Name ____ Period ______

GEOMETRY CHAPTER 3 Perpendicular and Parallel Lines

Section 3.1 Lines and Angles

GOAL 1: Relationship between lines

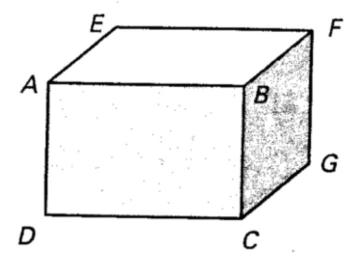
Two lines are _______________________ if they are coplanar and do not intersect.

Skew lines ____.

Two planes that do not intersect are called ____.

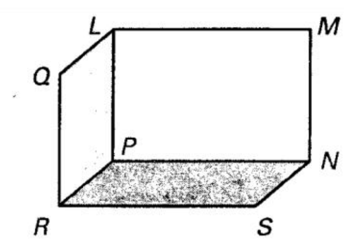
Ex. 1 Think of each segment in the diagram as part of a line. Fill in the blank with *parallel, skew,* or *perpendicular.*

1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are
2. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are
3. $\overleftrightarrow{BF}$ and $\overleftrightarrow{FG}$ are
4. $\overleftrightarrow{AB}$ and $\overleftrightarrow{FG}$ are



Ex. 2 Think of each segment in the diagram as part of a line. There may be more than one right answer.

5. Name a line parallel to $\overleftrightarrow{MN}$.
6. Name a line perpendicular to $\overleftrightarrow{PR}$.
7. Name a line skew to $\overleftrightarrow{SN}$.
8. Name a plane parallel to plane *RPL*.



Parallel and perpendicular Postulates

Postulate 13 Parallel Postulate
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

Postulate 14 Perpendicular Postulate
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

You can use a compass and a straightedge to construct the line that passes through a given point and is perpendicular to a given line.

GOAL 2: Identifying angles formed by transversals.

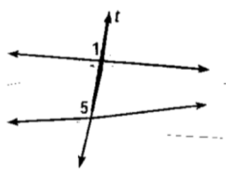
A ____________________ is a line that intersects two or more coplanar lines at different points.

Two angles are **corresponding angles** if ____.

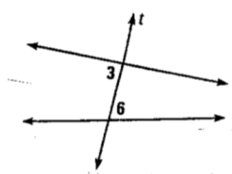
Two angles are **alternate interior angles** if ____ ______________________________________.

Two angles are **alternate exterior angles** if ____ ______________________________________.

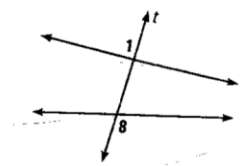
Two angles are **consecutive interior angles** if ____ ______________________________________.



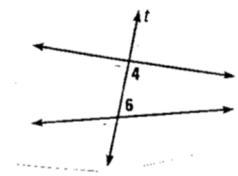
corresponding angles



alternate interior angles



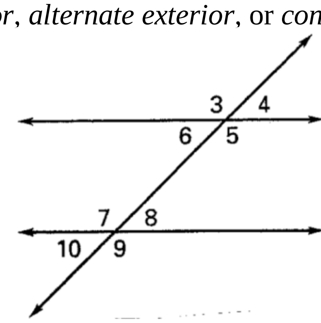
alternate exterior angles



consecutive interior angles

Ex. 3 Complete the statement with *corresponding, alternate interior*, *alternate exterior*, or *consecutive interior.*

9. $\angle 3$ and $\angle 7$ are _______ angles.
10. $\angle 4$ and $\angle 10$ are _______ angles.
11. $\angle 5$ and $\angle 8$ are _______ angles.
12. $\angle 8$ and $\angle 6$ are _______ angles.
13. $\angle 9$ and $\angle 5$ are _______ angles.
14. $\angle 5$ and $\angle 7$ are _______ angles.



Section 3.2 Proof and Perpendicular Lines

GOAL 1: Comparing types of proofs.

There is more than one way to write a proof. Here are three different ways:

1. **TWO-COLUMN PROOF:** This is the most formal type of proof. It lists numbered statements in the left column and a reason for each statement in the right column.
2. **PARAGRAPH PROOF:** This type of proof describes the logical argument with sentences.
3. **FLOW PROOF:** This type of proof used the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows.

GOAL 2: Proving results about perpendicular lines

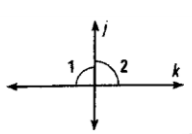
THEOREM 3.1 If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

THEOREM 3.2 If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

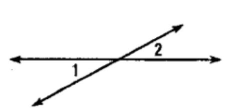
THEOREM 3.3 If two lines are perpendicular, then they intersect to form four right angles.

Ex. 1 Write the postulate of theorem that justifies the statement (a, b), in c., d., given that $g \perp h$.

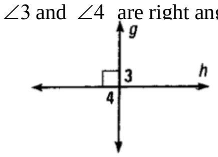
a.



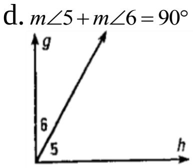
b.



c. $\angle 3$ and $\angle 4$ are right angles

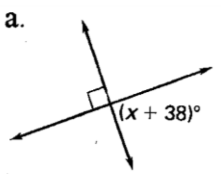


d. $m\angle 5 + m\angle 6 = 90°$

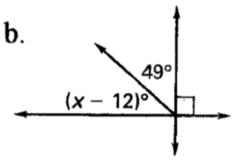


Ex. 2 Find the value of x.

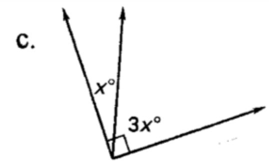
a.



b.



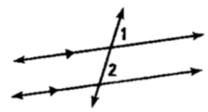
c.



SECTION 3.3 Parallel Lines and Transversals

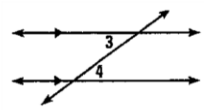
GOAL 1: Properties of Parallel lines

Postulate 15 Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then the pair of corresponding angles are congruent.



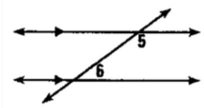
$\angle 1 \cong \angle 2$

Theorem 3.4 Alternate Interior Angles
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



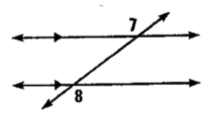
$\angle 3 \cong \angle 4$

Theorem 3.5 Consecutive Interior Angles
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



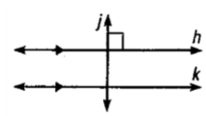
$m\angle 5 + m\angle 6 = 180°$

Theorem 3.6 Alternate Exterior Angles
If two parallel lines are cut by a transversal, then the Pairs of alternate exterior angles are congruent.



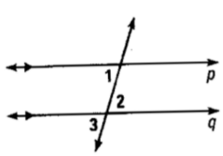
$\angle 7 \cong \angle 8$

Theorem 3.7 Perpendicular Transversal
If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



$j \perp k$

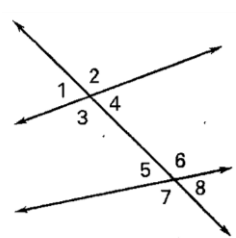
Ex. 1 Proving the Alternate Interior Angles Theorem
Given: $p \parallel q$
Prove: $\angle 1 \cong \angle 2$



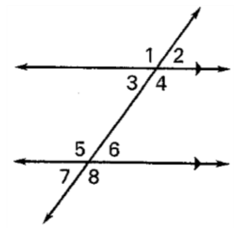
Statements	Reasons
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

Ex. 2 Name the relationship between the pair of angles.

1. $\angle 1$ and $\angle 5$
2. $\angle 2$ and $\angle 7$
3. $\angle 3$ and $\angle 6$
4. $\angle 8$ and $\angle 5$
5. $\angle 4$ and $\angle 6$
6. $\angle 8$ and $\angle 4$

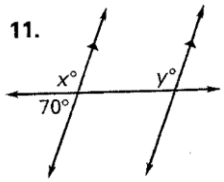


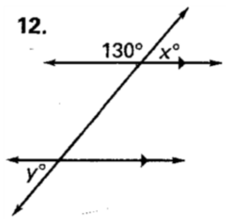
Ex. 3 State the postulate or theorem that justifies the statement.

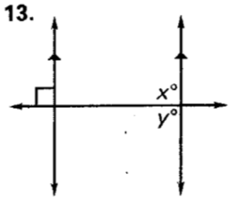


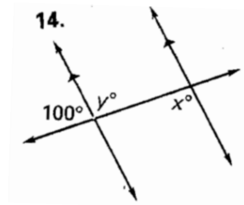
7. $\angle 3 \cong \angle 7$
8. $\angle 3 \cong \angle 6$
9. $\angle 2 \cong \angle 7$
10. $m\angle 4 + m\angle 6 = 180°$

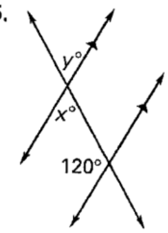
Ex. 4 Find the values of *x* and *y*.

11.


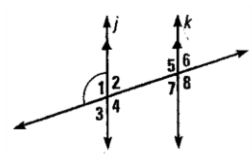
12.


13.


14.


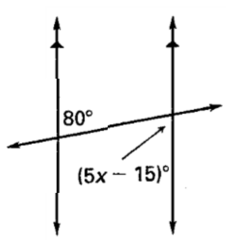
15.


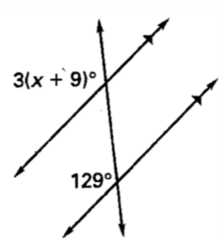
Ex. 5 Use the given information to find the measures of the other seven angles in the figure at the right.
Given: $j \parallel k$, $m\angle 1 = 110°$

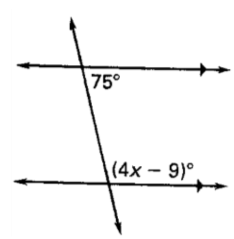


GOAL 2: Properties of Special Pairs of Angles

Ex. 6 Find the value of x.

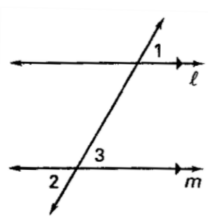
1.


2.


3.


Ex. 7 Complete the flow proof of the Alternate Exterior Angles Theorem.

Given: $\ell \parallel m$
Prove: $\angle 1 \cong \angle 2$



$\boxed{\ell \parallel m}$

a. ______________

$\boxed{\angle 1 \cong \angle 3}$ $\boxed{\angle 3 \cong \angle 2}$

b. ______________ **c.** ______________

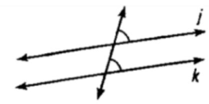
$\boxed{\angle 1 \cong \angle 2}$

d. ______________

SECTION 3.4 Proving Lines are Parallel

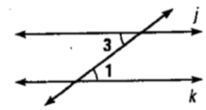
GOAL 1: Proving Lines are Parallel

Postulate 16 Corresponding Angles Converse
If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



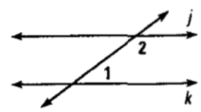
$j \parallel k$

Theorem 3.8 Alternate Interior Angles Converse
If two lines are cut by a transversal so the alternate interior\ angles are congruent, then the lines are parallel.



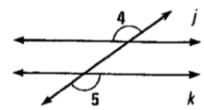
If $\angle 1 \cong \angle 3$, then $j \parallel k$.

Theorem 3.9 Consecutive Interior Angles Converse
If two lines are cut by a transversal so that consecutive angles angles are supplementary, then the lines are parallel.



If $m\angle 1 + m\angle 2 = 180°$, then $j \parallel k$.

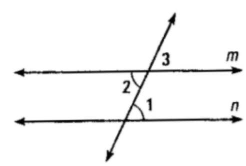
Theorem 3.10 Alternate Exterior Angles Converse
If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.



If $\angle 4 \cong \angle 5$, then $j \parallel k$.

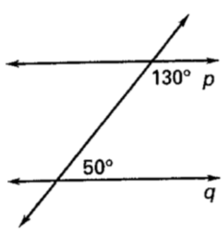
Ex. 1 Proof of the Alternate Interior Angles Converse.
Given: $\angle 1 \cong \angle 2$
Prove: $m \parallel n$

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2.
3. $\angle 1 \cong \angle 3$	3.
4. $m \parallel n$	4.

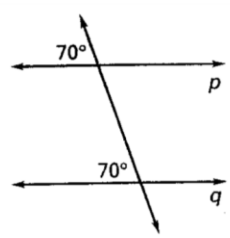


Ex. 2 Is it possible to prove that lines p and q are parallel? If so, state the postulate or theorem you would use.

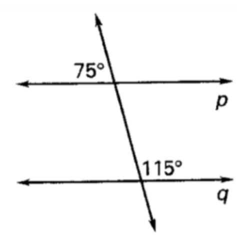
1.



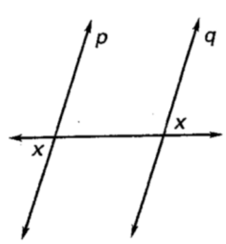
2.



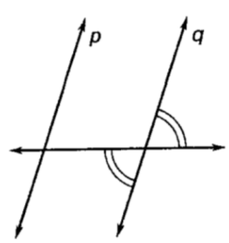
3.



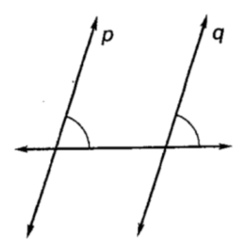
4.



5.

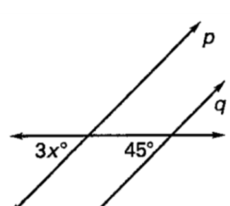


6.



Ex. 3 Find the value of x that makes $p \parallel q$.

7.



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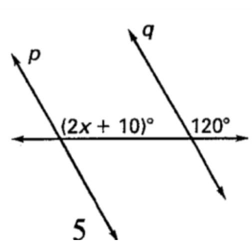

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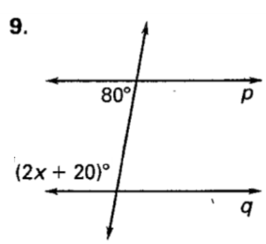
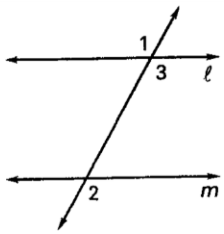


GOAL 2: Using the Parallel Converses

Ex. 4 Complete the two-column proof of the Alternate Exterior Angles Converse Theorem.

Given: $\angle 1 \cong \angle 2$

Prove: $l \parallel m$

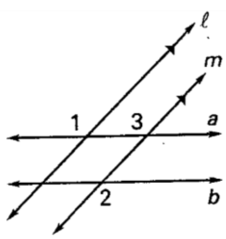


Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ______
2. $\angle 1 \cong \angle 3$	2. ______
3. $\angle 2 \cong \angle 3$	3. ______
4. $\ell \parallel m$	4. ______

Ex. 5 Complete the two-column proof.

Given: $l \parallel m$, $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

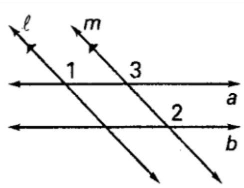


Statements	Reasons
1. $\ell \parallel m$	1. ______
2. $\angle 1 \cong \angle 3$	2. ______
3. $\angle 1 \cong \angle 2$	3. ______
4. $\angle 2 \cong \angle 3$	4. ______
5. $a \parallel b$	5. ______

Ex. 6 Write a two-column proof.

Given: $l \parallel m$, $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

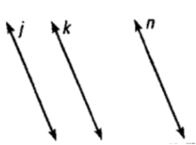


SECTION 3.5 Using Properties of Parallel Lines

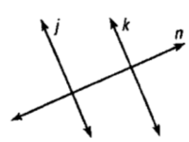
GOAL 1: Using Properties of Parallel Lines

Ex. 1 State the postulate or theorem that allows you to conclude that $j \parallel k$.

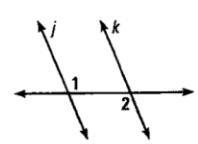
1. GIVEN ▸ $j \parallel n$, $k \parallel n$



2. GIVEN ▸ $j \perp n$, $k \perp n$

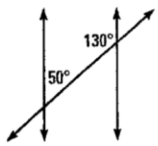


3. GIVEN ▸ $\angle 1 \cong \angle 2$



Ex. 2 Explain how you would show that $k \parallel j$. State any theorems or postulates that you would use.

4.



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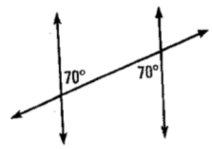

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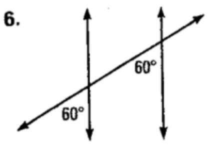
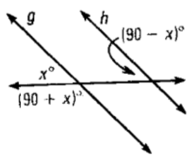


Ex. 3 Explain how you would show that $g \parallel h$

7.



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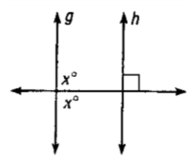

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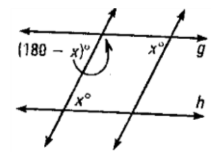
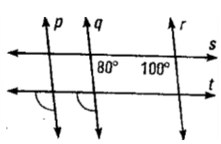


Ex. 4 Determine which lines, if any, must be parallel.

10.



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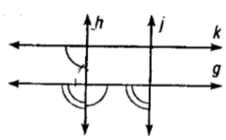

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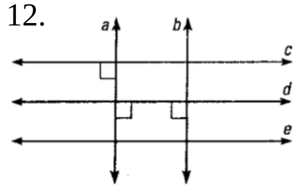

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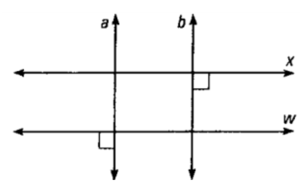


GOAL 2: Constructing Parallel Lines

Copy an angle.

Construct parallel lines.

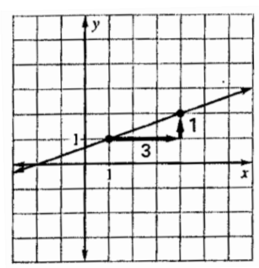
SECTION 3.6 Parallel Lines in the Coordinate Plane

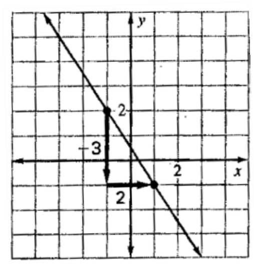
GOAL 1: Slope of Parallel Lines

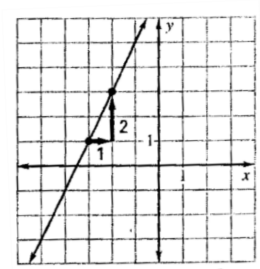
In algebra, you learned that the slope of a nonvertical line is the ratio of the vertical change (the rise) to the horizontal change (the run.) If the line passes through the points (x_1, y_1) and (x_2, y_2), then the slope is given by

$$\text{Slope} = \frac{\text{rise}}{\text{run}} \qquad \text{or} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

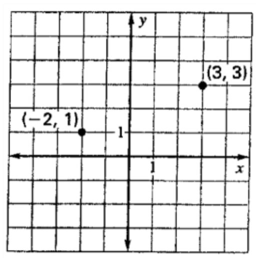
Ex. 1 Calculate the slope of the line shown.

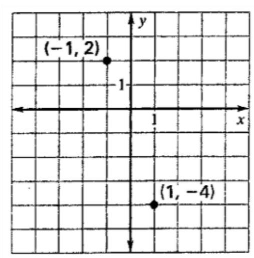
1. 

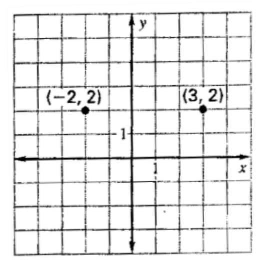
2. 

3. 

Ex. 2 Calculate the slope of the line that passes through the labeled points on the graph.

4. 

5. 

6. 

Ex. 3 Find the slope between the two points.

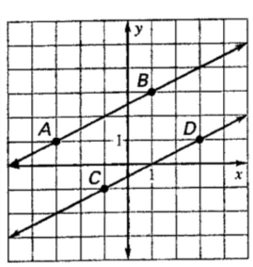
a. A(0, −6)
   B(2, 4)

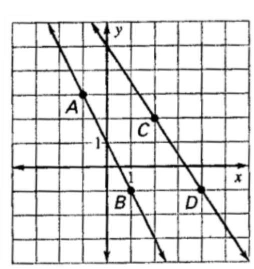
b. C(−4, 10)
   D(−8, −7)

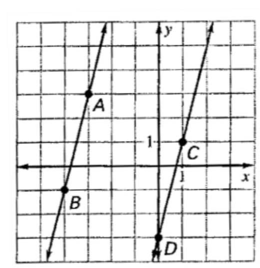
Postulate 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

Ex. 4 Find the slope of each line. Are the lines parallel?

7. 

8. 

9. 

GOAL 2: Writing Equations of Parallel Lines

In algebra, you learned that you can use the slope *m* of a nonvertical line to write an equation of the line in *slope-intercept form.*

Slope-intercept form: $y = mx + b$, where m = slope and b = y-intercept

The *y*-intercept is the y-coordinate of the point where the line crosses the y-axis.

Ex. 5

a. slope = 3

b. slope = ¾

c. slope = −½

y-intercept = 2 y-intercept = −5 y-intercept = 0

Ex. 6 Write an equation of the line that has a y-intercept of 3 and is parallel to the line whose equation is given.

a. $y = -6x + 2$ b. $y = x + 4$

Ex. 7 Write an equation on the line through the point (2, 3) that has a slope of 5.

Ex. 8 Write an equation on the line that passes through the given point P and has the given slope.

a. P(−3, 9), $m = -1$ b. P(2, −4), $m = 0$

Ex. 9 Write an equation on the line that passes through the given point (4, 6) and is parallel to $y = 4x - 3$

SECTION 3.7 Perpendicular Lines in the Coordinate Plane

GOAL 1: Slope of Perpendicular Lines

Postulate 18 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

Vertical and horizontal lines are perpendicular.

Ex. 1 The slopes of two lines are given. Are the lines perpendicular?

a. $m_1 = \frac{3}{4}, m_2 = \frac{4}{3}$ b. $m_1 = -\frac{1}{2}, m_2 = 2$ c. $m_1 = -\frac{2}{3}, m_2 = \frac{3}{2}$

d. $m_1 = 2, m_2 = \frac{1}{2}$ e. $m_1 = -1, m_2 = 1$ f. $m_1 = 4, m_2 = -\frac{1}{4}$

If a nonvertical line is perpendicular to another line, the slopes of the lines are negative reciprocals of one another.

Ex. 2 Lines *j* and *n* are perpendicular. The slope of line *j* is given. What is the slope of line *n*? Check your answer.

a. ½ b. 6 c. − ¾ d. − 4 e. ⅝ f. ⅓ g. − 1

Ex. 3 Decide whether the two lines are perpendicular.

a. line p_1: $y = 3x + 5$

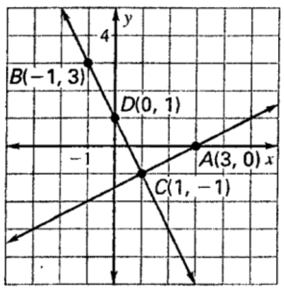
line p_1: $y = \frac{1}{3}x + 5$

b. line p_1: $3x + 5y = 12$

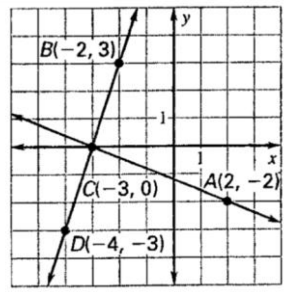
line p_1: $5x + 3y = 18$

Ex. 4 Find the slope of $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$. Decide whether they are perpendicular lines.

a.



b.



GOAL 2: Writing Equations of Perpendicular Lines

Ex. 5 Line *j* is perpendicular to the line with the given equation and line *j* passes through *P*. Write an equation of line *j*.

a. $y = \frac{1}{3}x + 4,\ P(0,5)$

b. $y = \frac{2}{3}x + 4,\ P(2,0)$

c. $y = -\frac{5}{6}x + 4,\ P(10,12)$

d. $y = 3x + 4,\ P(0,-2)$

Ex. 6 Decide whether the lines with the given equations are *parallel*, *perpendicular*, or *neither*.

a. $y = \frac{1}{3}x - 1$

$y = -3x + 2$

b. $y = -5x - 2$

$y = 5x + 2$

c. $$\begin{aligned} y &= \frac{5}{6}x + 8 \\ y &= -\frac{6}{5}x - 4 \end{aligned}$$

d. $$\begin{aligned} 2x - 5y &= 8 \\ 5x - 2y &= 2 \end{aligned}$$