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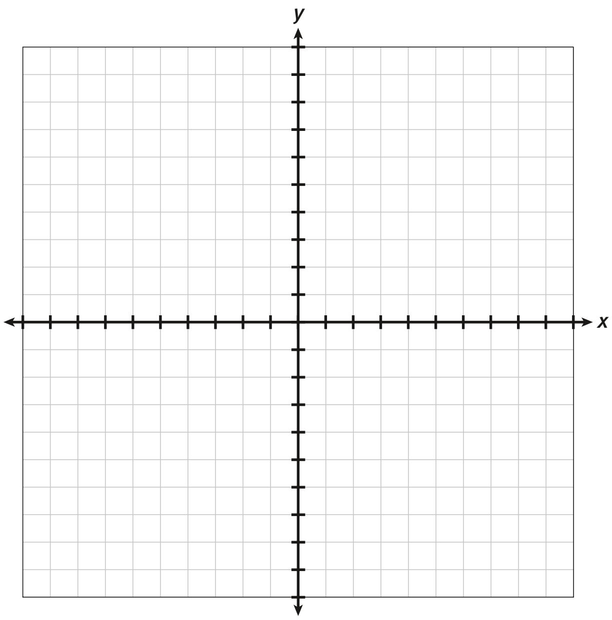
Precalculus Unit 5 Practice

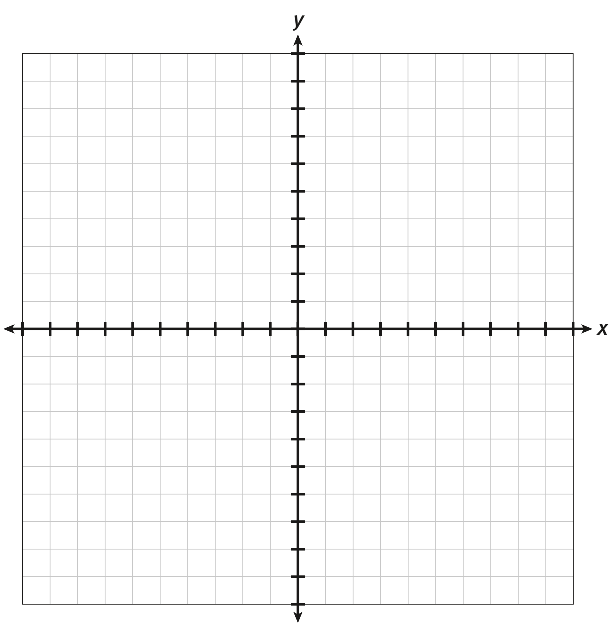
LESSON 26-1

For Items 1 and 2, graph each parabola. State the vertex and axis of symmetry.

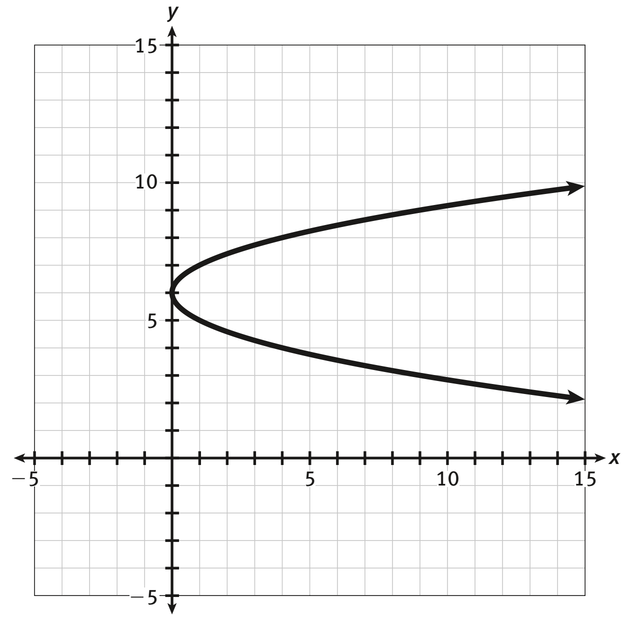
1. $y = (x + 1)^2 - 3$



2. $x = y^2 - 5$



3. **Make use of structure.** Consider the graph of the parabola shown below.



   a. Graph its inverse on the same graph of the given parabola.

   b. Write the equation of the inverse parabola and state the vertex and axis of symmetry.

   c. **Construct viable arguments.** Is the inverse a function? Explain.

4. What are the restrictions on the domain and range of the inverse of the function $y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$?

5. Consider Item 35 from Lesson 26-1. If a sign was placed on the directrix of the parabola, which of the following could be the coordinates of the sign post?

A. (0, 42)

B. (0, 50)

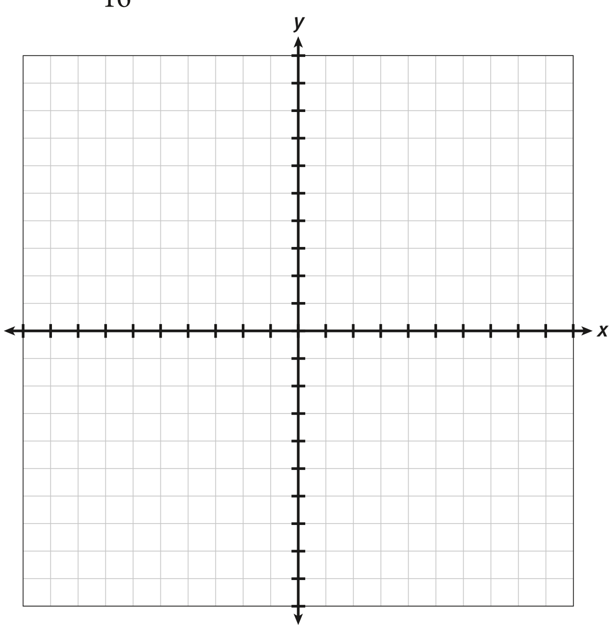
C. (58, 0)

D. (0, 58)

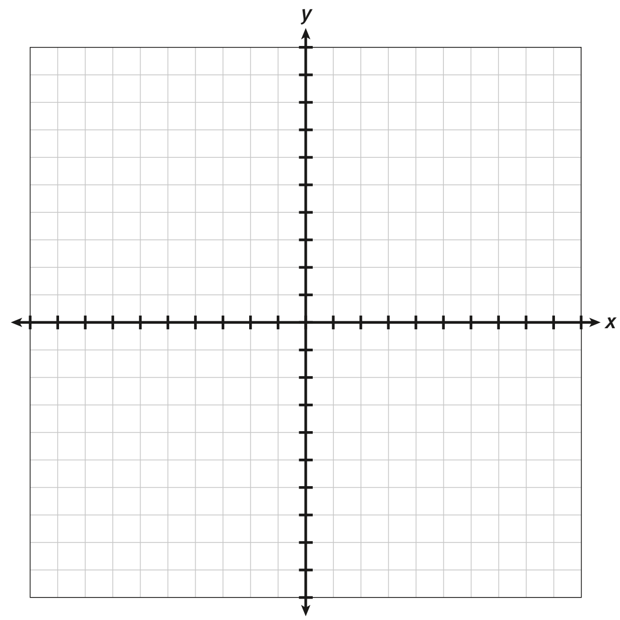
LESSON 26-2

Make use of structure. For Items 6 and 7, graph the parabola. State the vertex, axis of symmetry, focus, and directrix.

6. $y - 1 = \frac{1}{16}(x + 1)^2$



7. $x + 2 = \frac{1}{20}(y - 5)^2$



Attend to precision. For Items 8 and 9, find the standard form, vertex, focus, directrix, and axis of symmetry for the following parabolas.

8. $2y = -x^2$

9. $16x - 32 = 6(y + 9)^2$

10. Consider Item 14 from Lesson 26-2. What is the distance between the focus and directrix of the parabola (through the vertex)?

A. 4 cm

B. 40 cm

C. 78 cm

D. 156 cm

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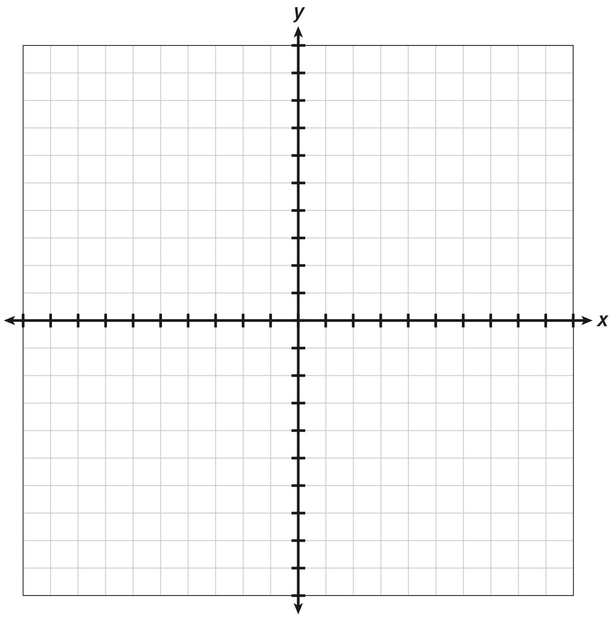
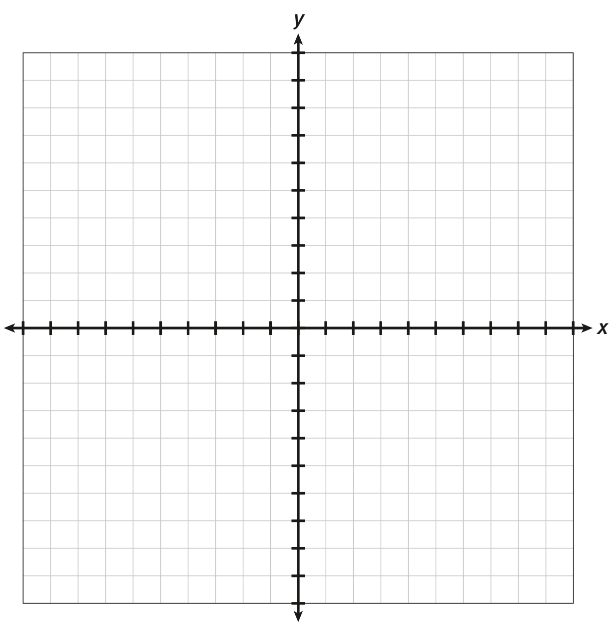
LESSON 27-1

Attend to precision. For Items 11–13, sketch the graph of each ellipse. Label the coordinates of the center, the foci, and the endpoints of the major and minor axes.

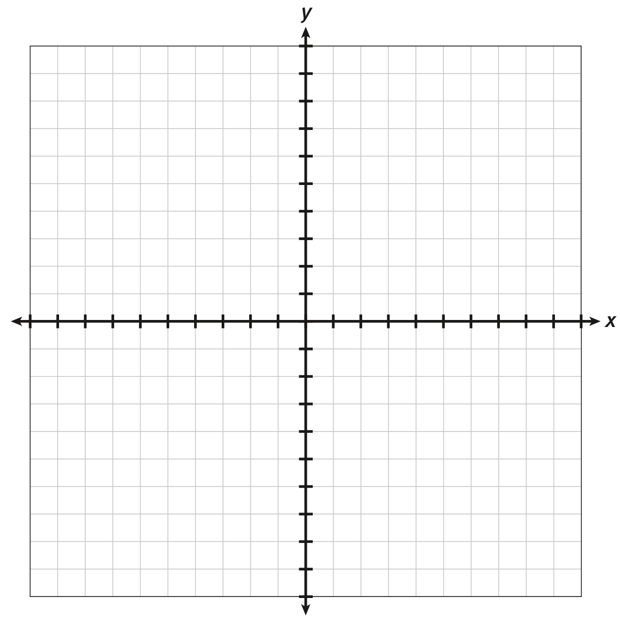
11. $\dfrac{(x+1)^2}{9} + \dfrac{y^2}{4} = 1$



12. $\dfrac{(x-2)^2}{16} + \dfrac{(y+3)^2}{25} = 1$



13. $5x^2 + 10x + 6y^2 - 48y + 71 = 0$



14. Given the standard form of the equation of an ellipse, $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, which condition needs to be true in order for the ellipse to be a circle?

A. $a > b$

B. $a = b$

C. $a = b = 0$

D. $a < b$

15. **Model with mathematics.** Refer to Item 19 in Lesson 27-1. Suppose a replica was created using the elliptical equation $\dfrac{x^2}{1000} + \dfrac{y^2}{625} = 1$, which condition needs to be true in order for the ellipse to be a circle?

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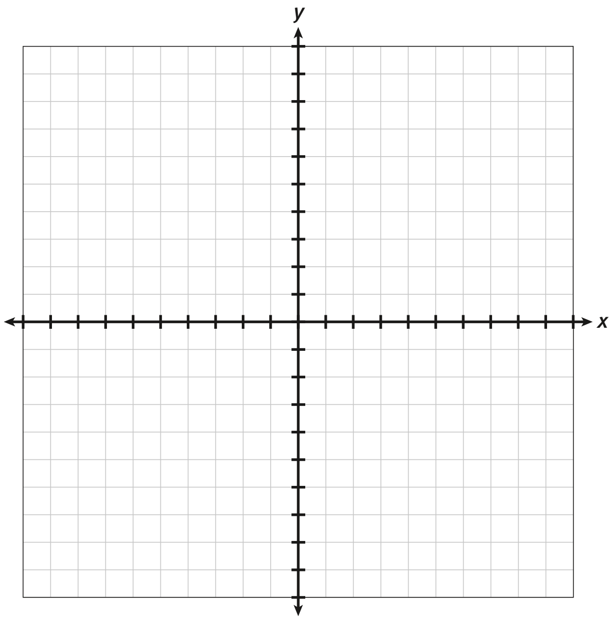
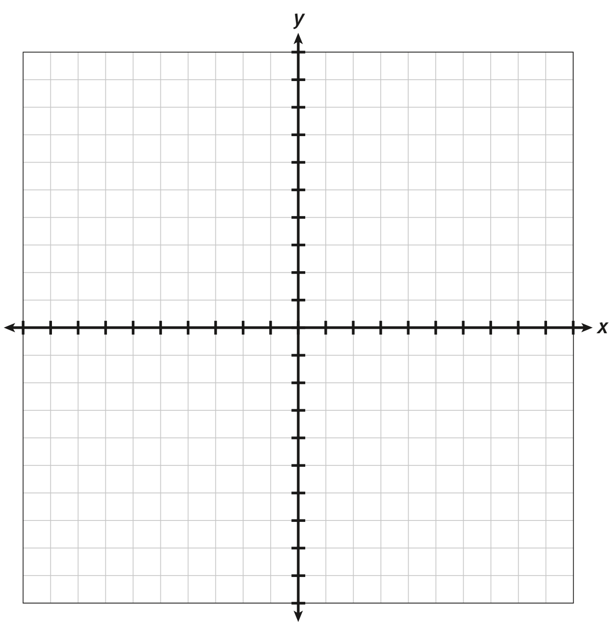
LESSON 27-2

Attend to precision. For Items 16–18, sketch the graph of each hyperbola. Label the coordinates of the center, the foci, the vertices, and the equations of the asymptotes.

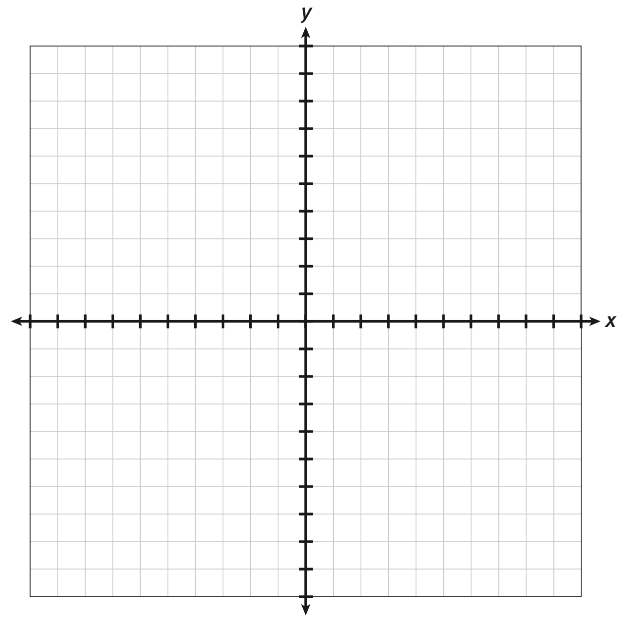
16. $x^2 - (y - 1)^2 = 1$



17. $\dfrac{(x-5)^2}{25} - \dfrac{(y+7)^2}{16} = 1$



18. $y^2 + 6y - x^2 + 8x - 3144 = 0$



19. Compare and contrast the standard forms for an ellipse and hyperbola.

20. Which of the following equations has a graph with a vertical transverse axis of 10?

A. $\dfrac{x^2}{25} - \dfrac{y^2}{100} = 1$

B. $\dfrac{x^2}{100} - \dfrac{y^2}{25} = 1$

C. $\dfrac{y^2}{25} - \dfrac{x^2}{100} = 1$

D. $\dfrac{y^2}{100} - \dfrac{x^2}{25} = 1$

LESSON 27-3

Model with mathematics. Refer to the LORAN model in Lesson 27-3. Suppose three different stations were located at $A(-80, -70)$, $B(-80, 60)$, and $C(-30, 60)$. The time difference for the radio signals transmitted by stations A and B is 1011.04 microseconds. The time difference for the radio signals transmitted by stations B and C is 183.11 microseconds. An airplane sends out a distress signal, indicating that it is northeast of the airport located at $(-50, 0)$.

21. Use the speed of light to convert the time difference of the signals for stations A and B, then for B and C, into a distance that denotes the difference in distance that the two stations are from the distressed airplane. Round to the nearest whole number.

22. The airplane lies on a hyperbola that contains the foci A and B. Write the equation for the hyperbola having foci at stations A and B that contains the distressed airplane. Round to the nearest whole number.

23. The airplane lies on a hyperbola that contains the foci B and C. Write the equation for the hyperbola having foci at stations B and C that contains the distressed airplane. Round to the nearest whole number.

24. Use a graphing calculator to find the approximate location of the airplane.

A. $(-37, 70)$

B. $(13, 70)$

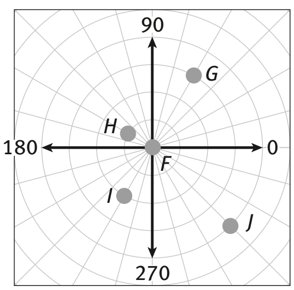
C. $(-50, 70)$

D. $(-80, 45)$

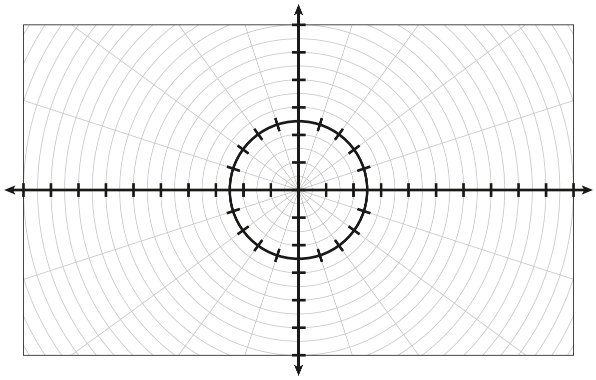
25. **Construct viable arguments.** The LORAN method can be an exact science. Describe some of the factors used in this problem that could contribute to inaccuracy.

LESSON 28-1

26. **Make use of structure.** Name the polar coordinates of each point on the given polar grid. Each concentric circle increases by 1 unit.



Attend to precision. For Items 27–29, plot and label the points given by the polar coordinates on the given polar grid below.



27. $K(-4, 165°)$

28. $L(3, 210°)$

29. $M(0, 360°)$

30. Which of the following points is equivalent to $(-3, 165°)$?

A. $(3, 345°)$

B. $(3, 195°)$

C. $(3, 255°)$

D. $(3, 15°)$

LESSON 28-2

31. **Attend to precision.** What are the rectangular coordinates of $(-4, 210°)$?

A. $(-3.94, -0.696)$

B. $(-3.94, 0.696)$

C. $(3.94, -0.696)$

D. $(3.94, 0.696)$

For Items 32 and 33, determine the rectangular coordinates of each point. Round to three decimal places.

32. $(11, 60°)$

33. $(-1, -285°)$

34. **Make use of structure.** Determine the polar coordinates of $(6, -15)$. Round to three decimal places.

35. A surveyor sights an object from her location in the direction of $(-133, -175)$. Determine the polar coordinates of the object.

LESSON 28-3

36. What are the polar coordinates of $(-5, 2)$?

A. $(-5.385, 158.199°)$

B. $(5.385, 68.199°)$

C. $(5.385, 158.199°)$

D. $(-5.385, 21.801°)$

37. **Use appropriate tools strategically.** Use a graphing calculator to convert $(-7, -13)$ to rectangular polar coordinates. Round each coordinate to three decimal places.

38. **Make use of structure.** Use the figure of the unit circle to find the value of each of the following. Transform the equation of the polar curve given by $r = -3 \cos \theta$ by writing it as an equation using the rectangular variables x and y.

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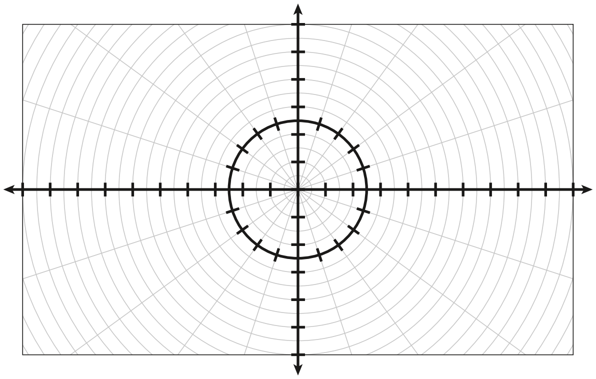
39. Graph the equation $r = -4 \sin \theta$ in polar coordinates. Then convert the equation to rectangular form.



40. A player is 3 yards to the west and 25 yards to the north of the quarterback. Given that the quarterback is at the origin, express the player's position in rectangular coordinates and in polar coordinates, rounded to the nearest tenth.

LESSON 29-1

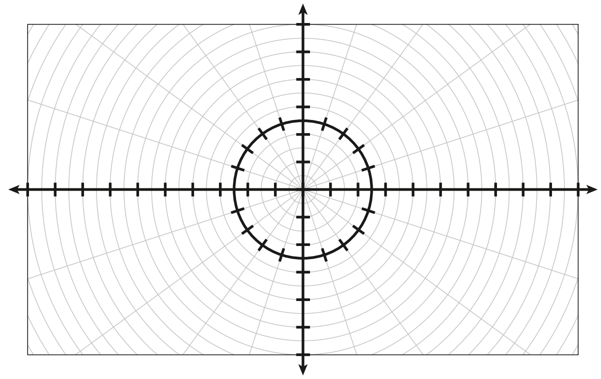
Make use of structure. List the characteristics of the graph and write the rectangular form for each polar equation.

41. $r = 9$

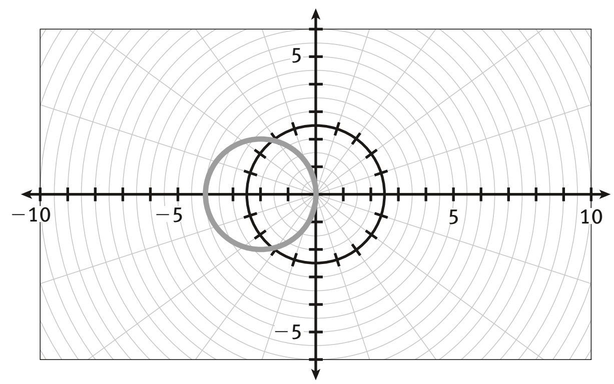
42. $\theta = 255°$

43. $\theta = 300°$

44. Sketch the graph of the equation $r = 7 \sin \theta$ on a rectangular coordinate plane. Then write the rectangular equation of the circle.



45. Attend to precision. Write the polar equation of the circle shown on the graph. Then write the rectangular equation of the circle.



LESSON 29-2

Make use of structure. For Items 46–47, list the characteristics of the graph and write the rectangular form for each polar equation.

46. $r = \dfrac{3}{-4 \cos \theta + 7 \sin \theta}$

47. $r = 5 \cos \theta + 6 \sin \theta$

For Items 48–49, convert each rectangular equation to polar form.

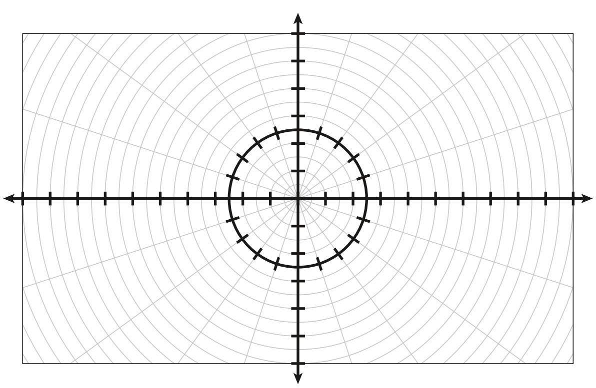
48. $y = -15$

49. $3x^2 + y^2 + 4x = 0$

50. a. Model with mathematics. Make a table of values for the polar equation $r = 5\cos(2\theta)$. Round to the nearest tenth.

θ	r
0°	
15°	
30°	
45°	
60°	
75°	
90°	
105°	
120°	
135°	
150°	
165°	
180°	

b. Then graph the equation on a polar grid.



LESSON 29-3

Use appropriate tools strategically. For Items 51–52, describe the symmetry of the graph of each polar equation.

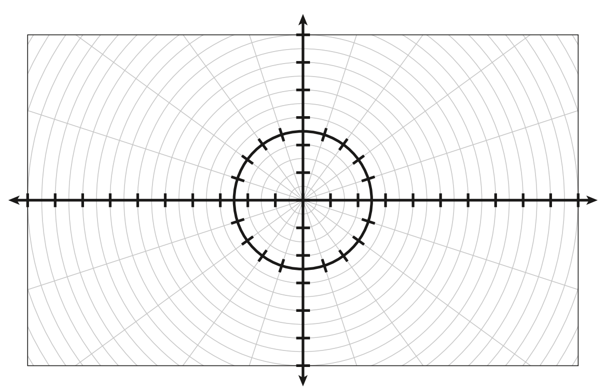
51. $r = -4\sin(5\theta)$

52. $r = 3\cos(6\theta)$

53. Write a polar equation for a rose curve that is symmetric to the y-axis and has 3 petals, each with length of 1 unit.

54. Use a graphing calculator to graph the polar equation $r = 4 + 5\sin\theta$. Find an interval for θ for which the graph is traced only once.

55. Make use of structure. Without using a graphing calculator, sketch the graph of the polar equation $r = -2\sin(7\theta)$.



LESSON 30-1

Model with mathematics. An air traffic controller notices two planes at the same altitude headed towards each other. Plane A moves on the radar screen from (10, 6) to (30, 16) in 1 second. Plane B moves from (40, 340) to (50, 300) in 1 second. The controller assumes that they will move at constant speeds along linear paths.

56. Write linear equations that model the paths.

57. What is the approximate point of intersection?

A. (100, 50)

B. (110, 56)

C. (130, 60)

D. (157, 71)

58. **Attend to precision.** Determine the speed of both planes in units per second. Round to the nearest whole number.

59. **a.** Make tables of values that show the east-west position (x) and north-south position (y) of Plane A and Plane B at time $t = 0, 1, 2, 3, 4,$ and 5 seconds.

Plane A		
t (in seconds)	East-West (x)	North-South (y)
0		
1		
2		
3		
4		
5		

Plane B		
t (in seconds)	East-West (x)	North-South (y)
0		
1		
2		
3		
4		
5		

b. Write rules for $x(t)$ and $y(t)$ to model the east-west and north-south positions of Planes A and B as functions of time.

60. Should the air traffic controller be concerned about a mid-air collision? Explain.

LESSON 30-2

Model with mathematics. Jacob is flying a model airplane and Jackson is flying a model helicopter on a field. Let $t = 0$ correspond to the number of seconds from the beginning of flight and suppose Jacob positions his plane according to the parametric equations $x(t) = 3t$ and $y(t) = 9t + 14$. Jackson's helicopter position is given by $x(t) = 2t$ and $y(t) = 4t + 50$.

61. Write linear equations in terms of x and y that model the path of both aircraft.

62. Find the point at which both aircraft may collide.

A. (14, 50)

B. (36, 122)

C. (2, 54)

D. (12, 14)

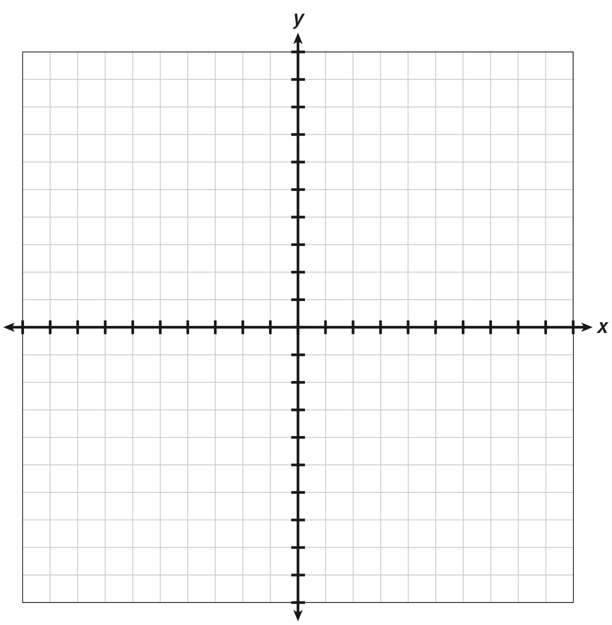
63. Find the amount of time it will take Jacob's model to reach the point of intersection and the time it will take Jackson's model to reach that point.

64. If Jackson's helicopter will run out of fuel after 60 seconds, is there still a risk that it might collide with Jacob's plane? Explain.

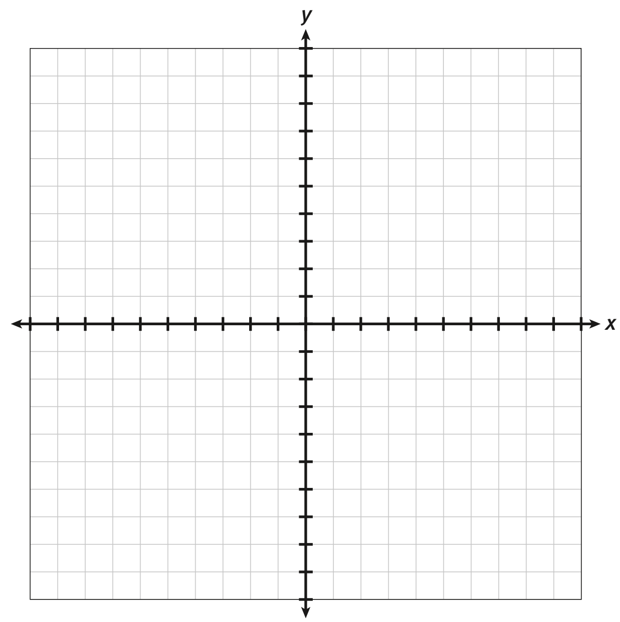
65. **Use appropriate tools strategically.** Use a graphing calculator in parametric mode to find the difference between the y-positions of both aircraft at 12 seconds of flight.

LESSON 30-3

66. **Model with mathematics.** The parametric equations $x(t) = 3t + 3$ and $y(t) = (3t - 2)^2 + 2$ model the position of a weather balloon on a rectangular grid in units of miles for the domain of $0 \le t \le 4$, where t is time in hours. Convert the parametric equations into a rectangular relation that represents the balloon's path. State the domain of the rectangular relation. Then graph the rectangular relation.



67. The parametric equations $x(t) = -5t$ and $y(t) = 7t - 1$ model the position of a girl running a football on a rectangular grid in units of yards for the domain of $0 \le t \le 2$, where t is time in seconds. Convert the parametric equations into a rectangular relation that represents the girl's path. State the domain of the rectangular relation. Then graph the rectangular relation.



68. The parametric equations $x(t) = \frac{1}{3}t^2$ and $y(t) = t^2 + \frac{1}{2}$ model the position of an aircraft on a rectangular grid in units of miles for the domain of $0 \le t \le 10$, where t is time in seconds. Which of the following is the domain of the rectangular relation?

A. $0 \le x \le 10$

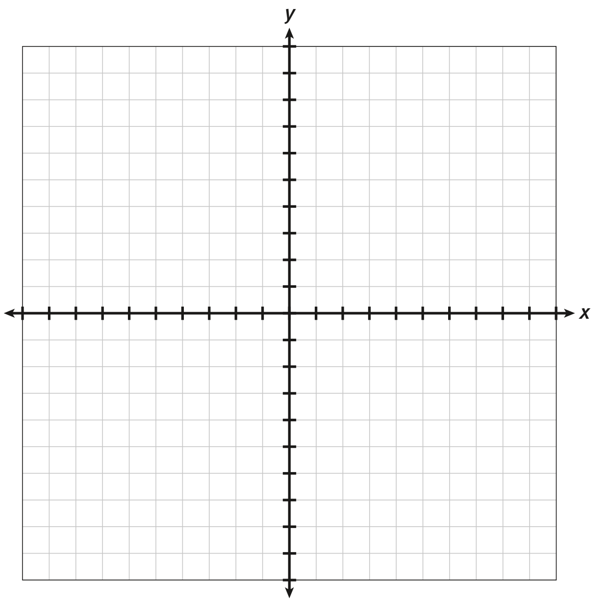
B. $0 \le x \le 100\frac{1}{2}$

C. $0 \le x \le 33\frac{1}{3}$

D. $\frac{1}{2} \le x \le 100\frac{1}{2}$

69. Make use of structure. Convert the parametric equations into a rectangular relation that represents the plane's path.

70. Graph the rectangular relation.



LESSON 31-1

Model with mathematics. A Ferris wheel at a traveling carnival has a radius of 23 meters and makes one revolution every 2 minutes.

71. Compute the angular and linear velocities of a rider on the rim of the Ferris wheel.

72. Locate the center of the Ferris wheel at the origin of a coordinate plane in units of meters, and let the position of a rider on the Ferris wheel be represented by a point (x, y). Which of the following is an equation for the rider's circular path as the Ferris wheel turns in terms of x and y?

A. $x^2 + y^2 = 46$

B. $x^2 + y^2 = 23$

C. $x^2 + y^2 = 46^2$

D. $x^2 + y^2 = 23^2$

73. When the x-coordinate for the location of the rider is 15, what are the possible y-coordinates?

Make use of structure. Suppose there is an inner circle of light bulbs 5 meters from the center of the Ferris wheel.

74. What are the angular and linear velocities of the light bulb as the Ferris wheel rotates?

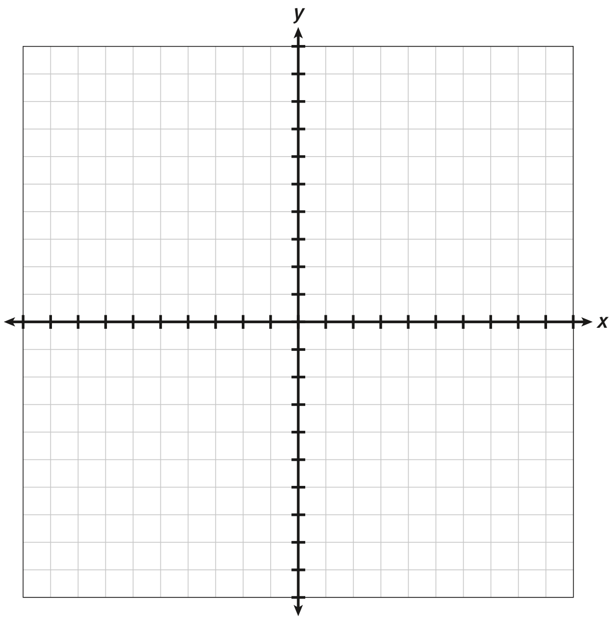
75. How does the linear velocity of the light bulb compare to the linear velocitiy of a rider on the Ferris wheel?

LESSON 31-2

A Ferris wheel at a traveling carnival has a radius of 23 meters and makes one revolution every 2 minutes. Assume that the wheel rotates counterclockwise.

76. Model with mathematics. Write the parametric equations that would model the motion of a rider that starts at the bottom of the wheel at time $t = 0$ at the point $(0, 0)$.

77. Use appropriate tools strategically. Graph the parametric equations on a graphing calculator. Then sketch a graph of the results. Indicate the direction of motion and the starting position of the rider.



78. In the first revolution, when will the rider be 40 meters off the ground?

A. $t = 0.7$

B. $t = 0.77$

C. $t = 40$

D. $t = 44$

79. What are the coordinates of the rider's position at 1.5 minutes?

80. What would the parametric equations be if the wheel rotated clockwise instead of counterclockwise?

LESSON 31-3

Players are shooting giant marbles on a giant game board from different points.

81. Model with mathematics. Let $x_0 = 0$. Write parametric equations to represent the path of the marble shot onto the game board by two different players given the following conditions.

Player A: $v_0 = 5$ ft/s, $\theta = 42°$, $y_0 = 101$ ft

Player B: $v_0 = 7.6$ ft/s, $\theta = 29°$, $y_0 = 80$ ft

82. If a marble lands in a hole at the position close to (8, 29), then the player receives a bonus score. Which player, if any, will get the bonus score and at what time?

83. Which conic section does $x(t) = 4t^2 + 5$, $y(t) = \frac{1}{2}t$ represent?

A. parabola

B. hyperbola

C. ellipse

D. none of the above

Use this information to for Items 84–85. The equations $x(t) = t - 1$ and $y(t) = 2t + 3$ model the path of one ship, and the equations $x(t) = 4t$ and $y(t) = 6t - 1$ model the path of a second ship for $t \geq 0$.

84. Make use of structure. Write the paths for each ship as a rectangular relation.

85. If the ships launch from their positions on the y-axis, will they ever collide? Explain.

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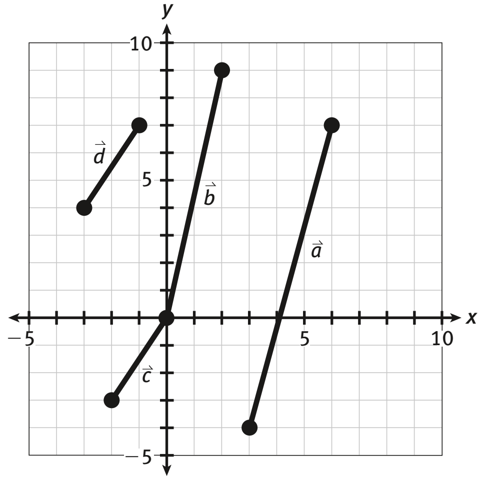
LESSON 32-1

Make use of structure. Use the following graph for Items 86 and 87.



86. Are $\vec{c}$ and $\vec{d}$ equal? Explain.

87. Which of the following is the component form of $\vec{a}$?

A. $\langle 3, -4 \rangle$

B. $\langle 6, 7 \rangle$

C. $\langle 3, 11 \rangle$

D. $\langle 3, 3 \rangle$

88. **Attend to precision.** What is the magnitude of $\vec{b}$?

Use this information for Items 89–90. Biologists continue to track the leopard using GPS technology from Item 10 in Lesson 32-1. The next day at 2:00 p.m., the leopard's coordinates are (15, 6). Let $\vec{q}$ represent the change in position of the leopard on the second day from its last known position on the first day.

89. **a.** What is the component form of $\vec{q}$?

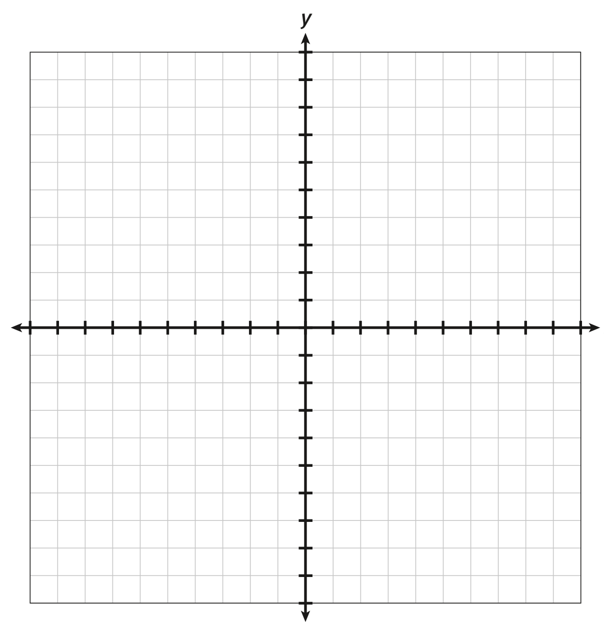
b. What is the magnitude of $\vec{q}$?

90. How do the component form and magnitude of $\vec{q}$ compare to those of $\vec{p}$?

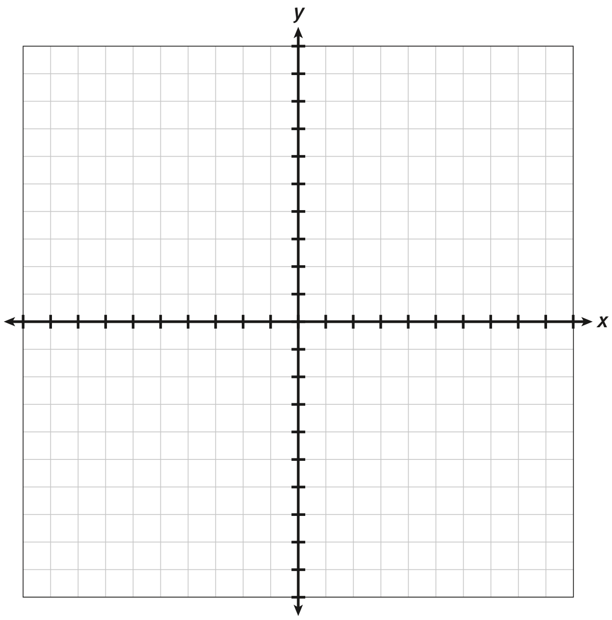
LESSON 32-2

Express regularity in repeated reasoning. Let $\vec{r} = \langle -1, -4 \rangle$ and $\vec{s} = \langle 3, 5 \rangle$. For Items 91–92, find each vector both graphically and symbolically.

91. $\vec{r} + \vec{s}$



92. $2\vec{r} - \vec{s}$



93. Which of the following is approximately $\left|3\vec{s} - 5\vec{r}\right|$?

A. 40

B. 4

C. 19

D. 1700

Model with mathematics. Use this information for Items 94–95. A coordinate grid is given in units of miles. Sunny's change in position on the grid during the first hour of a hike can be represented by $\vec{s} = \langle 4, 2 \rangle$. Her change in position during the second hour of the hike can be represented by $\vec{t} = \langle -3, 5 \rangle$.

94. a. What vector represents Sunny's overall change in position during the 2-hour hike?

b. Sunny plans to walk directly back to her starting position. To the nearest tenth of a mile, how far will she walk on the return trip?

95. How does Sunny's walk compare to Henry's from Item 20 in Lesson 32-2?

LESSON 32-3

Express regularity in repeated reasoning. For Items 96–97, compute the magnitude and direction of each vector. Let $\vec{j} = \langle -5, -8 \rangle$ and $\vec{k} = \langle 4, -6 \rangle$.

96. $\vec{j}$ and $\vec{k}$

97. $\vec{j} + \vec{k}$ and $\vec{j} - \vec{k}$

98. Which is the closest resolution to the components of a vector with a magnitude of 12 and direction of 300°?

A. (12, −15)

B. (6, −10)

C. (72, −125)

D. (−89, −144)

Attend to precision. Luciano leaves the cabin and drives a snowmobile 135° north of west for 2.1 km. Then he turns and drives 25°south of east for 4 km.

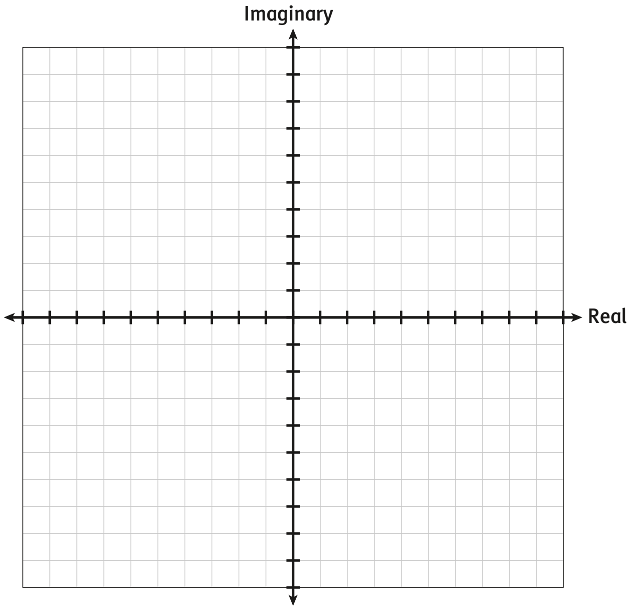
99. If Luciano turns and drives straight back to the cabin, what direction should he head? Round to the nearest tenth of a degree.

100. If Luciano averages 25 km/h on his return trip, how long will the return trip take? Round to the nearest minute.

LESSON 32-4

101. Make use of structure. Let $z = 2 + 7i$.

a. Draw a vector that represents z on the complex plane.



b. Determine arg(z).

c. Calculate $|z|$.

102. Given that $z = 4 - 11i$ and $w = -5 - 2i$, find each of the following.

a. $z + w$

b. $w - z$

c. wz

d. $\frac{z}{w}$

103. Let $a = -1 + 3i$ and $b = 9 - 10i$. What is the distance between a and b?

A. $\sqrt{5}$

B. $\sqrt{181}$

C. $\sqrt{269}$

D. $\sqrt{981}$

104. Let $f = \frac{1}{2} + \frac{\sqrt{3}}{2}i$.

a. Calculate $|f|$ and arg(f).

b. Determine f^2 and f^3.

c. Describe the geometric pattern that results from repeated multiplication of $f = \frac{1}{2} + \frac{\sqrt{3}}{2}i$.

105. Reason abstractly. What is the argument of $\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$? Explain your answer.

LESSON 32-5

Make use of structure. For Items 106–107, write each complex number in polar form.

106. $-3 + 19i$

107. $-1 - 10i$

108. Which of the following is the product of $2(\cos 25° + i \sin 25°) \cdot 5(\cos 45° + i \sin 45°)$ in polar form?

A. $7(\cos 20° + i \sin 20°)$

B. $7(\cos 70° + i \sin 70°)$

C. $10(\cos 70° + i \sin 70°)$

D. $10(\cos 20° + i \sin 20°)$

Express regularity in repeated reasoning. For Items 109–110, write each complex number in rectangular form.

109. $4(\cos 315° + i \sin 315°)$

110. $16(\cos 150° + i \sin 150°)$

LESSON 33-1

Jimmy moves across a 75-ft tightrope strung horizontally with the ground. The audience watches him walk from left to right at a steady pace of 0.4 ft/s. Let t represent the number of seconds since he stepped on the tightrope.

111. How long does it take him to get to the middle of the tightrope?

A. 93.75 seconds

B. 187.5 seconds

C. 30 seconds

D. 15 seconds

112. Model with mathematics. Write a vector for his velocity in feet per second.

113. When Jimmy reaches the middle of the rope he stops moving, and his partner Seth starts from the right side and walks toward him. Seth reaches Jimmy in 2 minutes. Write a vector for Seth's velocity in feet per second.

Make use of structure. Use the following information for Items 114 and 115.
Dave is hitting a pool ball. A wind is moving east to west to left at 3 ft/s. Dave is hitting the ball from south to north at a speed of 5 ft/s. Let t equal the number of seconds since Dave hits the ball. Assume he starts at (0, 0).

114. Write parametric equations that represent the motion of Dave's ball.

115. Write the component form of a vector that represents the position of Dave's pool ball compared to (0, 0) 2 seconds after the ball is hit.

LESSON 33-2

The vector $\vec{a} = \langle 4, 9 \rangle t + \langle 120, -10 \rangle$ represents the path of paper Plane A. The vector $\vec{b} = \langle -2, 5 \rangle t + \langle -30, 110 \rangle$ represents the path of paper Plane B. The positions of both planes are in time t hours on a coordinate grid in units of miles, where the positive y-axis points north.

116. Make sense of problems. What is Plane A's velocity vector? What is its speed to the nearest mile per hour?

117. What is Plane B's velocity vector? What is its speed to the nearest mile per hour?

118. What are the coordinates of Plane A's position at 1 minute?

A. (124, −1)

B. (120.067, −9.85)

C. (360, 530)

D. (120.033, −9.9925)

119. Write the component form of the displacement vector that describes Plane B's change in position from 0 to 1 minute.

120. If a paper plane could float indefinitely, how long would it take Plane B to reach (0, 0)?