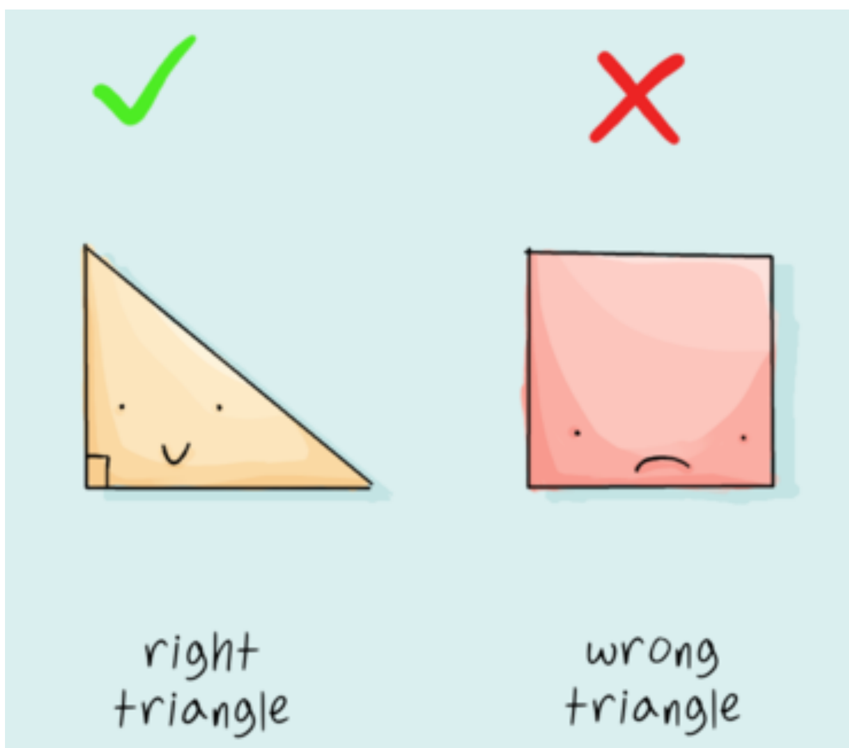
NAME: ____________________ **PERIOD: ___**

DATE: _______________ **PRE-CALCULUS**

MR. MELLINA

UNIT 2: TRIGONOMETRIC FUNCTIONS

- *Lesson 1: Right-Triangle Trigonometry*
- *Lesson 2: Trigonometric Applications*
- *Lesson 3: Angles and Radian Measure*
- *Lesson 4: Trigonometric Functions*
- *Lesson 5: Basic Trigonometric Identities*



Lesson 1: Right-Triangle Trigonometry

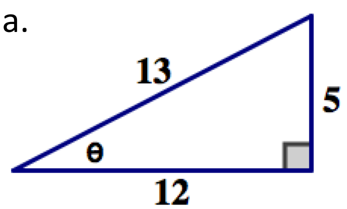
Objectives:

- *Define the six trigonometric ratios of an acute angle in terms of a right angle.*
- *Evaluate trigonometric ratios, using triangles and on a calculator.*

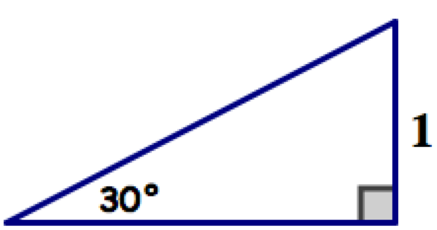
Warm Up

a. Write 35°15’27’’ in decimal form b. Write 48.3625° in DMS form

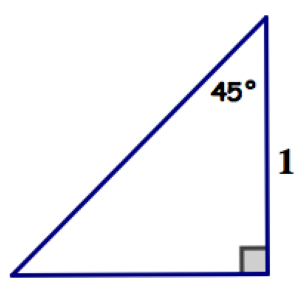
Example 1: Evaluating Trigonometric Ratios
Evaluate the six trigonometric ratios of the angle theta shown in the figure.
a.



Example 2: Special Right Triangles
Fill in the missing sides.
a.



b.



Example 3: Evaluating Trigonometric Ratios

Evaluate the six trigonometric ratios of the angle theta shown in the figure.

a. $30°$

b. $60°$

c. $45°$

Lesson 2: Trigonometric Applications

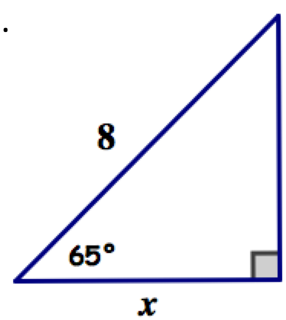
Objectives:

- *Solve triangles using trigonometric*
- *Solve applications using triangles*

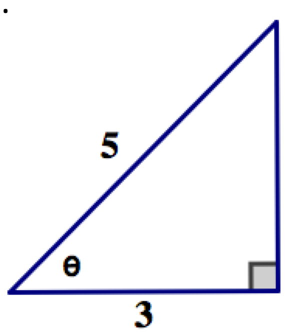
Warm Up

a. What does it mean to solve a triangle?

Example 1: Finding a Side of a Triangle
Find side *x* of the right triangle in the given triangle.
a.



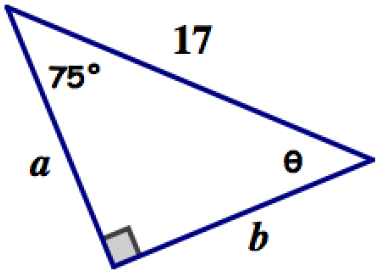
Example 2: Finding an Angle of a Triangle
Find the measure of angle θ in the given triangle.
a.



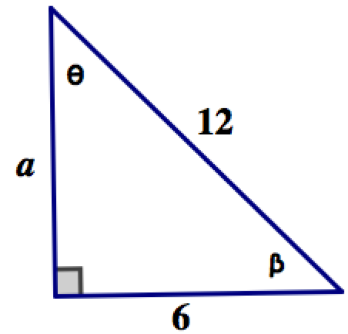
Example 3: Solving a Right Triangle

Solve the right triangle given.

a.



b.



Example 4: Applications

According to the safety sticker on a 20-foot ladder, the distance from the bottom of the ladder to the base of the wall on which it leans should be one-fourth of the length of the ladder: 5 ft.

a. How high up the wall will the ladder reach?

b. If the ladder is in this position, what angle does it make with the ground?

Example 5: Applications

A flagpole casts a 60-foot shadow when the angle of elevation from the edge of the shadow to the top of the flagpole is 35°.

a. Draw a diagram and find the height of the flagpole.

Example 6: Application

A wire needs to reach from the top of a building to a point on the ground. The building is 10m tall, and the angle of depression from the top of the building to the point on the ground is 22°.

a. How long should the wire be?

Example 7: Indirect Measurement

A person on the edge of a canal observes a lamp post on the other side with an angle of elevation of 12° to the top of the lamp post and an angle of depression of 7° to the bottom of the lamp post from eye level. The person's eye level is 152 cm (about 5ft).

a. Find the width of the canal.

b. Find the height of the lamp post.

Lesson 3: Angles and Radian Measure

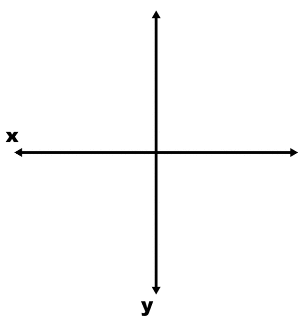
Objectives:

- *Use a rotating ray to extend the definition of angle measure to negative angles and angles greater than 180°.*
- *Define radian measure and convert angle measures between degrees and radians.*

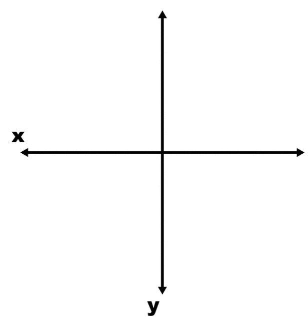
Warm Up

Graph the following angles in standard position. Label the vertex, intitial side, and terminal side.

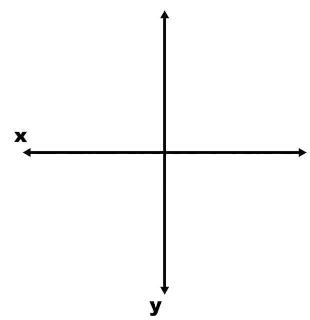
a. 75°



b. 210°



c. -270°



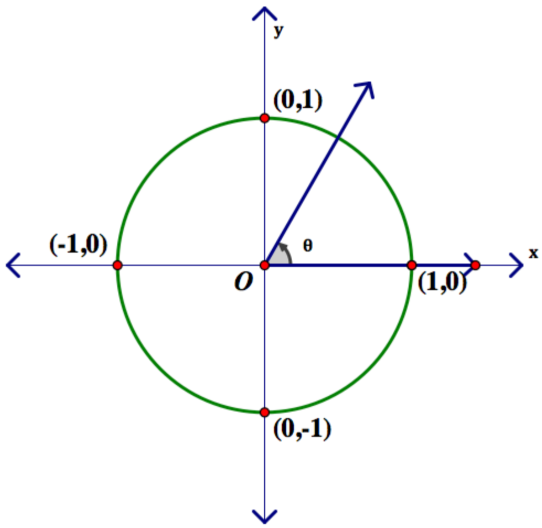
Example 1: Coterminal Angles

Find two angles, one positive and one negative, that are coterminal with the angle given in standard position.

a. 60°

b. -75°

A different way to measure Angles
The degree is commonly used in applications involving surveying and navigation. However, in the late 1800s, mathematicians began to see the need for another unit of measure, called a ________________, that would simplify certain mathematical and physical formulas.



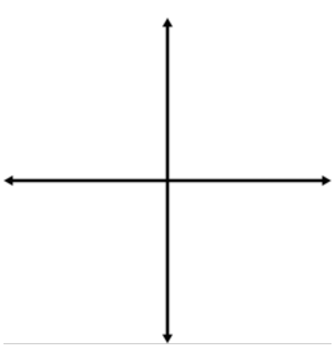
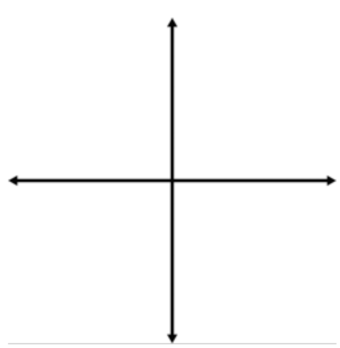
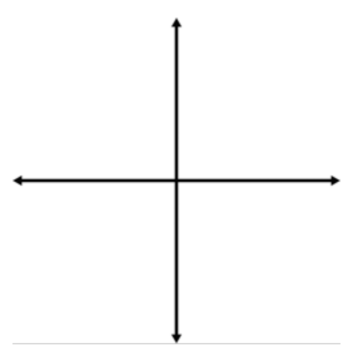
Radian Measure
The definition of a radian is based on the concept of a ___________ ____________, which is a circle of radius ___ unit with its center at the origin (0,0).

The radian measure of an angle is based on the length of an _____________ on the unit circle.

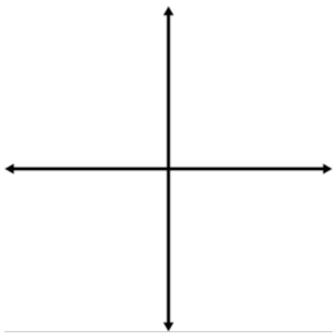
What is the Circumference of a Circle?

Example 2: Drawing angles in radian measure.
Draw the angle provided and provide one positive and negative coterminal angle.

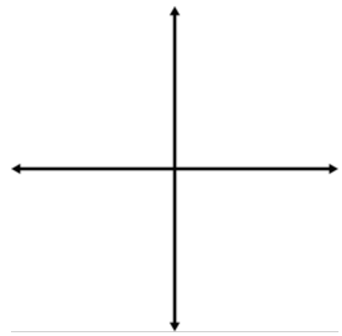
a. $\frac{\pi}{4}$ *radians* b. $\frac{3\pi}{4}$ *radians* c. $-\frac{5\pi}{4}$ *radians*

d. $-\pi$ *radians*



e. $-\frac{3\pi}{2}$ *radians*



Quadrantal Angles
The angle formed if the terminal side lies on one of the _______________ _________.

Example 3: Coverting between degrees and radians.
Use the following conversion factor to convert to radians.

Degrees to Radians

$$\_\_\_\_\_^{\circ}\left(\frac{\pi \ \ radians}{180^{\circ}}\right)$$

a. 45°

b. -225°

c. 450°

Use the following conversion factor to convert to degrees.

Radians to Degrees

$$\_\_\_\_\_ \ radians\left(\frac{180^{\circ}}{\pi \ \ radians}\right)$$

d. $\frac{\pi}{3}$ *radians*

e. $-\frac{3\pi}{4}$ *radians*

f. $-\frac{\pi}{12}$ *radians*

Arc Length and Area of a Sector

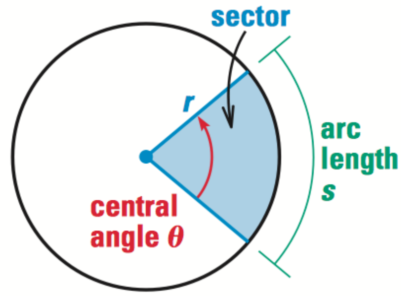
A <u>*Sector*</u> *is a region of a circle that is bounded by* ***two*** ________ *and an* _______ *of the circle.*

The <u>*Central Angle*</u> θ *of a sector is the angle formed by the* ***two*** __________.

The arc length ***s*** *and area* ***A*** *of a sector with radius* ***r*** *and central angle* θ *(measured in radians) are as follows:*

<u>*Arc Length*</u>*:*

<u>*Area*</u>*:*



Example 4: Finding Arc Length and Area

Find the arc length and the area of a sector with the given radius **r** and central angle θ.

a. $r = 3m,\ \theta = \dfrac{5\pi}{12}$

b. $r = 5m,\ \theta = \dfrac{3\pi}{4}$

Lesson 4: Trigonometric Functions

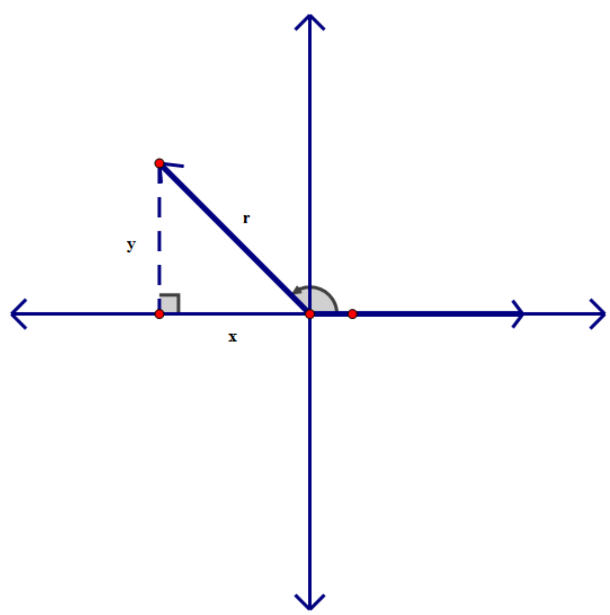
Objectives:

- *Define the trigonometric ratios in the coordinate plane.*
- *Define the trigonometric functions in terms of the unit circle.*

Warm Up 🔥

Find the sine, cosine, and tangent, of the angle θ, whose terminal side passes through the point $(-3, -2)$.

General Definitions of Trigonometric Functions
The trig functions of an angle is standard position may be defined in terms of the ordered pair for any point ____________ on its terminal side and the distance ________ between the point and the origin. By the Pythagorean theorem, ***r*** =



Trig Functions of an Angle in Standard Position:

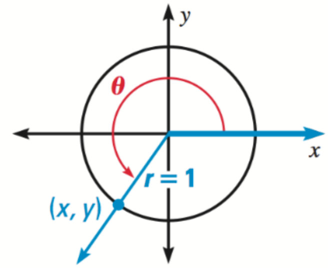
$\sin\theta = \text{——}$ $\quad\csc\theta = \text{——}$

$\cos\theta = \text{——}$ $\quad\sec\theta = \text{——}$

$\tan\theta = \text{——}$ $\quad\cot\theta = \text{——}$

The Unit Circle

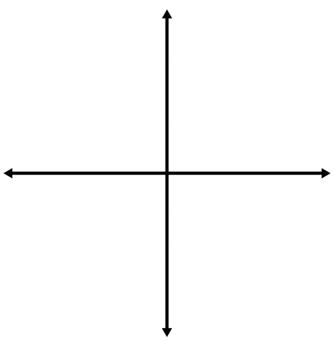
The unit circle has the center __________
and a radius of _____.



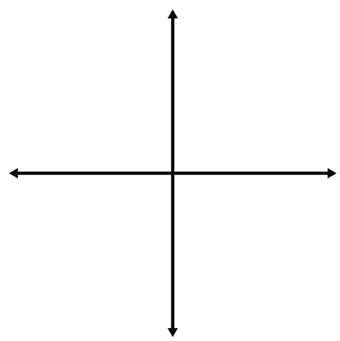
Example 1: Use the Unit Circle

Use the unit circle to evaluate the six trigonometric fucntions of the given angle.

a. $\theta = 270°$

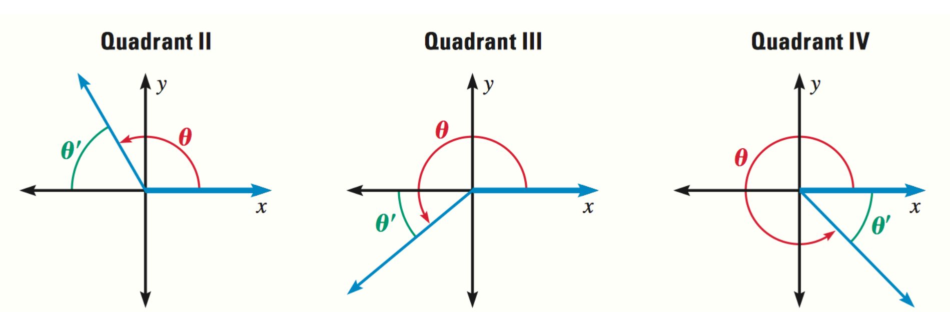


b. $\theta = 2\pi$



Reference Angle Relationships

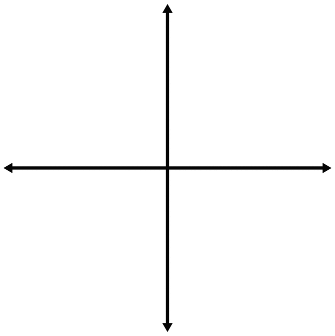
Let θ be an angle in standard position. The reference angle for θ is the ________________ angle θ' formed by the terminal side of θ and the _______ axis.



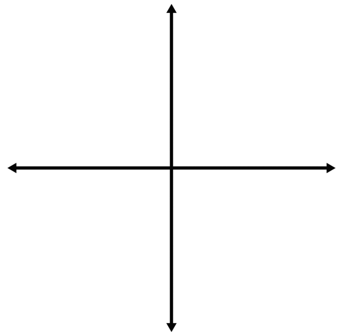
Example 2: Find Reference Angles.

Find the reference angle θ' for the given values of θ:

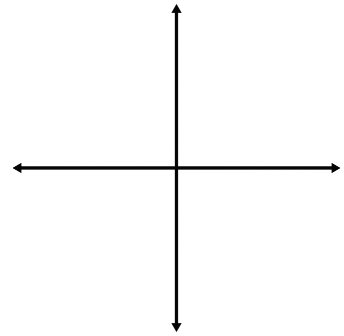
a. $\theta = 130°$



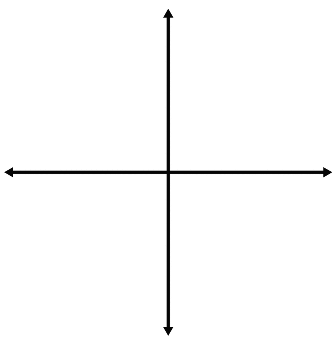
b. $\theta = -100°$



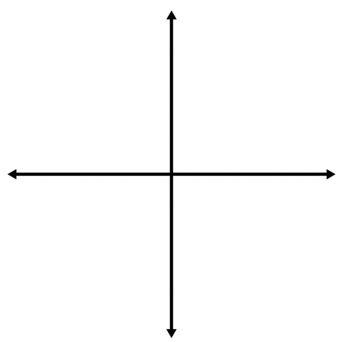
c. $\theta = -295°$



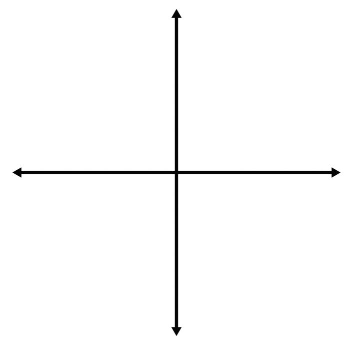
d. $\theta = \frac{3\pi}{4}$



e. $\theta = \frac{5\pi}{3}$

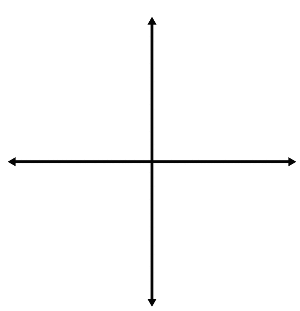


f. $\theta = -\frac{7\pi}{4}$

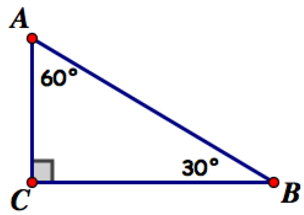


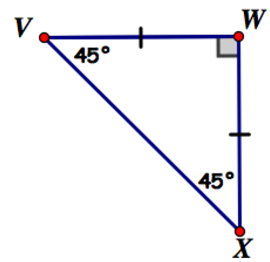
Evaluating Trigonometric Functions

Reference angles allow you to evaluate a trigonometric function for any angle θ. The sign of the trigonometric function depends on the quadrant in which θ lies.



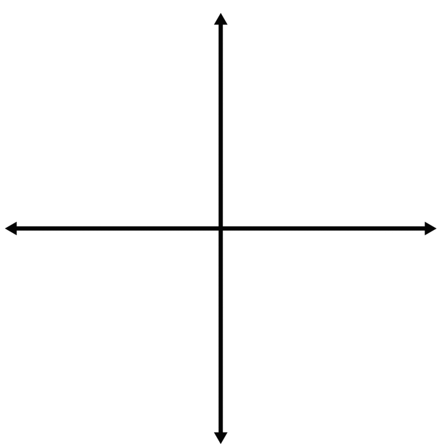
Example 3: Special Right Triangle Review.
Fill in the following two special right triangles if you are given that the shortest side of each triangle is 1 unit.



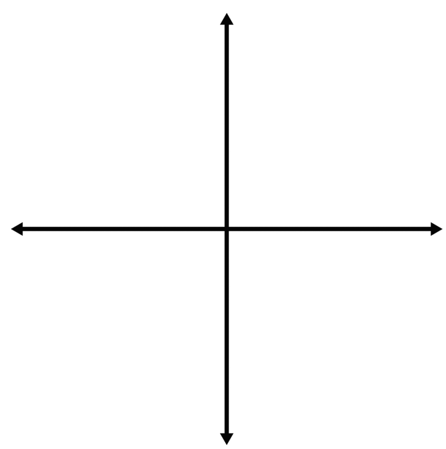


Example 4: Use Reference Angles to Evaluate Functions.
Find the exact value of each of the following trigonometric functions:

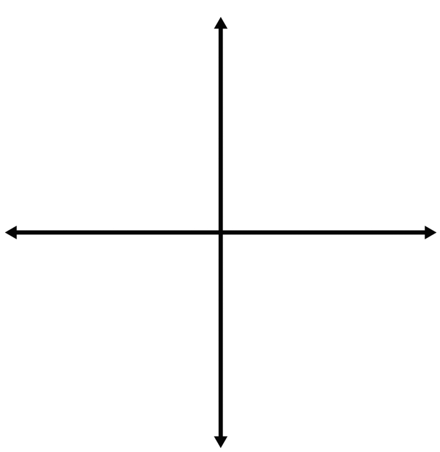
a. $\tan 300°$



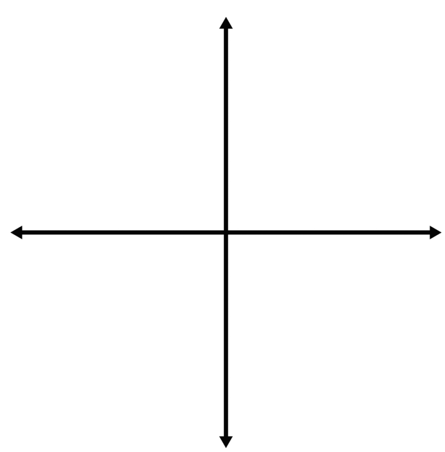
b. $\sin\frac{3\pi}{4}$



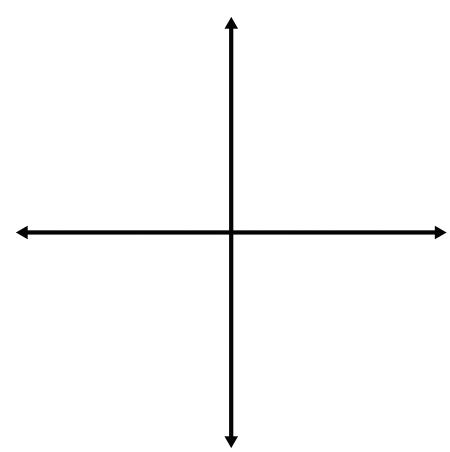
c. $\csc 240°$



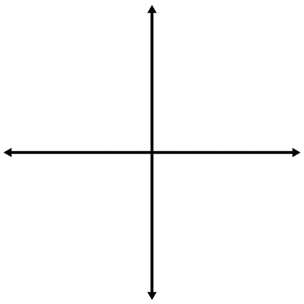
d. $\sin\left(-\frac{7\pi}{4}\right)$



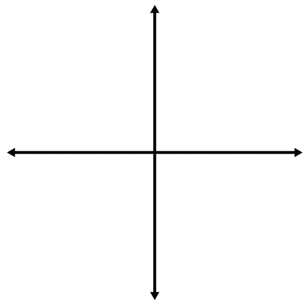
e. $\cos(-210°)$



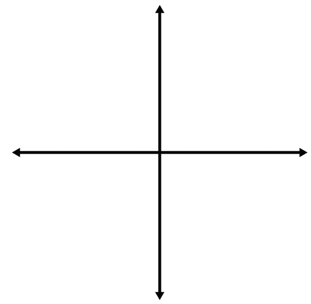
f. $\cot\left(-\frac{\pi}{6}\right)$



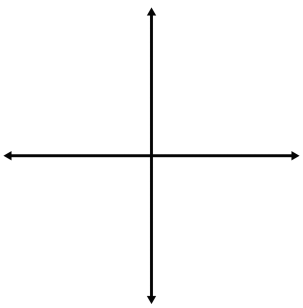
g. $\tan(270°)$



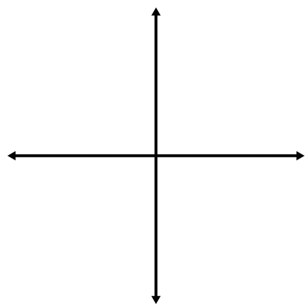
h. $\sec(-4\pi)$



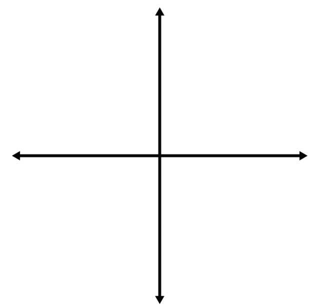
i. $\sin 240°$



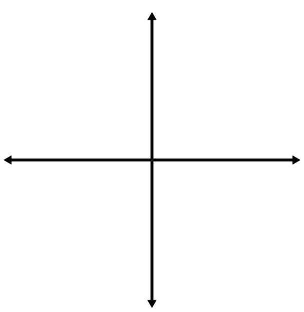
j. $\tan(150°)$



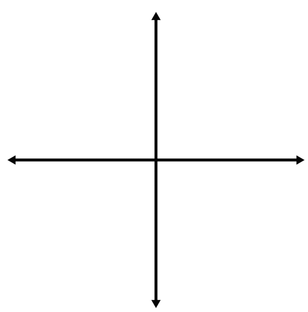
k. $\sec(-315°)$



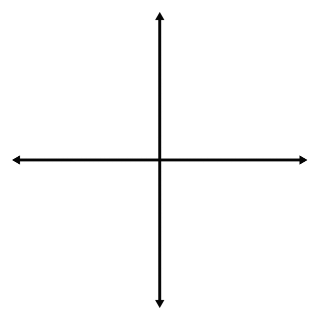
l. $\cot(-150°)$



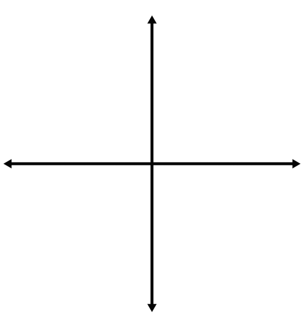
m. $\cos\left(-\frac{3\pi}{4}\right)$



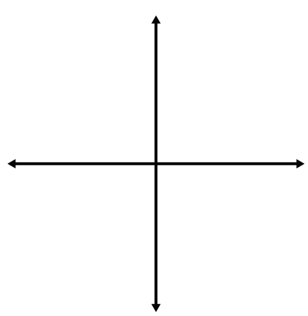
n. $\csc\frac{7\pi}{6}$



o. $\tan\left(\frac{8\pi}{3}\right)$



p. $\sin\left(-\frac{5\pi}{6}\right)$



Lesson 5: Basic Trigonometric Identities

Objectives:

- *Develop basic trigonometric identities.*

Warm Up

Identify the trigonometric function equivelant to the given function.

a. $\frac{1}{\tan\theta}$

b. $\frac{\sin\theta}{\cos\theta}$

c. $\frac{1}{\sec\theta}$

d. $\frac{1}{\csc\theta}$

Fundamental Trigonometric Identities

A trigonometric equation that is true for all values of the variables for which both sides of the equation are defined is called a ______________________ ________________.

Reciprocal Identities

$\frac{1}{\sin\theta} =$ $\frac{1}{\cos\theta} =$ $\frac{1}{\tan\theta} =$

Tangent and Cotangent Identities

$\frac{\sin\theta}{\cos\theta} =$ $\frac{\cos\theta}{\sin\theta} =$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta =$ $1 + \tan^2\theta =$ $1 + \cot^2\theta =$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) =$ $\cos\left(\frac{\pi}{2} - \theta\right) =$ $\tan\left(\frac{\pi}{2} - \theta\right) =$

Negative Angle Identities

$\sin(-\theta) =$ $\cos(-\theta) =$ $\tan(-\theta) =$

Example 1: Find Trigonometric Values

Use the given information to find the values of the other five trigonometric functions of θ.

a. $\sin\theta = \frac{4}{5}$ *and* $\frac{\pi}{2} < \theta < \pi$

b. $\tan\theta = \frac{5}{12}$ *and* $\pi < \theta < \frac{3\pi}{2}$

Example 2: Simplify a Trigonometric Expression

Simplify the expression

a. $\tan\left(\frac{\pi}{2}-\theta\right)\sin\theta$

b. $\tan(-\theta)\cos\theta$

c. $\csc\theta\cot^2\theta+\frac{1}{\sin\theta}$

d. $\sin\theta+\cos\theta\cot\theta$

e. $\sin x\cot x\sec x$

f. $\frac{\tan x \csc x}{\sec x}$

g. $\frac{\cos\left(\frac{\pi}{2}-\theta\right)-1}{1+\sin(-\theta)}$

Verifying Identities
You can use the fundamental identities from this chapter to ___________ new trigonometric identities. When verifying an identity, begin with the expression on one side. Use _____________ and trigonometric properties to manipulate the expression until it is __________________ to the other side.

Example 3: Verify a Trigonometric Identity
Verify the identity

a. $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

b. $\sec\theta - \cos\theta = \sin\theta\tan\theta$

c. $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$

d. $\cot(-\theta) = -\cot\theta$

e. $\csc^2 x(1 - \sin^2 x) = \cot^2 x$

f. $\cos x \csc x \tan x = 1$

g. $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

Unit 2 Lesson 5 - Verify Trigonometric Identities - Practice

For numbers 1 & 2: Find the values of the other five trigonometric functions of θ.

1. $\sin\theta = \frac{-15}{17},\ \pi < \theta < \frac{3\pi}{2}$

2. $\cot\theta = \frac{-2}{5},\ \frac{\pi}{2} < \theta < \pi$

3. $\tan\theta = \frac{3}{4},\ 0 < \theta < \frac{\pi}{2}$

4. $\cos\theta = \frac{5}{6},\ \frac{3\pi}{2} < \theta < 2\pi$

For numbers 3-8: Simplify the expression.

3. $\sec(-x)\cot(-x)\sin(-x)$

4. $\frac{\cos^2 x}{\sin x} + \sin x$

5. $\frac{1-\cos^2 x}{\cos^2 x}$

6. $\sin^3 x + \cos\left(\frac{\pi}{2} - x\right)\cos^2 x$

7. $\csc(-x) - \csc(-x)\cos^2 x$

For numbers 8-13: Verify the Identity.

8. $\cos x \sec x = 1$

9. $1 - \tan^2 x = 2 - \sec^2 x$

10. $\dfrac{\tan^2 x}{\sec x} = \sec x - \cos x$

11. $\tan\left(\dfrac{\pi}{2} - x\right)\sin x = \cos x$

12. $\dfrac{\cos^2 x}{1+\tan^2 x} + \dfrac{\sin^2 x}{\sec^2 x} = \cos^2 x$

13. $\dfrac{\sin\left(\dfrac{\pi}{2} - x\right) - 1}{1 - \cos(-x)} = -1$

For numbers 14-24: Simplify the expression.

14. $\dfrac{\sin(-\theta)}{\cos(-\theta)}$

15. $\cos\theta + \cos\theta\tan^2\theta$

16. $\dfrac{\cos\left(\frac{\pi}{2} - x\right)}{\csc x}$

17. $\sin\left(\frac{\pi}{2} - \theta\right)\sec\theta$

18. $\tan(-x)\csc x$

19. $-\cos x\tan(-x)$

20. $\sec x \tan^2 x + \sec x$

21. $\sin\left(\frac{\pi}{2} - x\right)\tan x$

22. $\frac{\sin(-\theta)}{\tan(-\theta)}$

23. $\cos^2 x + \sin^2 x - \csc^2 x$

24. $\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sec x}$

For numbers 25-31: Verify the Identity.

25. $\sin x \csc x = 1$

26. $\dfrac{\cos\left(\frac{\pi}{2} - \theta\right) + 1}{1 - \sin(-\theta)}$

27. $\dfrac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \sec^2 \theta$

28. $\sin x + \cos x \cot x = \csc x$

29. $\dfrac{\cot^2\theta}{\csc\theta} = \csc\theta - \sin\theta$

30. $\dfrac{\sin^2 x - 1}{\cot^2 x} = -\sin^2 x$

31. $\tan\left(\dfrac{\pi}{2} - x\right)\cot x = \csc^2 x - 1$