop. of $\cong$. $\overline{JM} \perp \overline{KL}$ is given. $\angle LMJ$ and $\angle KMJ$ are right $\measuredangle$ by the def. of perpendicular. Therefore, $\angle LMJ \cong \angle KMJ$ because all right $\measuredangle$ are $\cong$. So, $\triangle JMK \cong \triangle JML$ by ASA.

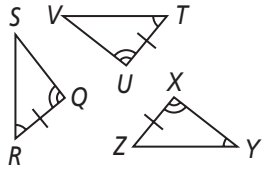
Name _____ Class _____ Date _____

# 4-3 Practice

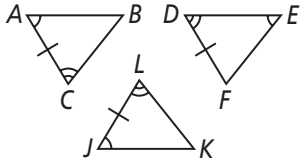
*Form K*

## Triangle Congruence by ASA and AAS

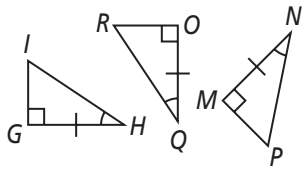
**Name the two triangles that are congruent by ASA.**

**1.** 


△*RQS* ≅ △*TUV*

**2.** 


△*ABC* ≅ △*JKL*

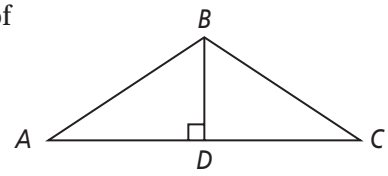
**3.** 


△*GHI* ≅ △*MNP* ≅ △*OQR*

**4. Developing Proof** Complete the two-column proof by filling in the blanks.

**Given:** $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\angle ABC$

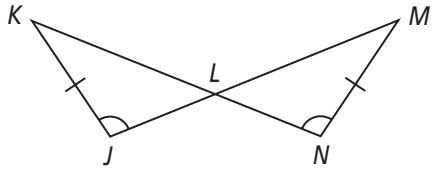
**Prove:** $\triangle ABD \cong \triangle CBD$




| Statements | Reasons |
|---|---|
| 1) $\overline{BD} \perp \overline{AC}$, $\overline{BD}$ bisects $\angle ABC$. | 1) Given |
| 2) ? ∠*ADB* and ∠*CDB* are right ∡s. | 2) Definition of perpendicular |
| 3) $\angle ADB \cong \angle CDB$ | 3) ? All right angles are ≅. |
| 4) $\angle ABD \cong \angle CBD$ | 4) ? Definition of ∠ bisector |
| 5) ? $\overline{BD} \cong \overline{BD}$ | 5) Reflexive Property of ≅ |
| 6) ? △*ABD* ≅ △*CBD* | 6) ASA |

**5. Given:** $\overline{KJ} \cong \overline{MN}$, $\angle KJL \cong \angle MNL$

**Prove:** $\triangle JKL \cong \triangle NML$



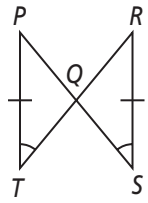

| Statements | Reasons |
|---|---|
| 1) $\overline{KJ} \cong \overline{MN}$, $\angle KJL \cong \angle MNL$ | 1) Given |
| 2) $\angle KLJ \cong \angle MLN$ | 2) ? Vertical ∡s are ≅. |
| 3) ? ∠*LKJ* ≅ ∠*LMN* | 3) Third Angles Theorem |
| 4) ? △*JKL* ≅ △*NML* | 4) ASA |

Name _____ Class _____ Date _____

# 4-3 Practice (continued) Form K

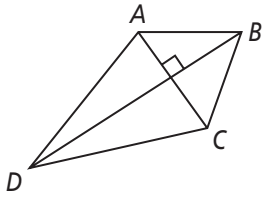
## Triangle Congruence by ASA and AAS

**6.** **Given:** $\overline{PT} \cong \overline{RS}$, $\angle PTR \cong \angle RSP$
**Prove:** $\triangle PQT \cong \triangle RQS$




| Statements | Reasons |
|---|---|
| 1) ? $\overline{PT} \cong \overline{RS}$, $\angle PTR \cong \angle RSP$ | 1) Given |
| 2) $\angle PQT \cong \angle RQS$ | 2) ? Vertical $\measuredangle$ are $\cong$. |
| 3) ? $\triangle PQT \cong \triangle RQS$ | 3) AAS |

**7.** **Given:** $\overline{BD}$ is the angle bisector of $\angle ABC$ and $\angle ADC$.
**Prove:** $\triangle ABD \cong \triangle CBD$

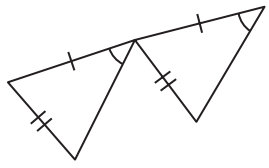



| Statements | Reasons |
|---|---|
| 1) ? $\overline{BD}$ is the angle bisector of $\angle ABC$ and $\angle ADC$. | 1) ? Given |
| 2) ? $\angle ABD \cong \angle CBD$, $\angle ADB \cong \angle CDB$ | 2) Definition of $\angle$ bisector |
| 3) $\angle BAD \cong \angle BCD$ | 3) ? Third Angles Theorem |
| 4) $\overline{BD} \cong \overline{BD}$ | 4) ? Reflexive Property of $\cong$ |
| 5) ? $\triangle ABD \cong \triangle CBD$ | 5) AAS |

**8.** **Reasoning** A student tells you that he can prove the AAS Theorem using the SAS Postulate and the Third Angles Theorem. Do you agree with him? Explain. (*Hint:* How many pairs of sides does the SAS Postulate use?)

No; answers may vary. Sample: the SAS Postulate requires two pairs of corresp. $\cong$ sides.

**9.** **Reasoning** Can you prove the triangles congruent? Justify your answer.



No; answers may vary. Sample. there are no included sides or included $\measuredangle$ that correspond in both $\triangle$, and you cannot use the Third Angles Theorem.

Name _____ Class _____ Date _____

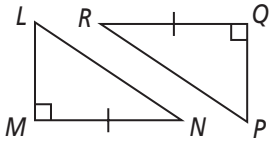
# 4-3 Standardized Test Prep

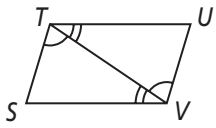
Triangle Congruence by ASA and AAS

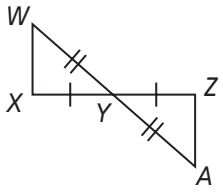
## Multiple Choice

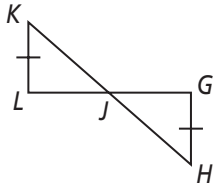
**For Exercises 1–4, choose the correct letter.**

**1.** Which pair of triangles can be proven congruent by the ASA Postulate? **C**

Ⓐ 


Ⓒ 


Ⓑ 


Ⓓ 


**2.** For the ASA Postulate to apply, which side of the triangle must be known? **F**

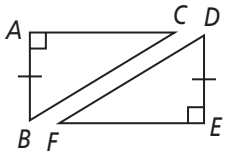
Ⓕ the included side

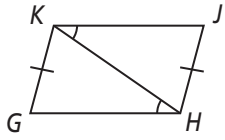
Ⓖ the longest side

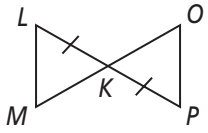
Ⓗ the shortest side

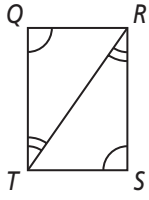
Ⓘ a non-included side

**3.** Which pair of triangles can be proven congruent by the AAS Theorem? **D**

Ⓐ 


Ⓒ 


Ⓑ 


Ⓓ 


**4.** For the AAS Theorem to apply, which side of the triangle must be known? **I**

Ⓕ the included side

Ⓖ the longest side

Ⓗ the shortest side

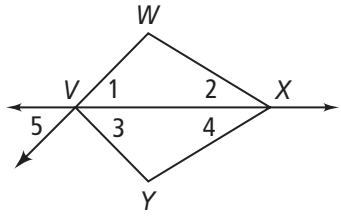
Ⓘ a non-included side

## Short Response

**5.** Write a paragraph proof.

**Given:** $\angle 3 \cong \angle 5, \angle 2 \cong \angle 4$

**Prove:** $\triangle VWX \cong \triangle VYX$




**[2]** $\angle 3 \cong \angle 5$ is given. $\angle 1 \cong \angle 5$ because vert. $\measuredangle$s are $\cong$. $\angle 3 \cong \angle 1$ by the Trans. Prop. of $\cong$. $\angle 2 \cong \angle 4$ is given. $\overline{VX} \cong \overline{VX}$ by the Refl. Prop. of $\cong$. $\triangle VWX \cong \triangle VYX$ by ASA.
**[1]** incomplete proof
**[0]** incorrect or no proof

Name _____ Class _____ Date _____

# 4-3 Enrichment

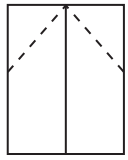
## Triangle Congruence by ASA and AAS

**Follow these steps to create a stunt plane from a sheet of $8\frac{1}{2}$-in.-by-11-in. paper.**

1. Fold the paper in half vertically. Using the definition of segment bisectors, note the congruent segments. **Check students' work.**

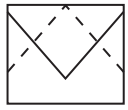
2. Fold the upper corners to lie on either side of the center line. Using the definitions of angle bisectors and right angles, note the angles that are congruent.
   a. What theorem allows you to conclude that the two triangles you created are congruent? **ASA**
   b. Identify the congruent angles and sides. **Check students' work.**

**Step 2**



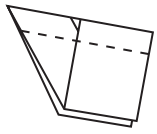
3. Fold the top point down along a line that is 1 in. below the bottom side of the triangles. Again, fold the upper corners to lie on either side of the center line.
   a. What theorem allows you to conclude that the two triangles you created are congruent? **ASA**
   b. Identify the congruent angles and sides. **Check students' work.**

**Step 3**



4. Fold the small triangle up along the line formed by the bottom sides of the triangles formed in Step 3. This exposes two more congruent triangles. The two triangles are congruent by ASA.
   a. Explain which angles are congruent and why. **Check students' work.**
   b. Name the congruent sides, and explain how you know the sides are congruent. **The included sides are congruent because the fold in Step 1 found the midpoint of the width of the paper, thus creating two equal segments.**

**Step 5**



5. Fold along the vertical fold line formed in Step 1 so that the triangles formed in Step 4 are on the outside of the airplane.

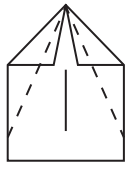
6. Turn the paper so that the long edge is at the bottom. To complete the paper airplane, fold each top edge down to meet the bottom edge, and crease.

**Follow the Steps to create a dart plane from a sheet of $8\frac{1}{2}$-in.-by-11-in. paper.**

7. Repeat Steps 1 and 2 above.

8. Fold the upper side corners to lie on either side of the center line.
   a. What theorem allows you to conclude that the two triangles you created are congruent? **ASA**
   b. Identify the congruent angles and sides. **Check students' work.**

**Step 8**



9. Turn the paper over. Fold the outer edges in so that they lie on either side of the center line.

10. Fold the paper in half along the original fold made in Step 7.

11. Adjust the wing flaps until they are at right angles with the body to complete the dart.

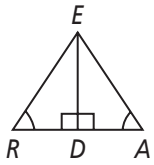
Name _____ Class _____ Date _____

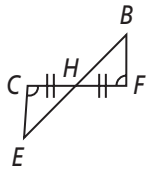
# 4-3 Reteaching

## Triangle Congruence by ASA and AAS

**Problem**

Can the ASA Postulate or the AAS Theorem be applied directly to prove the triangles congruent?




**a.** Because $\angle RDE$ and $\angle ADE$ are right angles, they are congruent. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property of $\cong$, and it is given that $\angle R \cong \angle A$. Therefore, $\triangle RDE \cong \triangle ADE$ by the AAS Theorem.

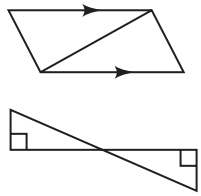
**b.** It is given that $\overline{CH} \cong \overline{FH}$ and $\angle F \cong \angle C$. Because $\angle CHE$ and $\angle FHB$ are vertical angles, they are congruent. Therefore, $\triangle CHE \cong \triangle FHB$ by the ASA Postulate.

## Exercises

**Indicate congruences.**

1. Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.

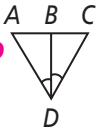
2. Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.



3. Draw and mark two triangles that are congruent by either the ASA Postulate or the AAS Theorem. **Check students' work.**

**What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?**

4. ASA Postulate
**$\angle ABD \cong \angle CBD$**




5. AAS Theorem
**$\angle JMK \cong \angle LKM$, $\angle JKM \cong \angle LMK$, $\angle JMK \cong \angle LMK$, or $\angle JKM \cong \angle LKM$**


6. ASA Postulate
**$\angle ZXY \cong \angle ZVU$**


7. AAS Theorem
**$\angle Y \cong \angle O$**

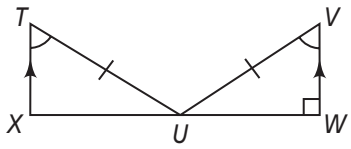

8. AAS Theorem
**$\angle P \cong \angle A$**


9. ASA Postulate
**$\angle CYL \cong \angle ALY$**

# 4-3 Reteaching (continued)

## Triangle Congruence by ASA and AAS

**10.** Provide the reason for each step in the two-column proof.

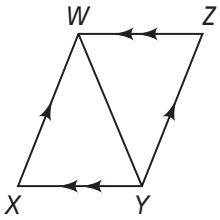



**Given:** $\overline{TX} \parallel \overline{VW}$, $\overline{TU} \cong \overline{VU}$, $\angle XTU \cong \angle WVU$, $\angle UWV$ is a right angle.

**Prove:** $\triangle TUX \cong \triangle VUW$

| Statements | Reasons |
|---|---|
| 1) $\angle UWV$ is a right angle. | 1) ? **Given** |
| 2) $\overline{VW} \perp \overline{XW}$ | 2) ? **Definition of perpendicular lines** |
| 3) $\overline{TX} \parallel \overline{VW}$ | 3) ? **Given** |
| 4) $\overline{TX} \perp \overline{XW}$ | 4) ? **Perpendicular Transversal Theorem** |
| 5) $\angle UXT$ is a right angle. | 5) ? **Definition of perpendicular lines** |
| 6) $\angle UWV \cong \angle UXT$ | 6) ? **All right angles are congruent.** |
| 7) $\overline{TU} \cong \overline{VU}$ | 7) ? **Given** |
| 8) $\angle XTU \cong \angle WVU$ | 8) ? **Given** |
| 9) $\triangle TUX \cong \triangle VUW$ | 9) ? **AAS Theorem** |

**11.** Write a paragraph proof.

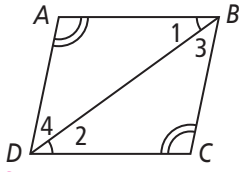



**Given:** $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \parallel \overline{XY}$

**Prove:** $\triangle WXY \cong \triangle YZW$

**It is given that $\overline{WX} \parallel \overline{ZY}$ and $\overline{WZ} \parallel \overline{XY}$, so $\angle XWY \cong \angle ZYW$ and $\angle XYW \cong \angle ZWY$, by the Alternate Interior $\measuredangle$ Thm. $\overline{WY} \cong \overline{YW}$ by the Reflexive Property of $\cong$. So, by ASA Post. $\triangle WXY \cong \triangle YZW$.**

**12. Developing Proof** Complete the proof by filling in the blanks.



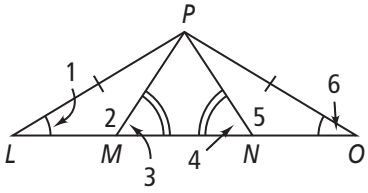

**Given:** $\angle A \cong \angle C$, $\angle 1 \cong \angle 2$

**Prove:** $\triangle ABD \cong \triangle CDB$

**Proof:** $\angle A \cong \angle C$ and $\angle 1 \cong \angle 2$ are given. $\overline{DB} \cong \overline{BD}$ by ? . **Refl. Prop. of Congruence**

So, $\triangle ABD \cong \triangle CDB$ by ? . **AAS**

**13.** Write a paragraph proof.




**Given:** $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$, $\overline{LP} \cong \overline{OP}$

**Prove:** $\triangle LMP \cong \triangle ONP$

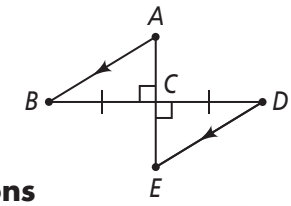
**$\angle 3 \cong \angle 4$ is given. Therefore, $m\angle 3 = m\angle 4$, by def. of $\cong$ $\angle$s. Because $\angle 2$ and $\angle 3$ are linear pairs, and $\angle 4$ and $\angle 5$ are linear pairs, the pairs of angles are suppl. Therefore, $\angle 2 \cong \angle 5$ by the Congruent Suppl. Thm. $\angle 1 \cong \angle 6$ and $\overline{LP} \cong \overline{OP}$ are given, so $\triangle LMP \cong \triangle ONP$, by the AAS Thm.**

Name _____ Class _____ Date _____

# 4-4 ELL Support

## Using Corresponding Parts of Congruent Triangles

**There are two sets of note cards below that show how to prove $\overline{BD}$ is the perpendicular bisector of $\overline{AE}$. The set on the left has the statements and the set on the right has the reasons. Write the statements and the reasons in the correct order.**




| Statements | Reasons |
|---|---|
| $\angle BAC \cong \angle DEC$ | Definition of the perpendicular bisector |
| $\overline{AC} \cong \overline{EC}$ | Angle-Angle-Side (AAS) Theorem |
| $\triangle ACB \cong \triangle ECD$ | When parallel lines are cut by a transversal, alternate interior angles are congruent. |
| $\overline{BD}$ is the perpendicular bisector of $\overline{AE}$. | Corresponding parts of congruent triangles are congruent. |
| $\overline{BC} \cong \overline{DC}$; $\angle ACB$ and $\angle ECD$ are right angles; $\overline{AB} \parallel \overline{DE}$ | Given |
| $\angle ACB \cong \angle ECD$ | All right angles are congruent. |

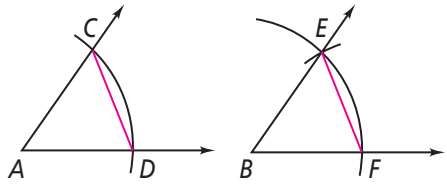
| Statements | Reasons |
|---|---|
| 1) $\overline{BC} \cong \overline{DC}$; $\angle ACB$ and $\angle ECD$ are right angles; $\overline{AB} \parallel \overline{DE}$ | 1) Given |
| 2) $\angle ACB \cong \angle ECD$ | 2) All right angles are congruent. |
| 3) $\angle BAC \cong \angle DEC$ | 3) When parallel lines are cut by a transversal, alternate interior angles are congruent. |
| 4) $\triangle ACB \cong \triangle ECD$ | 4) Angle-Angle-Side (AAS) Theorem |
| 5) $\overline{AC} \cong \overline{EC}$ | 5) Corresponding parts of congruent triangles are congruent. |
| 6) $\overline{BD}$ is the perpendicular bisector of $\overline{AE}$. | 6) Definition of the perpendicular bisector |

Name _____ Class _____ Date _____

# 4-4 Think About a Plan

## Using Corresponding Parts of Congruent Triangles

**Constructions** The construction of $\angle B$ congruent to given $\angle A$ is shown. $\overline{AD} \cong \overline{BF}$ because they are the radii of the same circle. $\overline{DC} \cong \overline{FE}$ because both arcs have the same compass settings. Explain why you can conclude that $\angle A \cong \angle B$.




## Understanding the Problem

1. What is the problem asking you to prove?

   $\angle A \cong \angle B$

2. Segments $\overline{DC}$ and $\overline{FE}$ are not drawn on the construction. Draw them in. What figures are formed by drawing these segments?

   two triangles

3. What information do you need to be able to use corresponding parts of congruent triangles?

   $\overline{AC}$ needs to be shown as congruent to $\overline{BE}$.

## Planning the Solution

4. To use corresponding parts of congruent triangles, which two triangles do you need to show to be congruent?

   $\triangle ACD$ and $\triangle BEF$

5. What reason can you use to state that $\overline{AC} \cong \overline{BE}$?

   They are congruent because they are radii of the same circle by construction.

## Getting an Answer

6. Write a paragraph proof that uses corresponding parts of congruent triangles to prove that $\angle A \cong \angle B$.

   $\overline{AC} \cong \overline{BE}$ and $\overline{AD} \cong \overline{BF}$ because they are both radii of the same circle. $\overline{DC} \cong \overline{FE}$ because they both have the same compass settings. Therefore, $\triangle ACD \cong \triangle BEF$ by SSS and $\angle A \cong \angle B$ by CPCTC.

Name _____ Class _____ Date _____

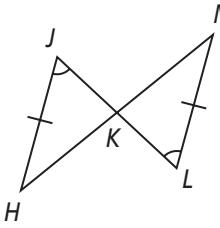
# 4-4 Practice

*Form G*

## Using Corresponding Parts of Congruent Triangles

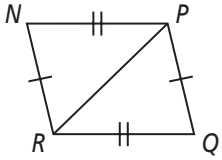
**For each pair of triangles, tell why the two triangles are congruent. Give the congruence statement. Then list all the other corresponding parts of the triangles that are congruent.**

**1.**




$\angle MKL \cong \angle HKJ$ because vertical angles are congruent, so $\triangle KJH \cong \triangle KLM$ by AAS. $\angle KML \cong \angle KHJ$, $\overline{MK} \cong \overline{HK}$, and $\overline{LK} \cong \overline{JK}$.

**2.**

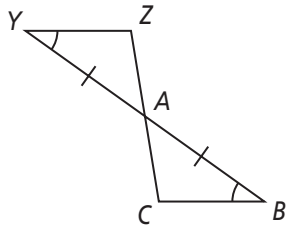



$\overline{PR} \cong \overline{RP}$ because the shared side of the two triangles is congruent to itself, so $\triangle PRQ \cong \triangle RPN$ by SSS. $\angle PRN \cong \angle RPQ$, $\angle NPR \cong \angle QRP$, and $\angle RNP \cong \angle PQR$.

**3.** Complete the proof.

**Given:** $\overline{YA} \cong \overline{BA}$, $\angle B \cong \angle Y$

**Prove:** $\overline{AZ} \cong \overline{AC}$




| Statements | Reasons |
|---|---|
| 1) $\overline{YA} \cong \overline{BA}$, $\angle B \cong \angle Y$ | 1) ? Given |
| 2) $\angle YAZ$ and $\angle BAC$ are vertical angles. | 2) Definition of vertical angles |
| 3) $\angle YAZ \cong \angle BAC$ | 3) ? Vertical angles are congruent. |
| 4) ? $\triangle AZY \cong \triangle ACB$ | 4) ? ASA |
| 5) ? $\overline{AZ} \cong \overline{AC}$ | 5) ? CPCTC |

**4. Open-Ended** Construct a figure that involves two congruent triangles. Set up given statements and write a proof that corresponding parts of the triangles are congruent. Check students' work.

Name _____ Class _____ Date _____

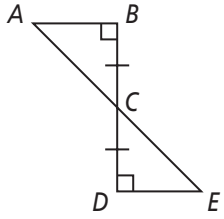
# 4-4 Practice (continued) Form G

## Using Corresponding Parts of Congruent Triangles

**5.** Complete the proof.

**Given:** $\overline{BD} \perp \overline{AB}$, $\overline{BD} \perp \overline{DE}$, $\overline{BC} \cong \overline{DC}$

**Prove:** $\angle A \cong \angle E$

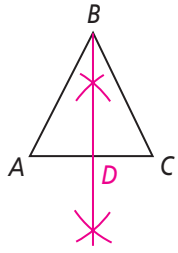



| Statements | Reasons |
|---|---|
| 1) $\overline{BD} \perp \overline{AB}$, $\overline{BD} \perp \overline{DE}$ | 1) ? **Given** |
| 2) $\angle CDE$ and $\angle CBA$ are right angles. | 2) Definition of right angles |
| 3) $\angle CDE \cong \angle CBA$ | 3) ? **All right angles are congruent.** |
| 4) ? **$\angle ECD \cong \angle ACB$** | 4) Vertical angles are congruent. |
| 5) $\overline{BC} \cong \overline{DC}$ | 5) ? **Given** |
| 6) ? **$\triangle CDE \cong \triangle CBA$** | 6) ? **ASA** |
| 7) $\angle A \cong \angle E$ | 7) ? **CPCTC** |

**6. Construction** Use a construction to prove that the two base angles of an isosceles triangle are congruent.

**Given:** Isosceles $\triangle ABC$ with base $\overline{AC}$

**Prove:** $\angle A \cong \angle C$




| Statements | Reasons |
|---|---|
| 1) $\triangle ABC$ is isosceles. | 1) ? **Given** |
| 2) $\overline{AB} \cong \overline{CB}$ | 2) Definition of isosceles triangle. |
| 3) Construct the midpoint of $\overline{AC}$ and call it $D$. Construct $\overline{DB}$. | 3) Construction |
| 4) ? **$\overline{AD} \cong \overline{CD}$** | 4) Definition of midpoint |
| 5) $\overline{BD} \cong \overline{BD}$ | 5) ? **Refl. Prop. of Congruence** |
| 6) $\triangle ABD \cong \triangle CBD$ | 6) ? **SSS** |
| 7) ? **$\angle A \cong \angle C$** | 7) ? **CPCTC** |

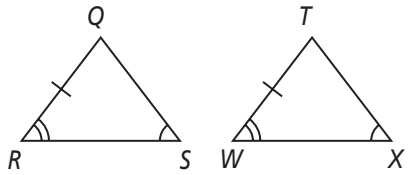
Name _____ Class _____ Date _____

# 4-4 Practice

*Form K*

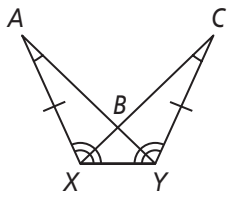
## Using Corresponding Parts of Congruent Triangles

1. **Developing Proof** State why the two triangles are congruent. Then list all other corresponding parts of the triangles that are congruent.



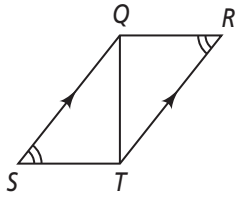

**AAS; $\triangle QRS \cong \triangle TWX$; $\angle Q \cong \angle T$, $\overline{RS} \cong \overline{WX}$**

2. **Developing Proof** State why $\triangle AXY$ and $\triangle CYX$ are congruent. Then list all other corresponding parts of the triangles that are congruent.




**Answers may vary. Sample: SAS; $\angle AYX \cong \angle CXY$, $\overline{AY} \cong \overline{CX}$**

3. **Given:** $\overline{QS} \parallel \overline{RT}$, $\angle R \cong \angle S$
**Prove:** $\angle QTS \cong \angle TQR$



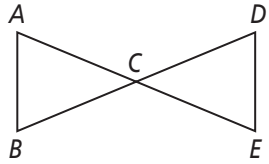

To start, determine how you can prove the triangles are congruent.

The triangles share a side and have a pair of congruent angles.

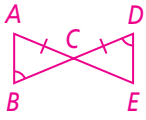
Because $\overline{QS} \parallel$ **$\overline{RT}$**, alternate interior angles $\angle SQT$ and **$\angle RTQ$** are congruent. The triangles can be proven congruent by AAS.

| Statements | Reasons |
|---|---|
| 1) ? **$\overline{QS} \parallel \overline{RT}$, $\angle R \cong \angle S$** | 1) Given |
| 2) ? **$\angle SQT \cong \angle RTQ$** | 2) Alternate interior $\measuredangle$ are $\cong$. |
| 3) ? **$\overline{QT} \cong \overline{TQ}$** | 3) Reflexive Property of Congruence |
| 4) ? **$\triangle STQ \cong \triangle RQT$** | 4) AAS |
| 5) ? **$\angle QTS \cong \angle TQR$** | 5) Corresp. parts of $\cong$ $\measuredangle$ are $\cong$. |

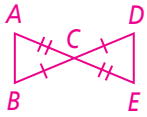
**Reasoning** **Copy and mark the figure to show the given information. Explain how you would prove $\overline{AB} \cong \overline{DE}$.**



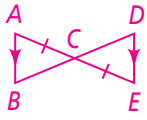

4. **Given:** $\overline{AC} \cong \overline{DC}$, $\angle B \cong \angle D$
**AAS and Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.**




5. **Given:** $\overline{AE}$ bisects $\overline{BD}$, $\overline{DB}$ bisects $\overline{AE}$
**SAS and Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.**




6. **Given:** $\overline{AB} \parallel \overline{DE}$, $AC = EC$
**Answers may vary. Sample: AAS and Corresp. parts of $\cong$ $\measuredangle$ are $\cong$.**

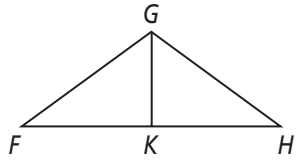


Name _____ Class _____ Date _____

# 4-4 Practice (continued) Form K

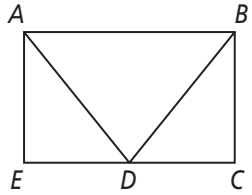
## Using Corresponding Parts of Congruent Triangles

**7.** **Given:** $\overline{GK}$ is the perpendicular bisector of $\overline{FH}$.
**Prove:** $\overline{FG} \cong \overline{HG}$




| Statements | Reasons |
|---|---|
| 1) $\overline{GK}$ is the perpendicular bisector of $\overline{FH}$. | 1) ? Given |
| 2) ? $\overline{KF} \cong \overline{KH}$ | 2) Def. of perpendicular bis. |
| 3) $\angle GKF \cong \angle GKH$ | 3) Def. of perpendicular bis.; all right $\triangle$ are $\cong$. |
| 4) ? $\overline{GK} \cong \overline{GK}$ | 4) Refl. Prop. of $\cong$ |
| 5) $\triangle FGK \cong \triangle HGK$ | 5) ? SAS |
| 6) ? $\overline{FG} \cong \overline{HG}$ | 6) Corresp. parts of $\cong$ $\triangle$ are $\cong$. |

**8.** **Given:** $ABCE$ is a rectangle.
$D$ is the midpoint of $\overline{CE}$.
**Prove:** $\overline{AD} \cong \overline{BD}$




| Statements | Reasons |
|---|---|
| 1) $ABCE$ is a rectangle. $D$ is the midpoint of $\overline{CE}$. | 1) Given |
| 2) $\angle AED \cong \angle BCD$ | 2) Definition of rectangle |
| 3) $\overline{AE} \cong \overline{BC}$ | 3) Definition of rectangle |
| 4) ? $\overline{ED} \cong \overline{CD}$ | 4) ? Definition of midpoint |
| 5) ? $\triangle AED \cong \triangle BCD$ | 5) ? SAS |
| 6) ? $\overline{AD} \cong \overline{BD}$ | 6) ? Corresp. parts of $\cong$ $\triangle$ are $\cong$. |

Name _____ Class _____ Date _____

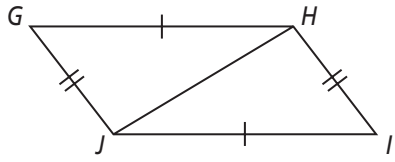
# 4-4 Standardized Test Prep

Using Corresponding Parts of Congruent Triangles

## Multiple Choice

**For Exercises 1–6, choose the correct letter.**

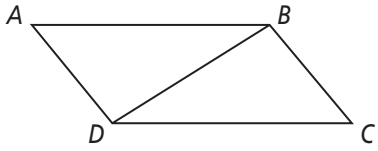
**1.** Based on the given information in the figure at the right, how can you justify that $\triangle JHG \cong \triangle HJI$? **B**




(A) ASA (C) AAS

(B) SSS (D) ASA

**2.** In the figure at the right the following is true: $\angle ABD \cong \angle CDB$ and $\angle DBC \cong \angle BDA$. How can you justify that $\triangle ABD \cong \triangle CDB$? **H**




(F) SAS (H) ASA

(G) SSS (I) CPCTC

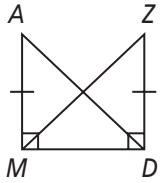
**3.** $\triangle BRM \cong \triangle KYZ$. How can you justify that $\overline{YZ} \cong \overline{RM}$? **A**

(A) CPCTC (B) SAS (C) ASA (D) SSS

**4.** Which statement *cannot* be justified given only that $\triangle PBJ \cong \triangle TIM$? **I**

(F) $\overline{PB} \cong \overline{TI}$ (G) $\angle B \cong \angle I$ (H) $\angle BJP \cong \angle IMT$ (I) $\overline{JP} \cong \overline{MI}$

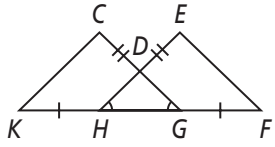
**5.** In the figure at the right, which theorem or postulate can you use to prove $\triangle ADM \cong \triangle ZMD$? **C**




(A) ASA (C) SAS

(B) SSS (D) AAS

**6.** In the figure at the right, which theorem or postulate can you use to prove $\triangle KGC \cong \triangle FHE$? **H**

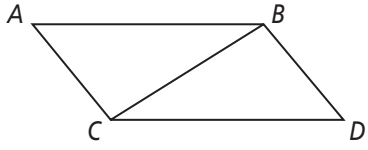



(F) ASA (H) SAS

(G) SSS (I) AAS

## Short Response

**7.** What would a brief plan for the following proof look like?




**Given:** $\overline{AB} \cong \overline{DC}$, $\angle ABC \cong \angle DCB$

**Prove:** $\overline{AC} \cong \overline{DB}$

**[2] $\overline{CB} \cong \overline{BC}$ by the Reflexive Property. $\triangle CBD \cong \triangle BCA$ by SAS. $\overline{AC} \cong \overline{DB}$ by CPCTC; [1] one step missing or one reason incorrect [0] incorrect or no response**

Name _____ Class _____ Date _____

# 4-4 Enrichment

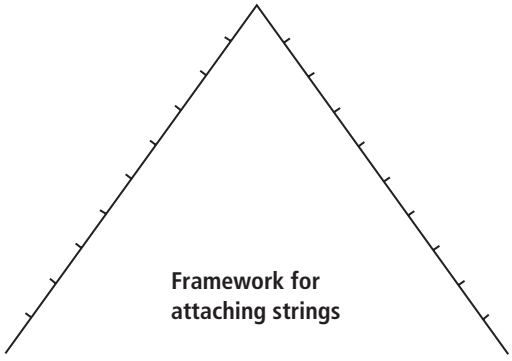
## Using Corresponding Parts of Congruent Triangles

### String Art

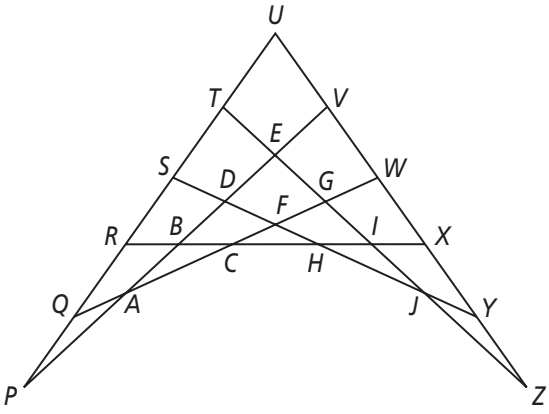
Artists have always used geometry, but in some cases geometry can become the driving force in art. In string art, for example, polygons combine to make interesting patterns. To make string art, you usually start with a frame with nails at equal intervals. The frame below left was used to make the art on the right. (The string wrapping around the nails has been omitted.)

**Use the diagram and information below for Exercise 1.**

**Given:** $PQ = QR = RS = ST = TU = UV = VW = WX = XY = YZ$



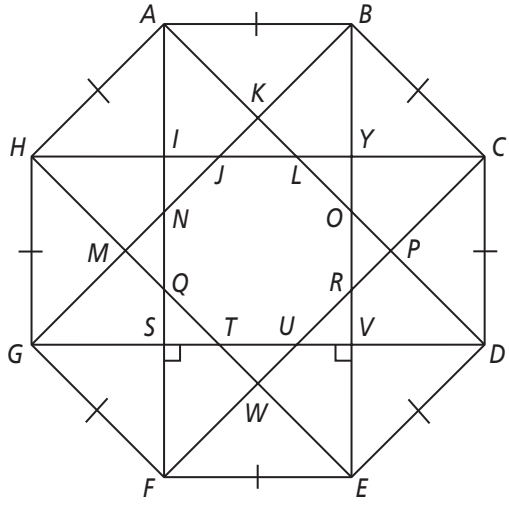
Framework for attaching strings




1. Find as many triangles as you can prove congruent in the above diagram. Name the postulate or theorem (SSS, SAS, ASA, or AAS) that justifies your answer. **Answers may vary. Sample: $\triangle TUZ \cong \triangle VUP$ (SAS); $\triangle SUY \cong \triangle WUQ$ (SAS); $\triangle QAP \cong \triangle YJZ$ (ASA); $\triangle TJS \cong \triangle VAW$ (SSS)**

**Use the diagram and information below for Exercises 2 and 3.**

**Given:** $QF = RE$, $\overline{QF} \perp \overline{SV} \perp \overline{RE}$, $\overline{FE} \parallel \overline{GD}$




2. Find three pairs of congruent triangles in the diagram. For each pair of triangles, name the postulate or theorem (SSS, SAS, ASA, or AAS) that justifies your answer. **Sample: $\triangle QWF \cong \triangle EWR$ (ASA); $\triangle QFE \cong \triangle REF$ (SAS); $\triangle FWE \cong \triangle FWQ$ (ASA)**
3. If $\overline{HC} \parallel \overline{AB} \parallel \overline{GD}$ and $\overline{HG} \parallel \overline{AF} \parallel \overline{CD}$, find as many congruent triangles as you can in the diagram, and give an example for each type of triangle. **Sample: 7 △ congruent to $\triangle IJN$, 23 △ congruent to $\triangle FWE$, 15 △ congruent to $\triangle QFE$, and 7 △ congruent to $\triangle ASD$**

Name _____ Class _____ Date _____

# 4-4 Reteaching

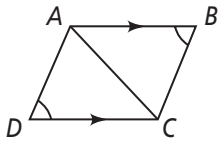
## Using Corresponding Parts of Congruent Triangles

If you can show that two triangles are congruent, then you can show that all the corresponding angles and sides of the triangles are congruent.

**Problem**

**Given:** $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$

**Prove:** $\overline{BC} \cong \overline{DA}$




In this case you know that $\overline{AB} \parallel \overline{DC}$. $\overline{AC}$ forms a transversal and creates a pair of alternate interior angles, $\angle BAC$ and $\angle DCA$.

You have two pairs of congruent angles, $\angle BAC \cong \angle DCA$ and $\angle B \cong \angle D$. Because you know that the shared side is congruent to itself, you can use AAS to show that the triangles are congruent. Then use the fact that corresponding parts are congruent to show that $\overline{BC} \cong \overline{DA}$. Here is the proof:

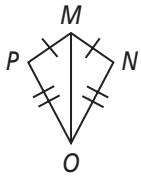
| Statements | Reasons |
|---|---|
| 1) $\overline{AB} \parallel \overline{DC}$ | 1) Given |
| 2) $\angle BAC \cong \angle DCA$ | 2) Alternate Interior Angles Theorem |
| 3) $\angle B \cong \angle D$ | 3) Given |
| 4) $\overline{AC} \cong \overline{CA}$ | 4) Reflexive Property of Congruence |
| 5) $\triangle ABC \cong \triangle CDA$ | 5) AAS |
| 6) $\overline{BC} \cong \overline{DA}$ | 6) CPCTC |

## Exercises

**1.** Write a two-column proof.

**Given:** $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$

**Prove:** $\angle N \cong \angle P$




| Statements | Reasons |
|---|---|
| 1) ? $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$ | 1) Given |
| 2) $\overline{MO} \cong \overline{MO}$ | 2) ? Reflexive Property of $\cong$ |
| 3) ? $\triangle MNO \cong \triangle MPO$ | 3) ? SSS |
| 4) $\angle N \cong \angle P$ | 4) ? CPCTC |

# 4-4 Reteaching (continued)

## Using Corresponding Parts of Congruent Triangles

**2.** Write a two-column proof.

**Given:** $\overline{PT}$ is a median and an altitude of $\triangle PRS$.

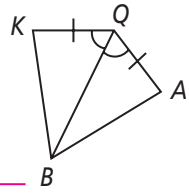
**Prove:** $\overline{PT}$ bisects $\angle RPS$.

| Statements | Reasons |
|---|---|
| 1) $\overline{PT}$ is a median of $\triangle PRS$. | 1) ? Given |
| 2) ? *T is the midpoint of $\overline{RS}$.* | 2) Definition of median |
| 3) ? *$\overline{RT} \cong \overline{ST}$* | 3) Definition of midpoint |
| 4) $\overline{PT}$ is an altitude of $\triangle PRS$. | 4) ? Given |
| 5) $\overline{PT} \perp \overline{RS}$ | 5) ? Definition of altitude |
| 6) $\angle PTS$ and $\angle PTR$ are right angles. | 6) ? Definition of perpendicular |
| 7) ? *$\angle PTS \cong \angle PTR$* | 7) All right angles are congruent. |
| 8) ? *$\overline{PT} \cong \overline{PT}$* | 8) Reflexive Property of Congruence |
| 9) ? *$\triangle PTS \cong \triangle PTR$* | 9) SAS |
| 10) $\angle TPS \cong \angle TPR$ | 10) ? CPCTC |
| 11) ? *$\overline{PT}$ bisects $\angle RPS$.* | 11) ? Definition of angle bisector |

**3.** Write a two-column proof.

**Given:** $\overline{QK} \cong \overline{QA}$; $\overline{QB}$ bisects $\angle KQA$.

**Prove:** $\overline{KB} \cong \overline{AB}$

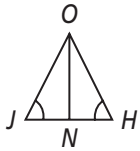



| Statements | Reasons |
|---|---|
| 1) *$\overline{QK} \cong \overline{QA}$; $\overline{QB}$ bisects $\angle KQA$.* | 1) Given |
| 2) *$\angle KQB \cong \angle AQB$* | 2) Def. of $\angle$ bis. |
| 3) *$\overline{BQ} \cong \overline{BQ}$* | 3) Refl. Prop. of Congruence |
| 4) *$\triangle KBQ \cong \triangle ABQ$* | 4) SAS |
| 5) *$\overline{KB} \cong \overline{AB}$* | 5) CPCTC |

**4.** Write a two-column proof.

**Given:** $\overline{ON}$ bisects $\angle JOH$, $\angle J \cong \angle H$

**Prove:** $\overline{JN} \cong \overline{HN}$



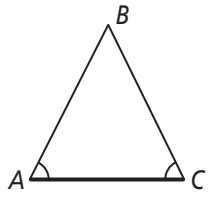
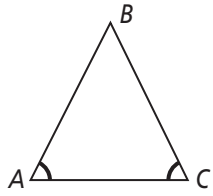
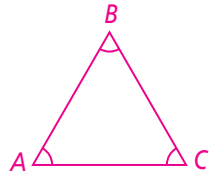
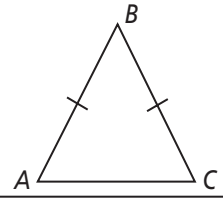
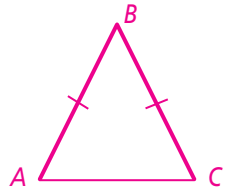
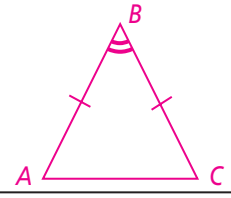

| Statements | Reasons |
|---|---|
| 1) *$\overline{ON}$ bisects $\angle JOH$, $\angle J \cong \angle H$* | 1) Given |
| 2) *$\angle JON \cong \angle HON$* | 2) Def. of $\angle$ bis. |
| 3) *$\overline{ON} \cong \overline{ON}$* | 3) Refl. Prop. of Congruence |
| 4) *$\triangle JON \cong \triangle HON$* | 4) AAS |
| 5) *$\overline{JN} \cong \overline{HN}$* | 5) CPCTC |

Name _____ Class _____ Date _____

# 4-5 ELL Support

## Isosceles and Equilateral Triangles

**Complete the vocabulary chart by filling in the missing information.**

| Word or Word Phrase | Definition | Picture or Example |
|---|---|---|
| base | The *base* of an isosceles triangle is the side included between the pair of congruent angles. |   |
| base angles | 1. The *base angles* are the two congruent angles of an isosceles triangle. |   |
| corollary | 2. A *corollary* is a theorem that can be proved easily by another theorem. | A corollary to the Isosceles Triangle Theorem is: If a triangle is equilateral, then the triangle is equiangular. |
| equiangular triangle | An *equiangular triangle* is a triangle with three congruent angles. The angles of an equiangular triangle all measure 60. | 3.   |
| isosceles triangle | 4. An *isosceles triangle* is a triangle with two congruent sides. |   |
| legs | The *legs* are the two congruent sides of an isosceles triangle. | 5.   |
| vertex angle | 6. The *vertex angle* is the angle formed by the legs of an isosceles triangle. | 7.   |

Name _____ Class _____ Date _____

# 4-5 Think About a Plan

## Isosceles and Equilateral Triangles

**Algebra** The length of the base of an isosceles triangle is $x$. The length of a leg is $2x - 5$. The perimeter of the triangle is 20. Find $x$.

### Know

1. What is the perimeter of a triangle?
   **the sum of the sides of a triangle**

2. What is an isosceles triangle?
   **a triangle with two sides the same length**

### Need

3. What are the sides of an isosceles triangle called?
   **base and leg**

4. How many of each type of side are there?
   **1 base and 2 legs**

5. The lengths of the base and one leg are given. What is the third side of the triangle called?
   **leg**

### Plan

6. Write an expression for the length of the third side. **$2x - 5$**

7. Write an equation for the perimeter of this isosceles triangle.
   **$x + (2x - 5) + (2x - 5) = 20$**

8. Solve the equation for $x$. Show your work.
   **$5x - 10 = 20$**
   **$5x = 30$**
   **$x = 6$**

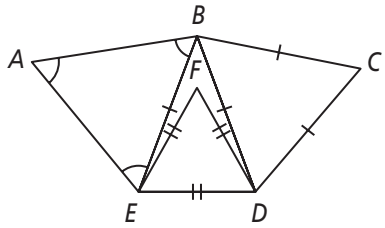
Name _____ Class _____ Date _____

# 4-5 Practice

*Form G*

## Isosceles and Equilateral Triangles

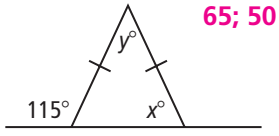
**Complete each statement. Explain why it is true.**



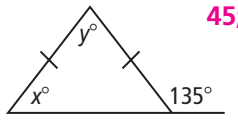

1. $\angle DBC \cong$ ? $\cong \angle CDB$
   **$\angle BCD$; all the angles of an equilateral triangle are congruent.**
2. $\angle BED \cong$ ?
   **$\angle BDE$; the base angles of an isosceles triangle are congruent.**
3. $\angle FED \cong$ ? $\cong \angle DFE$
   **$\angle EDF$; all the angles of an equilateral triangle are congruent.**
4. $\overline{AB} \cong$ ? $\cong \overline{BE}$
   **$\overline{EA}$; all the sides of an equilateral triangle are congruent.**

**Algebra Find the values of $x$ and $y$.**

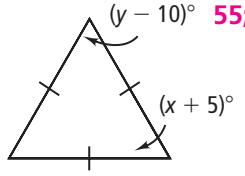
5. **65; 50**



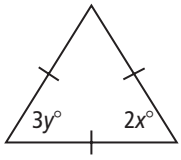

6. **45; 90**



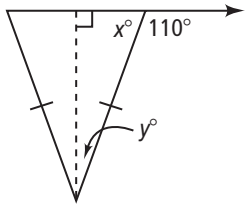

7. **55; 70**



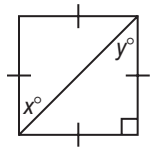

8. **30; 20**




9. **70; 20**

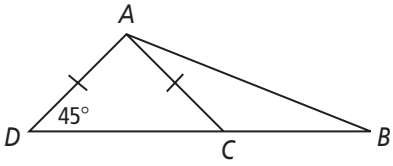



10. **45; 45**

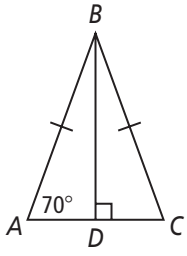



**Use the properties of isosceles and equilateral triangles to find the measure of the indicated angle.**

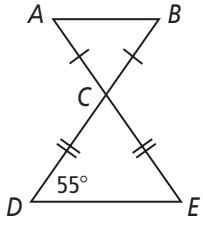
11. $m\angle ACB$ **135**




12. $m\angle DBC$ **20**




13. $m\angle ABC$ **55**



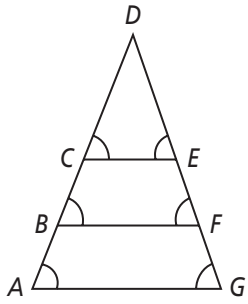

14. Equilateral $\triangle ABC$ and isosceles $\triangle DBC$ share side $BC$. If $m\angle BDC = 34$ and $BD = BC$, what is the measure of $\angle ABD$? (*Hint:* it may help to draw the figure described.) **172**

Name _____ Class _____ Date _____

# 4-5 Practice (continued) Form G

## Isosceles and Equilateral Triangles

**Use the diagram for Exercises 15–17 to complete each congruence statement. Explain why it is true.**




**15.** $\overline{DF} \cong$ ? **$\overline{DB}$; Converse of the Isosceles Triangle Theorem**

**16.** $\overline{DG} \cong$ ? **$\overline{DA}$; Converse of the Isosceles Triangle Theorem**

**17.** $\overline{DC} \cong$ ? **$\overline{DE}$; Converse of the Isosceles Triangle Theorem**

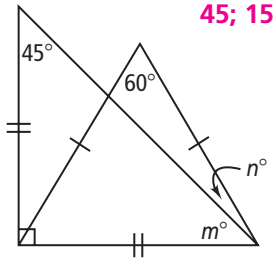
**18.** The wall at the front entrance to the Rock and Roll Hall of Fame and Museum in Cleveland, Ohio, is an isosceles triangle. The triangle has a vertex angle of 102. What is the measure of the base angles? **39**

**19. Reasoning** An exterior angle of an isosceles triangle has the measure 130. Find two possible sets of measures for the angles of the triangle.
**50, 50, and 80; 50, 65, and 65**

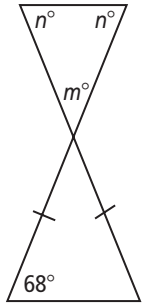
**20. Open-Ended** Draw a design that uses three equilateral triangles and two isosceles triangles. Label the vertices. List all the congruent sides and angles.
**Check students' work.**

**Algebra Find the values of *m* and *n*.**

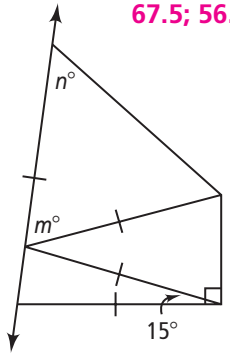
**21.** **45; 15**




**22.** **44; 68**




**23.** **67.5; 56.25**




**24. Writing** Explain how a corollary is related to a theorem. Use examples from this lesson in making your comparison.
**A theorem is a statement that is proven true by a series of steps. A corollary is a statement that can be taken directly from the conclusion of a theorem, usually by applying the theorem to a specific situation. For example, Theorems 4-3 and 4-4 are general statements about all isosceles triangles. Their corollaries apply the theorems to equilateral triangles.**

Name _____ Class _____ Date _____

# 4-5 Practice

*Form K*

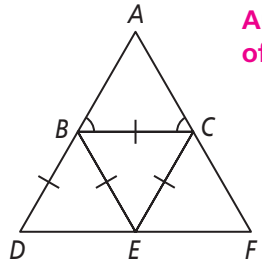
## Isosceles and Equilateral Triangles

**Complete each statement. Explain why it is true.**

1. $\overline{AB} \cong$ ? Answers may vary. Sample: $\overline{AC}$; the legs of an isosceles triangle are congruent.

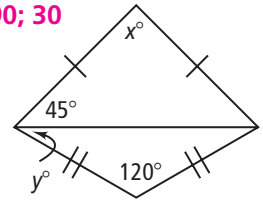
2. $\angle BDE \cong$ ? Answers may vary. Sample: $\angle BED$; the base angles of an isosceles triangle are congruent.

3. $\angle CBE \cong$ ? $\cong \angle BCE$ Answers may vary. Sample: $\angle BEC$; all the angles of an equilateral triangle are congruent.




**Algebra Find the values of $x$ and $y$.**

4. 90; 30

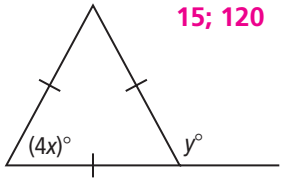



To start, determine what types of triangles are shown in the diagram. Then use an equation to find $x$.

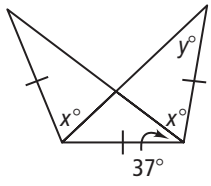
Because two sides are marked congruent in both triangles, the triangles are both ? . isosceles

$45 +$ 45 $+ x =$ 180

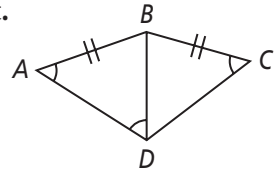
5. 15; 120




6. 69; 37




**Use the properties of isosceles triangles to complete each statement.**




7. If $m\angle ADB = 54$, then $m\angle CBD =$ ? . 72

8. If $AB = 8$, then $BD =$ ? . 8

9. You are asked to put a V-shaped roof on a house. The slope of the roof is 40°. What is the measure of the angle needed at the vertex of the roof? 100

10. **Reasoning** The measure of one angle of a triangle is 30. Of the two remaining angles, the larger angle is four times the size of the smaller angle. Is the triangle isosceles? Explain. Yes, because the measure of the smaller angle is 30.

Name _____ Class _____ Date _____

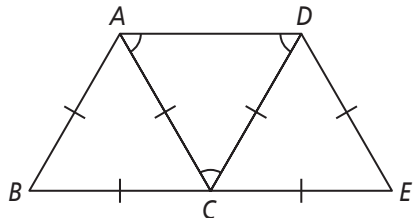
# 4-5 Practice (continued) Form K

## Isosceles and Equilateral Triangles

**For Exercises 11 and 12, use the diagram to complete each congruence statement. Then list the theorem or corollary that proves the statement. The first one has been done for you.**

$\angle B \cong$ ?

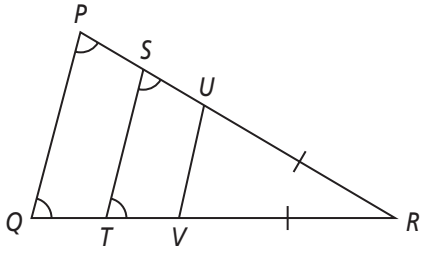
Answer: $\angle BAC$ (or $\angle ACB$); Corollary to Theorem 4-3




**11.** $\overline{AD} \cong$ ? **Answers may vary. Sample: $\overline{AC}$ or $\overline{DC}$; Corollary to Theorem 4-4**

**12.** $\angle E \cong$ ? **Answers may vary. Sample: $\angle DCE$ or $\angle CDE$; Corollary to Theorem 4-3**

**For Exercises 13–15, use the diagram to complete each congruence statement. Then list the theorem or corollary that proves the statement.**

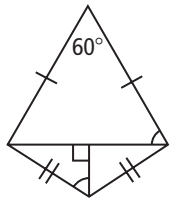



**13.** $\overline{PR} \cong$ ? **$\overline{QR}$; Converse of the Isosceles Triangle Theorem**

**14.** $\angle RUV \cong$ ? **$\angle RVU$; Isosceles Triangle Theorem**

**15.** $\overline{SR} \cong$ ? **$\overline{TR}$; Converse of the Isosceles Triangle Theorem**

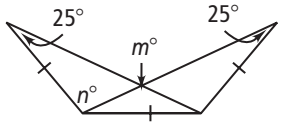
**16. Reasoning** An equilateral triangle and an isosceles triangle share a common side as shown at the right. What is the measure of the vertex angle? Explain.




**120; the congruent angles in the diagram both have a measure of 60. The base angles of the isosceles triangle have a measure of 30 because one is the other angle in a right triangle. The vertex angle must measure 120 if the base angles both measure 30.**

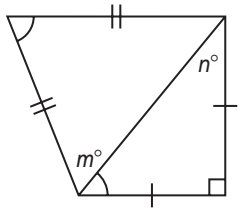
**Algebra Find the values of $m$ and $n$.**

**17.**




**130; 105**

**18.**




**67.5; 45**

Name _____ Class _____ Date _____

# 4-5 Standardized Test Prep

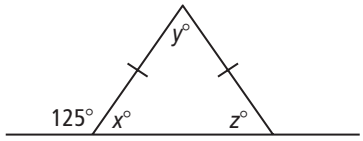
Isosceles and Equilateral Triangles

## Gridded Response

**Solve each exercise and enter your answer on the grid provided.**

**Refer to the diagram for Exercises 1–3.**

**1.** What is the value of $x$?




**2.** What is the value of $y$?

**3.** What is the value of $z$?

**4.** The measures of two of the sides of an equilateral triangle are $3x + 15$ in. and $7x - 5$ in. What is the measure of the third side in inches?

**5.** In $\triangle GHI$, $HI = GH$, $m\angle IHG = 3x + 4$, and $m\angle IGH = 2x - 24$. What is $m\angle HIG$?

## Answers

**1.** 55

**2.** 70

**3.** 55

**4.** 30

**5.** 40

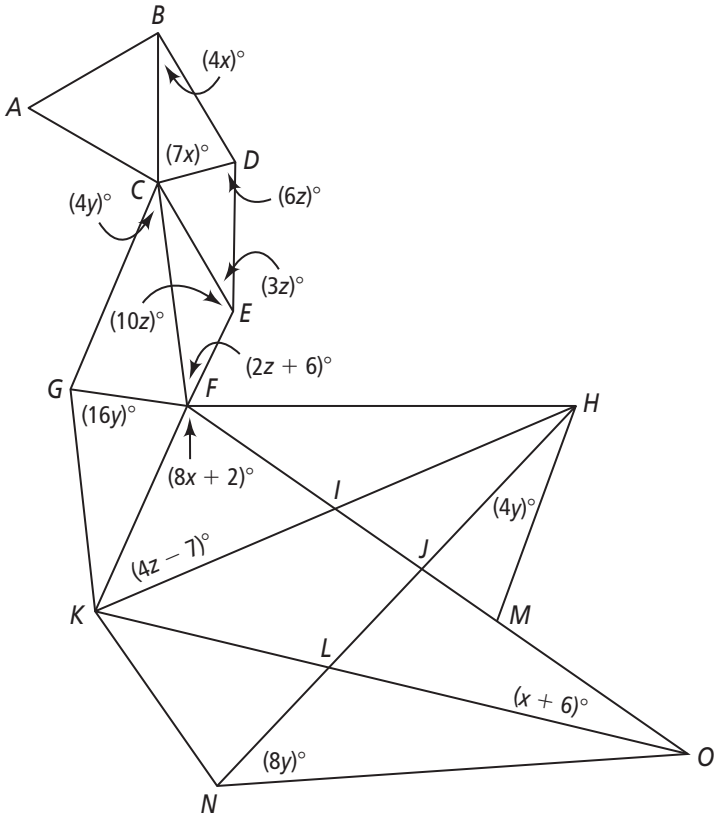
Name _____ Class _____ Date _____

# 4-5 Enrichment

## Isosceles and Equilateral Triangles

**The swan below is composed of several triangles. Use the given information and the figure to find each angle measure. Note: Figure not drawn to scale.**

**Given:** $\triangle ABC$ is equilateral; $\angle BCD \cong \angle BDC$; $\overline{DE} \cong \overline{CE} \cong \overline{EF}$; $\angle CGF \cong \angle CFG$; $\triangle GCF \cong \triangle GKF \cong \triangle JHM$; $\triangle KFH \cong \triangle KLH$; $\overline{KO} \cong \overline{FO}$; $\angle HKN \cong \angle HNK$; $\overline{JN} \cong \overline{JO}$




| | | | |
|---|---|---|---|
| **1.** $m\angle ABC$ **60** | **2.** $m\angle BCA$ **60** | **3.** $m\angle CAB$ **60** | **4.** $m\angle BCD$ **70** |
| **5.** $m\angle BDC$ **70** | **6.** $m\angle CBD$ **40** | **7.** $m\angle EDC$ **72** | **8.** $m\angle ECD$ **72** |
| **9.** $m\angle CED$ **36** | **10.** $m\angle ECF$ **30** | **11.** $m\angle EFC$ **30** | **12.** $m\angle CEF$ **120** |
| **13.** $m\angle CGF$ **80** | **14.** $m\angle CFG$ **80** | **15.** $m\angle GCF$ **20** | **16.** $m\angle KGF$ **80** |
| **17.** $m\angle KFG$ **80** | **18.** $m\angle GKF$ **20** | **19.** $m\angle FKH$ **41** | **20.** $m\angle FHK$ **23** |
| **21.** $m\angle KFH$ **116** | **22.** $m\angle KHL$ **23** | **23.** $m\angle HKL$ **41** | **24.** $m\angle KLH$ **116** |
| **25.** $m\angle HJM$ **80** | **26.** $m\angle HMJ$ **80** | **27.** $m\angle JHM$ **20** | **28.** $m\angle OFK$ **82** |
| **29.** $m\angle OKF$ **82** | **30.** $m\angle KOF$ **16** | **31.** $m\angle HKN$ **78.5** | **32.** $m\angle HNK$ **78.5** |
| **33.** $m\angle OKN$ **37.5** | **34.** $m\angle JNO$ **40** | **35.** $m\angle JON$ **40** | **36.** $m\angle NJO$ **100** |

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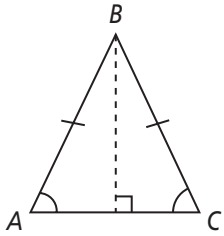
# 4-5 Reteaching

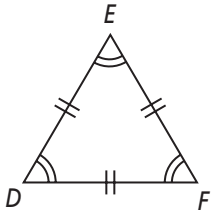
## Isosceles and Equilateral Triangles

Two special types of triangles are isosceles triangles and equilateral triangles.

An *isosceles* triangle is a triangle with two congruent sides. The base angles of an isosceles triangle are also congruent. An altitude drawn from the shorter base splits an isosceles triangle into two congruent right triangles.



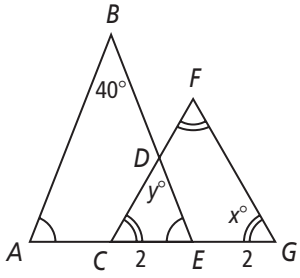

An *equilateral* triangle is a triangle that has three congruent sides and three congruent angles. Each angle measures 60°.




You can use the special properties of isosceles and equilateral triangles to find or prove different information about a given figure.

Look at the figure at the right.

You should be able to see that one of the triangles is equilateral and one is isosceles.




**Problem**

What is $m\angle A$?

$\triangle ABC$ is isosceles because it has two base angles that are congruent. Because the sum of the measures of the angles of a triangle is 180, and $m\angle B = 40$, you can solve to find $m\angle A$.

| | |
|---|---|
| $m\angle A + m\angle B + m\angle BEA = 180$ | There are 180° in a triangle. |
| $m\angle A + 40 + m\angle A = 180$ | Substitution Property |
| $2\,m\angle A + 40 = 180$ | Combine like terms. |
| $2\,m\angle A = 140$ | Subtraction Property of Equality |
| $m\angle A = 70$ | Division Property of Equality |

**Problem**

What is $FC$?

$\triangle CFG$ is equilateral because it has three congruent angles.

$CG = (2 + 2) = 4$, and $CG = FG = FC$.

So, $FC = 4$.

Name _____ Class _____ Date _____

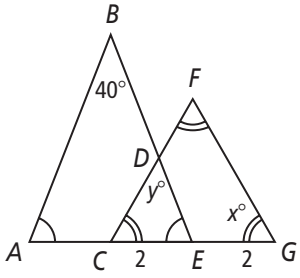
# 4-5 Reteaching (continued)

## Isosceles and Equilateral Triangles

**Problem**

What is the value of $x$?

Because $x$ is the measure of an angle in an equilateral triangle, $x = 60$.



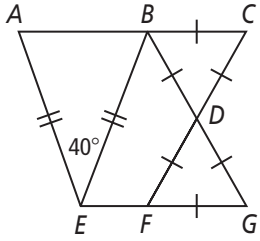

**Problem**

What is the value of $y$?

| | |
|---|---|
| $m\angle DCE + m\angle DEC + m\angle EDC = 180$ | There are 180° in a triangle. |
| $60 + 70 + y = 180$ | Substitution Property |
| $y = 50$ | Subtraction Property of Equality |

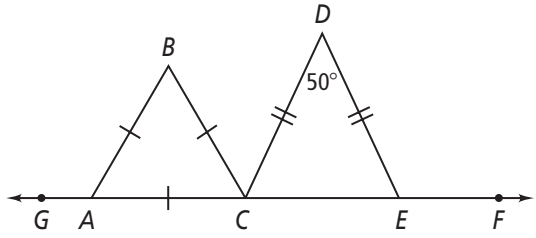
## Exercises

**Complete each statement. Explain why it is true.**



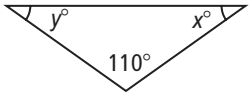

1. $\angle EAB \cong$ ? 
   **$\angle EBA$; base angles of an isosceles triangle are congruent.**
2. $\angle BCD \cong$ ? $\cong \angle DBC$
   **$\angle CDB$; the angles of an equilateral triangle are congruent.**
3. $\overline{FG} \cong$ ? $\cong \overline{DF}$
   **$\overline{GD}$; the sides of an equilateral triangle are congruent.**

**Determine the measure of the indicated angle.**

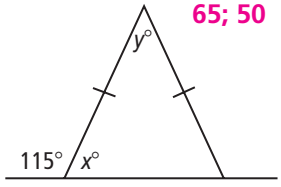



4. $\angle ACB$ **60**
5. $\angle DCE$ **65**
6. $\angle BCD$ **55**

**Algebra Find the value of $x$ and $y$.**

7. 

**35; 35**

8. 

**65; 50**

9. **Reasoning** An exterior angle of an isosceles triangle has a measure 140. Find two possible sets of measures for the angles of the triangle. **40, 40, 100; 40, 70, 70**

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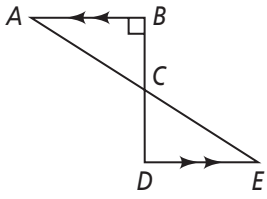
# 4-6 ELL Support

## Congruence in Right Triangles

**The column on the left shows the steps used to prove that $\overline{AB} \cong \overline{ED}$. Use the column on the left to answer each question in the column on the right.**

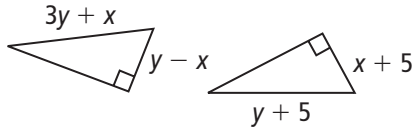
| | |
|---|---|
| **Problem** **$\cong$ in Right Triangles**<br>**Given: $C$ is the midpoint of $\overline{AE}$ and $\overline{BD}$.**<br>**Prove: $\overline{AB} \cong \overline{ED}$**<br>  <br> | **1.** What is the definition of the midpoint of a line segment?<br>The midpoint is halfway between the two endpoints. It divides the line segment into two equal halves. |
| 1) $C$ is the midpoint of $\overline{AE}$ and $\overline{BD}$; $\overline{AB} \parallel \overline{DE}$; $m\angle B = 90$<br>Given | **2.** How do you know that $\overline{AB} \parallel \overline{DE}$ and $m\angle B = 90$?<br>by reading the diagram |
| 2) $\overline{AC} \cong \overline{EC}$; $\overline{BC} \cong \overline{DC}$<br>Definition of midpoint | **3.** What does the symbol $\cong$ between two line segments mean?<br>The line segments are congruent. |
| 3) $\angle B \cong \angle D$<br>Alternate Interior Angles Theorem | **4.** What does the word *interior* mean?<br>Interior means on the inside of. |
| 4) $\angle B$ and $\angle D$ are right angles.<br>Definition of a right angle | **5.** What is the measure of $\angle D$?<br>90 |
| 5) $\triangle ABC \cong \triangle EDC$<br>Hypotenuse-Leg (HL) Postulate | **6.** What information is necessary to apply the HL Postulate?<br>Two right triangles must have congruent hypotenuses and one congruent leg. |
| 6) $\overline{AB} \cong \overline{ED}$<br>Corresponding parts of congruent triangles are congruent. | **7.** What are the corresponding angles and sides for $\triangle ABC$ and $\triangle EDC$?<br>$\angle B$ and $\angle D$, $\angle A$ and $\angle E$, $\angle BCA$ and $\angle DCE$, $\overline{AB}$ and $\overline{ED}$, $\overline{AC}$ and $\overline{EC}$, $\overline{BC}$ and $\overline{DC}$ |

Name _____ Class _____ Date _____

# 4-6 Think About a Plan

## Congruence in Right Triangles

**Algebra** For what values of $x$ and $y$ are the triangles congruent by HL?




### Know

1. For two triangles to be congruent by the Hypotenuse-Leg Theorem, there must be a pair of right angles, and the lengths of the hypotenuses and one of the legs of each triangle must be equal.

2. The length of the hypotenuse of the triangle on the left is $3y + x$ and the hypotenuse of the triangle on the right is $y + 5$.

3. The length of the leg of the triangle on the left is $y - x$ and the length of the leg of the triangle on the right is $x + 5$.

### Need

4. To solve the problem you need to find the values of $x$ and $y$.

### Plan

5. What system of equations can you use to find the values of $x$ and $y$?
   $3y + x = y + 5$; $y - x = x + 5$

6. What method(s) can you use to solve the system of equations?
   Sample: Solve one equation for $y$ and substitute into the other equation; graph both equations.

7. What is the value of $y$? What is the value of $x$? 3; −1

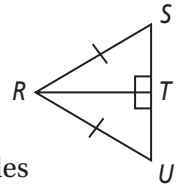
Name _____ Class _____ Date _____

# 4-6 Practice

*Form G*

## Congruence in Right Triangles

**1. Developing Proof** Complete the paragraph proof.



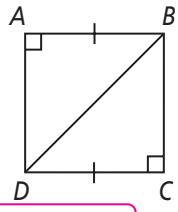

**Given:** $\overline{RT} \perp \overline{SU}$, $\overline{RU} \cong \overline{RS}$.

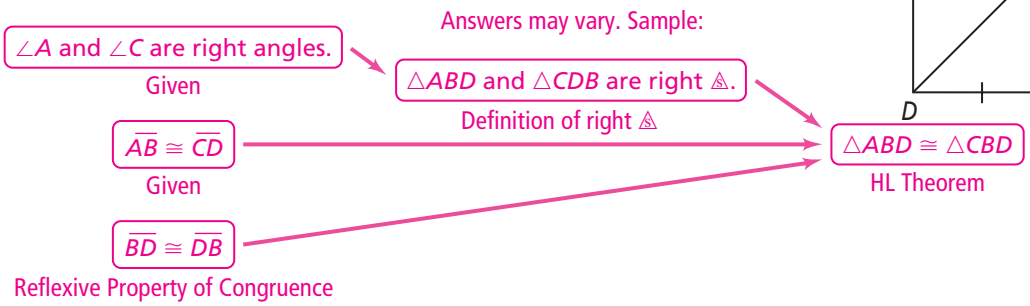
**Prove:** $\triangle RUT \cong \triangle RST$

**Proof:** It is given that $\overline{RT} \perp \overline{SU}$. So, ___∠RTS___ and ___∠RTU___ are ___right___ angles because perpendicular lines form ___right___ angles. ___$\overline{RT}$___ $\cong \overline{RT}$ by the Reflexive Property of Congruence. It is given that ___$\overline{RU}$___ $\cong \overline{RS}$. So, $\triangle RUT \cong \triangle RST$ by ___HL___.

**2.** Look at Exercise 1. If $m\angle RST = 46$, what is $m\angle RUT$? **46**

**3.** Write a flow proof. Use the information from the diagram to prove that $\triangle ABD \cong \triangle CDB$.

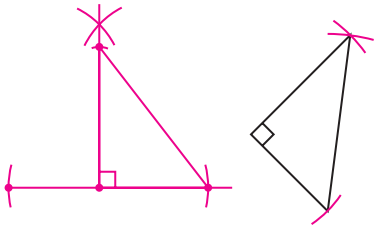


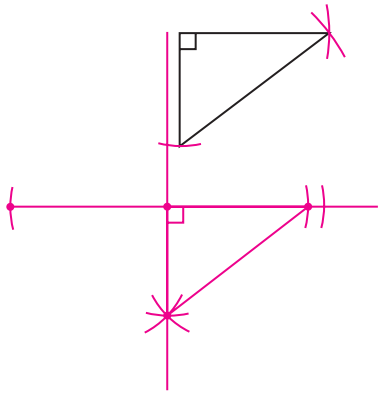

**4.** Look at Exercise 3. Can you prove that $\triangle ABD \cong \triangle CDB$ without using the Hypotenuse-Leg Theorem? Explain. **Yes; answers may vary. Sample: You know that $\overline{AB} \cong \overline{CD}$ from the diagram and $\overline{DB} \cong \overline{BD}$ by the Reflexive Property of Congruence. Because the triangles are right triangles, the sides are related by the Pythagorean Theorem. If we let the legs $= x$ and the hypotenuses $= y$, then the length of the other leg will be $\sqrt{y^2 - x^2}$ on both triangles. So, by SSS $\triangle ABD \cong \triangle CDB$.**

**Construct a triangle congruent to each triangle by the Hypotenuse-Leg Theorem.**

**5.**



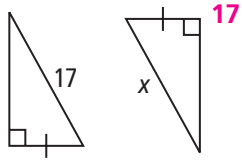
**6.**



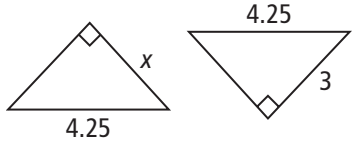
# 4-6 Practice (continued) Form G

## Congruence in Right Triangles

**Algebra** **For what values of $x$ or $x$ and $y$ are the triangles congruent by HL?**

**7.** 

17

**8.** 

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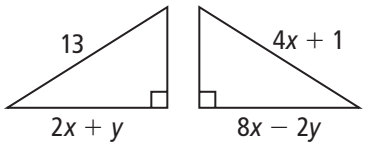
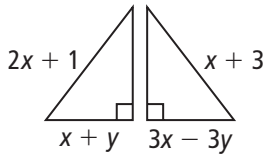
**9.** 

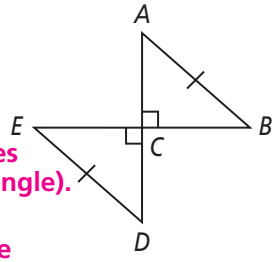
3; 6

**10.** 

2; 1

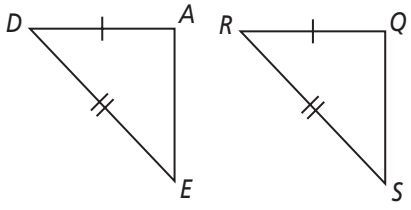
**11.** Write a paragraph proof.

**Given:** $\overline{AD}$ bisects $\overline{EB}$, $\overline{AB} \cong \overline{DE}$; $\angle ECD$, $\angle ACB$ are right angles.

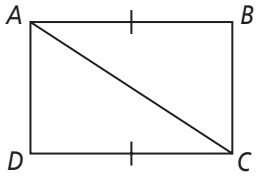



**Prove:** $\triangle ACB \cong \triangle DCE$ *$\triangle ACB$ and $\triangle DCE$ are right triangles because each contains a right angle (definition of a right triangle). It is given that $\overline{AB} \cong \overline{DE}$, so the hypotenuses of these right triangles are congruent. Because $\overline{AD}$ bisects $\overline{EB}$, point C is the midpoint of $\overline{EB}$. $\overline{EC} \cong \overline{BC}$ (definition of a midpoint), so the triangles have a pair of congruent legs. By HL, $\triangle ACB \cong \triangle DCE$.*

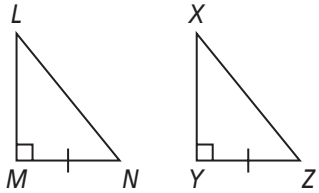
**What additional information would prove each pair of triangles congruent by the Hypotenuse-Leg Theorem?**

**12.** 

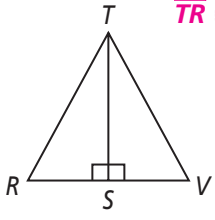
*$\angle A$ and $\angle Q$ are right angles.*

**13.** 

*$\angle B$ and $\angle D$ are right angles.*

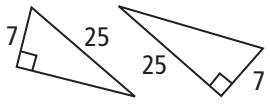
**14.** 

*$\overline{LN} \cong \overline{XZ}$*

**15.** 

*$\overline{TR} \cong \overline{TV}$*

**16. Reasoning** Are the triangles congruent? Explain.
*No; they are both right triangles, and one pair of legs is congruent, but the hypotenuse of one triangle is congruent to a leg of the other triangle.*



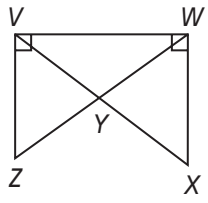
Name _____ Class _____ Date _____

# 4-6 Practice

*Form K*

## Congruence in Right Triangles

**1. Developing Proof** Complete the proof.




**Given:** $\angle WVZ$ and $\angle VWX$ are right angles.
$\overline{WZ} \cong \overline{VX}$

**Prove:** $\triangle WVZ \cong \triangle VWX$

To prove that right triangles $\triangle WVZ$ and $\triangle VWX$ are congruent, you must prove that the hypotenuses are congruent and that one ? is congruent.

leg

| Statements | Reasons |
|---|---|
| 1) ? $\angle WVZ$ and $\angle VWX$ are right $\measuredangle$. | 1) Given |
| 2) ? $\overline{WZ} \cong \overline{VX}$ | 2) Given |
| 3) ? $\overline{WV} \cong \overline{VW}$ | 3) Reflexive Property of Congruence |
| 4) ? $\triangle WVZ \cong \triangle VWX$ | 4) HL Theorem |

**2.** Look at Exercise 1. If $m\angle X = 54$, what is $m\angle Z$? 54

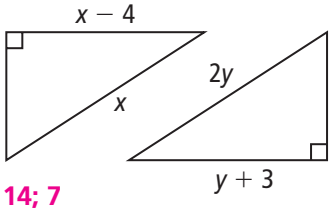
**3.** Look at Exercise 1. If $m\angle X = 54$, what is $m\angle VWZ$? 36

**4.** Study Exercise 1. Can you prove that $\triangle WVZ$ and $\triangle VWX$ are congruent without using the HL Theorem? Explain.

Yes; answers may vary. Sample: by using the Pythagorean Theorem to find another pair of congruent sides

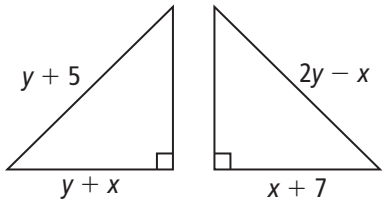
**Algebra** **For what values of $x$ and $y$ are the triangles congruent by HL?**

**5.**




14; 7

**6.**




2; 7

**7. Reasoning** The LL Theorem says that two right triangles are congruent if both pairs of legs are congruent. What theorem or postulate could be used to prove that the LL Theorem is true? Explain.

Answers may vary. Sample: The SAS Postulate could be used to prove the LL Theorem because the two right triangles would have two pairs of congruent legs and the included right angles would also be congruent.

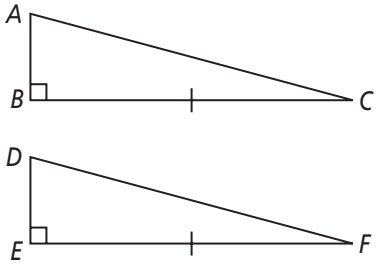
Name _____ Class _____ Date _____

# 4-6 Practice (continued) Form K

## Congruence in Right Triangles

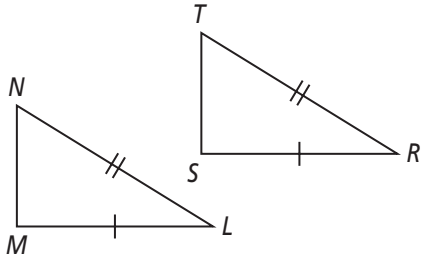
**What additional information would prove each pair of triangles congruent by the Hypotenuse-Leg Theorem?**

**8.**



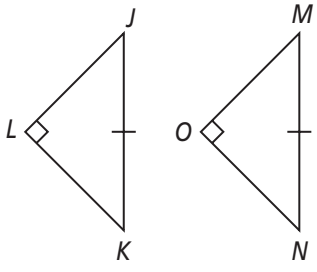

$\overline{AC} \cong \overline{DF}$

**9.**



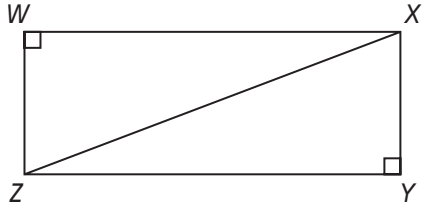

$\angle M$ and $\angle S$ are right angles.

**10.**



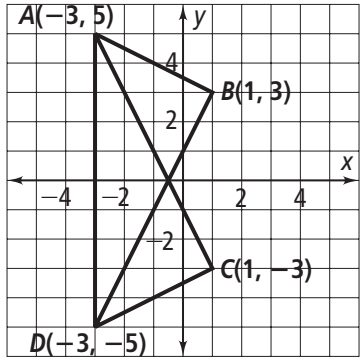

$\overline{JL} \cong \overline{MO}$ or $\overline{KL} \cong \overline{NO}$

**11.**




$\overline{WZ} \cong \overline{YX}$ or $\overline{WX} \cong \overline{YZ}$

**Coordinate Geometry** **Use the figure at the right for Exercises 12–14.**




**12.** Complete the paragraph proof that shows that $\overline{AC}$ and $\overline{CD}$ are perpendicular.

The slope of $\overline{AC}$ is $\boxed{-2}$. The slope of $\overline{CD}$ is $\boxed{0.5}$. The product of the two slopes is $\boxed{-1}$. Therefore the line segments are ? . perpendicular

**13.** How do you know that $AB = CD$?

The Distance Formula shows that both line segments have length $2\sqrt{5}$.

**14.** Complete the paragraph proof below that shows that $\triangle ACD \cong \triangle DBA$.

$\angle ACD$ is a ? right angle from Exercise 12. You can also use the product of slopes to show that $\angle ABD$ is a right angle. $\triangle ACD$ and $\triangle ABD$ share the same hypotenuse. You can use the ? Distance Formula to show that $AB = CD$. Therefore, by the ? HL Theorem, $\triangle ACD \cong \triangle DBA$.

Name _____ Class _____ Date _____

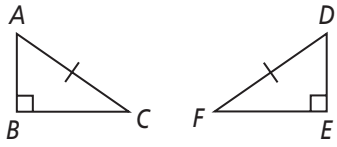
# 4-6 Standardized Test Prep

Congruence in Right Triangles

## Multiple Choice

**For Exercises 1–4, choose the correct letter.**

**1.** Which additional piece of information would allow you to prove that the triangles are congruent by the HL theorem? **C**



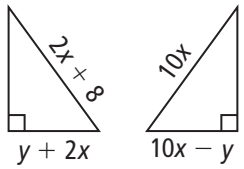

(A) $m\angle DFE = 40$

(B) $m\angle F = m\angle ABC$

(C) $\overline{AB} \cong \overline{DE}$

(D) $\overline{AC} \cong \overline{DF}$

**2.** For what values of $x$ and $y$ are the triangles shown congruent? **F**




(F) $x = 1, y = 4$

(G) $x = 2, y = 4$

(H) $x = 4, y = 1$

(I) $x = 1, y = 3$

**3.** Two triangles have two pairs of corresponding sides that are congruent. What else must be true for the triangles to be congruent by the HL Theorem? **D**

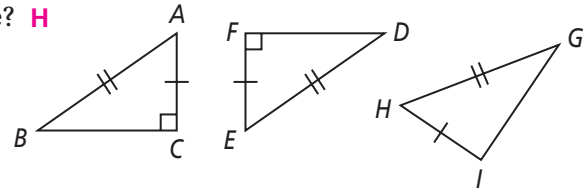
(A) The included angles must be right angles.

(B) They have one pair of congruent angles.

(C) Both triangles must be isosceles.

(D) There are right angles adjacent to just one pair of congruent sides.

**4.** Which of the following statements is true? **H**




(F) $\triangle BAC \cong \triangle GHI$ by SAS.

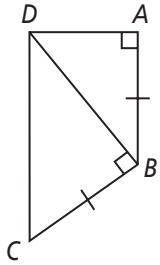
(G) $\triangle DEF \cong \triangle GHI$ by SAS.

(H) $\triangle BAC \cong \triangle DEF$ by HL.

(I) $\triangle DEF \cong \triangle GHI$ by HL.

## Extended Response

**5.** Are the given triangles congruent by the HL Theorem? Explain.




**[4] No; they are right triangles, and have a pair of congruent legs ($\overline{AB} \cong \overline{BC}$), but the hypotenuses, $\overline{DB}$ and $\overline{DC}$, are not congruent. So, the triangles only meet two of the three conditions for congruence by the HL Theorem.**
**[3] appropriate response plus a discussion of two of the three criteria for congruence [2] recognition only that the hypotenuses are not congruent [1] recognition that the triangles are not congruent [0] incorrect or no response**

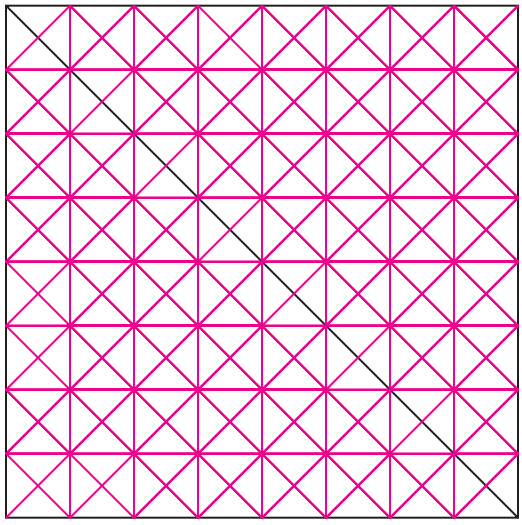
Name _____ Class _____ Date _____

# 4-6 Enrichment

Congruence in Right Triangles

## Right Triangle Patterns

An art student wants to make a painting with a simple geometric pattern. She starts with a square. She divides this square into two congruent triangles. Then she divides each of these triangles into two smaller congruent triangles. She repeats the process seven more times. What does her pattern look like in the end?



1. Show that the two triangles are congruent using the Hypotenuse-Leg Theorem.
   **Sample: Each is a right triangle. They have at least one pair of congruent legs and they have congruent hypotenuses.**
2. Use your knowledge of the Hypotenuse-Leg Theorem to divide each triangle in the figure above into two smaller congruent triangles. Repeat the process six more times. **Check students' work.**

3. How do you know that the triangles at each step are congruent?
   **Sample: Each is a right triangle, with equal legs and hypotenuses.**

4. How many triangles of the smallest size are shown? **256**

5. How many triangles are shown if they each contain 64 of the smallest-sized unit? **32**

6. How many triangles are shown if they each contain nine of the smallest-sized unit? **168**

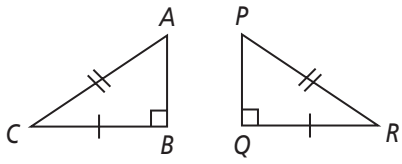
7. **Challenge** Find the sizes of all 16 different-sized triangles in the diagram.
   **1, 2, 4, 8, 9, 16, 18, 25, 32, 36, 49, 50, 64, 72, 98, 128**

Name _____ Class _____ Date _____

# 4-6 Reteaching

## Congruence in Right Triangles

Two right triangles are congruent if they have congruent hypotenuses and if they have one pair of congruent legs. This is the *Hypotenuse-Leg* (HL) *Theorem.*

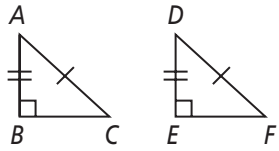



$\triangle ABC \cong \triangle PQR$ because they are both right triangles, their hypotenuses are congruent ($\overline{AC} \cong \overline{PR}$), and one pair of legs is congruent ($\overline{BC} \cong \overline{QR}$).

### Problem

How can you prove that two right triangles that have one pair of congruent legs and congruent hypotenuses are congruent (The Hypotenuse-Leg Theorem)?

Both of the triangles are right triangles.

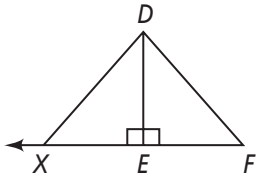



$\angle B$ and $\angle E$ are right angles.

$\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

How can you prove that $\triangle ABC \cong \triangle DEF$?

Look at $\triangle DEF$. Draw a ray starting at $F$ that passes through $E$. Mark a point $X$ so that $EX = BC$. Then draw $\overline{DX}$ to create $\triangle DEX$.



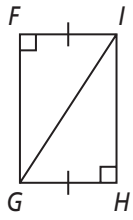

See that $\overline{EX} \cong \overline{BC}$. (You drew this.) $\angle DEX \cong \angle ABC$. (All right angles are congruent.) $\overline{DE} \cong \overline{AB}$. (This was given.) So, by SAS, $\triangle ABC \cong \triangle DEX$.

$\overline{DX} \cong \overline{AC}$ (by CPCTC) and $\overline{AC} \cong \overline{DF}$. (This was given.). So, by the Transitive Property of Congruence, $\overline{DX} \cong \overline{DF}$. Then, $\angle DEX \cong \angle DEF$. (All right angles are congruent.) By the Isosceles Theorem, $\angle X \cong \angle F$. So, by AAS, $\triangle DEX \cong \triangle DEF$.

Therefore, by the Transitive Property of Congruence, $\triangle ABC \cong \triangle DEF$.

### Problem

Are the given triangles congruent by the Hypotenuse-Leg Theorem? If so, write the triangle congruence statement.




$\angle F$ and $\angle H$ are both right angles, so the triangles are both right. $\overline{GI} \cong \overline{IG}$ by the Reflexive Property and $\overline{FI} \cong \overline{HG}$ is given.

So, $\triangle FIG \cong \triangle HGI$.

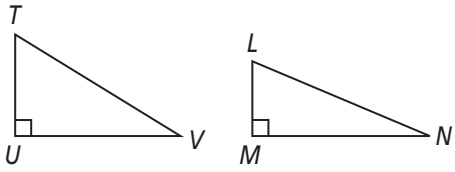
# 4-6 Reteaching (continued)

Congruence in Right Triangles

## Exercises

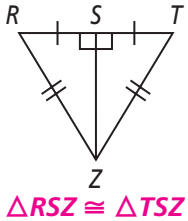
**Determine if the given triangles are congruent by the Hypotenuse-Leg Theorem. If so, write the triangle congruence statement.**

**1.**



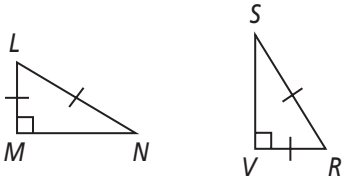

not congruent

**2.**



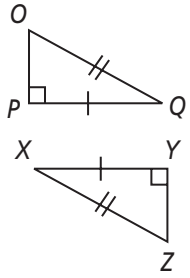

*△RSZ ≅ △TSZ*

**3.**




*△LMN ≅ △RVS*

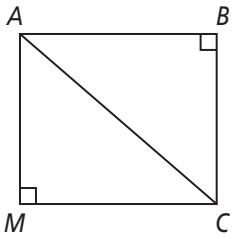
**4.**




*△OPQ ≅ △ZYX*

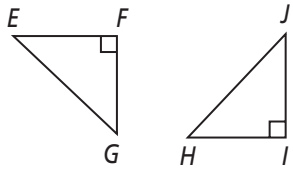
**Measure the hypotenuse and length of the legs of the given triangles with a ruler to determine if the triangles are congruent. If so, write the triangle congruence statement.**

**5.**




*△ABC ≅ △CMA*

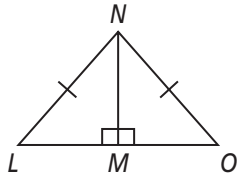
**6.**




*△EFG ≅ △HIJ*

**7.** Explain why $\triangle LMN \cong \triangle OMN$. Use the Hypotenuse-Leg Theorem.

**Because ∠NML and ∠NMO are right angles, both triangles are right triangles. It is given that their hypotenuses are congruent. Because they share a leg, one pair of corresponding legs is congruent. All criteria are met for the triangles to be congruent by the Hypotenuse-Leg Theorem.**




**8.** Visualize $\triangle ABC$ and $\triangle DEF$, where $AB = EF$ and $CA = FD$. What else must be true about these two triangles to prove that the triangles are congruent using the Hypotenuse-Leg Theorem? Write a congruence statement.

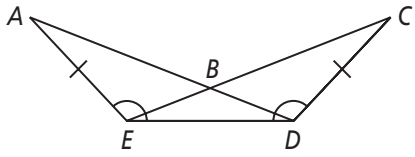
**∠B and ∠E are right angles, or ∠C and ∠D are right angles. △ABC ≅ △DEF or △ABC ≅ △FED.**

Name _____ Class _____ Date _____

# 4-7 ELL Support

## Congruence in Overlapping Triangles

**A student wanted to prove $\overline{EB} \cong \overline{DB}$ given $\angle AED \cong \angle CDE$ and $\overline{AE} \cong \overline{CD}$. She wrote the statements and reasons on note cards, but they got mixed up.**




| | | | |
|---|---|---|---|
| $\overline{EB} \cong \overline{DB}$ because corresponding parts of congruent triangles are congruent. | $\angle ABE \cong \angle CBD$ because vertical angles are congruent. | $\overline{ED} \cong \overline{DE}$ by the Reflexive Property of Congruence. | |
| $\triangle ABE \cong \triangle CBD$ by the Angle-Angle-Side (AAS) Theorem. | $\angle AED \cong \angle CDE$ and $\overline{AE} \cong \overline{CD}$ are given. | $\angle A \cong \angle C$ because corresponding parts of congruent triangles are congruent. | $\triangle AED \cong \triangle CDE$ by the Side-Angle-Side (SAS) Theorem. |

**Use the note cards to write the steps in order.**

1. First, $\angle AED \cong \angle CDE$ and $\overline{AE} \cong \overline{CD}$ are given.
2. Second, $\overline{ED} \cong \overline{DE}$ by the Reflexive Property of Congruence.
3. Third, $\triangle AED \cong \triangle CDE$ by the Side-Angle-Side (SAS) Theorem.
4. Next, $\angle A \cong \angle C$ because corresponding parts of congruent triangles are congruent.
5. Then, $\angle ABE \cong \angle CBD$ because vertical angles are congruent.
6. Then, $\triangle ABE \cong \triangle CBD$ by the Angle-Angle-Side (AAS) Theorem.
7. Finally, $\overline{EB} \cong \overline{DB}$ because corresponding parts of congruent triangles are congruent.

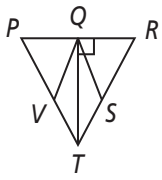
Name _____ Class _____ Date _____

# 4-7 Think About a Plan

## Congruence in Overlapping Triangles

**Given:** $\overline{QT} \perp \overline{PR}$, $\overline{QT}$ bisects $\overline{PR}$, $\overline{QT}$ bisects $\angle VQS$

**Prove:** $\overline{VQ} \cong \overline{SQ}$




### Know

1. What information are you given? What else can you determine from the given information and the diagram?

   $\overline{QT} \perp \overline{PR}$, $\overline{QT}$ bisects $\overline{PR}$, $\overline{QT}$ bisects $\angle VQS$; $\angle PQT$ and $\angle RQT$ are right angles, $PQ = QR$, $\angle VQT \cong \angle SQT$

2. To solve the problem, what will you need to prove?

   $\overline{VQ} \cong \overline{SQ}$

### Need

3. For which two triangles are $\overline{VQ}$ and $\overline{SQ}$ corresponding parts?

   $\triangle PQV$ and $\triangle RQS$ or $\triangle VQT$ and $\triangle SQT$

4. You need to use corresponding parts to prove the triangles from Exercise 3 congruent. Which two triangles should you prove congruent first, using the given information? Which theorem or postulate should you use?

   $\triangle PQT$ and $\triangle RQT$ by SAS

5. Which corresponding parts should you then use to prove that the triangles in Exercise 3 are congruent?

   Answers may vary. Sample: $\angle P$ and $\angle R$

### Plan

6. Use the space below to write the proof.

   Statements: 1) $\overline{QT} \perp \overline{PR}$; $\overline{QT}$ bisects $\overline{PR}$, $\overline{QT}$ bisects $\angle VQS$; 2) $m\angle PQT = m\angle RQT = 90$; 3) $PQ = QR$; 4) $QT = QT$; 5) $\triangle PQT \cong \triangle RQT$; 6) $\angle P \cong \angle R$; 7) $\angle PQV$ and $\angle VQT$ are compl. $\angle RQS$ and $\angle SQT$ are compl.; 8) $\angle VQT \cong \angle SQT$; 9) $\angle PQV \cong \angle RQS$; 10) $\triangle PQV \cong \triangle RQS$; 11) $\overline{VQ} \cong \overline{SQ}$; Reasons: 1) Given; 2) Definition of perpendicular lines; 3) Definition of bisector; 4) Reflexive Property of Congruence; 5) SAS; 6) CPCTC; 7) Definition of complementary angles; 8) Definition of angle bisector; 9) Complements of $\cong$ $\triangle$ are $\cong$; 10) ASA; 11) CPCTC

Name _____ Class _____ Date _____

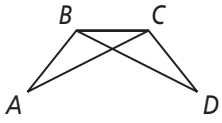
# 4-7 Practice

*Form G*

## Congruence in Overlapping Triangles

**For Exercises 1–6, separate and redraw the indicated triangles. Identify any common angles or sides.**

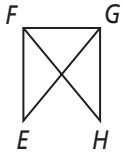
**1.** $\triangle ABC$ and $\triangle DCB$




$\overline{BC}$

Check students' work.

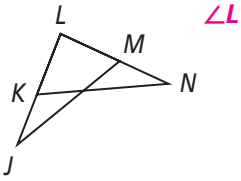
**2.** $\triangle EFG$ and $\triangle HGF$




$\overline{FG}$

Check students' work.

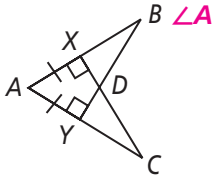
**3.** $\triangle JML$ and $\triangle NKL$




$\angle L$

Check students' work.

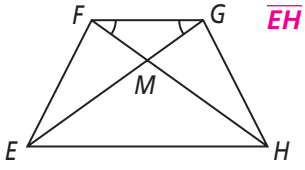
**4.** $\triangle BYA$ and $\triangle CXA$




$\angle A$

Check students' work.

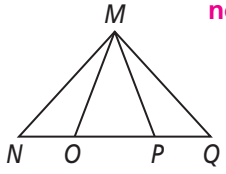
**5.** $\triangle GEH$ and $\triangle FEH$




$\overline{EH}$

Check students' work.

**6.** $\triangle MPN$ and $\triangle MOQ$

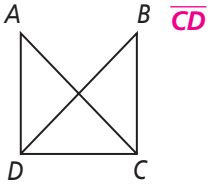



none

Check students' work.

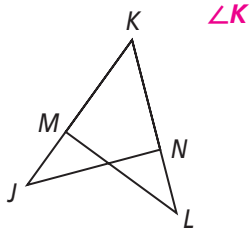
**In each diagram in Exercises 7–12 the given triangles are congruent. Identify their common side or angle.**

**7.** $\triangle ADC$ and $\triangle BCD$



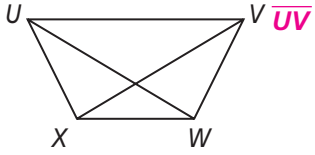

$\overline{CD}$

**8.** $\triangle KNJ$ and $\triangle KML$



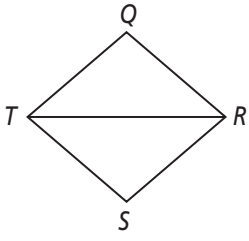

$\angle K$

**9.** $\triangle UXV$ and $\triangle VWU$



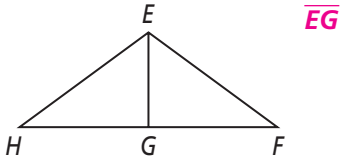

$\overline{UV}$

**10.** $\triangle QTR$ and $\triangle SRT$



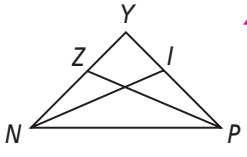

$\overline{RT}$

**11.** $\triangle EGH$ and $\triangle EGF$




$\overline{EG}$

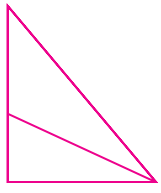
**12.** $\triangle YNI$ and $\triangle YPZ$




$\angle Y$

**13. Open-Ended** Draw a diagram of a pair of triangles that share a common angle and a common side.

Answers may vary. Check students' drawings. Sample:



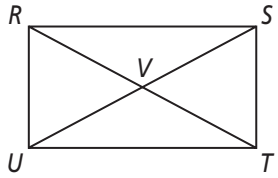
# 4-7 Practice (continued) Form G

## Congruence in Overlapping Triangles

**14.** Complete the following proof.

**Given:** $\overline{RU} \cong \overline{TS}$, $\angle RUT$ and $\angle UTS$ are right angles, $V$ is the midpoint of $\overline{US}$.

**Prove:** $\triangle RVU \cong \triangle TVS$

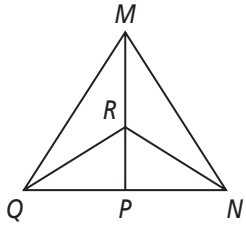



| Statements | Reasons |
|---|---|
| 1) $\overline{RU} \cong \overline{TS}$, $\angle RUT$ and $\angle UTS$ are right angles, $V$ is the midpoint of $\overline{US}$. | 1) ? Given |
| 2) $\overline{UT} \cong \overline{TU}$ | 2) ? Refl. Prop. of $\cong$ |
| 3) ? $\angle RUT \cong \angle STU$ | 3) All right angles are congruent |
| 4) ? $\triangle RUT \cong \triangle STU$ | 4) SAS |
| 5) $\angle RUS$ and $\angle SUT$ are complementary angles. | 5) ? Def. of compl. $\cong$ $\measuredangle$s |
| 6) ? $\angle STR$ and $\angle RTU$ are compl. $\measuredangle$s. | 6) Definition of complementary angles |
| 7) $\angle SUT \cong \angle RTU$ | 7) ? CPCTC |
| 8) $\angle RUS \cong \angle STR$ | 8) ? Compl. of $\measuredangle$s are $\cong$. |
| 9) ? $UV = SV$ | 9) Definition of midpoint |
| 10) $\angle RVU \cong \angle TVS$ | 10) ? Vert. $\measuredangle$s are $\cong$. |
| 11) $\triangle RVU \cong \triangle TVS$ | 11) ? ASA |

**15.** Write a paragraph proof.

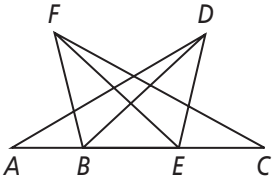
**Given:** $P$ is the midpoint of $\overline{QN}$, $\overline{MP} \perp \overline{QN}$

**Prove:** $\triangle MRQ \cong \triangle MRN$




Given that $P$ is the midpt. of $\overline{QN}$, $\overline{QP} \cong \overline{NP}$. $\overline{RP} \cong \overline{RP}$ by the Refl. Prop. of $\cong$. Given $\overline{MP} \perp \overline{QN}$, $\angle MPQ$ and $\angle MPN$ are rt. $\measuredangle$s by the def. of $\perp$ and $\angle MPQ \cong \angle MPN$ because all rt $\measuredangle$s are $\cong$. Therefore, $\triangle RPQ \cong \triangle RPN$ by SAS. Knowing CPCTC, $\overline{QR} \cong \overline{NR}$ and $\angle QRP \cong \angle NRP$. Because $\angle QRM$ and $\angle QRP$ are suppl. and $\angle NRM$ and $\angle NRP$ are also suppl., $\angle QRM \cong \angle NRM$, because suppl. of $\cong$ $\measuredangle$s are $\cong$. By the Refl. Prop. of $\cong$, it also follows that $\overline{MR} \cong \overline{MR}$. Therefore, $\triangle MRQ \cong \triangle MRN$ by SAS.

**16.** In the diagram at the right, $\angle A \cong \angle C$, $\overline{AB} \cong \overline{CE}$, and $\overline{DA} \cong \overline{FC}$. Which two triangles are congruent by SAS? Explain.




$\triangle ABD \cong \triangle CEF$; all necessary sides and angles given are congruent; or $\triangle AED \cong \triangle CBF$; $\angle A \cong \angle C$, $\overline{AB} \cong \overline{CE}$, and $\overline{DA} \cong \overline{FC}$ are given, and $\overline{EB} \cong \overline{BE}$ by the Reflexive Property. So, $\overline{CB} \cong \overline{AE}$ by segment addition, which gives $\triangle AED \cong \triangle CBF$ by SAS.

Name _____ Class _____ Date _____

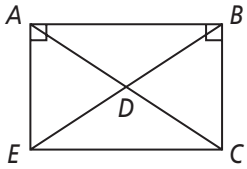
# 4-7 Practice

*Form K*

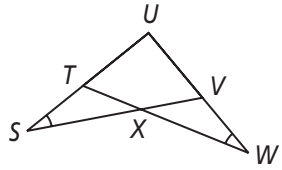
## Congruence in Overlapping Triangles

**In each diagram, the stated triangles are congruent. Identify their common side or angle.**

**1.** $\triangle BAE \cong \triangle ABC$ $\overline{AB}$

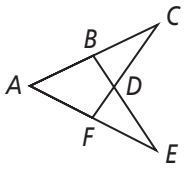



**2.** $\triangle SUV \cong \triangle WUT$ $\angle U$

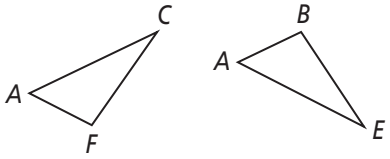



**Separate and redraw the indicated triangles. Identify any common angles or sides.**

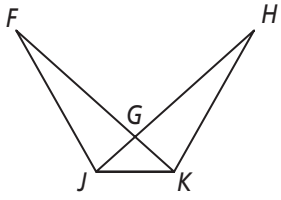
**3.** $\triangle ACF$ and $\triangle AEB$ $\angle A$



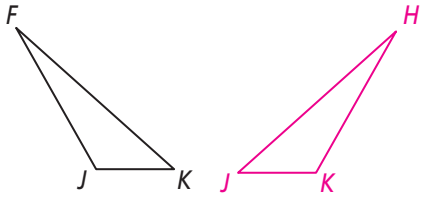

To start, redraw each triangle separately.




**4.** $\triangle FKJ$ and $\triangle HJK$ $\overline{JK}$



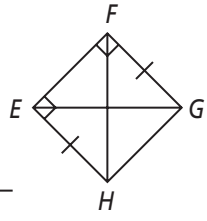

Complete the drawing to separate the triangles.




**5. Developing Proof** Complete the two-column proof.

**Given:** $m\angle FEH = m\angle GFE = 90$, $\overline{EH} \cong \overline{FG}$

**Prove:** $\overline{HF} \cong \overline{EG}$




| Statements | Reasons |
|---|---|
| 1) $m\angle FEH = m\angle GFE = 90$, $\overline{EH} \cong \overline{GF}$ | 1) Given |
| 2) $\angle FEH \cong \angle EFG$ | 2) ? **All right △ are ≅.** |
| 3) $\overline{EF} \cong \overline{FE}$ | 3) ? **Reflexive Prop. of ≅** |
| 4) ? **$\triangle HEF \cong \triangle GFE$** | 4) SAS |
| 5) $\overline{HF} \cong \overline{GE}$ | 5) ? **Corresp. parts of ≅ △ are ≅.** |

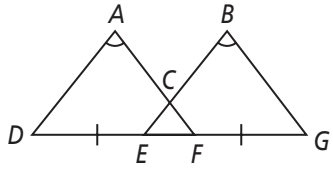
Name _____ Class _____ Date _____

# 4-7 Practice (continued) Form K

## Congruence in Overlapping Triangles

**6. Given:** $\triangle AFD$ and $\triangle BGE$ are equilateral triangles. $\angle A \cong \angle B$, $\overline{DE} \cong \overline{FG}$

**Prove:** $\overline{AD} \cong \overline{BE}$



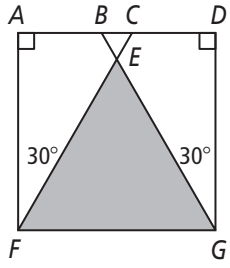

| Statements | Reasons |
|---|---|
| 1) $\triangle AFD$ and $\triangle BGE$ are equilateral $\triangle$. | 1) Given |
| 2) $\angle A \cong \angle D \cong \angle AFD$ | 2) ? Corollary to Theorem 4-3 |
| 3) $\angle B \cong \angle G \cong \angle BEG$ | 3) ? Corollary to Theorem 4-3 |
| 4) $\angle A \cong \angle B$ | 4) Given |
| 5) $\angle A \cong \angle D \cong \angle B \cong \angle G$ | 5) ? Transitive Prop. of $\cong$ |
| 6) $\overline{EF} \cong \overline{EF}$ | 6) ? Reflexive Prop. of $\cong$ |
| 7) $\overline{DE} \cong \overline{FG}$ | 7) Given |
| 8) $DE + EF = EF + FG$ | 8) ? Addition Prop. of = |
| 9) ? $\overline{DF} \cong \overline{EG}$ | 9) Segment Add. Post. |
| 10) ? $\triangle AFD \cong \triangle BGE$ | 10) AAS |
| 11) $\overline{AD} \cong \overline{BE}$ | 11) ? Corresp. parts of $\cong$ $\triangle$ are $\cong$. |

**Open-Ended Draw the diagram described.**

**7.** Draw a line segment on your paper. Then draw two overlapping, congruent triangles that share the segment as a common side.
Check students' work.

**8.** Draw two right triangles that share a common angle that is not a right angle.
Check students' work.

**9.** The pattern at the right has been designed for a square floor tile. Both $\triangle ACF$ and $\triangle DBG$ are 30°-60°-90° right triangles. Write a paragraph proof to prove that $\triangle FGE$ is an equilateral triangle.
Answers may vary. Sample: *ADGF* is a square, so $m\angle AFG = m\angle DGF = 90$. $m\angle EFG = m\angle EGF = 60$ because they are complements of 30° angles; $m\angle GEF = 60$ by the $\triangle$ Angle-Sum Thm., so $\triangle FGE$ is equilateral by Thm. 4-3.



Name _____ Class _____ Date _____

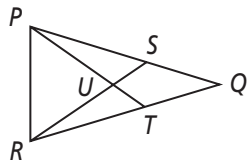
# 4-7 Standardized Test Prep

Congruence in Overlapping Triangles

## Multiple Choice

**For Exercises 1–5, choose the correct letter.**

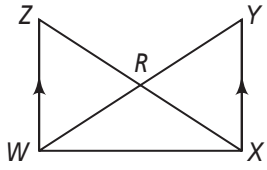
**1.** What is the common angle of $\triangle PQT$ and $\triangle RSQ$? **A**




(A) $\angle PQT$ (C) $\angle SRQ$

(B) $\angle SPT$ (D) $\angle SUT$

**Use the following information for Exercises 2–5.**




**Given:** $\triangle ZWX \cong \triangle YXW$, $\overline{ZW} \parallel \overline{YX}$

**Prove:** $\triangle ZWR \cong \triangle YRX$

**2.** Which corresponding parts statement is needed to prove $\triangle ZWR \cong \triangle YRX$? **H**

(F) $\angle ZWR \cong \angle YXR$ (H) $ZW = YX$

(G) $\angle Z \cong \angle R$ (I) $WX = WX$

**3.** A classmate writes the statement $\angle ZRW \cong \angle YRX$ to help prove the congruence of the triangles. What reason should the classmate give? **D**

(A) Given

(B) Angles cut by a bisector are congruent.

(C) Base angles of an isosceles triangle are congruent.

(D) Vertical angles are congruent.

**4.** After using the congruence statements from Exercises 2 and 3, which statement can be used to prove the triangles congruent? **F**

(F) $\angle Z \cong \angle Y$ (H) $\overline{WX} \cong \overline{WX}$

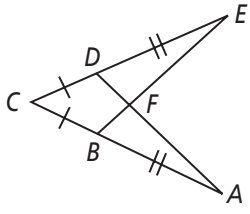
(G) $\angle ZWR \cong \angle RYX$ (I) $\overline{WR} \cong \overline{RX}$

**5.** Which theorem or postulate will prove $\triangle ZWR \cong \triangle YRX$? **D**

(A) SAS (B) SSS (C) ASA (D) AAS

## Short Response

**6.** In the diagram at the right, which two triangles should be proved congruent first to help prove $\triangle ABF \cong \triangle EDF$?




**[2] $\triangle ACD$ and $\triangle ECB$ [1] Correct △ named but vertices do not correspond. [0] incorrect △ named**

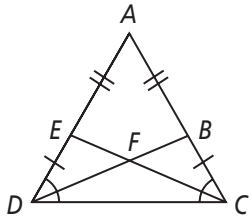
Name _____ Class _____ Date _____

# 4-7 Enrichment

## Congruence in Overlapping Triangles

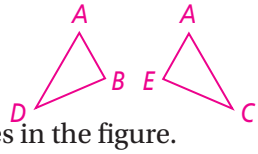
Many geometric figures and patterns involve congruent triangles. These figures are made by drawing overlapping congruent triangles or by drawing a figure and then drawing diagonals to create congruent triangles. These figures are used in items such as quilts, logos, stained glass windows, and architectural designs.

**Use the figure at the right to complete Exercises 1–5.**



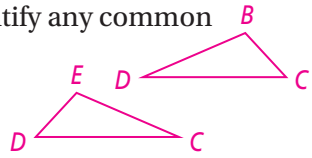

1. Name a pair of congruent triangles in the figure.
   **Sample: $\triangle ABD \cong \triangle AEC$**

2. Separate and redraw the triangles named in Exercise 1. Identify any common angles or sides.
   **common angle: $\angle A$**



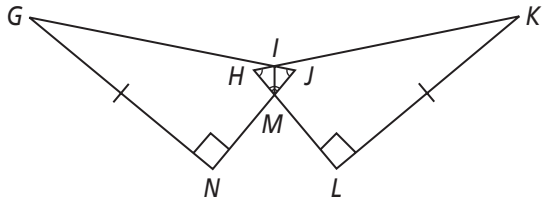

3. Name another pair of congruent triangles in the figure.
   **Sample: $\triangle DEC \cong \triangle CBD$**

4. Separate and redraw the triangles named in Exercise 3. Identify any common angles or sides.
   **common side: $\overline{DC}$**



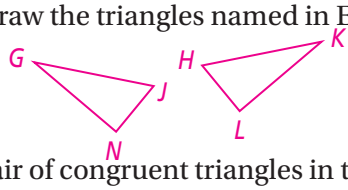

5. What is the total number of triangles in the figure? **8**

**Use the figure at the right to complete Exercises 6–10.**




6. Name a pair of congruent triangles in the figure.
   **Sample: $\triangle GNJ \cong \triangle KLH$**

7. Separate and redraw the triangles named in Exercise 6. Identify any common angles or sides.




8. Name another pair of congruent triangles in the figure.
   **Sample: $\triangle HIM \cong \triangle JIM$**

9. Separate and redraw the triangles named in Exercise 8. Identify any common angles or sides.
   **common side: $\overline{IM}$**


10. What is the total number of triangles in the figure? **4**

Name _____ Class _____ Date _____

# 4-7 Reteaching

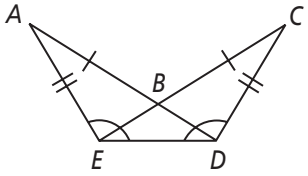
## Congruence in Overlapping Triangles

Sometimes you can prove one pair of triangles congruent and then use corresponding parts of those triangles to prove another pair congruent. Often the triangles overlap.

**Problem**

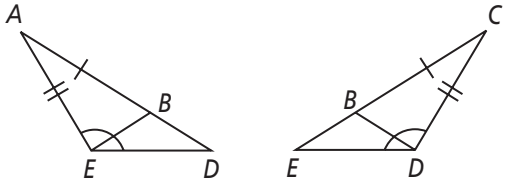
**Given:** $\overline{AB} \cong \overline{CB}$, $\overline{AE} \cong \overline{CD}$, $\angle AED \cong \angle CDE$

**Prove:** $\triangle ABE \cong \triangle CBD$




Think about a plan for the proof. Examine the triangles you are trying to prove congruent. Two pairs of sides are congruent. If the included angles, $\angle A$ and $\angle C$, were congruent, then the triangles would be congruent by SAS.

If the overlapping triangles $\triangle AED$ and $\triangle CDE$ were congruent, then the angles would be congruent by corresponding parts. When triangles overlap, sometimes it is easier to visualize if you redraw the triangles separately.




Now use the plan to write a proof.

**Given:** $\overline{AB} \cong \overline{CB}$, $\overline{AE} \cong \overline{CD}$, $\angle AED \cong \angle CDE$

**Prove:** $\triangle ABE \cong \triangle CBD$

| Statements | Reasons |
|---|---|
| 1) $\overline{AE} \cong \overline{CD}$, $\angle AED \cong \angle CDE$ | 1) Given |
| 2) $\overline{ED} \cong \overline{ED}$ | 2) Reflexive Property of $\cong$ |
| 3) $\triangle AED \cong \triangle CDE$ | 3) SAS |
| 4) $\angle A \cong \angle C$ | 4) CPCTC |
| 5) $\overline{AB} \cong \overline{CB}$ | 5) Given |
| 6) $\triangle ABE \cong \triangle CBD$ | 6) SAS |

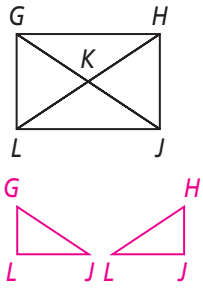
Name _____ Class _____ Date _____

# 4-7 Reteaching (continued)

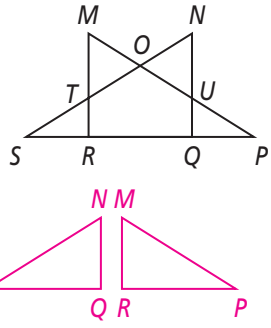
## Congruence in Overlapping Triangles

**Separate and redraw the overlapping triangles. Identify the vertices.**

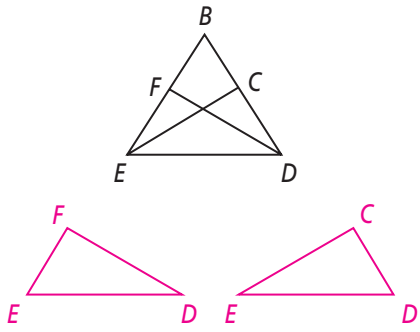
**1.** $\triangle GLJ$ and $\triangle HJL$




**2.** $\triangle MRP$ and $\triangle NQS$

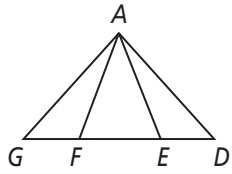



**3.** $\triangle FED$ and $\triangle CDE$




**Fill in the blanks for the two-column proof.**

**4. Given:** $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$
**Prove:** $\triangle AFG \cong \triangle AED$

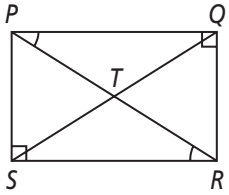



| Statements | Reasons |
|---|---|
| 1) $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$ | 1) ? **Given** |
| 2) ? **$\triangle AEG \cong \triangle AFD$** | 2) SAS |
| 3) $\overline{AG} \cong \overline{AD}$, $\angle G \cong \angle D$ | 3) ? **CPCTC** |
| 4) ? **$\overline{GE} \cong \overline{FD}$** | 4) Given |
| 5) $GE = FD$ | 5) ? **Def. of $\cong$** |
| 6) $GF + FE = GE$, $FE + ED = FD$ | 6) ? **Seg. Addition Post.** |
| 7) $GF + FE = FE + ED$ | 7) ? **Substitution Property** |
| 8) ? **$GF = ED$** | 8) Subtr. Prop. of Equality |
| 9) ? **$\triangle AFG \cong \triangle AED$** | 9) ? **SAS** |

**Use the plan to write a two-column proof.**

**5. Given:** $\angle PSR$ and $\angle PQR$ are right angles, $\angle QPR \cong \angle SRP$.

**Prove:** $\triangle STR \cong \triangle QTP$




Plan for Proof:

Prove $\triangle QPR \cong \triangle SRP$ by AAS. Then use CPCTC and vertical angles to prove $\triangle STR \cong \triangle QTP$ by AAS.

**Statements: 1) $\angle PSR$ and $\angle PQR$ are rt. $\measuredangle$; $\angle QPR \cong \angle SRP$; 2) $\angle PSR \cong \angle RQP$; 3) $\overline{PR} \cong \overline{RP}$; 4) $\triangle QPR \cong \triangle SRP$; 5) $\angle STR \cong \angle QTP$; 6) $\overline{PQ} \cong \overline{RS}$; 7) $\triangle STR \cong \triangle QTP$; Reasons: 1) Given; 2) Rt. $\measuredangle$ are congruent; 3) Refl. Prop. of $\cong$; 4) AAS; 5) Vert. $\measuredangle$ are $\cong$; 6) CPCTC; 7) AAS**

Name _____ Class _____ Date _____

# Chapter 4 Quiz 1

*Form G*

Lessons 4-1 through 4-3

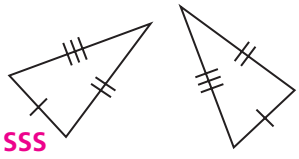
## Do you know HOW?

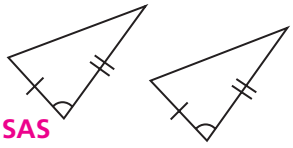
1. Two triangles have the following pairs of congruent sides: $\overline{BD} \cong \overline{FJ}$, $\overline{DG} \cong \overline{JM}$, and $\overline{GB} \cong \overline{MF}$. Write the congruence statement for the two triangles.
   $\triangle BDG \cong \triangle FJM$

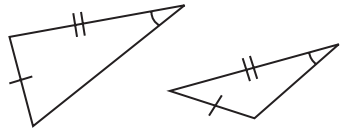
**$\triangle QRS \cong \triangle TUV$. Name the angle or side that corresponds to the given part.**

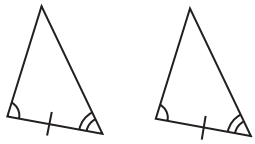
2. $\angle Q$ $\angle T$
3. $\overline{RS}$ $\overline{UV}$
4. $\angle S$ $\angle V$
5. $\overline{QS}$ $\overline{TV}$

**State the postulate or theorem that can be used to prove the triangles congruent. If you cannot prove the triangles congruent, write *not enough information.***

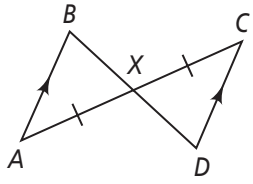
6. 
   SSS

7. 
   SAS

8. 
   not enough information

9. 
   ASA

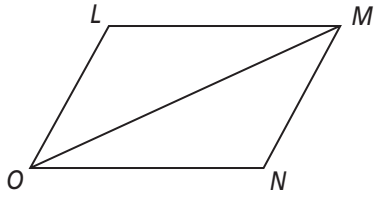
**Use the diagram below. Tell why each statement is true.**




10. $\angle A \cong \angle C$ Alternate Interior Angles Theorem
11. $\angle AXB \cong \angle CXD$ Vertical angles are congruent.
12. $\triangle ABX \cong \triangle CDX$ ASA

## Do you UNDERSTAND?

13. **Given:** $\overline{LM} \cong \overline{NO}$; $\angle LMO \cong \angle NOM$
    **Prove:** $\triangle LMO \cong \triangle NOM$
    $\overline{LM} \cong \overline{NO}$, $\angle LMO \cong \angle NOM$, given. $\overline{OM} \cong \overline{MO}$, Reflexive Property of Congruence; $\triangle LMO \cong \triangle NOM$, Side-Angle-Side Postulate




14. **Reasoning** Explain why it is not possible to have a Side-Side-Angle congruence postulate or theorem. Draw a picture if necessary.
    A triangle can have two sides and a not included angle congruent to another triangle, but the "hinge effect" of one side makes it possible for two different triangles to result.

Name _____ Class _____ Date _____

# Chapter 4 Quiz 2

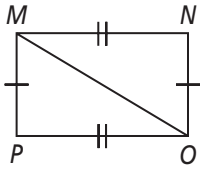
*Form G*

Lessons 4-4 through 4-7

## Do you know HOW?

**Explain how to use congruent triangles to prove each statement true.**

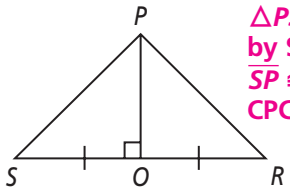
**1.**




$\triangle OMN \cong \triangle MOP$ by SSS. $\angle OMN \cong \angle MOP$ by CPCTC.

$\angle OMN \cong \angle MOP$

**2.**

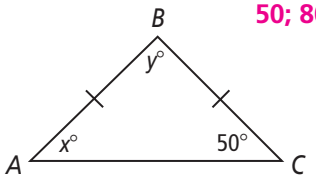



$\triangle PSO \cong \triangle PRO$ by SAS. $\overline{SP} \cong \overline{RP}$ by CPCTC.

$\overline{SP} \cong \overline{RP}$

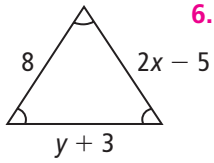
**Find the values of $x$ and $y$.**

**3.**




50; 80

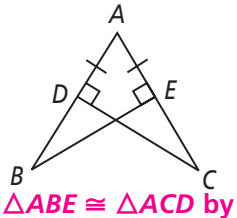
**4.**




6.5; 5

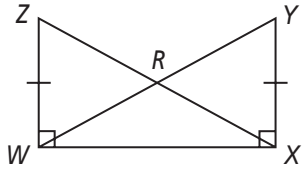
**Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.**

**5.**




$\triangle ABE \cong \triangle ACD$ by ASA

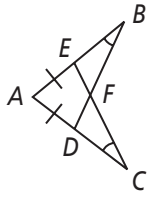
**6.**




$\triangle ZXW \cong \triangle YWX$ by SAS

## Do you UNDERSTAND?

**7. Reasoning** Complete the proof by filling in the missing statements and reasons.




**Given:** $\overline{AE} \cong \overline{AD}$, $\angle B \cong \angle C$

**Prove:** $\overline{EB} \cong \overline{DC}$

| Statements | Reasons |
|---|---|
| 1) $\overline{AE} \cong \overline{AD}$, $\angle B \cong \angle C$ | 1) ? Given |
| 2) ? $\angle A \cong \angle A$ | 2) Reflexive Property of Congruence |
| 3) $\triangle ABD \cong \triangle ACE$ | 3) ? AAS |
| 4) $\overline{AB} \cong \overline{AC}$ | 4) ? CPCTC |
| 5) ? $\overline{EB} \cong \overline{DC}$ | 5) Segment Addition Postulate |

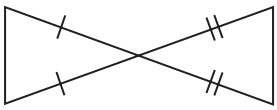
Name _____ Class _____ Date _____

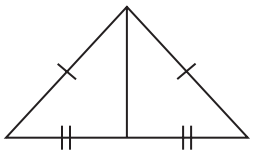
# Chapter 4 Test

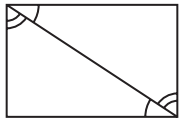
*Form G*

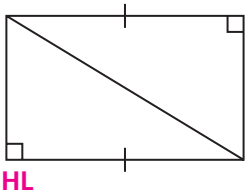
## Do you know HOW?

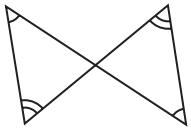
**State the postulate or theorem you would use to prove each pair of triangles congruent. If the triangles cannot be proven congruent, write *not enough information.***

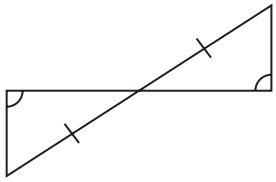
1. 
not enough information

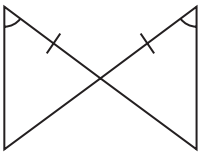
2. 
SSS

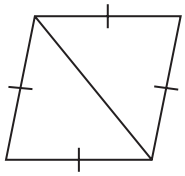
3. 
ASA

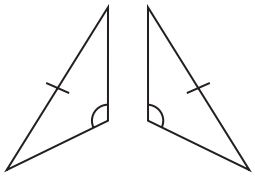
4. 
HL

5. 
not enough information

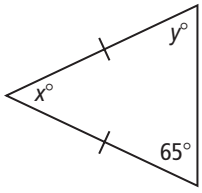
6. 
AAS

7. 
ASA

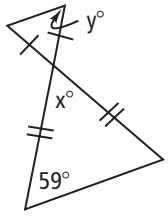
8. 
SSS

9. 
not enough information

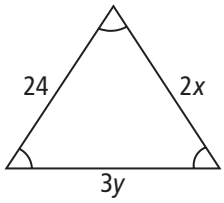
**Find the value of $x$ and $y$.**

10. 

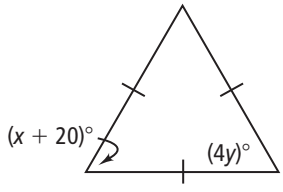
50; 65

11. 

62; 59

12. 

12; 8

13. 

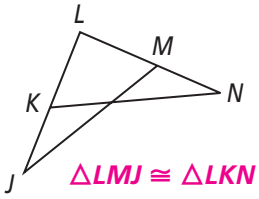
40; 15

14. $\triangle CGI \cong \triangle MPR$. Name all of the pairs of corresponding congruent parts.
$\angle C \cong \angle M$; $\angle G \cong \angle P$; $\angle I \cong \angle R$; $\overline{CG} \cong \overline{MP}$; $\overline{GI} \cong \overline{PR}$; $\overline{CI} \cong \overline{MR}$

# Chapter 4 Test (continued) *Form G*

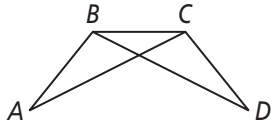
**Name a pair of overlapping congruent triangles in each diagram. State whether the triangles are congruent by SSS, SAS, ASA, AAS, or HL.**

**15.** Given: $\overline{LM} \cong \overline{LK}$; $\overline{LN} \cong \overline{LJ}$




**△LMJ ≅ △LKN by SAS**

**16.** Given: $\angle ABC \cong \angle DCB$; $\angle DBC \cong \angle ACB$

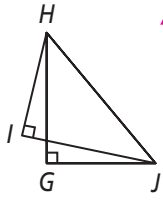



**△ABC ≅ △DCB by ASA**

**17.** Given: $\angle E \cong \angle D \cong \angle DCF \cong \angle EFC$


**△DCF ≅ △EFC by AAS**

**18.** Given: $\overline{HI} \cong \overline{JG}$



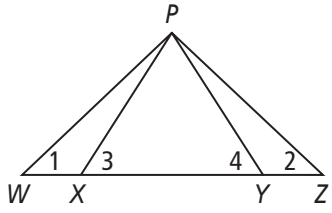

**△HIG ≅ △JGH by HL**

## Do you UNDERSTAND?

**19. Reasoning** Complete the following proof by providing the reason for each statement.

**Given:** $\angle 1 \cong \angle 2$; $\overline{WX} \cong \overline{ZY}$

**Prove:** $\angle 3 \cong \angle 4$

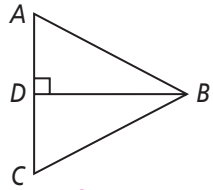



| Statements | Reasons |
|---|---|
| 1) $\angle 1 \cong \angle 2$; $\overline{WX} \cong \overline{ZY}$ | 1) ? **Given** |
| 2) $\overline{WP} \cong \overline{ZP}$ | 2) ? **Converse of Isosc. △ Thm.** |
| 3) $\triangle WXP \cong \triangle ZYP$ | 3) ? **SAS** |
| 4) $\overline{XP} \cong \overline{YP}$ | 4) ? **CPCTC** |
| 5) $\angle 3 \cong \angle 4$ | 5) ? **Isosceles Triangle Theorem** |

**20. Reasoning** Write a proof for the following:

**Given:** $\overline{BD} \perp \overline{AC}$, $D$ is the midpoint of $\overline{AC}$.

**Prove:** $\overline{BC} \cong \overline{BA}$




**$\overline{BD} \perp \overline{AC}$, $D$ is the midpoint of $\overline{AC}$ is given, so $\angle BDC \cong \angle BDA$ because perpendicular lines form congruent angles. Also, $\overline{AD} \cong \overline{CD}$ by definition of midpoint. $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of ≅. So, $\triangle BAD \cong \triangle BCD$ by SAS and $\overline{BC} \cong \overline{BA}$ by CPCTC.**

Name _____ Class _____ Date _____

# Chapter 4 Part A Test

*Form K*

Lessons 4-1 through 4-3

## Do you know HOW?

**Complete the following statements.**

1. **Given:** $\triangle FGH \cong \triangle WAX$

   a. $\overline{GH} \cong$ ? **$\overline{AX}$**

   b. $\angle W \cong$ ? **$\angle F$**

2. **Given:** $BIKE \cong PATH$

   a. $\angle T \cong$ ? **$\angle K$**

   b. $THPA \cong$ ? **$KEBI$**

3. In $\triangle HOT$ and $\triangle SUN$, $\angle O \cong \angle U$ and $\angle T \cong \angle N$.

   a. What is the relationship between $\angle S$ and $\angle H$? **$\angle S \cong \angle H$**

   b. If $m\angle O = 27$ and $m\angle T = 63$, what is $m\angle S$? **90**

4. In $\triangle RUG$, name the angle that is included between the given sides.

   a. $\overline{GR}$ and $\overline{RU}$ **$\angle R$**

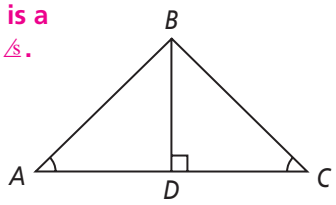
   b. $\overline{UG}$ and $\overline{GR}$ **$\angle G$**

5. In $\triangle PAD$, the given angle is included between which two sides?

   a. $\angle P$ **$\overline{AP}$ and $\overline{PD}$**

   b. $\angle D$ **$\overline{AD}$ and $\overline{DP}$**

**Use the diagram at the right. Tell why each statement is true.**

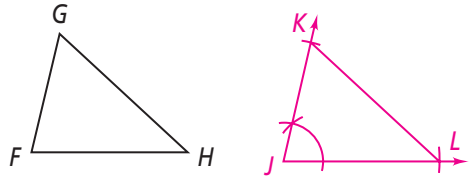



6. $m\angle ADB = 90$ **If two ∡s are suppl., and one is a right ∠, then both are right ∡s.**

7. $\overline{BD} \cong \overline{BD}$ **Refl. Prop. of ≅**

8. $\triangle ADB \cong \triangle CDB$ **AAS**

9. **Constructions** Construct $\triangle JKL$ congruent to $\triangle FGH$ using SAS.



Name _____ Class _____ Date _____

# Chapter 4 Part A Test (continued) *Form K*

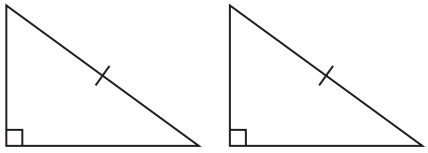
Lessons 4-1 through 4-3

**10.** In $\triangle ABC$, which side is included between $\angle B$ and $\angle C$? $\overline{BC}$

**11.** In $\triangle XYZ$, $\overline{YZ}$ is included between which two angles? $\angle Y$ and $\angle Z$

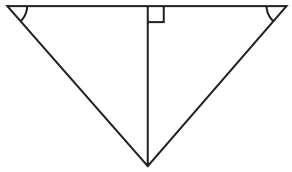
**State the postulate or theorem you can use to prove each pair of triangles congruent. If the triangles cannot be proven congruent, write *not enough information*.**

**12.**



not enough information

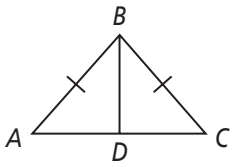
**13.**



Answers may vary. Sample: AAS or ASA

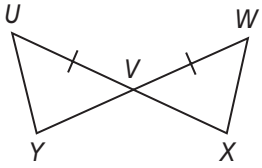
**Determine what other information you need to prove the two triangles congruent. Then write the congruence statement and name the postulate or theorem you would use.**

**14.**




Answers may vary. Sample: need $\overline{AD} \cong \overline{CD}$; $\triangle ADB \cong \triangle CDB$; SSS

**15.**



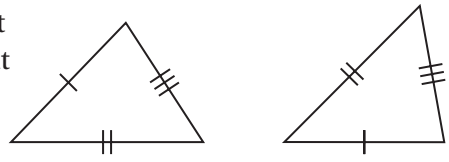

Answers may vary. Sample: need $\angle Y \cong \angle X$; $\triangle UVY \cong \triangle WVX$; AAS

## Do you UNDERSTAND?

**16. Reasoning** If two triangles are congruent, all their corresponding parts are congruent. Write the converse of this statement. Is the converse true? Explain.
Converse: If all the corresponding parts of two triangles are congruent, the triangles must be congruent; yes; by SSS, AAS, ASA, or SAS.

**17. Reasoning** The Third Angles Theorem can be applied to triangles that are not congruent. Explain.
Answers may vary. Sample: The sum of the measures of the angles of any triangle is 180.

**18. Error Analysis** Your classmate says the triangles at the right are not congruent by SSS. She explains that congruent sides do not correspond. Explain the error in her reasoning.



Answers may vary. Sample: The sides do correspond after flipping one of the triangles.

Name _____ Class _____ Date _____

# Chapter 4 Part B Test

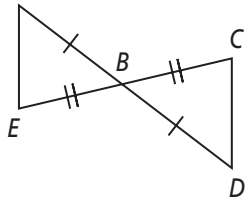
*Form K*

Lessons 4-4 through 4-7

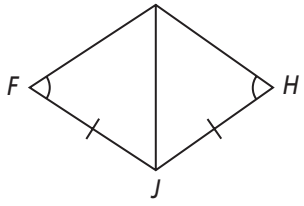
## Do you know HOW?

**State the postulate or theorem you can use to prove each pair of triangles congruent. If the triangles cannot be proven congruent, write *not enough information.***

**1.** SAS

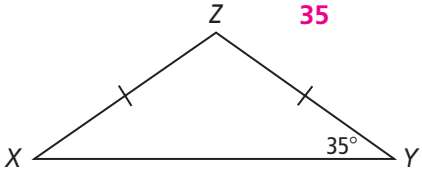



**2.** not enough information

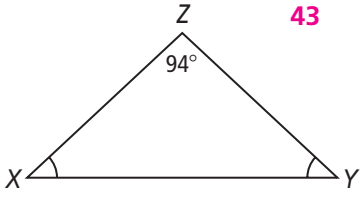



**3.** What is $m\angle X$?

**a.** 35

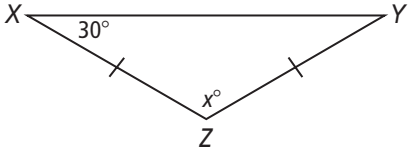



**b.** 43

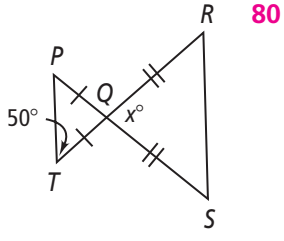



**4.** What is the value of $x$?

**a.** 120

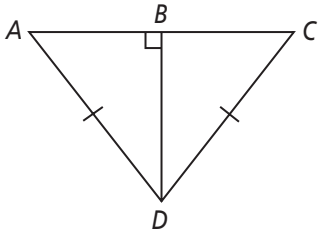



**b.** 80



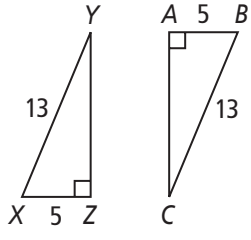

**Write a congruence statement for each pair of triangles. If the triangles cannot be proven congruent, write *not enough information.***

**5.**




$\triangle ABD \cong \triangle CBD$

**6.**




$\triangle XYZ \cong \triangle BCA$

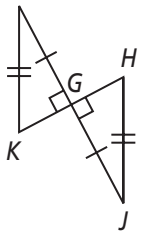
Name _____ Class _____ Date _____

# Chapter 4 Part B Test (continued) *Form K*

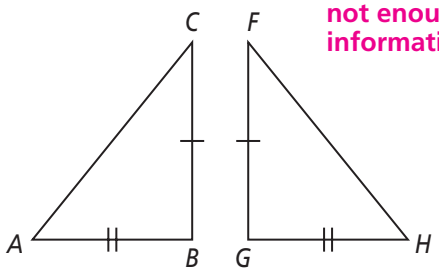
Lessons 4-4 through 4-7

**Write a congruence statement for each pair of triangles. If the triangles cannot be proven congruent, write *not enough information.***

**7.** $\triangle FGK \cong \triangle JGH$

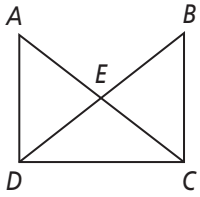



**8.** not enough information

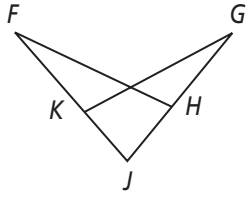



**Identify any common angles or sides for the indicated triangles.**

**9.** $\triangle ADC$ and $\triangle BDC$ $\overline{CD}$

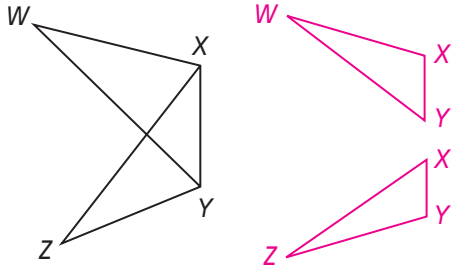



**10.** $\triangle FHJ$ and $\triangle GKJ$ $\angle J$

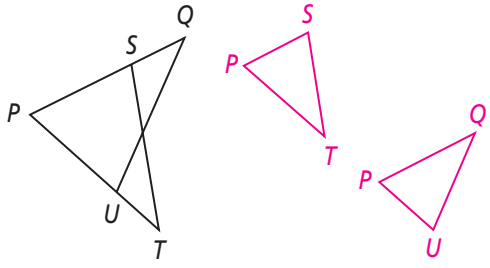



**Separate and redraw the indicated triangles.**

**11.**




**12.**




## Do you UNDERSTAND?

**13. Error Analysis** Your friend claims isosceles triangles are congruent if two corresponding sides are congruent. He explains there are only two different lengths of sides, so the third side must always be congruent. Explain the error in his reasoning. **Answers may vary. Sample: The ≅ sides can be corresp. legs, in which case the ∠ between them can be different.**

**14. Compare and Contrast** How can you use the Isosceles Triangle Theorem to prove that all equilateral triangles are also equiangular?
**Answers may vary. Sample: By using the Isosceles Triangle Thm. two times for two pairs of sides, you can show that all the ∡s of an equilateral △ are ≅.**

Name _____ Class _____ Date _____

# Performance Tasks

Chapter 4

## Task 1

Draw and label three pairs of triangles to illustrate the Side-Side-Side, Angle-Side-Angle, and Side-Angle-Side Postulates. One pair of triangles should share a common side. The figures should provide enough information to prove that they are congruent. Write the congruence statements for each pair.

**Check students' work.**

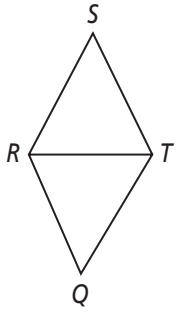
**[4] Student draws three pairs of triangles, and labels each pair. One pair of triangles shares a common side, and the three pairs demonstrate SSS, ASA, and SAS, respectively. [3] Student does three of the requirements above. [2] Student does two of the requirements above. [1] Student does one of the requirements above. [0] Student does none of the requirements above.**

## Task 2

A rhombus is a quadrilateral with four congruent sides.

**Given:** $RSTQ$ is a quadrilateral, $\angle SRT \cong \angle STR \cong \angle RTQ \cong \angle TRQ$.

**Prove:** $RSTQ$ is a rhombus.




**[4] $\angle SRT \cong \angle STR \cong \angle RTQ \cong \angle TRQ$ is given and the shared side $\overline{RT}$ is congruent to itself by the Refl. Prop. of Congruence. So, $\triangle STR \cong \triangle RTQ$ by ASA. $\triangle STR$ and $\triangle RTQ$ are also both isosceles by the Converse of the Isosc. Triangle Thm., so $\overline{ST} \cong \overline{TQ} \cong \overline{QR} \cong \overline{RS}$. So, this quadrilateral is a rhombus. [3] mostly complete proof [2] incomplete proof [1] proof missing many of the needed steps [0] incorrect or no proof**

# Performance Tasks (continued)

Chapter 4

## Task 3

You need to design a company logo. The requirements for the logo are as listed:

- The logo must include at least six triangles.
- Some of the triangles should overlap.
- Some of the triangles should share sides.
- One triangle needs to be isosceles.
- One triangle needs to be equilateral.
- At least two pairs of triangles should be congruent pairs.

Use a straightedge, compass, and protractor to aid in your design.

Label the vertices of the triangles and describe as many congruencies as you can (sides and angles).

Describe two pairs of congruent triangles in your design and justify how you know they are congruent. Include references to geometric theorems and postulates.

**Check students' work.**

**[4] Student's logo includes at least six triangles, some of which overlap. Some of the triangles share sides. At least one is isosceles, and at least one is equilateral. At least two pairs of triangles are congruent. Student has listed all possible congruencies and proved that the two congruent triangles are congruent. [3] Student has a complete logo but has not labeled diagrams accurately or proven that the congruent triangles are congruent. [2] Student logos meet most but not all of the six requirements above. [1] Student has an incomplete logo. [0] Student gives incorrect or no response.**

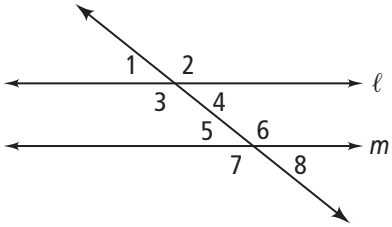
Name _____ Class _____ Date _____

# Cumulative Review

Chapters 1–4

## Multiple Choice

**Use the diagram for Exercises 1 and 2. Line $\ell$ is parallel to line $m$.**




**1.** Which best describes $\angle 1$ and $\angle 5$? **C**

(A) alternate interior angles

(B) alternate exterior angles

(C) corresponding angles

(D) same-side exterior angles

**2.** Which best describes $\angle 6$ and $\angle 7$? **F**

(F) vertical angles

(G) corresponding angles

(H) alternate exterior angles

(I) linear pair

**3.** If an animal is a mammal, then it has fur. What is the conclusion of this conditional? **B**

(A) An animal is a mammal.

(B) The animal has fur.

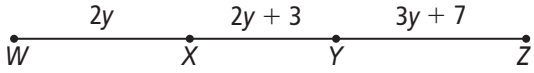
(C) Mammals have fur.

(D) Not all animals have fur.

**4.** Two of what geometric figure are joined at a vertex to form an angle? **H**

(F) points (G) planes (H) rays (I) lines

**5.** If $WZ = 80$, what is the value of $y$? **C**




(A) 8 (B) 9 (C) 10 (D) 11

**6.** If $\triangle ABC \cong \triangle DEF$, which is a correct congruence statement? **H**

(F) $\angle B \cong \angle D$ (G) $\overline{AB} \cong \overline{EF}$ (H) $\overline{CA} \cong \overline{FD}$ (I) $\angle A \cong \angle C$

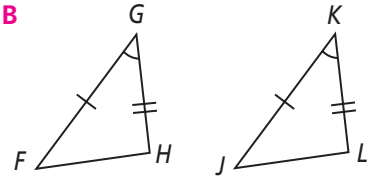
**7.** Which can be used to justify stating that $\triangle FGH \cong \triangle JKL$? **B**

(A) ASA

(B) SAS

(C) SSS

(D) AAS



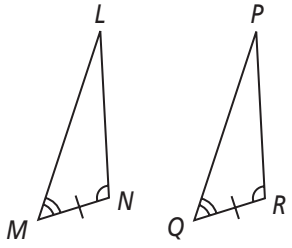

**8.** Which postulate can be used to justify stating that $\triangle LMN \cong \triangle PQR$? **F**

(F) ASA

(G) SAS

(H) SSS

(I) AAS



Name _____ Class _____ Date _____

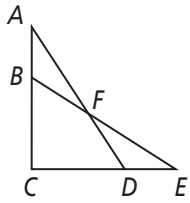
# Cumulative Review (continued)

Chapters 1–4

## Short Response

**9.** What is the midpoint of a segment with endpoints at $(-2, 2)$ and $(5, 10)$? **(1.5, 6)**

**Use the figure at the right for Exercises 10–12.**




Given: $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$

**10.** Which reason could you use to prove $\overline{AC} \cong \overline{EC}$?
**Segment Addition Postulate**

**11.** Which reason could you use to prove $\angle C \cong \angle C$?
**Reflexive Property of Congruence**

**12.** Which reason could you use to prove $\triangle ACD \cong \triangle ECB$? **SAS**

**13.** What is the slope of a line that passes through $(-3, 5)$ and $(4, 3)$? $-\frac{2}{7}$

**14.** What is the slope of a line that is perpendicular to the line that passes through $(-2, -2)$ and $(1, 3)$? $-\frac{3}{5}$

## Extended Response

**15.** Draw $\triangle ABC \cong \triangle EFG$. Write all six congruence statements. **Check students' work.**

**[4] Student draws and labels two triangles that look reasonably congruent. Student lists all six congruencies correctly. [3] Student draws and labels two triangles but lists only four congruencies correctly. [2] Student draws and labels two triangles but lists only three congruencies correctly, OR student does not draw the triangles but lists all six congruencies correctly. [1] Student draws and labels two triangles but lists only two congruencies correctly, OR student does not draw the triangles but lists four of the congruencies correctly. [0] Student gives incorrect or no response.**

**16.** The coordinates of rectangle $HIJK$ are $H(-4, 1)$, $I(1, 1)$, $J(1, -2)$, and $K(-4, -2)$. The coordinates of rectangle $LMNO$ are $L(-1, 3)$, $M(2, 3)$, $N(2, -3)$, and $O(-1, -3)$. Are these two rectangles congruent? Explain. If not, how could you change the coordinates of one of the rectangles to make them congruent?

**[4] No; answers may vary. Sample: *HIJK* has sides of length 3 and 5 while *LMNO* has sides of length 3 and 6. Change the coordinates for *N* and *O* to *N*(2, 8), *O*(−1, 8). [3] Student finds that the rectangles are not congruent, gives good replacement values to make them congruent, but does not fully explain. [2] Student either gives good explanation for why rectangles are not congruent or tells how to make them congruent, but not both. [1] Student finds that the rectangles are not congruent. [0] Student response is incorrect or blank.**

# Chapter 4 Project Teacher Notes: Tri, Tri Again

## About the Project

Students will explore how engineers use triangles to construct safe, strong, stable structures. Then they will apply these ideas to build their own bridges, using toothpicks or craft sticks.

## Introducing the Project

- Ask students whether they have ever built towers using playing cards. Ask them how they placed the first cards and why.
- Have students make towers using playing cards.

### Activity 1: Modeling

Students will discover that triangles are more stable or rigid than quadrilaterals. Discuss with students real-world examples in which triangles are used for stability, such as ironing boards, scaffolding, and frames of roofs.

### Activity 2: Observing

If students cannot find any local structures with exposed frameworks, suggest that they look in books or on the Internet for pictures of architecture or construction.

### Activity 3: Investigating

Have students work in groups, keeping a log of the different models they make in their attempt to find one that supports the weight of the geometry book. Have groups compare the successful models and discuss their similarities and differences.

## Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their products. Ask students to share how they selected their final bridge design. Ask students to submit their best models for a bridge-breaking competition, an event to which you could invite parents and the community.

Name _____ Class _____ Date _____

# Chapter 4 Project: Tri, Tri Again

## Beginning the Chapter Project

Have you ever wondered how bridges stay up? How do such frail-looking frameworks stretch through the air without falling? How can they withstand the twisting forces of hurricane winds and the rumbling weight of trucks and trains? Part of the answer lies in the natural strength of triangles.

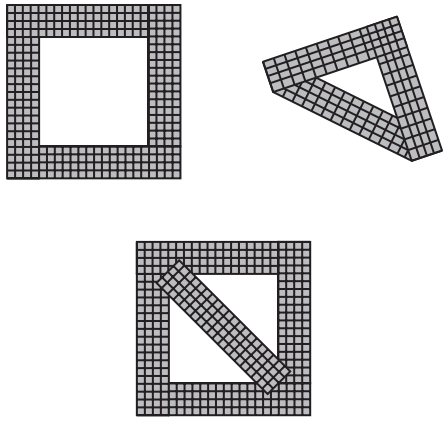
In your project for this chapter, you will explore how engineers use triangles to construct safe, strong, stable structures. You then will have a chance to apply these ideas as you design and build your own bridge with toothpicks or craft sticks. You will see how a simple shape often can be the strongest one.

## Activities

### Activity 1: Modeling

Many structures have straight beams that meet at joints. You can use models to explore ways to strengthen joints.

- Cut seven cardboard strips approximately 6 in. by $\frac{1}{2}$ in. Make a square frame and a triangular frame. Staple across the joints as shown.



- With your fingertips, hold each model flat on a desk or table, and try to Change its shape. Which shape is more stable? **triangle**
- Cut another cardboard strip, and use it to form a brace for the square frame. Is it more rigid? Why does the brace work? **Yes; the brace makes two rigid triangles.**

### Activity 2: Observing

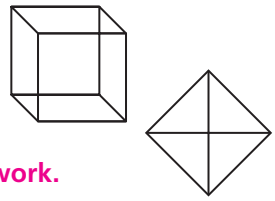
Visit local bridges, towers, or other structures that have exposed frameworks. Examine these structures for ideas you can use when you design and build a bridge later in this project. Record your ideas. Sketch or take pictures of the structures. On the sketches or photos, show where triangles are used for stability. **Check students' work.**

# Chapter 4 Project: Tri, Tri Again (continued)

### Activity 3: Investigating

In the first activity, you tested the strength of two-dimensional models. Now investigate the strength of three-dimensional models.

Use toothpicks or craft sticks and glue to construct a cube and a tetrahedron (a triangular pyramid).



- Which model is stronger? **tetrahedron**
- Describe how you could strengthen the weaker model.
  **Sample: You could add in diagonals of the cube. Check students' work.**

Use toothpicks or craft sticks and glue to construct a structure that can support the weight of your geometry book.

## Finishing the Project **Check students' work.**

Design and construct a bridge made entirely of glue and toothpicks or craft sticks. Your bridge must be at least 8 in. long and contain no more than 100 toothpicks or 30 craft sticks. With your classmates, decide how to test the strength of the bridge. Record the dimensions of your bridge, the number of toothpicks or craft sticks used, and the weight the bridge could support. Experiment with as many designs and models as you like—the more the better. Include a summary of your experiments with notes about how each one helped you improve your design.

### Reflect and Revise

Ask a classmate to review your project with you. Together, check to be sure that your bridge meets all the requirements and that your diagrams and explanations are clear. Have you tried several designs and kept a record of what you learned from each? Can your bridge be stronger or more pleasing to the eye? Can it be built using a more efficient design? Revise your work as needed.

### Extending the Project

Research architect R. Buckminster Fuller and geodesic domes. Design and build a geodesic structure, using toothpicks or other materials.

Name _____ Class _____ Date _____

# Chapter 4 Project Manager: Tri, Tri Again

## Getting Started

As you work on the project, you will need a sheet of cardboard, a stapler, 100 toothpicks or 30 craft sticks, and glue. Keep this Project Manager and all your work for the project in a folder or an envelope.

### Checklist

- ☐ Activity 1: cardboard frames
- ☐ Activity 2: observing bridges
- ☐ Activity 3: three-dimensional models
- ☐ toothpick bridge

### Suggestions

- ☐ Push or pull the models only along the plane of the frame.
- ☐ Look for small design features that are used repeatedly.
- ☐ Use glue that is strong but quick-drying.
- ☐ Test small parts of the bridge before building the entire structure. Also, decide in advance in what order you will assemble and glue the different sections.

## Scoring Rubric

**4** The toothpick bridge meets all specifications. The diagrams and explanations are clear. Geometric language was used appropriately and correctly. A complete account of the experiments was given, including how they led to improved designs.

**3** The toothpick bridge meets or comes close to meeting all specifications. The diagrams and explanations are understandable but may contain a few minor errors. Most of the geometric language is used appropriately and correctly. Evidence was shown of at least one experimental model prior to the finished model.

**2** The toothpick bridge does not meet specifications. Diagrams and explanations are misleading or hard to follow. Geometric terms are completely lacking, used sparsely, or often misused. The model shows little effort and no evidence of testing of preliminary designs.

**1** Major elements of the project are incomplete or missing.

**0** Project is not handed in or shows no effort.

**Your Evaluation of Project** Evaluate your work, based on the *Scoring Rubric.*

## Teacher's Evaluation of Project