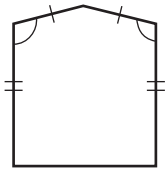
# Polygons

A **polygon** is a closed plane figure formed by three or more line segments that meet at points called vertices. You can classify a polygon by the number of sides and the number of angles that it has.

**Congruent** figures have the same size and shape. In a **regular polygon,** all sides are congruent and all angles are congruent.

| Polygon | Sides | Angles | Vertices |
|---|---|---|---|
| Triangle | 3 | 3 | 3 |
| Quadrilateral | 4 | 4 | 4 |
| Pentagon | 5 | 5 | 5 |
| Hexagon | 6 | 6 | 6 |
| Heptagon | 7 | 7 | 7 |
| Octagon | 8 | 8 | 8 |
| Nonagon | 9 | 9 | 9 |
| Decagon | 10 | 10 | 10 |

**Classify the polygon below.**



How many sides does this polygon have? 5 sides

How many angles does this polygon have? 5 angles

Name the polygon. pentagon

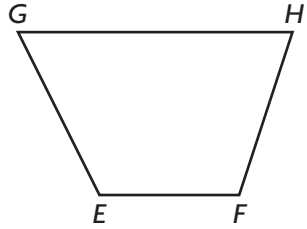
Are all the sides congruent? no

Are all the angles congruent? no

So, the polygon above is a pentagon. It is *not* a regular polygon.

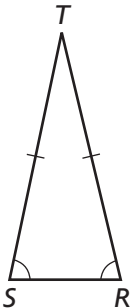
**Name each polygon. Then tell whether it is a *regular polygon* or *not a regular polygon*.**

**1.**



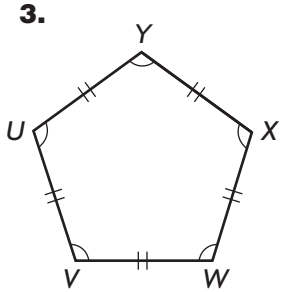

quadrilateral; not a regular polygon

**2.**



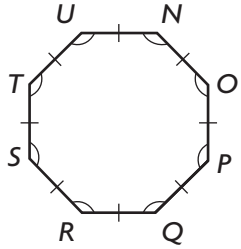

triangle; not a regular polygon

**3.**




pentagon; regular polygon

**4.**



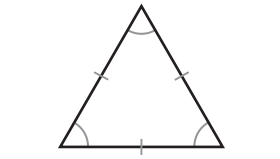

octagon; regular polygon

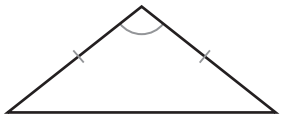
# Triangles

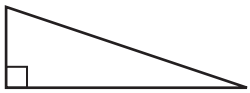
You can classify triangles by the length of their sides and by the measure of their angles. **Classify each triangle.**

Use a ruler to measure the side lengths.

- **equilateral triangle**
  All sides are the same length.
- **isosceles triangle**
  Two sides are the same length.
- **scalene triangle**
  All sides are different lengths.





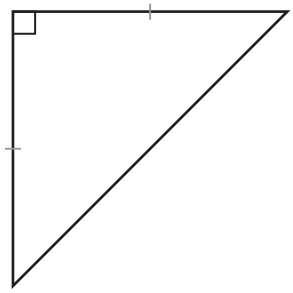


Use the corner of a sheet of paper to classify the angles.

- **acute triangle**
  All three angles are acute.
- **obtuse triangle**
  One angle is obtuse. The other two angles are acute.
- **right triangle**
  One angle is right. The other two angles are acute.

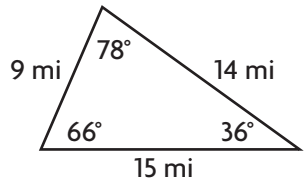
**Classify the triangle according to its side lengths.**
It has two congruent sides.
The triangle is an isosceles triangle.

**Classify the triangle according to its angle measures.**
It has one right angle.
The triangle is a right triangle.



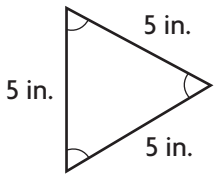
**Classify each triangle. Write *isosceles*, *scalene*, or *equilateral*. Then write *acute*, *obtuse,* or *right*.**

**1.**



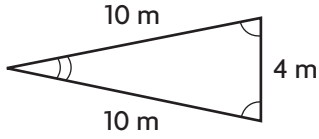

scalene; acute

**2.**



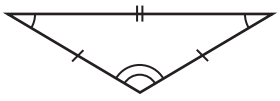

equilateral; acute

**3.**



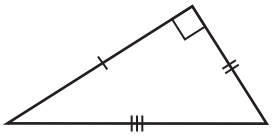

isosceles; acute

**4.**



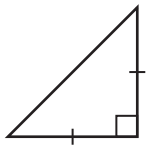
isosceles; obtuse

**5.**



scalene; right

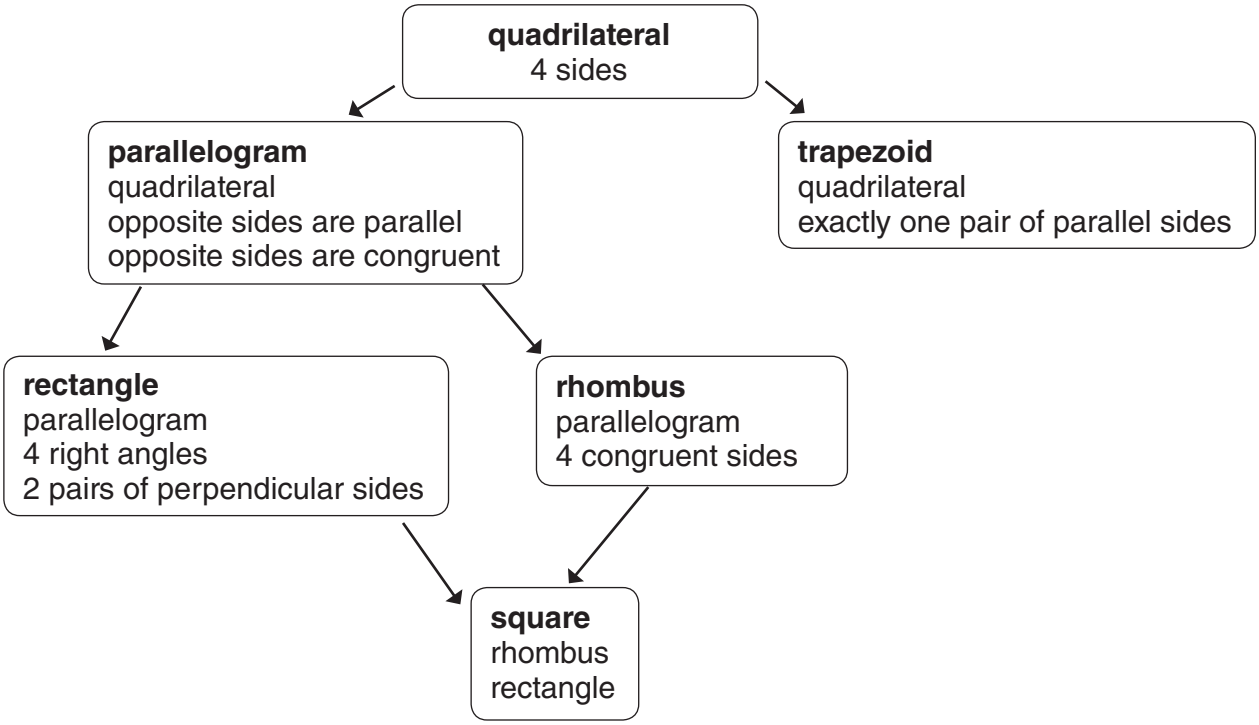
**6.**



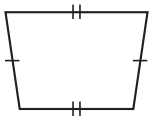
isosceles; right

# Quadrilaterals

You can use this chart to help you classify quadrilaterals.



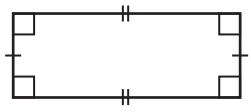

**Classify the figure.**



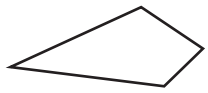
The figure has 4 sides, so it is a *quadrilateral*. The figure has exactly one pair of parallel sides, so it is a *trapezoid*.

quadrilateral, trapezoid

**Classify the quadrilateral in as many ways as possible. Write *quadrilateral*, *parallelogram*, *rectangle*, *rhombus*, *square*, or *trapezoid*.**

**1.** 

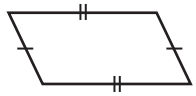
quadrilateral, parallelogram, rectangle

**2.** 

quadrilateral

**3.** 

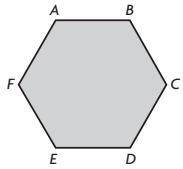
quadrilateral, trapezoid

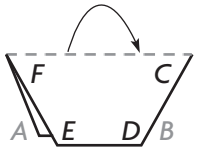
**4.** 

quadrilateral, parallelogram

# Problem Solving • Properties of Two-Dimensional Figures

Haley thinks hexagon *ABCDEF* has 6 congruent sides, but she does not have a ruler to measure the sides. Are the 6 sides congruent?




| Read the Problem | Solve the Problem |
|---|---|
| **What do I need to find?**<br>I need to determine if sides *AB*, *BC*, *CD*, *DE*, *EF*, and *FA* have the same length. | Trace the hexagon and cut out the shape.<br>**Step 1** Fold the hexagon to match the sides *AB* and *ED*, sides *FE* and *FA*, and sides *CD* and *CB*.<br>  <br><br>The sides match, so they are congruent.<br>**Step 2** Fold along the diagonal between *B* and *E* to match sides *BA* and *BC*, sides *AF* and *CD*, and sides *EF* and *ED*. Fold along the diagonal between *A* and *D* to match sides *AF* and *AB*, sides *FE* and *BC*, and sides *DE* and *DC*.<br>**Step 3** Use logic to match sides *AB* and *CD*, sides *AB* and *EF*, sides *BC* and *DE*, and sides *DE* and *FA*.<br>The sides match, so they are congruent. |
| **What information do I need to use?**<br>The figure is a hexagon with 6 sides and 6 congruent angles. | |
| **How will I use the information?**<br>I will act it out by tracing the figure and then folding the figure to match all the sides to see if they are congruent. | |

**1.** Justin thinks square *STUV* has 4 congruent sides, but he does not have a ruler to measure the sides. Are the sides congruent? **Explain.**

Possible answer: Yes. A square by definition has 4 congruent sides. If he folds the square in half both ways and along both diagonals, then the sides will match.

**2.** Esther knows octagon *OPQRSTUV* has 8 congruent angles. How can she determine whether the octagon has 8 congruent sides without using a ruler?

Possible answer: she could trace the octagon cut it out, and fold the figure to match the sides.

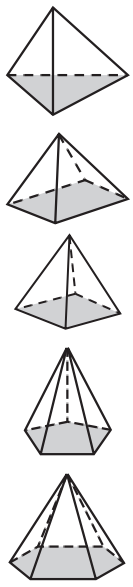
Name _____

# Three-Dimensional Figures

A **polyhedron** is a solid figure with faces that are polygons. You can identify a polyhedron by the shape of its faces.

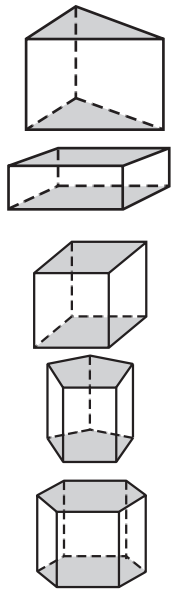
A **pyramid** is a polyhedron with one polygon base. The lateral faces of a pyramid are triangles that meet at a common vertex.

| | |
|---|---|
| **triangular pyramid** | The base and faces are triangles. |
| **rectangular pyramid** | The base is a rectangle. |
| **square pyramid** | The base is a square. |
| **pentagonal pyramid** | The base is a pentagon. |
| **hexagonal pyramid** | The base is a hexagon. |



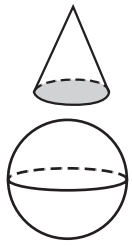
A **prism** is a polyhedron with two congruent polygons as bases. The lateral faces of a prism are rectangles.

| | |
|---|---|
| **triangular prism** | The two bases are triangles. |
| **rectangular prism** | All faces are rectangles. |
| **square prism or cube** | All faces are squares. |
| **pentagonal prism** | The two bases are pentagons. |
| **hexagonal prism** | The two bases are hexagons. |

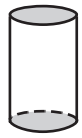


A solid figure with curved surfaces is **not a polyhedron**.

| | |
|---|---|
| **cone** | The one base is a circle. |
| **sphere** | There is no base. |



| | |
|---|---|
| **cylinder** | The two bases are circles. |

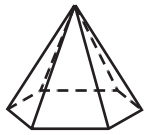


**Classify the solid figure. Write *prism, pyramid, cone, cylinder,* or *sphere.***

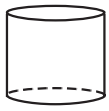
The solid figure has one base.

The rest of its faces are triangles.

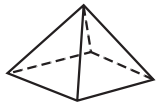
So, the solid figure is a pyramid.



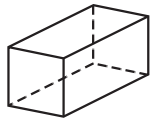
**Classify each solid figure. Write *prism, pyramid, cone, cylinder,* or *sphere.***

**1.** 

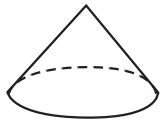
cylinder

**2.** 

pyramid

**3.** 

rectangular prism

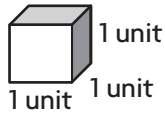
**4.** 

cone

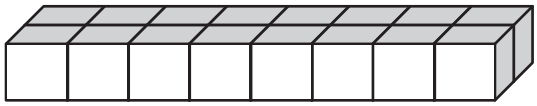
Name _____

# Unit Cubes and Solid Figures

A **unit cube** is a cube that has a length, width, and height of 1 unit. You can use unit cubes to build a rectangular prism.




**Count the number of cubes used to build the rectangular prism.**



The length of the prism is made up of 8 unit cubes.

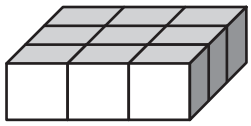
The width of the prism is made up of 2 unit cubes.

The height of the prism is made up of 1 unit cube.

The number of unit cubes used to build the rectangular prism is 16.

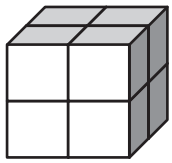
**Count the number of unit cubes used to build each solid figure.**

**1.**



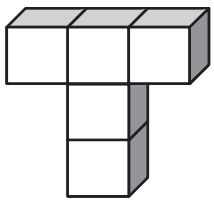
9 unit cubes

**2.**



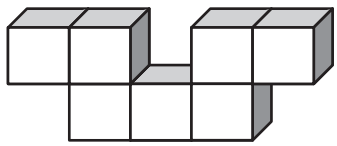
8 unit cubes

**3.**



5 unit cubes

**4.**



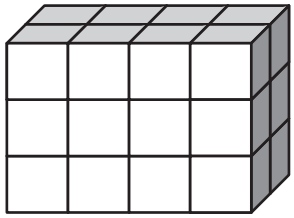
7 unit cubes

Name _____

# Understand Volume

The **volume** of a rectangular prism is equal to the number of unit cubes that make up the prism. Each unit cube has a volume of 1 cubic unit.

**Find the volume of the prism. 1 unit cube = 1 cubic inch**



**Step 1** Count the number of unit cubes in the bottom layer of the prism.

There are 4 unit cubes that make up the length of the first layer.

There are 2 unit cubes that make up the width of the first layer.

There is 1 unit cube that makes up the height of the first layer.

So, altogether, there are 8 unit cubes that make up the bottom layer of the prism.

**Step 2** Count the number of layers of cubes that make up the prism.

The prism is made up of 3 layers of unit cubes.

**Step 3** Find the total number of cubes that fill the prism.

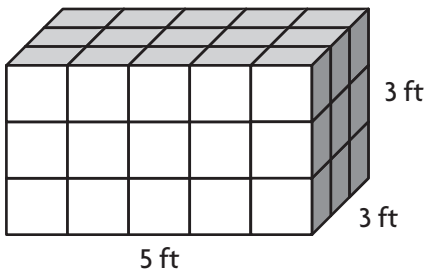
Multiply the number of layers by the number of cubes in each layer.

$3 \times 8 =$ 24 unit cubes

Each unit cube has a volume of 1 cubic inch. So, the volume of the prism is $24 \times 1$, or 24 cubic inches.

**Use the unit given. Find the volume.**

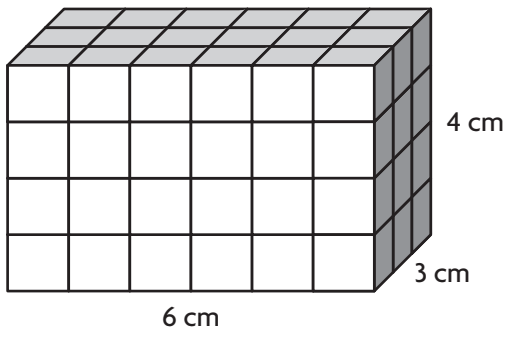
**1.**




Each cube = 1 cu ft

Volume = 45 cu ft

**2.**



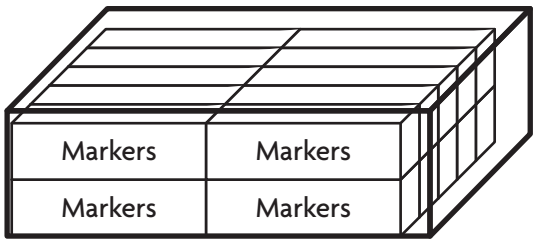

Each cube = 1 cu cm

Volume = 72 cu cm

# Estimate Volume

You can estimate the volume of a larger box by filling it with smaller boxes.

Mario packs boxes of markers into a large box. The volume of each box of markers is 15 cubic inches. Estimate the volume of the large box.




The volume of one box of markers is 15 cubic inches.

Use the box of markers to estimate the volume of the large box.

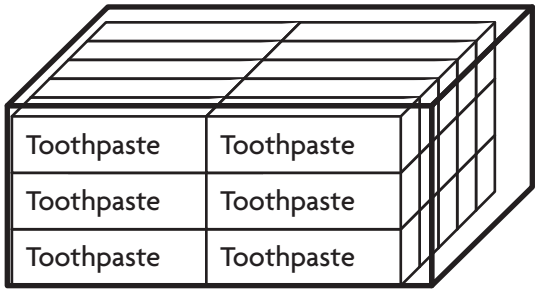
- The large box holds 2 layers of boxes of markers, a top layer and a bottom layer. Each layer contains 10 boxes of markers. So, the large box holds about 2 × 10, or 20 boxes of markers.
- Multiply the volume of 1 box of markers by the estimated number of boxes of markers that fit in the large box.

  20 × 15 = 300

So, the volume of the large box is about 300 cubic inches.

**Estimate the volume.**

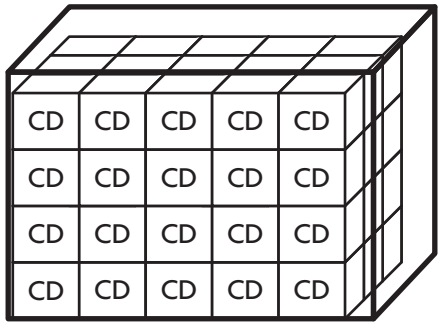
**1.** Each box of toothpaste has a volume of 25 cubic inches.




There are 30 boxes of toothpaste in the large box.

The estimated volume of the large box is 30 × 25 = 750 cubic inches.

**2.** Volume of CD case: 80 cu cm

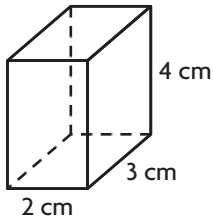



Volume of large box: about 4,800 cu cm

Name _____

# Volume of Rectangular Prisms

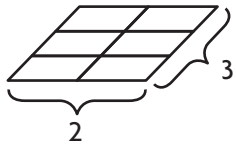
Jorge wants to find the volume of this rectangular prism. He can use cubes that measure 1 centimeter on each side to find the volume.




**Step 1** The base has a length of 2 centimeters and a width of 3 centimeters. Multiply to find the area of the base.

Base = 2 × 3

Base = 6 $cm^2$




**Step 2** The height of the prism is 4 centimeters. Add the number of cubes in each layer to find the volume.

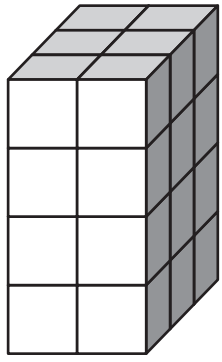
**Remember:** Each layer has 6 cubes.

**Step 3** Count the cubes. 24 cubes

Multiply the base and the height to check your answer.

Volume = 6 × 4

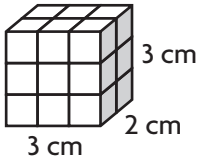
Volume = 24 cubic centimeters



So, the volume of Jorge's rectangular prism is 24 cubic centimeters.

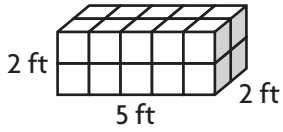
**Find the volume.**

**1.**



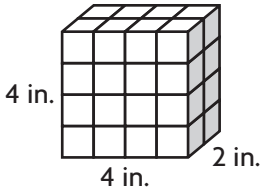

Volume: 18 $cm^3$

**2.**



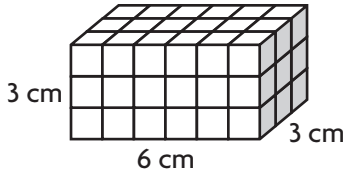

Volume: 20 $ft^3$

**3.**




Volume: 32 $in.^3$

**4.**




Volume: 54 $cm^3$

Name _____

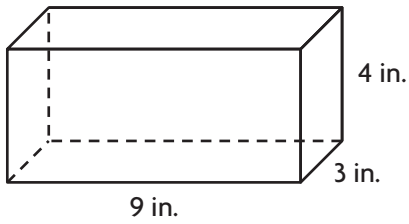
# Algebra • Apply Volume Formulas

You can use a formula to find the volume of a rectangular prism.

*Volume* = *length* × *width* × *height*

$V = (l \times w) \times h$

**Find the volume of the rectangular prism.**




**Step 1** Identify the length, width, and height of the rectangular prism.

length = 9 in. width = 3 in. height = 4 in.

**Step 2** Substitute the values of the length, width, and height into the formula.

$V = (l \times w) \times h$

$V = (\ 9\ \times\ 3\ ) \times\ 4$

**Step 3** Multiply the length by the width.

$V = (9 \times 3) \times 4$

$V = 27 \times 4$

**Step 4** Multiply the product of the length and width by the height.

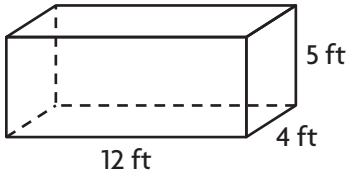
$V = 27 \times 4$

$= 108$

So, the volume of the rectangular prism is 108 cubic inches.

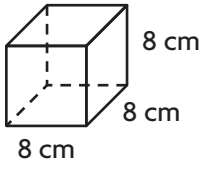
**Find the volume.**

**1.**




$V =$ 240 ft³

**2.**




$V =$ 512 cm³

# Problem Solving • Compare Volumes

A company makes aquariums that come in three sizes of rectangular prisms. The length of each aquarium is three times its width and depth. The depths of the aquariums are 1 foot, 2 feet, and 3 feet. What is the volume of each aquarium?

| Read the Problem | Solve the Problem |
|---|---|
| **What do I need to find?**<br>I need to find the volume of each aquarium. | **Think:** The depth of an aquarium is the same as the height of the prism formed by the aquarium |
| **What information do I need to use?**<br>I can use the formula for volume, $V = l \times w \times h$, or $V = B \times h$. I can use 1 ft, 2 ft, and 3 ft as the depths. I can use the clues the length is three times the width and depth. | |
| **How will I use the information?**<br>I will use the volume formula and a table to list all of the possible combinations of lengths, widths, and depths. | So, the volumes of the aquariums are 3 cubic feet, 24 cubic feet, and 81 cubic feet. |

| Length (ft) | Width (ft) | Depth, or Height (ft) | Volume (cu ft) |
|---|---|---|---|
| 3 | 1 | 1 | 3 |
| 6 | 2 | 2 | 24 |
| 9 | 3 | 3 | 81 |

**1.** Jamie needs a bin for her school supplies. A blue bin has a length of 12 inches, a width of 5 inches, and a height of 4 inches. A green bin has a length of 10 inches, a width of 6 inches, and a height of 5 inches. What is the volume of the bin with the greatest volume?

300 in.$^3$

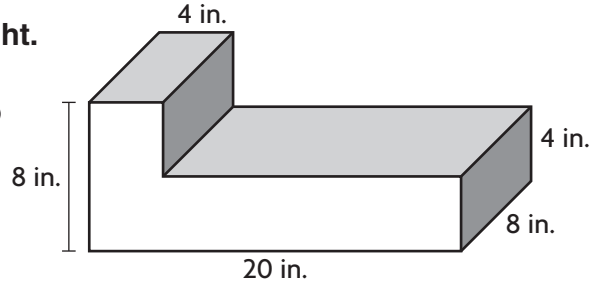
**2.** Suppose the blue bin that Jamie found had a length of 5 inches, a width of 5 inches, and a height of 12 inches. Would one bin have a greater volume than the other? **Explain.**

No. Both bins have the same volume; 300 in.$^3$

# Find Volume of Composed Figures

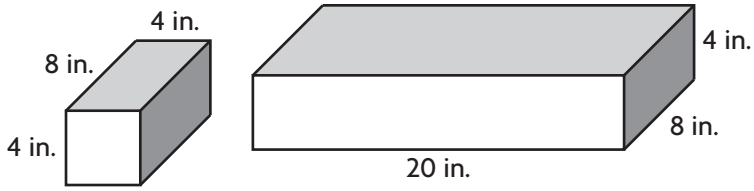
A composite figure is a solid made up of two or more solids. To find the volume of a composite figure, first find the volume of each solid that makes up the figure. Then find the sum of the volumes of the figures.

**Find the volume of the composite figure at right.**




**Step 1** Break apart the composite figure into two rectangular prisms. Label the dimensions of each prism.

**Prism 1** **Prism 2**




**Step 2** Find the volume of each prism.

**Prism 1**

$V = (l \times w) \times h$

$V =$ 4 $\times$ 8 $\times$ 4

$V = 128$ in.$^3$

**Prism 2**

$V = (l \times w) \times h$

$V =$ 20 $\times$ 8 $\times$ 4

$V = 640$ in.$^3$

**Step 3** Find the sum of the volumes of the two prisms.

Volume of Prism 1 + Volume of Prism 2 = Volume of Composite Figure

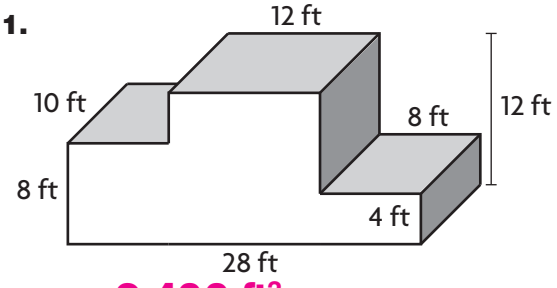
128 in.$^3$ + 640 in.$^3$ = Volume of Composite Figure

768 in.$^3$ = Volume of Composite Figure

So, the volume of the composite figure is 768 in.$^3$

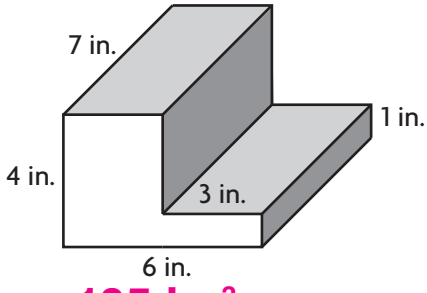
**Find the volume of the composite figure.**

**1.**




$V =$ 2,400 ft$^3$

**2.**




$V =$ 105 in.$^3$