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Study Guide

Student Edition
Pages 6–11

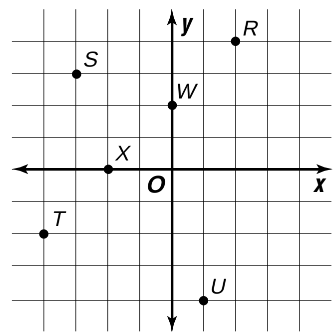
Integration: Algebra
The Coordinate Plane

Every point in the coordinate plane can be denoted by an ordered pair consisting of two numbers. The first number is the ***x*-coordinate**, and the second number is the ***y*-coordinate**.

> To determine the coordinates for a point, follow these steps.
> **1.** Start at the origin and count the number of units to the right or left of the origin. The *positive direction* is to the right, and the *negative direction* is to the left.
> **2.** Then count the number of units up or down. The positive direction is up, and the negative direction is down.
> *Note:* If you do not move either right or left, the *x*-coordinate is 0. If you do not move up or down, the *y*-coordinate is 0.

Example: Write the ordered pair for each point shown at the right.

The ordered pair for R is (2, 4).
The ordered pair for S is (−3, 3).
The ordered pair for T is (−4, −2).
The ordered pair for U is (1, −4).
The ordered pair for W is (0, 2).
The ordered pair for X is (−2, 0).

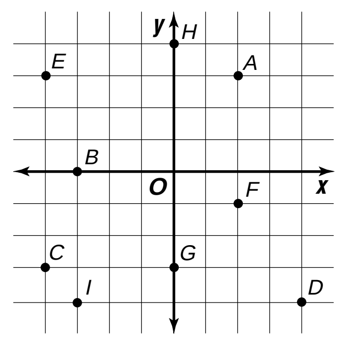


Write the ordered pair for each point shown at the right.

1. A **(2, 3)** **2.** B **(−3, 0)** **3.** C **(−4, −3)**

4. D **(4, −4)** **5.** E **(−4, 3)** **6.** F **(2, −1)**

7. G **(0, −3)** **8.** H **(0, 4)** **9.** I **(−3, −4)**



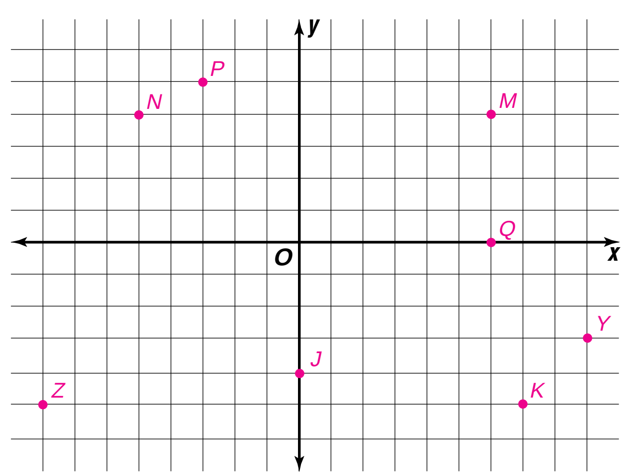
Graph each point on the coordinate plane.

10. $M(6, 4)$ **11.** $N(-5, 4)$

12. $P(-3, 5)$ **13.** $Q(6, 0)$

14. $J(0, -4)$ **15.** $K(7, -5)$

16. $Y(9, -3)$ **17.** $Z(-8, -5)$

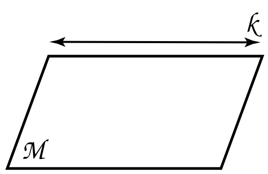


Points, Lines, and Planes

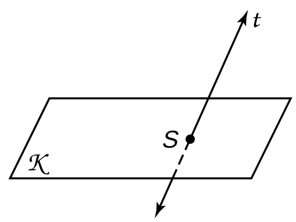
Points, lines, and planes can be related in many different ways. Figures can be used to show these relationships. When two figures have one or more points in common, the figures are said to **intersect**. When points lie on the same line, the points are said to be **collinear**. When points lie in the same plane, the points are said to be **coplanar**.

Example: Draw and label a figure for each relationship.

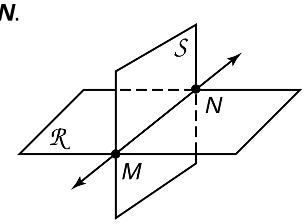
Line k does not intersect plane $\mathcal{M}$.



Line t intersects plane $\mathcal{K}$ at point ***S***.

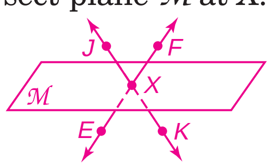


Planes $\mathcal{R}$ and $\mathcal{S}$ intersect in line ***MN***.

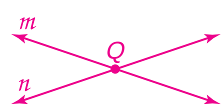


Draw and label a figure for each relationship.

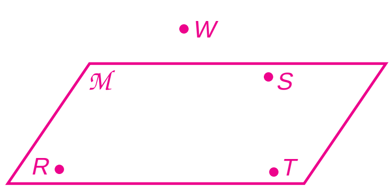
1. Lines JK and EF are not in plane $\mathcal{M}$, but intersect plane $\mathcal{M}$ at X.



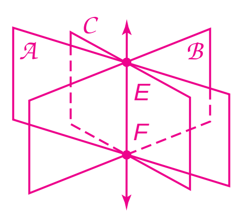
2. Lines m and n intersect at point Q.



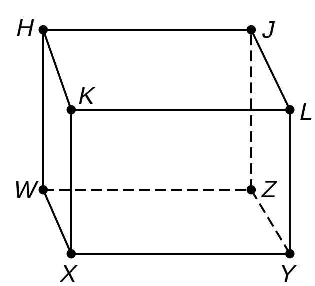
3. Points R, S, and T are in plane $\mathcal{M}$, but point W does not lie in plane $\mathcal{M}$.



4. The intersection of planes $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ is line EF.



Refer to the figure at the right to answer each question.



5. Are points H, J, K, and L coplanar? **yes**

6. Name three lines that intersect at X. $\overleftrightarrow{WX}$, $\overleftrightarrow{KX}$, $\overleftrightarrow{XY}$

7. What points do plane $WXYZ$ and HW have in common? **W**

8. Are points W, X, and Y collinear? **no**

9. List the possibilities for naming a line contained in plane $WXKH$. $\overleftrightarrow{HK}$, $\overleftrightarrow{KH}$, $\overleftrightarrow{HW}$, $\overleftrightarrow{WH}$, $\overleftrightarrow{WX}$, $\overleftrightarrow{XW}$, $\overleftrightarrow{XK}$, $\overleftrightarrow{KX}$, $\overleftrightarrow{KW}$, $\overleftrightarrow{WK}$, $\overleftrightarrow{HX}$, $\overleftrightarrow{XH}$

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Study Guide

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Integration: Algebra
Using Formulas

The following four-step plan can be used to solve any problem.

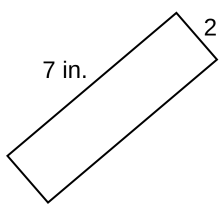
Problem-Solving Plan	
1. *Explore* the problem.	Identify what you want to know.
2. *Plan* the solution.	Choose a strategy.
3. *Solve* the problem.	Use the strategy to solve the problem.
4. *Examine* the solution.	Check your answer.

When finding a solution, it may be necessary to use a formula. Two useful formulas are the area formula and perimeter formula for a rectangle.

Area of a Rectangle	The formula for the area of a rectangle is $A = \ell w$, where A represents the area expressed in square units, ℓ represents the length, and w represents the width.
Perimeter of a Rectangle	The formula for the perimeter of a rectangle is $P = 2\ell + 2w$, where P represents the perimeter, ℓ represents the length and w represents the width.

Examples

1 Find the perimeter and area of the rectangle at the right.



$P = 2\ell + 2w$
$= 2(7) + 2(2)$
$= 14 + 4$ or 18
The perimeter is 18 inches.

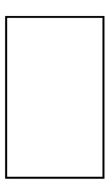
$A = \ell w$
$= 7 \cdot 2$ or 14
The area is 14 in^2.

2 Find the width of a rectangle whose area is 52 cm^2 and whose length is 13 cm.

$A = \ell w$
$\frac{52}{13} = \frac{13w}{13}$
$4 = w$
The width is 4 cm.

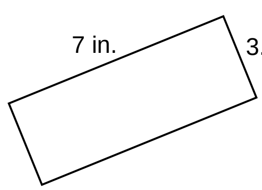
Find the perimeter and area of each rectangle.

1.



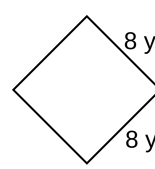
20 cm, 24 cm^2

2.



20.4 in., 22.4 in^2

3.



32 yd, 64 yd^2

Find the missing measure in each formula.

4. $\ell = 3, w = 7, P =$ __?__ 20

5. $w = 5.2, \ell = 6.5, A =$ __?__ 33.8

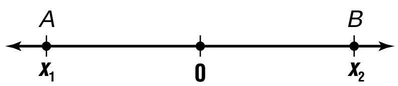
6. $w = 4, A = 36, \ell =$ __?__ 9

7. $P = 65, \ell = 18, w =$ __?__ 14.5

Measuring Segments

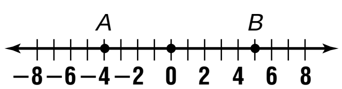
To find the distance between two points, there are two situations to consider.

Distance on a Number Line



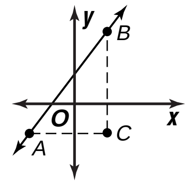
$AB = |x_2 - x_1|$

Example: Find AB on the number line shown below.



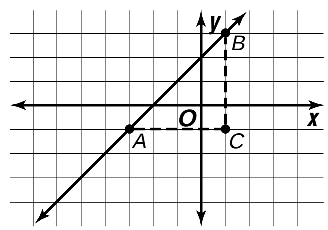
$$\begin{aligned} AB &= |5 - (^-4)| \\ &= |9| \\ &= 9 \end{aligned}$$

Distance in the Coordinate Plane



Pythagorean Theorem:
$(AB)^2 = (AC)^2 + (BC)^2$

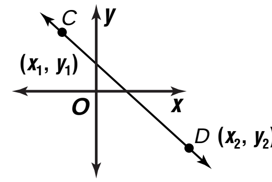
Example: Find the distance from $A(^-3, ^-1)$ to $B(1, 2)$ using the Pythagorean Theorem.



$AC = |1 - (^-3)|$ or 4
$BC = |2 - (^-1)|$ or 3

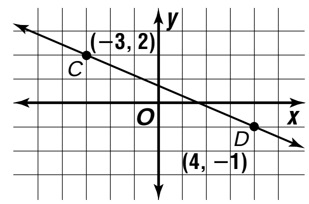
$$\begin{aligned} (AB)^2 &= 4^2 + 3^2 \\ &= 16 + 9 \text{ or } 25 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{25} \\ &= 5 \end{aligned}$$



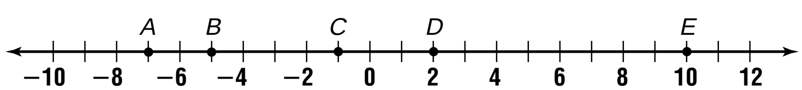
Distance Formula:
$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: Find the distance from $C(^-3, 2)$ to $D(4, ^-1)$ using the distance formula.



$$\begin{aligned} CD &= \sqrt{(^-3 - 4)^2 + [2 - (^-1)]^2} \\ &= \sqrt{(^-7)^2 + 3^2} \\ &= \sqrt{49 + 9} \\ &= \sqrt{58} \\ &\approx 7.62 \end{aligned}$$

Refer to the number line below to find each measure.



1. AC **6** **2.** BC **4** **3.** CD **3** **4.** AE **17**

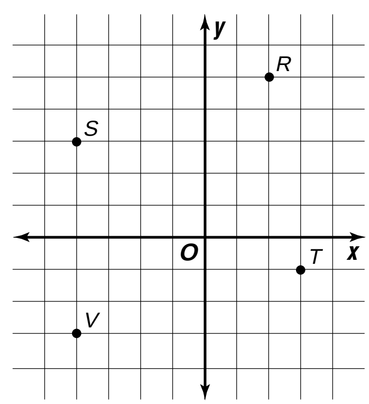
5. AB **2** **6.** DE **8** **7.** BE **15** **8.** CE **11**

Refer to the coordinate plane at the right to find each measure. Round your measures to the nearest hundredth.

9. RS **6.32** **10.** RT **6.08**

11. RV **10.00** **12.** VS **6.00**

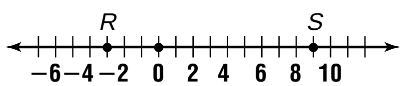
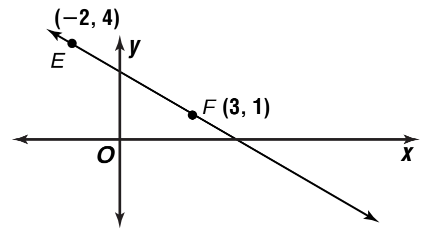
13. VT **7.28** **14.** ST **8.06**



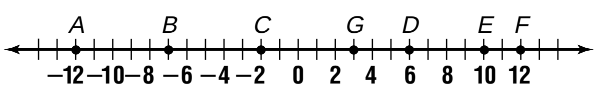
Study Guide

Midpoints and Segment Congruence

There are two situations in which you may need to find the midpoint of a segment.

Midpoint on a Number Line	Midpoint in the Coordinate Plane
The coordinate of the midpoint of a segment whose endpoints have coordinates a and b is $\frac{a+b}{2}$. ***Example:***  The coordinate of the midpoint of $\overline{RS}$ is $\frac{-3+9}{2}$ or 3.	The coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. ***Example:***  The coordinates of the midpoint of $\overline{EF}$ are $\left(\frac{-2+3}{2}, \frac{4+1}{2}\right)$ or $\left(\frac{1}{2}, \frac{5}{2}\right)$.

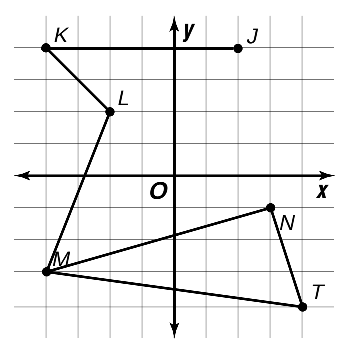
Use the number line below to find the coordinates of the midpoint of each segment.



1. $\overline{AB}$ $-\frac{19}{2}$ **2.** $\overline{BC}$ $-\frac{9}{2}$ **3.** $\overline{CE}$ 4 **4.** $\overline{DE}$ 8

5. $\overline{AE}$ −1 **6.** $\overline{FC}$ 5 **7.** $\overline{GE}$ $\frac{13}{2}$ **8.** $\overline{BF}$ $\frac{5}{2}$

Refer to the coordinate plane at the right to find the coordinates of the midpoint of each segment.



9. $\overline{JK}$ (−1, 4) **10.** $\overline{KL}$ (−3, 3)

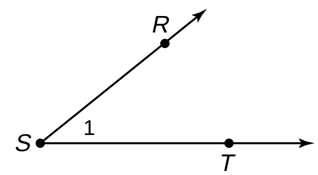
11. $\overline{LM}$ $\left(-3, -\frac{1}{2}\right)$ **12.** $\overline{MN}$ $\left(-\frac{1}{2}, -2\right)$

13. $\overline{NT}$ $\left(\frac{7}{2}, -\frac{5}{2}\right)$ **14.** $\overline{MT}$ $\left(0, -\frac{7}{2}\right)$

Study Guide

Exploring Angles

An angle is formed by two noncollinear rays with a common endpoint. You could name the angle in the figure at the right as $\angle S$, $\angle RST$, $\angle TSR$, or $\angle 1$.

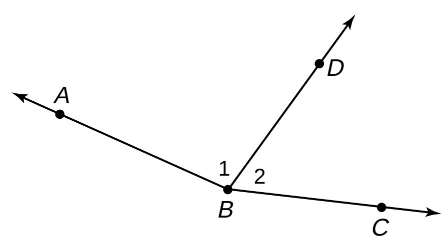


When two or more angles have a common vertex, you need to use either three letters or a number to name the angles. Make sure there is no doubt which angle your name describes.

A **right angle** is an angle whose measure is 90. Angles smaller than a right angle are **acute angles**. Angles larger than a right angle are **obtuse angles**. A **straight angle** has a measure of 180.

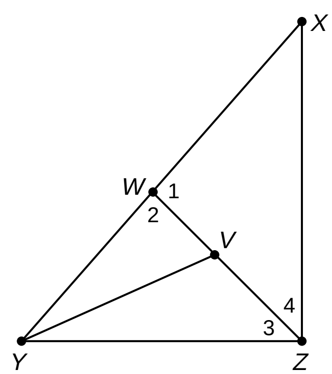
According to the Angle Addition Postulate, if D is in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$.

Example: In the figure at the right, $m\angle ABC = 160$, $m\angle 1 = x + 14$, and $m\angle 2 = 3x - 10$. Find the value of x.



$$m\angle 1 + m\angle 2 = m\angle ABC$$
$$(x + 14) + (3x - 10) = m\angle ABC$$
$$4x + 4 = 160$$
$$4x = 156$$
$$x = 39$$

For Exercises 1–5, refer to the figure at the right.



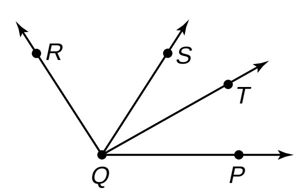
1. Do $\angle 3$ and $\angle Z$ name the same angle? Explain. **Not necessarily; *Z* is the vertex of three angles, ∠3, ∠4, and ∠*YZX*.**

2. List all the angles that have W as the vertex. **∠*YWX*, ∠*YWZ*, or ∠2, ∠*XWZ* or ∠1**

3. Name a straight angle. **∠*YWX* or ∠*WVZ***

4. If $m\angle WYV = 4x - 2$, $m\angle VYZ = 2x - 5$, and $m\angle WYZ = 77$, find the measurements of $\angle WYV$ and $\angle VYZ$. **54, 23**

5. Does $\angle YVW$ appear to be acute, obtuse, right, or straight? **acute**



6. In the figure at the right, if $\overrightarrow{QS}$ bisects $\angle RQP$, $m\angle RQS = 2x + 10$, and $m\angle SQP = 3x - 18$, find $m\angle SQR$. **66**

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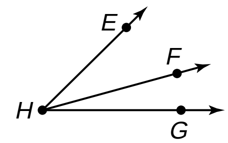
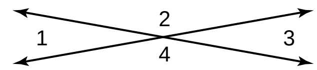
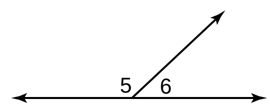
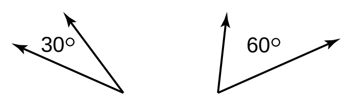
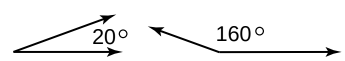
Angle Relationships

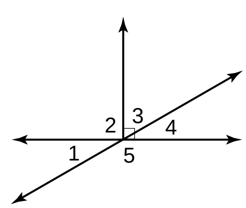
The following table identifies several different types of angles that occur in pairs.

Pairs of Angles		
Special Name	**Definition**	**Examples**
adjacent angles	angles in the same plane that have a common vertex and a common side, but no common interior points	 ∠*EHF* and ∠*FHG* are adjacent angles.
vertical angles	two nonadjacent angles formed by two intersecting lines (Vertical angles are congruent.)	 ∠1 and ∠3 are vertical angles. ∠2 and ∠4 are vertical angles. ∠1 ≅ ∠3, ∠2 ≅ ∠4
linear pair	adjacent angles whose noncommon sides are opposite rays	 ∠5 and ∠6 form a linear pair.
complementary angles	two angles whose measures have a sum of 90	
supplementary angles	two angles whose measures have a sum of 180	

Identify each pair of angles as adjacent, vertical, complementary, supplementary, and/or as a linear pair.

1. ∠1 and ∠2 **adjacent**

2. ∠1 and ∠4 **vertical**

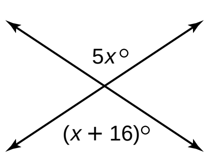


3. ∠3 and ∠4 **adjacent, complementary**

4. ∠1 and ∠5 **adjacent, supplementary, linear pair**

Find the value of x.

5.



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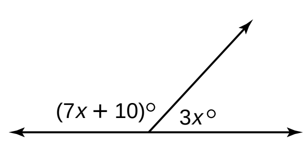
6.



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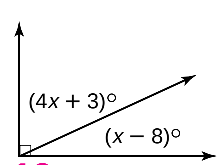
7.



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2–1

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Inductive Reasoning and Conjecturing

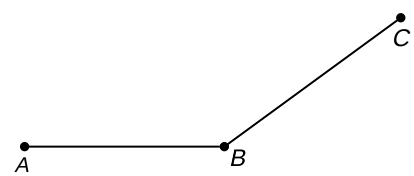
In daily life, you frequently look at several specific situations and reach a general conclusion based on these specific cases. For example, you might receive excellent service in a restaurant several times and conclude that the service is always good. Of course, you are not guaranteed that the service will be good when you return.

This type of reasoning, in which you look at several facts and then make an educated guess based on these facts, is called **inductive reasoning**. The educated guess is called a **conjecture**. Not all conjectures are true. When you find an example that shows the conjecture is false, this example is called a **counterexample**.

Example: Determine if the conjecture is *true* or *false* based on the given information. Explain your answer and give a counterexample if false.

Given: $\overline{AB} \cong \overline{BC}$
Conjecture: B is the midpoint of AC.

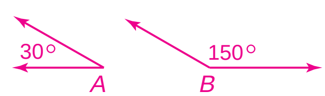
In the figure, $\overline{AB} \cong \overline{BC}$, but B is not the midpoint of $\overline{AC}$. So the conjecture is false.



Determine if each conjecture is true or false based on the given information. Explain your answer and give a counterexample for any false conjecture.

1. **Given:** Collinear points D, E, and F.
Conjecture: $DE + EF = DF$. **False; if *F* is between *D* and *E* (or if *D* is between *E* and *F*), then $DE + EF \neq DF$.**

2. **Given:** $\angle A$ and $\angle B$ are supplementary.
Conjecture: $\angle A$ and $\angle B$ are adjacent angles. **False; if $\angle A$ and $\angle B$ do not have the same vertex, they cannot be adjacent.**



3. **Given:** $\angle D$ and $\angle F$ are supplementary.
$\angle E$ and $\angle F$ are supplementary.
Conjecture: $\angle D \cong \angle E$ **True; if $m\angle F = 25$, them $m\angle D = 180 - 25$ or 155 and $m\angle E = 180 - 25$ or 155. Since $m\angle D = m\angle E$, then $\angle D \cong \angle E$.**

4. **Given:** $\overline{AB}$ is perpendicular to $\overline{BC}$.
Conjecture: $\angle ABC$ is a right angle. **True, if the sides of an angle are perpendicular, the angle is a right angle.**

Study Guide

If-Then Statements and Postulates

If-then statements are commonly used in everyday life. For example, an advertisement might say, "If you buy our product, then you will be happy." Notice that an if-then statement has two parts, a *hypothesis* (the part following "if") and a *conclusion* (the part following "then").

New statements can be formed from the original statement.

Statement	$p \rightarrow q$
Converse	$q \rightarrow p$
Inverse	$\sim p \rightarrow \sim q$
Contrapositive	$\sim q \rightarrow \sim p$

Example: Rewrite the following statement in if-then form. Then write the converse, inverse, and contrapositive.

All elephants are mammals.

If-then form:	If an animal is an elephant, then it is a mammal.
Converse:	If an animal is a mammal, then it is an elephant.
Inverse:	If an animal is not an elephant, then it is not a mammal.
Contrapositive:	If an animal is not a mammal, then it is not an elephant.

Identify the hypothesis and conclusion of each conditional statement.

1. If today is Monday, then tomorrow is Tuesday. **H: today is Monday; C: tomorrow is Tuesday**

2. If a truck weighs 2 tons, then it weighs 4000 pounds. **H: a truck weighs 2 tons; C: it weighs 4000 pounds**

Write each conditional statement in if-then form.

3. All chimpanzees love bananas. **If an animal is a chimpanzee, then it loves bananas.**

4. Collinear points lie on the same line. **If points are collinear, then they lie on the same line.**

Write the converse, inverse, and contrapositive of each conditional.

5. If an animal is a fish, then it can swim. **Converse: If an animal can swim, then it is a fish. Inverse: If an animal is not a fish, then it cannot swim. Contrapositive: If an animal cannot swim, then it is not a fish.**

6. All right angles are congruent. **Converse: If angles are congruent, then they are right angles. Inverse: If angles are not right angles, then they are not congruent. Contrapositive: If angles are not congruent, then they are not right angles.**

Study Guide

Deductive Reasoning

Two important laws used frequently in deductive reasoning are the **Law of Detachment** and the **Law of Syllogism**. In both cases you reach conclusions based on if-then statements.

Law of Detachment	Law of Syllogism
If $p \to q$ is a true conditional and p is true, then q is true.	If $p \to q$ and $q \to r$ are true conditionals, then $p \to r$ is also true.

Example: Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used.

(1) If you break an item in a store, you must pay for it.
(2) Jill broke a vase in Potter's Gift Shop.
(3) Jill must pay for the vase.

Yes, statement (3) follows from statements (1) and (2) by the Law of Detachment.

Determine if a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write no conclusion.

1. (1) If a number is a whole number, then it is an integer.
(2) If a number is an integer, then it is a rational number. **If a number is a whole number, then it is a rational number; syllogism.**

2. (1) If a dog eats Dogfood Delights, the dog is happy.
(2) Fido is a happy dog. **no conclusion**

3. (1) If people live in Manhattan, then they live in New York.
(2) If people live in New York, then they live in the United States. **If people live in Manhattan, then they live in the United States; syllogism.**

4. (1) Angles that are complementary have measures with a sum of 90.
(2) $\angle A$ and $\angle B$ are complementary. **$m\angle A + m\angle B = 90$; detachment**

5. (1) All fish can swim.
(2) Fonzo can swim. **no conclusion**

6. Look for a Pattern Find the next number in the list 83, 77, 71, 65, 59 and make a conjecture about the pattern. **53; each number is 6 less than the preceding one.**

Study Guide

Integration: Algebra
Using Proof in Algebra

Many rules from algebra are used in geometry.

Properties of Equality for Real Numbers	
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a may be replaced by b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$

Example: Prove that if $4x - 8 = {}^-8$, then $x = 0$.

Given: $4x - 8 = {}^-8$

Prove: $x = 0$

Proof:

Statements	Reasons
a. $4x - 8 = {}^-8$	**a.** Given
b. $4x = 0$	**b.** Addition Property (=)
c. $x = 0$	**c.** Division Property (=)

Name the property that justifies each statement.

1. Prove that if $\frac{3}{5}x = {}^-9$, then $x = {}^-15$.

Given: $\frac{3}{5}x = {}^-9$

Prove: $x = {}^-15$

Proof:

Statements	Reasons
a. $\frac{3}{5}x = {}^-9$	**a.** Given
b. $3x = {}^-45$	**b.** Multiplication Property (=)
c. $x = {}^-15$	**c.** Division Property (=)

2. Prove that if $3x - 2 = x - 8$, then $x = {}^-3$.

Given: $3x - 2 = x - 8$

Prove: $x = {}^-3$

Proof:

Statements	Reasons
a. $3x - 2 = x - 8$	**a.** Given
b. $2x - 2 = -8$	**b.** Subtraction Property (=)
c. $2x = {}^-6$	**c.** Addition Property (=)

Study Guide

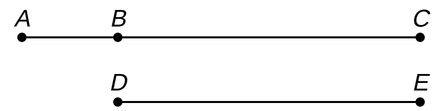
Verifying Segment Relationships

Proofs in geometry follow the same format that you used in Lesson 2–4. The steps in a two-column proof are arranged so that each step follows logically from the preceding one. The reasons can be given information, definitions, postulates of geometry, or rules of algebra. You may also use information that is safe to assume from a given figure.

Example: Write a two-column proof.

Given: $\overline{BC} \cong \overline{DE}$

Prove: $AC = AB + DE$



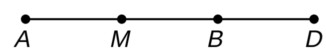
Proof:

Statements	Reasons
a. $\overline{BC} \cong \overline{DE}$	**a.** Given
b. $BC = DE$	**b.** Definition of congruent segments
c. $AC = AB + BC$	**c.** Segment Addition Postulate
d. $AC = AB + BC$	**d.** Substitution Property (=)

Complete each proof by naming the property that justifies each statement.

1. Given: M is the midpoint of $\overline{AB}$.
B is the midpoint of $\overline{MD}$.

Prove: $MD = 2MB$

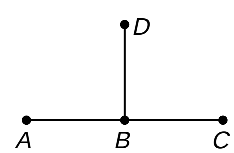


Proof:

Statements	Reasons
a. M is the midpoint of $\overline{AB}$. B is the midpoint of $\overline{MD}$.	**a.** Given
b. $AM = MB$, $MB = BD$	**b.** Definition of midpoint
c. $MD = MB + BD$	**c.** Segment Addition Postulate
d. $MD = MB + MB$	**d.** Substitution Property (=)
e. $MD = 2MB$	**e.** Substitution Property (=)

2. Given: A, B, and C are collinear.
$AB = BD$
$BD = BC$

Prove: B is the midpoint of $\overline{AC}$.



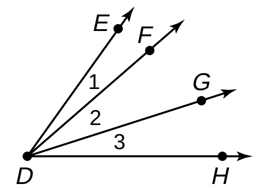
Proof:

Statements	Reasons
a. A, B, and C are collinear. $AB = BD$, $BD = BC$	**a.** Given
b. $AB = BC$	**b.** Transitive Property (=)
c. B is the midpoint of $\overline{AC}$.	**c.** Definition of midpoint

Study Guide

Verifying Angle Relationships

Many relationships involving angles can be proved by applying the rules of algebra, as well as the definitions and postulates of geometry.

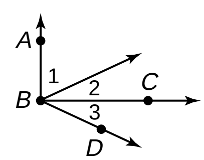


Example: **Given:** $\angle EDG \cong \angle FDH$
Prove: $m\angle 1 = m\angle 3$
Proof:

Statements	**Reasons**
a. $\angle EDG \cong \angle FDH$	**a.** Given
b. $m\angle EDG = m\angle FDH$	**b.** Definition of congruent angles
c. $m\angle EDG = m\angle 1 + m\angle 2$ $m\angle FDH = m\angle 2 + m\angle 3$	**c.** Angle Addition Postulate
d. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	**d.** Substitution Property (=)
e. $m\angle 1 = m\angle 3$	**e.** Subtraction Property (=)

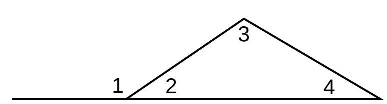
Complete the following proofs.

1. Given: $\overline{AB} \perp \overline{BC}$
$m\angle 2 = m\angle 3$
Prove: $m\angle 1 + m\angle 3 = 90$
Proof:



Statements	**Reasons**
a. $\overline{AB} \perp \overline{BC}$ $m\angle 2 = m\angle 3$	**a.** Given
b. ABC is a right angle.	**b.** Definition of perpendicular lines
c. $m\angle ABC = 90$	**c.** Definition of right angle
d. $m\angle ABC = m\angle 1 + m\angle 2$	**d.** Angle Addition Postulate
e. $m\angle 1 + m\angle 2 = 90$	**e.** Substitution Property (=)
f. $m\angle 1 + m\angle 3 = 90$	**f.** Substitution Property (=)

2. Given: $\angle 1$ and $\angle 2$ form a linear pair.
$m\angle 2 + m\angle 3 + m\angle 4 = 180$
Prove: $m\angle 1 = m\angle 3 + m\angle 4$
Proof:



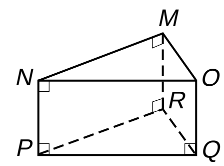
Statements	**Reasons**
a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 2 + m\angle 3 + m\angle 4 = 180$	**a.** Given
b. $\angle 1$ and $\angle 2$ are supplementary.	**b.** If 2 ∠s form a linear pair, they are supp.
c. $m\angle 1 + m\angle 2 = 180$	**c.** Definition of supplementary
d. $m\angle 1 + m\angle 2 =$ $m\angle 2 + m\angle 3 + m\angle 4$	**d.** Substitution Property (=)
e. $m\angle 1 = m\angle 3 + m\angle 4$	**e.** Subtraction Property (=)

Study Guide

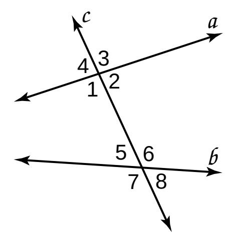
Parallel Lines and Transversals

When planes do not intersect, they are said to be **parallel**. Also, when lines in the same plane do not intersect, they are parallel. But when lines are not in the same plane and do not intersect, they are **skew**. A line that intersects two or more lines in a plane at different points is called a **transversal**. Eight angles are formed by a transversal and two lines. These angles and pairs of them have special names.

Example: Planes PQR and NOM are parallel.
Segments MO and RQ are parallel.
Segments MN and RQ are skew.



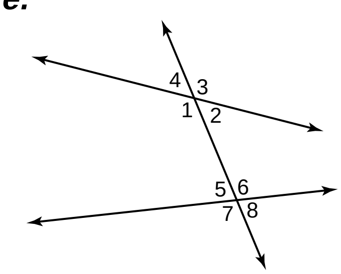
Example: Interior angles: $\angle 1$, $\angle 2$, $\angle 5$, $\angle 6$
Alternate interior angles: $\angle 1$ and $\angle 6$, $\angle 2$ and $\angle 5$
Consecutive interior angles: $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$
Exterior angles: $\angle 3$, $\angle 4$, $\angle 7$, $\angle 8$
Alternate exterior angles: $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Corresponding angles: $\angle 1$ and $\angle 7$,
$\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$



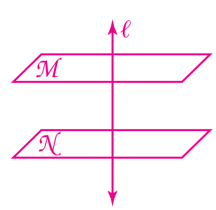
Refer to the figure in the first example.

1. Name two more pairs of parallel segments. **$\overline{NM}$ and $\overline{PR}$, $\overline{NO}$ and $\overline{PQ}$, $\overline{NP}$ and $\overline{MR}$, $\overline{NP}$ and $\overline{OQ}$, $\overline{MR}$ and $\overline{OQ}$**
2. Name two more segments skew to $\overline{NM}$. **$\overline{PQ}$, $\overline{OQ}$**
3. Name two transversals for parallel lines $\overleftrightarrow{NO}$ and $\overleftrightarrow{PQ}$. **$\overleftrightarrow{NP}$, $\overleftrightarrow{OQ}$**
4. Name a segment that is parallel to plane MRQ. **$\overline{NP}$**

Identify the special name for each pair of angles in the figure.



5. $\angle 2$ and $\angle 6$ **consecutive interior angles**
6. $\angle 4$ and $\angle 8$ **alternate exterior angles**
7. $\angle 4$ and $\angle 5$ **corresponding angles**
8. $\angle 2$ and $\angle 5$ **alternate interior angles**
9. Draw a diagram to illustrate two parallel planes with a line intersecting the planes.



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Study Guide

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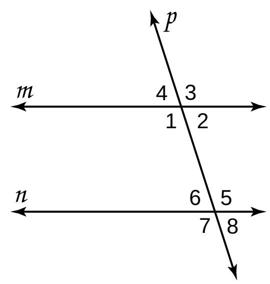
Angles and Parallel Lines

If two parallel lines are cut by a transversal, then the following pairs of angles are congruent.

corresponding angles   alternate interior angles   alternate exterior angles

If two parallel lines are cut by a transversal, then consecutive interior angles are supplementary.

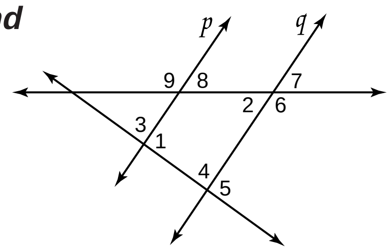
Example: In the figure $m \parallel n$ and p is a transversal. If $m\angle 2 = 35$, find the measures of the remaining angles.



Since $m\angle 2 = 35$, $m\angle 8 = 35$ (corresponding angles).
Since $m\angle 2 = 35$, $m\angle 6 = 35$ (alternate interior angles).
Since $m\angle 8 = 35$, $m\angle 4 = 35$ (alternate exterior angles).

$m\angle 2 + m\angle 5 = 180$. Since consecutive interior angles are supplementary, $m\angle 5 = 145$, which implies that $m\angle 3$, $m\angle 7$, and $m\angle 1$ equal 145.

In the figure at the right $p \parallel q$, $m\angle 1 = 78$, and $m\angle 2 = 47$. Find the measure of each angle.



1. $\angle 3$ **102**

2. $\angle 4$ **102**

3. $\angle 5$ **78**

4. $\angle 6$ **133**

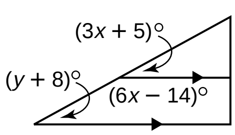
5. $\angle 7$ **47**

6. $\angle 8$ **47**

7. $\angle 9$ **133**

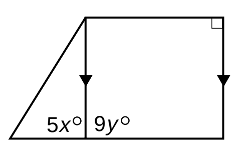
Find the values of x and y in each figure.

8.



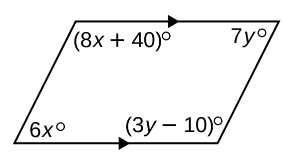
21, 60

9.



18, 10

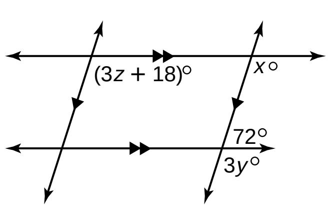
10.



10, 19

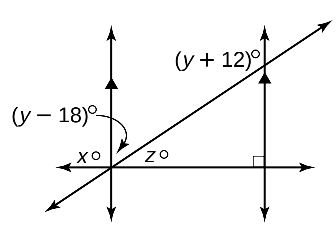
Find the values of x, y and z in each figure.

11.



108, 36, 30

12.



90, 93, 15

Study Guide

Integration: Algebra
Slopes of Lines

To find the slope of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2), use the following formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ where } x_1 \neq x_2$$

The slope of a vertical line, where $x_1 = x_2$, is undefined.

Two lines have the same slope if and only if they are parallel and nonvertical.

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Example: Find the slope of the line ℓ passing through $A(2, -5)$ and $B(-1, 3)$. State the slope of a line parallel to ℓ. Then state the slope of a line perpendicular to ℓ.

Let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (-1, 3)$.
Then $m = \frac{3 - (-5)}{-1 - 2} = -\frac{8}{3}$.

Any line in the coordinate plane parallel to ℓ has slope $-\frac{8}{3}$.

Since $-\frac{8}{3} \cdot \frac{3}{8} = -1$, the slope of a line perpendicular to the line ℓ is $\frac{3}{8}$.

Find the slope of the line passing through the given points.

1. $C(-2, -4)$, $D(8, 12)$ $\frac{8}{5}$

2. $J(-4, 6)$, $K(3, -10)$ $-\frac{16}{7}$

3. $P(0, 12)$, $R(12, 0)$ -1

4. $S(15, -15)$, $T(-15, 0)$ $-\frac{1}{2}$

5. $F(21, 12)$, $G(-6, -4)$ $\frac{16}{27}$

6. $L(7, 0)$, $M(-17, 10)$ $-\frac{5}{12}$

Find the slope of the line parallel to the line passing through each pair of points. Then state the slope of the line perpendicular to the line passing through each pair of points.

7. $I(9, -3)$, $J(6, -10)$ $\frac{7}{3}$, $-\frac{3}{7}$

8. $G(-8, -12)$, $H(4, -1)$ $\frac{11}{12}$, $-\frac{12}{11}$

9. $M(5, -2)$, $T(9, -6)$ -1, 1

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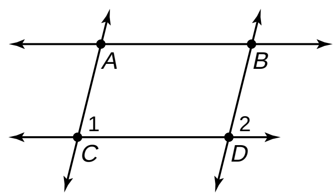
Proving Lines Parallel

Suppose two lines in a plane are cut by a transversal. With enough information about the angles that are formed, you can decide whether the two lines are parallel.

IF	THEN
Corresponding angles are congruent, Alternate interior angles are congruent, Alternate exterior angles are congruent, Consecutive interior angles are supplementary, The lines are perpendicular to the same line,	the lines are parallel.

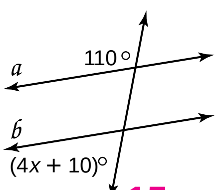
Example: If $\angle 1 = \angle 2$, which lines must be parallel? Explain.

$\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$ because a pair of corresponding angles are congruent.



Find the value of x so that a ∥ b.

1.



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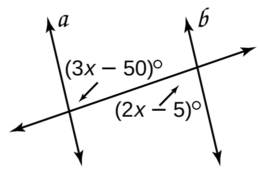
2.



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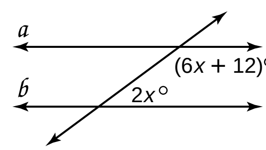
3.



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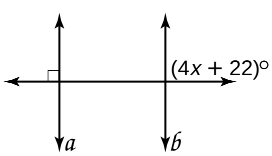
4.



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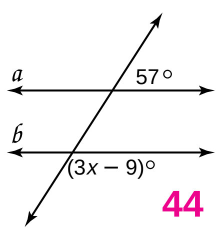
5.



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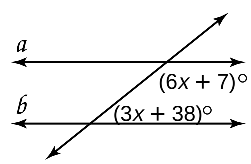
6.



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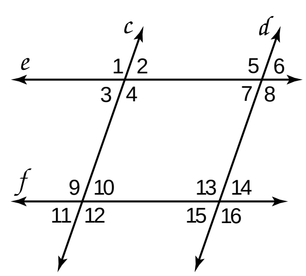
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



7. $\angle 1 \cong \angle 8$
c ∥ d, alternate exterior angles congruent

8. $\angle 4 \cong \angle 9$
e ∥ f, alternate interior angles congruent

9. $m\angle 7 + m\angle 13 = 180$
e ∥ f, consecutive interior angles supplementary

10. $\angle 9 \cong \angle 13$
c ∥ d, corresponding angles congruent

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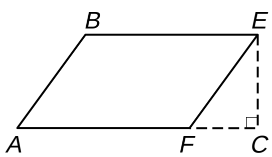
Parallels and Distance

The shortest segment from a point to a line is the perpendicular segment from the point to the line.

Distance Between a Point and a Line	The distance from a line to a point not on the line is the length of the segment perpendicular to the line from the point.
Distance Between Parallel Lines	The distance between two parallel lines is the distance between one of the lines and any point on the other line.

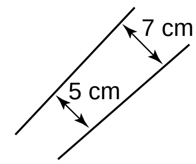
Example 1: Draw the segment that represents the distance indicated.

E to $\overline{AF}$



EC represents the distance from E to AF.

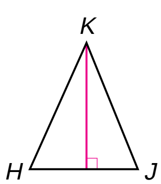
Example 2: Use a ruler to determine whether the lines are parallel.



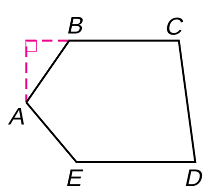
The lines are not everywhere equidistant, therefore they are not parallel.

Draw the segment that represents the distance indicated.

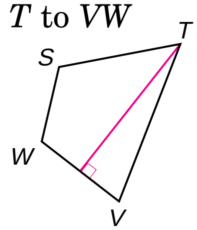
1. K to $\overline{HJ}$



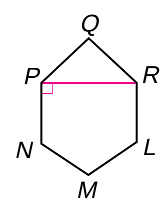
2. A to $\overline{BC}$



3. T to $\overline{VW}$

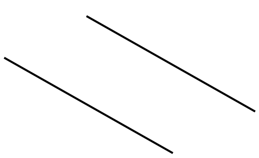


4. R to $\overline{NP}$



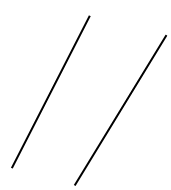
Use a ruler to determine whether the lines are parallel. Explain your reasoning.

5.



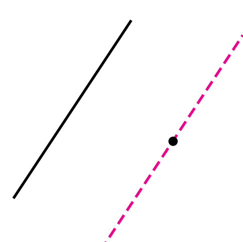
Yes, the lines are 1 cm apart.

6.



No, the lines are not everywhere equidistant.

7. Use a ruler to draw a line parallel to the given line through the given point.



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Study Guide

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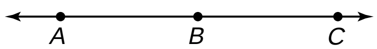
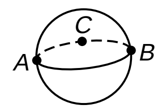
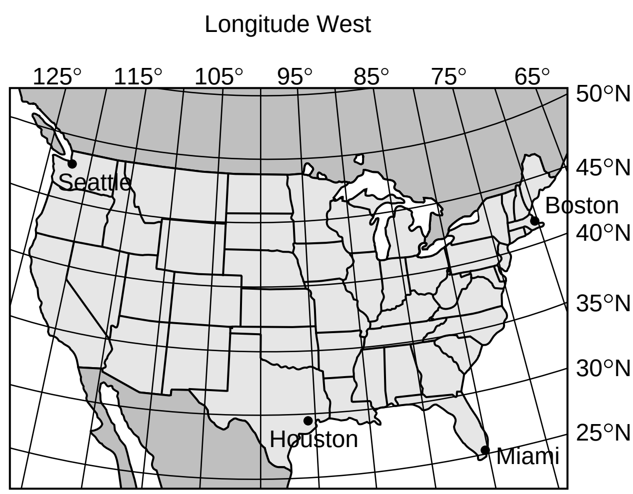
Integration: Non-Euclidean Geometry Spherical Geometry

Spherical geometry is one type of **non-Euclidean geometry**. A line is defined as a great circle of a sphere that divides the sphere into two equal half-spheres. A plane is the sphere itself.

Plane Euclidean Geometry Lines on the Plane	Spherical Geometry Great Circles (Lines) on the Sphere
1. A line segment is the shortest path between two points.	1. An arc of a great circle is the shortest path between two points.
2. There is a unique straight line passing through any two points.	2. There is a unique great circle passing through any pair of nonpolar points.
3. A straight line is infinite.	3. A great circle is finite and returns to its original starting point.
4. If three points are collinear, exactly one is between the other two.  *B* is between *A* and *C*.	4. If three points are collinear, any one of the three points is between the other two. *A* is between *B* and *C*. *B* is between *A* and *C*. *C* is between *A* and *B*. 

Latitude and **longitude**, measured in degrees, are used to locate places on a world map. Latitude provides the locations north or south of the equator. Longitude provides the locations east or west of the prime meridian (0°).



Example: Find a city located near the point with coordinates 29°N and 95°W.

The city near these coordinates is Houston, Texas.

Decide which statements from Euclidean geometry are true in spherical geometry. If false, explain your reasoning.

1. Given a point *Q* and a line *r*, where *Q* is not on *r*, exactly one line perpendicular to *r* passing through *Q* can be drawn. **True**

2. Two lines equidistant from each other are parallel. **False; there are no parallel lines in spherical geometry.**

Use a globe or world map to name the latitude and longitude of each city.

3. Havana, Cuba
23°N, 82°W

4. Beira, Mozambique
18°S, 35°E

5. Kabul, Afghanistan
35°N, 69°E

Use a globe or world map to name the city located near each set of coordinates.

6. 39°N, 73°W
New York City, USA

7. 59°N, 18°E
Stockholm, Sweden

8. 42°S, 146°E
Hobart, Tasmania

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Pages 180–187

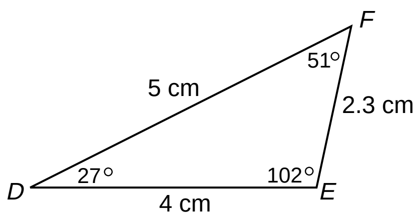
Classifying Triangles

Triangles are classified in two different ways, either by their angles or by their sides.

Classification of Triangles			
Angles		**Sides**	
acute	three acute angles	scalene	no two sides congruent
obtuse	one obtuse angle	isosceles	at least two sides congrent
right	one right angle	equilateral	three sides congruent
equiangular	three congruent angles		

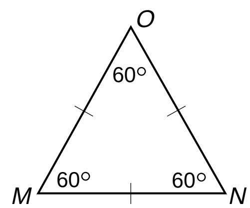
Examples: Classify each triangle by its angles and by its sides.

1



$\triangle DEF$ is obtuse and scalene.

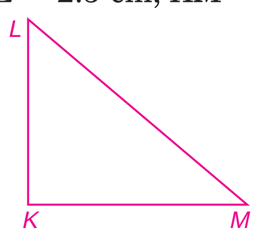
2



$\triangle MNO$ is equiangular and equilaterial.

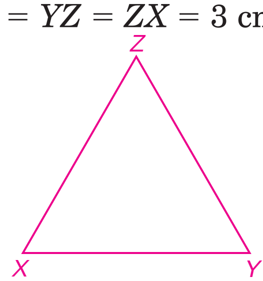
Use a protractor and ruler to draw triangles using the given conditions. If possible, classify each triangle by the measures of its angles and sides.

1. $\triangle KLM$, $m\angle K = 90$, $KL = 2.5$ cm, $KM = 3$ cm



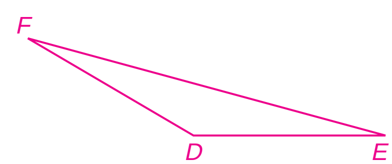
right, scalene

2. $\triangle XYZ$, $m\angle X = 60$, $XY = YZ = ZX = 3$ cm



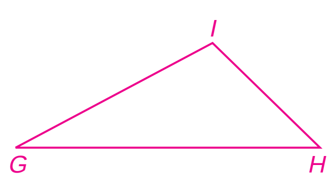
equiangular, equilateral

3. $\triangle DEF$, $m\angle D = 150$, $DE = DF = 1$ inch



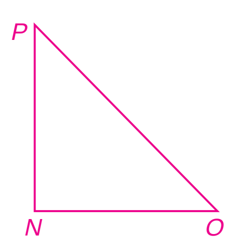
obtuse, isoceles

4. $\triangle GHI$, $m\angle G = 30$, $m\angle H = 45$, $GH = 4$ cm



obtuse, scalene

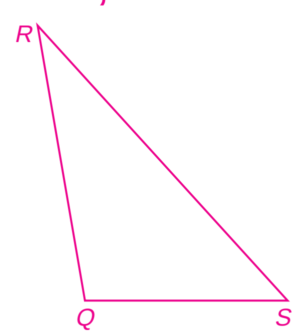
5. $\triangle NOP$, $m\angle N = 90$, $NO = NP = 2.5$ cm



right, isosceles

6. $\triangle QRS$, $m\angle Q = 100$, $QS = 1$ inch, $QR = 1\frac{1}{2}$ inches

obtuse, scalene



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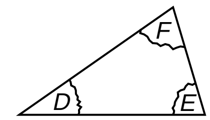
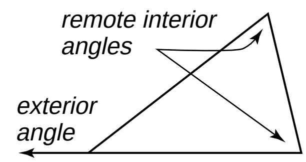
Measuring Angles in Triangles

On a separate sheet of paper, draw a triangle of any size. Label the three angles D, E, and F. Then tear off the three corners and rearrange them so that the three vertices meet at one point, with $\angle D$ and $\angle F$ each adjacent to $\angle E$. What do you notice?



The sum of the measures of the angles of a triangle is 180.

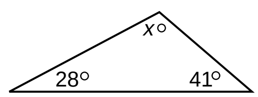
When one side of a triangle is extended, the angle formed is called the **exterior angle**. In a triangle, the angles not adjacent to an exterior angle are called **remote interior angles**.



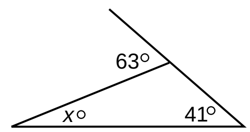
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Examples: Find the value of x in each figure.

1



2

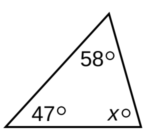


$$28 + 41 + x = 180$$
$$69 + x = 180$$
$$x = 111$$

$$x + 41 = 63$$
$$x = 22$$

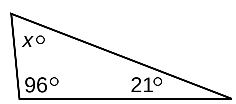
Find the value of x.

1.



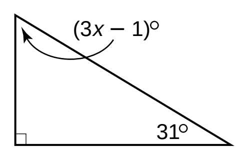
75

2.



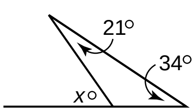
63

3.



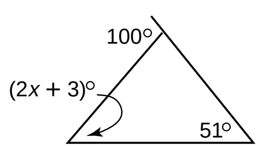
20

4.



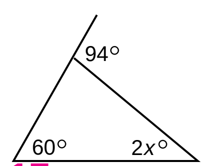
55

5.



23

6.



17

Find the measure of each angle.

7. $\angle 1$
112

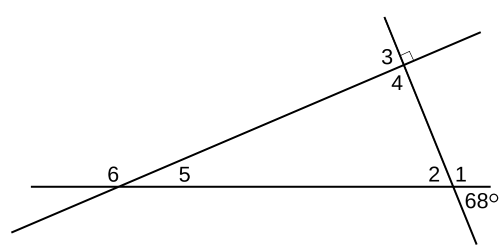
8. $\angle 2$
68

9. $\angle 3$
90

10. $\angle 4$
90

11. $\angle 5$
22

12. $\angle 6$
158

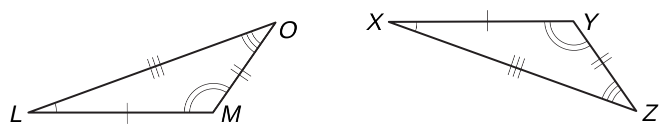


Study Guide

Exploring Congruent Triangles

When two figures have exactly the same shape and size, they are said to be congruent. For two congruent triangles there are three pairs of corresponding (matching) sides and three pairs of corresponding angles. To write a correspondence statement about congruent triangles, you should name corresponding angles in the same order. Remember that congruent parts are marked by identical markings.

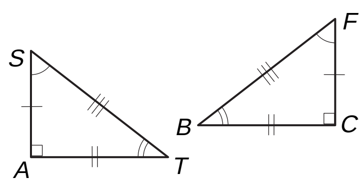
Example: Write a correspondence statement for the triangles in the diagram.



$\triangle LMO \cong \triangle XYZ$

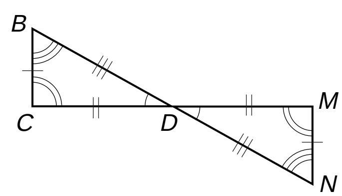
Complete each correspondence statement.

1.



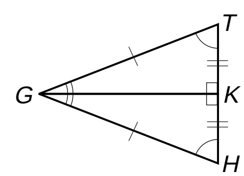
$\triangle SAT \cong \triangle$ _?_ **FCB**

2.



$\triangle BCD \cong \triangle$ _?_ **NMD**

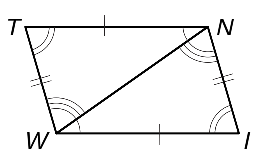
3.



$\triangle GHK \cong \triangle$ _?_ **GTK**

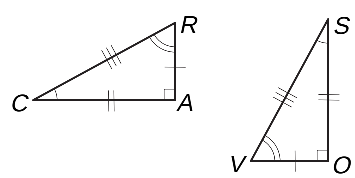
Write a congruence statement for each pair of congruent triangles.

4.



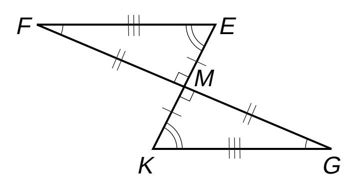
△TWN ≅ △INW

5.



△CAR ≅ △SOV

6.



△FEM ≅ △GKM

Draw triangles △EDG and △QRS. Label the corresponding parts if △EDG ≅ △QRS. Then complete each statement.

7. $\angle E \cong$ _?_ **∠ Q**

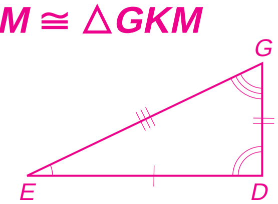
8. $\overline{DG} \cong$ _?_ $\overline{\textbf{RS}}$

9. $\angle EDG \cong$ _?_ **∠ QRS**

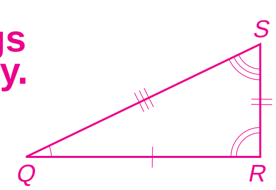
10. $\overline{GE} \cong$ _?_ $\overline{\textbf{SQ}}$

11. $\overline{ED} \cong$ _?_ $\overline{\textbf{QR}}$

12. $\angle EGD \cong$ _?_ **∠ QSR**



Drawings may vary.



Study Guide

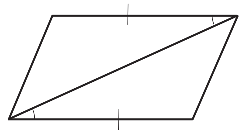
Proving Triangles Congruent

You can show two triangles are congruent with the following:

SSS Postulate (Side–Side–Side)	Three sides of one triangle are congruent to the sides of a second triangle.
SAS Postulate (Side–Angle–Side)	Two sides and the included angle of one triangle are congruent to two sides and an included angle of another triangle.
ASA Postulate (Angle–Side–Angle)	Two angles and the included side of one triangle are congruent to two angles and the included side of another triangle.

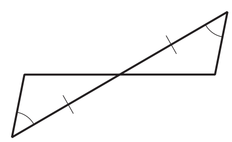
Examples: Determine whether each pair of triangles are congruent. If they are congruent, indicate the postulate that can be used to prove their congruence.

1



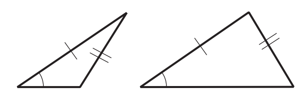
SAS Postulate

2



ASA Postulate

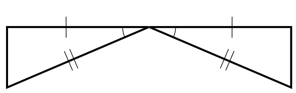
3



not congruent

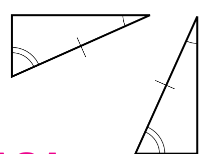
Determine which postulate can be used to prove the triangles are congruent. If it is not possible to prove that they are congruent, write <u>not possible</u>.

1.



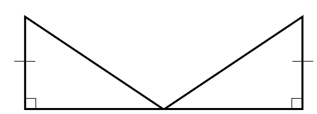
SAS

2.



ASA

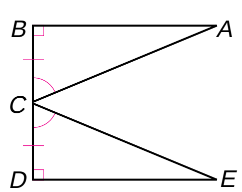
3.



not possible

Mark all congruent parts in each figure, complete the prove statement, and identify the postulate that proves their congruence.

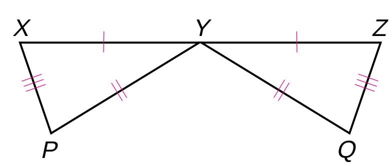
4.



Given: $\angle BCA \cong \angle DCE$
$\angle B$ and $\angle D$ are right angles.
$\overline{BC} \cong \overline{CD}$

Prove: $\triangle CAB \cong$ ***△CED***
ASA

5.



Given: $\overline{XY} \cong \overline{YZ}$
$\overline{PY} \cong \overline{QY}$
$\overline{XP} \cong \overline{ZQ}$

Prove: $\triangle XYP \cong$ ***△ZYQ***
SSS

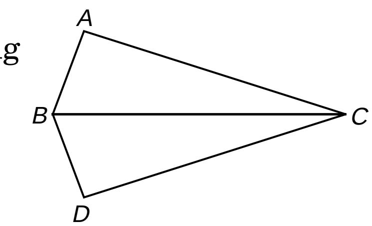
More Congruent Triangles

In the previous lesson, you learned three postulates for showing that two triangles are congruent: Side–Side–Side (SSS), Side–Angle–Side (SAS), and Angle–Side–Angle (ASA).

Another test for triangle congruence is the Angle–Angle–Side theorem (AAS).

If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a side of a second triangle, the two triangles are congruent.

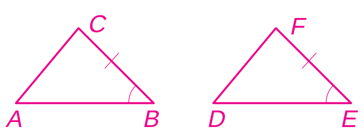
Example: In $\triangle ABC$ and $\triangle DBC$, $\overline{AC} \cong \overline{DC}$, and $\angle ACB \cong \angle DCB$. Indicate the additional pair of corresponding parts that would have to be proved congruent in order to use AAS to prove $\triangle ACB \cong \triangle DCB$.



You would need to prove $\angle ABC \cong \angle DBC$ in order to prove that $\triangle ACB \cong \triangle DCB$.

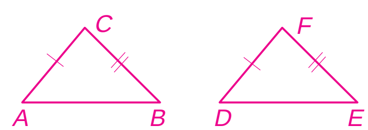
Draw and label triangles ABC and DEF. Indicate the additional pairs of corresponding parts that would have to be proved congruent in order to use the given postulate or theorem to prove the triangles congruent. **Drawings will vary.**

1. $\angle B \cong \angle E$ and $\overline{BC} \cong \overline{EF}$ by ASA



$\angle C \cong \angle F$

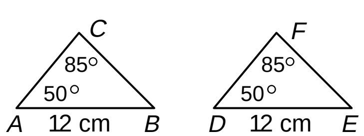
2. $\overline{AC} \cong \overline{DF}$ and $\overline{CB} \cong \overline{FE}$ by SSS



$\overline{AB} \cong \overline{DE}$

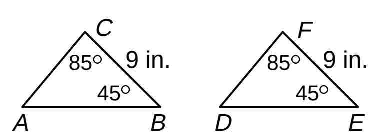
Eliminate the possibilities. Determine which postulates show that the triangles are congruent.

3.



AAS

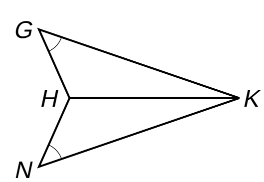
4.



ASA

Write a paragraph proof.

5. Given: $\overline{HK}$ bisects $\angle GKN$.
$\angle G \cong \angle N$
Prove: $\overline{GK} \cong \overline{NK}$



We are given that $\overline{HK}$ bisects $\angle GKN$. So $\angle GKH \cong \angle NKH$. We also are given that $\angle G \cong N$. $HK \cong HK$ since congruence of segments is reflexive. Therefore, $\triangle GKH \cong \triangle NKH$ by AAS. So, $\overline{GK} \cong \overline{NK}$ by the definition of congruent triangles (CPCTC).

Analyzing Isosceles Triangles

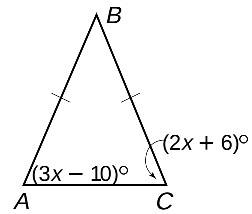
Remember that two sides of an isosceles triangle are congruent. Two important theorems about isosceles triangles are as follows.

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example: Find the value of x.

Since $\overline{AB} \cong \overline{BC}$, the angles opposite $\overline{AB}$ and $\overline{BC}$ are congruent. So $m\angle A = m\angle C$.

Therefore, $3x - 10 = 2x + 6$
$x = 16$



Find the value of x.

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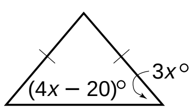

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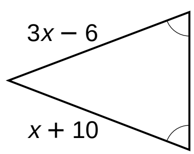
2.


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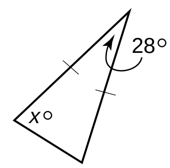

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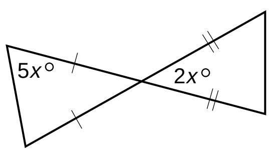

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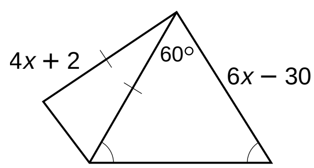

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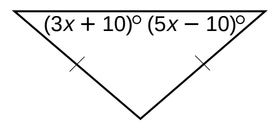

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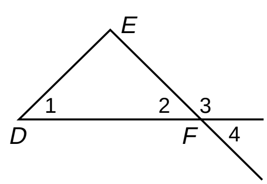
Write a two-column proof.

7. Given: $\angle 1 \cong \angle 4$
Prove: $\overline{DE} \cong \overline{FE}$



Proof:

Statements	Reasons
a. $\angle 1 \cong \angle 4$	a. Given
b. $\angle 2 \cong \angle 4$	b. Vertical angles are congruent.
c. $\angle 1 \cong \angle 2$	c. Congruence of angles is transitive.
d. $\overline{DE} \cong \overline{FE}$	d. If two angles are congruent, then the sides opposite those angles are congruent.

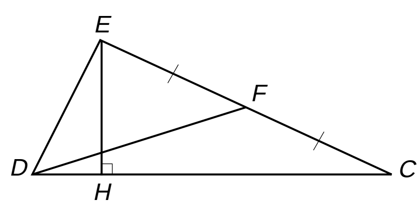
Special Segments in Triangles

Four special types of segments are associated with triangles.

- A **median** is a segment that connects a vertex of a triangle to the midpoint of the opposite side.
- An **altitude** is a segment that has one endpoint at a vertex of a triangle and the other endpoint on the line containing the opposite side so that the altitude is perpendicular to that line.
- An **angle bisector** of a triangle is a segment that bisects an angle of the triangle and has one endpoint at the vertex of that angle and the other endpoint on the side opposite that vertex.
- A **perpendicular bisector** is a segment or line that passes through the midpoint of a side and is perpendicular to that side.

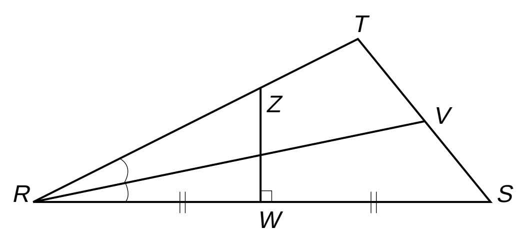
Examples:

1



$\overline{DF}$ is a median of $\triangle DEC$.
$\overline{EH}$ is an altitude of $\triangle DEC$.

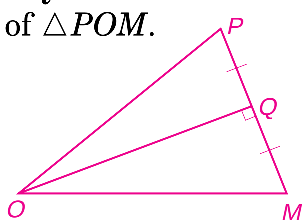
2



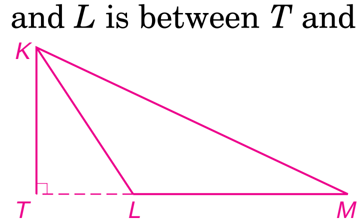
$\overline{RV}$ is an angle bisector of $\triangle RST$.
$\overline{WZ}$ is a perpendicular bisector of side $\overline{RS}$.

Draw and label a figure to illustrate each situation. **Sample answers are given.**

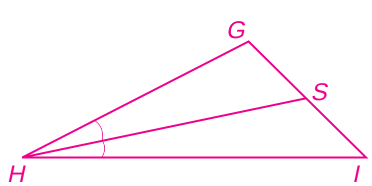
1. $\overline{OQ}$ is a median and an altitude of $\triangle POM$.



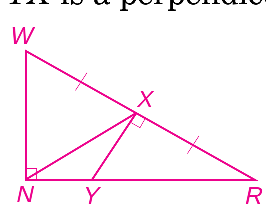
2. $\overline{KT}$ is an altitude of $\triangle KLM$, and L is between T and M.



3. $\overline{HS}$ is an angle bisector of $\triangle GHI$, and S is between G and I.



4. $\triangle NRW$ is a right triangle with right angle at N. $\overline{NX}$ is a median of $\triangle NRW$. $\overline{YX}$ is a perpendicular bisector of $\overline{WR}$.



5. $\triangle TRE$ has vertices $T(3, 6)$, $R(-3, 10$, and $E(-9, 4)$. Find the coordinates of point M if $\overline{TM}$ is a median of $\triangle TRE$. **(−6, 7)**

NAME ______________________________ DATE ____________

Study Guide

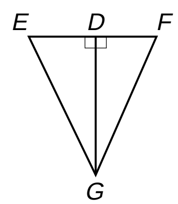
Right Triangles

Two right triangles are congruent if one of the following conditions exist.

Theorem 5-5 LL	If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.
Theorem 5-6 HA	If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.
Theorem 5-7 LA	If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.
Postulate 5-1 HL	If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

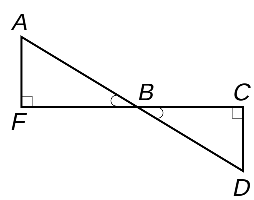
State the additional information needed to prove each pair of triangles congruent by the given theorem or postulate.

1. HL



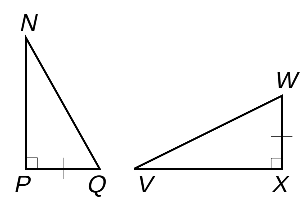
$\overline{EG} \cong \overline{FG}$

2. HA



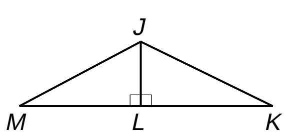
$\overline{AB} \cong \overline{DB}$

3. LL



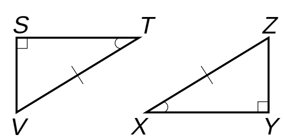
$\overline{NP} \cong \overline{VX}$

4. LA



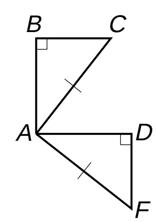
$\angle M \cong \angle K$ or $\angle MJL \cong \angle KJL$

5. HA



no extra information needed

6. LA



$\overline{BC} \cong \overline{DF}$ or $AB \cong AD$, and $\angle BAC \cong \angle DAF$ or $\angle C \cong \angle F$

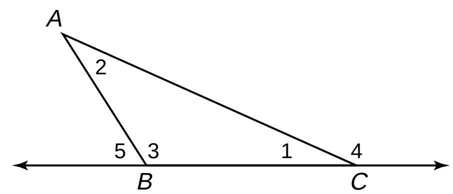
Indirect Proof and Inequalities

A type of proof called **indirect proof** is sometimes used in geometry. In an indirect proof you assume that the conclusion is false and work backward to show that this assumption leads to a contradiction of the original hypothesis or some other known fact, such as a postulate, theorem, or corollary.

The following theorem can be proved by an indirect proof. (See page 253 in your book.)

Exterior Angle Inequality Theorem	If an angle is an exterior angle of a triangle, then its measure is greater than the measure either of its corresponding remote interior angles.

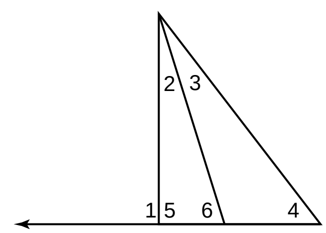
Example: Use the figure at the right to complete the statement with either $<$ or $>$.



$m\angle 1 \underline{\ ?\ } m\angle 5$

Since $\angle 5$ is an exterior angle of $\triangle ABC$ and $\angle 1$ and $\angle 2$ are the corresponding remote interior angles, you know that $m\angle 1 < m\angle 5$ by the Exterior Angle Inequality Theorem.

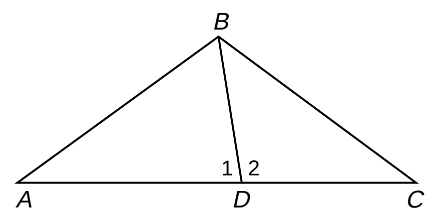
Use the figure at the right to complete each statement with $<$ ***or*** $>$***.***



1. $m\angle 1$ ___>___ $m\angle 6$ **2.** $m\angle 2$ ___<___ $m\angle 1$

3. $m\angle 6$ ___>___ $m\angle 3$ **4.** $m\angle 4$ ___<___ $m\angle 6$

5. Use the problem-solving strategy of working backward to complete the indirect proof in paragraph form.
Given: $m\angle 1 \neq m\angle 2$
Prove: BD is not an altitude of $\triangle ABC$.



Proof:

a. Assume that $\overline{BD}$ is an altitude of $\triangle ABC$.

b. Then $\overline{BD} \perp \overline{AC}$ by definition of altitude.

c. Since perpendicular segments form four right angles, $\angle 1$ and $\angle 2$ are right angles.

d. Since all right angles are congruent, $\angle 1 \cong \angle 2$.

e. Since $\angle 1 \cong \angle 2$, $m\angle 1 =$ $m\angle 2$.

f. But it is given that $m\angle 1 \neq m\angle 2$.

g. So our assumption is incorrect. Therefore, $\overline{BD}$ is not an altitude of $\triangle ABC$.

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Study Guide

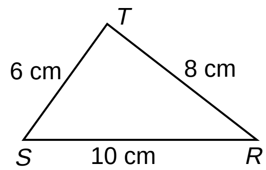
Student Edition
Pages 259–265

Inequalities for Sides and Angles of a Triangle

Two theorems are very useful for determining relationships between sides and angles of triangles.

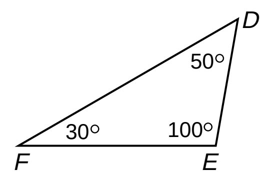
- If one side of a triangle is longer than another side, then the angle opposite the longer side is greater than the angle opposite the shorter side.
- If one angle of a triangle is greater than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Examples: ***1*** List the angles in order from least to greatest measure.



∠R, ∠S, ∠T

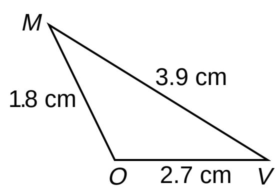
2 List the sides in order from shortest to longest.



$\overline{DE}$, $\overline{FE}$, $\overline{FD}$

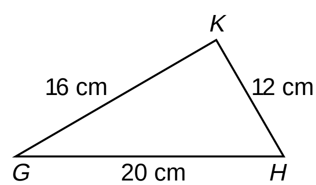
For each triangle, list the angles in order from least to greatest measure.

1.



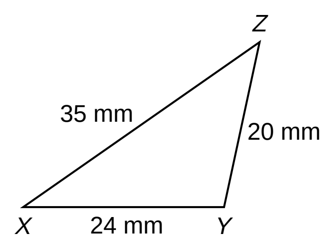
∠V, ∠M, ∠O

2.



∠G, ∠H, ∠K

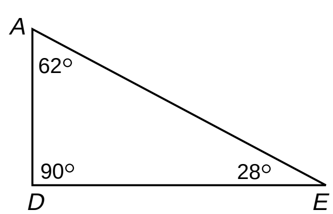
3.



∠X, ∠Z, ∠Y

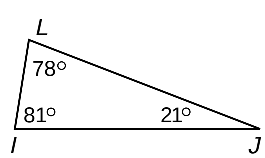
For each triangle, list the sides in order from shortest to longest.

4.



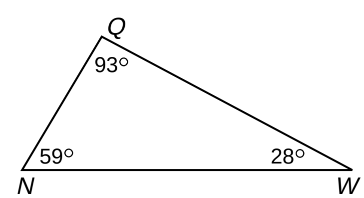
$\overline{AD}$, $\overline{DE}$, $\overline{AE}$

5.



$\overline{IL}$, $\overline{IJ}$, $\overline{LJ}$

6.



$\overline{NQ}$, $\overline{QW}$, $\overline{NW}$

List the sides of △ABC in order from longest to shortest if the angles of △ABC have the indicated measures.

7. $m\angle A = 5x + 2$, $m\angle B = 6x - 10$, $m\angle C = x + 20$

$\overline{AC}$, $\overline{BC}$, $\overline{AB}$

8. $m\angle A = 10x$, $m\angle B = 5x - 17$, $m\angle C = 7x - 1$

$\overline{BC}$, $\overline{AB}$, $\overline{AC}$

The Triangle Inequality

If you take three straws that are 8 inches, 4 inches, and 3 inches in length, can you use these three straws to form a triangle? Without actually trying it, you might think it is possible to form a triangle with the straws. If you try it, however, you will notice that the two smaller straws are too short. This example illustrates the following theorem.

Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example: If the lengths of two sides of a triangle are 7 centimeters and 11 centimeters, between what two numbers must the measure of the third side fall?

Let x = the length of the third side.

By the Triangle Inequality Theorem, each of these inequalities must be true.

$x + 7 > 11$ $\quad$ $x + 11 > 7$ $\quad$ $11 + 7 > x$

$x > 4$ $\quad$ $x > -4$ $\quad$ $18 > x$

Therefore, x must be between 4 centimeters and 18 centimeters.

Determine whether it is possible to draw a triangle with sides of the given measures. Write <u>yes</u> or <u>no</u>.

1. 15, 12, 9 **yes**

2. 23, 16, 7 **no**

3. 20, 10, 9 **no**

4. 8.5, 6.5, 13.5 **yes**

5. 47, 28, 70 **yes**

6. 28, 41, 13 **no**

The measures of two sides of a triangle are given. Between what two numbers must the measure of the third side fall?

7. 9 and 15 **6 and 24**

8. 11 and 20 **9 and 31**

9. 23 and 14 **9 and 37**

10. Suppose you have three different positive numbers arranged in order from greatest to least. Which sum is it most crucial to test to see if the numbers could be the lengths of the sides of a triangle? **the sum of the two smaller numbers**

Study Guide

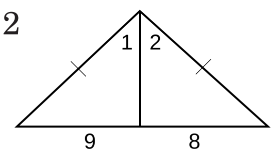
Inequalities Involving Two Triangles

The following two theorems are useful in determining relationships between sides and angles in triangles.

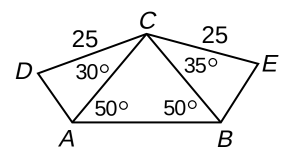
SAS Inequality (Hinge Theorem)	If two sides of one triangle are congruent to two sides of another triangle, and the included angle in one triangle is greater than the included angle in the other, then the third side of the first triangle is longer than the third side in the second triangle.
SSS Inequality	If two sides of one triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.

Examples: Refer to each figure to write an inequality relating the given pair of angle or segment measures.

1 $m\angle 1, m\angle 2$



2 AD, BE

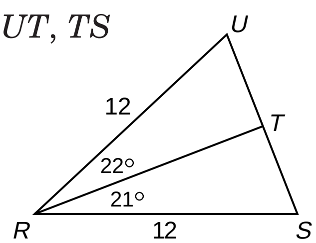


By SSS, $m\angle 1 > m\angle 2$.

By SAS, $AD < BE$.

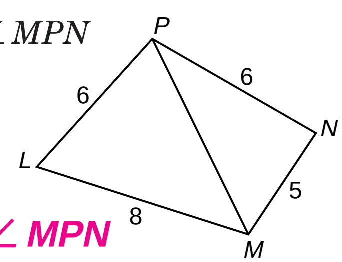
Refer to each figure to write an inequality relating the given pair of angle or segment measures.

1. UT, TS



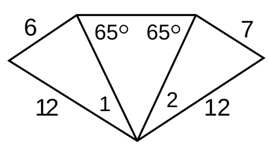
$UT > TS$

2. $m\angle LPM, m\angle MPN$



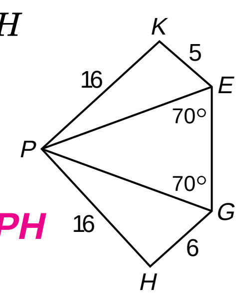
$m\angle LPM > m\angle MPN$

3. $m\angle 1, m\angle 2$



$m\angle 1 < m\angle 2$

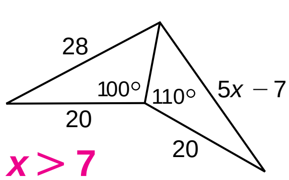
4. $m\angle KPE, m\angle GPH$



$m\angle KPE < m\angle GPH$

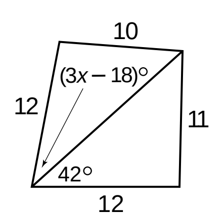
Write an inequality or pair of inequalities to describe the possible values of x.

5.



$x > 7$

6.



$6 < x$ and $x < 20$

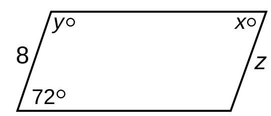
Parallelograms

Any four-sided polygon is called a **quadrilateral**. A segment joining any two nonconsecutive vertices in a quadrilateral is called a **diagonal**. A special kind of quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram**.

The following theorems all concern parallelograms.

- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Consecutive angles in a parallelogram are supplementary.
- The diagonals of a parallelogram bisect each other.

Example: If the quadrilateral in the figure is a parallelogram, find the values of x, y, and z.



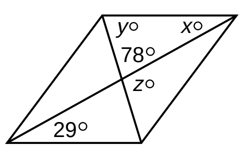
Since opposite angles of a parallelogram are congruent, $x = 72$.

Since consecutive angles of a parallelogram are supplementary, $y + 72 = 180$. Therefore, $y = 108$.

Since opposite sides of a parallelogram are congruent, $z = 8$.

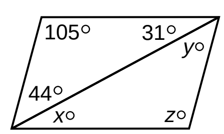
If each quadrilateral is a parallelogram, find the values of x, y, and z.

1.



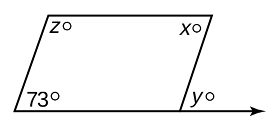
29, 73, 102

2.



31, 44, 105

3.



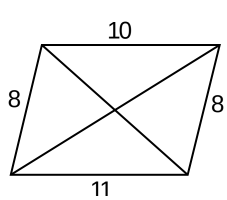
73, 73, 107

4. In parallelogram $ABCD$, $m\angle A = 3x$ and $m\angle B = 4x + 40$. Find the measure of angles A, B, C, and D.
$m\angle A = 60$, $m\angle B = 120$,
$m\angle C = 60$, $m\angle D = 120$

5. In parallelogram $RSTV$, diagonals $\overline{RT}$ and $\overline{VS}$ intersect at Q. If $RQ = 5x + 1$ and $QT = 3x + 15$, find QT. **$QT = 36$**

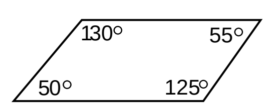
Explain why it is impossible for each figure to be a parallelogram.

6.



Each pair of opposite sides should be congruent.

7.



The opposite angles should be congruent.

NAME____ DATE ______________

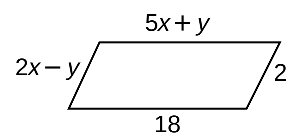
Study Guide

Tests for Parallelograms

You can show that a quadrilateral is a parallelogram if you can show that one of the following is true.

1. Both pairs of opposite sides are parallel.
2. Both pairs of opposite sides are congruent.
3. Diagonals bisect each other.
4. Both pairs of opposite angles are congruent.
5. A pair of opposite sides are both parallel and congruent.

Example: Find the values of x and y that ensure the quadrilateral is a parallelogram.

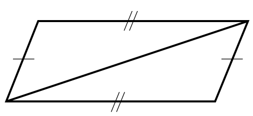


Since opposite sides of a parallelogram must be congruent, then $5x + y = 18$ and $5x - y = 2$.

Solving the system of two equations, you get $x = 2$ and $y = 8$.

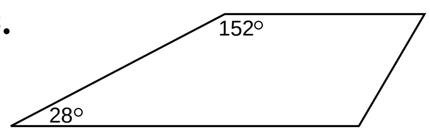
Determine if each quadrilateral is a parallelogram. Justify your answer.

1.



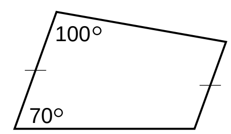
Yes; both pairs of opposite sides are congruent.

2.



No; the top and bottom sides are parallel, but the other pair may not be.

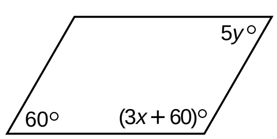
3.



No; top and bottom sides are not parallel.

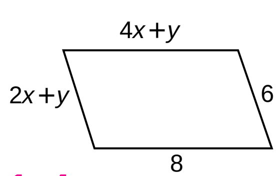
Find the values of x and y that ensure each quadrilateral is a parallelogram.

4.



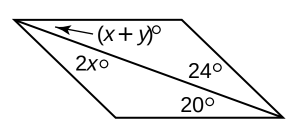
20, 12

5.



1, 4

6.



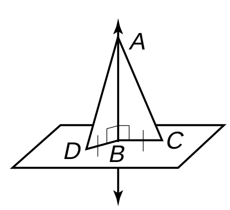
12, 8

7. Identify the subgoals you would need to accomplish to complete the proof.

Given: $\overleftrightarrow{AB} \perp$ plane BCD.
$\overline{DB} \cong \overline{CB}$

Prove: $\angle DAB \cong \angle CAB$

Prove $\triangle ADB \cong \triangle ACB$, then show $\angle DAB \cong \angle CAB$.



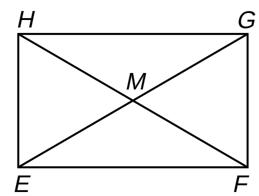
Study Guide

Rectangles

A **rectangle** is a quadrilateral with four right angles. Since both pairs of opposite angles are congruent, a rectangle is a parallelogram and has all the properties of a parallelogram. The following list summarizes the properties of a rectangle.

- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- All four angles are right angles.
- Diagonals are congruent.

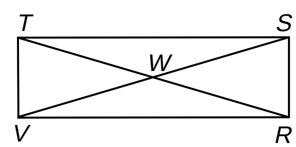
Example: Quadrilateral $EFGH$ is a rectangle. If $EM = 5x + 1$ and $HF = 42$, find the value of x.



Since the diagonals of a rectangle bisect each other and are congruent, you know that $5x + 1 = \frac{1}{2}(42)$.

$$5x + 1 = 21$$
$$5x = 20$$
$$x = 4$$

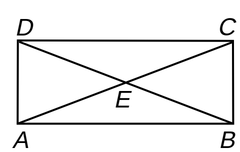
Quadrilateral RSTV is a rectangle. Find the values of x and y.



1. $VW = 2x + y$
$WS = 36$
$RS = x - y$
$VT = 9$ **15, 6**

2. $VR = y$
$TS = x + 11$
$VT = y - 3x$
$RS = x + 2$ **3, 14**

Quadrilateral ABCD is a rectangle. Find the value of x.



3. $m\angle DAC = 4x + 8$
$m\angle CAB = 5x - 8$ **10**

4. $AC = x^2$
$DB = 6x - 8$ **2 or 4**

Determine whether ABCD is a rectangle. Justify your answer.

5. $A(10, 4)$, $B(10, 8)$, $C(-4, 8)$, $D(-4, 4)$
Yes; opposite sides are parallel and all angles are right angles.

6. $A(3, 7)$, $B(10, 7)$, $C(11, 12)$, $D(4, 12)$
no; not all right angles

Squares and Rhombi

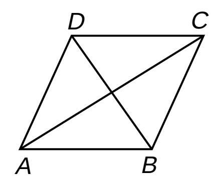
A **rhombus** is a quadrilateral with four congruent sides. A **square** is a quadrilateral with four right angles and four congruent sides.

The diagonals of a rhombus have two special relationships.

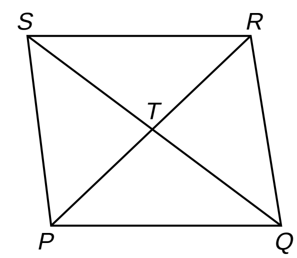
- The diagonals of a rhombus are perpendicular.
- Each diagonal of a rhombus bisects a pair of opposite angles.

Example: *ABCD* is a rhombus. If $m\angle ADB = 27$, find $m\angle ADC$.

Since each diagonal of a rhombus bisects a pair of opposite angles, $m\angle ADC = 2(m\angle ADB)$. So $m\angle ADC = 2(27)$ or 54.



Use rhombus PQRS and the given information to find each value.



1. If $ST = 13$, find SQ. **26**

2. If $m\angle PRS = 17$, find $m\angle QRS$. **34**

3. Find $m\angle STR$. **90**

4. If $SP = 4x - 3$ and $PQ = 18 + x$, find the value of x. **7**

Determine whether each quadrilateral with the given vertices is a parallelogram, a rectangle, a rhombus, or a square. List all that apply.

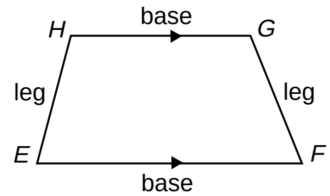
5. $M(1, 5)$, $N(6, 5)$, $O(6, 10)$, $P(1, 10)$
parallelogram, rectangle, rhombus, square

6. $W(^-4, ^-2)$, $X(5, ^-2)$, $Y(8, 4)$, $Z(^-1, 4)$
parallelogram

7. $D(^-7, 3)$, $E(^-2, 3)$, $F(1, 7)$, $G(^-4, 7)$
parallelogram, rhombus

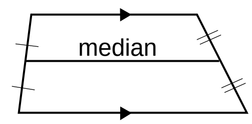
8. $R(0, 0)$, $E(10, 0)$, $S(10, 5)$, $T(0, 5)$
parallelogram, rectangle

Trapezoids



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**, and the nonparallel sides are called **legs**. In trapezoid $EFGH$, $\angle E$ and $\angle F$ are called **base angles**. $\angle H$ and $\angle G$ form the other pair of base angles.

A trapezoid is an **isosceles trapezoid** if its legs are congruent.



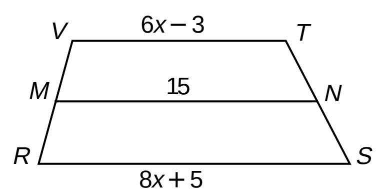
The **median** of a trapezoid is the segment that joins the midpoints of the legs.

The following theorems about trapezoids can be proved.

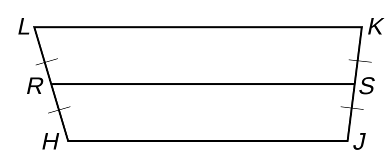
- Both pairs of base angles of an isosceles trapezoid are congruent.
- The diagonals of an isosceles trapezoid are congruent.
- The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

Example: Given trapezoid $RSTV$ with median $\overline{MN}$, find the value of x.

$$MN = \frac{1}{2}(VT + RS)$$
$$15 = \frac{1}{2}(6x - 3 + 8x + 5)$$
$$15 = \frac{1}{2}(14x + 2)$$
$$15 = 7x + 1$$
$$14 = 7x$$
$$2 = x$$



HJKL is an isosceles trapezoid with bases $\overline{HJ}$ and $\overline{LK}$, and median RS. Use the given information to solve each problem.



1. If $LK = 30$ and $HJ = 42$, find RS. **36**

2. If $RS = 17$ and $HJ = 14$, find LK. **20**

3. If $RS = x + 5$ and $HJ + LK = 4x + 6$, find RS. **7**

4. If $m\angle LRS = 66$, find $m\angle KSR$. **66**

5. Find the length of the median of a trapezoid with vertices at $C(3, 1)$, $D(10, 1)$, $E(7, 9)$, and $F(5, 9)$. $\frac{9}{2}$