Name ______________________________ Class ______________ Date ______________

7-1 ELL Support

Ratios and Proportions

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Description	Picture or Example
ratio	A ratio is a comparison of two quantities by division.	2 to 9, 2 : 9, or $\frac{2}{9}$
proportion	**1.** A proportion is an equation that states that two ratios are equal.	$\frac{3}{21} = \frac{2}{14}$
extremes	**2.** The extremes are the first and last numbers in a proportion.	$\frac{3}{21} = \frac{2}{14}$
means	The means are the middle two numbers in a proportion.	**3.** $\frac{3}{21} = \frac{2}{14}$
extended ratio	**4.** An extended ratio compares three or more numbers.	An isosceles right triangle has angle measures that are in the extended ratio 45 : 45 : 90.
Cross Products Propertry	In a proportion, the product of the extremes equals the product of the means.	**5.** $\frac{3}{21} = \frac{2}{14}$ $3 \cdot 14 = 2 \cdot 21$ $42 = 42$

Name ______________________ Class ____________ Date ____________

7-1 Think About a Plan

Ratios and Proportions

Reasoning The means of a proportion are 4 and 15. List all possible pairs of positive integers that could be the extremes of the proportion.

Understanding the Problem

1. What is a proportion? **an equation that states two ratios are equal**

2. What are some of the forms in which a proportion can be written?

Answers may vary. Samples: $a : b = c : d$ or $\frac{a}{b} = \frac{c}{d}$

3. Explain the difference between the means and the extremes of a proportion. Use an example in your explanation.

When you write a proportion in the form $a : b = c : d$, the first and last numbers are the extremes and the middle numbers are the means. In this example *a* and *d* are the extremes and *b* and *c* are the means.

Planning the Solution

4. How can you write the proportion described in the problem, using variables for the extremes? Should you use the same variable for the extremes or different variables?

$a : 4 = 15 : b$; use different variables because the extremes may be different numbers.

5. How can you rewrite the proportion as equivalent fractions? **$\frac{a}{4} = \frac{15}{b}$**

6. How do you solve for variables in a proportion? Apply this to the proportion you wrote in Step 5. **Find the cross-products; $ab = 60$.**

Getting an Answer

7. Look at the equation you wrote in Step 6. How do the two variables on the one side of the equation relate to the value on the other side?

The value is the product of the two variables.

8. How can you use factoring to find all the positive integers that could represent the values of the variables?

List all factor pairs for 60. These are the possible values for the extremes.

9. Find the solution to the problem.

1 and 60; 2 and 30; 3 and 20; 4 and 15; 5 and 12; 6 and 10

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7-1 Practice

Form G

Ratios and Proportions

Write the ratio of the first measurement to the second measurement.

1. diameter of a salad plate: 8 in. diameter of a dinner plate: 1 ft $\frac{2}{3}$
2. weight of a cupcake: 2 oz weight of a cake: 2 lb 2 oz $\frac{1}{17}$
3. garden container width: 2 ft 6 in. garden container length: 8 ft $\frac{5}{16}$
4. width of a canoe: 28 in. length of a canoe: 12 ft 6 in. $\frac{14}{75}$
5. height of a book: 11 in. height of a bookshelf: 3 ft 3 in. $\frac{11}{39}$
6. The perimeter of a rectangle is 280 cm. The ratio of the width to the length is 3 : 4. What is the length of the rectangle? **80 cm**
7. The ratio of country albums to jazz albums in a music collection is 2 : 3. If the music collection has 45 albums, how many are country albums? **18**
8. The lengths of the sides of a triangle are in the extended ratio 3 : 6 : 8. The triangle's perimeter is 510 cm. What are the lengths of the sides? **90 cm, 180 cm, 240 cm**

Algebra **Solve each proportion.**

9. $\frac{x}{4} = \frac{13}{52}$ **1**
10. $\frac{x}{2x + 1} = \frac{16}{40}$ **2**
11. $\frac{9}{10} = \frac{9x}{70}$ **7**
12. $\frac{2}{7} = \frac{b + 1}{56}$ **15**
13. $\frac{11}{y} = \frac{9}{27}$ **33**
14. $\frac{3}{34} = \frac{m}{51}$ **4.5**

Use the proportion $\frac{x}{z} = \frac{6}{5}$. Complete each statement. Justify your answer.

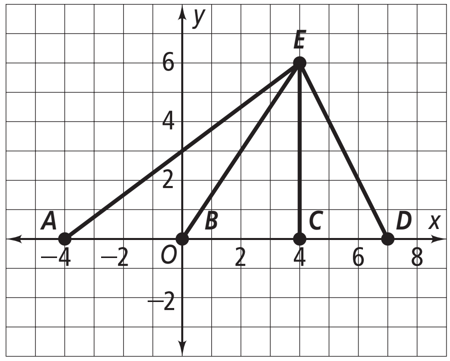
15. $\frac{x}{6} = \frac{\square}{\square}$ **$\frac{z}{5}$; Prop. of Proportions (2)**
16. $\frac{x + z}{z} = \frac{\square}{\square}$ **$\frac{11}{5}$; Prop. of Proportions (3)**
17. $\frac{z}{x} = \frac{\square}{\square}$ **$\frac{5}{6}$; Prop. of Proportions (1)**
18. $5x = \square$ **6*z*; Cross Products Property**
19. The measures of two consecutive angles in a parallelogram are in the ratio 4 : 11. What are the measures of the four angles of the parallelogram?
48, 48, 132, 132

7-1 Practice (continued)

Form G

Ratios and Proportions

Coordinate Geometry **Use the graph. Write each ratio in simplest form.**



20. $\frac{AB}{BD}$ $\frac{4}{7}$

21. $\frac{AE}{EC}$ $\frac{5}{3}$

22. $\frac{EC}{BC}$ $\frac{3}{2}$

23. $\frac{\text{slope of } \overline{BE}}{\text{slope of } \overline{AE}}$ $\frac{2}{1}$ or 2

24. A band director needs to purchase new uniforms. The ratio of small to medium to large uniforms is $3 : 4 : 6$.

a. If there are 260 total uniforms to purchase, how many will be small? **60**

b. How many of these uniforms will be medium? **80**

c. How many of these uniforms will be large? **120**

25. The measures of two complementary angles are in the ratio $2 : 3$. What is the measure of the smaller angle? **36**

26. The measures of two supplementary angles are in the ratio $4 : 11$. What is the measure of the larger angle? **132**

27. The means of a proportion are 4 and 17. List all possible pairs of positive integers that could be the extremes of the proportion. **1 and 68, 2 and 34, 4 and 17**

28. The extremes of a proportion are 5 and 14. List all possible pairs of positive integers that could be the means of the proportion. **1 and 70, 2 and 35, 5 and 14, 7 and 10**

Algebra **Solve each proportion.**

29. $\frac{(x - 1)}{(x + 1)} = \frac{10}{14}$ **6**

30. $\frac{7}{50} = \frac{x}{30}$ **4.2**

31. Writing Explain why solving proportions is an important skill for solving geometry problems. **Answers may vary. Sample: Many geometric properties involve ratios. You can use proportions to model them and solve problems.**

32. Draw a triangle that satisfies this condition: The ratio of the interior angles is $7 : 11 : 12$. **Triangle should have angles that measure 42, 66, and 72.**

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7-1 Practice

Form K

Ratios and Proportions

Write the ratio of the first measurement to the second measurement.

1. length of car: 14 ft 10 in.
length of model car: 8 in.

$\dfrac{14\text{ ft }10\text{ in.}}{8\text{ in.}} = \dfrac{\boxed{178}\text{ in.}}{8\text{ in.}} = \dfrac{\boxed{89}}{\boxed{4}}$

2. weight of car: 2900 lb
weight of model car: 8 oz

$\dfrac{2900\text{ lb}}{8\text{ oz}} = \dfrac{2900\text{ lb}}{\boxed{}\text{ lb}} = \dfrac{\boxed{1}}{\boxed{\frac{1}{2}}}$

5800

3. diameter of car tire: 40 cm
diameter of toy car tire: 18 mm **200 : 9**

4. height of car: 4 ft 8 in.
height of toy car: 3 in. **56 : 3**

5. There are 238 juniors at a high school. The ratio of girls to boys in the junior class is 3 : 4. How many juniors are girls? How many are boys? **102; 136**

6. The sides of a rectangle are in the ratio 2 : 5. The perimeter of the rectangle is 70 cm. What is the width of the rectangle? **10 cm**

7. The measures of the angles of a triangle are in the extended ratio 6 : 1 : 5. What is the measure of the largest angle? **90**

Algebra **Solve each proportion. To start, use the Cross Products Property.**

8. $\frac{3}{5} = \frac{x}{25}$ **15**

9. $\frac{x}{4} = \frac{9}{2}$ **18**

10. $\frac{x-2}{8} = \frac{3}{4}$ **8**

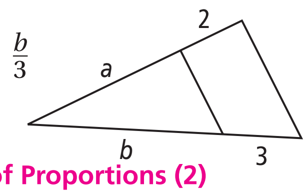
11. $\frac{y}{3} = \frac{y+6}{8}$ **3.6**

In the diagram, $\frac{a}{b} = \frac{2}{3}$. Complete each statement. Justify your answer.

12. $\frac{b}{a} = \dfrac{\boxed{3}}{\boxed{2}}$

Prop. of Proportions (1)

13. $\dfrac{\boxed{a}}{\boxed{2}} = \frac{b}{3}$



Prop. of Proportions (2)

14. $\frac{a+b}{b} = \dfrac{\boxed{5}}{\boxed{3}}$

Prop. of Proportions (3)

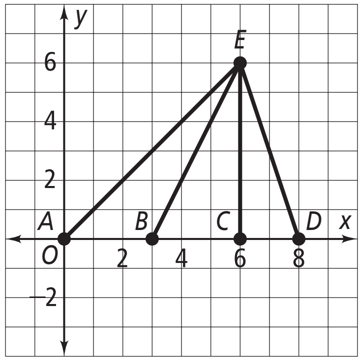
15. $\dfrac{b}{\boxed{a}} = \dfrac{3}{\boxed{2}}$

Prop. of Proportions (1)

7-1 Practice (continued) Form K

Ratios and Proportions

Coordinate Geometry **Use the graph. Write each ratio in simplest form.**



16. $\frac{AC}{AD} = \frac{\boxed{6}}{\boxed{8}}$; simplified to $\frac{\boxed{3}}{\boxed{4}}$.

17. $\frac{AB}{EC}$ $\frac{1}{2}$

18. slope of $\overline{ED}$ -3

19. You are helping to hang balloons in the gym for a school dance. There are a total of 175 balloons. Some of the balloons are gold and the rest are silver. If the ratio of gold to silver is 3 : 2, how many gold balloons are there? **105**

20. The ratio of the width to the height of a window is 2 : 7. The width of the window is 3 ft. Write and solve a proportion to find the height.
$\frac{2}{7} = \frac{3}{x}$; **10.5 ft**

21. The sides of a triangle are in the extended ratio of 3 : 4 : 10. If the length of the shortest side is 9 in., what is the perimeter of the triangle? **51 in.**

22. Write a proportion that has means 4 and 15 and extremes 6 and 10.
Answers may vary. Sample: $\frac{6}{4} = \frac{15}{10}$

Algebra **Solve each proportion.**

23. $\frac{x}{4} = \frac{77}{28}$ **11**

24. $\frac{3}{4y} = \frac{9}{138}$ **11.5**

25. $\frac{6}{d+5} = \frac{3}{d+1}$ **3**

26. $\frac{8}{2y-3} = \frac{6}{y+4}$ **12.5**

27. Writing Explain how the Cross Products Property can be used to show that $\frac{2}{x-3} = \frac{4}{2x+1}$ is not a true proportion.
Answers may vary. Sample: When you multiply the means and the extremes and simplify, you get $2 = -12$, which is not true.

Name ______________________ Class ____________ Date ____________

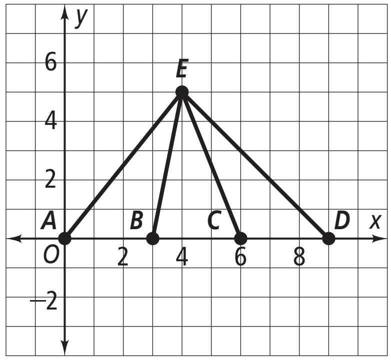
7-1 Standardized Test Prep

Ratios and Proportions

Gridded Response

Solve each exercise and enter your answer on the grid provided.

Use the graph at the right for Exercises 1 and 2.



1. What is $\frac{AD}{AB}$ in simplest form?

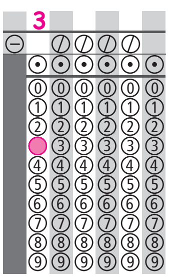
2. What is $\dfrac{\text{slope of } \overline{BE}}{\text{slope of } \overline{AE}}$ in simplest form?

3. What is the value of x in the proportion $\frac{(x - 1)}{5} = \frac{(4x + 2)}{35}$?

4. What is the value of x in the proportion $\frac{x + 1}{x + 3} = \frac{15}{21}$?

5. The lengths of the sides of a triangle are in the extended ratio 3 : 10 : 12. The perimeter is 400 cm. What is the length of the longest side in centimeters?

Answers

1.


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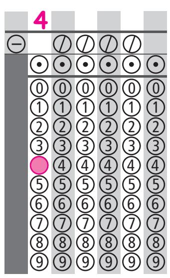

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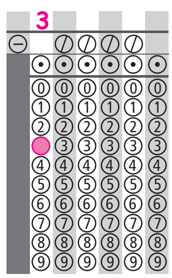

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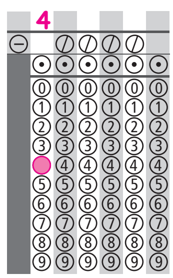
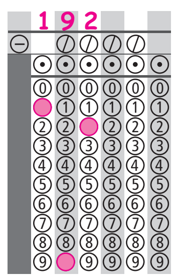


5.


7-1 Enrichment

Ratios and Proportions

Ratios and proportions occur frequently in everyday situations. Some involve linear equations, such as those concerning menu planning and recipes, whereas others, often involving geometry, require quadratic equations.

Use ratios and proportions to solve each problem.

1. A meatloaf recipe uses 4 lb of hamburger to feed 6 people. How many pounds of hamburger will be used to feed 15 people? **10 lb**

2. The tenth grade at Milford High School has a dance every year. Last year there were 80 students in the tenth grade, and the party cost $200. This year there are 100 students in the tenth grade. How much should they plan to spend? **$250**

3. If it costs $200,000 to build a sidewalk around a rectangular field whose dimensions are 200 yd by 800 yd, how much will it cost to build a sidewalk around a rectangular field whose dimensions are 300 yd by 900 yd? **$240,000**

4. The cost of buying a plot of land in Happy Valley depends on the area of the plot. If a rectangular plot of land whose dimensions are 200 yd by 800 yd costs $100,000, what is the cost of a rectangular plot of land whose dimensions are 300 yd by 900 yd? **$168,750**

5. If it costs $5980 to have a picket fence installed around a rectangular lot that is 110 ft by 150 ft, how much will it cost to have a picket fence installed around a rectangular lot that is 125 ft by 170 ft? **$6785**

6. At Pools-a-Plenty it costs $165 for a swimming pool cover for a round, aboveground pool that is 30 ft in diameter. How much will a cover for a pool that is 24 ft in diameter cost at Pools-a-Plenty? Use 3.14 for π. **$105.60**

7. Fencing was purchased for two rectangular plots of land. The first plot measured 80 yd by 100 yd, and the cost of the fencing was $900. The cost of the fencing for the second plot was $2400, and one of the dimensions of the plot was 120 yd. What was the other dimension? **360 yd**

8. If 3 hens lay 10 eggs in 5 days, how many eggs will 3 hens lay in 20 days? **40 eggs**

9. If 8 hens lay 14 eggs in 6 days, how many eggs will 24 hens lay in 6 days? **42 eggs**

10. If 4 hens lay 7 eggs in 3 days, how many eggs will 12 hens lay in 9 days? **63 eggs**

11. If Jenna can walk 6 mi in 2 h, how many miles could she walk in 2.5 h, assuming she keeps the same pace? **7.5 mi**

12. Suppose 2.5 lb of grass seed can cover a plot of land that is 30 ft by 30 ft. How much grass seed is needed to cover a plot of land 45 ft by 60 ft? **7.5 lb**

Name ________________________ Class ____________ Date ____________

7-1 Reteaching

Ratios and Proportions

Problem

About 15 of every 1000 light bulbs assembled at the Brite Lite Company are defective. If the Brite Lite Company assembles approximately 13,000 light bulbs each day, about how many are defective?

Set up a proportion to solve the problem. Let x represent the number of defective light bulbs per day.

$\frac{15}{1000} = \frac{x}{13,000}$	
$15(13,000) = 1000x$	Cross Products Property
$195,000 = 1000x$	Simplify.
$\frac{195,000}{1000} = x$	Divide each side by 1000.
$195 = x$	Solve for the variable.

About 195 of the 13,000 light bulbs assembled each day are defective.

Exercises

Use a proportion to solve each problem.

1. About 45 of every 300 apples picked at the Newbury Apple Orchard are rotten. If 3560 apples were picked one week, about how many apples were rotten? **534**
2. A grocer orders 800 gal of milk each week. He throws out about 64 gal of spoiled milk each week. Of the 9600 gal of milk he ordered over three months, about how many gallons of spoiled milk were thrown out? **768**
3. Seven of every 20 employees at V & B Bank Company are between the ages of 20 and 30. If there are 13,220 employees at V & B Bank Company, how many are between the ages of 20 and 30? **4627**
4. About 56 of every 700 picture frames put together on an assembly line have broken pieces of glass. If 60,000 picture frames are assembled each month, about how many will have broken pieces of glass? **4800**

Algebra Solve each proportion.

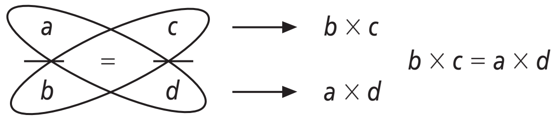
5. $\frac{300}{1600} = \frac{x}{4800}$ **900**
6. $\frac{40}{140} = \frac{700}{x}$ **2450**
7. $\frac{x}{2000} = \frac{17}{400}$ **85**
8. $\frac{35}{x} = \frac{150}{2400}$ **560**
9. $\frac{x}{1040} = \frac{290}{5200}$ **58**
10. $\frac{x}{42,000} = \frac{87}{500}$ **7308**
11. $\frac{x}{380} = \frac{180}{5700}$ **12**
12. $\frac{1200}{90,000} = \frac{270}{x}$ **20,250**
13. $\frac{325}{x} = \frac{7306}{56,200}$ **2500**

Name ______________________________ Class ____________ Date ____________

7-1 Reteaching (continued)

Ratios and Proportions

In a proportion, the products of terms that are diagonally across the equal sign from each other are the same. This is called the *Cross Products Property* because the products cross at the equal sign.



Proportions have other properties:

Property (1) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{b}{a} = \frac{d}{c}$. Use reciprocals of the ratios.

Property (2) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}$. Switch *b* and *c* in the proportion.

Property (3) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a+b}{b} = \frac{c+d}{d}$. Add the denominator to the numerator.

Problem

How can you use the Cross Products Property to verify Property (3)?

$\frac{a}{b} = \frac{c}{d}$ is equivalent to $ad = bc$.

$\frac{a+b}{b} = \frac{c+d}{d}$ is equivalent to $(a+b)d = b(c+d)$. Cross Products Property

$ad + bd = bc + bd$ Distributive Property

$ad = bc$ Subtraction Property of Equality

So, $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a+b}{b} = \frac{c+d}{d}$.

Exercises

Use the proportion $\frac{x}{10} = \frac{2}{z}$. Complete each statement. Justify your answer.

14. $\frac{x}{2} = \frac{\square}{\square}$ **$\frac{10}{z}$, Prop. of Proportions (2)**

15. $\frac{10}{x} = \frac{\square}{\square}$ **$\frac{z}{2}$, Prop. of Proportions (1)**

16. $\frac{x+10}{10} = \frac{\square}{\square}$ **$\frac{2+z}{z}$, Prop. of Proportions (3)**

17. The ratio of width to length of a rectangle is 7 : 10. The width of the rectangle is 91 cm. Write and solve a proportion to find the length. **$\frac{7}{10} = \frac{91}{x}$; 130 cm**

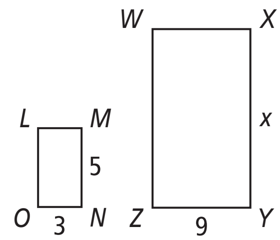
18. The ratio of the two acute angles in a right triangle is 5 : 13. What is the measure of each angle in the right triangle? **25, 65, 90**

Name ______________________ Class ____________ Date ____________

7-2 ELL Support

Similar Polygons

There are two sets of note cards below that show how to solve for x, given $LMNO \sim WXYZ$, in the diagram at the right. The set on the left explains the thinking and the set on the right shows the steps. Write the thinking and the steps in the correct order.



Think Cards	**Write Cards**
Use the Cross Products Property.	$15 = x$
Substitute the values in the proportion.	$45 = 3x$
Divide each side by 3.	$\frac{MN}{XY} = \frac{ON}{ZY}$
Set up a proportion.	$\frac{5}{x} = \frac{3}{9}$

Think	**Write**
First, you should set up a proportion.	Step 1 $\frac{MN}{XY} = \frac{ON}{ZY}$
Second, you should substitute the values in the proportion.	Step 2 $\frac{5}{x} = \frac{3}{9}$
Next, you should use the Cross Products Property.	Step 3 $45 = 3x$
Finally, you should divide each side by 3.	Step 4 $15 = x$

Name ______________________ Class ____________ Date ____________

7-2 Think About a Plan

Similar Polygons

Sports Choose a scale and make a scale drawing of a rectangular soccer field that is 110 yd by 60 yd.

1. What is a scale drawing? How does a figure in a scale drawing relate to an actual figure?

Answers may vary. Sample: A scale drawing is enlarged or reduced proportionally to the actual figure. A figure in a scale drawing and the actual figure are similar figures.

2. What is a scale? What will the scale of your drawing compare? Write a ratio to represent this.

Answers may vary. Sample: a ratio of the actual size to the size in the drawing; the soccer field's actual length to the length in the drawing; actual length : length of drawing

3. To select a scale you need to choose a unit for the drawing. Assuming you are going to make your drawing on a typical sheet of paper, which customary unit of length should you use? **inches**

4. You have to choose how many yards each unit you chose in Step 3 will represent. The soccer field is 110 yd long. What is the least number of yards each unit can represent and still fit on an 8.5 in.-by-11 in. sheet of paper? Explain. Does this scale make sense for your scale drawing?

The least number of yards each inch can represent is 10 yd. If the scale is 1 in. = 10 yd, the scale drawing will be 11 in. long, which is the length of the paper. It might make sense to use a scale that makes the drawing smaller.

5. Choose the scale of your drawing. **Answers may vary. Sample: 1 in. = 20 yd**

6. How can you use the scale to write a proportion to find the length of the field in the scale drawing? Write and solve a proportion to find the length of the soccer field in the scale drawing.

Answers may vary. Sample: Make a proportion using the actual length of the soccer field, the length in the drawing, and the scale factor. $\frac{110\text{ yd}}{\ell\text{ in.}} = \frac{20\text{ yd}}{1\text{ in.}}$**; 5.5 in.**

7. Write and solve a proportion to find the width of the soccer field in the scale drawing. **Answers may vary. Sample:** $\frac{60\text{ yd}}{w\text{ in.}} = \frac{20\text{ yd}}{1\text{ in.}}$**; 3 in.**

8. Use a ruler to create the scale drawing on a separate piece of paper.
Check students' work.

Name ______________________ Class ____________ Date ____________

7-2 Practice

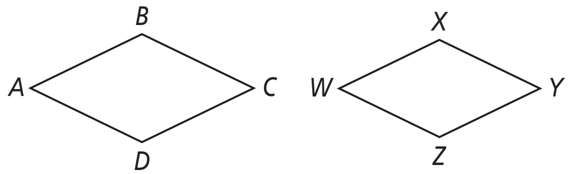
Form G

Similar Polygons

List the pairs of congruent angles and the extended proportion that relates the corresponding sides for the similar polygons.

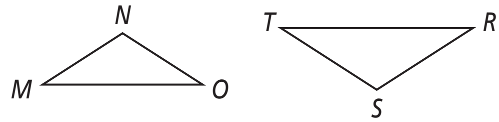
1. $ABCD \sim WXYZ$

$\angle A \cong \angle W$, $\angle B \cong \angle X$,
$\angle C \cong \angle Y$, $\angle D \cong \angle Z$;
$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW}$



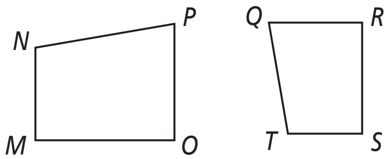
2. $\triangle MNO \sim \triangle RST$

$\angle M \cong \angle R$, $\angle N \cong \angle S$, $\angle O \cong \angle T$;
$\frac{MN}{RS} = \frac{NO}{ST} = \frac{OM}{TR}$



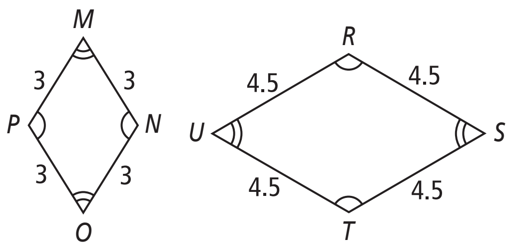
3. $NPOM \sim TQRS$

$\angle N \cong \angle T$, $\angle P \cong \angle Q$;
$\angle O \cong \angle R$, $\angle M \cong \angle S$;
$\frac{NP}{TQ} = \frac{PO}{QR} = \frac{OM}{RS} = \frac{MN}{ST}$



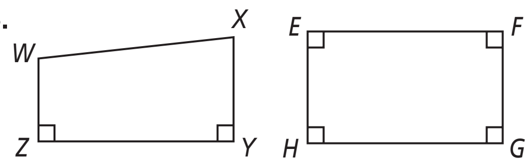
Determine whether the polygons are similar. If so, write a similarity statement and give the scale factor. If not, explain.

4.



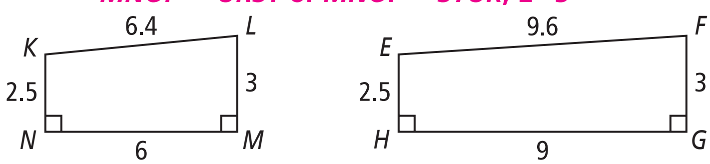
$MNOP \sim URST$ or $MNOP \sim STUR$; 2 : 3

5.



not similar; corresponding ∠s not ≅

6.



not similar; corresponding sides not proportional

Determine whether the polygons are similar.

7. an equilateral triangle with side length 6 and an equilateral triangle with side length 15 **yes**

8. a square with side length 4 and a rectangle with width 8 and length 8.5 **no**

9. a triangle with side lengths 3 cm, 4 cm, and 5 cm, and a triangle with side lengths 18 cm, 19 cm, and 20 cm **no**

10. a rhombus with side lengths 8 and consecutive angles 50° and 130°, and a rhombus with side lengths 13 and consecutive angles 50° and 130° **yes**

Name ______________________ Class ____________ Date ____________

7-2 Practice (continued) — Form G

Similar Polygons

11. An architect is making a scale drawing of a building. She uses the scale 1 in. = 15 ft.

a. If the building is 48 ft tall, how tall should the scale drawing be? **3.2 in.**

b. If the building is 90 ft wide, how wide should the scale drawing be? **6 in.**

12. A scale drawing of a building was made using the scale 15 cm = 120 ft. If the scale drawing is 45 cm tall, how tall is the actual building? **360 ft**

Determine whether each statement is *always*, *sometimes*, or *never* true.

13. Two squares are similar. **always**

14. Two hexagons are similar. **sometimes**

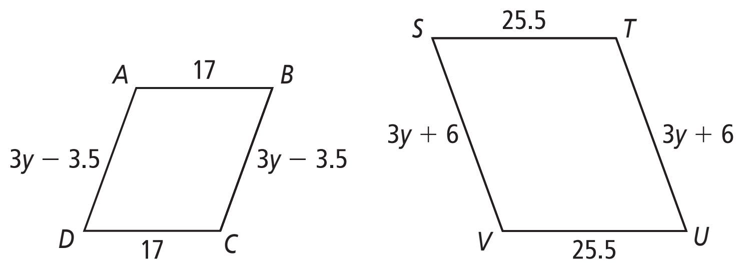
15. Two similar triangles are congruent. **sometimes**

16. A rhombus and a pentagon are similar. **never**

Algebra Find the value of y. Give the scale factor of the polygons.

17. $ABCD \sim TSVU$

7.5; 2 : 3



18. The scale factor of $RSTU$ to $VWXY$ is 14 : 3. What is the scale factor of $VWXY$ to $RSTU$? **3 : 14**

In the diagram below, $\triangle PRQ \sim \triangle DEF$. Find each of the following.

19. the scale factor of $\triangle PRQ$ to $\triangle DEF$ **5 : 6**

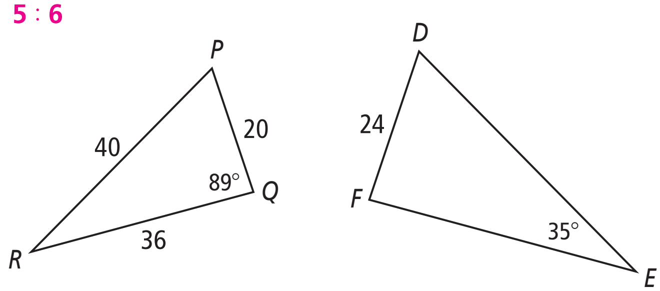
20. $m\angle D$ **56**

21. $m\angle R$ **35**

22. $m\angle P$ **56**

23. DE **48**

24. FE **43.2**



25. Writing Explain why all isosceles right triangles are similar, but not all scalene right triangles are similar. **Answers may vary. Sample: All isosceles right triangles have angle measures 45-45-90, the legs of the triangle will always be congruent, and the hypotenuses are always about 1.4 times the length of the leg. Scalene right triangles can have any pair of angle measures that adds up to 90 for the non-right angles, so they are not all similar.**

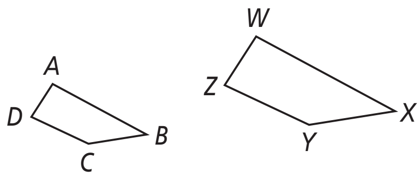
Name ______________________ Class ____________ Date ____________

7-2 Practice *Form K*

Similar Polygons

List the pairs of congruent angles and the extended proportion that relates the corresponding sides for the similar polygons.

1. $ABCD \sim WXYZ$

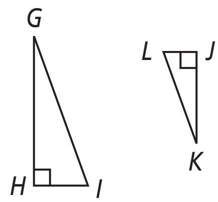


$\angle A \cong \angle W$ $\quad \angle B \cong \boxed{\angle X}$

$\angle C \cong \boxed{\angle Y}$ $\quad \angle D \cong \boxed{\angle Z}$

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{\boxed{CD}}{\boxed{YZ}} = \frac{\boxed{DA}}{\boxed{ZW}}$$

2. $\triangle GHI \sim \triangle KJL$



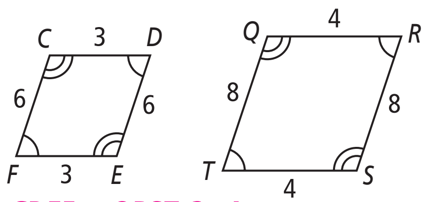
$\angle G \cong \boxed{\angle K}$ $\quad \angle H \cong \boxed{\angle J}$

$\angle I \cong \boxed{\angle L}$

$$\frac{GH}{KJ} = \frac{\boxed{HI}}{\boxed{JL}} = \frac{\boxed{IG}}{\boxed{LK}}$$

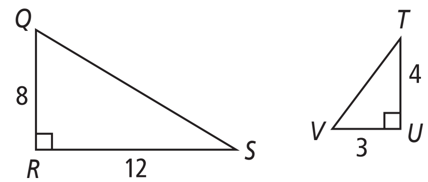
Determine whether the polygons are similar. If so, write a similarity statement and give the scale factor. If not, explain.

3.



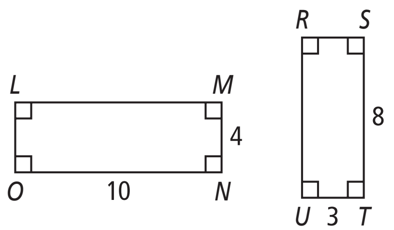
CDEF ~ *QRST*; 3 : 4

4.



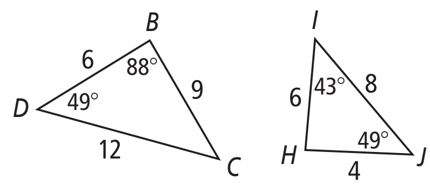
no; corresponding sides not proportional

5.



no; corresponding sides not proportional

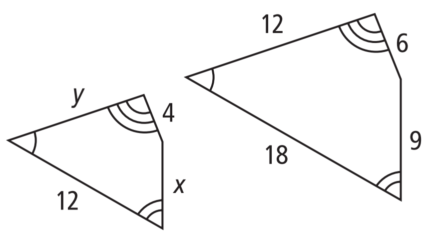
6.



$\triangle DBC \sim \triangle JHI$; 3 : 2

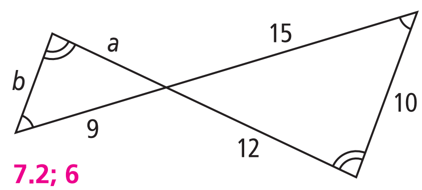
Algebra **The polygons are similar. Find the value of each variable.**

7.



6; 8

8.



7.2; 6

7-2 Practice (continued) — Form K

Similar Polygons

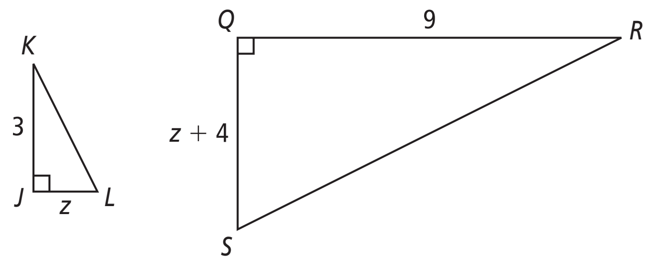
9. You want to enlarge a 3 in-by-5 in. photo. The paper you will print on is 8.5 in.-by-14 in. What is the largest size the photo can be? **8.4 in.-by-14 in.**

10. For art class, you need to make a scale drawing of the Parthenon using the scale 1 in. = 5 ft. The Parthenon is 228 ft long. How long should you make the building in your scale drawing? **45.6 in.**

11. Ella is reading a map with a scale of 1 in. = 20 mi. On the map, the distance Ella must drive is 4.25 in. How many miles is this? **85 mi**

Algebra Find the value of *z*. Give the scale factor of the polygons.

12. $\triangle JKL \sim \triangle QRS$ **2; 1 : 3**



13. The scale factor of *ABCD* to *EFGH* is 7 : 20. What is the scale factor of *EFGH* to *ABCD*? **20 : 7**

In the diagram below, $\triangle NOP \sim \triangle WXY$. Find each of the following.

14. the scale factor of $\triangle NOP$ to $\triangle WXY$ **2 : 5**

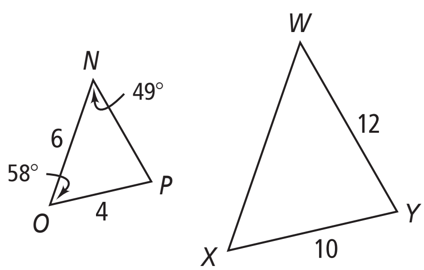
15. $m\angle X$ **58**

16. $m\angle Y$ **73**

17. $\frac{NP}{WY}$ **$\frac{2}{5}$**

18. WX **15**

19. NP **4.8**



20. A company makes rugs. Their smallest rug is a 2 ft-by-3 ft rectangle. Their largest rug is a similar rectangle. If one side of their largest rug is 18 ft, what are the possible dimensions of their largest rug? **18 ft-by-27 ft or 12 ft-by-18 ft**

Name ______________________ Class ____________ Date ____________

7-2 Standardized Test Prep

Similar Polygons

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. You make a scale drawing of a tree using the scale 5 in. = 27 ft. If the tree is 67.5 ft tall, how tall is the scale drawing? **D**

(A) 10 in. (B) 11.5 in. (C) 12 in. (D) 12.5 in.

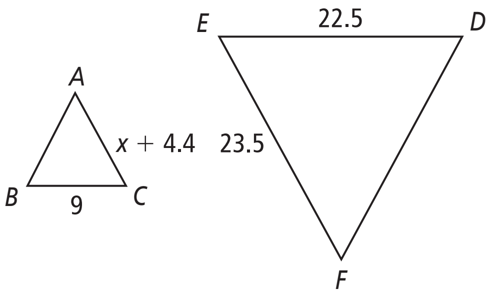
2. You make a scale drawing of a garden plot using the scale 2 in. = 17 ft. If the length of a row of vegetables on the drawing is 3 in., how long is the actual row? **G**

(F) 17 ft (G) 25.5 ft (H) 34 ft (I) 42.5 ft

3. The scale factor of $\triangle RST$ to $\triangle DEC$ is 3 : 13. What is the scale factor of $\triangle DEC$ to $\triangle RST$? **D**

(A) 3 : 13 (B) 1 : 39 (C) 39 : 1 (D) 13 : 3

4. $\triangle ACB \sim \triangle FED$. What is the value of x? **I**



(F) 4 (G) 4.2 (H) 4.5 (I) 5

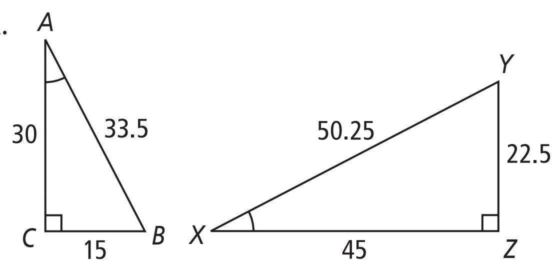
5. $MNOP \sim QRST$ with a scale factor of 5 : 4. MP = 85 mm. What is the value of QT? **B**

(A) 60 mm (B) 68 mm (C) 84 mm (D) 106.25 mm

Short Response

6. Are the triangles at the right similar? Explain.

[2] Yes; corresponding angles are congruent and lengths of corresponding sides are proportional. [1] recognition that corresponding angles are congruent or corresponding side lengths are proportional. [0] No explanation given.



Name ______________________ Class ____________ Date ____________

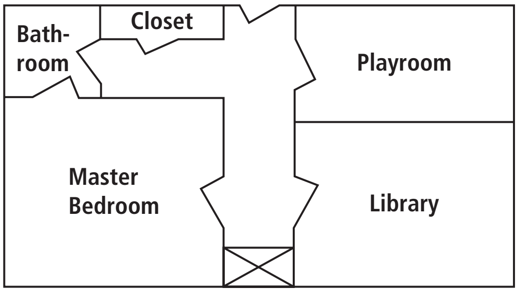
7-2 Enrichment

Similar Polygons

Floor Plans

Architects, engineers, and other professionals make scale drawings to design or present building plans. A floor plan of the second floor of a house is shown below. Use the scale to find the actual dimensions of each room.

1. playroom **18 ft by 10 ft**
2. library **18 ft by 14 ft**
3. master bedroom **18 ft by 16 ft**
4. bathroom **8 ft by 8 ft**
5. closet **3 ft by 10 ft**



Scale: 1 in. = 16 ft

Someone who wants to rearrange a room can make use of a scale drawing of the room that includes furniture. Two-dimensional shapes can represent the objects that sit on the floor in the room.

Make a scale drawing of a room in which you spend a lot of time, such as your classroom or bedroom, including any objects that take up floor space.

6. Choose an appropriate scale so the drawing covers most of an 8.5 in.-by-11 in. piece of paper. What scale did you choose?
   Answers may vary. Sample: 1 in. = 3 ft

7. What shape is the room? Measure the dimensions of the room and draw the shape to represent the room's outline.
   Answers may vary. Sample: rectangle; 15 ft by 24 ft

8. List three objects that take up floor space. Measure the dimensions of each object, then determine their dimensions in the scale drawing. You can round to the nearest millimeter or quarter of an inch.

Object	Actual Dimensions	Scale Factor	Dimensions on Drawing
Sample: table	**Sample: 4 ft by 8 ft**	**192 : 1**	**Sample: 0.25 in. by 0.50 in.**

9. Complete the scale drawing. Remember to measure the distance between objects so that this is accurately represented in the drawing. **Check students' work.**

Name ______________________ Class ____________ Date ____________

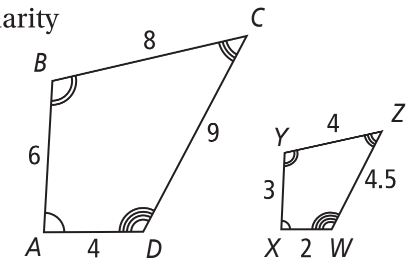
7-2 Reteaching

Similar Polygons

Similar polygons have corresponding angles that are congruent and corresponding sides that are proportional. An **extended proportion** can be written for the ratios of corresponding sides of similar polygons.

Problem

Are the quadrilaterals at the right similar? If so, write a similarity statement and an extended proportion.



Compare angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$.
$\angle C \cong \angle Z$, $\angle D \cong \angle W$

Compare ratios of sides: $\frac{AB}{XY} = \frac{6}{3} = 2$ $\quad \frac{CD}{ZW} = \frac{9}{4.5} = 2$

$\frac{BC}{YZ} = \frac{8}{4} = 2$ $\quad \frac{DA}{WX} = \frac{4}{2} = 2$

Because corresponding sides are proportional and corresponding angles are congruent, $ABCD \sim XYZW$.

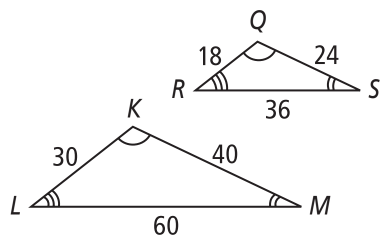
The extended proportion for the ratios of corresponding sides is:

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZW} = \frac{DA}{WX}$$

Exercises

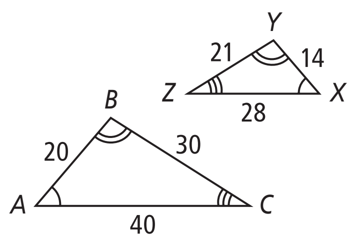
If the polygons are similar, write a similarity statement and the extended proportion for the ratios of corresponding sides. If the polygons are not similar, write *not similar.*

1.



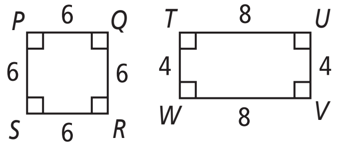
$KML \sim QSR$, $\frac{KM}{QS} = \frac{ML}{SR} = \frac{LK}{RQ}$

2.



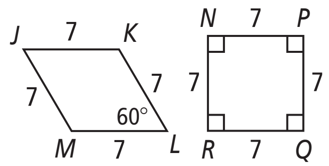
$BCA \sim YZX$, $\frac{BC}{YZ} = \frac{CA}{ZX} = \frac{AB}{XY}$

3.



not similar

4.



not similar

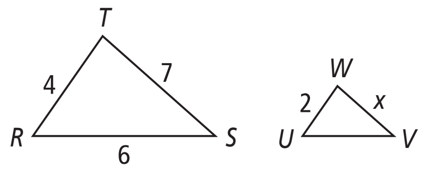
7-2 Reteaching (continued)

Similar Polygons

Problem

$\triangle RST \sim \triangle UVW$. What is the scale factor?

What is the value of x?



Identify corresponding sides: $\overline{RT}$ corresponds to $\overline{UW}$, $\overline{TS}$ corresponds to $\overline{WV}$, and $\overline{SR}$ corresponds to $\overline{VU}$.

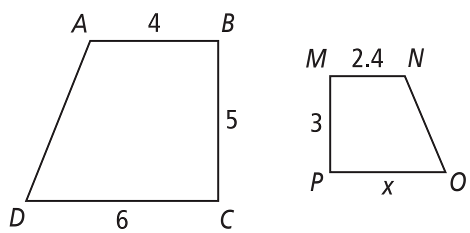
$\frac{RT}{UW} = \frac{TS}{WV}$	Compare corresponding sides.
$\frac{4}{2} = \frac{7}{x}$	Substitute.
$4x = 14$	Cross Products Property
$x = 3.5$	Divide each side by 4.

The scale factor is $\frac{4}{2} = \frac{7}{3.5} = 2$. The value of x is 3.5.

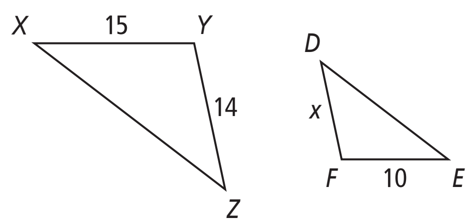
Exercises

Give the scale factor of the polygons. Find the value of x. Round answers to the nearest tenth when necessary.

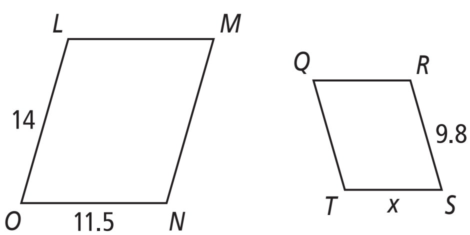
5. $ABCD \sim NMPO$ **5 : 3; 3.6**



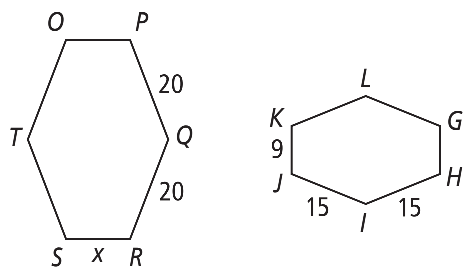
6. $\triangle XYZ \sim \triangle EFD$ **3 : 2; 9.3**



7. $LMNO \sim RQTS$ **10 : 7; 8.1**



8. $OPQRST \sim GHIJKL$ **4 : 3; 12**

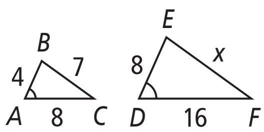


Name ______________________ Class ____________ Date ____________

7-3 ELL Support

Proving Triangles Similar

The column on the left shows the steps used to solve a proportion. Use the column on the left to answer each question in the column on the right.

Problem **SAS ~ Theorem** **Use a proportion to find x.**  	**1.** Look at the diagram. What measure are you trying to find? The length of $\overline{EF}$ is unknown.
Verify that the triangles are similar. $\frac{4}{8} = \frac{8}{16}$ $\angle A \cong \angle D$ $\triangle ABC \sim \triangle DEF$ by SAS $\sim$ Theorem.	**2.** What is the SAS ~ Theorem? If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.
Write a proportion. $\frac{4}{8} = \frac{7}{x}$	**3.** What is a proportion? A proportion is an equation that states that two ratios are equal.
Write the cross products. $4 \cdot x = 8 \cdot 7$ Simplify. $4x = 56$	**4.** What does writing the cross products mean? The cross products of a proportion are equal. First multiply the extremes of the proportion to get the product for one side of the equation, and then multiply the means to get the product of the other side of the equation.
Divide both sides by 4. $x = 14$	**5.** Why do you divide both sides by 4? To isolate the variable x.

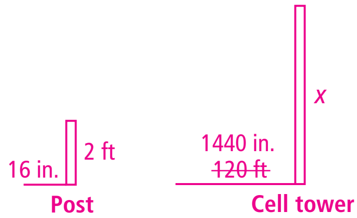
Name ______________________ Class ____________ Date ____________

7-3 Think About a Plan

Proving Triangles Similar

Indirect Measurement A 2-ft vertical post casts a 16-in. shadow at the same time a nearby cell phone tower casts a 120-ft shadow. How tall is the cell phone tower?

Answers may vary. Sample:



Know

1. Draw a sketch of the situation described in the problem. Label the sketch with information from the problem and assign a variable to represent the unknown.

2. If you connect the top of each figure to the end of its shadow, what kind of polygons have you formed? How are these polygons related?
   right triangles; they are similar.

3. Which parts of the polygons are corresponding?
   Answers may vary. Sample: the lengths of the shadows are corresponding and the heights of the objects are corresponding.

Need

4. In your diagram, which corresponding parts have different units?
   the lengths of the post's shadow and the cell phone tower's shadow

5. What must you do so that corresponding parts have the same units? Which unit does it make the most sense to change? Explain.
   Answers may vary. Sample: convert in. to ft or ft to in.; change 120 ft to inches; easier because you don't need to use fractions or decimals.

6. Change the units and update your diagram.
   Check students' work.

Plan

7. Write a proportion in words that compares the corresponding parts.
   $\frac{\text{height of post}}{\text{length of post's shadow}} = \frac{\text{height of tower}}{\text{length of tower's shadow}}$

8. Use information from the diagram to write and solve a numerical proportion. What is the height of the cell phone tower?
   $\frac{2 \text{ ft}}{16 \text{ in.}} = \frac{x}{1440 \text{ in.}}$; **180 ft**

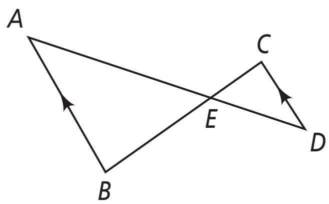
7-3 Practice

Form G

Proving Triangles Similar

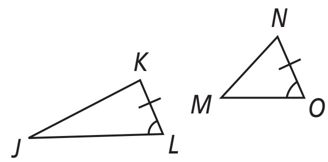
Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

1.



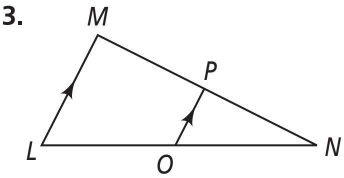
$\triangle ABE \sim \triangle DCE$ by the AA ~ Postulate

2.



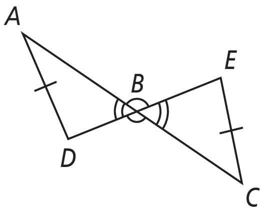
not similar; only one side and one angle ≅

3.



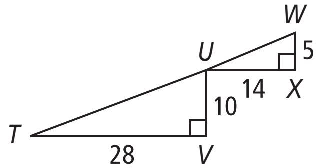
$\triangle LMN \sim \triangle OPN$ by the AA ~ Postulate

4.



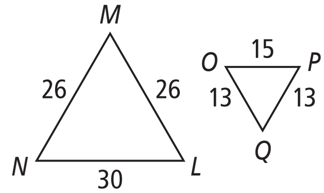
not similar; only one angle and one side ≅

5.



$\triangle TUV \sim \triangle UWX$ by the SAS ~ Theorem

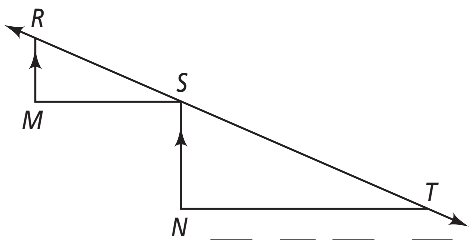
6.



$\triangle MNL \sim \triangle QOP$; SSS ~ Theorem

7. Given: $\overline{RM} \parallel \overline{SN}$, $\overline{RM} \perp \overline{MS}$, $\overline{SN} \perp \overline{NT}$

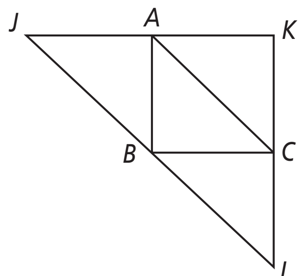
Prove: $\triangle RSM \sim \triangle STN$



Statements: 1) $\overline{RM} \parallel \overline{SN}$; $\overline{RM} \perp \overline{MS}$, $\overline{SN} \perp \overline{NT}$ 2) $\angle MRS \cong \angle NST$; 3) $\angle M$ and $\angle N$ are rt. ∠s; 4) $\angle M \cong \angle N$; 5) $\triangle RSM \sim \triangle STN$;
Reasons: 1) Given; 2) Corresp. ∠s Post.; 3) Perp. lines form rt. ∠s; 4) All rt. ∠s are ≅; 5) AA ~ Post.

8. Given: A bisects $\overline{JK}$, C bisects $\overline{KL}$, B bisects $\overline{JL}$

Prove: $\triangle JKL \sim \triangle CBA$



It is given that A, C, and B are the midpoints of $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$. Therefore, according to the Midsegment Theorem, $\overline{AB}$ is half the length of $\overline{KL}$, $\overline{BC}$ is half the length of $\overline{JK}$, and $\overline{AC}$ is half the length of $\overline{JL}$. It follows then that $\triangle JKL \sim \triangle CBA$ by the SSS ~ Theorem.

9. A 1.4-m tall child is standing next to a flagpole. The child's shadow is 1.2 m long. At the same time, the shadow of the flagpole is 7.5 m long. How tall is the flagpole? 8.75 m

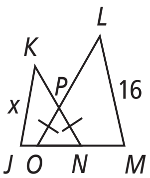
Name ______________________ Class ____________ Date ____________

Practice (continued) — Form G

Proving Triangles Similar

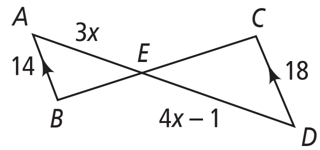
Explain why the triangles are similar. Then find the value of *x*.

10. $\overline{OP} \cong \overline{NP}$, $KN = 15$, $LO = 20$, $JN = 9$, $MO = 12$



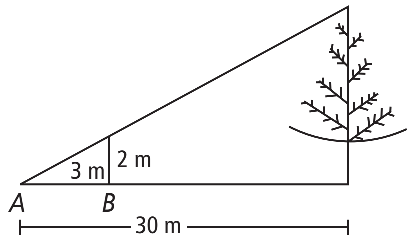
$\overline{OP} \cong \overline{NP}$ means that $\angle PON \cong \angle PNO$ because if two sides of a $\triangle$ are $\cong$, the $\angle$s opposite those sides are $\cong$. $\frac{KN}{LO} = \frac{15}{20} = \frac{3}{4}$ and $\frac{JN}{MO} = \frac{9}{12} = \frac{3}{4}$. So, $\triangle JKN \sim \triangle MLO$ by SAS $\sim$ Thm. $\frac{x}{16} = \frac{3}{4}$; 12

11.

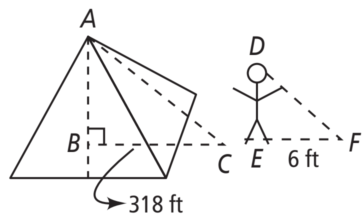


$\triangle ABE \sim \triangle DCE$ by the AA $\sim$ Post. $\overline{AB} \parallel \overline{CD}$ means that $\angle A \cong \angle D$ by the Alt. Int. $\angle$s Thm. For the same reason, $\angle B \cong \angle C$. $\frac{3x}{4x-1} = \frac{14}{18}$; 7

12. A stick 2 m long is placed vertically at point *B*. The top of the stick is in line with the top of a tree as seen from point *A*, which is 3 m from the stick and 30 m from the tree. How tall is the tree? 20 m

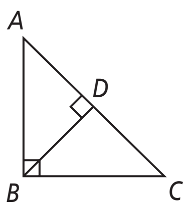


13. Thales was an ancient philosopher familiar with similar triangles. One story about him says that he found the height of a pyramid by measuring its shadow and his own shadow at the same time. If the person is 5-ft tall, what is the height of the pyramid in the drawing? 265 ft



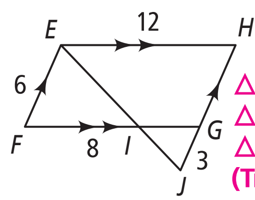
Identify the similar triangles in each figure. Explain.

14.



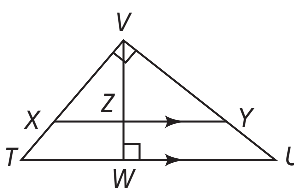
$\triangle ADB \sim \triangle ABC$ (AA $\sim$); $\triangle BDC \sim \triangle ABC$ (AA $\sim$); $\triangle ADB \sim \triangle BDC$ (Trans. Prop. of $\sim$ $\triangle$s)

15.



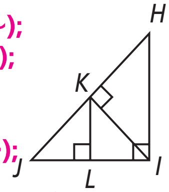
$\triangle EIF \sim \triangle JIG$ (AA $\sim$); $\triangle JIG \sim \triangle JEH$ (AA $\sim$); $\triangle JEH \sim \triangle EIF$ (Trans. Prop. of $\sim$ $\triangle$s)

16.



$\triangle TUV \sim \triangle VUW$ (AA $\sim$); $\triangle TUV \sim \triangle XYV$ (AA $\sim$); $\triangle VUW \sim \triangle XYV$ (Trans. Prop. of $\sim\triangle$s); $\triangle TUV \sim \triangle TVW$ (AA $\sim$); $\triangle TVW \sim \triangle VUW$ (Trans. Prop. of $\sim\triangle$s); $\triangle TVW \sim \triangle XYV$ (Trans. Prop. of $\sim\triangle$s); $\triangle XVZ \sim \triangle TVW$ (AA $\sim$); $\triangle VYZ \sim \triangle VUW$ (AA $\sim$); $\triangle VYZ \sim \triangle XYV$ (AA $\sim$); $\triangle XVZ \sim \triangle XYV$ (AA $\sim$); $\triangle VYZ \sim \triangle XVZ$ (Trans. Prop. of $\sim\triangle$s); $\triangle TUV \sim \triangle VYZ$ (Trans. Prop. of $\sim\triangle$s); $\triangle TUV \sim \triangle XVZ$ (Trans. Prop. of $\sim\triangle$s); $\triangle TVW \sim \triangle VYZ$ (Trans. Prop. of $\sim\triangle$s); $\triangle VUW \sim \triangle XVZ$ (Trans. Prop. of $\sim\triangle$s)

17.



$\triangle JKL \sim \triangle JHI$ (AA $\sim$); $\triangle JKL \sim \triangle JIK$ (AA $\sim$); $\triangle JIK \sim \triangle KIL$ (AA $\sim$); $\triangle IHK \sim \triangle JHI$ (AA $\sim$); $\triangle IHK \sim \triangle JKL \sim \triangle JIK \sim \triangle KIL \sim \triangle JHI$ (Trans. Prop. of $\sim\triangle$s)

Name ______________________ Class ____________ Date ____________

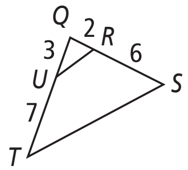
7-3 Practice

Form K

Proving Triangles Similar

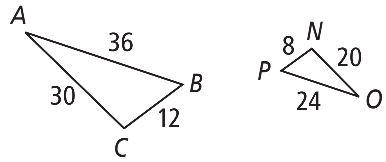
Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

1.



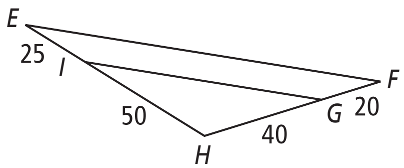
not similar; corresp. sides not proportional

2.



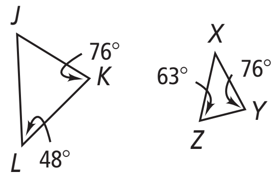
$\triangle ABC \sim \triangle OPN$; SSS ~ Thm.

3.



$\triangle EFH \sim \triangle IGH$; SAS ~ Thm.

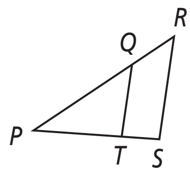
4.



not similar; corresp. angles not ≅

5. Given: $PQ = \frac{3}{4}PR$, $PT = \frac{3}{4}PS$

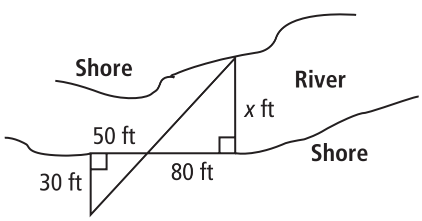
Prove: $\triangle PQT \sim \triangle PRS$



Statements	Reasons
1) $PQ = \frac{3}{4}PR$ and $PT = \frac{3}{4}PS$	1) _?_ Given
2) $\frac{PQ}{PR} = \frac{3}{4}$ and $\frac{PT}{PS} = \frac{3}{4}$	2) _?_ Division Property of =
3) $\frac{\boxed{PQ}}{PR} = \frac{\boxed{PT}}{PS}$	3) _?_ Transitive Property of =
4) $\angle P \cong \boxed{\angle P}$	4) _?_ Reflexive Property of ≅
5) _?_ $\triangle PQT \sim \triangle PRS$	5) _?_ SAS ~ Theorem

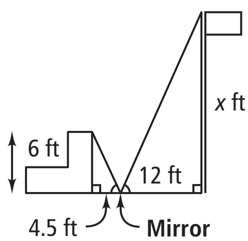
Explain why the triangles are similar. Then find the distance represented by *x*.

6.



AA ~ Post.; 48 ft

7.



AA ~ Post.; 16 ft

7-3 Practice (continued) *Form K*

Proving Triangles Similar

8. A 1.6-m-tall woman stands next to the Eiffel Tower. At this time of day, her shadow is 0.5 m long. At the same time, the tower's shadow is 93.75 m long. How tall is the Eiffel Tower? **300 m**

9. At 4:00 P.M. Karl stands next to his house and measures his shadow and the house's shadow. Karl's shadow is 8 ft long. The house's shadow is 48 ft long. If Karl is 6 ft tall, how tall is his house? **36 ft**

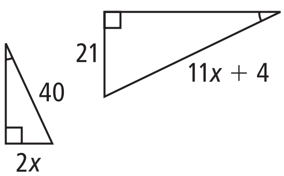
10. Error Analysis Jacob wants to use indirect measurement to find the height of his school. He knows the basketball pole next to the school is 13 ft high. He measures the length of the pole's shadow. At the same time of day, he measures the length of the school's shadow. Then he writes a proportion:

$$\frac{13\text{ ft}}{\text{school height}} = \frac{\text{school shadow}}{\text{pole shadow}}.$$

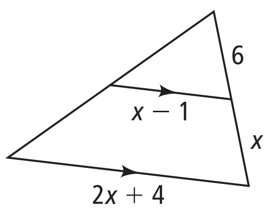
What error has Jacob made?
The proportion should compare corresp. sides of ~ △s: $\frac{\text{pole height}}{\text{school height}} = \frac{\text{pole shadow}}{\text{school shadow}}$.

11. Reasoning Explain why there is an AA Similarity Postulate but not an AA Congruence Postulate. **Answers may vary. Sample: If two pairs of ∠s in two △s are ≅, the third pair of ∠s are determined, so you can prove the △s are ~. But to prove △s are ≅, you must show at least one pair of corresp. sides are ≅.**

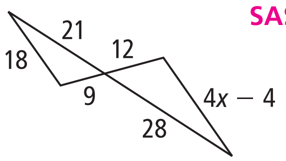
Algebra **Explain why the triangles are similar. Then find the value of *x*.**

12. 

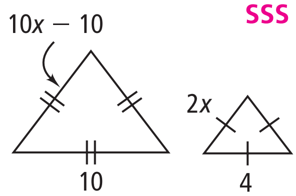
AA ~ Post.; 6

13. 

AA ~ Post.; 10

14. 

SAS ~ Thm.; 7

15. 

SSS ~ Thm.; 2

16. Think About a Plan A right triangle has legs 3 cm and 4 cm and a hypotenuse 5 cm. Another right triangle has a 12-cm leg. Find all the possible lengths of the second leg that would make the triangles similar. For each possible length, find the corresponding length of the hypotenuse. **9 cm and 15 cm; 16 cm and 20 cm**

- To which measures must you compare the 12-cm leg? **to the 3-cm leg and 4-cm leg**
- How can you find the measure of the hypotenuse? **Use a proportion.**

Name ______________________ Class ____________ Date ____________

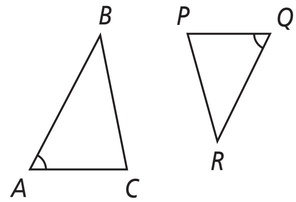
7-3 Standardized Test Prep

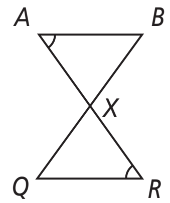
Proving Triangles Similar

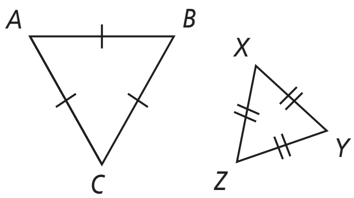
Multiple Choice

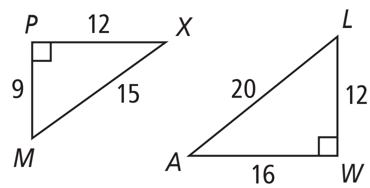
For Exercises 1–3, choose the correct letter.

1. Which pair of triangles can be proven similar by the AA ~ Postulate? **C**

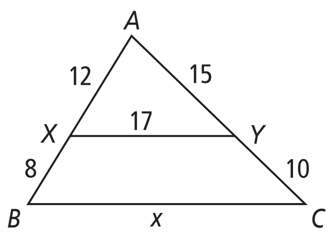
A


C


B


D


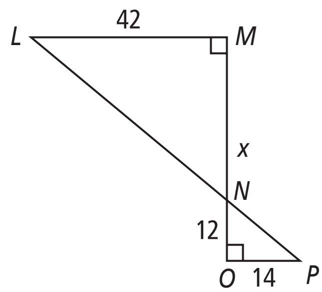
2. $\triangle AXY \sim \triangle ABC$. What is the value of x? **I**



F $10\frac{1}{5}$ H $11\frac{1}{3}$

G 19 I $28\frac{1}{3}$

3. $\triangle LMN \sim \triangle PON$. What is the value of x? **A**

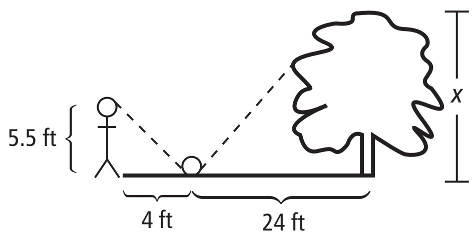


A 36 C 25

B 20 D $28\frac{1}{3}$

Short Response

4. Irene places a mirror on the ground 24 ft from the base of an oak tree. She walks backward until she can see the top of the tree in the middle of the mirror. At that point, Irene's eyes are 5.5 ft above the ground, and her feet are 4 ft from the mirror. How tall is the oak tree? Explain.



[2] Set up the extended proportion: $\frac{5.5}{x} = \frac{4}{24}$. Solve for x. The oak tree is 33 ft tall.
[1] incorrect proportion or error in calculation [0] incorrect proportion and error in calculation

Name ______________________ Class ______________ Date ______________

7-3 Enrichment

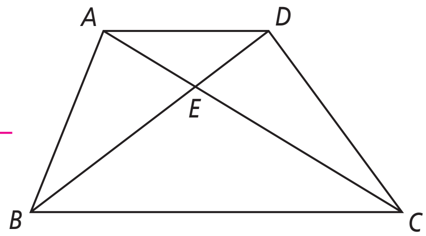
Proving Triangles Similar

Similarity Proofs

Write two-column proofs for Exercises 1 and 2.

1. Given: *ABCD* is a trapezoid.

Prove: $\triangle AED \sim \triangle CEB$



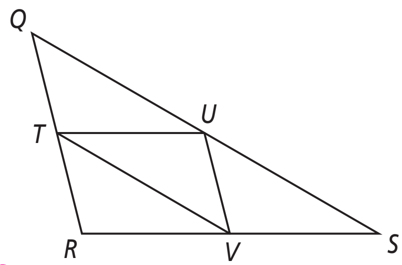
Statements	Reasons
1) *ABCD* is a trapezoid.	1) Given
2) $\overline{AD} \parallel \overline{BC}$	2) Definition of a trapezoid
3) $\angle AED \cong \angle CEB$	3) Vertical angles are $\cong$.
4) $\angle EBC \cong \angle EDA$	4) Alt. Int. Angles Theorem
5) $\triangle CEB \sim \triangle AED$	5) AA $\sim$ Post.

2. Given: *T* is the midpoint of $\overline{QR}$.

U is the midpoint of $\overline{QS}$.

V is the midpoint of $\overline{RS}$.

Prove: $\triangle QRS \sim \triangle VUT$



Statements	Reasons
1) *T, U,* and *V* are midpoints.	1) Given
2) $\overline{TU} \parallel \overline{RV}$, $\overline{UV} \parallel \overline{TR}$, and $\overline{TV} \parallel \overline{US}$	2) Triangle Midsegment Theorem
3) *TUVR* and *TUSV* are parallelograms.	3) Definition of a parallelogram
4) $\angle TUV \cong \angle R$	4) Opposite $\angle$s of a parallelogram are $\cong$.
5) $\angle UTV \cong \angle S$	5) Opposite $\angle$s of a parallelogram are $\cong$.
6) $\triangle QRS \sim \triangle VUT$	6) AA $\sim$ Post.

Write a paragraph proof for Exercise 3.

3. Graph $\triangle ABC$ and $\triangle TBS$ with vertices $A(-2, -8)$, $B(4, 4)$, $C(-2, 7)$, $T(0, -4)$, and $S(0, 6)$. Then prove: $\triangle ABC \sim \triangle TBS$.

Check students' graphs. Use the distance formula to find the side lengths. $AB = 6\sqrt{5}$, $BC = 3\sqrt{5}$, $CA = 15$, $ST = 10$, $TB = 4\sqrt{5}$, and $BS = 2\sqrt{5}$. If the corresponding sides are proportional, then the two triangles are similar. Substitute the values found previously into the extended proportion $\frac{CA}{ST} = \frac{AB}{TB} = \frac{BC}{BS}$. $\frac{15}{10} = \frac{6\sqrt{5}}{4\sqrt{5}} = \frac{3\sqrt{5}}{2\sqrt{5}} = \frac{3}{2}$, so $\triangle ABC \sim \triangle TBS$ by the SSS $\sim$ Theorem.

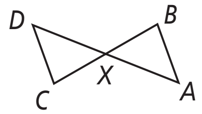
Name ________________ Class ________ Date ________

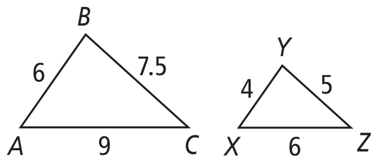
7-3 Reteaching

Proving Triangles Similar

Problem

Are the triangles similar? How do you know? Write a similarity statement.





Given: $\overline{DC} \parallel \overline{BA}$

Because $\overline{DC} \parallel \overline{BA}$, $\angle A$ and $\angle D$ are alternate interior angles and are therefore $\cong$. The same is true for $\angle B$ and $\angle C$. So, by AA $\sim$ Postulate, $\triangle ABX \sim \triangle DCX$.

Compare the ratios of the lengths of sides:

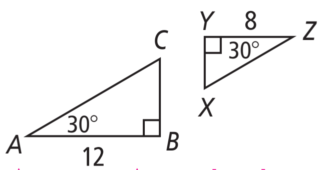
$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX} = \frac{3}{2}$

So, by SSS $\sim$ Theorem, $\triangle ABC \sim \triangle XYZ$.

Exercises

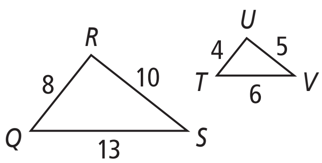
Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

1.



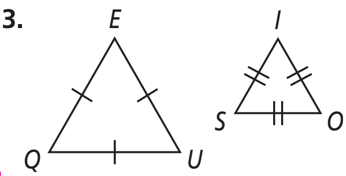
$\triangle ABC \sim \triangle ZYX$ by the AA $\sim$ Postulate.

2.



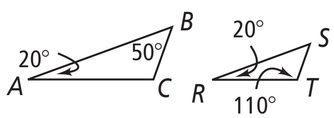
Not similar; not all corresponding sides are proportional.

3.



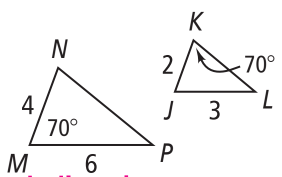
$\triangle QEU \sim \triangle SIO$ by the SSS $\sim$ Theorem.

4.



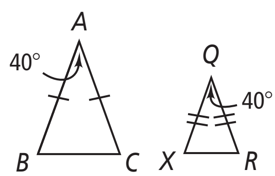
$\triangle ABC \sim \triangle RST$ by the AA $\sim$ Postulate.

5.



Not similar; the congruent angles are not corresponding.

6.



$\triangle BAC \sim \triangle XQR$ by the SAS $\sim$ Postulate.

7. Are all equilateral triangles similar? Explain.
Yes; by the SSS $\sim$ Theorem or by the AA $\sim$ Postulate

8. Are all isosceles triangles similar? Explain.
No; the vertex angles may differ in measure.

9. Are all congruent triangles similar? Are all similar triangles congruent? Explain.
All congruent triangles are similar, with ratio 1 : 1. Similar triangles are not necessarily congruent.

7-3 Reteaching (continued)

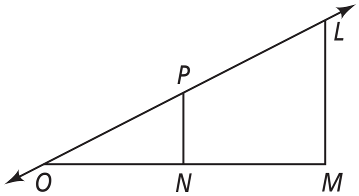
Proving Triangles Similar

10. Provide the reason for each step in the two-column proof.

Given: $\overline{LM} \perp \overline{MO}$

$\overline{PN} \perp \overline{MO}$

Prove: $\triangle LMO \sim \triangle PNO$



Statements	Reasons
1) $\overline{LM} \perp \overline{MO}$, $\overline{PN} \perp \overline{MO}$	1) ? **Given**
2) $\angle PNO$ and $\angle LMO$ are right $\angle$s.	2) ? **Definition of a perpendicular line**
3) $\angle PNO \cong \angle LMO$	3) ? **All right $\angle$s are $\cong$.**
4) $\angle O \cong \angle O$	4) ? **Reflexive Property of $\cong$**
5) $\triangle LMO \sim \triangle PNO$	5) ? **AA $\sim$ Postulate**

11. Developing Proof Complete the proof by filling in the blanks.

Given: $\overline{AB} \parallel \overline{EF}$, $\overline{AC} \parallel \overline{DF}$

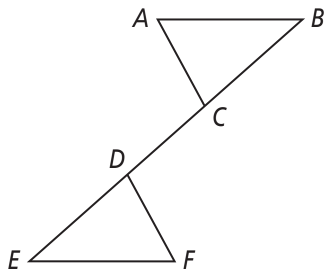
Prove: $\triangle ABC \sim \triangle FED$

Proof: $\overline{AB} \parallel \overline{EF}$ and $\overline{AC} \parallel \overline{DF}$ are given. $\overline{EB}$ is a transversal by ? . **Definition of a transversal**

$\angle E \cong \angle B$ by ? . **Alt. Int. $\angle$s Thm.**

Similarly, $\angle EDF \cong \angle BCA$ by ? . **Alt. Ext. $\angle$s Thm.**

So, $\triangle ABC \sim \triangle FED$ by ? . **AA $\sim$ Postulate**



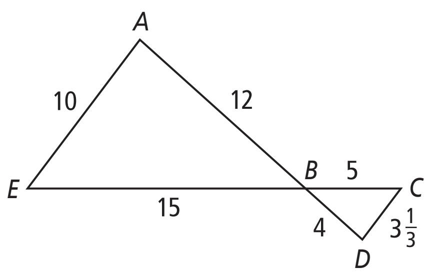
12. Write a paragraph proof.

Given: $\overline{AD}$ and $\overline{EC}$ intersect at B.

Prove: $\triangle ABE \sim \triangle DBC$

Answers may vary. Sample:

$\frac{BD}{AB} = \frac{4}{12} = \frac{1}{3} = \frac{CB}{EB} = \frac{5}{15} = \frac{1}{3} = \frac{CD}{EA} = \frac{3\frac{1}{3}}{10}$. **So,** $\frac{BD}{AB} = \frac{CB}{EB} = \frac{CD}{AE}$. **Therefore, the sides of $\triangle ABE$ and $\triangle DBC$ are proportional and $\triangle ABE \sim \triangle DBC$, by SSS $\sim$ Theorem.**



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7-4 ELL Support

Similarity in Right Triangles

You can use a proportion to find the geometric mean by finding the cross products and simplifying.

Problem What is the geometric mean of 4 and 8?

$\frac{4}{x} = \frac{x}{8}$	Set up a proportion.
$32 = x^2$	Use the Cross Products Property.
$\sqrt{32} = \sqrt{x^2}$	Take the positive square root.
$\sqrt{16 \cdot 2} = \sqrt{x^2}$	Factor the perfect square 16.
$4\sqrt{2} = x$	Simplify.

1. Solve: $\sqrt{324}$ **18**

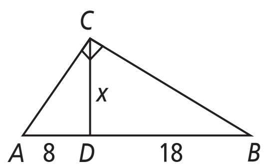
2. Circle the *geometric mean* of 6 and 12. $6\sqrt{2}$ 9 $9\sqrt{2}$

3. Circle the *geometric mean* of 10 and 40. $10\sqrt{3}$ 20 25

4. Find the geometric mean of 50 and 75. **$25\sqrt{6}$**

You can use the geometric mean to find the altitude of a right triangle.

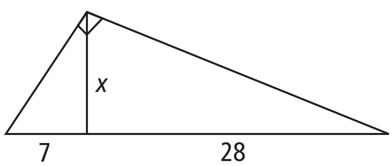
Problem What is the value of x?



$\frac{AD}{CD} = \frac{CD}{DB}$	Set up a proportion.
$\frac{8}{x} = \frac{x}{18}$	Substitute.
$144 = x^2$	Use the Cross Products Property.
$\sqrt{144} = \sqrt{x^2}$	Take the positive square root.
$12 = x$	Simplify.

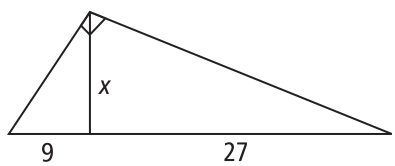
Solve for x.

5.



14

6.



$9\sqrt{3}$

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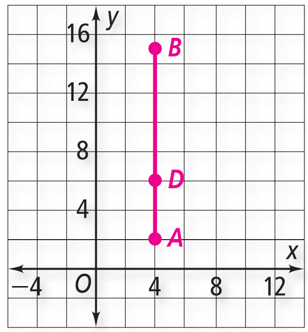
7-4 Think About a Plan

Similarity in Right Triangles

Coordinate Geometry $\overline{CD}$ is the altitude to the hypotenuse of right $\triangle ABC$. The coordinates of A, D, and B are (4, 2), (4, 6), and (4, 15), respectively. Find all possible coordinates of point C.

Understanding the Problem



1. What is an altitude? **a segment from a vertex to the opposite side, ⊥ to the opposite side**

2. Plot the points given in the problem on the grid. Which side of the triangle must $\overline{AB}$ be? Explain.

$\overline{AB}$ is the hypotenuse because $\overline{CD}$ is the altitude to the hypotenuse, and C lies opposite $\overline{AB}$.

3. What is the special relationship between the altitude to a hypotenuse of a right triangle and the lengths of the segments it creates? **The length of the altitude is the geometric mean of the lengths of the segments of the hypotenuse it creates.**

4. What does the phrase "Find all possible coordinates of point C" tell you about the problem? **There may be more than one correct answer.**

Planning the Solution

5. How can you find the geometric mean of a pair of numbers?

Write a proportion in which the geometric mean is each mean of the proportion, and the known pair of numbers are the extremes, or vice versa.

6. For which numbers or lengths are you finding the geometric mean? How can you determine the geometric mean? **4 and 9; you can determine the geometric mean by setting up and solving the proportion $\frac{4}{x} = \frac{x}{9}$.**

Getting an Answer

7. Find the geometric mean. $\frac{4}{x} = \frac{x}{9}$; $x^2 = 36$; $x = 6$

8. What does your answer represent? ***CD***

9. Why is there more than one possible correct answer?

The altitude can extend 6 units to the right of *D* or 6 units to the left of *D*.

10. What are the possible coordinates of point C? **(−2, 6) and (10, 6)**

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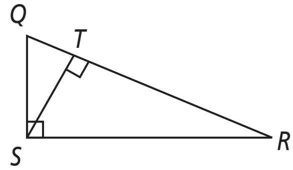
7-4 Practice

Form G

Similarity in Right Triangles

Identify the following in right $\triangle QRS$.



1. the hypotenuse $\overline{QR}$

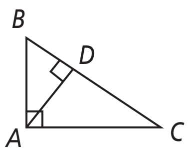
2. the segments of the hypotenuse $\overline{QT}$ and $\overline{TR}$

3. the altitude $\overline{ST}$

4. the segment of the hypotenuse adjacent to leg $\overline{QS}$ $\overline{QT}$

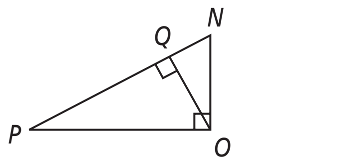
Write a similarity statement relating the three triangles in the diagram.

5.



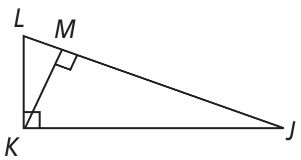
$\triangle ABC \sim \triangle DBA \sim \triangle DAC$

6.



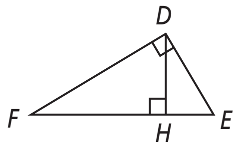
$\triangle NOP \sim \triangle OQP \sim \triangle NQO$

7.



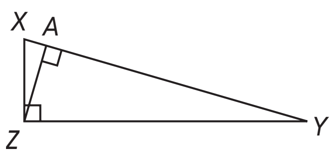
$\triangle JKL \sim \triangle JMK \sim \triangle KML$

8.



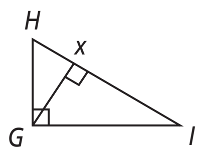
$\triangle DEF \sim \triangle HDF \sim \triangle HED$

9.



$\triangle XYZ \sim \triangle XZA \sim \triangle ZYA$

10.



$\triangle GHI \sim \triangle XGI \sim \triangle XHG$

Algebra **Find the geometric mean of each pair of numbers.**

11. 9 and 4 6

12. 14 and 6 $2\sqrt{21}$

13. 9 and 30 $3\sqrt{30}$

14. 25 and 49 35

15. 4 and 120 $4\sqrt{30}$

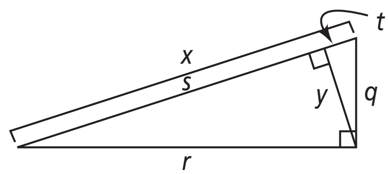
16. 9 and 18 $9\sqrt{2}$

17. 16 and 64 32

18. 5 and 25 $5\sqrt{5}$

19. 12 and 16 $8\sqrt{3}$

Use the figure at the right to complete each proportion.



20. $\frac{q}{r} = \frac{\boxed{t}}{y}$

21. $\frac{s}{y} = \frac{\boxed{y}}{t}$

22. $\frac{t}{q} = \frac{q}{\boxed{x}}$

23. $\frac{q}{x} = \frac{t}{\boxed{q}}$

24. $\frac{s}{r} = \frac{\boxed{y}}{q}$

25. $\frac{\boxed{s}}{r} = \frac{r}{x}$

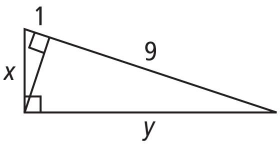
Name ______________________ Class ____________ Date ____________

7-4 Practice (continued) — *Form G*

Similarity in Right Triangles

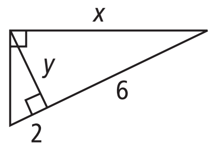
Algebra **Solve for the value of the variables in each right triangle.**

26.



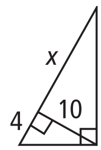
$\sqrt{10}$; $3\sqrt{10}$

27.



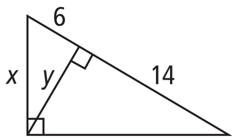
$4\sqrt{3}$; $2\sqrt{3}$

28.



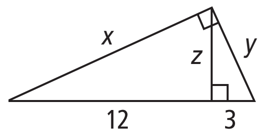
25

29.



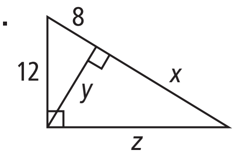
$2\sqrt{30}$; $2\sqrt{21}$

30.



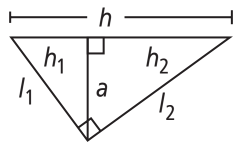
$6\sqrt{5}$; $3\sqrt{5}$; 6

31.



10; $4\sqrt{5}$; $6\sqrt{5}$

The diagram shows the parts of a right triangle with an altitude to the hypotenuse. For the two given measures, find the other four. Use simplest radical form.



32. $h = 12, h_1 = 4$

$h_2 = 8$; $a = 4\sqrt{2}$; $\ell_1 = 4\sqrt{3}$; $\ell_2 = 4\sqrt{6}$

33. $a = 6, h_2 = 9$

$h_1 = 4$; $h = 13$; $\ell_1 = 2\sqrt{13}$; $\ell_2 = 3\sqrt{13}$

34. $\ell_1 = 6\sqrt{3}, h_2 = 3$

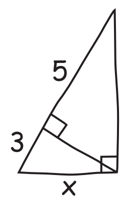
$h_1 = 9$; $a = 3\sqrt{3}$; $\ell_2 = 6$; $h = 12$

35. $h_1 = 5, \ell_2 = 2\sqrt{51}$

$a = 2\sqrt{15}$; $h_2 = 12$; $h = 17$; $\ell_1 = \sqrt{85}$

36. The altitude of the hypotenuse of a right triangle divides the hypotenuse into 45 in. and 5 in. segments. What is the length of the altitude? **15 in.**

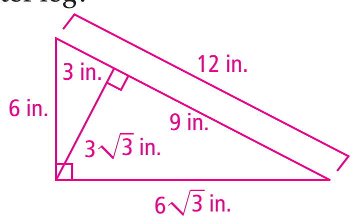
37. Error Analysis A classmate writes an incorrect proportion to find *x*. Explain and correct the error.



$\cancel{\frac{3}{x} = \frac{x}{5}}$

The value of *x* is the geometric mean of the adjacent segment of the hypotenuse, 3, and the entire hypotenuse, 3 + 5, or 8. So the correct proportion is $\frac{3}{x} = \frac{x}{8}$.

38. Draw a Diagram The sides of a right triangle measure $6\sqrt{3}$ in., 6 in., and 12 in. If an altitude is drawn from the right angle to the hypotenuse, what is the length of the segment of the hypotenuse adjacent to the shorter leg? What is the length of the altitude? **3; $3\sqrt{3}$**



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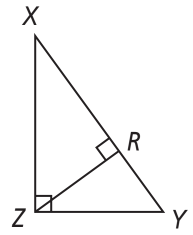
7-4 Practice

Form K

Similarity in Right Triangles

Identify the following in right $\triangle XYZ$.



1. the hypotenuse $\overline{XY}$

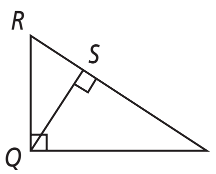
2. the segments of the hypotenuse $\overline{XR}$ and $\overline{RY}$

3. the altitude to the hypotenuse $\overline{ZR}$

4. the segment of the hypotenuse adjacent to leg $\overline{ZY}$ $\overline{RY}$

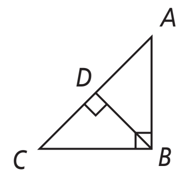
Write a similarity statement relating the three triangles in each diagram.

5.



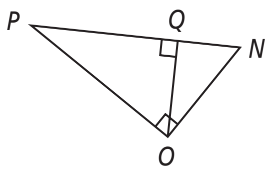
$\triangle QRT \sim \triangle SQT \sim \triangle SRQ$

6.



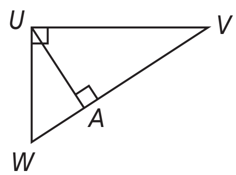
$\triangle ABC \sim \triangle BDC \sim \triangle ADB$

7.



$\triangle PNO \sim \triangle POQ \sim \triangle ONQ$

8.



$\triangle WVU \sim \triangle WUA \sim \triangle UVA$

Algebra **Find the geometric mean of each pair of numbers.**

9. 4 and 9 $\quad \frac{4}{x} = \frac{\boxed{x}}{9} \rightarrow x^2 = \boxed{36} \rightarrow x = \boxed{6}$

10. 6 and 12 $\quad \frac{6}{y} = \frac{y}{\boxed{12}} \rightarrow y^2 = \boxed{72} \rightarrow y = \boxed{}$ $6\sqrt{2}$

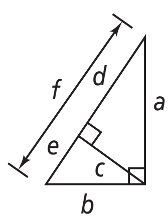
11. 14 and 12 $2\sqrt{42}$

12. 6 and 500 $10\sqrt{30}$

13. 4.2 and 10 $\sqrt{42}$

14. $\sqrt{50}$ and $\sqrt{2}$ 10

Use the figure at the right to complete each proportion.



15. $\frac{d}{c} = \frac{c}{\boxed{e}}$

16. $\frac{\boxed{f}}{b} = \frac{b}{e}$

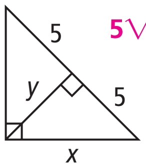
17. $\frac{f}{\boxed{a}} = \frac{\boxed{a}}{d}$

18. $\frac{f}{b} = \frac{b}{\boxed{e}}$

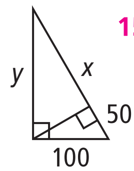
7-4 Practice (continued) *Form K*

Similarity in Right Triangles

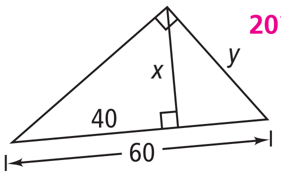
Algebra **Solve for *x* and *y*.**

19. 

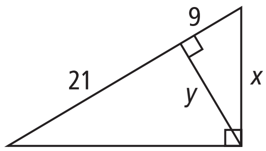
$5\sqrt{2}$; 5

20. 

150; $100\sqrt{3}$

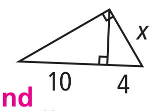
21. 

$20\sqrt{2}$; $20\sqrt{3}$

22. 

$3\sqrt{30}$; $3\sqrt{21}$

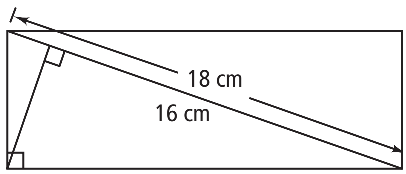
23. Error Analysis A classmate writes an incorrect proportion to find *x*. Explain and correct the error.



$\cancel{\frac{x}{4} = \frac{4}{10}}$

Find *x* using the geometric mean of the hypotenuse and the segment of the hypotenuse adjacent to the leg; $\frac{4}{x} = \frac{x}{14}$.

24. A quilter sews three right triangles together to make the rectangular quilt block at the right. What is the area of the rectangle? **$72\sqrt{2}$ cm^2**

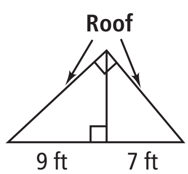


- How can you find the dimensions of the rectangle? **Use Corollary 1 to Theorem 7-3 and Corollary 2 to Theorem 7-3.**
- What is the formula for the area of a rectangle? ***A* = *bh***

25. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 9 in. and 12 in. long. Find the length of the altitude to the hypotenuse. **$6\sqrt{3}$ in.**

26. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 4 in. long and 12 in. long. What are the lengths of the other legs of the triangle? **$8\sqrt{3}$ in.; 8 in.**

27. A carpenter is framing a roof for a shed. What is the length of the longer slope of the roof? **12 ft**



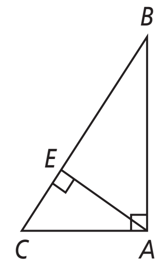
Name ______________________ Class ____________ Date ____________

7-4 Standardized Test Prep

Similarity in Right Triangles

Multiple Choice

For Exercises 1–5, choose the correct letter.



1. Which segment of the hypotenuse is adjacent to $\overline{AB}$? **D**

Ⓐ $\overline{EC}$ Ⓑ $\overline{AC}$ Ⓒ $\overline{AE}$ Ⓓ $\overline{BE}$

2. What is the geometric mean of 7 and 12? **H**

Ⓕ $1\frac{5}{7}$ Ⓖ 9.5 Ⓗ $2\sqrt{21}$ Ⓘ $4\sqrt{21}$

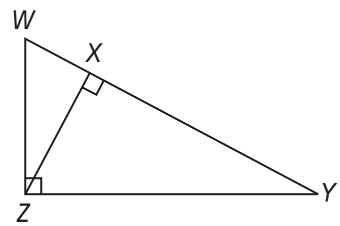
3. Which similarity statement is true? **B**

Ⓐ $\triangle WYZ \sim \triangle XZW \sim \triangle XYZ$

Ⓑ $\triangle WYZ \sim \triangle WZX \sim \triangle ZYX$

Ⓒ $\triangle YZW \sim \triangle XZW \sim \triangle XZY$

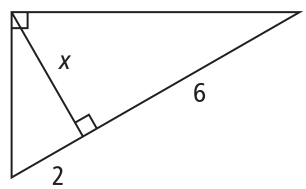
Ⓓ $\triangle YZW \sim \triangle ZXW \sim \triangle ZYX$



4. What is the value of x? **F**

Ⓕ $2\sqrt{3}$ Ⓗ 4

Ⓖ $4\sqrt{3}$ Ⓘ 6



5. The altitude of the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 14 and 8. What is the length of the altitude? **B**

Ⓐ $2\sqrt{77}$ Ⓑ $4\sqrt{7}$ Ⓒ $4\sqrt{11}$ Ⓓ 11

Extended Response

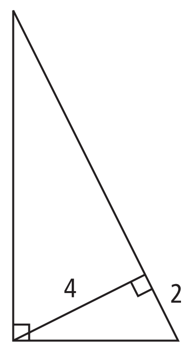
6. What is the perimeter of the large triangle shown at the right? Show your work.

[4] $10 + 6\sqrt{5}$; $h_2 = \frac{2}{4} = \frac{4}{x} = 8$; $\ell_1 : \frac{2}{x} = \frac{x}{10}$, so $x^2 = \sqrt{20} = 2\sqrt{5}$; $\ell_2 : \frac{8}{x} = \frac{x}{10}$; $x^2 = \sqrt{80} = 4\sqrt{5}$; perimeter $= 8 + 2 + 2\sqrt{5} =$ $10 + 6\sqrt{5}$

[3] appropriate methods, but with one computational error

[2] appropriate methods, but with several computational errors

[1] correct perimeter only [0] inappropriate methods, several computational errors, and incorrect perimeter



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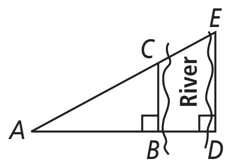
7-4 Enrichment

Similarity in Right Triangles

You can use geometry to measure distances that cannot be measured directly. Geometry provides a way to make these measurements indirectly through the use of the proportionality ratios that exist in similar triangles. This is the ancestor of trigonometry—the study of measurements using triangles.

Right triangles are exceptionally important in trigonometry because of the following: Two right triangles that contain congruent nonright angles are similar.

1. Why is this true?



A standard surveying problem involves measuring the distance across a wide river. B and D represent points on opposite banks of the river. The most straightforward way to measure this distance indirectly is to align stakes so that both $\triangle ABC$ and $\triangle ADE$ are right triangles. $\overline{AB}$ and $\overline{BC}$ both can be measured because they are on the same side of the river. $\overline{DE}$ also can be measured.

The triangles have two congruent angles, AA Postulate.

Use the information and diagram above to answer Exercises 2–5.

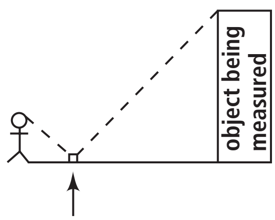
2. Use similar triangles to obtain a proportion involving the known distances and the unknown distance BD. $\frac{AB + BD}{AB} = \frac{DE}{BC}$

3. Solve this equation for BD. $BD = \frac{(DE - BC)\,AB}{BC}$

4. Suppose DE is 500 yd, AB is 200 yd, and BC is 100 yd. What is the distance across the river? **800 yd**

5. Suppose BC is 1000 ft, DE is 2500 ft, and AB is 3000 ft. What is the distance across the river? **4500 ft**

Another application of similar right triangles can be used to measure the heights of objects that are difficult to measure directly. If you place a mirror on the ground between you and the object you are measuring and then position yourself so that you can see the top of the object in the mirror, you can use similar triangles to estimate the height of the object.



6. A student places a mirror between herself and a school building. She places the mirror 60 ft from the base of the school building and 7 ft from her on the ground. If the student is 5 ft tall, how tall is the school building to the nearest foot? **43 ft**

7. A 10th grader is 1.8 m tall. To measure the height of a flagpole, he places a mirror 20 m from the base of the flagpole and 4 m from his feet. To the nearest meter, how tall is the flagpole? **9 m**

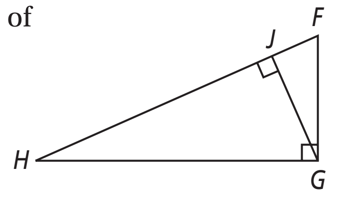
Name ______________________________ Class ____________ Date ____________

7-4 Reteaching

Similarity in Right Triangles

Theorem 7-3

If you draw an altitude from the right angle to the hypotenuse of a right triangle, you create three similar triangles. This is Theorem 7-3.



$\triangle FGH$ is a right triangle with right $\angle FGH$ and the altitude of the hypotenuse $\overline{JG}$. The two triangles formed by the altitude are similar to each other and similar to the original triangle. So, $\triangle FGH \sim \triangle FJG \sim \triangle GJH$.

Two corollaries to Theorem 7-3 relate the parts of the triangles formed by the altitude of the hypotenuse to each other by their geometric mean.

The geometric mean, x, of any two positive numbers a and b can be found with the proportion $\frac{a}{x} = \frac{x}{b}$.

Problem

What is the geometric mean of 8 and 12?

$$\frac{8}{x} = \frac{x}{12}$$

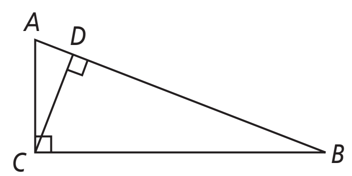
$$x^2 = 96$$

$$x = \sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$$

The geometric mean of 8 and 12 is $4\sqrt{6}$.

Corollary 1 to Theorem 7-3

The altitude of the hypotenuse of a right triangle divides the hypotenuse into two segments. The length of the altitude is the geometric mean of these segments.



Since $\overline{CD}$ is the altitude of right $\triangle ABC$, it is the geometric mean of the segments of the hypotenuse $\overline{AD}$ and $\overline{DB}$:

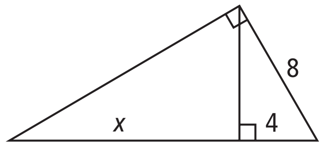
$$\frac{AD}{CD} = \frac{CD}{DB}.$$

7-4 Reteaching (continued)

Similarity in Right Triangles

Corollary 2 to Theorem 7-3

The altitude of the hypotenuse of a right triangle divides the hypotenuse into two segments. The length of each leg of the original right triangle is the geometric mean of the length of the entire hypotenuse and the segment of the hypotenuse adjacent to the leg. To find the value of *x*, you can write a proportion.



$\dfrac{\text{segment of hypotenuse}}{\text{adjacent leg}} = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$ $\quad \dfrac{4}{8} = \dfrac{8}{4+x}$ Corollary 2

$4(4 + x) = 64$ Cross Products Property

$16 + 4x = 64$ Simplify.

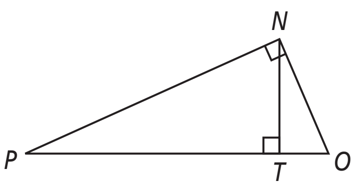
$4x = 48$ Subtract 16 from each side.

$x = 12$ Divide each side by 4.

Exercises

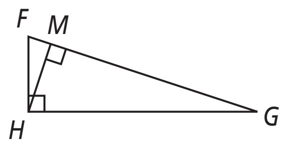
Write a similarity statement relating the three triangles in the diagram.

1.



$\triangle NOP \sim \triangle TNP \sim \triangle TON$

2.



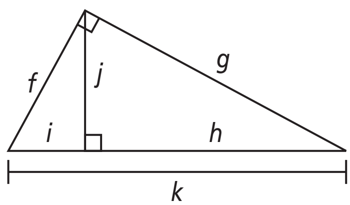
$\triangle FHG \sim \triangle HMG \sim \triangle FMH$

Algebra **Find the geometric mean of each pair of numbers.**

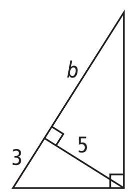
3. 2 and 8 **4** **4.** 4 and 6 $2\sqrt{6}$ **5.** 8 and 10 $4\sqrt{5}$ **6.** 25 and 4 **10**

Use the figure to complete each proportion.

7. $\dfrac{i}{\boxed{f}} = \dfrac{f}{k}$ **8.** $\dfrac{i}{j} = \dfrac{j}{\boxed{h}}$ **9.** $\dfrac{\boxed{i}}{f} = \dfrac{f}{\boxed{k}}$



10. Error Analysis A classmate writes the proportion $\frac{3}{5} = \frac{5}{(3 + b)}$ to find *b*. Explain why the proportion is incorrect and provide the right answer.



The altitude is the geometric mean for the two segments of the hypotenuse, not for one segment and the entire hypotenuse. $\frac{3}{5} = \frac{5}{b}$

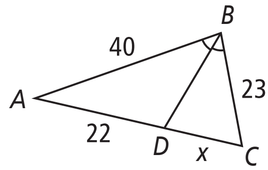
Name ______________________ Class __________ Date __________

7-5 ELL Support

Proportions in Triangles

Problem

What is the value of x in the diagram at the right?

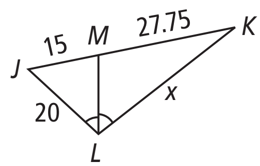


Explain	Work	Justify
First, use the Triangle-Angle-Bisector Theorem to write a proportion.	$\frac{AB}{BC} = \frac{AD}{DC}$	Triangle-Angle-Bisector Theorem
Next, substitute corresponding side lengths in the proportion.	$\frac{40}{23} = \frac{22}{x}$	Substitution
Then, use the Cross Products Property.	$40x = 506$	Cross Products Property
Finally, divide by 40 to get the answer of 12.65.	$x = 12.65$	Division Property of Equality

Solution
12.65

Exercise

What is the value of x in the diagram at the right?



Explain	Work	Justify
First, use the Triangle-Angle-Bisector Theorem to write a proportion.	$\frac{JL}{LK} = \frac{JM}{MK}$	Triangle-Angle-Bisector Theorem
Next, substitute corresponding side lengths in the proportion.	$\frac{20}{x} = \frac{15}{27.75}$	Substitution
Then, use the Cross Products Property.	$15x = 555$	Cross Products Property
Finally, divide each side by 15.	$x = 37$	Division Property of Equality

Solution
37

Name ________________________________ Class ____________ Date ____________

7-5 Think About a Plan

Proportions in Triangles

An angle bisector of a triangle divides the opposite side of the triangle into segments 5 cm and 3 cm long. A second side of the triangle is 7.5 cm long. Find all possible lengths for the third side of the triangle.

1. What is the Triangle-Angle-Bisector Theorem? What relationships does it specifically describe?

It states that the angle bisector of a triangle divides the opposite side of the triangle into proportional segments. So the ratio of one segment and its adjacent side is proportional to the ratio of the second segment and its adjacent side.

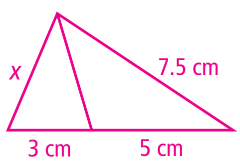
2. What information is given in this problem? What information is not given?

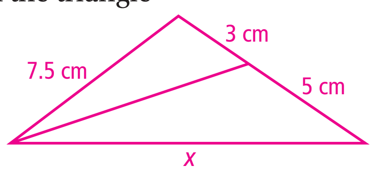
You know that the lengths of the two segments created by the angle bisector are 5 cm and 3 cm long. You know the length of the second side is 7.5 cm, but you don't know to which segment this side is adjacent.

3. What does the phrase "all possible lengths" tell you about the problem?

There may be more than one answer.

4. In the space below, draw all the possible representations of the triangle described in the problem.





5. How can proportions be used to solve this problem?

Write a proportion comparing the ratio of each segment to its adjacent side.

6. How many proportions will you need to set up? Explain.

You will need to set up two proportions because you don't know the segment that is adjacent to the 7.5 cm side.

7. Use the space below to write and solve the proportions.

$\frac{3}{7.5} = \frac{5}{x}$
$3x = 37.5$
$x = 12.5$

$\frac{3}{x} = \frac{5}{7.5}$
$5x = 22.5$
$x = 4.5$

8. What are the possible lengths for the third side of the triangle?

12.5 cm or 4.5 cm

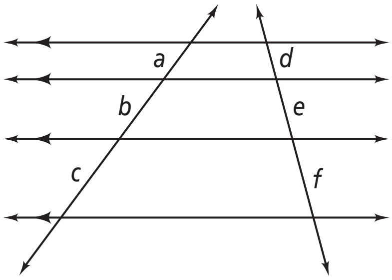
Name ______________________ Class ____________ Date ____________

7-5 Practice

Form G

Proportions in Triangles

Use the figure at the right to complete each proportion.



1. $\frac{a}{c} = \frac{\boxed{d}}{f}$

2. $\frac{f}{e} = \frac{c}{\boxed{b}}$

3. $\frac{\boxed{b}}{c} = \frac{e}{f}$

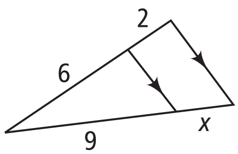
4. $\frac{a}{\boxed{d}} = \frac{b}{e}$

5. $\frac{a}{b} = \frac{\boxed{d}}{e}$

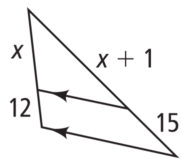
6. $\frac{e}{\boxed{b}} = \frac{f}{c}$

Algebra **Solve for x.**

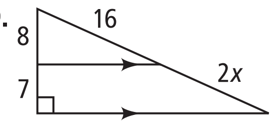
7. 3



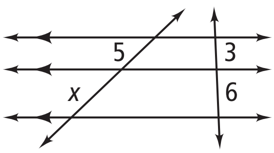
8. 4



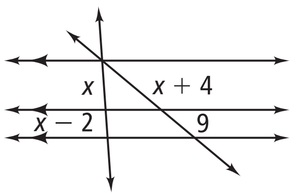
9. 7



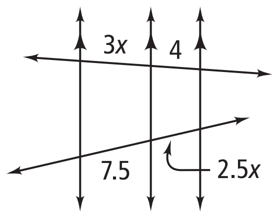
10. 10



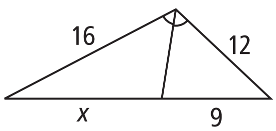
11. 8



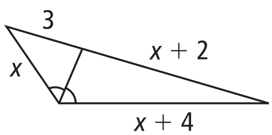
12. 2



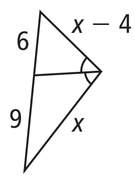
13. 12



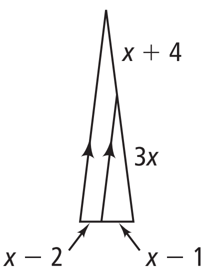
14. 4



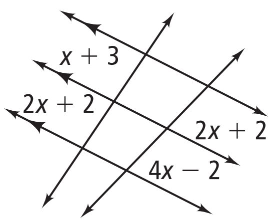
15. 12



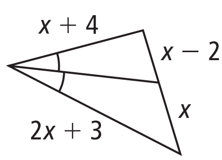
16. 4



17. 5



18. 6



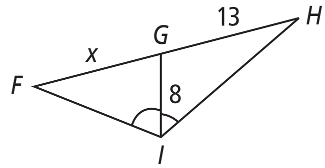
19. Compare and Contrast How is the Triangle-Angle-Bisector Theorem similar to Corollary 2 of Theorem 7-3? How is it different?

Answers may vary. Sample: Both relate to a line that intercepts an angle of a triangle and its opposite side. In both, the segments created by the intersecting line are related proportionally to the sides of the triangle. Corollary 2 of Theorem 7-3 is only true of right triangles with an altitude to the hypotenuse. The Triangle-Angle-Bisector Theorem relates to all triangles that contain an angle bisector that intersects the opposite side.

20. Reasoning In $\triangle FGH$, the bisector of $\angle F$ also bisects the opposite side. The ratio of each half of the bisected side to each of the other sides is 1 : 2. What type of triangle is $\triangle FGH$? Explain.

$\triangle FGH$ is an equilateral triangle. Because the side has been bisected, each segment is the same length. So, their sum is: $x + x = 2x$. This is the same as the length of a side.

21. Error Analysis Your classmate says you can use the Triangle-Angle-Bisector Theorem to find the value of x in the diagram. Explain what is wrong with your classmate's statement.

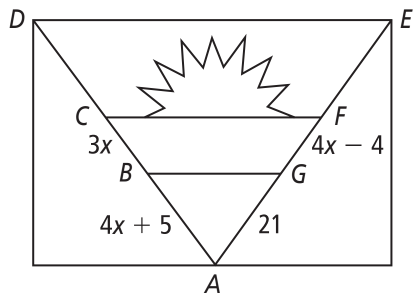


The classmate is confusing this Theorem with Corollary 1 to Theorem 7-3. You could only find the value of x if $\triangle FHI$ were a right triangle with right $\angle I$, and $\overline{IG}$ were an altitude to the hypotenuse.

22. Reasoning An angle bisector of a triangle divides the opposite side of the triangle into segments 3 in. and 6 in. long. A second side of the triangle is 5 in. long. Find the length of the third side of the triangle. Explain how you arrived at the correct length.

10 in.; The other possible side length is 2.5 in., but because 2.5 in. + 5 in. < 9 in., it violates the Triangle Inequality Theorem.

23. The flag of Antigua and Barbuda is like the image at the right. In the image, $\overline{DE} \parallel \overline{CF} \parallel \overline{BG}$.



a. An artist has made a sketch of the flag for a mural. The measures indicate the length of the lines in feet. What is the value of x? **4**

b. What type of triangle is $\triangle ACF$? Explain.

$\triangle ACF$ is isosceles. Because $x = 4$, $\overline{CB} \cong \overline{FG}$ and $\overline{BA} \cong \overline{GA}$. Because $CA = CB + BA$ and $FA = FG + GA$, by substitution $\overline{CA} \cong \overline{FA}$.

c. Given: $\overline{DE} \parallel \overline{CF} \parallel \overline{BG}$

Prove: $\triangle ABG \sim \triangle ACF \sim \triangle ADE$

Statements: 1) $\overline{DE} \parallel \overline{CF} \parallel \overline{BG}$; 2) $\angle EDC \cong \angle FCB \cong \angle GBA$; 3) $\angle DEF \cong \angle CFG \cong \angle BGA$; 4) $\triangle ABG \sim \triangle ACF \sim \triangle ADE$; Reasons: 1) Given; 2) If lines are $\parallel$, corresponding $\angle$s are $\cong$; 3) If lines are $\parallel$, corresponding $\angle$s are $\cong$; 4) AA $\sim$

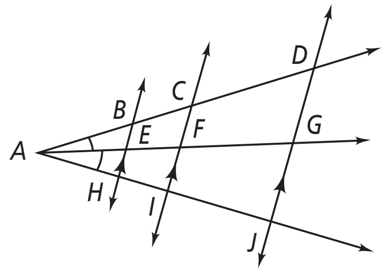
Name ____________________ Class __________ Date __________

7-5 Practice

Form K

Proportions in Triangles

Use the figure at the right to complete each proportion.



1. $\dfrac{CF}{\boxed{FI}} = \dfrac{AC}{AI}$

2. $\dfrac{AB}{BC} = \dfrac{\boxed{AH}}{HI}$

3. $\dfrac{\boxed{CD}}{IJ} = \dfrac{BC}{HI}$

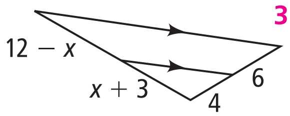
4. $\dfrac{JG}{\boxed{AJ}} = \dfrac{GD}{AD}$

5. $\dfrac{FG}{EF} = \dfrac{CD}{\boxed{BC}}$

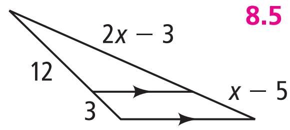
6. $\dfrac{AC}{AI} = \dfrac{\boxed{CD}}{IJ}$

Algebra **Solve for *x*.**

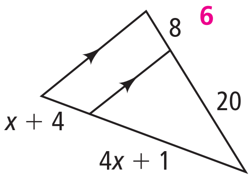
7. 3



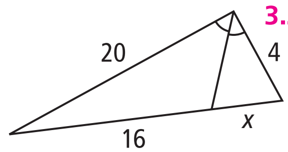
8. 8.5



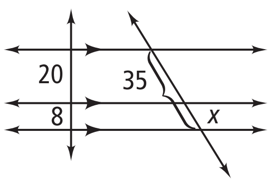
9. 6



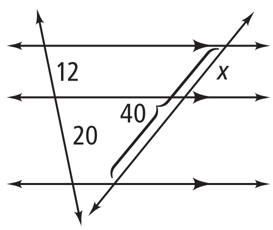
10. 3.2



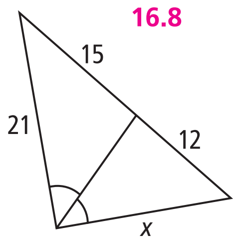
11. 10



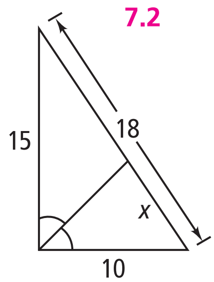
12. 15



13. 16.8



14. 7.2

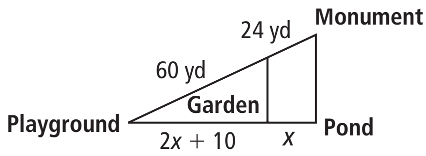


7-5 Practice (continued)

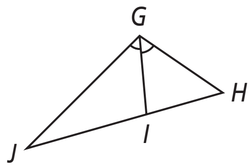
Form K

Proportions in Triangles

15. The map at the right shows the walking paths at a local park. The garden walkway is parallel to the walkway between the monument and the pond. How long is the path from the pond to the playground? **70 yd**

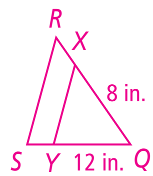


16. Error Analysis A classmate says you can use the Triangle-Angle-Bisector Theorem to find the length of GI. Explain what is wrong with your classmate's statement.
Answers may vary. Sample: The Triangle-Angle-Bisector Thm. states that the segments formed when the bisector divides a side are proportional to the other sides. It cannot be used to find the length of the bisector.



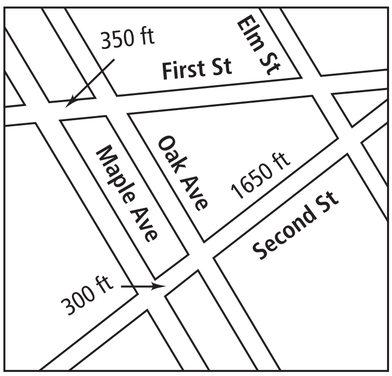
17. Triangle QRS has line XY parallel to side RS. The length of QY is 12 in. The length of QX is 8 in.

a. Draw a picture to represent the problem.
Answers may vary. Sample:



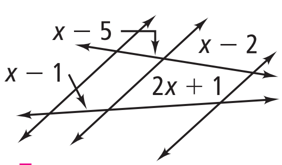
b. If the length of XR is 5 in., what is the length of QS? **19.5 in.**

18. The business district of a town is shown on the map below. Maple Avenue, Oak Avenue, and Elm Street are parallel. How long is the section of First Street from Elm Street to Maple Avenue? **2275 ft**



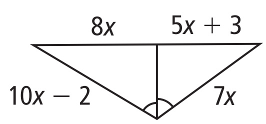
Algebra Solve for x.

19.



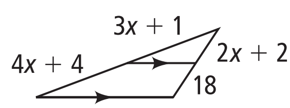
7

20.



3 or $\frac{1}{3}$

21.



5 or $-\frac{1}{4}$

Name ______________________ Class __________ Date __________

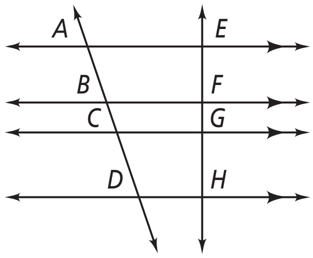
7-5 Standardized Test Prep

Proportions in Triangles

Multiple Choice

For Exercises 1–5, choose the correct letter.

For Exercises 1 and 2, use the diagram at the right.



1. Which makes the proportion true? $\frac{AB}{\square} = \frac{EF}{GH}$ C

Ⓐ AD Ⓒ CD

Ⓑ DH Ⓓ BC

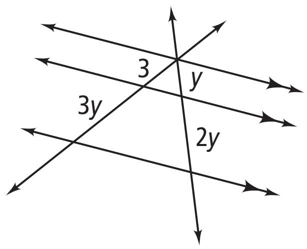
2. Which proportion is *not* true? I

Ⓕ $\frac{BC}{CD} = \frac{FG}{GH}$ Ⓖ $\frac{AC}{CD} = \frac{EG}{GH}$ Ⓗ $\frac{BD}{FH} = \frac{AD}{EH}$ Ⓘ $\frac{AB}{AE} = \frac{EF}{BF}$

3. What is the value of y? A

Ⓐ 2 Ⓒ 3

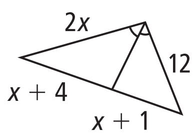
Ⓑ 4 Ⓓ 6



4. What is the value of x? G

Ⓕ 3 Ⓗ 6

Ⓖ 8 Ⓘ 12



5. In $\triangle DEF$, the bisector of $\angle F$ divides the opposite sides into segments that are 4 and 9 in. long. The side of the triangle adjacent to the 4 in. segment is 6 in. long. To the nearest tenth of an inch, how long is the third side of the triangle? D

Ⓐ 2.7 in. Ⓑ 6 in. Ⓒ 13 in. Ⓓ 13.5 in.

Short Response

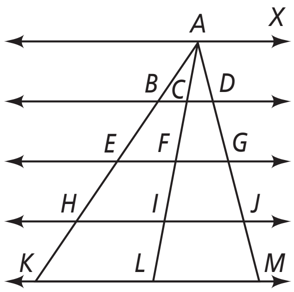
6. In $\triangle QRS$, $\overline{XY} \parallel \overline{SR}$. $\overline{XY}$ divides $\overline{QR}$ and $\overline{QS}$ into segments as follows: $\overline{SX} = 3$, $\overline{XQ} = 2x$, $\overline{RY} = 4.5$, and $\overline{YQ} = 7.5$. Write a proportion to find x. What is the length of $\overline{QS}$? [2] $\frac{7.5}{4.5} = \frac{2x}{3}$; $x = 2.5$; 8 [1] incorrect proportion or error in calculation [0] incorrect proportion and error in calculation

Name ______________________ Class ____________ Date ____________

7-5 Enrichment

Proportions in Triangles

Great Geometers



One of the first great women mathematicians was born in ancient Greece in the middle of the 4th century CE. Taught by her father, she was also a preeminent astronomer and philosopher. She became the head of the Neoplatonist school of philosophy and was widely known. She wrote commentaries on several mathematical works and was consulted in the construction of an astrolabe and hydroscope. She was eventually murdered by religious zealots who disagreed with her philosophical teachings.

To find the name of this mathematician and astronomer, consider the diagram, and use the information given below to find the lengths of all the segments.

$\overline{AX} \parallel \overline{BC} \parallel \overline{EF} \parallel \overline{HI} \parallel \overline{KL}$

$\overline{AL}$ bisects $\angle KAM$

$AK = 40$, $EF = 12$, $BE = 16$, $HK = 4$, $AD = 4$, $AC = 6$, and $AE = 24$.

Compute the lengths associated with the various letters, and place the letters in the appropriate spaces below.

Letter	Segment Length	Numerical length	Letter	Segment Length	Numerical length
1. I	HI	?	**2.** H	$IL - JM$	?
3. A	$EK + IL$	?	**4.** P	IL	?
5. O	DG	?	**6.** A	$FI - CD$	?
7. A	$LM + BC$	?	**8.** F	IJ	?
9. X	$FI + HK$	?	**10.** A	BC	?
11. T	$IL + JM$	?	**12.** I	GJ	?
13. E	CF	?	**14.** R	$IJ + DG$	?
15. Y	JM	?	**16.** A	LM	?
17. L	$IL + JM + GJ$	?	**18.** N	$FG + FI$	?
19. D	DM	?			

H	Y	P	A	T	I	A		O	F
1	2	3	4	5	6	7		8	9

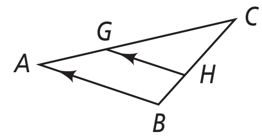
A	L	E	X	A	N	D	R	I	A
10	11	12	13	14	15	16	17	18	19

7-5 Reteaching

Proportions in Triangles

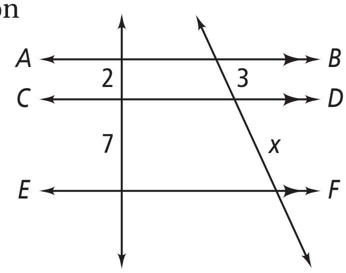
The Side-Splitter Theorem states the proportional relationship in a triangle in which a line is parallel to one side while intersecting the other two sides.

Theorem 7-4: Side-Splitter Theorem



In $\triangle ABC$, $\overline{GH} \parallel \overline{AB}$. $\overline{GH}$ intersects $\overline{BC}$ and $\overline{AC}$. The segments of $\overline{BC}$ and $\overline{AC}$ are proportional: $\frac{AG}{GC} = \frac{BH}{HC}$

The corollary to the Side-Splitter Theorem extends the proportion to three parallel lines intercepted by two transversals.



If $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$, you can find x using the proportion:

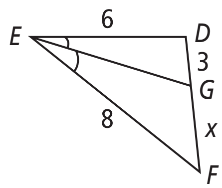
$$\frac{2}{7} = \frac{3}{x}$$

$2x = 21$ Cross Products Property

$x = 10.5$ Solve for *x*.

Theorem 7-5: Triangle-Angle-Bisector Theorem

When a ray bisects the angle of a triangle, it divides the opposite side into two segments that are proportional to the other two sides of the triangle.



In $\triangle DEF$, $\overline{EG}$ bisects $\angle E$. The lengths of $\overline{DG}$ and $\overline{GF}$ are proportional to their adjacent sides $\overline{DE}$ and $\overline{EF}$: $\frac{DG}{DE} = \frac{GF}{EF}$.

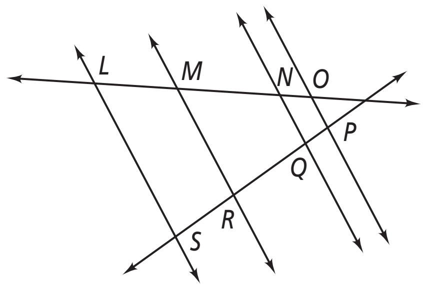
To find the value of x, use the proportion $\frac{3}{6} = \frac{x}{8}$.

$$6x = 24$$

$$x = 4$$

Exercises

Use the figure at the right to complete each proportion.



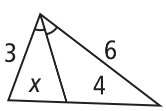
1. $\frac{\boxed{RQ}}{MN} = \frac{SR}{LM}$

2. $\frac{NO}{\boxed{QP}} = \frac{LM}{SR}$

3. $\frac{MN}{RQ} = \frac{\boxed{NO}}{QP}$

4. $\frac{SQ}{LN} = \frac{RP}{\boxed{MO}}$

Algebra **Solve for *x*.**

5. 

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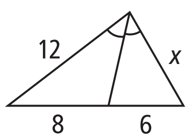
6. 

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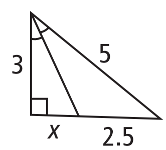
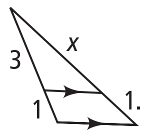
7. 

1.5

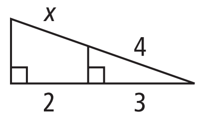
7-5 Reteaching (continued)

Proportions in Triangles

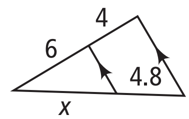
Algebra **Solve for *x*.**

8. 

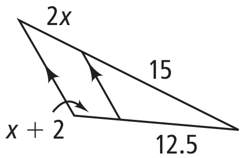
4.5

9. 

$2\frac{2}{3}$

10. 

7.2

11. 

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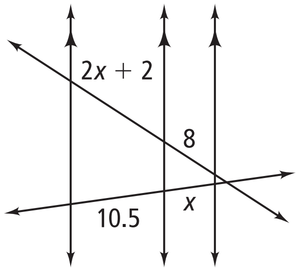
12. 

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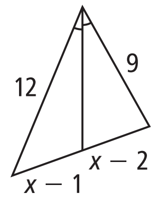
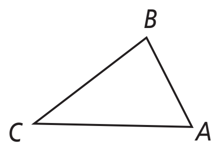
13. 

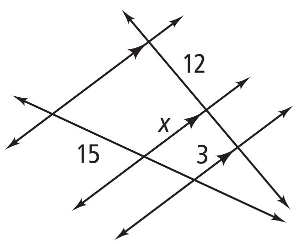
5

In $\triangle ABC$, $AB = 6$, $BC = 8$, and $AC = 9$.



14. The bisector of $\angle A$ meets $\overline{BC}$ at point N. Find BN and CN. $BN = 3\frac{1}{5}$, $CN = 4\frac{4}{5}$

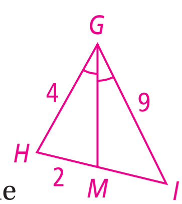
15. $\overline{XY} \parallel \overline{CA}$. Point X lies on $\overline{BC}$ such that $BX = 2$, and Y is on $\overline{BA}$. Find BY. **1.5**



16. Error Analysis A classmate says you can use the Corollary to the Side-Splitter Theorem to find the value of x. Explain what is wrong with your classmate's statement.
The corollary states that the segments on the transversal, not the segments on the parallel lines, are proportional.

17. An angle bisector of a triangle divides the opposite side of the triangle into segments 6 and 4 in. long. The side of the triangle adjacent to the 6-in. segment is 9 in. long. How long is the third side of the triangle? **6 in.**

18. Draw a Diagram $\triangle GHI$ has angle bisector $\overline{GM}$, and M is a point on $\overline{HI}$. $GH = 4$, $HM = 2$, $GI = 9$. Solve for MI. Use a drawing to help you find the answer. **4.5**



19. The lengths of the sides of a triangle are 7 mm, 24 mm, and 25 mm. Find the lengths to the nearest tenth of the segments into which the bisector of each angle divides the opposite side.
5.6 mm and 19.4 mm; 3.4 mm and 3.6 mm; 5.3 mm and 18.8 mm

Name ______________________________ Class ____________ Date ____________

Chapter 7 Quiz 1

Form G

Lessons 7-1 through 7-3

Do you know HOW?

1. The lengths of two sides of a polygon are in the ratio 2 : 3. Write expressions for the measures of the two sides in terms of the variable x.
   measure of side 1 = 2*x*; measure of side 2 = 3*x*
2. $\triangle HJK \sim \triangle RST$. Complete each statement. $\angle K \cong \boxed{\angle T}$ and $\frac{JK}{ST} = \frac{\boxed{HK}}{RT}$

Solve each proportion.

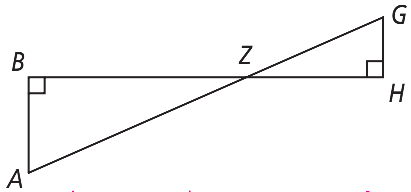
3. $\frac{z}{15} = \frac{45}{75}$ **9**

4. $\frac{5}{8} = \frac{x + 2}{5}$ $\mathbf{\frac{9}{8}}$

5. To the nearest inch, a door is 75 in. tall and 35 in. wide. What is the ratio of the width to the height? **35 : 75 or 7 : 15**

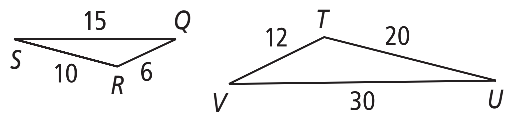
In Exercises 6–9, are the triangles similar? If yes, write a similarity statement and explain how you know they are similar. If not, explain.

6.



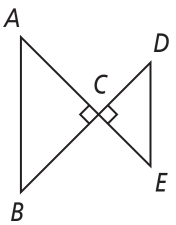
yes; △*ABZ* ∼ △*GHZ*; AA ∼ Theorem

7.



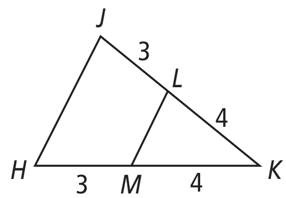
yes; △*QRS* ∼ △*VTU*; SSS ∼ Theorem

8.



No; Answers may vary. Sample: Only one pair of angles is always congruent.

9.



yes; △*HJK* ∼ △*MLK*; SAS ∼ Theorem

Do you UNDERSTAND?

10. **Vocabulary** What is a proportion that has means 9 and 10 and extremes 6 and 15? $\frac{6}{9} = \frac{10}{15}$

11. **Reasoning** To prove that any two isosceles triangles are similar you only need to show that the vertex angles are congruent or a pair of corresponding base angles is congruent. Explain.
    Answers may vary. Sample: An isosceles triangle has two congruent angles, so knowing the measure of a base angle or the measure of a vertex angle provides enough information to find the measures of the other two angles.

Name ______________________________ Class ____________ Date ____________

Chapter 7 Quiz 2

Form G

Lessons 7-4 through 7-5

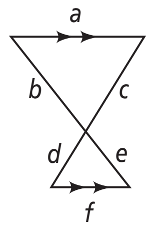
Do you know HOW?

Find the geometric mean of each pair of numbers.

1. 4 and 25 **10**   **2.** 9 and 12 **$6\sqrt{3}$**   **3.** 2 and 8 **4**

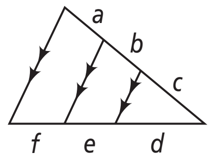
4. 5 and 45 **15**   **5.** 18 and 50 **30**   **6.** 6 and 15 **$3\sqrt{10}$**

Use the figure at the right to complete each proportion.



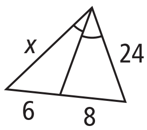
7. $\frac{a}{f} = \frac{\boxed{b}}{e}$ **8.** $\frac{a}{f} = \frac{c}{\boxed{d}}$

Use the figure at the right to complete each proportion.



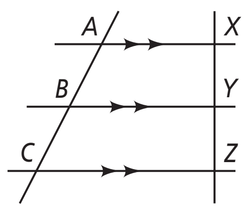
9. $\frac{a}{b} = \frac{\boxed{f}}{e}$ **10.** $\frac{c}{a+b} = \frac{\boxed{d}}{e+f}$

11. What is the value of x in the figure?



18

Use the figure at the right to complete each proportion.

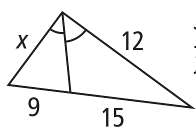


12. $\frac{AB}{AC} = \frac{\boxed{XY}}{XZ}$ **13.** $\frac{AB}{XY} = \frac{BC}{\boxed{YZ}}$

Do you UNDERSTAND?

14. Compare and Contrast How are the Triangle-Angle-Bisector-Theorem and Corollary 2 to Theorem 7-3 alike? How are they different?
Answers may vary. Sample: Both break a side into two segments and give proportions to find the lengths of the segments; the △-∠-Bis.-Thm does not create similar triangles; Corollary 2 does.

15. Error Analysis A classmate writes an incorrect proportion to find x. Explain and correct the error.



$\cancel{\frac{x}{15} = \frac{9}{12}}$

Answers may vary. Sample: The classmate created ratios using sides that do not correspond; $\frac{x}{12} = \frac{9}{15}$.

16. In $\triangle DEF$, the angle bisector of $\angle D$ is perpendicular to $\overline{EF}$. What type of triangle is $\triangle DEF$? Explain your reasoning. **Because the angle bisector of $\angle D$ is perpendicular to $\overline{EF}$, it makes right angles. So, $\angle E \cong \angle F$ by the Third Angles Theorem. Because $\triangle DEF$ has two congruent angles, it is an isosceles triangle.**

Chapter 7 Test

Form G

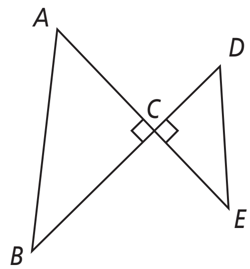
Do you know HOW?

Algebra **Solve each proportion.**

1. $\frac{y}{4} = \frac{15}{20}$ **3**

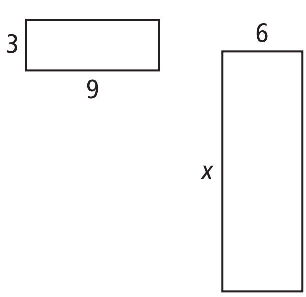
2. $\frac{6}{z-3} = \frac{8}{5}$ $\mathbf{\frac{27}{4}}$

3. Determine whether the polygons at the right are similar. If so, write a similarity statement and give the scale factor. If not, explain.
No; answers may vary. Sample: Only one pair of angles is definitely congruent.

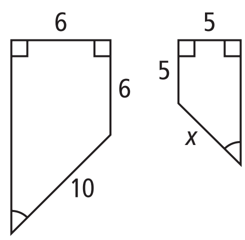


Algebra **The polygons are similar. Find the value of each variable.**

4. **18**

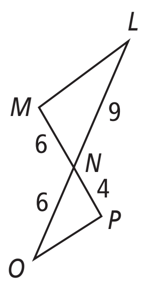


5. $\mathbf{\frac{25}{3}}$



Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

6. **Yes; △*LMN* ~ △*OPN***
SAS ~ Theorem



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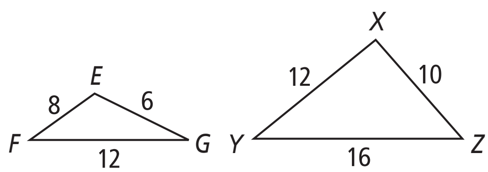


No; answers may vary. Sample: Ratios between pairs of sides are not the same.

Find the geometric mean of each pair of numbers.

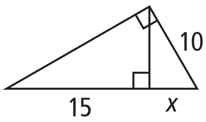
8. 8 and 12 $\mathbf{4\sqrt{6}}$

9. 20 and 6 $\mathbf{2\sqrt{30}}$

10. **Coordinate Geometry** Plot $A(0, 0)$, $B(1, 0)$, $C(1, 2)$, $D(2, 0)$, and $E(2, 4)$. Then sketch $\triangle ABC$ and $\triangle ADE$. Use SAS ~ to prove $\triangle ABC \sim \triangle ADE$. **Sample: Check students' drawings; $\angle A \cong \angle A$ (Reflexive Prop. of Congruence); $\frac{AC}{AE} = \frac{AB}{AD} = \frac{1}{2}$**

11. Reasoning Name two different pairs of whole numbers that have a geometric mean of 4. Name a pair of positive numbers that are not whole numbers that have a geometric mean of 4. How many pairs of positive numbers have a geometric mean of 4? Explain. **Answers may vary. Sample: 1 and 16, 2 and 8; $\frac{3}{2}$ and $\frac{32}{3}$; infinitely many; any two real positive numbers whose product is 16 will have a geometric mean of 4.**

Algebra **Find the value of *x*.**

12. 

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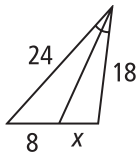
13. 

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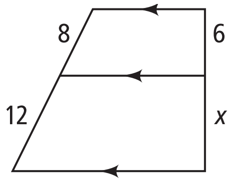
14. 

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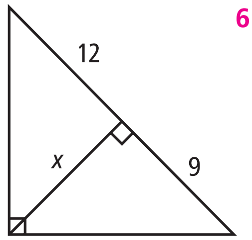
15. 

$6\sqrt{3}$

Do you UNDERSTAND?

16. Reasoning $\triangle ABC \sim \triangle HJK$ and $\triangle HJK \sim \triangle XYZ$. Furthermore, the ratio between the sides of $\triangle ABC$ and $\triangle HJK$ is $a : b$. Finally, the ratio of the sides between $\triangle HJK$ and $\triangle XYZ$ is $b : a$. What can you conclude about $\triangle ABC$ and $\triangle XYZ$? Explain.
They are congruent; they are similar by the Transitive Property. Because the ratio of their sides is $a : a$, they are congruent.

17. Compare and Contrast How are Corollary 1 to Theorem 7-3 and Corollary 2 to Theorem 7-3 alike? How are they different?
Answers may vary. Sample: They both involve the altitude of right triangles and segments of the hypotenuse. Corollary 1 involves the length of the altitude, while Corollary 2 involves the lengths of the legs.

18. Error Analysis A student says that since all isosceles right triangles are similar, all isosceles triangles that are similar must be right triangles. Is the student right? Explain.
No; answers may vary. Sample: A counterexample is any pair of equilateral triangles, which are isosceles and similar, but are not right triangles.

Determine whether each statement is *always, sometimes,* or *never* true.

19. Two equilateral triangles are similar. **always**

20. The angle bisector of a triangle divides the triangle into two similar triangles. **sometimes**

21. A rectangle is similar to a rhombus. **sometimes**

Name ______________________ Class ____________ Date ____________

Chapter 7 Quiz 1

Form K

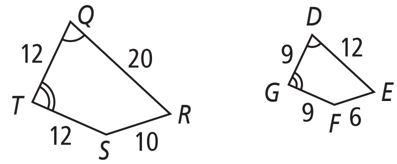
Lessons 7-1 through 7-3

Do you know HOW?

1. Twyla's pet cat weighs 8 lb. Her pet hamster weighs 12 ounces. What is the ratio of her cat's weight to her hamster's weight? **32 : 3**

2. The non-right angles of a right triangle are in the ratio 1 : 5. Write an equation that could be used to find the measure of each angle. **$x + 5x = 90$**

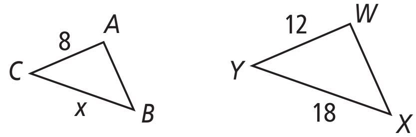
3. Are the quadrilaterals similar? If so, write a similarity statement and give the scale factor. If not, explain.
No; not all pairs of corresponding sides are proportional.



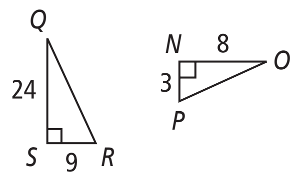
4. What is the value of x in the proportion $\frac{2}{5} = \frac{10}{x}$? **25**

5. If $QRST \sim DEFG$, what would make the proportion $\frac{ST}{FG} = \frac{RS}{\boxed{EF}}$ true?

6. $\triangle ABC \sim \triangle WXY$. What is the value of x? **12**



7. Are the triangles at the right similar? If yes, write a similarity statement and explain how you know. If not, explain.
$\triangle QRS \sim \triangle OPN$; SAS ~ Thm.



Do you UNDERSTAND?

8. Vocabulary Explain how similar triangles can be used to measure an object indirectly. Give a specific example.
Check students' work.

9. Compare and Contrast What is the difference between proving that a set of quadrilaterals are similar and proving that a set of triangles are similar?
Answers may vary. Sample: To prove quadrilaterals similar, you must prove all four angles are ≅ and all four sides are in proportion. To prove △ ~, you need to prove either that two ∠ pairs are ≅, that one ∠ pair is ≅ and the adjacent sides are in proportion, or that all corresp. sides are in proportion.

10. Reasoning Are all parallelograms similar? Are any types of parallelograms always similar? Explain.
No; yes; all squares are similar.

Name ______________________ Class ____________ Date ____________

Chapter 7 Quiz 2

Form K

Lessons 7-4 and 7-5

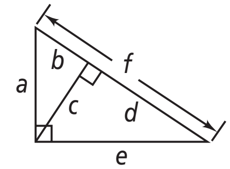
Do you know HOW?

Find the geometric mean of each pair of numbers.

1. 2 and 32 **8**

2. 5 and 18 $\mathbf{3\sqrt{10}}$

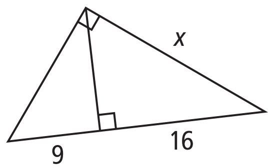
Use the figure at the right to complete each proportion.



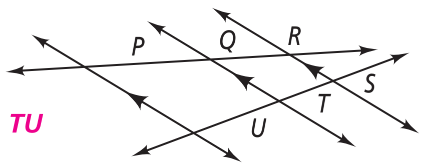
3. $\frac{b}{c} = \frac{\boxed{c}}{d}$

4. $\frac{b}{a} = \frac{a}{\boxed{f}}$

5. What is the value of x? **20**



Use the figure at the right to complete each proportion.

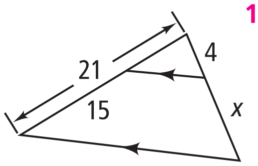


6. $\frac{RQ}{QP} = \frac{ST}{\boxed{TU}}$

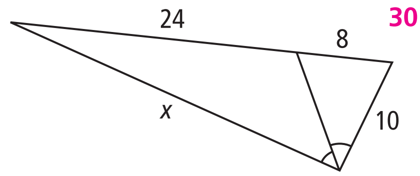
7. $\frac{QP}{RQ + QP} = \frac{TU}{\boxed{}}$ **$ST + TU$**

What is the value of x in each figure?

8. **10**



9. **30**

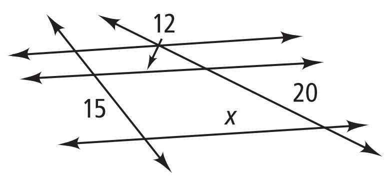


Do you UNDERSTAND?

10. Right $\triangle ABC$ with right angle A has altitude to the hypotenuse $\overline{AD}$. Which parts of the triangle are geometric means to other parts?

$\overline{AD}$ is the mean of $\overline{BD}$ and $\overline{DC}$. Leg $\overline{AB}$ is the mean of $\overline{BC}$ and $\overline{BD}$. Leg $\overline{AC}$ is the geometric mean of $\overline{BC}$ and $\overline{DC}$.

11. Error Analysis A classmate says you can use the Corollary to the Side-Splitter Theorem to find x. Explain your classmate's error.



The Side-Splitter Thm. states that the sections of the transversals are proportional, not the sections of the parallel lines.

Name ______________________ Class __________ Date __________

Chapter 7 Test

Form K

1. An adult female panda weighs 200 lb. Its newborn baby weighs only $\frac{1}{4}$ lb. What is the ratio of the weight of the adult to the weight of the baby panda? **800 : 1**

2. An animal shelter has 104 cats and dogs. The ratio of cats to dogs is 5 : 3. How many cats are at the shelter? **65**

3. The sides of a triangle are in the extended ratio of 3 : 2 : 4. If the length of the shortest side is 6 cm, what is the length of the longest side? **12 cm**

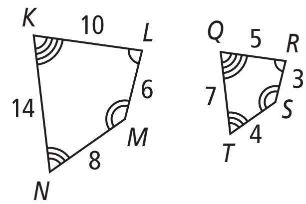
Solve each proportion.

4. $\frac{12}{x} = \frac{4}{7}$ **21**

5. $\frac{x}{10} = \frac{7}{20}$ **3.5**

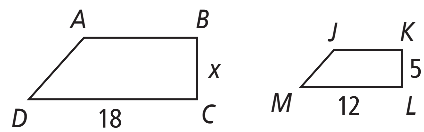
6. $\frac{x}{x+5} = \frac{5}{7}$ **12.5**

7. Are the polygons similar? If they are, write a similarity statement and give the scale factor. If not, explain.
***KLMN* ~ *QRST*; 2 : 1**



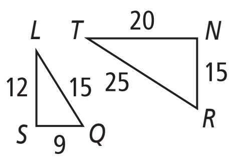
8. The scale of a map is 1 in. = 25 mi. On the map, the distance between two cities is 5.25 in. What is the actual distance? **131.25 mi**

9. $ABCD \sim JKLM$. What is the value of x? **7.5**



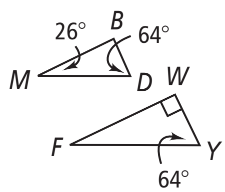
Determine whether the triangles are similar. If so, write the similarity statement and name the postulate or theorem you used. If not, explain.

10.



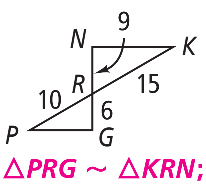
△*TNR* ~ △*LSQ*; SSS ~ Thm.

11.



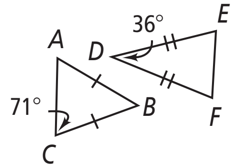
△*MBD* ~ △*FWY*; AA ~ Postulate

12.



△*PRG* ~ △*KRN*; SAS ~ Thm.

13.



No; included angles are not ≅.

14. A person 2 m tall casts a shadow 5 m long. At the same time, a building casts a shadow 24 m long. How tall is the building? **9.6 m**

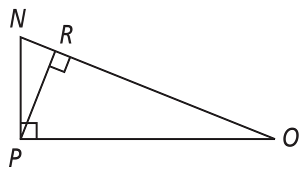
Find the geometric mean of each pair of numbers.

15. 9 and 25 **15**

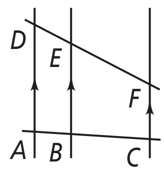
16. 10 and 12 $2\sqrt{30}$

17. A pie shop sold a total of 117 pies one day. The pies were apple, cherry, and blueberry. The ratio of apple pies sold to cherry pies to blueberry pies was 6 : 2 : 5. How many cherry pies were sold? **18**

18. Write a similarity statement relating the three triangles in the diagram. $\triangle NPO \sim \triangle NRP \sim \triangle PRO$



Use the figure at the right to complete each proportion.

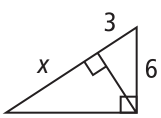


19. $\frac{DF}{DE} = \frac{\boxed{AC}}{AB}$

20. $\frac{EF}{DE} = \frac{\boxed{BC}}{AB}$

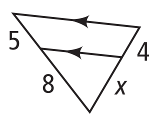
Find the value of *x*.

21.



9

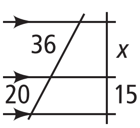
22.



6.4

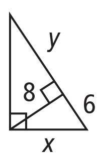
Find the values of the variables.

23.



27

24.



10; $\frac{32}{3}$

25. Find the length of the altitude to the hypotenuse of a right triangle whose sides have lengths 6.8 and 10. The altitude to the hypotenuse separates the hypotenuse into two parts. The smaller part is 3.8. Round your answer to the nearest tenth. **5.6**

Name ______________________ Class ____________ Date ____________

Performance Tasks

Chapter 7

Task 1

Prove three different pairs of triangles are similar using the following postulates and theorems. Sketch each pair of triangles on your own paper in your explanations.

a. Postulate 7-1 Angle-Angle (AA $\sim$) Similarity Postulate

b. Theorem 7-1 Side-Angle-Side Similarity (SAS $\sim$) Theorem

c. Theorem 7-2 Side-Side-Side Similarity (SSS $\sim$) Theorem

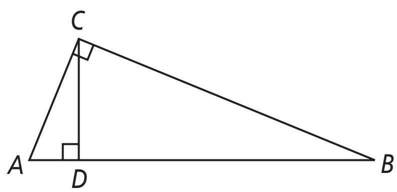
[4] The student implements appropriate strategies to produce correct proofs. [3] The student's strategy could lead to correct proofs but the student misunderstands part of the problem or ignores important information. [2] The student chooses an appropriate strategy but implements it incorrectly, or the student chooses an inappropriate strategy but shows some understanding. [1] The student shows some understanding but with incorrect strategies and little progress toward correct proofs. [0] Student gives incorrect or no response.

Task 2

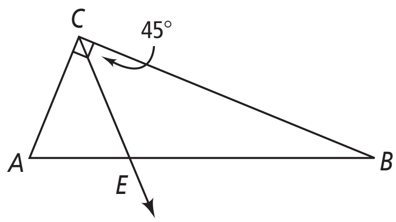
There are three claims made about the right triangles below. Evaluate each claim.

Use the sketches at the right of each claim to help you.

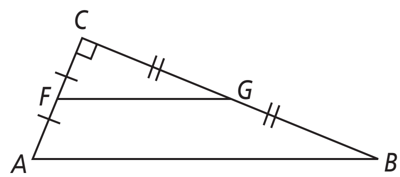
a. The altitude to the hypotenuse forms similar triangles. In the drawing at the right, is $\triangle ABC \sim \triangle ACD \sim \triangle CBD$?



b. The angle bisector of the right angle forms triangles with pairs of proportionate sides. In the drawing at the right, is $\triangle ACE \sim \triangle BCE$?



c. The midsegment connecting the legs forms a triangle similar to the original triangle. In the drawing at the right, is $\triangle CFG \sim \triangle CAB$?



Check students' work. Claim (a) is true, Claim (b) is false, and Claim (c) is true. [4] Student gives correct answers and provides logical reasons to support the answers. [3] Student gives mostly correct answers, but the work may contain minor errors or omissions. [2] Student gives answers or explanations that contain significant errors. [1] Student makes little or no progress toward the answers or explanations. [0] Student gives incorrect or no response.

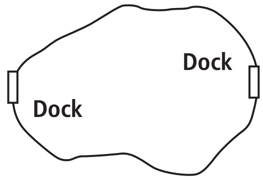
Name ______________________ Class ____________ Date ____________

Performance Tasks (continued)

Chapter 7

Task 3

The drawing at the right shows two docks on opposite sides of a lake. You want to find the distance across the lake between the two docks, but you can only measure distances on land. Use indirect measurement and similar triangles to find the distance.



a. Devise and explain your plan for finding the distance.

b. Describe the indirect measurements you need to take.

c. Sketch the similar triangles you need.

d. Find the distance.

e. Devise and explain another way to find the distance.

Sample: One method is to pick any point A on land. Let B and C be the centers of the two docks. Measure AB and AC. Then select a fraction and mark X and Y halfway from A to B and C, respectively. Then measure XY. Then $BC = 2XY$.
[4] Student gives mathematically correct procedures and explanations. [3] Student gives a procedure and explanation that may contain minor errors. [2] Student gives an invalid procedure or a valid procedure with little or no explanation. [1] Student makes little or no progress toward the correct procedure or explanation. [0] Student gives incorrect or no response.

Task 4

When Sarah was 6 months old, she was 21 in. tall and her head had a diameter of 4.5 in. Now, at 20 years old, she is 6 ft 1 in. tall and her head has a diameter of 7.65 in. If Sarah is typical, should a person at 20 years of age be considered "similar" to herself at 6 months of age?

a. Find the ratio of height-to-head diameter for Sarah at 6 months old. If the ratio were the same when she was 20 years old, what diameter would her head have?

b. Find the ratio of height-to-head diameter for Sarah at 20 years old. If the ratio were the same when she was 6 months old, how tall would she be at 6 months?

c. How close are the two ratios you found in Part (a) and Part (b)?

d. If Sarah were taller or shorter either at 6 months of age or 20 years of age, would this make her more or less "similar" at those two ages? Explain.

[4] Student gives correct ratios at 6 months (about 4.7 : 1) and at 20 years (9.54 : 1), and correctly predicts head size at 20 years (15.6 in.) and height at 6 months (42.9 in.); student provides sound analysis. [3] Student gives correct answers but explanation contains minor errors or omissions. [2] Student gives some correct answers but explanations contain significant errors or omissions. [1] Student makes little progress toward the correct solution or explanation. [0] Student gives incorrect or no response.

Name ______________________ Class ____________ Date ____________

Cumulative Review

Chapters 1–7

Multiple Choice

1. Which of the following never contains an angle with a measure of 90°? C

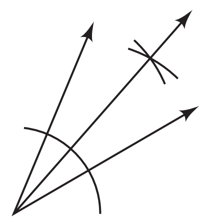
Ⓐ a right triangle Ⓒ an equilateral triangle

Ⓑ an isosceles triangle Ⓓ an isosceles trapezoid

2. What type of construction is shown at the right? F

Ⓕ angle bisector Ⓗ perpendicular bisector

Ⓖ congruent angles Ⓘ congruent segments



3. Two lines intersect to form four congruent angles. You can conclude the lines are B

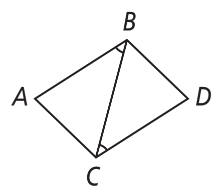
Ⓐ skew Ⓒ parallel

Ⓑ perpendicular Ⓓ not perpendicular

4. Which of the following facts would be sufficient to prove $\triangle ABC \cong \triangle DCB$? F

Ⓕ $\overline{BD} \parallel \overline{AC}$ Ⓗ $\angle ABC \cong \angle ACB$

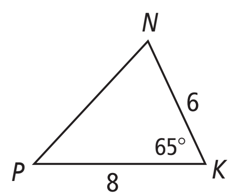
Ⓖ $\overline{AB} \parallel \overline{CD}$ Ⓘ $\overline{BD} \cong \overline{CD}$



5. What can you conclude from the diagram? A

Ⓐ $m\angle P < m\angle N$ Ⓒ $m\angle N < 65$

Ⓑ $m\angle P > 65$ Ⓓ $NK < NP$



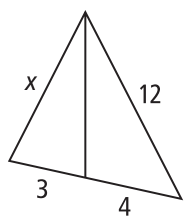
6. By which theorem or postulate does $x = 9$? G

Ⓕ Side-Splitter Theorem

Ⓖ Triangle-Angle-Bisector Theorem

Ⓗ SAS Similarity Theorem

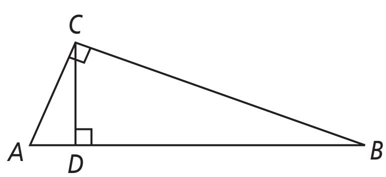
Ⓘ SSS Similarity Theorem



7. Which angle is complementary to $\angle ABC$? D

Ⓐ $\angle ACB$ Ⓒ $\angle ADC$

Ⓑ $\angle ACD$ Ⓓ $\angle DAC$



Cumulative Review (continued)

Chapters 1–7

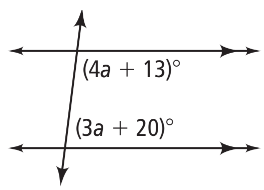
8. On a map, Jenia draws a segment from her home to her school. She measures this segment and finds it is 3 cm. She knows her home is 0.5 mi from school. What is the scale of her map in cm/mi? **F**

(F) 6 (G) 3 (H) $\frac{1}{3}$ (I) $\frac{1}{6}$

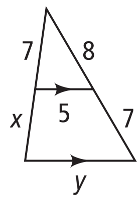
Gridded Response

9. In pentagon $ABCDE$, $\angle A \cong \angle B \cong \angle C \cong \angle D$ and the ratio of $m\angle A$ to $m\angle E$ is 2 : 1. What is $m\angle E$? **60**

10. What is the value of a in the figure at the right? **21**

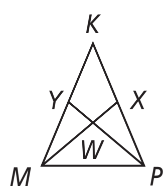


11. What is the value of x in the figure below? $\frac{49}{8}$



Extended Response

12. $\triangle KMP$ is isosceles with $KM = KP$. $\overline{MX}$ and $\overline{PY}$ are angle bisectors.



a. Is there enough information to prove $\triangle WMP$ is an isosceles triangle? Explain. **Yes; $\angle M \cong \angle P$. Therefore, $\angle PMW \cong \angle MPW$.**

b. Can you conclude that $\overline{MX}$ and $\overline{PY}$ are medians? **No**

c. What one additional piece of information would allow you to prove that $\overline{MX}$ and $\overline{PY}$ are altitudes? **Answers may vary. Sample: $\angle K \cong \angle M$**

d. Why is it impossible for $\triangle WMP$ to be an equilateral triangle?
The measure of the base angles of $\triangle WMP$ is half the measure of the base angles of $\triangle KMP$. So, the measure of both base angles of $\triangle KMP$ would have to be 120. That is not possible.

Chapter 7 Project Teacher Notes: Miami Models

About the Project

Students will investigate proportionality of scale models using a representation of a satellite image of downtown Miami to create model buildings. They may also use actual satellite images found on the Internet. A city other than Miami can be substituted.

Introducing the Project

- Ask students where they have seen scale models of three-dimensional objects (museums, toy stores, etc.).
- Ask students what objects they have seen modeled and how they compare to the buildings they will be modeling.

Activity 1: Height

Discuss how present-day Miami may differ from the map.

Activity 2: Area

Students will use geometric formulas to approximate the area of the footprint, or base, of each building.

Activity 3: Modeling

Students should build their models of these buildings using sturdy materials. With the class, determine a scale that everyone will use to allow their projects to be displayed in your classroom.

Finishing the Project

You may wish to have students display their completed three-dimensional models in your classroom. Students may be interested in making some additional models to make the miniature city look more lively (miniature trees, cars, roads, beach, smaller buildings, etc.). Ask students to share any insights they gained while building their models.

Name ______________________ Class ____________ Date ____________

Chapter 7 Project: Miami Models

Beginning the Chapter Project

Miami, Florida, has one of the largest skylines in the world. Through the early 2000s the city has built more skyscrapers than just about anywhere else in the United States. As of 2008, Miami had 56 buildings over 400 ft and another eight buildings projected to be higher than 400 ft when completed.

In your project for this chapter, you will use similarity to find the height and location of individual buildings, and to build a model of one of these buildings, helping your class recreate the Miami skyline.

Activities

Activity 1: Height

The map on page 65 shows 15 of the tallest buildings in Miami.

The table beside the map lists the names of the buildings in the image, and some of their heights and labels.

Use the shadows of the buildings to find the height and the name of each building on the map.

Activity 2: Area

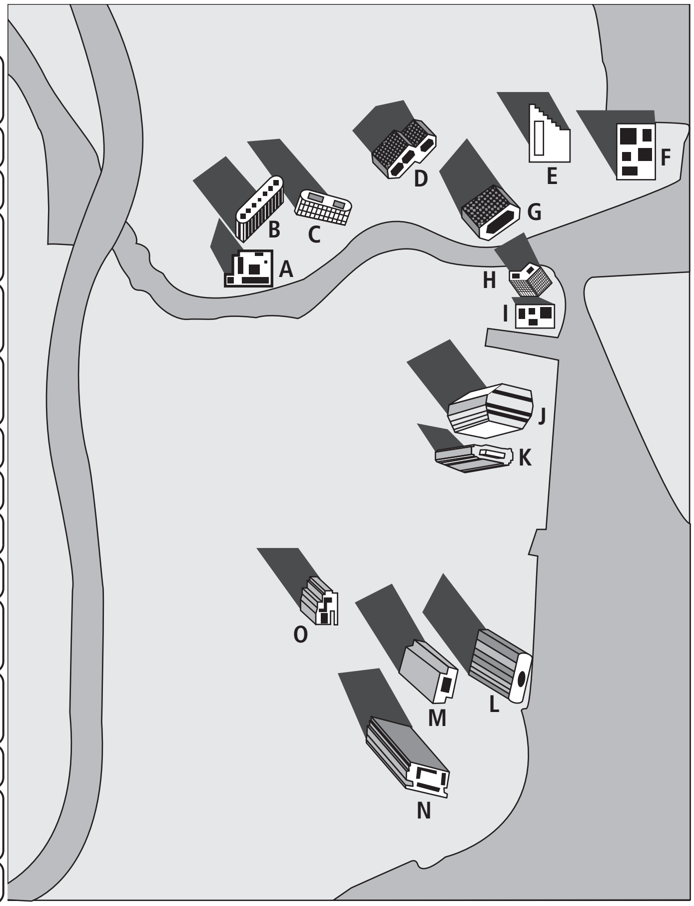
The map shows the aerial view of these downtown buildings. If the footprint of a building is the amount of land it covers, what is the area of each footprint? Measure carefully and round your answer to the nearest 1000 ft^2. **A 29,000; B 27,000; C 22,000; D 18,000; E 30,000; F 29,000; G 36,000; H 38,000; I 26,000; J 22,000; K 19,000; L 21,000; M 19,000; N 23,000; O 16,000**

Activity 3: Models

- With a partner, choose one of the 15 buildings in the map as the basis for your three-dimensional model.
- Use your answers from Activities 1 and 2 to help you determine the amount of material you will need to build your model.
- Do research to find more images of your building and to check your measurements so that your model will look like the actual building.
- Decide with your partner what you will use to build the model, and then build it according to the scale that your class and your teacher determine will work best.
- Take the time to work with your partner to build the model to scale and decorate it to look like the actual building.

Chapter 7 Project: Miami Models (continued)

Name	Label	Height
Bank of America Tower	D	625 ft
Epic	G	500 ft
Espirito Santo Plaza	M	487 ft
Four Seasons Hotel & Tower	N	789 ft
Icon Brickell North Tower	H	586 ft
Icon Brickell South Tower	I	586 ft
Infinity at Brickell	O	630 ft
Jade at Brickell Bay	L	528 ft
Miami Center	F	484 ft
Mint at Riverfront	A	631 ft
Plaza on Brickell Tower I	K	610 ft
Plaza on Brickell Tower II	J	525 ft
The Ivy	B	512 ft
Wachovia Financial Center	E	764 ft
Wind	C	501 ft



Finishing the Project

Write how you found the dimensions you used for this project, and provide a copy of any images (or tell exactly where you found the images) that you used, other than the one on this page.

Reflect and Revise

Measure your model carefully and make sure that it matches the measurements you intended. Make your model as sturdy as possible.

Extending the Project

Find another building in Miami (or somewhere else) that you would like to model. Research its dimensions and its appearance. Build a model using the same scale your class is using for the Miami buildings.

Name ______________________________ Class ____________ Date ____________

Chapter 7 Project Manager: Miami Models

Getting Started

Read about the project. As you work on it, you will need a calculator, a ruler, and building materials for your model. Keep all of your work for the project together along with this Project Manager.

Checklist	**Suggestions**
☐ Activity 1: Height	☐ Measure and draw carefully.
☐ Activity 2: Area	☐ Estimate area using triangles and rectangles.
☐ Activity 3: Modeling	☐ Use sturdy building materials.

Scoring Rubric

4 All elements of the project are clearly and accurately presented. Your models are well constructed and your explanations are clear and use geometric language appropriately.

3 Your models and estimations are adequate. Some elements of the project are unclear or inaccurate.

2 Significant portions of the project are unclear or inaccurate.

1 Major elements of the project are incomplete or missing.

0 Project is not handed in or shows no effort.

Your Evaluation of Project Evaluate your work, based on the *Scoring Rubric.*

Teacher's Evaluation of Project