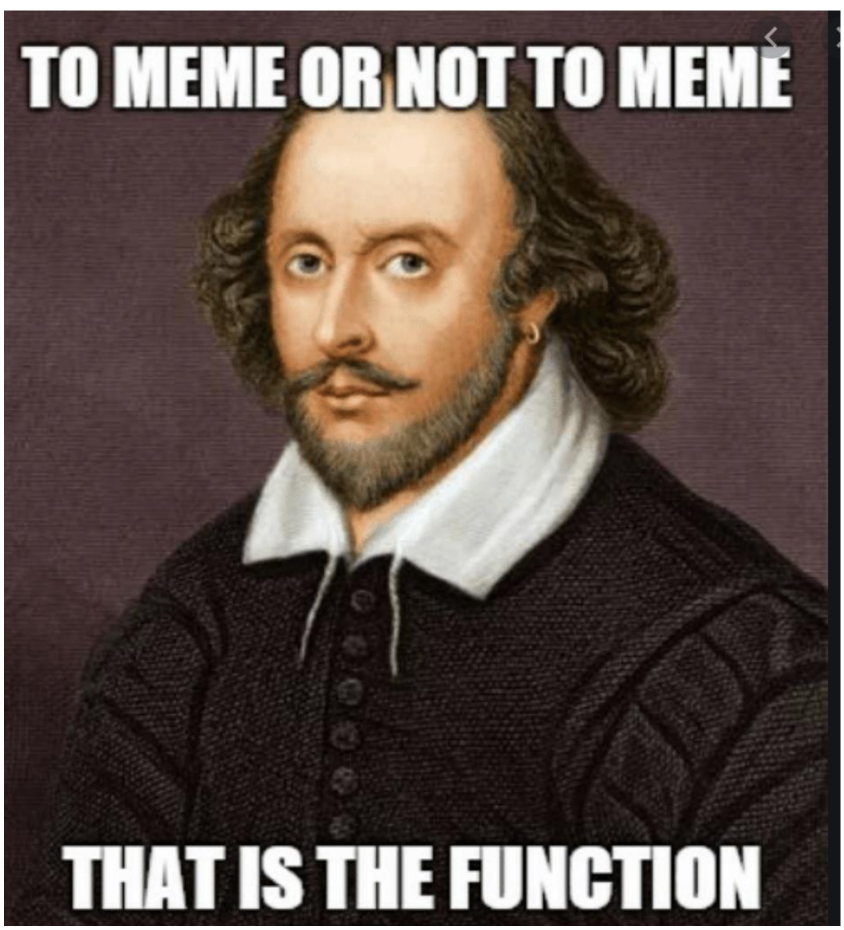
Name: ___________________________ **Pre-Calculus**

Date: ____________________ **Mr. Mellina**

Unit 1: Review of Algebra
Part A: Functions and Their Graphs

- *3.1 – Functions*
- *3.2 – Graphs of Functions*
- *3.3 – Quadratic Functions*
- *3.4 – Graphs & Transformations*
- *3.5 – Operations on Functions*
- *3.6 – Inverse Functions*
- *3.7 – Rates of Change*
- *3.8 – Systems of Equations*



3.1 – Functions

Objectives:

- Determine whether a relation is a function
- Find the domain of functions
- Evaluate piecewise-defined and greatest integer functions

Example 1: Determining Inputs and Outputs of Functions

Describe the set of inputs, the set of outputs, and the rule for the following functions.

a. The amount of income tax you pay depends on your income.

Set of Inputs *Set of Outputs* *Function Rule*

____________________ ____________________ ____________________

b. Suppose a rock is dropped straight down from a high place. Physics tells us that the distance traveled by the rock in *t* seconds is $16t^2$ feet.

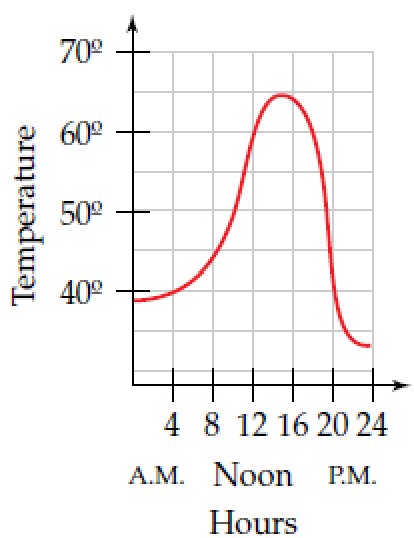
Set of Inputs *Set of Outputs* *Function Rule*

____________________ ____________________ ____________________

c. The weather bureau records the temperature over a 24-hour period in the form of a graph shown. The graph shows the temperature that corresponds to each given item.

Set of Inputs *Set of Outputs* *Function Rule*

____________________ ____________________ ____________________



A *function* consists of

- a set of inputs, called the *domain*
- a rule by which each input determines one and only one output
- a set of outputs, called the *range*

The phrase "one and only one" means that for each input (element of the domain), the rule of a function determines exactly one output (element of the range). However, different inputs may produce the same output.

Example 2: Determining Whether a Relation is a Function

The tables below list the inputs and outputs for two relations. Determine whether each relation is a function.

a.

Inputs	1	1	2	3	3
Outputs	5	6	7	8	9

b.

Inputs	1	3	5	7	9
Outputs	5	5	7	8	5

Example 3: Evaluating a Function

Find the indicated values of $f(x) = \sqrt{x^2 + 1}$.

a. $f(3)$

b. $f(-5)$

c. $f(0)$

d. $f(x) = 4$ when $x =$ ______

Example 4: Finding a Difference Quotient
For $f(x) = x^2 - x + 2$ and $h \neq 0$, find each output.

a. $f(x + h)$

b. $f(x + h) - f(x)$

c. $\frac{f(x+h)-f(x)}{h}$

Example 5: Determining if an Equation Defines a Function
Determine whether each equation defines y as a function of x.

a. $4x - 2y^3 + 5 = 0$

b. $y^2 - x + 1 = 0$

Domains

When the rule of a function is given by a formula, as in Examples 3–6, its domain (set of inputs) is determined by the following convention.

Unless information to the contrary is given, the domain of a function f consists of every real number input for which the function rule produces a real number output.

Thus, the domain of a polynomial function such as $f(x) = x^3 - 4x + 1$ is the set of all real numbers, since $f(x)$ is defined for every value of x. However, in cases where applying the rule of a function leads to one of the following, the domain may not consist of all real numbers.

- division by zero
- the square root of a negative number (or nth root, where n is even)

Example 6: Finding Domains of Functions

Find the domain for each function given below.

a. $k(x) = \frac{x^2-6x}{x-1}$

b. $f(a) = \sqrt{a+2}$

Example 7: Finding the Domain of a Profit Function

A glassware factory has fixed expenses (mortgage, taxes, machinery, etc.) of $12,000 per week. In addition, it costs 80 cents to make one cup (labor, materials, shipping). A cup sells for $1.95. At most, 18,000 cups can be manufactured each week. Let x represent the number of cups made per week.

a. Express the weekly revenue R as a function of x.

b. Express the weekly cost C as a function of x.

c. Find the rule and the domain of the weekly profit function P.

Piecewise-Defined and Greatest Integer Functions

A piecewise-defined function is one whose rule includes several formulas. The formula for each piece of the function is applied to certain values of the domain, as specified in the definition of the function.

Example 8: Evaluating a Piecewise-Defined Function

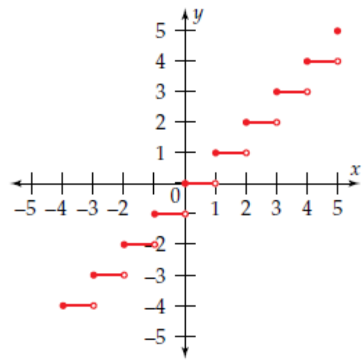
For the piecewise-defined function below, find the following.

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < 4 \\ x^2 - 1 & \text{if } 4 \le x \le 10 \end{cases}$$

a. $f(-5)$ b. $f(8)$ c. The domain of f

The **greatest integer function** is a piecewise-defined function with infinitely many pieces.

$$f(x) = \begin{cases} \vdots \\ -3 & \text{if } -3 \le x < -2 \\ -2 & \text{if } -2 \le x < -1 \\ -1 & \text{if } -1 \le x < 0 \\ 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x < 2 \\ 2 & \text{if } 2 \le x < 3 \\ \vdots \end{cases}$$



The rule can be written in words as follows:

For any number x, round *down* to the nearest integer less than or equal to x.

The domain of the greatest integer function is all real numbers, and the range is the set of integers. It is written as $f(x) = [x]$.

Example 9: Evaluating the Greatest Integer Function

Let $f(x) = [x]$. Evaluate the following:

a. $f(-4.7)$

b. $f(-3)$

c. $f(0)$

d. $f\left(\frac{5}{4}\right)$

e. $f(\pi)$

Practice Odd Numbered Questions as Necessary. Answers are at the back of the textbook.

Exercises 3.1

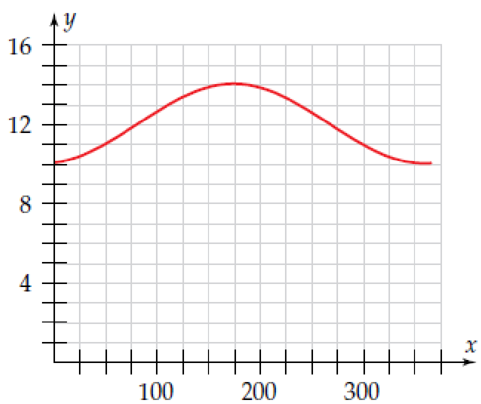
In Exercises 1–4, describe the set of inputs, the set of outputs, and the rule for each function.

1. The amount of your paycheck before taxes is a function of the number of hours worked.

2. Your shoe size is a function of the length of your foot.

3. In physics, the pressure P of a gas kept at a constant volume is a function of the temperature T, related by the formula $P \cdot T = k$ for some constant k.

4. The number of hours of daylight at a certain latitude is a function of the day of the year. The following graph shows the number of hours of daylight that corresponds to each day.



In Exercises 5–12, determine whether the equation defines y as a function of x.

5. $y = 3x^2 - 12$

6. $y = 2x^4 + 3x^2 - 2$

7. $y^2 = 4x + 1$

8. $5x - 4y^4 + 64 = 0$

9. $3x + 2y = 12$

10. $y - 4x^3 - 14 = 0$

11. $x^2 + y^2 = 9$

12. $y^2 - 3x^4 + 8 = 0$

Exercises 13–34 refer to the functions below. Find the indicated value of the function.

$$f(x) = \sqrt{x+3} - x + 1$$
$$g(t) = t^2 - 1$$
$$h(x) = x^2 + \frac{1}{x} + 2$$

13. $f(0)$

14. $f(1)$

15. $f(\sqrt{2})$

16. $f(\sqrt{2} - 1)$

17. $f(-2)$

18. $f\left(-\frac{3}{2}\right)$

19. $h(3)$

20. $h(-4)$

21. $h\left(\frac{3}{2}\right)$

22. $h(\pi + 1)$

23. $h(a + k)$

24. $h(-x)$

25. $h(2 - x)$

26. $h(x - 3)$

27. $g(3)$

28. $g(-2)$

29. $g(0)$

30. $g(x)$

31. $g(s + 1)$

32. $g(1 - r)$

33. $g(-t)$

34. $g(t + h)$

In Exercises 35–42, compute and simplify the difference quotient (shown below). Assume $h \neq 0$.

$$\frac{f(x+h) - f(x)}{h}$$

35. $f(x) = x + 1$

36. $f(x) = -10x$

37. $f(x) = 3x + 7$

38. $f(x) = x^2$

39. $f(x) = x - x^2$

40. $f(x) = x^3$

41. $f(x) = \sqrt{x}$

42. $f(x) = \frac{1}{x}$

In Exercises 43–56, determine the domain of the function according to the domain convention.

43. $f(x) = x^2$

44. $g(x) = \frac{1}{x^2} + 2$

45. $h(t) = |t| - 1$

46. $k(u) = \sqrt{u}$

47. $k(x) = |x| + \sqrt{x} - 1$

48. $h(x) = \sqrt{(x+1)^2}$

49. $g(u) = \frac{|u|}{u}$

50. $h(x) = \frac{\sqrt{x-1}}{x^2 - 1}$

51. $g(y) = [-y]$

52. $f(t) = \sqrt{-t}$

53. $g(u) = \frac{u^2 + 1}{u^2 - u - 6}$

54. $f(t) = \sqrt{4 - t^2}$

55. $f(x) = -\sqrt{9 - (x-9)^2}$

56. $f(x) = \sqrt{-x} + \frac{2}{x+1}$

In Exercises 57–62, find the following:

a. $f(0)$ b. $f(1.6)$
c. $f(-2.3)$ d. $f(5 - 2\pi)$
e. The domain of f

57. $f(x) = [x]$

58. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

59. $f(x) = \begin{cases} x^2 + 2x & \text{if } x < 2 \\ 3x - 5 & \text{if } 2 \leq x \leq 20 \end{cases}$

60. $f(x) = \begin{cases} x + 5 & \text{if } -3 < x \leq 0 \\ 3x & \text{if } \quad 0 < x \leq 5 \end{cases}$

61. $f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ |x| - 5 & \text{if } -1 \leq x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$

62. $f(x) = \begin{cases} x^2 & \text{if } -4 \leq x < -2 \\ x - 3 & \text{if } -2 \leq x \leq 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

63. Find an equation that expresses the area A of a circle as a function of its
a. radius r b. diameter d

64. Find an equation that expresses the area A of a square as a function of its
a. side s b. diagonal d

65. A box with a square base of side x is four times higher than it is wide. Express the volume V of the box as a function of x.

66. The surface area of a cylindrical can of radius r and height h is $2\pi r^2 + 2\pi rh$. If the can is twice as high as the diameter of its top, express its surface area S as a function of r.

67. A rectangular region of 6000 square feet is to be fenced in on three sides with fencing that costs \$3.75 per foot and on the fourth side with fencing that costs \$2.00 per foot.
a. Express the cost of the fence as a function of the length x of the fourth side.
b. Find the domain of the function.

68. A box with a square base measuring $t \times t$ ft is to be made of three kinds of wood. The cost of the wood for the base is \$0.85 per square foot; the wood for the sides costs \$0.50 per square foot, and the wood for the top \$1.15 per square foot. The volume of the box must be 10 cubic feet.
a. Express the total cost of the box as a function of the length t.
b. Find the domain of the function.

69. A man walks for 45 minutes at a rate of 3 mph, then jogs for 75 minutes at a rate of 5 mph, then sits and rests for 30 minutes, and finally walks for 90 minutes at a rate of 3 mph.
a. Write a piecewise-defined function that expresses his distance traveled as a function of time.
b. Find the domain of the function.

70. Average tuition and fees in private four-year colleges in recent years were as follows. (Source: The College Board)

Year	Tuition & fees
1995	\$12,432
1996	\$12,823
1997	\$13,664
1998	\$14,709
1999	\$15,380
2000	\$16,332

a. Use linear regression to find the rule of a function f that gives the approximate average tuition in year x, where $x = 0$ corresponds to 1990.
b. Find $f(6)$, $f(8)$, and $f(10)$. How do they compare with the actual data?
c. Use f to estimate tuition in 2003.

71. Suppose that a state income tax law reads as follows:

Annual income	Amount of tax
less than \$2000	0
\$2000–\$6000	2% of income over \$2000
more than \$6000	\$80 plus 5% of income over \$6000

Write a piecewise-defined function that represents the income tax law. What is the domain of the function?

3.2 – Graphs of Functions

Objectives:

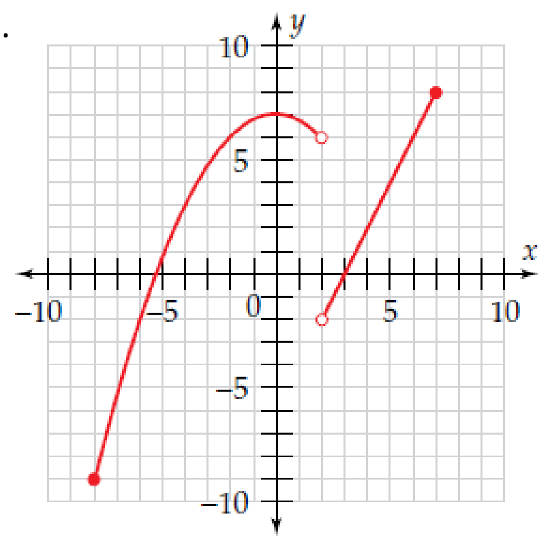
- Determine whether a graph represents a function
- Analyze graphs to determine domain and range, local maxima and minima, inflection points, and intervals where they are increasing, decreasing, concave up, and concave down.

Functions Defined by Graphs

A graph may be used to define a function or relation. Suppose that f is a function defined by a graph in the coordinate plane. If the point (x, y) is on the graph of f, then y is the output produced by the input x, or $y = f(x)$.

Example 1: A function Defined by a Graph

The graph of $f(x)$ is given below. Determine the following.



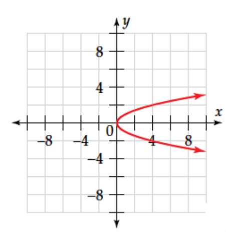
a. $f(0)$ b. $f(3)$

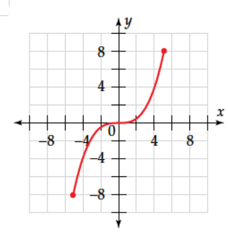
c. The domain of f

d. The range of f

The Vertical Line Test

If a graph represents a function, then each input determines one and only one output. Thus, no two points can have the same x-coordinate and different y-coordinates. Since any two such points would lie on the same vertical line, this fact provides a useful test for determining whether a graph represents a function.



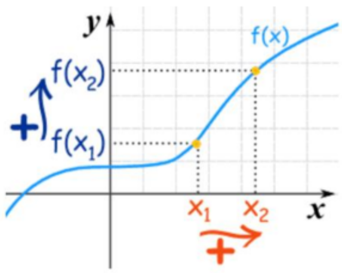


A graph in a coordinate plane represents a function if and only if no vertical line intersects the graph more than once.

Increasing vs. Decreasing

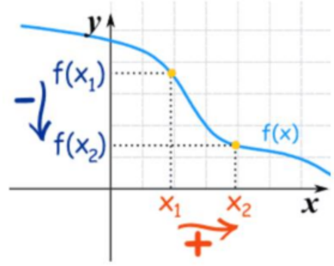
*A function is **increasing** when the **y-value increases** as the **x-value increases**.*

Ex:



*A function is **decreasing** when the **y-value decreases** as the **x-value increases.***

Ex:

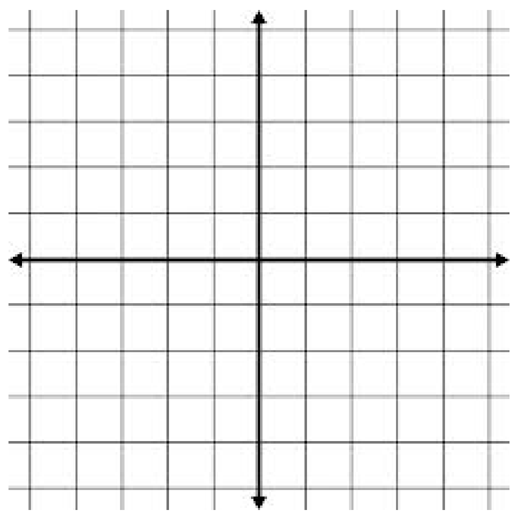


We can use interval notation to indicate the x-values at which a function is increasing or decreasing.

*If a function is not increasing or decreasing over an interval, it must be **constant**.*

Example 3: Where a Function is Increasing/Decreasing

Graph $f(x) = |x| + |x - 2|$



a. On what interval(s) is $f(x)$ increasing?

b. On what interval(s) is $f(x)$ decreasing?

c. On what interval(s) is $f(x)$ constant?

Example 4: Finding Local Maxima and Minima

Graph $f(x) = x^3 - 1.8x^2 + x + 1$ in your calculator.

a. Find all local maxima

b. Find all local minima

Concavity and Inflection Points

Concavity is used to describe the way that a curve bends. For any two points in a given interval that lie on a curve, if the segment that connects them is *above* the curve, then the curve is said to be **concave up** over the given interval. If the segment is *below* the curve, then the curve is said to be **concave down** over the interval (see Figure 3.2-10). A straight line is neither concave up nor concave down. A point where the curve changes concavity is called an **inflection point.**

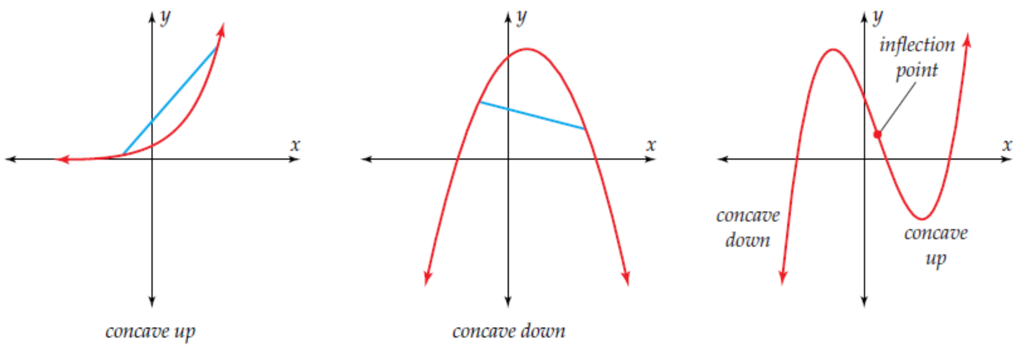


Figure 3.2-10

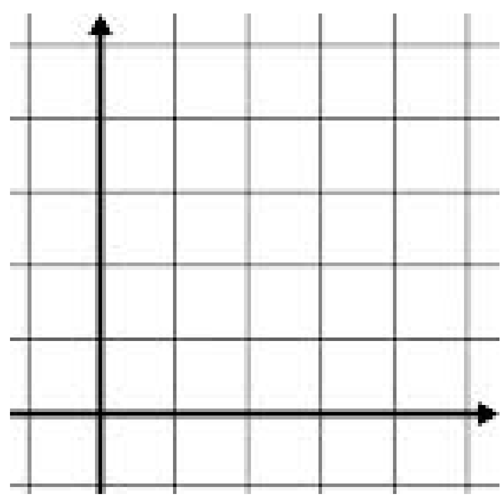
Example 5: Analyzing a Graph

Graph the function $f(x) = -2x^3 + 6x^2 - x + 3$ and estimate the following, using the graphing calculator as necessary.

a. All local maxima

b. All local minima

c. Intervals where the function is increasing



d. Intervals where the function is decreasing

e. All points of inflection

f. Intervals where the function is concave up

g. Intervals where the function is concave down

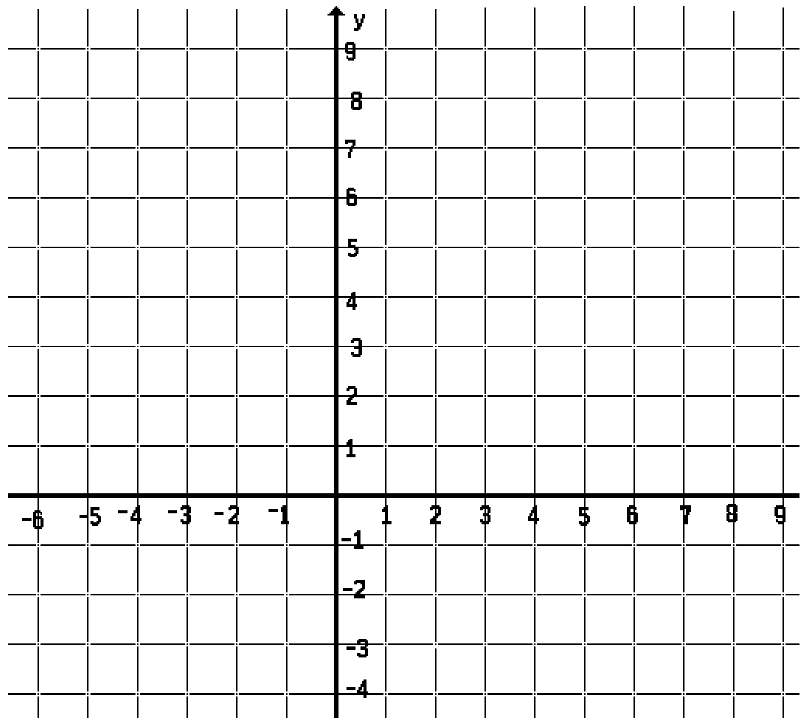
Graphs of Piecewise-Defined and Greatest Integer Functions

The graphs of piecewise-defined functions are often discontinuous, that is, they commonly have jumps or holes. To graph a piecewise-defined function, graph each piece separately.

Example 6: Graphing a Piecewise-Defined Function

Graph the piecewise-defined function below.

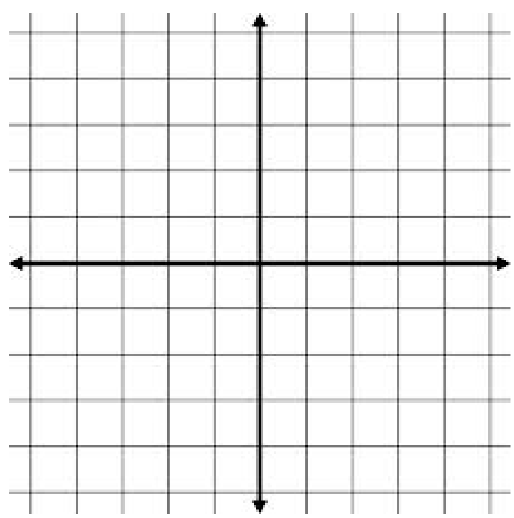
$$f(x) = \begin{cases} x^2 & if\ x \leq 1 \\ x + 2 & if\ 1 < x \leq 4 \\ [x] & if\ x > 4 \end{cases}$$



Example 7: The Absolute-Value Function

Rewrite $f(x) = |x|$ as a piecewise-defined function and graph.

a. $f(x) =$



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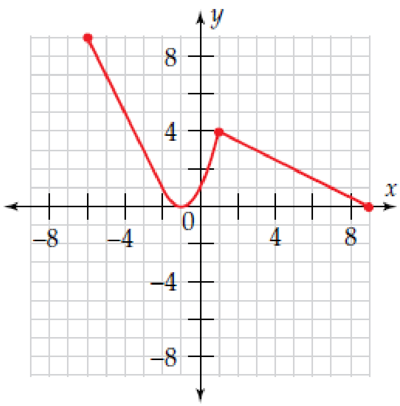
Exercises 3.2

In Exercises 1–4, the graph below defines a function, f. Determine the following:



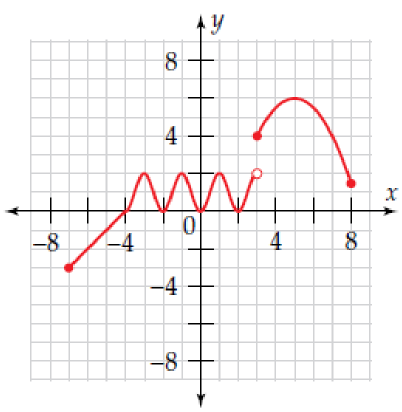
1. $f(-5)$

2. $f(1)$

3. the domain of f

4. the range of f

In Exercises 5–8, the graph below defines a function, g. Determine the following:



5. $g(1)$

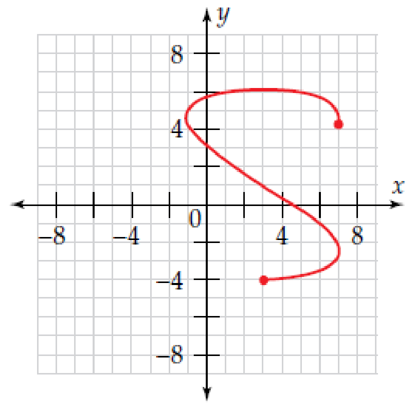
6. $g(5)$

7. the domain of g

8. the range of g

In Exercises 9–14, use the Vertical Line Test to determine whether the graph defines a function. If not, give an example of an input value that corresponds to more than one output value.

9.



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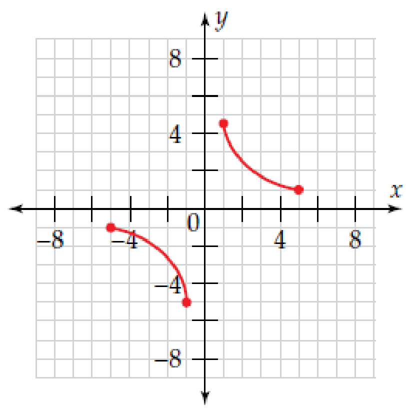

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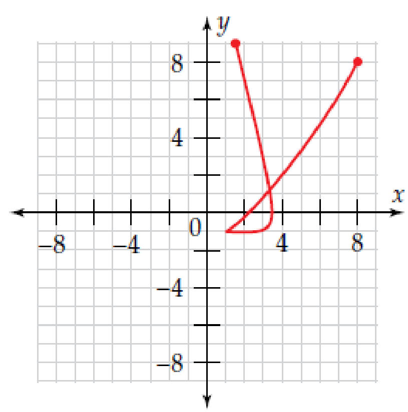

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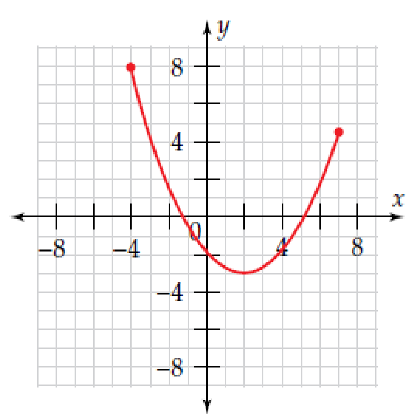

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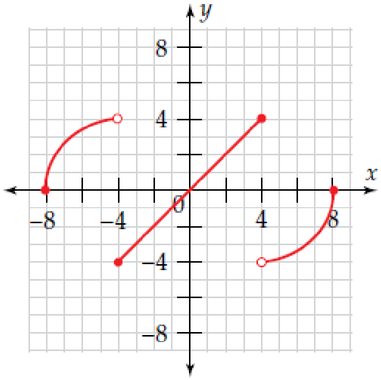

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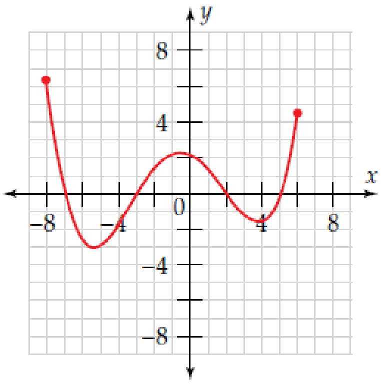
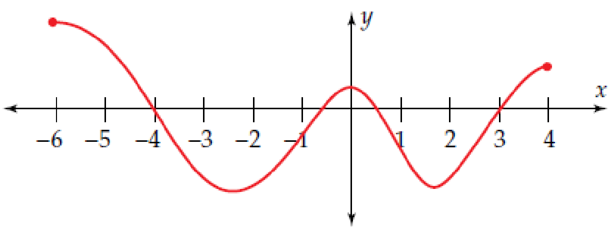


In Exercises 15 and 16, the graph of a function is shown. Find the approximate intervals on which the function is increasing and on which it is decreasing.

15.



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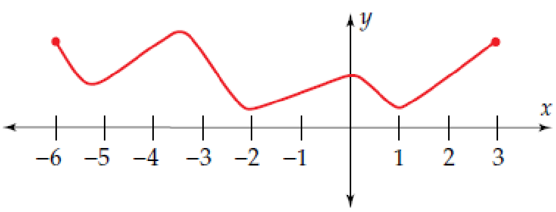


In Exercises 17–22, graph each function. Find the approximate intervals on which the function is increasing, decreasing, and constant.

17. $f(x) = |x - 1| - |x + 1|$

18. $g(x) = |x - 1| + |x + 2|$

19. $f(x) = -x^3 - 8x^2 + 8x + 5$

20. $f(x) = x^4 - 0.7x^3 - 0.6x^2 + 1$

21. $g(x) = 0.2x^4 - x^3 + x^2 - 2$

22. $g(x) = x^4 + x^3 - 4x^2 + x - 1$

In Exercises 23–28, graph each function. Estimate all local maxima and minima of the function.

23. $f(x) = x^3 - x$

24. $g(t) = -\sqrt{16 - t^2}$

25. $h(x) = \dfrac{x}{x^2 + 1}$

26. $k(x) = x^3 - 3x + 1$

27. $f(x) = x^3 - 1.8x^2 + x + 2$

28. $g(x) = 2x^3 + x^2 + 1$

29. a. A rectangle has a perimeter of 100 inches, and one side has length x. Express the area of the rectangle as a function of x.

b. Use the function in part a to find the dimensions of the rectangle with perimeter 100 inches and the largest possible area.

30. a. A rectangle has an area of 240 in^2, and one side has length x. Express the perimeter of the rectangle as a function of x.

b. Use the function in part a to find the dimensions of the rectangle with area 240 in^2 and the smallest possible perimeter.

31. a. A box with a square base has a volume of 867 in^3. Express the surface area of the box as a function of the length x of a side of the base. (Be sure to include the top of the box.)

b. Use the function in part a to find the dimensions of the box with volume 867 in^3 and the smallest possible surface area.

32. a. A cylindrical can has a surface area of 60 in^2. Express the volume of the can as a function of the radius r.

b. Use the function in part a to find the radius and height of the can with surface area 60 in^2 and the largest possible volume.

In Exercises 33–36, graph each function. Find the approximate intervals on which the function is concave up and concave down, and estimate all inflection points.

33. $f(x) = x^3$

34. $f(x) = x^3 - 2x$

35. $h(x) = x^4 - 2x^2$

36. $g(x) = x^3 - 3x^2 + 2x + 1$

In Exercises 37–40,

a. Graph each function.

b. Find the approximate intervals on which the function is increasing, decreasing, and constant.

c. Estimate all local maxima and minima.

d. Find the approximate intervals on which the function is concave up and concave down.

e. Estimate the coordinates of any inflection points.

37. $f(x) = x^2 - 2x + 1$ **38.** $f(x) = -x^2 - 4x - 3$

39. $g(x) = x^3 - 3x^2 + 2$ **40.** $g(x) = -x^3 + 4x - 2$

In Exercises 41–44, sketch the graph of the function. Be sure to indicate which endpoints are included and which are excluded.

41. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } x \geq -1 \end{cases}$

42. $g(x) = \begin{cases} |x| & \text{if } x < 1 \\ -3x + 4 & \text{if } x \geq 1 \end{cases}$

43. $k(u) = \begin{cases} -2u - 2 & \text{if } u < -3 \\ u - [u] & \text{if } -3 \leq u \leq 1 \\ 2u^2 & \text{if } u > 1 \end{cases}$

44. $f(x) = \begin{cases} x^2 & \text{if } x < -2 \\ x & \text{if } -2 \leq x < 4 \\ \sqrt{x} & \text{if } x \geq 4 \end{cases}$

In Exercises 45–49,

a. Use the fact that the absolute-value function is piecewise-defined (see Example 7) to write the rule of the given function as a piecewise-defined function whose rule does not include any absolute value bars.

b. Graph the function.

45. $f(x) = |x| + 2$ **46.** $g(x) = |x| - 4$

47. $h(x) = \dfrac{|x|}{2} - 2$ **48.** $g(x) = |x + 3|$

49. $f(x) = |x - 5|$

In Exercises 50–53, sketch the graph of the function. Be sure to indicate which endpoints are included and which are excluded.

50. $f(x) = -[x]$

51. $g(x) = [-x]$ (This is not the same function as in Exercise 50.)

52. $h(x) = [x] + [-x]$ **53.** $f(x) = 2[x]$

54. A common mistake is to graph the function f in Example 6 by graphing both $y = x^2$ and $y = x + 2$ on the same screen, with no restrictions on x. Explain why this graph could not possibly be the graph of a function.

55. Show that the function $f(x) = |x| + |x - 2|$ is constant on the interval $[0, 2]$. *Hint:* Use the piecewise definition of absolute value in Example 7 to compute $f(x)$ when $0 \leq x \leq 2$.

In Exercises 56–59, use your calculator to estimate the domain and range of the function by tracing its graph.

56. $g(x) = x^2 - 4$ **57.** $h(x) = \sqrt{x^2 - 4}$

58. $k(x) = \sqrt{x^2 + 4}$ **59.** $f(x) = 3x - 2$

In Exercises 60 and 61, draw the graph of a function f that satisfies the given conditions. The function does not need to be given by an algebraic rule.

60.
- $f(-1) = 2$
- $f(x) \geq 2$ when x is in the interval $\left(-1, \frac{1}{2}\right)$
- $f(x)$ starts decreasing when $x = 1$
- $f(3) = 3 = f(0)$
- $f(x)$ starts increasing when $x = 5$

61.
- domain $f = [-2, 4]$
- range $f = [-5, 6]$
- $f(-1) = f(3)$
- $f\left(\frac{1}{2}\right) = 0$

3.3 – Quadratic Functions

Objectives:

- Define three forms for quadratic functions
- Find the vertex and intercepts of a quadratic function and sketch its graph
- Convert one form of a quadratic function to another

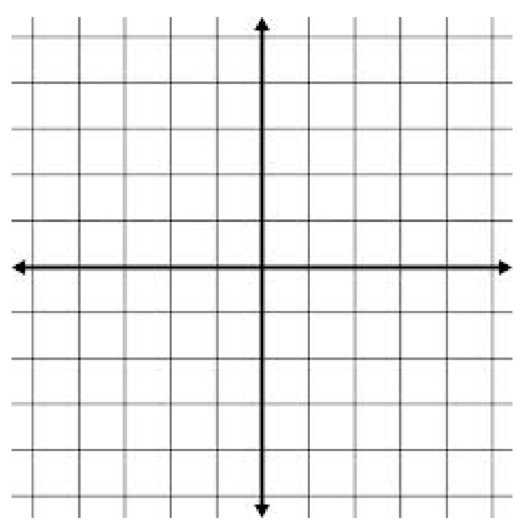
Summary of Quadratic Forms

Below is a summary of the basic forms and the important points of a quadratic function. You should memorize the highlighted items.

Name	Transformation	Polynomial	x-Intercept
Form	$f(x) = a(x-h)^2 + k$	$f(x) = ax^2 + bx + c$	$f(x) = a(x-s)(x-t)$
Vertex	(h, k)	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	$\left(\frac{s+t}{2}, f\left(\frac{s+t}{2}\right)\right)$
x-Intercepts	$h + \sqrt{\frac{-k}{a}}$ and $h - \sqrt{\frac{-k}{a}}$	$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$	s and t
y-Intercept	$ah^2 + k$	c	ast

Example 1: Transformation Form (Vertex Form)

For the function $f(x) = 2(x-3)^2 - 4$, find the following then sketch the graph



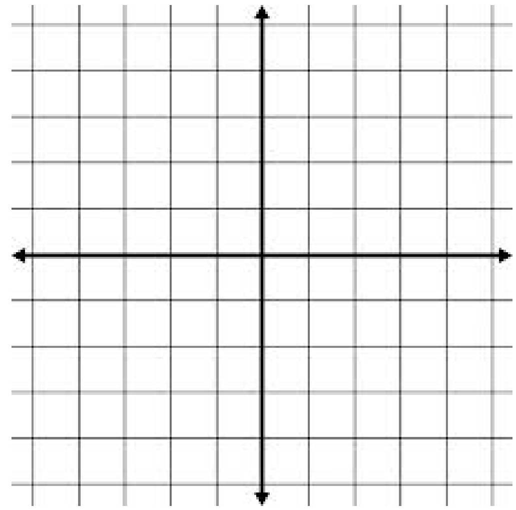
a. Vertex

b. x-intercept(s)

c. y-intercept

Example 2: Polynomial Form (Standard Form)
For the function $f(x) = x^2 + 2x + 3$, find the following.

a. Vertex

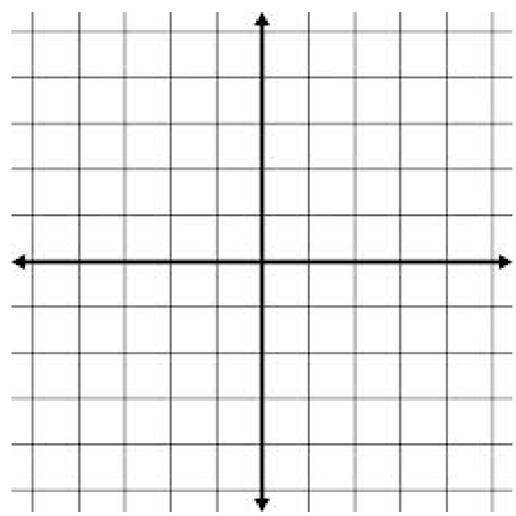


b. x-intercept(s)

c. y-intercept

Example 3: X-Intercept Form (Intercept Form)
For the function $f(x) = -\frac{1}{2}(x-4)(x+2)$, find the following.

a. Vertex



b. x-intercept(s)

c. y-intercept

Example 6: Maximum Area for a Fixed Perimeter
Find the dimensions of a rectangular field that can be enclosed with 3000 feet of fence and that has the largest possible area.

Practice Odd Numbered Questions as Necessary. Answers are at the back of the textbook.

Exercises 3.3

In Exercises 1–4, determine the vertex of the given quadratic function and state whether its graph opens upward or downward.

1. $f(x) = 3(x - 5)^2 + 2$ **2.** $g(x) = -6(x - 2)^2 - 5$

3. $f(x) = -(x - 1)^2 + 2$ **4.** $h(x) = -x^2 + 1$

In Exercises 5–8, determine the y-intercept of the given quadratic function and state whether its graph opens upward or downward.

5. $f(x) = x^2 - 6x + 3$ **6.** $g(x) = x^2 + 8x - 1$

7. $h(x) = -3x^2 + 4x + 5$ **8.** $g(x) = 2x^2 - x - 1$

In Exercises 9–12, determine the x-intercepts of the given quadratic function and state whether its graph opens upward or downward.

9. $f(x) = (x - 2)(x + 3)$

10. $h(x) = -2(x + 3)(x + 1)$

11. $g(x) = \frac{1}{3}\left(x - \frac{3}{4}\right)\left(x - \frac{1}{2}\right)$

12. $f(x) = -0.4(x + 2.1)(x - 0.7)$

In Exercises 13–21, determine the vertex and x- and y-intercepts of the given quadratic function, and sketch a graph.

13. $f(x) = 2(x + 3)^2 - 4$ **14.** $g(x) = -\frac{1}{2}(x + 4)^2 + 2$

15. $h(x) = (x + 1)^2 + 4$ **16.** $f(x) = x^2 - 6x + 3$

17. $h(x) = -x^2 + 8x - 2$ **18.** $f(x) = 2x^2 - 4x + 2$

19. $g(x) = (x + 1)(x - 3)$ **20.** $h(x) = -(x + 2)(x + 6)$

21. $g(x) = 2(x - 3)(x + 4)$

Write the following functions in polynomial form.

22. $f(x) = 3(x - 2)(x + 1)$

23. $g(x) = -2(x - 5)(x - 2)$

24. $h(x) = 3(x - 4)^2 - 47$

25. $f(x) = -\frac{1}{2}(x + 4)^2 - 5$

Write the following functions in x-intercept form.

26. $f(x) = x^2 - 3x - 4$ **27.** $h(x) = -2x^2 + 13x + 7$

28. $g(x) = 3(x + 3)^2 - 3$ **29.** $f(x) = 6\left(x - \frac{2}{3}\right)^2 - \frac{2}{3}$

Write the following functions in transformation form.

30. $g(x) = x^2 + 4x - 5$ **31.** $f(x) = -3x^2 + 6x - 1$

32. $h(x) = -(x - 4)(x + 2)$

33. $g(x) = 2(x - 1)(x + 6)$

34. Write a rule in transformation form for the quadratic function whose graph is the parabola with vertex at the origin that passes through (2, 12).

35. Write a rule in transformation form for the quadratic function whose graph is the parabola with vertex (0, 1) that passes through (2, −7).

36. Find the number c such that the vertex of $f(x) = x^2 + 8x + c$ lies on the x-axis.

37. If the vertex of $f(x) = x^2 + bx + c$ is at (2, 4), find b and c.

38. If the vertex of $f(x) = -x^2 + bx + 8$ has y-coordinate 17 and is in the second quadrant, find b.

39. Find the number b such that the vertex of $f(x) = x^2 + bx + c$ lies on the y-axis.

40. If the vertex of $f(x) = a(x - s)(x - 4)$ has x-coordinate 7, find s.

41. If the y-intercept of $f(x) = a(x - 3)(x + 2)$ is 3, find a.

42. Find two numbers whose sum is −18 and whose product is the maximum.

43. Find two numbers whose difference is 4 and whose product is the minimum.

44. The sum of the height h and the base b of a triangle is 30. What height and base will produce a triangle of maximum area?

45. A field bounded on one side by a river is to be fenced on three sides to form a rectangular enclosure. If the total length of fence is 200 feet, what dimensions will give an enclosure of maximum area?

46. A salesperson finds that her sales average 40 cases per store when she visits 20 stores per week. If she visits an additional store per week, her average sales per store decrease by one case. How many stores per week should she visit to maximize her sales?

47. A potter can sell 120 bowls per week at \$4 per bowl. For each 50¢ decrease in price, 20 more bowls are sold. What price should be charged in order to maximize revenue?

48. When a basketball team charges \$4 per ticket, average attendance is 400 people. For each 20¢ decrease in ticket price, average attendance increases by 40 people. What should the ticket price be to maximize revenue?

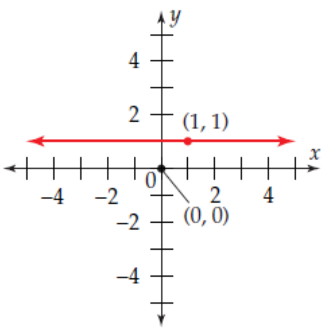
3.4 – Graphs and Transformations

Objectives:

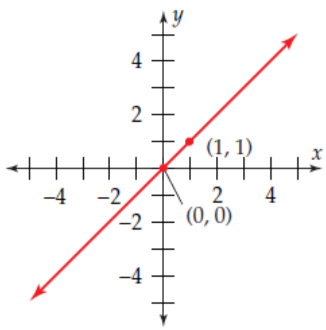
- Define parent functions
- Transform graphs of parent functions

Parent Functions

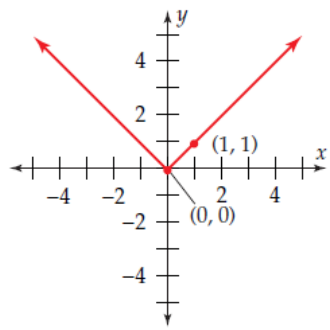
The functions on the next page are often called **parent functions.** A parent function is a function with a certain shape that has the simplest algebraic rule for that shape. For example, $f(x) = x^2$ is the simplest rule for a parabola. You should memorize the basic shapes of the parent functions.



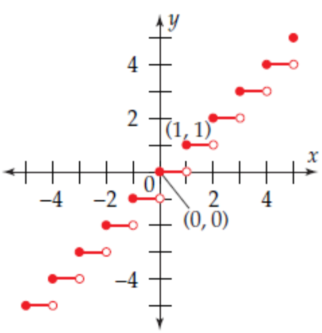
$f(x) = 1$
Constant function



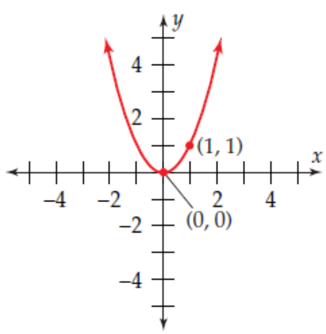
$f(x) = x$
Identity function



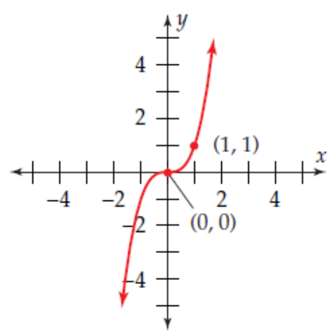
$f(x) = |x|$
Absolute-value function



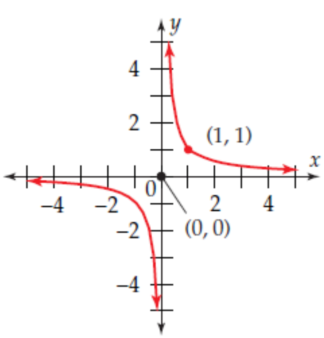
$f(x) = [x]$
Greatest integer function



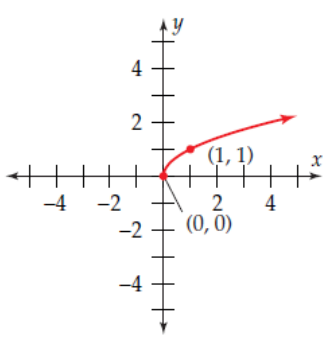
$f(x) = x^2$
Quadratic function



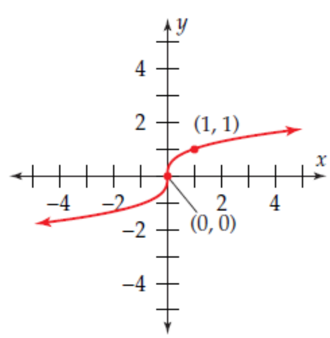
$f(x) = x^3$
Cubic function



$f(x) = \frac{1}{x}$
Reciprocal function



$f(x) = \sqrt{x}$
Square root function



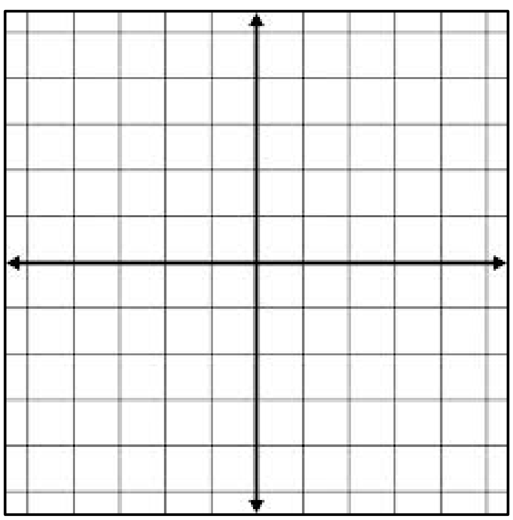
$f(x) = \sqrt[3]{x}$
Cube root function

Transformation Rules

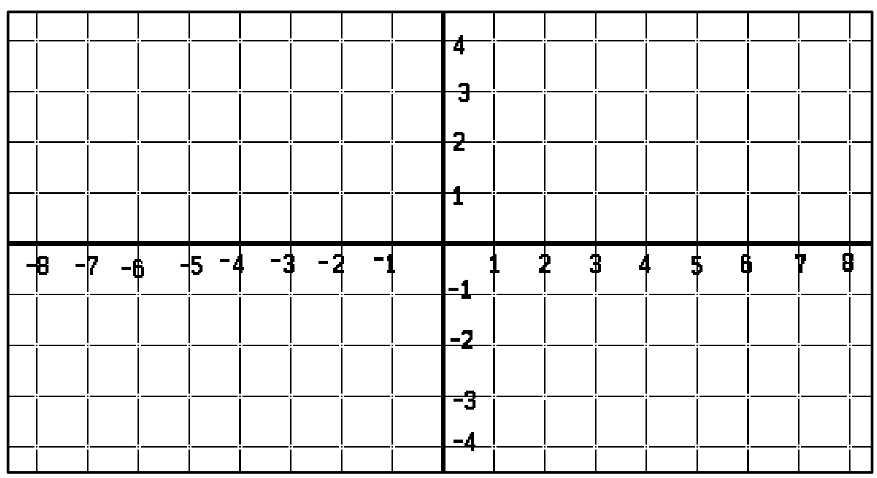
The graph of $g(x) = f(x) - c$ is the graph of f shifted downward c units.
The graph of $g(x) = f(x + c)$ is the graph of f shifted c units to the left.
The graph of $g(x) = f(x - c)$ is the graph of f shifted c units to the right.
The graph of $g(x) = -f(x)$ is the graph of f reflected across the x-axis.
The graph of $g(x) = f(-x)$ is the graph of f reflected across the y-axis.
The graph of $g(x) = c \cdot f(x)$ is the graph of f stretched or compressed vertically by a factor of c.
The graph of $g(x) = f(c \cdot x)$ is the graph of f stretched or compressed horizontally by a factor of $\frac{1}{c}$.

Example 1: Transformations
Identify the transformations from the parent function and graph.

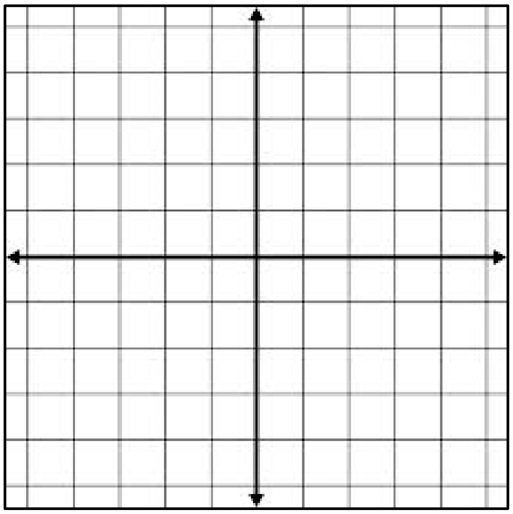
a. $f(x) = \frac{1}{2}x^2$



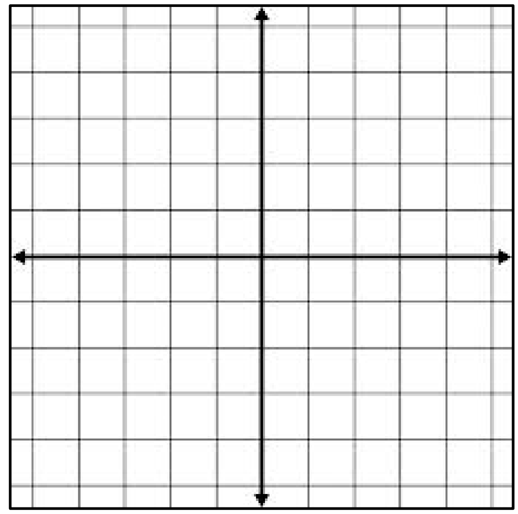
b. $k(x) = \sqrt[3]{4x}$



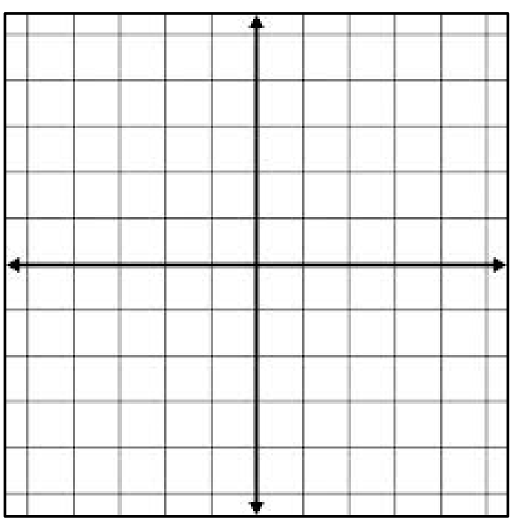
c. $h(x) = [-x] - 1$



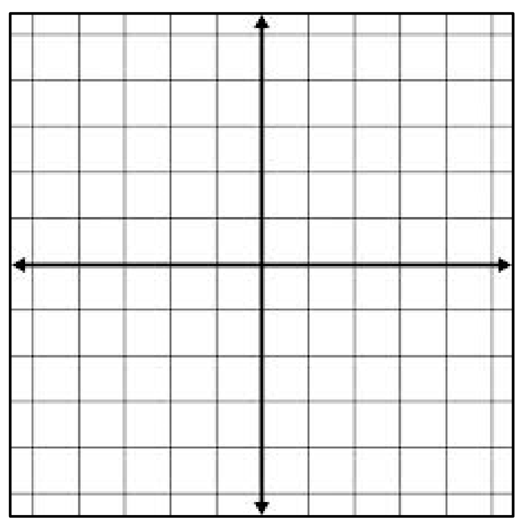
d. $f(x) = -\sqrt{x+4} + 5$



e. $j(x) = -|2x - 1| + 4$



f. $g(x) = \frac{3}{4-2x}$



Practice Odd Numbered Questions as Necessary. Answers are at the back of the textbook.

Exercises 3.4

In Exercises 1–9, identify the parent function that can be used to graph each function. Do not graph the function.

1. $f(x) = -(x + 2)^3$
2. $g(x) = 4|x| - 3$
3. $f(x) = 7$
4. $h(x) = \sqrt{-2x + 4}$
5. $g(x) = \dfrac{2}{x - 1} + 4$
6. $f(x) = -3x + 2$
7. $h(x) = -6\left[\dfrac{1}{2}x + 3\right] + \dfrac{2}{3}$
8. $g(x) = 2\sqrt[3]{x} + 5$
9. $h(x) = 2(3 - x)^2 + 4$

In Exercises 10–21, graph each function and its parent function on the same set of axes.

10. $f(x) = -x^2$
11. $h(x) = -\dfrac{1}{x}$
12. $g(x) = |-x|$
13. $h(x) = \sqrt[3]{-x}$
14. $f(x) = x - 5$
15. $g(x) = [x] + 1$
16. $f(x) = \sqrt{x + 4}$
17. $h(x) = |x - 2|$
18. $g(x) = 3x$
19. $f(x) = \dfrac{1}{2}$
20. $g(x) = \left(\dfrac{1}{4}x\right)^3$
21. $h(x) = \dfrac{1}{3x}$

In Exercises 22–27, write a rule for the function whose graph can be obtained from the given parent function by performing the given transformations.

22. parent function: $f(x) = x^3$
    transformations: shift the graph 5 units to the left and upward 4 units
23. parent function: $f(x) = \sqrt{x}$
    transformations: reflect the graph across the x-axis and shift it upward 3 units
24. parent function: $f(x) = [x]$
    transformations: shift the graph 6 units to the right, stretch it vertically by a factor of 2, and shift it downward 3 units
25. parent function: $f(x) = |x|$
    transformations: shift the graph 3 units to the left, reflect it across the x-axis, and shrink it vertically by a factor of $\dfrac{1}{2}$
26. parent function: $f(x) = \dfrac{1}{x}$
    transformations: shift the graph 2 units to the right, stretch it horizontally by a factor of 2, and shift it upward 2 units
27. parent function: $f(x) = x^2$
    transformations: shift the graph 3 units to the left, reflect it across the y-axis, and stretch it vertically by a factor of 1.5

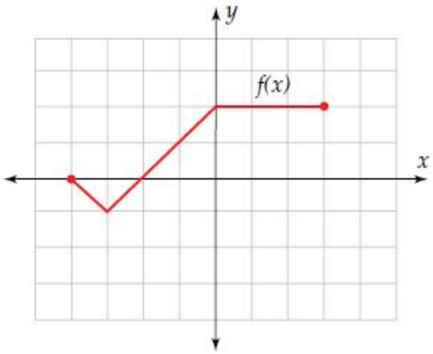
In Exercises 28–33, describe a sequence of transformations that transform the graph of the parent function f into the graph of the function g. Do not graph the functions.

28. $f(x) = \sqrt{x}$ $\quad g(x) = -\sqrt{-\dfrac{1}{2}x + 3}$
29. $f(x) = x$ $\quad g(x) = -3(x - 4) + 1$
30. $f(x) = [x]$ $\quad g(x) = 2\left[\dfrac{1}{3}x\right] + 5$
31. $f(x) = x^3$ $\quad g(x) = 4(2 - x)^3 - 3$
32. $f(x) = \dfrac{1}{x}$ $\quad g(x) = \dfrac{3}{4 - 2x}$
33. $f(x) = \sqrt[3]{x}$ $\quad g(x) = \sqrt[3]{1.3x - 4.2} + 0.4$

In Exercises 34–41, graph each function and its parent function on the same graph.

34. $f(x) = -2(x + 1)^2 + 3$
35. $g(x) = \dfrac{1}{4}\sqrt[3]{x + 3} - 1$
36. $f(x) = -\dfrac{3}{4}\left[-\dfrac{1}{3}x\right]$
37. $h(x) = 3(x - 1) + 5$
38. $g(x) = |-x + 5| - 3$
39. $f(x) = \dfrac{-3}{2 - x} + 4$
40. $h(x) = \sqrt{-4x + 3} - 1$
41. $g(x) = \dfrac{2}{5}(5 - x)^3 - \dfrac{3}{5}$

In Exercises 42–45, use the graph of the function f in the figure to sketch the graph of the function g.



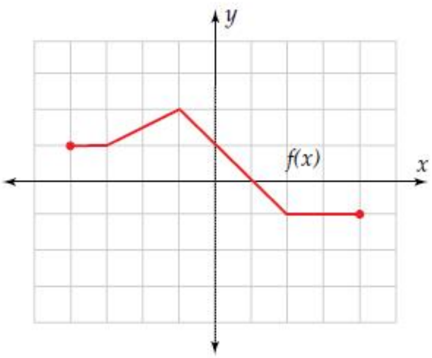
42. $g(x) = f(x) - 1$

43. $g(x) = 3f(x)$

44. $g(x) = 0.25f(x)$

45. $g(x) = f(x) + 3$

In Exercises 46–49, use the graph of the function f in the figure to sketch the graph of the function h.



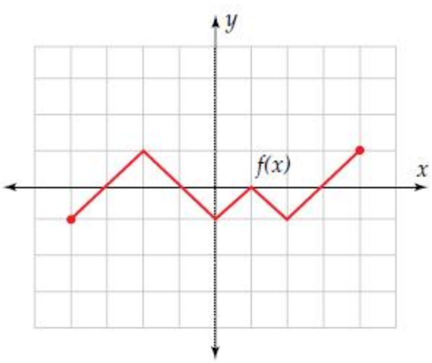
46. $h(x) = -4f(x)$

47. $h(x) = -f(x)$

48. $h(x) = f(-x)$

49. $h(x) = f(-2x)$

In Exercises 50–55, use the graph of the function f in the figure to sketch the graph of the function g.



50. $g(x) = f(x - 2)$

51. $g(x) = f(x - 2) + 3$

52. $g(x) = f(x + 1) - 3$

53. $g(x) = 2 - f(x)$

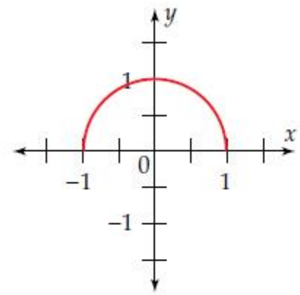
54. $g(x) = f(-x) + 2$

55. $g(x) = f(x + 3)$

Exercises 56–61 refer to the parent function

$$f(x) = \sqrt{1 - x^2}$$

The graph of f is a semicircle with radius 1, as shown below.



Use the graph of f to sketch the graph of the function g.

56. $g(x) = -\sqrt{1 - x^2}$

57. $g(x) = \sqrt{1 - x^2} + 4$

58. $g(x) = \sqrt{1 - (x - 3)^2}$

59. $g(x) = 3\sqrt{1 - x^2}$

60. $g(x) = \sqrt{1 - (x + 2)^2} + 1$

61. $g(x) = 5\sqrt{1 - \left(\frac{1}{5}x\right)^2}$

62. In 2002, the cost of sending first-class mail was \$0.37 for a letter weighing less than 1 ounce, \$0.60 for a letter weighing at least one ounce, but less than 2 ounces, \$0.83 for a letter weighing at least 2 ounces, but less than 3 ounces, and so on.
   a. Write a function $c(x)$ that gives the cost of mailing a letter weighing x ounces (see Example 7).
   b. Graph $c(x)$ and interpret the result.

63. A factory has a linear cost function $f(x) = ax + b$, where b represents fixed costs and a represents the labor and material costs of making one item, both in thousands of dollars.
   a. If property taxes (part of the fixed costs) are increased by \$28,000 per year, what effect does this have on the graph of the cost function?
   b. If labor and material costs for making 100,000 items increase by \$12,000, what effect does this have on the graph of the cost function?

3.5 – Operations on Functions

Objectives:

- Form sum, difference, product, and quotient functions and find their domains
- Form composite functions and find their domains

Example 1: Sum and Difference Functions

Let $f(x) = \sqrt{9 - x^2}$ and $g(x) = \sqrt{x - 2}$.

a. Find $(f + g)(x)$ and state its domain

Example 2: Product and Quotient Functions

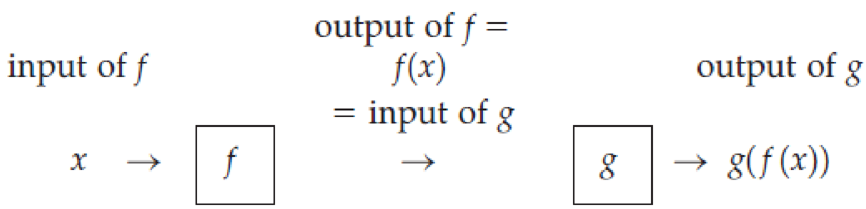
Let $f(x) = \sqrt{3x}$ and $g(x) = \sqrt{x^2 - 1}$

a. $(f \cdot g)(x)$ and state its domain

b. $\left(\frac{f}{g}\right)(x)$ and state its domain

Composition of Functions

Another way of combining functions is to use the output of one function as the input of another. This operation is called **composition of functions.** The idea can be expressed in function notation as shown below.



If f and g are functions, then the *composite function* of f and g is

$$(g \circ f)(x) = g(f(x))$$

The expression $g \circ f$ is read "g circle f" or "f followed by g." Note the order carefully; the functions are applied right to left.

Example 3: Composite Functions

Let $f(x) = 4x^2 + 1$ and $g(x) = \frac{1}{x+2}$, find the following.

a. $(g \circ f)(2)$

b. $(f \circ g)(-1)$

c. $(g \circ f)(x)$

d. $(f \circ g)(x)$

Domains of Composite Functions

The domain of $g \circ f$ is determined by the following convention.

Let f and g be functions. The domain of $g \circ f$ is the set of all real numbers x such that

- x is in the domain of f
- $f(x)$ is in the domain of g

Example 4: Finding the Domain of a Composite Function

Let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5$

a. Find $g \circ f$ and state the domain of the composite function

b. Find $f \circ g$ and state the domain of the composite function

Example 5: Writing a Function as a Composite

Let $h(x) = \sqrt{3x^2 + 1}$

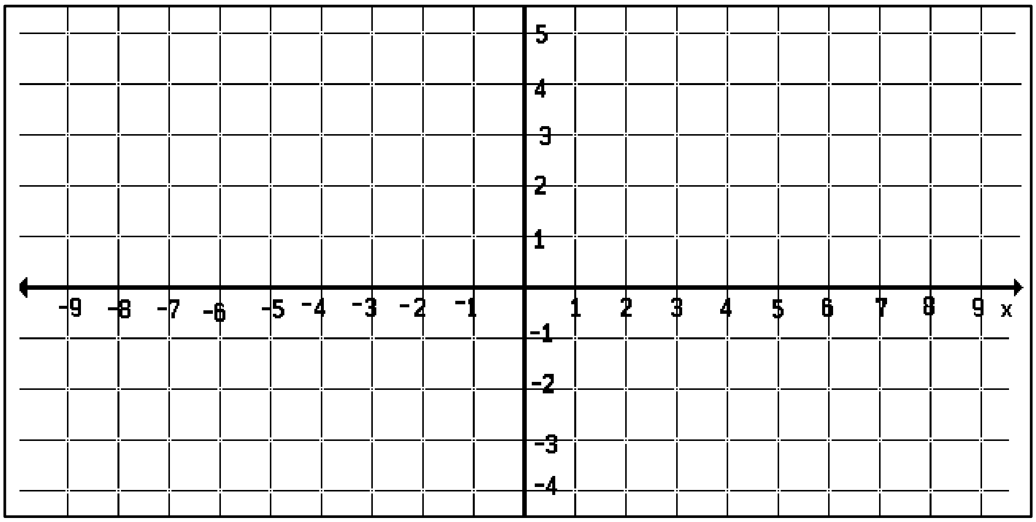
a. Write h as a composition of functions in two different ways.

Example 6: Compositions with Absolute-Value Functions

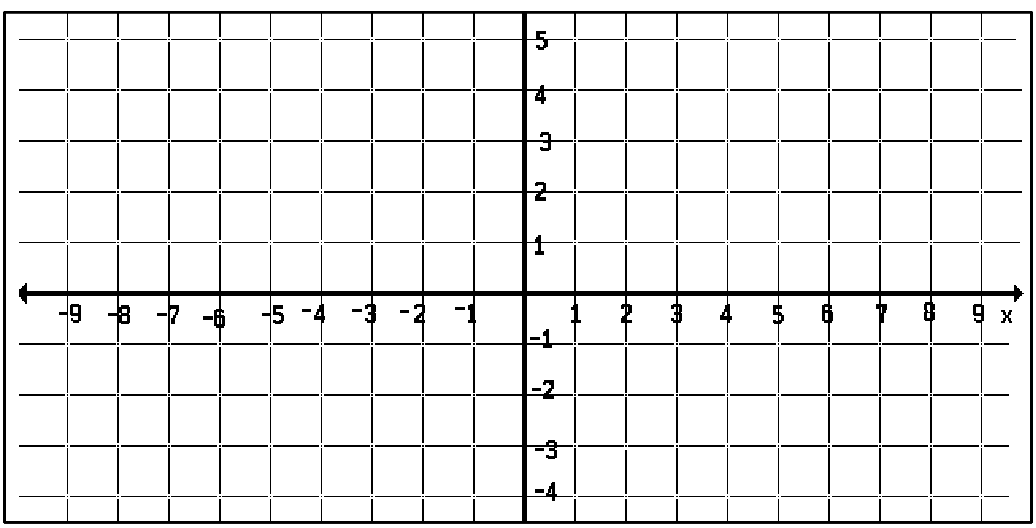
Let $g(x) = |x|$ and $f(x) = \sqrt{x}$

a. Graph $f(x)$ and the composition $(f \circ g)(x)$, and describe the relationship between the graphs of f and $(f \circ g)(x)$ in terms of transformations

$f(x)$



$(f \circ g)(x)$



Practice Odd Numbered Questions as Necessary. Answers are at the back of the textbook.

Exercises 3.5

In Exercises 1–4, find $(f+g)(x)$, $(f-g)(x)$, $(g-f)(x)$, and their domains.

1. $f(x) = -3x + 2 \quad g(x) = x^3$

2. $f(x) = x^2 + 2 \quad g(x) = -4x + 7$

3. $f(x) = \frac{1}{x} \quad g(x) = x^2 + 2x - 5$

4. $f(x) = \sqrt{x} \quad g(x) = x^2 + 1$

In Exercises 5–7, find $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, and $\left(\frac{g}{f}\right)(x)$.

5. $f(x) = -3x + 2 \quad g(x) = x^3$

6. $f(x) = 4x^2 + x^4 \quad g(x) = \sqrt{x^2 + 4}$

7. $f(x) = \sqrt{x^2 - 1} \quad g(x) = \sqrt{x - 1}$

In Exercises 8–11, find the domains of fg and $\frac{f}{g}$.

8. $f(x) = x^2 + 1 \quad g(x) = \frac{1}{x}$

9. $f(x) = x + 2 \quad g(x) = \frac{1}{x+2}$

10. $f(x) = \sqrt{4 - x^2} \quad g(x) = \sqrt{3x + 4}$

11. $f(x) = 3x^2 + x^4 + 2 \quad g(x) = 4x - 3$

In Exercises 12–14, find $(g \circ f)(3)$, $(f \circ g)(1)$, and $(f \circ f)(0)$.

12. $f(x) = 3x - 2 \quad g(x) = x^2$

13. $f(x) = |x + 2| \quad g(x) = -x^2$

14. $f(x) = x^2 - 1 \quad g(x) = \sqrt{x}$

In Exercises 15–18, find the indicated values, where $g(t) = t^2 - t$ and $f(x) = 1 + x$.

15. $g(f(0)) + f(g(0))$

16. $(f \circ g)(3) - 2f(1)$

17. $g(f(2) + 3)$

18. $f(2g(1))$

In Exercises 19–22, find the rule of the function $g \circ f$ and its domain and the rule of $f \circ g$ and its domain.

19. $f(x) = x^2 \quad g(x) = x + 3$

20. $f(x) = -3x + 2 \quad g(x) = x^3$

21. $f(x) = \frac{1}{x} \quad g(x) = \sqrt{x}$

22. $f(x) = \frac{1}{2x+1} \quad g(x) = x^2 - 1$

In Exercises 23–26, find the rules of the functions ff and $f \circ f$.

23. $f(x) = x^3$

24. $f(x) = (x - 1)^2$

25. $f(x) = \frac{1}{x}$

26. $f(x) = \frac{1}{x-1}$

In Exercises 27–30, verify that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ for the given functions f and g.

27. $f(x) = 9x + 2 \quad g(x) = \frac{x-2}{9}$

28. $f(x) = \sqrt[3]{x - 1} \quad g(x) = x^3 + 1$

29. $f(x) = \sqrt[3]{x} + 2 \quad g(x) = (x - 2)^3$

30. $f(x) = 2x^3 - 5 \quad g(x) = \sqrt[3]{\frac{x+5}{2}}$

In Exercises 31–36, write the given function as the composite of two functions, neither of which is the identity function, $f(x) = x$. (There may be more than one possible answer.)

31. $f(x) = \sqrt[3]{x^2 + 2}$

32. $g(x) = \sqrt{x + 3} - \sqrt[3]{x + 3}$

33. $h(x) = (7x^3 - 10x + 17)^7$

34. $k(x) = \sqrt[3]{(7x - 3)^2}$

35. $f(x) = \frac{1}{3x^2 + 5x - 7}$

36. $g(t) = \frac{3}{\sqrt{t} - 3} + 7$

In Exercises 37 and 38, graph both $g \circ f$ and $f \circ g$ on the same screen. Use the graphs to show that $g \circ f \neq f \circ g$.

37. $f(x) = x^5 - x^3 - x \quad g(x) = x - 2$

38. $f(x) = x^3 + x \quad g(x) = \sqrt[3]{x - 1}$

For Exercises 39–42, complete the given tables by using the values of the functions f and g given below.

x	$f(x)$
1	3
2	5
3	1
4	2
5	3

x	$g(x)$
1	5
2	4
3	4
4	3
5	2

39.

x	$(g \circ f)(x)$
1	4
2	?
3	5
4	?
5	?

40.

x	$(f \circ g)(x)$
1	?
2	2
3	?
4	?
5	?

41.

x	$(f \circ f)(x)$
1	?
2	?
3	3
4	?
5	?

42.

x	$(g \circ g)(x)$
1	?
2	?
3	?
4	4
5	?

In Exercises 43–46, let $g(x) = |x|$. Graph the function f and the composite function $g \circ f = |f(x)|$ on the same graph.

43. $f(x) = 0.5x^2 - 5$

44. $f(x) = x^3 - 4x^2 + x + 3$

45. $f(x) = x + 3$

46. $f(x) = |x| - 2$

3.5 – Inverse Functions

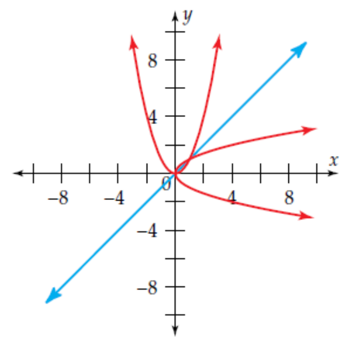
Objectives:

- Define Inverse Relations and Functions
- Find Inverse Relations from tables, graphs, and equations
- Determine whether an inverse relation is a function
- Verify inverses using composition

The result of exchanging the input and output values of a function or relation is called an **inverse relation.** If the inverse is a function, it is called the **inverse function.**

Graphs of Inverse Relations

Suppose that (x, y) is a point on the graph of a function. Then (y, x) is a point on its inverse function or relation. This fact can be used to graph the inverse of a function.



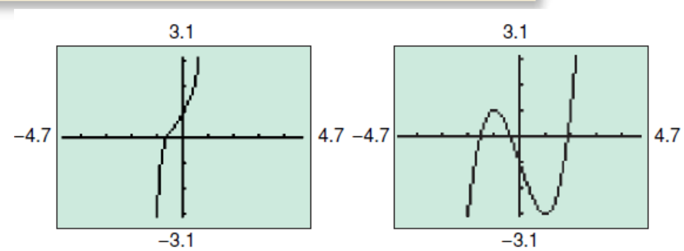
Example 3: Finding an Inverse from an Equation

Find the inverse of the given function.

a. $f(x) = 3x - 2$

b. $f(x) = x^2 + 4x$

A function f is one-to-one if and only if no horizontal line intersects the graph of f more than once.



Let f be a function. The following statements are equivalent.

- The inverse of f is a function.
- f is one-to-one.
- The graph of f passes the Horizontal Line Test.

The inverse function, if it exists, is written as f^{-1}, where

$$\text{if } y = f(x), \text{ then } x = f^{-1}(y).$$

The notation f^{-1} does not mean $\frac{1}{f}$.

Restricting the Domain

For a function that is not one-to-one, it is possible to produce an inverse function by considering only a part of the function that is one-to-one. This is called **restricting the domain.**

Example 6: Restricting the Domain

Find an interval on which the function $f(x) = x^2$ is one-to-one, and find f^{-1} on that interval.

Inverse Functions

$f(x)$ and $g(x)$ are inverses if $\boldsymbol{f(g(x)) = g(f(x)) = x}$

Example 7: Verifying the Inverse of a Function

Let $f(x) = \frac{5}{2x-4}$ and $g(x) = \frac{4x+5}{2x}$

a. Use composition to verify that f and g are inverses of each other.

3.6 – Average Rate of Change

Objectives:

- Define and model how to use a graph or table to evaluate the average rate of change of a linear or non-linear function on a specific interval.
- Model how to algebraically evaluate the average rate of change of a linear or non-linear function on a specific interval.

Average Rate of Change
Is defined as the change in y divided by the change in x.

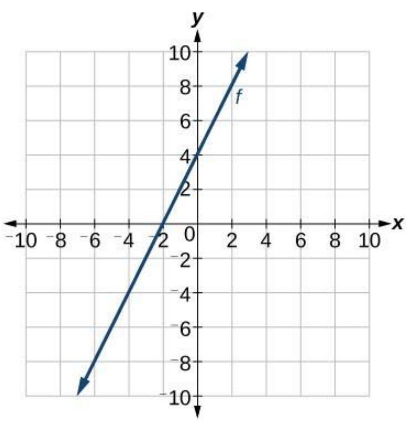
$$\textit{Average Rate of Change on a closed interval } a \le x \le b = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

Graphically, the average rate of change is the same as the slope between two points.
It is a measure of how much the function changed per unit, on average, over that interval.

Example 1: Finding the AROC

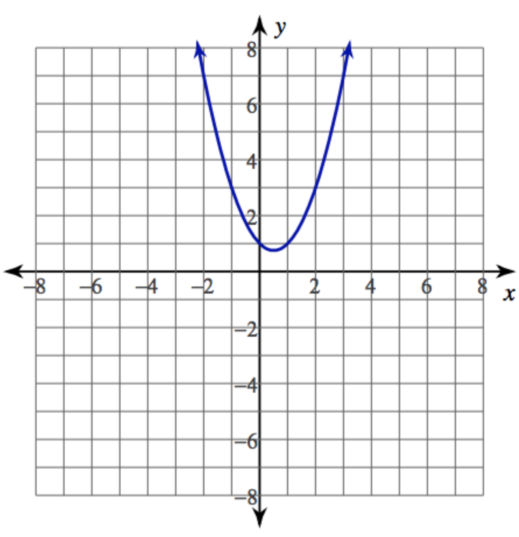
Find the average rate of change for the following functions on the given intervals.

a. $-4 \le x \le 0$



b. $f(x) = -3(x - 1)$ on $1 \le x \le 5$

c. $-2 \le x \le 0$



d. $f(x) = x^2 - 4x - 12$ on $0 \le x \le 6$

Let f be a function. The average rate of change of f over the interval from x to $x + h$ is given by the difference quotient.

$$\frac{f(x+h) - f(x)}{h}$$

Example 2: Difference Quotient
Find the Difference Quotient for the given function.

a. $f(x) = \frac{x^3}{55}$

Practice Odd Numbered Questions as Necessary. Answers are at the back of the textbook.

Exercises 3.7

1. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table below. Find the average speed of the car over each interval.
a. 0 to 10 seconds
b. 10 to 20 seconds
c. 20 to 30 seconds
d. 15 to 30 seconds

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	20	140	400	680	1400	1800

2. The yearly profit of a small manufacturing firm is shown in the tables below. What is the average rate of change of profits over the given time span?
a. 1996–2000 **b.** 1996–2003
c. 1999–2002 **d.** 1998–2002

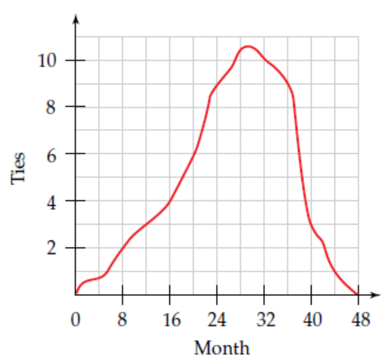
Year	1996	1997	1998	1999
Profit	\$5000	\$6000	\$6500	\$6800

Year	2000	2001	2002	2003
Profit	\$7200	\$6700	\$6500	\$7000

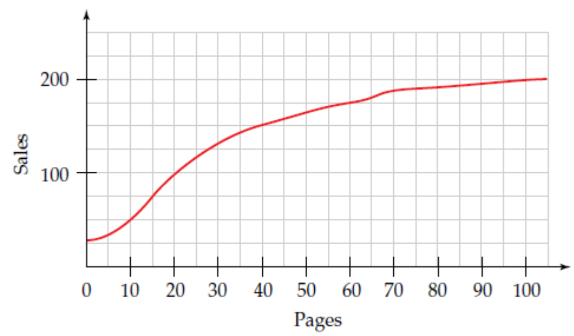
3. Find the average rate of change of the volume of the balloon in Example 2 as the radius increases
a. from 2 to 5 inches. **b.** from 4 to 8 inches.

4. Find the average rate of change of cost for the company in Example 3 when production increases from
a. 5 to 25 desks. **b.** 0 to 40 desks.

5. The graph in the figure shows the monthly sales of floral pattern ties (in thousands of ties) made by a company over a 48-month period. Sales are very low when the ties are first introduced; then they increase significantly, hold steady for a while, and then drop off as the ties go out of fashion. Find the average rate of change of sales (in ties per month) over the given interval.
a. 0 to 12 **b.** 8 to 24
c. 12 to 24 **d.** 20 to 28
e. 28 to 36 **f.** 32 to 44
g. 36 to 40 **h.** 40 to 48



6. A certain company has found that its sales are related to the amount of advertising it does in trade magazines. The graph in the figure shows the sales (in thousands of dollars) as a function of the amount of advertising (in number of magazine ad pages). Find the average rate of change of sales when the number of ad pages increases from
a. 10 to 20. **b.** 20 to 60.
c. 60 to 100. **d.** 0 to 100.
e. Is it worthwhile to buy more than 70 pages of ads if the cost of a one-page ad is \$2000? if the cost is \$5000? if the cost is \$8000?



7. When blood flows through an artery (which can be thought of as a cylindrical tube) its velocity is greatest at the center of the artery. Because of friction along the walls of the tube, the blood's velocity decreases as the distance r from the center of the artery increases, finally becoming 0 at the wall of the artery. The velocity (in centimeters per second) is given by the function

$$v = 18{,}500(0.000065 - r^2)$$

where r is measured in centimeters. Find the average rate of change of the velocity as the distance from the center changes from
a. $r = 0.001$ to $r = 0.002$.
b. $r = 0.002$ to $r = 0.003$.
c. $r = 0$ to $r = 0.025$.

8. A car is stopped at a traffic light and begins to move forward along a straight road when the light turns green. The distance (in feet) traveled by the car in t seconds is given by $s(t) = 2t^2$ for $0 \leq t \leq 30$ What is the average speed of the car from
a. $t = 0$ to $t = 5$? **b.** $t = 5$ to $t = 10$?
c. $t = 10$ to $t = 30$? **d.** $t = 10$ to $t = 10.1$?

In Exercises 9–14, find the average rate of change of the function f over the given interval.

9. $f(x) = 2 - x^2$
from $x = 0$ to $x = 2$

10. $f(x) = 0.25x^4 - x^2 - 2x + 4$
from $x = -1$ to $x = 4$

11. $f(x) = x^3 - 3x^2 - 2x + 6$
from $x = -1$ to $x = 3$

12. $f(x) = -\sqrt{2x^2 - x + 4}$
from $x = 0$ to $x = 3$

13. $f(x) = \sqrt{x^3 + 2x^2 - 6x + 5}$
from $x = 1$ to $x = 2$

14. $f(x) = \dfrac{x^2 - 3}{2x - 4}$
from $x = 3$ to $x = 6$

In Exercises 15–22, compute the difference quotient of the function.

15. $f(x) = x + 5$

16. $f(x) = 7x + 2$

17. $f(x) = x^2 + 3$

18. $f(x) = x^2 + 3x - 1$

19. $f(t) = 160{,}000 - 8000t + t^2$

20. $V(x) = x^3$

21. $A(r) = \pi r^2$

22. $V(p) = \dfrac{5}{p}$

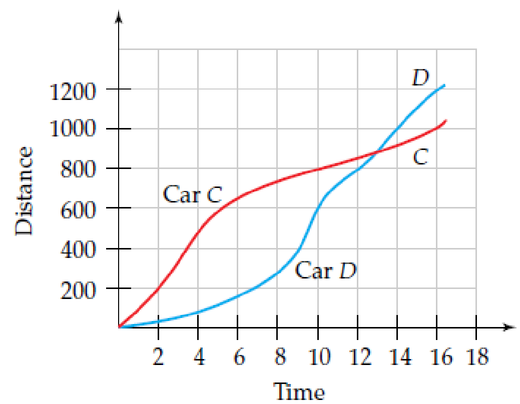
23. Water is draining from a large tank. After t minutes there are $160{,}000 - 8000t + t^2$ gallons of water in the tank.
   a. Use the results of Exercise 19 to find the average rate at which the water runs out in the interval from 10 to 10.1 minutes.
   b. Do the same for the interval from 10 to 10.01 minutes.
   c. Estimate the rate at which the water runs out after exactly 10 minutes.

24. Use the results of Exercise 20 to find the average rate of change of the volume of a cube whose side has length x as x changes from
   a. 4 to 4.1. b. 4 to 4.01. c. 4 to 4.001.
   d. Estimate the rate of change of the volume at the instant when $x = 4$.

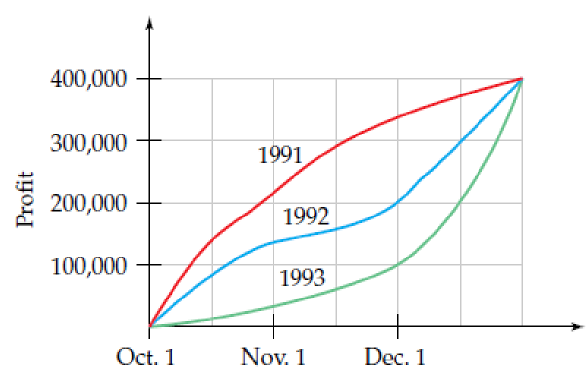
25. Use the results of Exercise 21 to find the average rate of change of the area of a circle of radius r as r changes from
   a. 3 to 3.5. b. 3 to 3.2. c. 3 to 3.1.
   d. Estimate the rate of change at the instant when $r = 3$.
   e. How is your answer in part d related to the circumference of a circle of radius 3?

26. Under certain conditions, the volume V of a quantity of air is related to the pressure p (which is measured in kilopascals) by the equation $V = \frac{5}{p}$. Use the results of Exercise 22 to estimate the rate at which the volume is changing at the instant when the pressure is 50 kilopascals.

27. Two cars race on a straight track, beginning from a dead stop. The distance (in feet) each car has covered at each time during the first 16 seconds is shown in the figure below.
   a. What is the average speed of each car during this 16-second interval?
   b. Find an interval beginning at $t = 4$ during which the average speed of car D was approximately the same as the average speed of car C from $t = 2$ to $t = 10$.
   c. Use secant lines and slopes to justify the statement "car D traveled at a higher average speed than car C from $t = 4$ to $t = 10$."



28. The figure below shows the profits earned by a certain company during the last quarter of three consecutive years.
   a. Explain why the average rate of change of profits from October 1 to December 31 was the same in all three years.
   b. During what month in what year was the average rate of change of profits the greatest?



29. The graph in the figure shows the chipmunk population in a certain wilderness area. The population increases as the chipmunks reproduce, but then decreases sharply as predators move into the area.

3.8 – Systems of Equations

Objectives:

- Solving Linear Systems Algebraically
- Modeling Linear Systems
- Solve Systems in the Calculator
- Solve other Systems
- Solve using Matrices
- Solve and Model Linear Systems of Inequalities

> ***Solving Systems of Equations***
> *To solve a system of equations, find the point(s) of intersection. This can be done algebraically, graphically, or using matrices.*

Example 1: Solving Systems

Solve the system given algebraically and verify graphically.

a. $\begin{cases} 4x+2y=16 \\ 3x+3y=15 \end{cases}$

b. $\begin{cases} 3x+y=7 \\ 2x+2y=6 \end{cases}$

c. Alexandra purchases two doughnuts and three cookies at a doughnut shop and is charged \$3.30. Briana purchases five doughnuts and two cookies at the same shop for \$4.95. All the doughnuts have the same price and all the cookies have the same price. Find the cost of one doughnut and find the cost of one cookie.

Example 2: Other Systems

Use a Graphing Calculator to solve the following systems graphically.

a.

1. absolute linear $y=\|x\|$ $y=x+2$	2. absolute quadratic $y=\|x\|-2$ $y=-x^2+4$	3. quadratic piecewise $y=-x^2+4$ $y=\begin{cases} x+2 & x<0 \\ x-2 & x\geq 0 \end{cases}$	4. quadratic quadratic $y=x^2+5x+5$ $y=-x^2-5x-3$	5. absolute quadratic $y=-\|x\|+2$ $y=x^2$

b. Solve number 4 above algebraically.

c. Solve algebraically: $\begin{cases} y=3x-30 \\ x^2+y^2=100 \end{cases}$

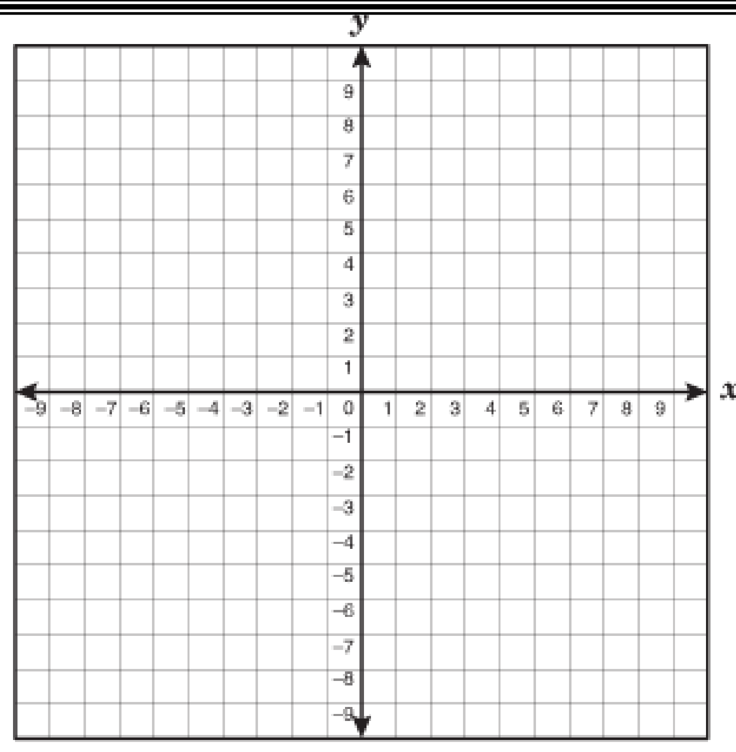
d. $-x - 5y - 5z = 2$
$4x - 5y + 4z = 19$
$x + 5y - z = -20$

Solving Systems of Inequalities
To solve a system of inequalities, first find the point(s) of intersection. This can be done algebraically, graphically, or using matrices. Then find the solution set by testing points in the regions created.

Example 11: Systems of Inequalities

a.

Graph the system: $4y \geq 6x$
$-3x + 6y \leq -6$



b. A high school drama club is putting on their annual theater production. There is a maximum of 800 tickets for the show. The costs of the tickets are $6 before the day of the show and $9 on the day of the show. To meet the expenses of the show, the club must sell at least $5,000 worth of tickets.

a) Write a system of inequalities that represent this situation.

b) The club sells 440 tickets before the day of the show. Is it possible to sell enough additional tickets on the day of the show to at least meet the expenses of the show? Justify your answer.