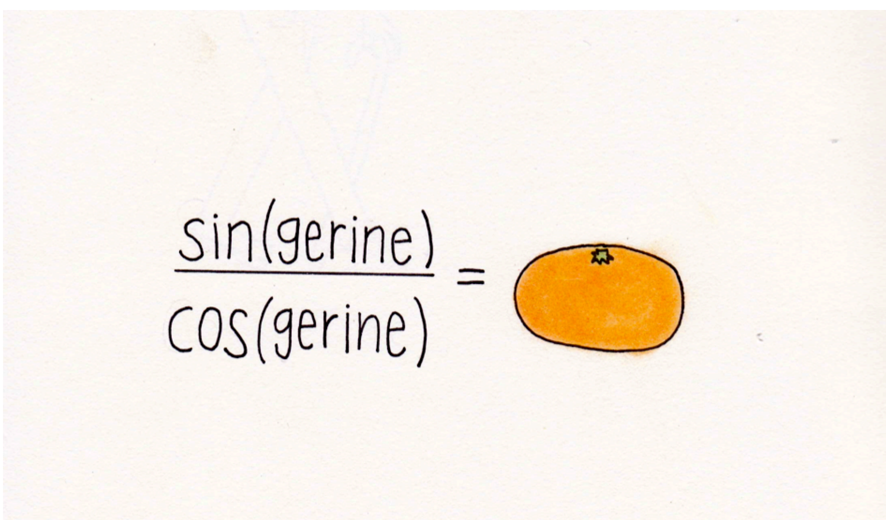
NAME: ____________________ **PERIOD: ___**

DATE: ______________ **PRE-CALCULUS**

MR. MELLINA

UNIT 4: SOLVING TRIGONOMETRIC EQUATIONS

- *Lesson 1: Graphical Solutions to Trigonometric Equations*
- *Lesson 2: Inverse Trigonometric Functions*
- *Lesson 3: Algebraic Solutions of Trigonometric Equations*
- *Lesson 4: Simple Harmonic Motion & Modeling*



Lesson 1: Graphical Solutions to Trigonometric Equations

Objectives:

- *Solve trigonometric equations graphically.*
- *State the complete solution of a trigonometric equation.*

Warm Up 🔥

An equation that involves a single trigonometric function set equal to a number is called a basic equation. Solve the following basic equations graphically using a graphing utility.

a. $\tan x = 2$ b. $\sin x = -0.75$

Solving Trigonometric Equations Graphically

- *Write the equation in the form* $f(x) = 0$.
- *Determine the period, p, of* $f(x)$.
- *Graph* $f(x)$ *over an interval of length p.*
- *Use a calculator's zero finder to determine the x-intercepts of the graph in this interval.*
- *For each x-intercept u, all of the numbers*

  $u + kp$ *where k is any integer are solutions of the equation.*

Example 1: Other Trigonometric Equations

Solve for x (in radians).

a. $3\sin^2 x - \cos x - 2 = 0$ b. $\tan x = 2\sin 2x$

Example 2: Trigonometric Equations in Degree Measure

Solve for x (in degrees)

a. $2\sin^2\theta - 3\sin\theta - 3 = 0$

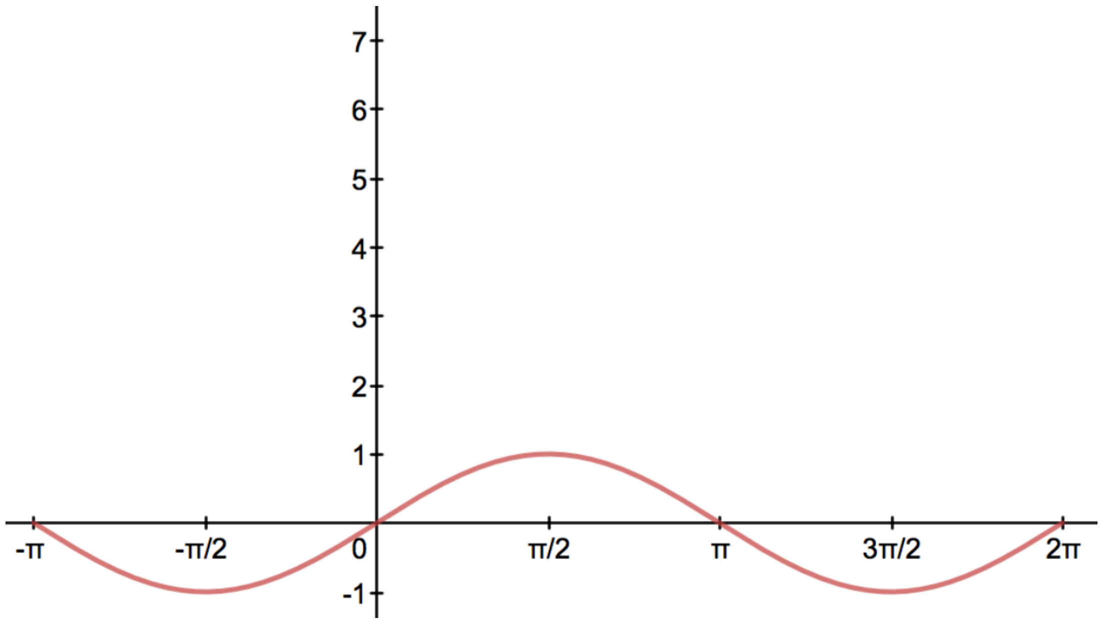
Lesson 2: Inverse Trigonometric Equations

Objectives:

- *Define the domain and range of the inverse trigonometric functions.*
- *Use inverse trigonometric function notation.*

Warm Up 🔥

The following graph is $\sin x$. Graph the inverse $\sin^{-1} x$.



Inverse Sine Function

For each v with $-1 \leq v \leq 1$, $\sin^{-1} v$ is the unique number v in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is v, that is,

$\sin^{-1} v = u$ *exactly when* $\sin u = v$

Example 1: Special Values

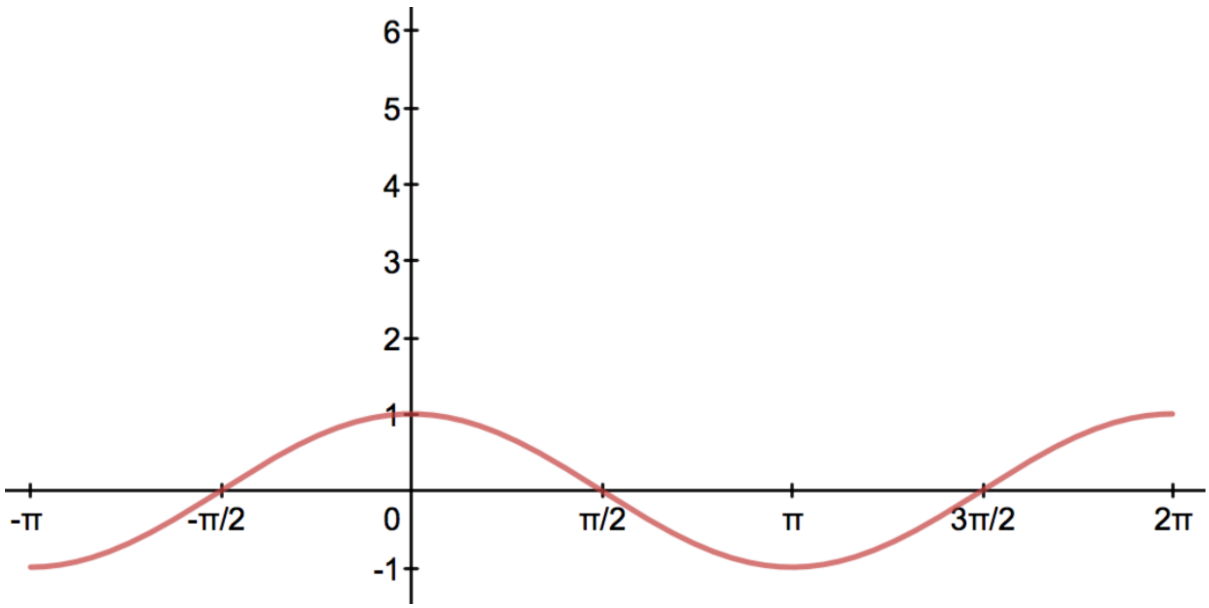
Evaluate

a. $\sin^{-1}\left(\frac{1}{2}\right)$

b. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Example 2: Investigating Cosine

The following graph is $\cos x$. Graph the inverse $\cos^{-1} x$.



Inverse Cosine Function

For each v with $-1 \leq v \leq 1$, $\cos^{-1} v$ is the unique number v in the interval $[0, \pi]$ whose cosine is v, that is,

$$\cos^{-1} v = u \qquad \textit{exactly when} \qquad \cos u = v$$

Example 3: Evaluating Inverse Cosine Expressions

Evaluate

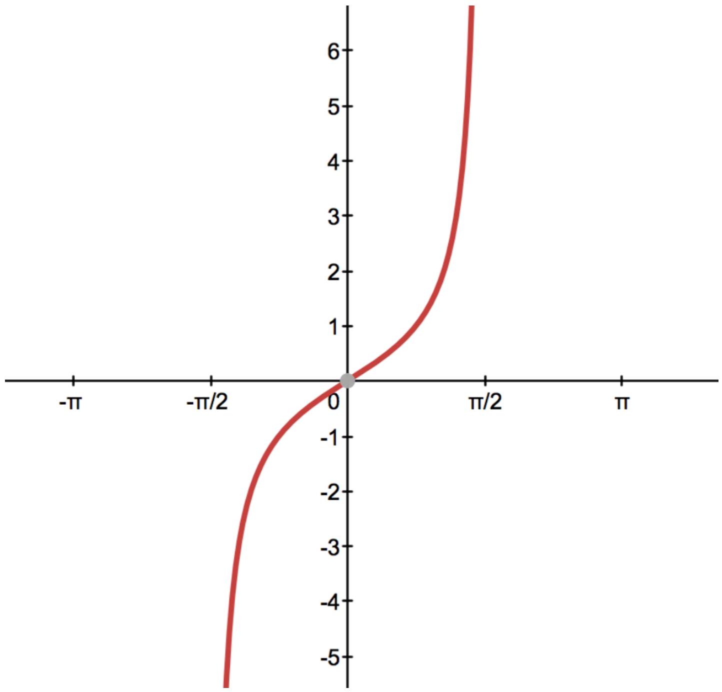
a. $\cos^{-1}\left(\frac{1}{2}\right)$

b. $\cos^{-1} 0$

c. $\cos^{-1}(-0.63)$

Example 4: Investigating Tangent

The following graph is $\tan x$. Graph the inverse $\tan^{-1} x$.



Inverse Tangent Function

For each real number v, $\tan^{-1} v$ *is the unique number v in the interval* $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *whose tangent is v, that is,*

$\tan^{-1} v = u$ *exactly when* $\tan u = v$

Example 5: Evaluating Inverse Tangent Expressions

Evaluate

a. $\tan^{-1}(1)$

b. $\tan^{-1} 136$

Evaluating 6: Exact Values

Find the exact value.

a. $\cos\left(\tan^{-1}\frac{\sqrt{5}}{2}\right)$

Lesson 3: Algebraic Solutions of Trigonometric

Objectives:

- *Define the domain and range of the inverse trigonometric functions.*
- *Use inverse trigonometric function notation.*

Warm Up 🔥

Solve the equation.

a. $3x^2 - 5x - 2 = 0$

Example 1: Solving Basic Equations

Solve

a. $\cos x = 0.6$

b. $\sin x = -0.75$

c. $\tan x = 3$

Example 2: Solving Basic Equations with Special Values

Solve

a. $\sin x = \frac{\sqrt{2}}{2}$

b. $\sin 2x = \frac{\sqrt{2}}{2}$

c. $3\sin^2 x - \sin x - 2 = 0$ on $[-\pi, \pi]$

d. $\tan x \cos^2 x = \tan x$

e. $-10\cos^2 x - 3\sin x + 9 = 0$

f. $\sec^2 x + 5\tan x = -2$

Lesson 4: Simple Harmonic Motionand Modeling

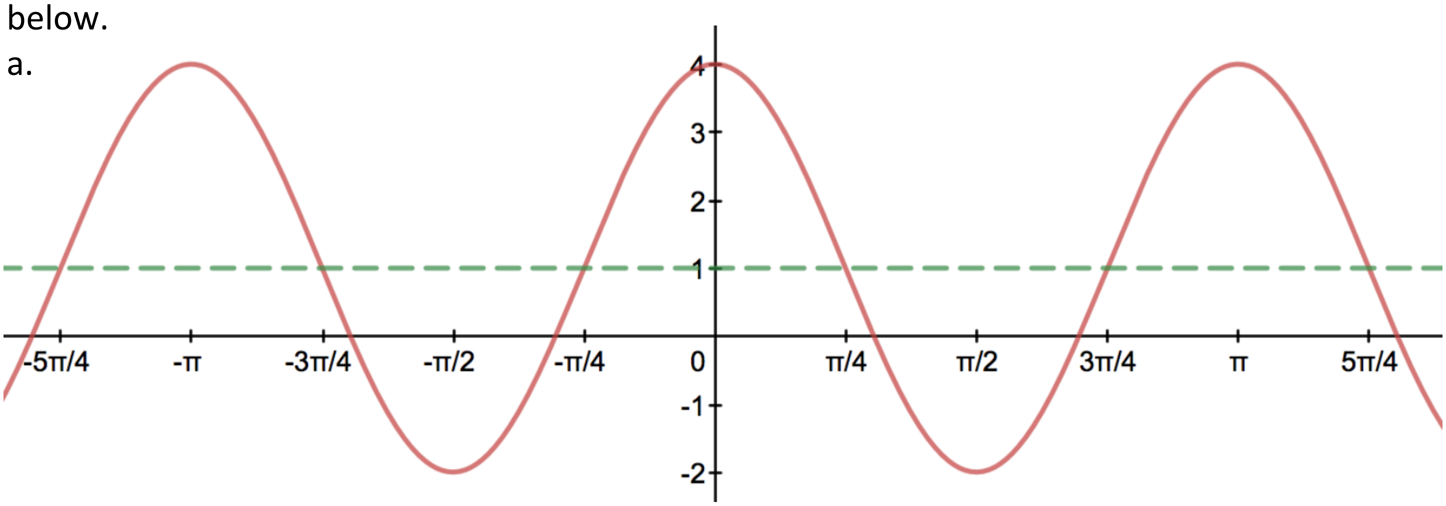
Objectives:

- *Write a sinusoidal function whose graph resembles a given graph.*
- *Write a sinusoidal function to represent a given simple harmonic motion, and use the function to solve problems.*
- *Find a sinusoidal model for a set of data, and use the model to make predictions.*

Warm Up 🔥

Write a sine function and a cosine function whose graph resembles the sinusoidal graph below.

a.



Simple Harmonic Motion

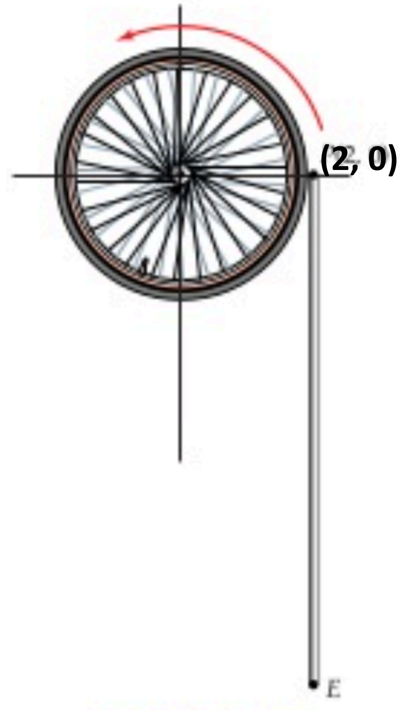
Motion that can be described by a function of the form

$$f(t) = a\sin(bt + c) + d \quad \text{or} \quad g(t) = a\cos(bt + c) + d$$

*is called **simple harmonic motion**. Many kinds of physical motion are simple harmonic motions.*

Example 1: Rotating Wheel

A wheel with a radius of 2 centimeters is rotating counterclockwise at 3 radians per second. A free-hanging rod 10 centimeters long is connected to the edge of the wheel at point *P* and remains vertical as the wheel rotates.



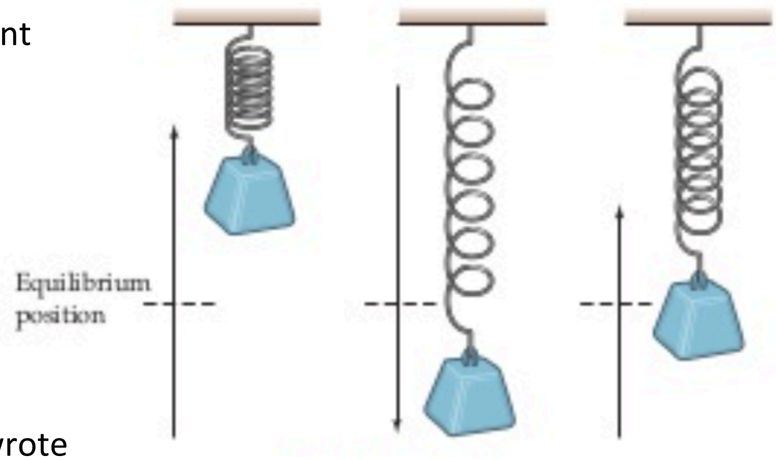
a. Assuming that the center of the wheel is at the origin and that *P* is at $(2,0)$ at time $t = 0$, find a function that describes the y −coordinate of the tip *E* of the rod at time t.

b. What is the first time that the tip *E* of the rod will be at height of -9 centimeters.

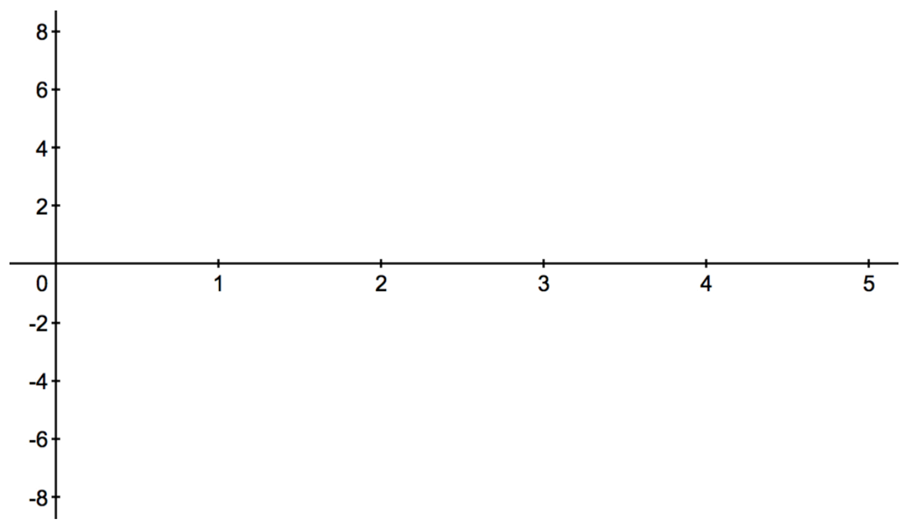
Example 2: Bouncing Spring

Suppose that a weight hanging from a spring is set in motion by an upward push. It takes 5 seconds for it to complete one cycle of moving from its equilibrium position to 8 centimeters above, then dropping to 8 centimeters below, and finally returning to its equilibrium position. (This is an idealized situation in which the spring has perfect elasticity, and friction, air resistance, etc., are negligible.

a. Find a sinusoidal function to represent the motion of the moving spring.



b. Sketch a graph of the function you wrote in part **a**.



c. Use the function from part **a** to predict the height of the weight after 3 seconds.

d. In the first 5 seconds, when will the height of the weight be 6 centimeters below the quilibrium postion?

Example 3: Temperature Data

The following table shows the average monthly temperature in Cleveland, Ohio, based on data from 1961 to 1990. Because average temperatures are not likely to vary much from year to year, the data essentially repeats the same pattern in subsequent years. So, a period model is appropriate.

Month	Temperature (°F)	Month	Temperature (°F)
Jan.	25.7	July	71.9
Feb.	28.4	Aug.	70.2
Mar.	37.5	Sep.	63.3
Apr.	47.6	Oct.	52.2
May	58.5	Nov.	41.8
June	67.5	Dec.	31.1

a. Make a scatter plot of the data and use the sine regression feature on a calculator to find another sinusoidal model for the data.

b Use the model to predict time(s) of year in which the average temperature is 45°F.