

9-1

Skills Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{21x^3y}{14x^2y^2}$

2. $\frac{5ab^3}{25a^2b^2}$

3. $\frac{(x^6)^3}{(x^3)^4}$

4. $\frac{8y^2(y^6)^3}{4y^{24}}$

5. $\frac{18}{2x - 6}$

6. $\frac{x^2 - 4}{(x - 2)(x + 1)}$

7. $\frac{3a^2 - 24a}{3a^2 + 12a}$

8. $\frac{3m}{2n} \cdot \frac{n^3}{6}$

9. $\frac{24e^3}{5f^2} \cdot \frac{10(ef)^3}{8e^5f}$

10. $\frac{5s^2}{s^2 - 4} \cdot \frac{s + 2}{10s^5}$

11. $\frac{7g}{y^2} \div 21g^3$

12. $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5}$

13. $\frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4}$

14. $\frac{q^2 + 2q}{6q} \div \frac{q^2 - 4}{3q^2}$

15. $\frac{w^2 - 5w - 24}{w + 1} \cdot \frac{w^2 - 6w - 7}{w + 3}$

16. $\frac{t^2 + 19t + 84}{4t - 4} \cdot \frac{2t - 2}{t^2 + 9t + 14}$

17. $\frac{x^2 - 5x + 4}{2x - 8} \div (3x^2 - 3x)$

18. $\frac{16a^2 + 40a + 25}{3a^2 - 10a - 8} \div \frac{4a + 5}{a^2 - 8a + 16}$

19. $\frac{\frac{c^2}{2d^2}}{\frac{c^6}{5d}}$

20. $\frac{\frac{a^2 - b^2}{4a}}{\frac{a + b}{2a}}$

9-1

Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{9a^2b^3}{27a^4b^4c}$

2. $\frac{(2m^3n^2)^3}{-18m^5n^4}$

3. $\frac{10y^2 + 15y}{35y^2 - 5y}$

4. $\frac{2k^2 - k - 15}{k^2 - 9}$

5. $\frac{25 - v^2}{3v^2 - 13v - 10}$

6. $\frac{x^4 + x^3 - 2x^2}{x^4 - x^3}$

7. $\frac{-2u^3y}{15xz^5} \cdot \frac{25x^3}{14u^2y^2}$

8. $\frac{a + y}{6} \cdot \frac{4}{y + a}$

9. $\frac{n^5}{n - 6} \cdot \frac{n^2 - 6n}{n^8}$

10. $\frac{a - y}{w + n} \cdot \frac{w^2 - n^2}{y - a}$

11. $\frac{x^2 - 5x - 24}{6x + 2x^2} \cdot \frac{5x^2}{8 - x}$

12. $\frac{x - 5}{10x - 2} \cdot \frac{25x^2 - 1}{x^2 - 10x + 25}$

13. $\frac{a^5y^3}{wy^7} \div \frac{a^3w^2}{w^5y^2}$

14. $\left(\frac{2xy}{w^2}\right)^3 \div \frac{24x^2}{w^5}$

15. $\frac{x + y}{6} \div \frac{x^2 - y^2}{3}$

16. $\frac{3x + 6}{x^2 - 9} \div \frac{6x^2 + 12x}{4x + 12}$

17. $\frac{2s^2 - 7s - 15}{(s + 4)^2} \div \frac{s^2 - 10s + 25}{s + 4}$

18. $\frac{9 - a^2}{a^2 + 5a + 6} \div \frac{2a - 6}{5a + 10}$

19. $\frac{\frac{2x + 1}{x}}{4 - x}$

20. $\frac{\frac{x^2 - 9}{4}}{3 - x}$
8

21. $\frac{\frac{x^3 + 2^3}{x^2 - 2x}}{(x + 2)^3}$
 $x^2 + 4x + 4$

22. **GEOMETRY** A right triangle with an area of $x^2 - 4$ square units has a leg that measures $2x + 4$ units. Determine the length of the other leg of the triangle.

23. **GEOMETRY** A rectangular pyramid has a base area of $\frac{x^2 + 3x - 10}{2x}$ square centimeters and a height of $\frac{x^2 - 3x}{x^2 - 5x + 6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid.

9-2

Skills Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. $12c, 6c^2d$

2. $18a^3bc^2, 24b^2c^2$

3. $2x - 6, x - 3$

4. $5a, a - 1$

5. $t^2 - 25, t + 5$

6. $x^2 - 3x - 4, x + 1$

Simplify each expression.

7. $\frac{3}{x} + \frac{5}{y}$

8. $\frac{3}{8p^2q} + \frac{5}{4p^2q}$

9. $\frac{2c - 7}{3} + 4$

10. $\frac{2}{m^2n} + \frac{5}{n}$

11. $\frac{12}{5y^2} - \frac{2}{5yz}$

12. $\frac{7}{4gh} + \frac{3}{4h^2}$

13. $\frac{2}{a + 2} - \frac{3}{2a}$

14. $\frac{5}{3b + d} - \frac{2}{3bd}$

15. $\frac{3}{w - 3} - \frac{2}{w^2 - 9}$

16. $\frac{3t}{2 - x} + \frac{5}{x - 2}$

17. $\frac{m}{m - n} - \frac{m}{n - m}$

18. $\frac{4z}{z - 4} + \frac{z + 4}{z + 1}$

19. $\frac{1}{x^2 + 2x + 1} + \frac{x}{x + 1}$

20. $\frac{2x + 1}{x - 5} - \frac{4}{x^2 - 3x - 10}$

21. $\frac{n}{n - 3} + \frac{2n + 2}{n^2 - 2n - 3}$

22. $\frac{3}{y^2 + y - 12} - \frac{2}{y^2 + 6y + 8}$

9-2 Practice

Adding and Subtracting Rational Expressions

Find the LCM of each set of polynomials.

1. x^2y, xy^3

2. a^2b^3c, abc^4

3. $x + 1, x + 3$

4. $g - 1, g^2 + 3g - 4$

5. $2r + 2, r^2 + r, r + 1$

6. $3, 4w + 2, 4w^2 - 1$

7. $x^2 + 2x - 8, x + 4$

8. $x^2 - x - 6, x^2 + 6x + 8$

9. $d^2 + 6d + 9, 2(d^2 - 9)$

Simplify each expression.

10. $\frac{5}{6ab} - \frac{7}{8a}$

11. $\frac{5}{12x^4y} - \frac{1}{5x^2y^3}$

12. $\frac{1}{6c^2d} + \frac{3}{4cd^3}$

13. $\frac{4m}{3mn} + 2$

14. $2x - 5 - \frac{x - 8}{x + 4}$

15. $\frac{4}{a - 3} + \frac{9}{a - 5}$

16. $\frac{16}{x^2 - 16} + \frac{2}{x + 4}$

17. $\frac{2 - 5m}{m - 9} + \frac{4m - 5}{9 - m}$

18. $\frac{y - 5}{y^2 - 3y - 10} + \frac{y}{y^2 + y - 2}$

19. $\frac{5}{2x - 12} - \frac{20}{x^2 - 4x - 12}$

20. $\frac{2p - 3}{p^2 - 5p + 6} - \frac{5}{p^2 - 9}$

21. $\frac{1}{5n} - \frac{3}{4} + \frac{7}{10n}$

22. $\frac{2a}{a - 3} - \frac{2a}{a + 3} + \frac{36}{a^2 - 9}$

23. $\frac{\frac{2}{x - y} + \frac{1}{x + y}}{\frac{1}{x - y}}$

24. $\frac{\frac{r + 6}{r} - \frac{1}{r + 2}}{\frac{r^2 + 4r + 3}{r^2 + 2r}}$

25. **GEOMETRY** The expressions $\frac{5x}{2}$, $\frac{20}{x + 4}$, and $\frac{10}{x - 4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle.

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r + 2$ represents her rate with the current, and $r - 2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t = \frac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip.

9-3 Skills Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{3}{x^2 - 2x - 8}$

2. $f(x) = \frac{10}{x^2 - 13x + 36}$

3. $f(x) = \frac{x + 12}{x^2 + 10x - 24}$

4. $f(x) = \frac{x - 1}{x^2 - 4x + 3}$

5. $f(x) = \frac{x^2 + 8x + 12}{x + 2}$

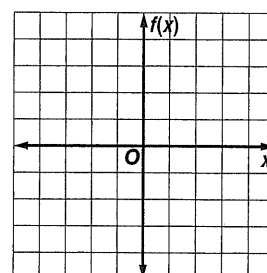
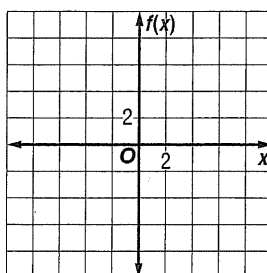
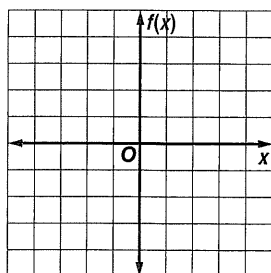
6. $f(x) = \frac{x^2 + x - 12}{x - 3}$

Graph each rational function.

7. $f(x) = \frac{-3}{x}$

8. $f(x) = \frac{10}{x}$

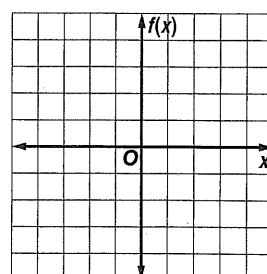
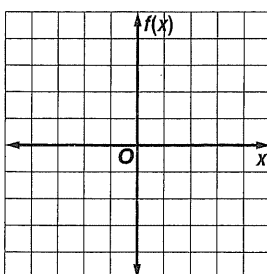
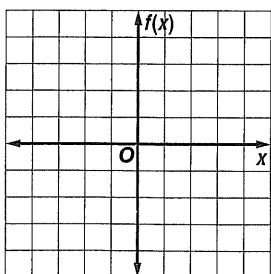
9. $f(x) = \frac{-4}{x}$



10. $f(x) = \frac{2}{x - 1}$

11. $f(x) = \frac{x}{x + 2}$

12. $f(x) = \frac{x^2 - 4}{x - 2}$



9-3 Practice

Graphing Rational Functions

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{6}{x^2 + 3x - 10}$

2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$

3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$

4. $f(x) = \frac{x^2 - 100}{x + 10}$

5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$

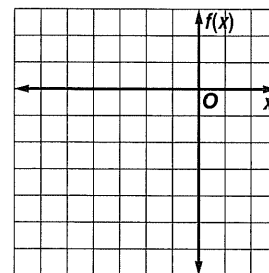
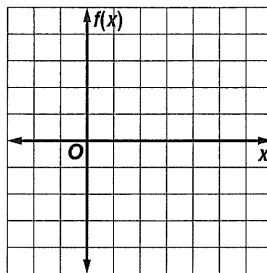
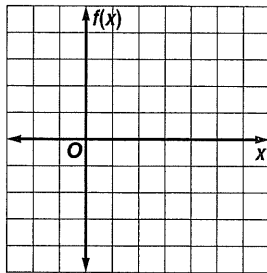
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$

Graph each rational function.

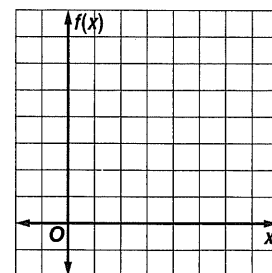
7. $f(x) = \frac{-4}{x - 2}$

8. $f(x) = \frac{x - 3}{x - 2}$

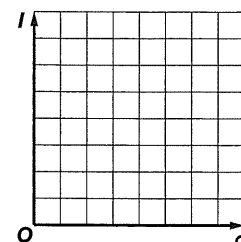
9. $f(x) = \frac{3x}{(x + 3)^2}$



10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x \geq 0, y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour?



11. **LIGHT** The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source?



9-4

Skills Practice

Direct, Joint, and Inverse Variation

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $c = 12m$

2. $p = \frac{4}{q}$

3. $A = \frac{1}{2}bh$

4. $rw = 15$

5. $y = 2rst$

6. $f = 5280m$

7. $y = 0.2s$

8. $uz = -25$

9. $t = 16rh$

10. $R = \frac{8}{w}$

11. $\frac{a}{b} = \frac{1}{3}$

12. $C = 2\pi r$

Find each value.

13. If y varies directly as x and $y = 35$ when $x = 7$, find y when $x = 11$.

14. If y varies directly as x and $y = 360$ when $x = 180$, find y when $x = 270$.

15. If y varies directly as x and $y = 540$ when $x = 10$, find x when $y = 1080$.

16. If y varies directly as x and $y = 12$ when $x = 72$, find x when $y = 9$.

17. If y varies jointly as x and z and $y = 18$ when $x = 2$ and $z = 3$, find y when $x = 5$ and $z = 6$.

18. If y varies jointly as x and z and $y = -16$ when $x = 4$ and $z = 2$, find y when $x = -1$ and $z = 7$.

19. If y varies jointly as x and z and $y = 120$ when $x = 4$ and $z = 6$, find y when $x = 3$ and $z = 2$.

20. If y varies inversely as x and $y = 2$ when $x = 2$, find y when $x = 1$.

21. If y varies inversely as x and $y = 6$ when $x = 5$, find y when $x = 10$.

22. If y varies inversely as x and $y = 3$ when $x = 14$, find x when $y = 6$.

23. If y varies inversely as x and $y = 27$ when $x = 2$, find x when $y = 9$.

24. If y varies directly as x and $y = -15$ when $x = 5$, find x when $y = -36$.

9-4 Practice***Direct, Joint, and Inverse Variation***

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$

2. $p = 4s$

3. $L = \frac{5}{k}$

4. $xy = 4.5$

5. $\frac{C}{d} = \pi$

6. $2d = mn$

7. $\frac{1.25}{g} = h$

8. $y = \frac{3}{4x}$

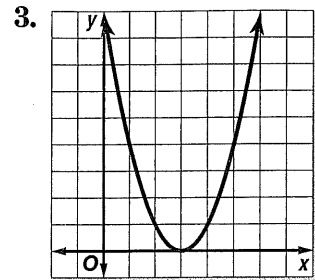
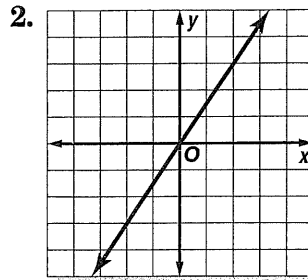
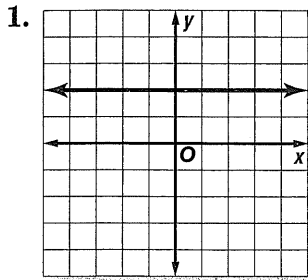
Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$.
10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$.
11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$.
12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$.
13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$.
14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$.
15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$.
16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$.
17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$.
18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$.
19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$.
20. **GASES** The volume V of a gas varies inversely as its pressure P . If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury.
21. **SPRINGS** The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?
22. **GEOMETRY** The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters?

9-5 Skills Practice

Classes of Functions

Identify the type of function represented by each graph.



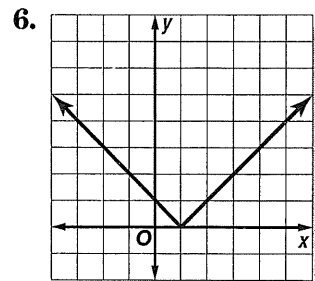
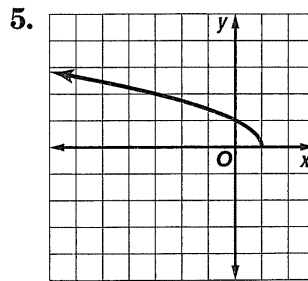
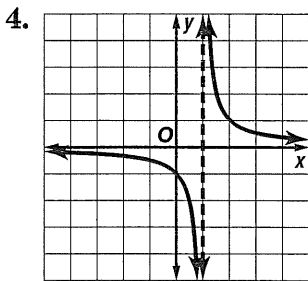
Match each graph with an equation below.

A. $y = |x - 1|$

B. $y = \frac{1}{x - 1}$

C. $y = \sqrt{1 - x}$

D. $y = [x] - 1$

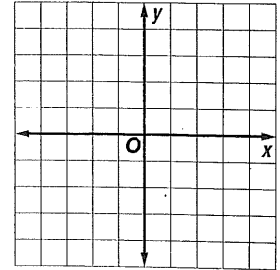
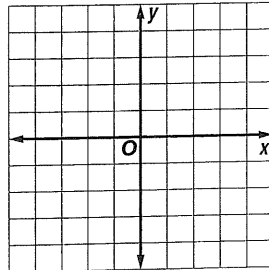
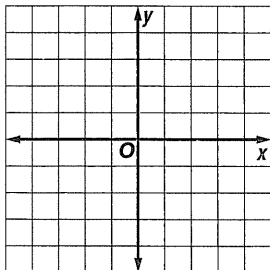


Identify the type of function represented by each equation. Then graph the equation.

7. $y = \frac{2}{x}$

8. $y = 2[x]$

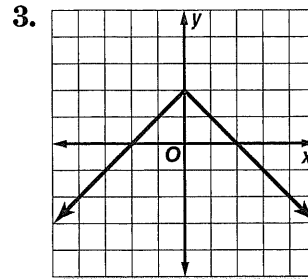
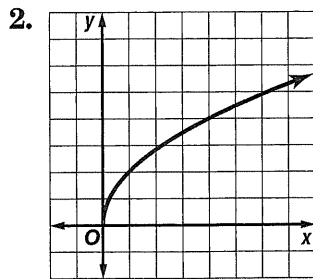
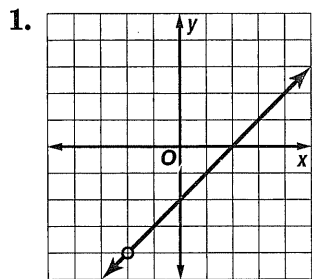
9. $y = -3x$



9-5 Practice

Classes of Functions

Identify the type of function represented by each graph.



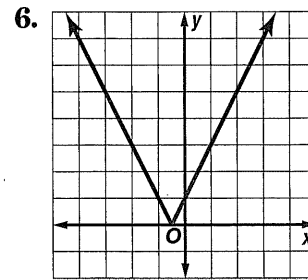
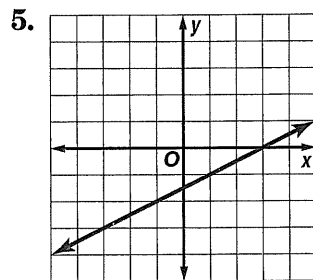
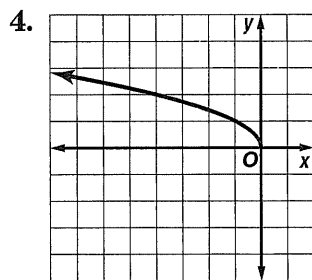
Match each graph with an equation below.

A. $y = |2x + 1|$

B. $y = \lfloor 2x + 1 \rfloor$

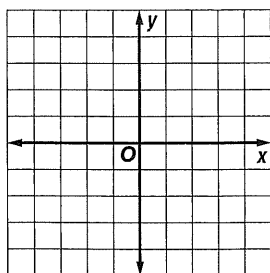
C. $y = \frac{x - 3}{2}$

D. $y = \sqrt{-x}$

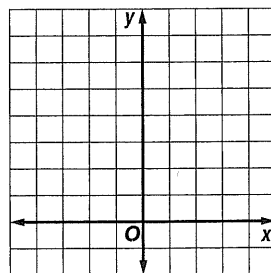


Identify the type of function represented by each equation. Then graph the equation.

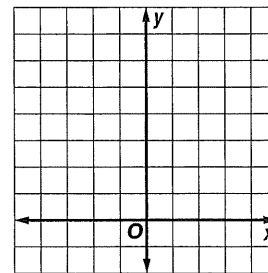
7. $y = -3$



8. $y = 2x^2 + 1$



9. $y = \frac{x^2 + 5x + 6}{x + 2}$



10. **BUSINESS** A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function.

11. **PARKING** A parking lot charges \$10 to park for the first day or part of a day. After that, it charges an additional \$8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days.

9-6

Skills Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-1} = \frac{1}{2}$

2. $2 = \frac{4}{n} + \frac{1}{3}$

3. $\frac{9}{3x} = \frac{-6}{2}$

4. $3 - z = \frac{2}{z}$

5. $\frac{2}{d+1} = \frac{1}{d-2}$

6. $\frac{s-3}{5} = \frac{8}{s}$

7. $\frac{2x+3}{x+1} = \frac{3}{2}$

8. $-\frac{12}{y} = y - 7$

9. $\frac{x-2}{x+4} = \frac{x+1}{x+10}$

10. $\frac{3}{k} - \frac{4}{3k} > 0$

11. $2 - \frac{3}{v} < \frac{5}{v}$

12. $n + \frac{3}{n} < \frac{12}{n}$

13. $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$

14. $\frac{1}{2x} < \frac{2}{x} - 1$

15. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$

16. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$

17. $2 = \frac{5}{2q} + \frac{2q}{q+1}$

18. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$

19. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$

20. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

21. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

22. $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$

23. $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$

24. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$

9-6 Practice**Solving Rational Equations and Inequalities**

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$

7. $\frac{5}{t} < \frac{9}{2t+1}$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$

9. $\frac{4}{w-2} = \frac{-1}{w+3}$

10. $5 - \frac{3}{a} < \frac{7}{a}$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$

12. $8 + \frac{3}{y} > \frac{19}{y}$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$

17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$

18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$

19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$

20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$

21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$

22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$

23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$

24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$

25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$

26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%?

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters?